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UNDEREMPLOYMENT AND CAPITAL IRREVERSIBILITY IN A UNIONIZED
OVERLAPPING GENERATIONS ECONOMY

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Abstract

In a unionized OLG model, it is shown that steady-state underutilization of labour and equipment can be due to the combination of the two following elements: (i) Irreversibility of capital, technology and skill decisions, (ii) Firm specific shocks on productivity. The presence of unions is neither sufficient nor necessary for having unemployment. The result of Devereux and Lockwood (1991) that union power affect positively the capital stock in general equilibrium does not always hold under capital irreversibility.

Key Words

Unemployment; Underutilization; Irreversibility; Investment; Union power.

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Introduction

The fact that both labour and capital can remain underutilized over very long periods of time is one of the most striking stylized facts in macroeconomics. On the one hand, the explanation of unemployment has been tackled by a large strand of the New Keynesian literature. These models often rely on non-competitive wage and price mechanism in short-run frameworks, i.e., without including any growth process. On the other hand, growth theory expanded quickly this last decade, trying to explain long-run accumulation process on the basis that all markets are perfectly competitive. This approach neglects the fact that the labour market seems to be characterised by persistent underemployment but also the fact that equipment could remain underutilised at length. These two facts should affect the accumulation process, and therefore the long-run movements of the economy. In this paper, we present a growth model with persistent unemployment and underutilization of equipment. The aim is to make a first step towards a complete treatment of underemployment with the growth theory toolbox and to show that this does not require any price rigidity assumption. It will be shown that long-run underutilization of productive factors can be due to the combination of the two following elements: (i) Irreversibility of capital, technology, and skill decisions (implying firm-specific skills), (ii) firm specific shocks on productivity. Underemployment will result from the fact that the irreversible skill decisions of the households and the investment decisions of the firms are taken without knowing with certainty the firms productivity for the next period. This implies that agents will invest their human and physical capital in some firms that will be hit by a negative shock, generating underemployment of their resources, while other firms are hit by a positive shock, but are unable to benefit fully from it, because there is a lack of capital and skills in their micro-market.

In this framework we are also able to analyse the effects of unions on accumulation in the presence of underemployment and underutilization of capital. Devereux and Lockwood (1991) have shown that the analysis of the effect of unions on capital accumulation in a general equilibrium perspective gives very different results than the one in partial equilibrium, as in Grout (1984). This is basically due to the fact that the interest rate is made endogenous. An interesting feature of unions overlapping generation model is that the bargaining between workers and firms is also, in this context, a bargaining between young saver workers and old dissaver capitalists. In this paper, the interactions between unions and long-run unemployment are studied. In particular it is shown that the presence of unions is neither sufficient nor necessary for having unemployment.

An important result of Devereux and Lockwood (1991) is that union power affects positively the capital stock in both the binding and non-binding contract case. This is because an increase in union power increases the wages and the savings of the young generation. However, it is shown in this paper that their result depends crucially on their assumption of zero depreciation of the capital stock. We assume here a full depreciation of the capital stock, which is consistent with our irreversibility assumption. In this case the effect of a rise in union power is ambiguous.

The recent literature on quantity rationing models show some examples of the existence of unemployment and underutilization of capacities at equilibrium. Sneessens (1987), Licandro (1992c) and Arnsperger and de la Croix (1993) show that, in a general equilibrium model with monopolistic competition and wage bargaining, the existence of complemen-

tarity in the production technology and of firm specific uncertainty gives an explanation to the natural rate of unemployment. In a partial equilibrium model, Licandro (1992a) and (1992b) shows that, under these assumptions, capacities are generally underemployed, even at steady state. de la Croix (1992) analyses the dynamic of unemployment and the degree of capacity utilization in a dynamic model of this type. Our model is built on these previous work, after having abandoned the assumptions related to nominal rigidities and monopolistic competition on the goods market.

Our framework has little in common with the new branch of dynamic disequilibrium models represented e.g. by Van Marrewijk and Verbeek (1993) and Weddepohl and Yildirim (1993) in the sense that our unemployment is not generated by price rigidities (real and/or nominal) but by technical rigidities and uncertainty, which seems more appropriate in a growth model.

The main assumptions under which the model is built are the following. First, it is a two period overlapping generation model, as in Diamond (1965), with perfect competition on the goods market and wage bargaining in the labor market, as in Devereux and Lockwood (1991). Secondly, there is irreversibility of technological choices (a putty-clay technology). As it is standard in OLG models, the capital stock is decided one period before. Associated with this capital stock there is a given technology, i.e., even if the ex-ante production function is Cobb-Douglas, the choice of the capital-labor ratio is taken at the same time that the equipment is bought. Third, the factors of production are firm specific, i.e., the labor market is segmented and investment is irreversible. Finally, it is assumed that the productivity of capital is random and that the firm faces some uncertainty when it is choosing the capital stock and the capital/labor ratio.

The timing, even if it is relatively standard, is relevant in generating underemployment of production factors. As stated before, technological and accumulation choices are made one period in advance under uncertainty. Wage and employment are decided at the beginning of each period, after the realization of the technological shock, by an efficient bargain at the firm level. Consumption, savings and production takes place simultaneously under full-information.

1 The model

It is a standard OLG model, where N_t represents the members of generation t and n the growth rate of population. Individuals live three periods. They are kids in the first period, young workers in the second period and old capitalists in the third period. Kids do not work and do not consume at all in their first period of life; they only must choose a firm specific human capital i .¹ Since *expected* labor incomes are the same for all types of human capital, kids select randomly their qualification implying that they are uniformly over the different segments of the labor market. The young-worker consumes, saves for future consumption and offers inelastically one unit of labor. The old-capitalist, who holds shares and debts issued by firms, only consumes. At the end of each period, old-capitalists sell their shares to young-workers.

¹The introduction of kids in the model is a simple way to rationalize the skill decision. This does not modify the formal structure of the model, so that it is comparable to the standard two-period Diamond model. A more refined analysis of this decision should be an interesting extension of the model.

Firms live infinitely, hire labor, buy capital and produce. To finance their investments firms borrow from individuals. Any pure profits at equilibrium are distributed to the share-holders.

Each machine, once installed, has a given productivity for both capital and labor, i.e., the technology is putty-clay. Decisions about the capital stock and productivities are taken by the firm one period ahead. At the time of the decision, there is uncertainty concerning the productivity of capital. However, this uncertainty is only firm specific so that there is no aggregate uncertainty. Investment is irreversible, i.e., it is specific to a particular firm and can not be valuable anywhere.

There is a continuum of labor markets with specific human capital in the interval $[0, 1]$. Each segment of this market is denoted by i . The number of workers in each segment of the market at period t is equal to N_t . We think as if there was one firm in each labour market (returns to scale are assumed constant). Since skills are labor market specific, the workers cannot move from one segment of the labor market to another.

Finally, in each segment of the labor market the N_t young-workers are organized in a union. The union and the firm bargain over wages and employment as in the standard "efficient bargaining" model of McDonald and Solow (1981).

1.1 The consumer problem

Individuals have preferences over consumption when young c_{1t} and consumption when old c_{2t+1} . These preferences can be represented by a Cobb-Douglas utility function, which is the same for all individuals from all generations. However, individuals from the same generation have a different labor endowment, since labor is firm specific. An individual of generation t , with specific labor endowment i , solves the following problem

$$\max_{c_{1it}, c_{2it+1}} c_{1it}^\theta c_{2it+1}^{1-\theta} \quad 0 < \theta < 1$$

subject to

$$c_{1it} + \frac{c_{2it+1}}{1 + r_{t+1}} = w_{it}l_{it} \quad l_{it} \in \{0, 1\}.$$

The real wage w_{it} and the interest rate r_t are given, and employment l_{it} can be 1 if employed or 0 if unemployed.

The first order conditions for this problem are $c_{1t} = \theta w_{it}l_{it}$ and $c_{2t+1} = (1 - \theta)(1 + r_{t+1})w_{it}l_{it}$. The corresponding individual savings s_{it} are

$$s_{it} = (1 - \theta)w_{it}l_{it}. \quad (1)$$

The indirect utility function of a young worker of generation t is proportional to

$$w_{it}(1 + r_{t+1})^{1-\theta}, \quad (2)$$

if he works and it is equal to zero if the worker is unemployed.

1.2 The putty-clay technology

The firm's technology is putty-clay and the ex-ante production function of firm i is supposed to be Cobb-Douglas, i.e.,

$$Y_{it} = L_{it}^{\alpha} K_{it}^{1-\alpha}, \quad (3)$$

where Y represents production, L is employment and K the capital stock. The parameter α satisfies $0 < \alpha < 1$.

Defining the ex-ante capital-labour ratio as x_{it} , the ex-post average labour productivity is $a_{it} = x_{it}^{1-\alpha}$, and the corresponding ex-post average capital productivity is $b_{it} = x_{it}^{-\alpha} \mu_{it}$. The ex-post production function is Leontief, i.e.,

$$Y_{it} = \min \{ x_{it}^{1-\alpha} L_{it}, x_{it}^{-\alpha} K_{it} \mu_{it} \}. \quad (4)$$

The average productivity of capital is assumed stochastic, where μ_{it} is an idiosyncratic productivity shock coming from a known distribution $F(\mu; \xi)$, the same for all i and t . ξ is a vector of known parameters. We assume for simplicity that the productivity of labor is not stochastic.

Given that in each segment of the labor market employment should be lower or equal to labour supply N_t , equation (4) implies that:

$$Y_{it} \leq \min \{ x_{it}^{1-\alpha} N_t, x_{it}^{-\alpha} K_{it} \mu_{it} \}. \quad (5)$$

This says that the firm faces two constraints for producing: (i) the *full-employment constraint* which is equal to the number of labor-suppliers in this segment of the labor market N_t multiplied by their average productivity, and (ii) the *capacity constraint* which is equal to the capital stock K_{it} times its average productivity. All the components of both constraints were decided in the previous period.

Assuming that μ_{it} is lognormally distributed with unit mean and variance σ^2 , and using the result of Lambert (1988), expected production can be approximated by a CES function of the two expected constraints:

$$E(Y_{it}) = \left((x_{it}^{1-\alpha} N_t)^{-\rho} + (x_{it}^{-\alpha} K_{it})^{-\rho} \right)^{-1/\rho}, \quad (6)$$

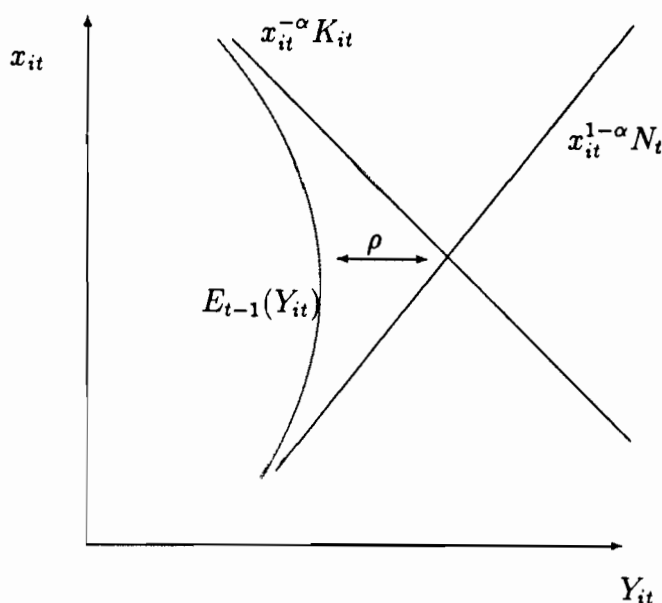
where ρ is a function of σ^2 , the variance of the shock:

$$\rho = -1 + \frac{2}{\sigma} \frac{f(-\sigma/2)}{F(-\sigma/2)},$$

where F is the standard normal distribution and f the corresponding density function.

The interpretation of equation (6) is made clearer by considering Figure 1: The upward sloping line describes the known full-employment output as a function of the ex-ante capital/labour ratio. This ratio affects the constraints positively through its effect on the ex-post productivity of labour. The downward sloping curve describes the *expected* capacity constraint. If the location of this constraint was known with certainty output and expected output coincide and would simply be equal to the minimum of the two constraints. However, this is not the case since the capacity constraint is affected by a random term. Lambert's result says that expected output can be approximated by a CES function of the two constraints, which is the smooth curve of Figure 1. The distance between the CES and the lines is positively affected by the variance of the shock. An increase in σ (i.e. in $1/\rho$) moves the CES to the West.

Figure 1: Expected production



1.3 The Efficient Bargaining

As stated before, the negotiation is decentralized. In each segment of the labor market, firm and union bargain once the capital stock has been installed by the firm, which corresponds to the non-binding contract of Grout (1984). In this case, the firm must take into account in the previous period the effect of the capital stock and technological decisions on the wage and employment level. We solve therefore the model backwards, starting with the bargaining problem and ending with the capital and technological choice. The bargaining process is modeled using the generalized Nash bargaining solution. The utility of the union is the sum of the indirect utility (2) of its members, where the fallback utility is zero.

The objective of the firm is to maximize the expected discounted value of the flow of profits. The capital stock is fully depreciated after one period.² Time t profits are

$$\pi_{it} = Y_{it} - w_{it}L_{it} - (1 + r_t)K_{it}.$$

The firm maximises

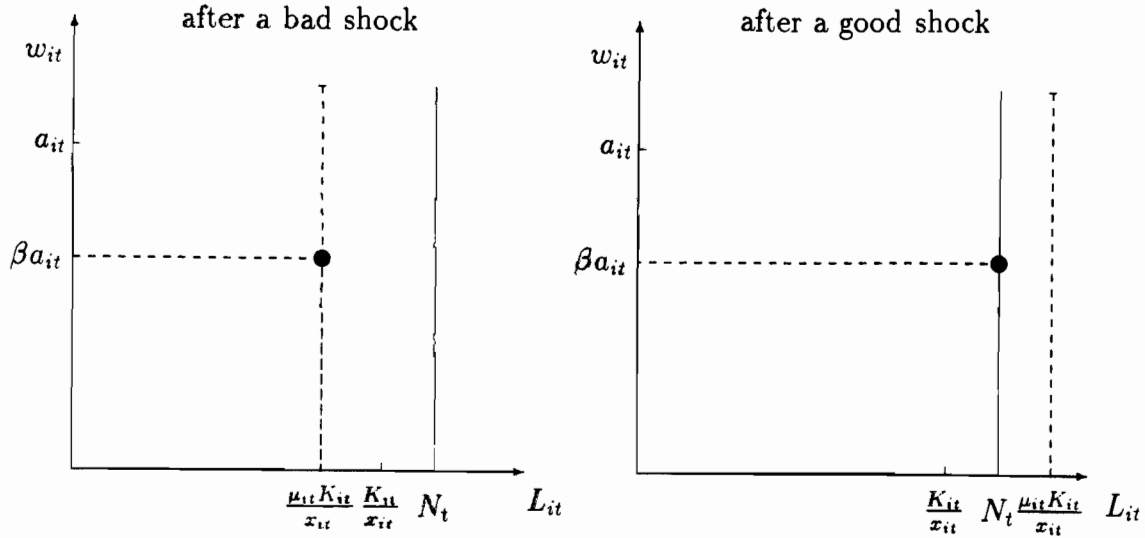
$$E_{t-1} \left[\sum_{\tau=t}^{\infty} \delta_{\tau} \pi_{i\tau} \right],$$

where the discount rate is $\delta_{\tau} = \prod_{s=t+1}^{\tau} (1 + r_s)^{-1}$.

Assuming that once installed the capital stock can not be sold to other firms, even in

²This is consistent with our irreversibility assumption and with the real time-horizon of two-period overlapping generation models, say 30 years. Devereux and Lockwood (1991) assume a zero depreciation rate. It will be shown that their assumption is crucial in generating their result concerning the role of union power on the capital stock.

Figure 2: The efficient bargaining



case of breakdown in the negotiation, the expected fall-back profit is

$$-(1 + r_t)K_{it} + E_{t-1} \left[\sum_{\tau=t+1}^{\infty} \delta_{\tau} \pi_{i\tau} \right].$$

Therefore, the profit net of its fall-back is simply $Y_{it} - w_{it}L_{it}$.

Each union-firm couple maximizes

$$\max_{w_{it}, L_{it}} (L_{it} w_{it} (1 + r_{t+1})^{1-\theta})^{\beta} (Y_{it} - L_{it} w_{it})^{1-\beta}, \quad 0 < \beta < 1$$

subject to equations (4) and (5). The parameter β represents the relative power of the union in the negotiation. Given that the interest rate is exogenous at the firm level,³ the first order conditions for this problem are

$$w_{it} = \beta a_{it} \quad (7)$$

$$Y_{it} = a_{it} L_{it} \quad (8)$$

$$L_{it} = \min \left\{ N_t, \frac{K_{it}}{x_{it}} \mu_{it} \right\}. \quad (9)$$

The outcome of the efficient bargain is represented in Figure 2 for the two possible branches of the min function in (9). In both cases, the wage is a share β of average labour productivity. Notice that if the labour market was characterized by perfect competition, wages would differ across firms: If the productivity shock was such that capacities are smaller than full-employment output, the real wage would then be equal to the reservation wage (here 0) and there would be voluntary unemployment ($L_{it} < N_t$). If the productivity

³The internalisation of the capital market equilibrium by a centralized union in OLG models creates or increases dynamic inefficiency generating unemployment, see Cahuc (1991).

shock was large enough to ensure that capacities were greater than full-employment output, the wage would be equal to marginal productivity and employment would be equal to N_t .

The solution for L is a corner solution. If the productivity shock is *bad* (i.e., if $\mu_{it} \leq \frac{N_t x_{it}}{K_{it}}$), capacities are smaller than full-employment output, the capacity constraint determines employment and there is *ex-post* involuntary unemployment (in the sense that unemployed persons are willing to work at the equilibrium wage). If the productivity shock is *good* (i.e., if $\mu_{it} \geq \frac{N_t x_{it}}{K_{it}}$), full-employment occurs in this segment of the labor market. Notice that employment is not affected by union power, which is a standard result of efficient bargaining models when the workers are risk neutral (see Svejnar (1986)).

1.4 The Technological Choice

At period $t - 1$ there is uncertainty concerning the average productivity of capital for the next periods. The firm chooses the capital stock and the capital/labor ratio for time t by maximising the expected flow of profits, subject to the outcome of the future negotiation given by equations (7) to (9). Given the fact that the capital stock lives one period, the problem of the firm is a one period problem. Using (6), (7) and (8), the optimal capital stock and the optimal capital/labour ratio are given by:

$$\max_{K_{it}, x_{it}} (1 - \beta) \left((x_{it}^{1-\alpha} N_t)^{-\rho} + (x_{it}^{-\alpha} K_{it})^{-\rho} \right)^{-\frac{1}{\rho}} - (1 + r_t) K_{it}.$$

The first order condition for the capital/labour ratio is:

$$x_{it} = \left(\frac{1 - \alpha}{\alpha} \right)^{\frac{1}{\rho}} \frac{K_{it}}{N_t}, \quad (10)$$

Equation (10) says that the optimal capital/labour ratio is proportional to the capital stock per-capita. Following Lambert, the weighted probability of being capacity constrained is given by

$$P_w[Y_{it} = x_{it}^{-\alpha} K_{it} \mu_{it}] = \left(\frac{E_{t-1}(Y_{it})}{x_{it}^{-\alpha} K_{it}} \right)^{\rho}.$$

Computing the value of this probability with equations (6) and (10), it can be shown that the optimally chosen weighted probability of being capacity constrained is $1 - \alpha$. The ratio of expected production to expected capacities, denoted by d_{it} , verifies

$$d_{it} = \frac{E_{t-1}(Y_{it})}{x_{it}^{-\alpha} K_{it}} = (1 - \alpha)^{\frac{1}{\rho}}.$$

In the same way, it can be shown that the weighted probability of being constrained by labour supply is α .

The first order condition for the capital stock merged with (10) gives:

$$1 + r_t = (1 - \beta)(1 - \alpha)^{1 + \frac{1}{\rho}} x_{it}^{-\alpha}. \quad (11)$$

The marginal cost of capital, at the right hand side of equation (12), is equal to the expected marginal productivity of capital times the firm power on the negotiation, times d_{it} :

$$1 + r_t = (1 - \beta) d_{it} (1 - \alpha) x_{it}^{-\alpha}.$$

The marginal cost of capital should be smaller than that the expected marginal productivity of capital, i.e., $(1 - \alpha)x_{it}^{-\alpha}$.

2 The aggregate equilibrium

Since all firms are ex-ante identical (uncertainty is i specific and there is no aggregate uncertainty) they will all choose the same capital stock and the same capital/labour ratio. Conditions (10) and (11) verify at the aggregate, i.e.,

$$1 + r_t = (1 - \beta)(1 - \alpha)^{1+1/\rho} x_t^{-\alpha} \quad (12)$$

and

$$x_t = \left(\frac{1 - \alpha}{\alpha} \right)^{\frac{1}{\rho}} k_t, \quad (13)$$

where $k_t = \frac{K_t}{N_t}$ is the capital stock per-capita.

Moreover, since x_t is the same for all firms, all union-firm couples set the same real wage

$$w_t = \beta x_t^{1-\alpha}. \quad (14)$$

Finally, aggregate output and aggregate employment are equal to firm's expected output and employment, implying that

$$l_t = \left(1 + \left(\frac{k_t}{x_t} \right)^{-\rho} \right)^{-1/\rho} \quad (15)$$

and

$$y_t = x_t^{1-\alpha} l_t, \quad (16)$$

where $l_t = \frac{L_t}{N_t}$ is the employment rate and $y_t = \frac{Y_t}{N_t}$ is per-capita production.

2.1 Underemployment of production factors

Before closing the model by writing down the capital market equilibrium conditions, we are already able to prove our first results which are that capital and labor are underutilized. Combining (13) and (15) we derive the equilibrium value for aggregate employment

$$L_t = \alpha^{\frac{1}{\rho}} N_t \leq N_t. \quad (17)$$

For all period t , employment is smaller than the labor supply. This result comes from the aggregation of heterogeneous situations. In the economy, firms facing a good productivity shock are able to hire all the workers in their segment of the labor market. At the same time, firms facing a bad shock are unable to hire all the workers in the corresponding segments of the labor market. At the aggregate, the weighted proportion of firms being constrained by labor supply is equal to α . Notice that this weighted proportion is equal to $(L/N)^\rho$ for all t and is equal to the weighted probability of full-employment constraint defined in section 1.4. In this economy, heterogeneity is related to uncertainty and it is at the basis of the existence of unemployment.

The corresponding unemployment rate is independent of time and it is given by

$$u_t = \bar{u} = 1 - \alpha^{\frac{1}{\rho}} \geq 0 \quad \forall t. \quad (18)$$

It is zero only when $\rho \rightarrow \infty$, in which case both uncertainty and heterogeneity vanish. Underemployment results from the fact that the irreversible skill decisions of the households and the investment decisions of the firms are taken without knowing with certainty the firms productivity for the next period.

The degree of capacity utilization d_t is defined as the ratio of aggregate production to aggregate capacities, i.e.,

$$d_t = \frac{Y_t}{x_t^{-\alpha} K_t}.$$

Using (13), (15) and (16) it can be shown that d_t is independent of time and that it takes the following value at equilibrium

$$d_t = \bar{d} = (1 - \alpha)^{\frac{1}{\rho}} \leq 1 \quad \forall t. \quad (19)$$

The degree of capacity utilization is generally smaller than one. As for the unemployment rate, there is full-utilization of capacities only when $\rho \rightarrow \infty$, i.e., when uncertainty and heterogeneity vanish.

The economy exhibits unemployment and underutilization of capacity at equilibrium. In this simple version of the model d and u take the same values for all periods. This is due to the Cobb-Douglas specification that has been chosen for the production function. More general functional forms would lead to a dynamic pattern of unemployment and capacity utilisation.

Notice that the existence of underemployment does not depend on union's power. It is only linked to heterogeneity and irreversibility. Although, if households were risk averse and their indirect utility function would be concave in labor income, leading to an effect of union power on employment.

Both heterogeneity and uncertainty play a crucial role in this economy. When uncertainty and heterogeneity disappear (if $\rho \rightarrow \infty$), capacities become full-employed and the unemployment rate goes to zero. When there is uncertainty but there is not heterogeneity, i.e., if uncertainty is not idiosyncratic, all firms will be in the same situation at equilibrium. Under these conditions two types of equilibrium could be possible, full-employment with underutilization of capacities or full-capacity with unemployment. To generate simultaneously unemployment and underutilization of capacities the existence of heterogeneity is necessary.

2.2 The capital/labor ratio

In this framework we have three different capital/labor ratio: x represents the optimal capital/labor ratio which is incorporated in the existing machines, k represents the capital stock per-capita, and k/l represents the effective capital/labor ratio. Using equations (13), (17) and (19) we know that

$$x_t = \bar{d} \left(\frac{k_t}{l_t} \right).$$

The effective capital/labor ratio is greater than the optimal one because some units of capital are not employed at equilibrium.

These three definitions of the capital/labour ratio are related by the following expression (coming from equation (15)):

$$\left(\frac{k_t}{l_t}\right)^\rho = k_t^\rho + x_t^\rho.$$

This means that the effective capital/labour ratio (the one which is observed at the macroeconomic level) is a weighted average of (i) the capital stock per capital which is the capital/labour ratio prevailing in firms with a *good* productivity shock and (ii) the ex- ante capital/labour ratio which is the effective ratio prevailing in the firms with a *bad* productivity shock.

2.3 The capital market equilibrium

As stated in Section 1, individuals can put their savings in two different assets, debts and shares, both issued by firms. *Intragenerational* borrowing and lending is zero at equilibrium, because all individuals are identical.

Even if expected profits are equal among firms, profits are different from one firm to one another depending on the realization of the idiosyncratic shock. To avoid uncertainty in individual's problem, let us assume that individuals buy a share of the market portfolio composed by all the firm shares.⁴ The return of the market portfolio is equal to the expected return of firms. As stated before, each share pays at t as dividend the time t profits. Since there is no aggregate uncertainty, the aggregate flow of profits is known and individuals can forecast correctly the share price q_t for all future periods. Expected profits are the same for all firms and equal to aggregate profits $\pi_{t+1} = Y_{t+1} - w_{t+1}L_{t+1}$. The expected return on a share is

$$\frac{E_t(q_{t+1} + \pi_{t+1})}{q_t} = \frac{q_{t+1} + \pi_{t+1}}{q_t}.$$

For both shares and physical capital to be held at equilibrium, the *arbitrage condition* must hold:

$$\frac{q_{t+1} + \pi_{t+1}}{q_t} = 1 + r_{t+1} \quad (20)$$

Computing π_{t+1} with the aggregate equilibrium conditions (12) to (16) and the equilibrium values for \bar{u} , one has

$$\frac{\pi_{t+1}}{N_t} = (1 - \beta)x_{t+1}^{1-\alpha}(1 - \bar{u})^{1+\rho}. \quad (21)$$

The arbitrage condition becomes

$$z_{t+1} = (1 - \beta)x_{t+1}^{-\alpha} \left(\frac{\bar{d}^{1+\rho}}{1+n} z_t - (1 - \bar{u})^{1+\rho} x_{t+1} \right), \quad (22)$$

⁴Notice that the utility function is concave in c_2 , implying that individuals are risk-averse and that they optimally like to diversify their investments. The asset market equilibrium predicts that individuals buy the market portfolio.

where z_t is equal to the value of shares per person q_t/N_t .

The equality between savings and investment can be written as

$$(1 - \theta)w_t l_t = k_{t+1}(1 + n) + z_t.$$

Using the conditions (12) to (16), it becomes

$$x_{t+1} = \frac{(1 - \theta)\beta}{1 + n} \bar{d} x_t^{1-\alpha} - \frac{\bar{d}}{(1 + n)(1 - \bar{u})} z_t. \quad (23)$$

Equations (22) and (23) characterize a dynamic system in z and x .

The optimal capital/labor ratio follows a process (23) which is very similar to the one obtained in the standard Diamond model. There are however three main differences with the Diamond model: One difference comes from the wage bargaining process (represented by the parameter β). Considering (23) at given z_t , union power has a positive effect on the capital-labor ratio as in Devereux and Lockwood (1991). This is because an increase in union power increases the wages of the young workers, increasing therefore their savings and reducing the rate of return. The second difference comes from the presence of uncertainty and heterogeneity (represented by the parameter ρ). An increase in the variance of the productivity shock reduces the value of ρ . This increase in uncertainty leads to an increase in the probability of non-utilisation of the equipment, i.e., a decrease in the marginal productivity of capital. Finally, the fact that firms obtain pure profits under efficient bargaining makes them valuable. This requires the existence of a market for shares whose arbitrage condition introduces a second dimension in the dynamic system. In this case, the analysis of the model is more complicated than in the neo-classical case.

To have a better understanding of the model let us first compute the steady state behind (22) and (23) before analysing the dynamics more carefully.

3 The steady state

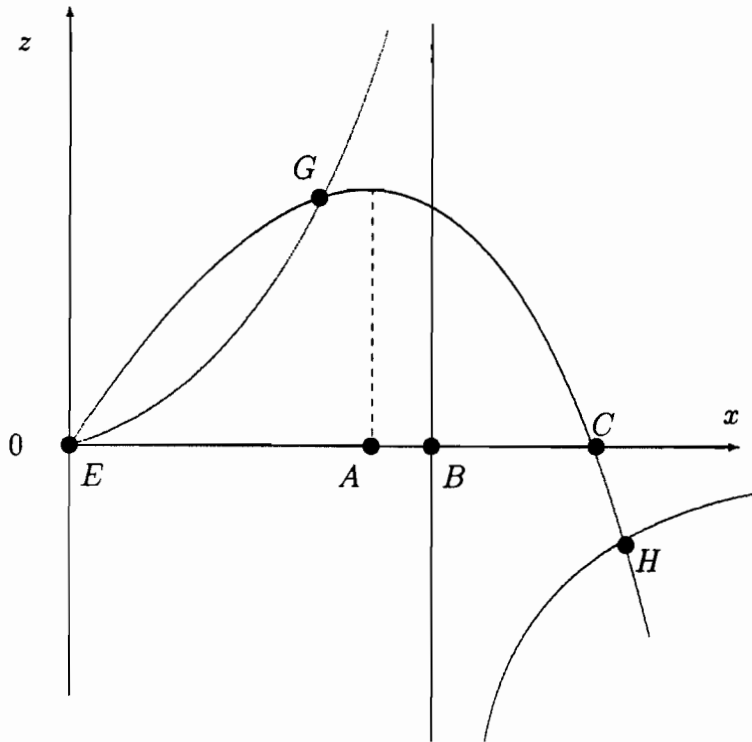
The steady state value for x and z should satisfy the following system computed from (22) and (23):

$$\begin{aligned} z &= \frac{(1 + n)(1 - \bar{u})}{\bar{d}} \left[\frac{(1 - \theta)\beta\bar{d}}{1 + n} x^{1-\alpha} - x \right] \\ z &= [(1 - \beta)x(1 - \bar{u})^{1+\rho}] \left[\frac{(1 - \beta)\bar{d}^{1+\rho}}{1 + n} - x^\alpha \right]^{-1} \end{aligned}$$

These two equations are drawn in Figure 3. The first equation defines a function $z(x)$ which passes through the origin. The second equation defines a function $z(x)$, which also passes through the origin and is not defined at point B where

$$x_B = \left[\frac{(1 - \beta)\bar{d}^{1+\rho}}{1 + n} \right]^{1/\alpha}.$$

Figure 3: The steady state



As it is shown in the appendix, the first function $z(x)$ is concave and start from 0 with an infinite slope. The second function is convex at the left of the discontinuity point, with a positive and finite slope at 0 and an infinite slope at x_B . There is therefore an intersection G of these two functions at the left of the discontinuity point, ensuring the existence of a steady state with a positive x and z .⁵

Notice that if we assume zero depreciation of capital, the analysis is quite simpler. In this case, equation (12) becomes

$$r_t = (1 - \beta)(1 - \alpha)^{1+1/\rho} x_t^{-\alpha}$$

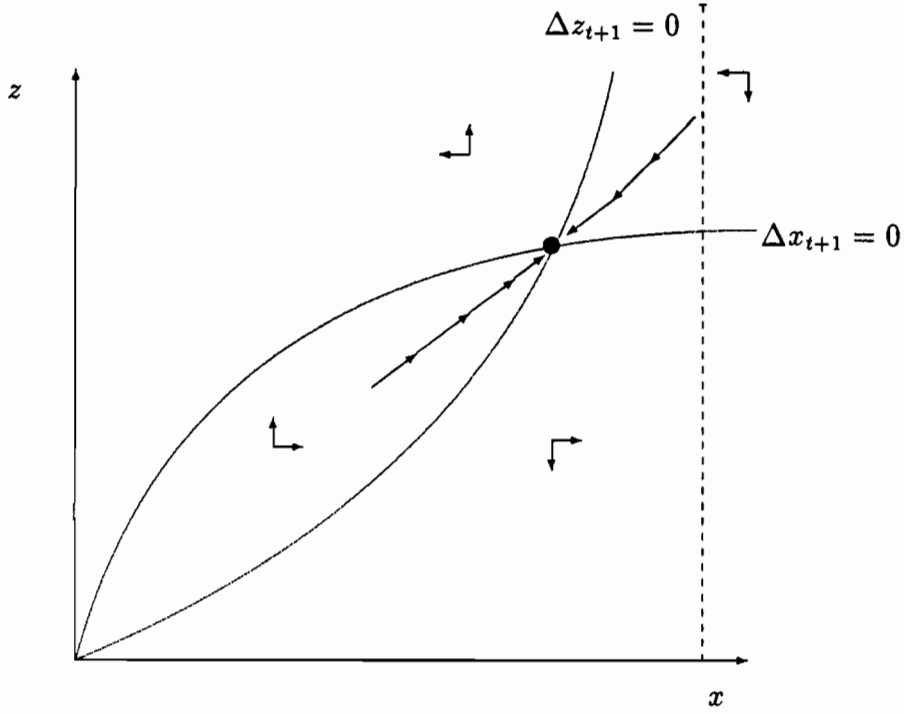
and the system for computing the steady state is:

$$z = \frac{(1+n)(1-\bar{u})}{\bar{d}} \left[\frac{(1-\theta)\beta\bar{d}}{1+n} x^{1-\alpha} - x \right]$$

$$z = \frac{(1+n)(1-\bar{u})^{1+\rho}}{\bar{d}^{1+\rho}} x$$

⁵Let us illustrate this with a numerical example. Taking $\alpha = 0.35$, $\rho = 10$ (which corresponds to a standard-error of the shock of 15% by (10)), $\theta = .6$, $\beta = .5$ and $n = 1.01^{30}$ (a yearly growth of population of 1%), the unemployment rate is 10%, the degree of utilisation of capacities is 96 %, the capital/labour ratio is 0.0007 implying through (16) an yearly real interest rate of 4.7%.

Figure 4: The phase diagram



In this case, the second equation defines a linear relation between z and x and is no longer affected by union power. The steady state equilibrium is

$$x^* = \left[\frac{(1 - \theta)\beta \bar{d}^{1+\rho}}{1 + n} \right]^{\frac{1}{\alpha}},$$

which is equal to the standard Diamond equilibrium when $\beta = 1$ and $\bar{d} = 1$.

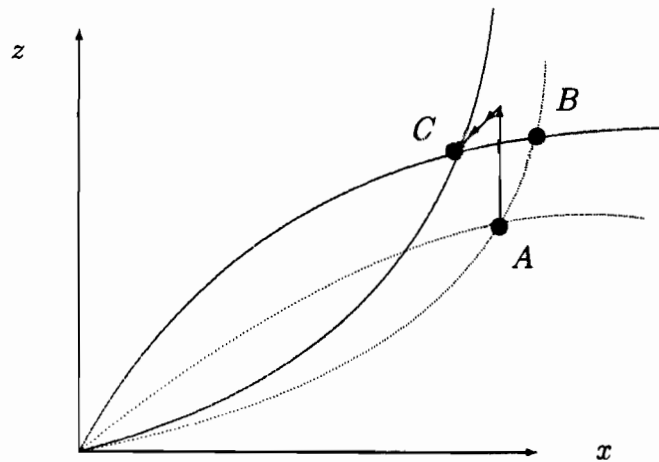
4 Dynamic analysis

Replacing x_{t+1} from (22) in (23), the dynamic system gives now the change in x_{t+1} and z_{t+1} as a function of their past levels.

$$\begin{aligned} \Delta x_{t+1} &= \frac{(1 - \theta)\beta}{1 + n} \bar{d} x_t^{1-\alpha} - x_t - \frac{\bar{d}}{(1 + n)(1 - \bar{u})} z_t \\ \Delta z_{t+1} &= z_t \frac{(1 - \beta)\bar{d}^{1+\rho}}{1 + n} \left[\frac{(1 - \theta)\beta}{1 + n} \bar{d} x_t^{1-\alpha} - \frac{\bar{d}}{(1 + n)(1 - \bar{u})} z_t \right]^{-\alpha} \\ &\quad - (1 - \beta) \left[\frac{(1 - \theta)\beta}{1 + n} \bar{d} x_t^{1-\alpha} - \frac{\bar{d}}{(1 + n)(1 - \bar{u})} z_t \right]^{1-\alpha} (1 - \bar{u})^{1+\rho} \end{aligned}$$

The phase diagram for this system is presented in Figure 4. The concave phaseline $\Delta x_{t+1} = 0$ coming from the first equation describe the equilibrium locus where savings

Figure 5: Union power and the capital/labour ratio



are equal to investment in shares and physical capital. The convex phaseline $\Delta z_{t+1} = 0$ coming from the second equation describes the equilibrium locus where the return to physical capital is equal to the dividend per share plus the capital gain. The arrows indicate the directions of motion. As in standard growth models, the loglinearization of the dynamic system around the steady state confirms that there is a unique saddle path converging (locally) to the steady-state. Under rational expectations the economy is always located on the saddle path.

Let us now consider the effect of a rise in union power on the steady-state capital/labour ratio and on its dynamics.

Considering first the concave phaseline $\Delta x_{t+1} = 0$, it is clear that this will move to the North-West. This first effect is the one present in Devereux and Lockwood (1991). Since in their model, the second phaseline is not affected by union power, the equilibrium moves from point *A* in Figure 5 to point *B*. The increase in union power increases the wages of the young workers, increasing therefore their savings and the capital stock and reduces the rate of return.

Considering now the convex phase line $\Delta z_{t+1} = 0$ it can be shown that it moves also to the North West. Through (12) and (21), at given capital/labor ratio, a rise in union power cuts the dividends π less than it cuts the interest rate r on physical capital (as long as the depreciation rate is non-zero). This requires a rise in the value of the firm to maintain the arbitrage condition. This rise in the value of the firm could be achieved by an increase in the probability of using the capital stock implying a reduction in physical capital.

Therefore, in the presence of capital irreversibility, there is a second effect of union power on the stock of capital which goes in the opposite direction of the first one. The new steady state is in *C* with an ambiguous net effect on the capital/labor ratio (remember that through (13) there is a monotonous relation between the stock of capital per head and the capital/labour ratio).

The dynamics related to a rise in union power goes as follows. In the case depicted in Figure 5, the net effect on the capital stock is negative. In this case, at the time of the change in union power, the economy jumps on the new saddle path and the value of the firm rises above its steady state level. Then the capital stock starts declining, so does the value of the firm, in order to reach the new steady state at C . If the net effect of the rise in union power on the capital stock at steady state is positive, there is no overshooting of the value of the firm and both variables increase monotonically to their new steady state value.

5 Conclusion

In this model, which is a particular case of a more general model where the utility function is homotetic and the ex-ante production function has constant returns to scale, we show that there is underemployment of production factors at equilibrium, even at the steady state. The result depends crucially on the putty-clay technology, the existence of firm-specific skill (labor market segmentation) and irreversible investment and on technological uncertainty.

The existence of unemployment and of capacity underutilisation is independent from the presence of unions. Because of capital depreciation (linked with irreversibility), the role of unions in the model is different from the one in Devereux and Lockwood (1991). In particular their result that union power affects positively the capital stock is no longer true. In the presence of capital irreversibility (or more generally, when there is a non-zero depreciation rate), the effect of union power on the stock of capital is ambiguous.

Both heterogeneity and uncertainty play a crucial role in this economy. When uncertainty and heterogeneity disappear, capacities become full-employed and the unemployment rate goes to zero. When there is uncertainty but there is not heterogeneity, i.e., if uncertainty is not idiosyncratic, all firms will be in the same situation at equilibrium. Under these conditions two types of equilibrium could be possible, full-employment with underutilization of capacities or full-capacity with unemployment. To generate simultaneously unemployment and underutilization of capacities the existence of heterogeneity is necessary.

Underemployment results from the fact that the irreversible skill decisions of the households and the investment decisions of the firms are taken without knowing with certainty the firms productivity for the next period. Once the shock has occurred, it is too late to revise its plan and to move to more favorable micro-markets. This result can be interpreted, as in Sneessens and Drèze (1986), as inducing some "mismatch" in the economy. The "mismatch" depends on the existence of firm specific uncertainty and heterogeneity.

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Appendix

The steady state is given by:

$$z = \frac{(1+n)(1-\bar{u})}{\bar{d}} \left(\frac{(1-\theta)\beta\bar{d}}{1+n} x^{1-\alpha} - x \right) \quad (I)$$

$$z = (1-\beta)x^{1-\alpha}(1-\bar{u})^{1+\rho} \left[\frac{(1-\beta)\bar{d}^{1+\rho}}{1+n} x^{-\alpha} - 1 \right]^{-1}. \quad (II)$$

Let us first consider the function $z(x)$ given by (I). This function has two roots, one at point E of Figure 3 (where the capital labour ratio is equal to 0) and one at C where

$$x = \left[\frac{(1-\theta)\beta\bar{d}}{1+n} \right]^{1/\alpha}.$$

The function attains its maximum at A where

$$x = \left[\frac{(1-\alpha)(1-\theta)\beta\bar{d}}{1+n} \right]^{1/\alpha}.$$

Its first derivative is:

$$z' \propto \frac{(1-\alpha)(1-\theta)\beta\bar{d}}{1+n} x^{-\alpha} - 1,$$

which is positive as long as

$$x < \left[\frac{(1-\alpha)(1-\theta)\beta\bar{d}}{1+n} \right]^{1/\alpha}.$$

The value of z' when $x = 0$ is

$$z'(0) \propto \infty.$$

The second-derivative is:

$$z'' \propto -\alpha x^{-\alpha-1},$$

which is always negative.

Let us now consider the function $z(x)$ given in (II). Its first derivative is

$$z' \propto \frac{(1-\beta)\bar{d}^{1+\rho}}{1+n} - (1-\alpha)x^\alpha \left(\frac{(1-\beta)\bar{d}^{1+\rho}}{1+n} - x^\alpha \right)^{-2},$$

which is positive as long as

$$x < \left[\frac{(1-\beta)\bar{d}^{1+\rho}}{(1-\alpha)(1+n)} \right]^{1/\alpha}.$$

The value of this derivative when $x = 0$ is

$$z'(0) \propto \frac{1+n}{(1-\beta)\bar{d}^{1+\rho}},$$

which is finite. The second derivative of this function is:

$$z' \propto \left(\frac{(1-\beta)\bar{d}^{1+\rho}}{1+n} - x^\alpha \right) \left((1+\alpha) \frac{(1-\beta)\bar{d}^{1+\rho}}{1+n} - (1-\alpha)x^\alpha \right),$$

which is positive as long as

$$x < \left[\frac{(1-\beta)\bar{d}^{1+\rho}}{(1+n)} \right]^{1/\alpha} \quad \text{or} \quad x > \left[\frac{(1-\beta)\bar{d}^{1+\rho}(1+\alpha)}{(1-\alpha)(1+n)} \right]^{1/\alpha}.$$

The first function $z(x)$ is concave and start from 0 with an infinite slope. The second function has a vertical asymptote, is convex at the left of the asymptote and start from 0 with a finite slope. There is therefore an intersection of these two functions at the left of the asymptote, ensuring the existence of a steady state.