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THE WELFARE COST OF TAXATION AND ENDOGENOUS GROWTH

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Abstract

The marginal efficiency costs of different taxes is analyzed in three models with endogenous growth, and the values are compared with those found in standard models. The models analyze how taxes affect (i) the trade-off between human capital accumulation and leisure, (ii) the intertemporal trade-off in consumption, and (iii) the trade-offs in a two-sector model. In general, the efficiency cost in models with endogenous growth may be greater or lower than in models with exogenous growth. When the value of the efficiency cost is very large, it is found to be very sensitive to the specification of the model, and it is reduced dramatically when government expenditures are a production input. In the two-sector model, the only tax which has a very high efficiency cost is the tax on time spent for human capital accumulation, and it may not be empirically important. It is verified that a positive impact of a tax reform on the long-term growth rate is not indicative of welfare improvement.

Key Words
Welfare cost of taxation; endogenous growth; fiscal policy.

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1 Introduction

Recent studies have analyzed mechanisms in which an economy's long-term growth rate is endogenous to individual actions, (*e.g.*, Romer (1986), Lucas (1988), and their references). These models have the property that economic policy has an impact on long-term growth, (Jones and Manuelli (1990), Rebelo (1991), Rebelo and Stokey (1992)). Perhaps because compounded growth rates may produce very large differences in the long-run, it has been suggested that in models of endogenous growth, the efficiency cost of taxation may be much larger than previously thought. For example, King and Rebelo (1990) argue that when the income tax rate is raised from, say, 20 to 30 percent, the welfare impact is 15 to 60 times larger in a model of endogenous growth than in the standard neo-classical model. The purpose of this paper is to analyze carefully the efficiency cost of taxation in models of endogenous growth. It will be shown that for plausible parameter values the difference between the efficiency cost of taxation in models of exogenous growth and endogenous growth is not very large. This cost may even be smaller in the latter case.

Since there are no externalities, the standard principles of Public Finance apply. These principles state that the welfare cost of taxation increases with the degrees of substitutability in consumption and production. In a previous paper, I emphasized that "the welfare cost of (capital income) taxation depends essentially on the elasticity of substitution between capital and labor in the production function" (Chamley, (1981)). I compared the result of Levhari and Sheshinski (1972) who assumed a linear model, with the value of the efficiency cost of taxation in a model of growth with a Cobb-Douglas production function. The welfare cost of taxation was found to be about 40 times higher in the linear model of Levhari-Sheshinski than in the Cobb-Douglas model[1]. The one sector linear model of King and Rebelo (1990) and Rebelo (1991) is the same as that of Levhari and Sheshinski (1972). In view of the previous studies, the claim of a large welfare cost of taxation is not new. This paper emphasizes again that the reason for such high values is the (assumed) high degree of intertemporal substitution in the production function.

High values of the elasticities of substitution are neither necessary nor sufficient to generate the property of endogenous growth. Therefore, in general, the efficiency cost of taxation may be higher or lower in the case of endogenous growth than in standard models with exogenous technological change.

Since policies have an impact on the long-run growth rate, they may generate very significant differences in the long-run. However, the long-run impact of a policy is not an appropriate criterion for policy evaluation, as it is well known in the standard models of growth. It will be verified here that there exist simple revenue neutral tax reforms which improve the long-run growth rate, and lower welfare.

In this paper, I focus on the marginal efficiency cost of taxes (MEC), which is defined as the ratio between the welfare impact of a small change of the rate of taxation (in terms of a wealth equivalent), and the impact on revenues. In some cases, there is a simple relation between the MEC and the welfare impact of a change of the tax rate. In other cases, this simple relation does not hold because of incidence shifts. The criterion of the marginal efficiency cost is more appropriate than the welfare impact: a small impact on welfare may be due to a small tax base; in some cases a reduction of tax rates may induce a welfare improvement and a revenue

[1]See the case where $\sigma = 1$, Table 1, Chamley (1981).

increase. Such a situation cannot be established by considering the welfare impact only.

In order to analyze the effect of tax distortions on the intertemporal allocation of resources, I consider three separate models; each focuses on a specific trade-off: (i) leisure today versus consumption of physical goods in the future, (ii) the consumption of physical goods today versus their consumption in the future, (iii) the allocation of physical and human capital in a model with two sectors of production.

Comparisons between models of exogenous and endogenous growth are subject to obvious pitfalls: an arbitrarily chosen structural model generates, not surprisingly, an arbitrary value of the efficiency cost of taxation. However, arbitrary structures may also generate arbitrary responses, say, of savings or labor to tax reforms. There is currently less consensus on the long-run dynamic properties of economies[2], than on the responses to taxation in the short-run. I suggest that in comparing different models, one should at least verify that they generate plausible responses of consumption or labor to tax changes.

The paper is organized as follows. In the first model which presented in Section 2, agents choose between leisure and the production of human capital which produces output. This model generates the property of endogenous growth but it does not require a high degree of intertemporal substitutability. The model is a simple extension of the standard (static) model with a leisure-labor trade-off. The results of the two models can thus be compared when their structural parameters are chosen such that they generate the same response of labor to the net wage rate. The efficiency cost of income taxation is found to be lower in the model with endogenous growth than in the simple static model.

In the model of Section 3, physical capital is the only variable input, and the labor supply is fixed. The formulation of the model encompasses

[2]For some empirical studies see Barro (1989), Mulligan and Sala-i-Martin (1992).

the special cases of the standard model with exogenous growth and of the linear model of King and Rebelo (1990), which has endogenous growth. It is found that a general income tax has a smaller efficiency cost in the basic model largely because three quarters of the tax falls on labor which is in fixed supply. When the tax is raised only on the income of physical capital, it may be higher or lower in the linear model than in the standard model.

This model is extended as in Barro (1990) to take into account the role of government expenditures in the aggregate production function. These expenditures are financed by distortionary taxation, and their level is optimized in the second-best. For the computation of the marginal efficiency cost of taxation, one can assume, by the envelope theorem, that government expenditures are fixed. Since the growth rate of these expenditures determines the growth rate of the economy, it follows immediately that the MEC is the same as in an economy with exogenous growth. However, the MEC is significantly lower in this case than when government expenditures do not affect individuals' decisions because publicly provided goods reduce the possibilities of intertemporal substitution of the private sector. For the parameters of King and Rebelo, this reduction is by a factor of 20 when the share of government expenditures in total output is equal to 25 percent.

The one sector model takes a broad view of capital, which in the words of Barro is a "composite of physical and human capital". However, most systems of taxation have different impacts on the accumulation of physical and human capital. This has been highlighted by Lucas (1990), who notes that in a model where the production of human capital requires only human capital, a wage tax has no effect on the incentive for human capital investment: it affects both the return and the opportunity cost in the same proportion. A realistic evaluation of the efficiency cost of taxation requires therefore a model with two sectors. Such a model is analyzed in Section 4. The first sector produces physical goods for consumption or investment, and the second produces human capital. The model extends those of Uzawa

(1965), Lucas (1988) and King and Rebelo (1991). Both sectors have different capital intensities[3], and may have elasticities of substitution that are different from one. The impact of taxation on each of the four factors is analyzed separately.

The following conclusions are robust to the choice of the structural parameters of the model: (i) as expected, the MEC of the labor income tax in the sector of physical goods is very low ; (ii) the efficiency cost of the tax on capital goods is of the same order of magnitude in both sectors, and its value is also of the same order of magnitude as in the standard neo-classical model; (iii) the only tax which has an MEC of a higher order of magnitude is the taxation of the use of time for human capital formation. This tax may not be empirically important, however: the specific taxation of time spent at school is not a feature of actual tax systems.

Section 4 analyzes also the impact of taxes on the long-term growth rate. It is verified that an efficient tax reform may lower the long-term growth rate.

Concluding remarks and suggestions for future work are presented in the last section.

2 Output Taxation and the Trade-Off Growth versus Leisure

2.1 The Model

The model in this section focuses on the impact of the income tax on the trade-off between leisure and human capital accumulation: time is used either for leisure or for the production of human capital, which determines the level of output in goods. There is no intertemporal trade-off in physical goods.

[3]When capital intensities are identical, tax reforms induce a massive reallocation of resources between the two sectors which is not plausible.

Hence, there is no physical capital and all physical goods are consumed. Note that in actual economies, the income of human capital dwarfs that of physical capital. Hence, the model can be justified as a stylized description which is compared to other models which focus on the accumulation of physical capital.

There is one representative consumer with the utility function

$$U = \frac{1}{1-\sigma} \int_0 \infty e^{-\rho t} u^{1-\sigma}(C_t, h_t(1-\ell_t))dt, \quad \sigma \neq 1.$$

where u is homogenous of degree one[4]. The variables C, h, and ℓ represent the levels of consumption, human capital, and time for physical production, respectively. Effective labor is the product of human capital and time. In order to focus on the trade-off between leisure and the production of human capital, the amount of time which is devoted to physical production is assumed to be fixed and normalized to 1.

The production function for goods is linear,

$$y_t = Ah_t,$$

and the production function for human capital is the same as in Lucas (1988):

$$\dot{h}_t = h_t g(\ell_t).$$

Withouth loss of generality, human capital at time 0 is normalized: $h_0 = 1$.

The fiscal instrument is a tax on physical output at the rate θ. Tax revenues finance government expenditures. The mechanism by which these expenditures have a impact on social welfare need not be made explicit here. It is assumed that these expenditures do not affect individuals' choice. This assumption will be relaxed in Section 3.4.

[4]This function is used in Rebelo (1991). Other utility functions could be considered such as $u = Log(C) - B(\ell)$, with B concave.

2.2 The Equilibrium

A standard exercise shows that the competitive equilibrium is characterized by the first order conditions:

$$\begin{cases} u^{-\sigma}\Big(A(1-\theta)u_1 + (1-\ell_t)u_2\Big) + g(\ell_t)\lambda_t & = & -\dot{\lambda}_t + \rho\lambda_t, \\ -u^{-\sigma}h_t u_2 + \lambda_t h_t g'(\ell_t) & = & 0, \end{cases}$$

where λ is the current private marginal value of human capital. Since the function u is homogenous of degree one, the values of $u_1 = u_1(Ah, (1-\ell)h)$ and $u_2 = u_2(Ah, (1-\ell)h)$ depend only on ℓ. Define the variable $\mu_t = \lambda_t h_t^\sigma$. By the first of the two previous conditions, μ_t is constant:

$$\mu_t = \mu = \frac{u^{-\sigma}\Big(A(1-\theta)u_1 + (1-\ell_t)u_2\Big)}{\rho + \sigma g - g},$$

and the level of ℓ is also constant and determined by the equation

$$u_2 = \frac{g'}{\rho + \sigma g - g}(A(1-\theta)u_1 + (1-\ell)u_2). \tag{1}$$

Since the value of ℓ determines the growth rate $g(\ell)$, the growth rate is constant over time, and its value depends on the tax rate θ. The term $r = \rho + \sigma g$ is the intertemporal rate of substitution in consumption; it determines the rate of return in this economy where there is no physical capital.

Define the variables w and $\bar{w}$ as:

$$w = \frac{g'}{\rho + \sigma g - g}(A + (1-\ell)\frac{u_2}{u_1}), \quad \text{and } \bar{w} = w - \tau, \quad \text{with } \tau = \frac{g'}{\rho + \sigma g - g}A\theta.$$

These variables can be interpreted as the gross and the net of tax marginal productivity of labor, respectively, and τ is the tax wedge. With this notation, the first order condition (1) which determines the labor supply takes the form

$$u_2 = \bar{w}u_1.$$

This equation would be the same in a standard model with leisure-labor choice and the proper notation. The comparison between the two models will be discussed further at the end of Section 2.3.

2.3 The Marginal Efficiency Cost of Taxation

In order to define the marginal efficiency cost of taxation, assume that the economy is on a path with a constant growth rate, and that the ratio between government expenditures and output is constant and equal to q. Taxes cover expenditures and $\theta = q$. At time zero, the rate θ is reduced permanently by $\Delta\theta$. The government is assumed to maintain the same stream of expenditures and to meet the shortfall of revenues by lump-sum taxation[5]. Call ΔB the present value of these lump-sum tax revenues. The reduction of the tax rate θ has a positive effect on welfare which has a wealth equivalent of ΔJ. The marginal efficiency cost of taxation is defined as the ratio $MEC = \Delta J/\Delta B$. The impacts of the tax change on welfare and tax revenues are now analyzed separately.

Welfare

Since the level of utility on the growth path with a tax rate θ is equal to

$$U = \frac{1}{1-\sigma}\frac{u^{1-\sigma}(A, 1-\ell)}{\rho + (\sigma - 1)g(\ell)} \quad ,$$

one finds, using $u = u_1 + (1-\ell)u_2$ and (1), that the welfare impact of the tax change is

$$\Delta U = \frac{\hat{u}^{-\sigma}u_1}{\rho - g(1-\sigma)}\tau\Delta\ell. \tag{2}$$

Its wealth equivalent is ΔJ, where

$$\Delta J = \frac{\tau\Delta\ell}{r-g}, \quad \text{and } r = \rho + \sigma g. \tag{3}$$

It is equal to the present value of the product of the variation of time for human capital investment and of the difference between the social and the private marginal return, i.e., the tax wedge.

[5] It will be shown in Section 3.4 that this is the correct method of computation when government expenditures are endogenous in the second-best.

Revenues

The assumption made at the beginning of this subsection implies that the present value of lump-sum taxes which is required to meet the government's budget constraint is equal to

$$\Delta B = \Delta(\frac{\theta - q}{r - g}) = \frac{\Delta\theta}{r - g}. \tag{4}$$

The Marginal Efficiency Cost of Taxation

The value of the marginal efficiency cost of taxation follows immediately from the expressions (3) and (4):

$$MEC = \tau\frac{\Delta\ell}{\Delta\theta}. \tag{5}$$

It is interesting to compare now the previous results with those of the static model which focuses on the trade-off between leisure and work. In that model, the consumption good c is produced by labor ℓ and the technology of production is linear: $c = w\ell$, where w is fixed. The utility function of the representative individual is of the form $u(c, 1-\ell)$. The income equivalent of the welfare impact of a change of the tax rate is equal to $\tilde{\Delta J} = w\theta\Delta\ell$. This formula is equivalent to (3). The impact of the tax change on revenues is equal to $\tilde{\Delta B} = w\ell\Delta\theta + w\theta\Delta\ell$, and the marginal welfare cost of revenues is equal to

$$M\tilde{E}C = \frac{\tau\Delta\ell}{w\ell\Delta\theta + w\theta\Delta\ell}. \tag{6}$$

The sign of $\Delta\ell$ is the opposite of that of $\Delta\theta$ because the tax change is compensated. Hence, for given values of the variations of the tax rate $\Delta\theta$, and of the labor supply $\Delta\ell$, $MEC < M\tilde{E}C$.

Suppose that changes of the tax rate have the same effect on the choice of leisure in the model with endogenous growth and in the static model. Then, the marginal efficiency cost of taxation is smaller in the first model than in the second.

The explanation of this remarkable result follows from the equations (5) and (6): changes of the tax rate have the same effect on leisure and welfare in the two models. The difference is that (i) in the dynamic model with endogenous growth, the output tax falls on both the return of new investments after time zero and on the existing stock at time zero, which is inelastic; (ii) in the static model, the second effect does not occur.

3 Consumption versus Investment in the One Sector Model

In this section, the model is tailored to highlight the impact of taxation on the trade-off between consumption and saving of produced goods. A one sector model is sufficient for the analysis which emphasizes the role of the intertemporal substitution. It will be extended to a two-sector model in Section 4.

3.1 The Model

Population is constant and labor-augmenting technological change occurs at a constant rate of μ[6]. There is one produced good which can be used for consumption or as an input for production. The level of output per unit of labor, y, is a function of the capital labor ratio k:

$$y = f(k) \quad \text{with } f''(k) < 0, \quad Lim_{k \to 0} f'(k) = \infty, \quad Lim_{k \to \infty} f'(k) = 0.$$

Consumers are represented by a single agent who has the utility function

$$U = \frac{1}{1-\sigma} \int_0^\infty e^{-\rho t} C_t^{1-\sigma} dt, \qquad (\sigma \neq 1),$$

[6] One could also assume that population is growing. In this case, μ should be the sum of the growth rates of population and productivity.

where C_t is the level of consumption. When the consumption per unit of efficient labor is represented by c_t, the utility function takes the form

$$U = \frac{1}{1-\sigma}\int_0^\infty e^{-(\rho+\sigma\mu-\mu)t} c_t^{1-\sigma} dt, \qquad (\sigma \neq 1).$$

The tax rate on capital income is equal to θ. As a first step, it is assumed that government expenditures do not affect the decisions of the private sector. It is well known that the dynamics of this model are determined by (i) the equation of capital accumulation, (ii) the Euler equation, and (iii) the convergence to a stationary solution for the variables per unit of human capital. The dynamic system is therefore defined by

$$(DS) \quad \begin{cases} \dot{k} = f(k) - c - \mu k, \\ \dot{c} = c(\frac{(1-\theta)r - \rho}{\sigma} - \mu), \quad \text{with } r = f'(k). \end{cases}$$

3.2 Tax Reform

The methology is the same as in the previous section. Initially the economy is on a balanced growth path with a constant tax rate θ, and a flow of government expenditures equal to that of revenues. After time 0, the tax rate is changed by a small value $\Delta\theta$. Changes of tax revenues are met by lump-sum transfers. I consider first the impact on welfare.

3.2.1 Welfare

Define by Δc_t the perturbation of private consumption at time t, *i.e.*, the difference at time t between the level of consumption under the new program of taxes and the level of consumption if there had been no reform. The wealth equivalent ΔJ of the welfare impact of the tax change is measured by the sum of the consumption changes valued at the consumer prices, (which are proportional to the marginal utilities of consumption):

$$\Delta J = \int_0^\infty e^{-(\bar{r}-\mu)t} \Delta c_t dt. \tag{7}$$

This expression[7] can be transformed to highlight the role of capital accumulation. Define by $y_t = f(k_t) - \mu k_t$ and $s_t = y_t - c_t$ the levels of output and of saving, net of the accumulation for growth. As for consumption, Δy_t and Δs_t are the perturbations of y_t and s_t for $t \geq 0$. Since $\Delta c_t = \Delta y_t - \Delta s_t$, the expression (7) can be rewritten $\Delta J = \int_0^\infty e^{-(\bar{r}-\mu)t}(\Delta y_t - \Delta s_t)dt$. Using the same notation, $\Delta y_t = (r-\mu)\Delta k_t = (r-\mu)\int_0^t \Delta s_\tau d\tau$. By substitution in the previous equation, one finds that

$$\Delta J = \int_{t\geq 0} e^{-(\bar{r}-\mu)t}\Big((r-\mu)\int_0^t \Delta s_\tau d\tau - \Delta s_t\Big)dt,$$

which is equivalent to

$$\Delta J = \int_{t\geq 0} e^{-(\bar{r}-\mu)t}\Big(\frac{(r-\mu)}{(\bar{r}-\mu)} - 1\Big)\Delta s_t dt. \tag{8}$$

This expression has a simple interpretation. The saving of one unit of goods generates a permanent income stream of $r-\mu$ on the balanced growth path, which is discounted by the private sector at the net rate $\bar{r}$ since this rate determines the relative prices of incomes at different dates. Because of growth, the value of the income stream is therefore equal to $(r-\mu)/(\bar{r}-\mu)$. The social gain of the additional saving Δs_t is the difference between this value and the cost of the reduced consumption. The total impact on welfare is the discounted sum of the impacts on saving for all instants.

The wealth equivalent ΔJ is very sensitive to the difference between the rate of return and the growth rate. We may use therefore in its stead a consumption equivalent ΔM, which is defined as the permanent variation of the level of consumption per unit of human capital which generates the same variation of welfare:

$$\Delta J = \frac{\Delta M}{(\bar{r}-\mu)}, \quad \text{with } \Delta M = r\theta \int_{t\geq 0} e^{-(\bar{r}-\mu)t}\Delta s_t dt, \quad \text{and } \bar{r} = r(1-\theta). \tag{9}$$

[7]Because of the economy's resource constraint, $0 = \int_0^\infty e^{-(r-\mu)t}\Delta c_t dt$, the wealth equivalent is also equal to the well known expression $\Delta J = \int_0^\infty (e^{-(\bar{r}-\mu)t} - e^{-(r-\mu)t})e^{\mu t}\Delta c_t dt$, which is the sum of the products of the tax wedges and the consumption variations.

A tax change has an impact on welfare which is equivalent to a permanent change of consumption equal to the gap between the gross and the net rate of return, multiplied by the present value of additional savings, measured at the consumer prices.

The previous formula which is similar to (3) in the previous section, emphasizes the simple point that the efficiency gain of the tax change depends on the product of two factors, the tax wedge and the response of savings to the tax changes.

For later use a simple transformation of the previous expression will be useful:

$$\Delta J = r\theta \int_{t\geq 0} e^{-(\bar{r}-\mu)t} \Delta k_t dt. \tag{10}$$

In the standard growth model with a strictly concave production function, the economy converges in the long-run to a steady state: at time 0, the level of saving jumps up by Δs_0. The growth rate of the economy also jumps up. After time 0, on the transition path, the level of saving and the growth rate are higher than in the long-term. Both values decrease gradually to the long-run levels which do not depend on the tax policy. The rate of convergence is asymptotically constant and equal to some value λ. Note that the property of a constant convergence rate may also be closely approximated by other models such as the overlapping generation model of Auerbach *et al.* (1986). Since the variation of saving Δs_t is approximated by $\Delta s_0 e^{-\lambda t}$, the consumption equivalent in (9) is equal to

$$\Delta M = \Delta s_0 \left(\frac{r\theta}{\bar{r} - \mu + \lambda} \right). \tag{11}$$

In this expression the welfare effect of taxation depends only on the initial response of saving to a tax change and on the rate of convergence to the path of constant growth.

Exogenous versus Endogenous Growth

Assume now that the production function f is linear. This is the limit case of the previous model when the concavity of f becomes vanishingly small. The new steady state, after the tax reform, is postponed to the indefinite future and the rate of convergence of the intensive variables (*e.g.* y, k), becomes vanishingly small. During the transition, which is now permanent, the growth rate depends on the tax policy. This is the linear model model of endogenous growth which is analyzed by King and Rebelo (1990).

We can use the previous analysis to find immediately an expression of the welfare impact of taxation. In equation (11), the rate of convergence is λ is equal to zero, and the value of the consumption equivalent becomes

$$\Delta M^* = \Delta s_0 \left(\frac{r\theta}{\bar{r} - \mu} \right). \tag{12}$$

The welfare impacts for the cases of exogenous and endogenous growth are measured in the expressions (11) and (12), respectively. These expressions use empirically observable variables; they provide a simple tool for the analysis of the differences between the cases of exogenous and endogenous growth.

Suppose that the structures of the two models are such that the initial responses of saving Δs_0, to a change of θ is the same in the two cases. The impacts on welfare in the two models are related by

$$\Delta M^* = \Delta M \frac{\bar{r} - \mu + \lambda}{\bar{r} - \mu}.$$

As an example, suppose that $\bar{r} = .04$, $\mu = .02$ and $\lambda = .04$. The welfare impact is three times larger in the model with endogenous growth.

The previous formula enables one to interpret in general, how a given rate change has a different impact on welfare in the cases of endogenous and exogenous growth, respectively. A large difference between the two cases must depend on the choice of the values of the net rate of return, the growth rate and the convergence rate λ, or on the initial response of the savings rate[8].

[8]King and Rebelo (1991) choose $\bar{r} = .032$, $\mu = .02$, a depreciation rate of .1, a capital

3.2.2 Revenues

In the standard model, a general income tax falls on both capital and labor. Since the second production factor is inelastically supplied, its taxation is not distortionary. The only distortion on the intertemporal allocation of resources is created by the tax on capital income. In order to analyze carefully the determinants of the efficiency cost of taxation, we have to consider the capital income tax and the general income tax separately.

The Capital Income Tax

The impact on tax revenues is measured as in Section 2. Call q the level of revenues before the reform: $q = \theta r k$. The interpretation of the expression is easier if we assume that the rate θ is increased at time 0: $\Delta\theta > 0$. Three effects take place: First, the higher value of θ increases revenues through a direct effect. Second, the tax increase is partially shifted to labor. Third, the level of capital falls gradually after time 0, thus decreasing the tax base and the amount of revenues. This decomposition is now made more specific.

The impact on revenues is equal to

$$\Delta B_C = \Delta \int_0^\infty e^{-(1-\theta)R(t)+\mu t}(\theta r_t k_t - q)dt, \tag{13}$$

where $R(t)$ is the rate of return between 0 and t: $R(t) = \int_0^t r_\tau d\tau$. In the original position, $\theta r k - q = 0$. Therefore, the impact through the discount factor $e^{-(1-\theta)R(t)+\mu t}$ is nil, and the previous expression is equivalent to

$$\Delta B_C = \Delta\theta \int_0^\infty e^{-(\bar{r}-\mu)t} r k dt + \theta \int_0^\infty e^{-(\bar{r}-\mu)t} k \Delta r_t dt + \theta \int_0^\infty e^{-(\bar{r}-\mu)t} r \Delta k_t dt.$$

share of $\alpha = .33$ in the case of exogenous growth, and 1 in the case of endogenous growth; for $\alpha = .33$, $\lambda = .13$. The ratio $\Delta M^* / \Delta M = (\bar{r} - \mu + \lambda)/(\bar{-}\mu)$ is therefore equal to about 12. Note however, that a convergence rate of .13 is very high: it implies a half-period of about 4 years. In addition, the response of net saving is 4.3 times higher in the endogenous growth model than in the standard case. The product of the two terms is about 50.5. Given the sensitivity of this number (see below), it is well within the same range of 41, which is the number of King and Rebelo (65.4/1.6 in their Table 4).

Since $k\Delta r_t = -\Delta w_t$, we find

$$\Delta B_C = \Delta\theta\frac{rk}{\bar{r}-\mu} - \theta\int_0^\infty e^{-(\bar{r}-\mu)t}\Delta w_t dt - \Delta J. \tag{14}$$

The three terms correspond to the direct effect, the variation of the incidence on labor, and the incentive effect on saving, which is the same as the welfare effect (in wealth equivalence). The second term is positive if $\Delta\theta$ is positive because an increase of taxation reduces the capital stock and the wage rate[9]. The last term is equal to the welfare cost of taxation ΔJ ($\Delta J > 0$), and, with the current sign convention, is the opposite of the value in equation (10).

The Income Tax

The income tax is levied on the incomes of both capital and labor. Revenues are equal to $q = \theta y$, and the impact of a rate change on revenues is equal to

$$\Delta B_I = \Delta\theta\frac{y}{\bar{r}-\mu} - \Delta J, \tag{15}$$

In this case the shift to labor (the second term in (14)), does not appear because the tax is on all factors.

3.3 The Marginal Efficiency Cost of Taxation: A Comparison between Exogenous and Endogenous Growth

As in the previous section, the marginal efficiency cost is the ratio between the marginal effects on welfare and on revenues: $MEC = \Delta J/\Delta B$. For the income tax, the marginal efficiency cost is equal to

$$MEC_I = \frac{\Delta J}{\Delta\theta\frac{y}{\bar{r}-\mu} - \Delta J}, \quad \text{with } \Delta\theta > 0, \quad \text{and } \Delta J > 0.$$

[9] A straightforward exercise shows that it is equal to $(\lambda/(\bar{r}-\mu+\lambda))(\theta/(1-\theta))rk(\Delta\theta/(\bar{r}-\mu)$.

and the marginal efficiency of the capital income tax is therefore equal to

$$MEC_C = \frac{\Delta J}{\Delta\theta\frac{rk}{\bar{r}-\mu} - \theta\int_0^\infty e^{-(\bar{r}-\mu)t}\Delta w_t dt - \Delta J}.$$

Numerical values of the marginal efficiency costs of the capital income tax and of the general income tax are presented in Tables 1.a-c for different choices of the structural parameters. In all the tables the initial value of the tax rate is equal to $\theta = .2$, the total growth rate in the initial position is $\mu = .02$, and the production function is of the Cobb-Douglas type with a capital share equal to α. For each choice of σ, the third line presents an approximation of the initial response of consumption to the tax change (in proportional terms). It is computed as follows: the marginal proportional reduction of consumption due to a tax decrease is multiplied by the initial value of $\theta = .2$. The value thus obtained provides an approximation of the order of magnitude of the response of consumption or saving to an abolishion of the tax.

The Capital Income Tax

Two differences appear between the standard model with exogenous growth ($\alpha = .25$), and the linear model ($\alpha = 1$), which has endogenous growth. First, the tax base in the former model is only one fourth of the base in the latter. This tends to increase the efficiency cost of taxation. Second, some of the tax incidence is shifted to the inelastic labor in the first model. This effect tends to decrease the efficiency cost of taxation. It is stronger when there is less intertemporal substitution and the incidence is shifted more rapidly. In general, the sum of the two effects is ambiguous. The first line of Table 1.a shows an example of plausible parameter values, in which *the marginal efficiency cost of the capital income tax is higher in the standard model than in the model with endogenous growth.*

Other parameter values show that the order of magnitude of the marginal efficiency cost of the capital income tax is not altered by the choice of the rate of return, or of the growth rate if σ is sufficiently large. However, an increase of the depreciation rate reduces the efficiency cost by an order of magnitude *in the standard model.* In comparing Tables 1.a and 1.b, a value of $\alpha = .25$ with no depreciation (first column in Table 1.a), corresponds roughly to a value of (the gross capital income share) $\alpha = .33$ when the depreciation rate is equal to .1, (column 2 in Table 1.b). Note that an increase of the depreciation rate reduces also the initial response of consumption. In the linear model, depreciation has no impact because it does not affect the net production function. As in Chamley (1981), the intertemporal elasticity of substitution in consumption has little impact in the standard model and a proportional impact in the linear model. For $\sigma = .5$ and $\theta \doteq .2$, a reduction of the tax rate improves revenues and welfare.

These remarks imply that the choice of the depreciation rate and of σ affects in an important way the difference in the efficiency costs of the capital income tax between the standard and the linear models.

The General Income Tax

We have seen that for a given value of the tax rate θ, the efficiency cost of the general income tax is much smaller than that of the capital income tax in the standard model because its base includes that of the (fixed) labor. The MEC of the income tax is about one fourth of the MEC of the capital income tax. In the linear model however, all income is generated by capital and the MEC of the two taxes are obviously identical.

Finally, the parameter values of King and Rebelo are used for Table 1.c. They chose a value of σ which is one. The marginal efficiency cost of the income tax is 60 times larger when $\alpha = 1$ than when $\alpha = .33$. This factor is higher than their number (about 40), because they consider the impact on welfare and not the marginal efficiency cost. As explained before, the large difference between the two cases has a simple explanation. In the standard model, the major part of the income tax falls on labor which is supplied inelastically. When one considers only the capital income tax, the gap between the values in the two cases is reduced significantly.

As emphasized in the discussion following (9), large values of the efficiency cost are associated to large responses of savings to the tax changes. Since numerous studies have found some difficulties in establishing a relation between taxes and savings, such large values of the response of consumption to tax changes or of the efficiency cost of taxation do not seem to be plausible.

3.4 Government Spending as a Production Input

Public expenditures on investment for infrastructure, maintenance, or services may be arguments in the production function or the utility function of consumers. They may thus affect the interaction between saving and the rate of return. We have seen that this interaction is an important determinant of the efficiency cost of taxation. A model of government spending with endogenous growth was first proposed by Barro (1991). I use here the same model and I focus on the case of public spending as a production input.

The production function takes the form $Y = F(K, L, P)$, where P is the level of public consumption. The extension to public investment would be straightforward. The function is assumed to be Cobb-Douglas:

$$F(K, L, P) = K^{\alpha} L^{\beta} P^{\gamma},$$

where α, β and γ are positive parameters. L represent some fixed input which may be labor, and which has a fixed growth rate μ_0:

$$L_t = L_0 e^{\mu_0 t}, \quad \text{with } L_0 \geq 0, \mu_0 \geq 0.$$

For simplicity, the production function has constant returns to scale in the variable inputs: $\alpha + \gamma = 1$.

Government expenditures are financed either by a tax on capital income or a general income tax, at the rate θ. Note that there are constant returns to scale in capital and government spending and that the growth rate is endogenous to the fiscal policy. In the second-best, the government optimizes both the level of the tax rate θ and the stream of expenditures. A simple exercise shows that the dynamic path under the efficient fiscal policy is a balanced growth path with a growth rate $\mu \geq \mu_0$. In the second-best, the level of welfare is obviously smaller than if public spending could be financed by lump-sum taxation. Some numerical comparisons were presented by Barro (1991).

Let us now consider the following experiment. In the initial state of the economy, government spending is financed only by the capital income tax or the income tax and the value of the tax rate θ is optimized. At time zero, a lump-sum tax is introduced which generates a revenue of one unit. The tax rate θ and the level of government expenditures are then reoptimized. By the envelope theorem, the impact on welfare is the same as when government expenditures do not change, and revenues of the distortionary tax are reduced by the amount of the lump-sum tax. This is the experiment which was conducted in the previous sections. For the computation of the marginal efficiency cost, one can therefore assume that the level of government spending is a fixed input, and that the production function takes the form

$$y = Ak^{\alpha},$$

where α is equal to one minus the share of government expenditures.

Therefore, *the welfare cost of taxation is the same as in the case of "exogenous growth" in which the share of capital is replaced by one minus the share of government expenditures.*

Although the growth rate is endogenous to policy, the efficiency cost of taxation is the same as that of a model where growth is exogenous. This property illustrates that whether growth is endogenous or not is largely irrelevant for the determination of the efficiency cost of taxation.

In general, the introduction of a small share of government expenditures in the production is sufficient to reduce dramatically the efficiency cost, whenever this cost is very high with a linear technology. For example, in the case of the parametric values considered by King and Rebelo, MEC_I is reduced by a factor of 6 when the share of government expenditures in production increase from 0 to 0.1, (from 2.0 to .3526 in the line $\sigma = 1$ in Table 1.c). If the share of government expenditures is equal to .75 (which is the value considered in Barro (1989)), MEC_I is reduced by a factor of 18. Such large variations show that whenever the efficiency cost is much higher in the case of endogenous growth than in the standard.neoclassical model, its value is extremely sensitive to the structural parameters.

4 A Two Sector Model

Any evaluation of a tax reform has to deal with the relative magnitudes of the efficiency costs of the taxes on capital and labor (or human capital), respectively. The analysis of this problem requires a model which distinguishes the two types of capital. A moderate degree of empirical realism suggests also that the productions of physical and human capital use different technologies. These issues are now analyzed in a model where physical goods and human capital are produced in two different sectors.

4.1 The Model

As in Section 3, there is one type of physical goods and its production technology is represented by the function $Y = F(K_1, H_1)$, which has constant returns to scale, and where K_1 and H_1 are the inputs of physical and human capital, respectively. All production functions are net of depreciation.

The output of human capital is determined by another function, which has also constant returns to scale. Since human capital is not consumed, its production is equal to its accumulation:

$$\dot{H} = G(K_2, H_2). \tag{16}$$

The accumulation of physical capital is the same as in the previous section:

$$\dot{K} = F(K_1, H_1) - C, \tag{17}$$

where C is total consumption, and consumers have also the same representation.

The resources constraints are determined by the two previous equations and

$$K_1 + K_2 = K, \quad H_1 + H_2 = H.$$

For simplicity, time subscripts are omitted whenever possible.

In this section, the role of government expenditures is neglected and all taxes are refunded in lump-sum fashion. There are four tax instruments, one on each of the factors of production in the two sectors. The tax rates on the incomes of K_i and L_i are equal to θ_{ri} and θ_{wi}, respectively.

The equations of inter-temporal arbitrage are

$$\dot{C} = \frac{C}{\sigma}((1 - \theta_{r1})F_1 - \rho),$$

and

$$\dot{p} = p(1 - \theta_{r1})F_1 - (1 - \theta_{w1})F_2,$$

where p is the price of human capital in terms of physical capital.

The equations of intra-temporal arbitrage are

$$(1 - \theta_{r1})F_1 - p(1 - \theta_{r2})G_1 = 0,$$

and

$$(1 - \theta_{w1})F_2 - p(1 - \theta_{w2})G_2 = 0.$$

These 8 equations and the convergence to a balanced growth path determine a unique path for the variables K_1,H_1, K_2, H_2, K, H, C and p. The dynamic properties of this model have been analyzed by Mino (1992)[10]. For further discussion it is assumed that both production functions are Cobb-Douglas:

$$F(K_1, H_1) = K_1^{\alpha_1} H_1^{\beta_1}, \quad \text{with } \alpha_1 + \beta_1 = 1,$$

and

$$G(K_2, H_2) = K_2^{\alpha_2} H_2^{\beta_2}, \quad \text{with } \alpha_2 + \beta_2 = 1.$$

4.2 Impacts of Tax Changes

The marginal efficiency cost of taxation is defined as in the previous sections. For its computation, one could express all variables as ratios with respect to the level of human capital, H. In this formulation, the dynamic system has one state variable which is the ratio K/H. Therefore, the algorithm of the previous section could be used here[11]. For reasons of generality however, a different procedure will be followed. The impact of a tax change on welfare and revenues is computed when $\alpha_1 + \beta_1 < 1$ and $\alpha_2 + \beta_2 < 1$. In this case, there are two state variables K and H, and two stable eigenvalues which determine two rates of convergence. The method of approximations of integrals that was used in the previous section is extended to take this into account. Then, the parameter values are chosen such that $\alpha_1 + \beta_1$ and $\alpha_2 + \beta_2$ are vanishingly close to 1.

Numerical results are presented in Tables 2-5 for different choices of structural parameters and fiscal instruments. The base case is presented in Table 2 with a net rate of return of 2 percent, a growth rate (population plus technological change) of 2 percent, an elasticity of the marginal utility σ equal

[10] Mino assumed strictly concave production functions. The dynamics of the linear case were analyzed in the original paper of Uzawa (1965) which inspired many of the recent studies, (*e.g.*, Lucas (1988), Caballé and Santos (1991), Chamley (1991), Mulligan and Sala-i-Martin (1992)).

[11] This property is exploited by Mulligan and Sala-i-Martin (1991) for the exact computation of the dynamic path.

to 2, and no depreciation of capital. The shares of physical capital in the two sectors are equal to .25 and .15, respectively. The first of these two numbers is standard. There is no consensus on the second number, but it is smaller than the first because the production of human capital is probably more intensive in human capital than the production of physical goods.

In Table 2.a, the economy is at time 0 on a balanced growth path with one tax rate set at .2 and all other rates set at 0. Each tax reform is a variation of the unique rate that is different from 0. In Table 2.b, all tax rates are equal to .2 in the initial position, and each tax reform changes only one tax rate.

The term $\Delta J/J$ in the first line measures the welfare impact of a tax reduction expressed as an equivalent proportional increase of consumption (or wealth), on the balanced growth path. It is computed as a marginal welfare change, but in order to facilitate its interpretation the marginal change is multiplied by the tax rate $\theta = .2$ in the tables. The presentation of the initial response of consumption $(\Delta C_0)/C$ to the tax reform is the same as in the previous section. The term $(\Delta G_0)/G$ is the response of investment in human capital to the tax reform at time zero. The term $\Delta\mu$ represents the impact of the tax reform on the growth rate in the long-run. As for $(\Delta C_0)/C$, both $\Delta G_0/G$ and $\Delta\mu$ are presented as the product of the marginal impact of a tax change and of the initial value of the tax rate ($\theta = .2$).

4.2.1 The Marginal Efficiency Cost of Taxation

Factor Taxation in the Production of Physical Goods (Sector 1)

The marginal efficiency cost of **the wage tax** in Sector 1 is positive because the tax reduces the incentive for the utilization of physical capital in sector 2. Note that if the production of human capital would not require any physical capital, the wage tax would not be distortionary, because it affects the return and the cost of human capital production in the same way, (Lucas

(1989)). This is verified in Tables 3.a and 3.b for the case $\alpha_2 = 0$. The benchmark case in which $\alpha_2 = .15$, is not significantly different from the case that was considered by Lucas, hence the small marginal efficiency cost of the wage tax. When the share of labor in Sector 2 is increased to 1, the MEC increases as expected, but its level remains moderate, with maxima of 0.1053 and 0.2399 in Tables 3.a and 3.b, respectively. The impact of the wage tax through the leisure-human capital formation has been analyzed in Section 2.

The marginal efficiency cost of **the capital income tax** is of the same order of magnitude as in the basic model of exogenous growth. It is equal to 0.2334 in the latter model, (Table 1.a), and to 0.3220 in Table 2.a, with identical values of α_1, σ and of the discount rate r^*. Note that the MEC is lower and equal to .0846 when all other tax rates are equal to .2 (Table 2.b): we have seen in the previous section that changes of the tax rates produce smaller changes of the tax base in this case, and therefore larger changes of revenues.

Factor Taxation in the Production of Human Capital (Sector 2)

The striking feature in all the tables is the high value of the MEC of **the wage tax** in Sector 2. In a few cases, incentive effects are so strong that a reduction of the tax rate from its initial value of .2 increases both welfare and revenues (for $\sigma \leq .5$ in Table 4a, and $\sigma \leq 1$ in Table 4.b). The MEC is especially high when the initial position is defined by a uniform tax on all incomes, and it is very sensitive to the choice of the intertemporal elasticity of substitution.

The MEC of **the capital income tax** in Sector 2 has a fairly high value but it is stable with respect to the parameters that are considered here. As for the wage tax, the MEC is particularily high when other taxes are in place.

Other Policies

A tax on all factors in the same sector is equivalent to a sales tax. The impact of changing the rates of the sales taxes in the two sectors are presented in Columns 5 and 6 of the tables. One can verify that there are the averages of the impacts of the factor taxes. Finally the case of the uniform tax on all factors (which is equivalent to a uniform sales tax), is presented in the last column. Note that it is the same as in the one sector model. This was pointed already by King and Rebelo (1990).

The case where government expenditures enter the production functions could be analyzed in the previous section. Numerical results are omitted here because they are similar to those of the one sector model. The introduction of these expenditures may reduce dramatically the MEC of the wage tax when it is high.

4.2.2 The Responses of Consumption and Capital Allocation

The values of initial responses of physical consumption and human capital investment bear consideration because they help explaining why some values of the MEC are so high. Some of the cases should be excluded because they generate responses which are not realistic. Consider first the wage tax in Sector 2. Its MEC is very high when σ is not strictly greater than 1, (Tables 4.a and 4.b). In some cases, a rate reduction even improves revenues. A low value of σ corresponds to a high intertemporal elasticity of substitution. The tables show that the initial response of consumption is indeed very high. A reduction of the tax rate from .2 to 0 would bring a massive reallocation of resources. Consumption falls by about 20 to 30 percent. This order of magnitude does not seem to be plausible.

The large impact on the reallocation of capital between the two sectors is not associated to a large efficiency cost. This is best seen in Tables 3.a and 3.b, where the share of the income of physical capital varies in Sector 2. When the capital intensities of the two sector becomes similar, small tax changes induce huge a huge reallocation on the margin[12], but the MEC varies continuously in terms of the capital intensities.

4.2.3 Impact on the Growth Rate in the Long-run

Tax policies affect the long-run growth rate because they have an effect on the rate of accumulation of human capital in the long-run. It is well known that in the standard model of growth, long-run effects do not provide good criteria for policy evaluation. The same remark applies here. Positive effects on the long-run level of output or consumption may be generated by tax changes which are not welfare improving.

In general, there is no simple relation between the impacts of tax reform on welfare, tax revenues and the long-term growth rate. It is therefore possible to find reforms which increase both welfare and revenues and decrease the long-term growth rate. This can be verified in the following case. Suppose that in the initial state, there is a uniform income tax on all factors and that the share of physical capital in Sector 2 is very small, albeit not zero. This is the case which is presented in Table 3.b for $\alpha_2 \approx 0$. The tax rate on capital in Sector 2 has a vanishingly small impact on the long-term growth rate, but its MEC does not tend to zero as its base becomes very small. Since its MEC is greater than that of the wage tax in the same sector, (1.33 *versus* 1.1592), there is a revenue neutral shift of taxation from capital to labor in Sector 2 which improves welfare. Because the wage tax reduces the long-term growth rate (last line of the table), the tax shift has a negative impact on long-term growth.

[12]Note that the case of identical capital intensities which was the only one considered by King and Rebelo is in general not compatible with an interior solution for the equilibrium.

5 Conclusion

This paper began with an investigation whether a property of endogenous growth should induce us to revise our view on the efficiency cost of taxation. The general conclusion is that the standard principles of public finance still apply: one should consider carefully the impact of taxes on specific trade-offs, and the assumptions made on the structure of these trade-offs. The property of endogenous growth is *per se* not important for the order of magnitude of the efficiency cost.

It has been found that in models of endogenous growth, most taxes have a efficiency cost of the same order of magnitude as in standard models: these include taxes on the trade-offs between leisure and production (either for physical goods or for human capital), taxes on the income of physical capital, and taxes on consumption.

The only tax which may have a high MEC was found to be the tax on human capital which is used for the production of human capital. This high value of the MEC is very sensitive to the assumptions which define the stucture of the model (*e.g.*, on the role of public expenditures). It depends on the (untested) assumption of a high intertemporal elasticity of substitution, but it does not depend on the property of endogenous growth.

For future research, a more precise evaluation of the MEC of taxation would require first, an explicitly estimated model which focuses on the values of the elasticities of substitution of the available trade-offs, second, a careful examination of how the tax system affects specific trade-offs.

The taxation of human capital input for the production of human capital can be viewed as a particularily strong form of the taxation of intermediate goods[13], which is known to be inefficient in many situations (Diamond and Mirrlees (1970)). However, it is doubtful whether this form of taxation is

[13]Note that in the first model with the trade-off between leisure and human capital formation, the output tax in not on an intermeditate good.

empirically important: One does not observe the taxation of time that is used for studying, or the of the output of research in universities. On the contrary, many activities which are involved in the formation of human capital are subsidized.

Such subsidies are in general motivated by a belief that externalities occur in human capital formation, and that they cannot be internalized by markets. This issue was not addressed here because we maintained the framework of a second-best without externalities. The presence of externalities raise two issues.

First, is the welfare cost of missing markets greater in models of endogenous growth than in standard models? The answer seems to be similar to the one given here for the MEC of taxation. This can be seen easily by considering the first model in which agents optimize their leisure decision according to the market wage rate. Should this wage rate be different from the social marginal value of labor, the social costs due to the misallocation of time would be, for a given elasticity of the labor supply, of the same order of magnitude in the static model and in the model of endogenous growth.

Second, do externalities change the dynamic pattern of efficient taxes on, say, labor? As we have seen, under some condition, a wage tax *at a constant rate* is neutral on human capital accumulation, (Lucas (1990)). However, a cycling pattern of tax rates may have an effect on the average level of human capital accumulation in the long-run. This problem, which may has implications for the design of optimal taxation or the impact of exogenous cycles, is analyzed in forthcoming work, (Chamley, (1992)).

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Table 1.a
The Marginal Efficiency Cost of Taxation
$(r(1-\theta) = .04, \quad \delta = 0)$

Capital Share	0.25	0.33	0.75	0.8	0.85	0.90	1.0
$\sigma = 3.0$							
MEC_C	0.2105	0.2109	0.2134	0.2121	0.2102	0.2076	0.2000
MEC_I	0.0526	0.0696	0.1600	0.1697	0.1787	0.1869	0.2000
$\Delta C_0/C$	-0.0556	-0.0636	-0.0966	-0.0998	-0.1029	-0.1058	-0.1111
$\sigma = 2.0$							
MEC_C	0.2334	0.2403	0.2967	0.3047	0.3126	0.3202	0.3333
MEC_I	0.0583	0.0793	0.2225	0.2437	0.2657	0.2881	0.3333
$\Delta C_0/C$	-0.0724	-0.0839	-0.1373	-0.1434	-0.1493	-0.1552	-0.1667
$\sigma = 1.0$							
MEC_C	0.2667	0.2850	0.4951	0.5481	0.6153	0.7034	1.0000
MEC_I	0.0667	0.0940	0.3713	0.4385	0.5230	0.6331	1.0000
$\Delta C_0/C$	-0.1111	-0.1312	-0.2421	-0.2579	-0.2747	-0.2927	-0.3333
$\sigma = 0.5$							
MEC_C	0.2927	0.3213	0.7688	0.9427	1.2294	1.7964	***
MEC_I	0.0732	0.1060	0.5766	0.7542	1.0450	1.6167	***
$\Delta C_0/C$	-0.1667	-0.1994	-0.4079	-0.4436	-0.4844	-0.5323	-0.6667

*** A reduction of θ increases both welfare and revenues.

Table 1.b
The Marginal Efficiency Cost of Taxation
$(r(1-\theta) = .04, \quad \delta = 0.1)$

Capital Share	0.25	0.33	0.75	0.8	0.85	0.90	1.0
$\sigma = 3.0$							
MEC_C	0.0837	0.0906	0.1593	0.1727	0.1863	0.1985	0.2000
MEC_I	0.0094	0.0143	0.0830	0.1017	0.1242	0.1504	0.2000
$\Delta C_0/C$	-0.0245	-0.0301	-0.0709	-0.0781	-0.0860	-0.0945	-0.1111
$\sigma = 2.0$							
MEC_C	0.0862	0.0941	0.1856	0.2085	0.2364	0.2694	0.3333
MEC_I	0.0098	0.0149	0.0973	0.1236	0.1584	0.2048	0.3333
$\Delta C_0/C$	-0.0307	-0.0379	-0.0948	-0.1060	-0.1188	-0.1334	-0.1667
$\sigma = 1.0$							
MEC_C	0.0894	0.0988	0.2270	0.2690	0.3301	0.4267	1.0000
MEC_I	0.0102	0.0157	0.1203	0.1612	0.2235	0.3270	1.0000
$\Delta C_0/C$	-0.0448	-0.0557	-0.1510	-0.1726	-0.1991	-0.2325	-0.3333
$\sigma = 0.5$							
MEC_C	0.0918	0.1021	0.2615	0.3232	0.4243	0.6222	***
MEC_I	0.0105	0.0163	0.1399	0.1955	0.2902	0.4816	***
$\Delta C_0/C$	-0.0647	-0.0808	-0.2329	-0.2708	-0.3199	-0.3870	-0.6667

Table 1.c
The Marginal Efficiency Cost of Taxation
$(r(1-\theta) = .032, \quad \delta = 0.1)$

Capital Share	0.25	0.33	0.75	0.8	0.85	0.90	1.0
$\sigma = 3.0$							
MEC_C	0.0748	0.0822	0.1705	0.1935	0.2215	0.2534	0.2857
MEC_I	0.0075	0.0115	0.0835	0.1081	0.1414	0.1857	0.2857
$\Delta C_0/C$	0.0220	0.0273	0.0723	0.0818	0.0929	0.1056	0.1333
$\sigma = 2.0$							
MEC_C	0.0762	0.0842	0.1897	0.2220	0.2660	0.3275	0.5000
MEC_I	0.0076	0.0118	0.0935	0.1249	0.1709	0.2414	0.5000
$\Delta C_0/C$	0.0273	0.0341	0.0949	0.1088	0.1258	0.1465	0.2000
$\sigma = 1.0$							
MEC_C	0.0780	0.0868	0.2175	0.2654	0.3402	0.4731	2.0000
MEC_I	0.0078	0.0123	0.1083	0.1509	0.2211	0.3523	2.0000
$\Delta C_0/C$	0.0395	0.0496	0.1472	0.1721	0.2042	0.2475	0.4000
$\sigma = 0.5$							
MEC_C	0.0793	0.0887	0.2390	0.3006	0.4058	0.6279	***
MEC_I	0.0080	0.0126	0.1200	0.1724	0.2663	0.4729	***
$\Delta C_0/C$	0.0567	0.0715	0.2225	0.2639	0.3198	0.4006	0.8000

Table 2.a
The Marginal Efficiency Cost of Taxation
(One instrument in the original position)
(All initial impacts follow a reduction of the tax rate(s))

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$(\Delta J)/J$	0.0175	0.0054	0.0045	0.0664	0.0201	0.0746	0.0833
MEC	0.3220	0.2342	0.0254	0.9674	0.0945	0.9489	0.3330
$(\Delta C_0)/C$	-0.0070	-0.0273	-0.0226	-0.0820	-0.0324	-0.1013	-0.1636
$(\Delta G_0)/G$	-0.7561	-0.0343	-0.0398	1.4318	-1.0019	1.4166	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050

In the initial position, one tax instrument is different from zero. $\theta_1 = \theta_{r1} = \theta_{w1}$. $\theta = \theta_{r1} = \theta_{w1} = \theta_{r2} = \theta_{w2}$.
Sum of productivity and population growth rates $\mu = 2\%$; net long-term net rate of return $r^* = \rho + \sigma\mu = 4\%$; $\sigma = 2$.

Table 2.b
The Marginal Efficiency Cost of Taxation
(Uniform income tax in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$(\Delta J)/J$	0.0039	0.0104	0.0104	0.0585	0.0144	0.0689	0.0833
MEC	0.0846	1.0941	0.0747	1.0751	0.0772	1.0779	0.3330
$(\Delta C_0)/C$	-0.0242	-0.0216	-0.0216	-0.0962	-0.0458	-0.1178	-0.1636
$(\Delta G_0)/G$	-1.0106	-0.0145	-0.0145	1.4724	-1.0251	1.4580	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050

Table 3.a
Variation of the Capital Income Share in Sector 2
(One instrument in the original position)
(All initial impacts follow a reduction of the tax rate(s))

	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\alpha_2 = 0$							
MEC	0.2763	0.2500	0.0000	1.0904	0.0707	1.0904	0.3251
$(\Delta C_0)/C$	-0.0418	-0.0000	-0.0000	-0.0696	-0.0372	-0.0696	-0.1414
$(\Delta G_0)/G$	-0.1910	0.0000	0.0000	0.5774	-0.2547	0.5774	0.3788
$\Delta\mu$	0.0000	0.0000	0.0000	0.0050	0.0000	0.0050	0.0050
$\alpha_2 = .15$							
MEC	0.3220	0.2342	0.0254	0.9674	0.0945	0.9489	0.3330
$(\Delta C_0)/C$	-0.0070	-0.0273	-0.0226	-0.0820	-0.0324	-0.1013	-0.1636
$(\Delta G_0)/G$	-0.7561	-0.0343	-0.0398	1.4318	-1.0019	1.4166	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\alpha_2 = .26$							
MEC	0.3354	-0.8520	-0.5429	0.8487	0.1090	0.8214	0.3333
$(\Delta C_0)/C$	0.0027	-0.3115	-0.2628	-0.0775	-0.0308	-0.1101	-0.1667
$(\Delta G_0)/G$	-1.9128	0.3479	0.2934	3.0976	-12.2333	15.8208	0.0001
$\Delta\mu$	0.0003	0.0009	0.0009	0.0028	0.0012	0.0038	0.0050
$\alpha_2 = .75$							
MEC	0.3077	0.2318	0.0877	0.4158	0.1286	0.3636	0.3333
$(\Delta C_0)/C$	-0.0119	-0.0750	-0.0667	-0.0187	-0.0816	-0.0821	-0.1667
$(\Delta G_0)/G$	-0.2137	0.0897	0.0797	0.1355	-0.2176	0.2361	0.0000
$\Delta\mu$	0.0006	0.0019	0.0019	0.0006	0.0025	0.0025	0.0050
$\alpha_2 = .99$							
MEC	0.3006	0.2369	0.1053	0.3159	0.1378	0.2581	0.3333
$(\Delta C_0)/C$	-0.0159	-0.0842	-0.0760	-0.0007	-0.0961	-0.0742	-0.1667
$(\Delta G_0)/G$	-0.1541	0.1181	0.1066	0.0040	-0.1107	0.1079	0.0000
$\Delta\mu$	0.0007	0.0021	0.0021	0.0000	0.0028	0.0022	0.0050

Table 3.b
Variation of the Capital Income Share in Sector 2
(Uniform Income Tax in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\alpha_2 = 0$							
MEC	0.0807	1.3300	0.0000	1.1592	0.0197	1.1592	0.3251
$(\Delta C_0)/C$	-0.0505	-0.0000	-0.0000	-0.0909	-0.0505	-0.0909	-0.1414
$(\Delta G_0)/G$	-0.2576	0.0000	0.0000	0.6364	-0.2576	0.6364	0.3788
$\Delta\mu$	0.0000	0.0000	0.0000	0.0050	0.0000	0.0050	0.0050
$\alpha_2 = .15$							
MEC	0.0846	1.0941	0.0747	1.0751	0.0772	1.0779	0.3330
$(\Delta C_0)/C$	-0.0242	-0.0216	-0.0216	-0.0962	-0.0458	-0.1178	-0.1636
$(\Delta G_0)/G$	-1.0106	-0.0145	-0.0145	1.4724	-1.0251	1.4580	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\alpha_2 = 0.25$							
MEC	0.1111	1.0000	0.1111	1.0000	0.1111	1.0000	0.3333
$(\Delta C_0)/C$	-0.0104	-0.0313	-0.0313	-0.0937	-0.0417	-0.1250	-0.1667
$(\Delta G_0)/G$	-530.9	-27.9	-27.9	586.9	-558.88	559.01	0.2499
$\Delta\mu$	0.0003	0.0009	0.0009	0.0028	0.0012	0.0038	0.0050
$\alpha_2 = .75$							
MEC	0.2143	0.7500	0.2143	0.7500	0.2143	0.7500	0.3333
$(\Delta C_0)/C$	-0.0208	-0.0625	-0.0625	-0.0208	-0.0833	-0.0833	-0.1667
$(\Delta G_0)/G$	-0.3437	0.0938	0.0938	0.1562	-0.2500	0.2500	0.0000
$\Delta\mu$	0.0006	0.0019	0.0019	0.0006	0.0025	0.0025	0.0050
$\alpha_2 = .99$							
MEC	0.2399	0.6864	0.2399	0.6864	0.2399	0.6864	0.3333
$(\Delta C_0)/C$	-0.0237	-0.0711	-0.0711	-0.0007	-0.0948	-0.0718	-0.1667
$(\Delta G_0)/G$	-0.2410	0.1182	0.1182	0.0046	-0.1228	0.1228	0.0000
$\Delta\mu$	0.0007	0.0021	0.0021	0.0000	0.0028	0.0022	0.0050

Table 4.a
Variation of the Intertemporal Elasticity of Substitution in Consumption
(One instrument in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\sigma = .5$							
$(\Delta J)/J$	0.0181	0.0101	0.0084	0.1869	0.0270	0.2418	0.3321
MEC	0.3359	0.5497	0.0486	***	0.1312	***	305.188
$(\Delta C_0)/C$	-0.0776	-0.1109	-0.0918	-0.3919	-0.1742	-0.4323	-0.6189
$(\Delta G_0)/G$	-0.6666	0.1675	0.1466	2.0939	-0.7486	2.3146	1.6783
$\Delta\mu$	0.0008	0.0025	0.0025	0.0142	0.0033	0.0167	0.0200
$\sigma = 1.0$							
$(\Delta J)/J$	0.0177	0.0070	0.0058	0.1066	0.0224	0.1304	0.1663
MEC	0.3266	0.3245	0.0330	2.9254	0.1065	3.8476	0.9967
$(\Delta C_0)/C$	-0.0309	-0.0568	-0.0470	-0.1877	-0.0811	-0.2163	-0.3211
$(\Delta G_0)/G$	-0.7259	0.0347	0.0237	1.6556	-0.9159	1.7224	0.8567
$\Delta\mu$	0.0004	0.0013	0.0013	0.0071	0.0017	0.0083	0.0100
$\sigma = 2.0$							
$(\Delta J)/J$	0.0175	0.0054	0.0045	0.0664	0.0201	0.0746	0.0833
MEC	0.3220	0.2342	0.0254	0.9674	0.0945	0.9489	0.3330
$(\Delta C_0)/C$	-0.0070	-0.0273	-0.0226	-0.0820	-0.0324	-0.1013	-0.1636
$(\Delta G_0)/G$	-0.7561	-0.0343	-0.0398	1.4318	-1.0019	1.4166	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\sigma = 3.0$							
$(\Delta J)/J$	0.0175	0.0049	0.0040	0.0530	0.0193	0.0560	0.0555
MEC	0.3204	0.2067	0.0228	0.6676	0.0905	0.5983	0.1999
$(\Delta C_0)/C$	0.0011	-0.0171	-0.0142	-0.0462	-0.0158	-0.0619	-0.1097
$(\Delta G_0)/G$	-0.7663	-0.0577	-0.0613	1.3565	-1.0308	1.3132	0.2896
$\Delta\mu$	0.0001	0.0004	0.0004	0.0024	0.0006	0.0028	0.0033

Table 4.b
Variation of the Intertemporal Elasticity of Substitution in Consumption
(Uniform income tax in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\sigma = .5$							
$(\Delta J)/J$	0.0156	0.0416	0.0416	0.2332	0.0572	0.2748	0.3321
MEC	0.4499	***	0.3835	***	0.3996	***	305.188
$(\Delta C_0)/C$	-0.0930	-0.0818	-0.0818	-0.3621	-0.1749	-0.4440	-0.6189
$(\Delta G_0)/G$	-0.8839	0.1462	0.1462	2.2698	-0.7377	2.4160	1.6783
$\Delta\mu$	0.0008	0.0025	0.0025	0.0142	0.0033	0.0167	0.0200
$\sigma = 1.0$							
$(\Delta J)/J$	0.0078	0.0209	0.0209	0.1168	0.0287	0.1377	0.1663
MEC	0.1845	***	0.1612	***	0.1670	***	0.9967
$(\Delta C_0)/C$	-0.0477	-0.0424	-0.0424	-0.1886	-0.0901	-0.2310	-0.3211
$(\Delta G_0)/G$	-0.9675	0.0402	0.0402	1.7438	-0.9273	1.7840	0.8567
$\Delta\mu$	0.0004	0.0013	0.0013	0.0071	0.0017	0.0083	0.0100
$\sigma = 2.0$							
$(\Delta J)/J$	0.0039	0.0104	0.0104	0.0585	0.0144	0.0689	0.0833
MEC	0.0846	1.0941	0.0747	1.0751	0.0772	1.0779	0.3330
$(\Delta C_0)/C$	-0.0242	-0.0216	-0.0216	-0.0962	-0.0458	-0.1178	-0.1636
$(\Delta G_0)/G$	-1.0106	-0.0145	-0.0145	1.4724	-1.0251	1.4580	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\sigma = 3.0$							
$(\Delta J)/J$	0.0026	0.0070	0.0070	0.0390	0.0096	0.0459	0.0555
MEC	0.0549	0.5347	0.0486	0.5278	0.0502	0.5288	0.1999
$(\Delta C_0)/C$	-0.0162	-0.0145	-0.0145	-0.0646	-0.0307	-0.0791	-0.1097
$(\Delta G_0)/G$	-1.0252	-0.0330	-0.0330	1.3807	-1.0582	1.3478	0.2896
$\Delta\mu$	0.0001	0.0004	0.0004	0.0024	0.0006	0.0028	0.0033

Table 5.a
Variation of the Net of Rate of Return
(One instrument in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\rho^* = .025$							
MEC	0.3693	0.2635	0.0847	1.5290	0.1930	1.6156	1.6659
$(\Delta C_0)/C$	-0.0095	-0.0622	-0.0516	-0.1471	-0.0558	-0.1921	-0.2742
$(\Delta G_0)/G$	-0.3170	-0.1456	-0.1487	0.9970	-0.5322	0.8763	0.2744
$\Delta\mu$	0.0001	0.0004	0.0004	0.0022	0.0005	0.0026	0.0031
$\rho^* = .03$							
MEC	0.3419	0.2337	0.0457	1.0120	0.1280	0.9892	0.5996
$(\Delta C_0)/C$	-0.0070	-0.0404	-0.0334	-0.1080	-0.0408	-0.1389	-0.2109
$(\Delta G_0)/G$	-0.4643	-0.1061	-0.1101	1.1413	-0.6895	1.0564	0.3272
$\Delta\mu$	0.0002	0.0005	0.0005	0.0027	0.0006	0.0031	0.0038
$\rho^* = .04$							
MEC	0.3220	0.2342	0.0254	0.9674	0.0945	0.9489	0.3330
$(\Delta C_0)/C$	-0.0070	-0.0273	-0.0226	-0.0820	-0.0324	-0.1013	-0.1636
$(\Delta G_0)/G$	-0.7561	-0.0343	-0.0398	1.4318	-1.0019	1.4166	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\rho^* = .05$							
MEC	0.3124	0.2471	0.0183	1.1038	0.0829	1.1232	0.2628
$(\Delta C_0)/C$	-0.0075	-0.0224	-0.0185	-0.0721	-0.0294	-0.0864	-0.1441
$(\Delta G_0)/G$	-1.0461	0.0333	0.0266	1.7236	-1.3128	1.7767	0.5387
$\Delta\mu$	0.0003	0.0008	0.0008	0.0044	0.0010	0.0052	0.0063

Table 5.b
Variation of the Net of Rate of Return
(Uniform income tax in the original position)

Variable	θ_{r1}	θ_{r2}	θ_{w1}	θ_{w2}	θ_1	θ_2	θ
$\rho^* = .025$							
MEC	0.4474	3.8995	0.4343	3.8445	0.4375	3.8527	1.6659
$(\Delta C_0)/C$	-0.0264	-0.0386	-0.0386	-0.1705	-0.0651	-0.2092	-0.2742
$(\Delta G_0)/G$	-0.4859	-0.1105	-0.1105	0.9813	-0.5964	0.8709	0.2744
$\Delta\mu$	0.0001	0.0004	0.0004	0.0022	0.0005	0.0026	0.0031
$\rho^* = .03$							
MEC	0.1679	1.3809	0.1577	1.3623	0.1603	1.3651	0.5996
$(\Delta C_0)/C$	-0.0249	-0.0292	-0.0292	-0.1276	-0.0541	-0.1568	-0.2109
$(\Delta G_0)/G$	-0.6622	-0.0774	-0.0774	1.1442	-0.7396	1.0668	0.3272
$\Delta\mu$	0.0002	0.0005	0.0005	0.0027	0.0006	0.0031	0.0038
$\rho^* = .04$							
MEC	0.0846	1.0941	0.0747	1.0751	0.0772	1.0779	0.3330
$(\Delta C_0)/C$	-0.0242	-0.0216	-0.0216	-0.0962	-0.0458	-0.1178	-0.1636
$(\Delta G_0)/G$	-1.0106	-0.0145	-0.0145	1.4724	-1.0251	1.4580	0.4329
$\Delta\mu$	0.0002	0.0006	0.0006	0.0035	0.0008	0.0042	0.0050
$\rho^* = .05$							
MEC	0.0636	1.2118	0.0535	1.1878	0.0560	1.1914	0.2628
$(\Delta C_0)/C$	-0.0240	-0.0183	-0.0183	-0.0836	-0.0422	-0.1019	-0.1441
$(\Delta G_0)/G$	-1.3566	0.0466	0.0466	1.8021	-1.3100	1.8487	0.5387
$\Delta\mu$	0.0003	0.0008	0.0008	0.0044	0.0010	0.0052	0.0063