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NONLINEAR ERROR CORRECTION, ASYMMETRIC ADJUSTMENT AND COINTEGRATION

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Abstract

This paper has three main components. First, it outlines a model of non-linear error correction (NEC) in which the linear error correction term $\alpha'X_t$ (the vector time series X_t is cointegrated, α is the cointegrating vector) is replaced by the nonlinear term $g(\alpha'X_t)$, where $g(.)$ is a nonlinear function. Second, several types of asymmetries are discussed. The NEC model is shown to have an underlying structural model in the form of an adjustment cost model with asymmetric adjustment costs. The implications for the NEC model of trending targets are explained. Third, it is shown that nonlinear error correction is present in a trivariate series of UK employment, wage, and capital stock.

Key words: Nonlinear Error Correction; Asymmetric Adjustment Costs; Dynamic Labor Demand; Cointegration.

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1. INTRODUCTION

Nonlinear error correction basically refers to nonlinear adjustment to longrun equilibrium economic relationships. In this paper we show that the concept of nonlinear error correction (NEC) models (Escribano (1987), Granger and Lee (1990)) has its structural counterpart in the form of optimizing a decision process under uncertainty over an infinite horizon where the decision variable is quasi-fixed and bears asymmetric costs of adjustment (Pfann and Palm (1988), Pfann and Verspagen (1989)). The concept of asymmetry implies that the costs of adjusting to a higher target level are not necessarily marginally equivalent to the costs of adjusting to a lower target level.

This paper has three main components. First, it outlines a model of nonlinear error correction (NEC), in which the linear error correction term $\alpha'X_t$ (the vector time series X_t is cointegrated, α is the cointegrating vector) is replaced by the nonlinear term $g(\alpha'X_t)$, where $g(.)$ is a nonlinear function. Second, several types of asymmetries are discussed. The implications for the NEC model of trending targets are explained. Third, it is shown that nonlinear error correction is present in a trivariate series of UK employment, wage, and capital stock.

The paper is organized as follows. In section 2 the nonlinear error correction representation is derived from a general nonlinear autoregressive distributed lag model, and issues of integration and cointegration (Engle and Granger (1987)) are passed in review. Section 3 presents the linear partial adjustment model. In section 4 the characteristics of the asymmetric adjustment model are linked with the concept of nonlinear error correction. In section 5 the implications of variables having trends in mean with respect to NEC models are discussed. Section 6 presents several specifications of asymmetries in the NEC model, that can be found in the literature. It is shown that these nonlinear error corrections are special cases of the general formulation presented in the paper. An empirical application is given in section 7 where the nonlinear relationship is investigated between UK time series data on employment real wage costs and the stock of capital goods. Finally, in section 8 conclusions are drawn.

2. NONLINEAR ERROR CORRECTION AND COINTEGRATION

Let X_t be an (Nx1)-vector of economic variables, and suppose that we have T observations of each individual series of X_t. Let $E(X_t) = \mu_t$ be an (Nx1)-vector whose element can be constant terms, deterministic trends etc., and define $\bar{X}_t = X_t - \mu_t$. If we decompose $\bar{X}_t = (\bar{Q}_t, \bar{P}_t)'$, where $\bar{Q}_t$ is one dimensional and $\bar{P}_t$ is an ((N-1)x1)-vector, we can factorize the joint density of $\bar{X}_t$ into the conditional and the marginal, see for example Engle et al. (1983),[1]

$$D(\bar{X}_t|\bar{X}_{t-1},\bar{X}_{t-2},\ldots,\bar{X}_0,\theta) = D(\bar{Q}_t|\bar{P}_t,\bar{X}_{t-1},\bar{X}_{t-2},\ldots,\bar{X}_0,\theta_1)D(\bar{P}_t|\bar{X}_{t-1},\bar{X}_{t-2},\ldots,\bar{X}_0,\theta_2) \quad (2.1)$$

If the parameters of interest ψ are a function of the parameters θ_1, $\psi = f(\theta_1)$, and if $\bar{P}_t$ is <u>weakly exogenous</u> for the parameter of interest ψ, we can make inference on ψ based on the conditional density without any loss of relevant information. In particular we will be interested in the conditional expectation $E(\bar{Q}_t|\bar{P}_t,\bar{X}_{t-1},\bar{X}_{t-2},\ldots,\bar{X}_0,\theta_1)$.
Let $\epsilon_t = Q_t - E(\bar{Q}_t|\bar{P}_t,\bar{X}_{t-1},\ldots,\bar{X}_0,\theta_1)$ so that ϵ_t is a martingale difference sequence relative to the γ-algebra generated by $(\bar{P}_t,\bar{X}_{t-1},\ldots,\bar{X}_0)$. For simplicity we will assume that ϵ_t has a constant variance equal to σ^2_ϵ. Suppose we can approximate the conditional expectation by a finite <u>autoregressive distributed lag model with a nonlinear term</u>, see Escribano (1987),

$$E(\bar{Q}_t|\bar{P}_t,\bar{X}_{t-1},\bar{X}_{t-2},\ldots,\bar{X}_0,\theta_1) = -\phi^1(B)\bar{Q}_{t-1} - \theta(B)P_t - g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}). \quad (2.2)$$

Then we can write the equation for $\bar{Q}_t$ as

$$\phi(B)\bar{Q}_t + \theta(B)\bar{P}_t = -g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) + \epsilon_t,$$

where $\phi(B)$ is a finite lag polynomial in the lag operator B, with $\phi(0) = 1$, $\theta(B)$ is a (1x(N-1))-vector of finite polynomials in the lag operator B. The lag operator B is such that $B^k\bar{X}_t = \bar{X}_{t-k}$ and $\theta(0)$ is a (1x(N-1))-vector whose elements are not all equal to 0 so that in euqation (2.1) there are some contemporaneous weakly exogenous variables. The nonlinear function g is such that $|g(Z)| \leq aZ$ where $a < 1$.

If both $\phi(B)$ and $\theta(B)$ have a unit root then $\bar{Q}_t$ and $\bar{P}_t$ are weakly integrated of order one, I(1). In this case we can obtain different, but observationally equivalent, representations from (2.2). Taking Taylor series expansions of $\phi(B)$ and $\theta(B)$ around the point B = 1 we get

$$\phi(B) = \phi(1) + \phi^*(B)(1-B) \tag{2.3}$$

and

$$\theta(B) = \theta(1) + \theta^*(B)(1-B) \tag{2.4}$$

where $\phi^*(B)$ and $\theta^*(B)$ have all roots outside the unit circle. Substituting (2.3) and (2.4) in equation (2.2) and rearranging terms we obtain

$$\phi(1)\bar{Q}_t + \theta(1)\bar{P}_t = -\phi^*(B)(1-B)\bar{Q}_t - \theta^*(B)(1-B)\bar{P}_t - g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) + \epsilon_t. \tag{2.5}$$

Now decompose $\phi(1) = \Gamma_1\alpha_1$ and $\theta(1) = \Gamma_1\alpha_2$, and divide (2.5) by the scalar $\Gamma_1\alpha_1$, we normalize (2.5) as

$$\bar{Q}_t = \alpha\bar{P}_t - \phi^{-1}(1)\phi^*(B)(1-B)\bar{Q}_t - \phi^{-1}(1)\theta^*(B)(1-B)\bar{P}_t - \phi^{-1}(1)g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) + \phi^{-1}(1)\epsilon_t, \tag{2.6}$$

which is a nonlinear version of Bewley's representation, (Bewley (1979)), with $\alpha = -\phi^{-1}(1)\theta(1)$. Notice that Bewley's linear representation is obtained from (2.6) by setting $g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) = 0$. See Hylleberg and Mizon (1989) for an alternative procedure to derive this representation.

If we add and subtract $\phi(1)B$ and $\theta(1)B$ to (2.3) and (2.4) respectively, we can rewrite $\phi(B)$ and $\theta(B)$ as follows

$$\phi(B) = \phi(1)B + [\phi^*(B) + \phi(1)](1-B) = \phi(1)B + \phi^{**}(B)(1-B) \tag{2.7}$$

and

$$\theta(B) = \theta(1)B = [\theta^*(B) + \theta(1)](1-B) = \theta(1)B + \theta^{**}(B)(1-B), \tag{2.8}$$

where $\phi^{**}(B)$ and $\theta^{**}(B)$ have all roots outside the unit circle.

Substituting (2.7) and (2.8) into (2.2) we obtain a <u>nonlinear error correction representation</u>

$$\phi^{**}(B)(1-B)\bar{Q}_t + \theta^{**}(B)(1-B)\bar{P}_t = -\phi(1)\bar{Q}_{t-1} - \theta(1)\bar{P}_{t-1} - g(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) + \epsilon_t. \tag{2.9}$$

Decomposing the long-term components as $\phi(1) = \Gamma_1\alpha_1$, $\theta(1) = \Gamma_1\alpha_2$, and dividing by the scalar α_1 we can normalize (2.9) getting a more explicit representation

$$\phi_\alpha(B)(1-B)\bar{Q}_t + \theta_\alpha(B)(1-B)\bar{P}_t = -\Gamma_1(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) - g_\alpha(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1}) + \epsilon_{\alpha t} \tag{2.10}$$

where $\phi_\alpha(B) + (1/\alpha_1)\phi^{**}(B)$, $\theta_\alpha(B) = (1/\alpha_1)\theta^{**}(B)$, $g_\alpha(.) = (1/\alpha_1)g(.)$, and $\epsilon_{\alpha t} = (1/\alpha_1)\epsilon_t$. If $g_\alpha(.) = 0$, we obtain the <u>linear error correction model</u>.

In general the function $g_\alpha(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1})$ incorporates all departures from the linear and symmetric error correction term, $\Gamma_1(\bar{Q}_{t-1} - \alpha\bar{P}_{t-1})$. For models (2.6) and (2.10) to be well specified, $\bar{Q}_t - \alpha\bar{P}_t$ must be I(0) since $\bar{Q}_t$ and $\bar{P}_t$ are both I(1), with $(1,-\alpha)'$ being the <u>cointegrating vector</u>. Also it must hold that a nonlinear function $g_\alpha(I(0))$ is still I(0). This last condition is generally satisfied under α-mixing conditions, see Escribano (1987b).

The error correction and Bewley's representations are observationally equivalent although in practice one can be preferred over the other. Wickens and Breusch (1988) mentioned that Bewley's representation has the advantage of giving the correct standard errors from the longrun coefficients at the costs of requiring instrumental variables estimation (IV), since the error term ϵ_t is correlated with the regressor $(1-B)\bar{Q}_t$. On the other hand, the error correction representation can be estimated by OLS, and the standard errors of the longrun coefficients may be obtained after some calculations (Dolado et al. (1990)) or by nonlinear least squares (Stock (1987)).

3. <u>THE LINEAR PARTIAL ADJUSTMENT MODEL</u>

In the linear partial adjustment model a representative economic agent is assumed to construct a contingency plan at time t for a purely nondeterministic quasi-fixed decision variable Q [1] in order to minimize the expected real present value of a quadratic loss-function over an infinite time horizon. The optimization problem is as follows

$$\underset{Q}{\text{MINIMIZE}}\ E[\Sigma_{i=0}^{\infty}\beta^{i}((Q_{t+i} - Q_{t+i}^{*})^{2} + \gamma((1-B)Q_{t+i})^{2})|\Omega_{t}], \qquad (3.1)$$

where E is the mathematical expectations operator, Ω_t is the conditioning set of available information at time t, β is a real discount value lying between zero and one, γ is a constant positive parameter measuring the adjustment costs of changing the level of Q over time. Q^* is the target level of Q, and is assumed to be linearly related to the firms purely nondeterministic forcing variables P_t and a stochastic zero mean shock u_t

$$Q_t^* = \alpha P_t - u_t \qquad (3.2)$$

where α' is a ((N-1)x1)-vector of constant parameters. If $Q_t = Q_t^*$, equation (3.2) can be interpreted as the longrun equilibrium relation between Q and P, also known as the cointegration relationship, with $(1,-\alpha)$ being the cointegration vector.
The first order condition for (3.1) at time t is

$$Q_t + \gamma(1-B)Q_t - \beta\gamma E[(1-B)Q_{t+1}|\Omega_t] = Q_t^* \qquad (3.3)$$

or

$$E[(1-B)Q_{t+1}|\Omega_t] = \beta^{-1}(1-B)Q_t + (\beta\gamma)^{-1}(Q_t - Q_t^*). \qquad (3.4)$$

The left-hand side of (3.4) cannot be observed as such, but the forward looking closed form solution for the inhomogeneous second order linear difference equation is well-known in literature, and can be written as the <u>partial adjustment representation</u> (cf. Nickell (1985))

$$(1-\lambda B)Q_t = (1-\lambda)(1-\beta\lambda)\Sigma_{i=0}^{\infty}(\beta\lambda)^{i}E[Q_{t+i}^{*}|\Omega_t] \qquad (3.5)$$

1) In section 5 we generalize this approach to the case of variables having trends in the means.

where λ is the root of the characteristic equation

$$\lambda^2 - (1+\beta^{-1}+(\beta\gamma)^{-1})\lambda + \beta^{-1} = 0, \qquad (3.6)$$

that lies within the unit circle, being

$$\lambda = \tfrac{1}{2}(1+\beta^{-1}+(\beta\gamma)^{-1}) - \tfrac{1}{2}((1+\beta^{-1}+(\beta\gamma)^{-1})^2-4\beta^{-1})^{\frac{1}{2}} \qquad (3.7)$$

since both roots are real and lie on either side of the unit circle (cf. Palm and Pfann (1991)). Without loss of generality we may assume that the generating process of P_t is an autoregressive process, where T(B) is the corresponding autoregressive lag polynomial. Then we substitute (3.2) into (3.5) and obtain (cf. Hansen and Sargent (1980))

$$(1-\lambda B)Q_t = \alpha(1-\lambda)(1-\beta\lambda)(1-\beta\lambda B^{-1}))^{-1}(T(\beta\lambda)-\beta\lambda B^{-1}T(B))P_t + (1-\lambda)(1-\beta\lambda)u_t \qquad (3.8)$$

which may be simplified into the unique closed form solution of (3.1)

$$Q_t = \lambda Q_{t-1} + \alpha(1-\lambda)T^*(B)P_t + (1-\lambda)(1-\beta\lambda)u_t \qquad (3.9)$$

where $T^*(B) = (1-\beta\lambda)(T(\beta\lambda)(1-\beta\lambda B^{-1}))^{-1}(T(\beta\lambda) - \beta\lambda B^{-1}T(B))$.
$T^*(B)P_t$ is known as the forward looking target of the linear partial adjustment model. The zero mean process of stochastic shocks u_t is predominantly found to follow an autoregressive process in the empirical literature on flexible adjustment mechanisms. Then the resulting autocorrelation in the residual error of (3.9) is eliminated applying the Koyck transformation procedure, transforming (3.9) into

$$(1-\lambda B)\phi(B)Q_t = \alpha(1-\lambda)\theta(B)P_t + \eta_t \qquad (3.10)$$

where η_t is a white noise innovation. If, in accordance with section 2, Q_t as well as P_t have unit roots the partial adjustment model (3.10) can be written as a linear error correction model with $g_\alpha(.) = 0$ (see also Nickell (1985))

$$\phi_\alpha(B)(1-B)Q_t = -(1-\lambda)(Q_{t-1} - \alpha P_{t-1}) + \alpha(1-\lambda)\theta_\alpha(B)(1-B)P_t + \eta_t \qquad (3.11)$$

where $\phi_\alpha(B)$ and $\theta_\alpha(B)$ have all roots outside the unit circle. Define $\phi_\lambda(B) = (1-\lambda B)\phi(B)$. Now we can write (3.10) as

$$\phi_\lambda(B)Q_t = \alpha(1-\lambda)\theta(B)P_t + \eta_t. \tag{3.12}$$

Decomposing the polynomials $\phi_\lambda(B)$ and $\theta(B)$ according to equations (2.3) and (2.4) we get

$$\phi_\lambda(1)Q_t - \alpha(1-\lambda)\theta(1)P_t = -\phi_\lambda^*(B)(1-B)Q_t + \alpha(1-\lambda)\theta^*(B)(1-B)P_t + \eta_t. \tag{3.13}$$

Deviding equation (3.13) by $\phi_\lambda(1)$, we obtain Bewley's representation

$$Q_t = \alpha^* P_t - \phi_\lambda(1)^{-1}\phi_\lambda^*(B)(1-B)Q_t + \phi_\lambda^{-1}\alpha(1-\lambda)\theta^*(B)(1-B)P_t + \phi_\lambda^{-1}(1)\eta_t \tag{3.14}$$

where $\alpha^* = \phi_\lambda^{-1}(1)\alpha(1-\lambda)$.

4. ASYMMETRIC ADJUSTMENT MODEL AND NONLINEAR ERROR CORRECTION

In this section we implement the asymmetric adjustment costs flexible functional form proposed by Pfann and Verspagen (1989) into the structural partial adjustment model. The economic agent chooses a contingency plan at time t for a quasi-fixed decision variable Q in order to minimize the expected real present value of a nonlinear loss-function over an infinite time horizon. The optimization problem with asymmetric adjustment costs (AAC) is as follows

$$\underset{Q}{\text{Min}}\ E(\Sigma_{i=0}^{\infty}\beta^i((Q_i-Q_{t-i}^*) + AAC((1-B)Q_{t+i})|\Omega_t) \tag{4.1}$$

$$\text{with } AAC((1-B)Q_t) = \gamma((1-B)Q_t)^2 + 2(\exp(\delta(1-B)Q_t) - (1+\delta((1-B)Q_t)). \tag{4.2}$$

The constant parameter δ measures the difference in costs between an increase in Q and a decrease in Q. If δ is positive, costs of increasing Q exceed costs of reducing Q, and vice versa. Under the restriction of δ being equal to zero (4.1) is just the linear-quadratic optimization problem discussed in the previous section.

Hence, the symmetric linear partial adjustment model is nested in the asymmetric model (4.1). We note that the asymmetric specification is strictly convex under the standard assumption of γ being positive. The exponential AAC also encompasses polynomial approximations of many nonlinear functions. This is shown in the sequel of the paper.
The first order necessary conditions for (4.1) are as follows

$$\beta E(\gamma(1-B)Q_{t+1} + \delta(\exp(\delta(1-B)Q_{t+1})-1)|\Omega_t) = \gamma(1-B)Q_t + (Q_t-Q_t^*) + \delta(\exp(\delta(1-B)Q_t)-1). \quad (4.3)$$

According to our knowledge a closed form solution for equation (4.3) is not (yet) known. One possible way to circumvent the absence of a reduced form model is to estimate the Euler equations with Hansen's GMM-estimation technique. This approach has been followed in Pfann and Palm (1988).
Hamilton's (1989) approach is to transform the data into discrete Markov processes, arguing nonlinearities in the data are generated by stochastic processes that are subject to discrete shifts in regime. We believe that valuable information being present in the data will be lost by Hamilton's transformation method. Novales (1990) proposed a solving technique for nonlinear models positting stochastic processes for the decision variable Q_t in order to solve the model for the forcing variables. This method is untractable with respect to our approach, since the parameters of asymmetry have to be chosen a priori in Novales' method. Yet, a suitable approximation of the closed form solution may exist, and using additional information more efficient estimates of the structural parameters may be obtained.
Granger and Lee (1990) considered error correction models where the positive residual error of the longrun relationship, $\max(Q_{t-1} - \alpha P_{t-1};0)$, and the negative residual error of the longrun relationship, $\min(Q_{t-1} - \alpha P_{t-1};0)$, have been introduced into the model as separate regressors.

The optimization model with asymmetries in adjustment costs (4.1) is the structural counterpart of the asymmetric error correction model. To measure the asymmetric error correction we introduce the following concepts.

Positive error correction movements are characterized by positive differences between two subsequent measurement points of the longrun equilibrium error

$$(Q_t-\alpha P_t)^+ = \begin{cases} (Q_t-\alpha P_t) & \text{iff } (1-B)(Q_t-\alpha P_t) > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4.4.1}$$

Negative error correction movements are characterized by negative differences between two subsequent measurement points of the longrun equilibrium error

$$(Q_t-\alpha P_t)^- = \begin{cases} (Q_t-\alpha P_t) & \text{iff } (1-B)(Q_t-\alpha P_t) < 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4.4.2}$$

The nonlinear function $g_\alpha(.)$ introduced in (2.10) is hence expressed as follows (see figure A.1) [2)]

$$\Gamma_1(Q_t-\alpha P_t) + g_\alpha(Q_t-\alpha P_t) = -(1-\lambda_1)(Q_t-\alpha P_t)^- - (1-\lambda_2)(Q_t-\alpha P_t)^+, \tag{4.5}$$

since $(1-B)(Q_t-\alpha P_t) = (Q_t-\alpha P_t)^+ + (Q_t-\alpha P_t)^-$, whereas λ_1 and λ_2 correspond with λ in equation (3.11) such that $\frac{1}{2}(\lambda_1+\lambda_2) = \lambda$.

The corresponding asymmetric error correction model can be obtained substituting (4.5) for $-(1-\lambda)(Q_{t-1}-\alpha P_{t-1})$ in (3.11). This gives

$$\begin{aligned}\phi_\alpha(B)(1-B)Q_t = &-(1-\lambda_1)(Q_{t-1}-\alpha P_{t-1})^- - (1-\lambda_2)(Q_{t-1}-\alpha P_{t-1})^+ \\ &+ \alpha(1-\lambda)\theta_\alpha(b)(1-B)P_t + \eta_t.\end{aligned} \tag{4.6}$$

Equation (4.6) can be analyzed using a two step estimation technique, as proposed by Engle and Granger (1987). First, one estimates the cointegrating vector α by OLS, $\hat{\alpha}$. Second, equation (4.6) can be estimated with $(Q_{t-1}-\hat{\alpha}P_{t-1})^-$ and $(Q_{t-1}-\hat{\alpha}P_{t-1})^+$ as separate regressors identifying λ_1 and λ_2.

Intuitively, one expects the adjustment speed parameters, λ_1 and λ_2, and the parameter of asymmetric adjustment costs, δ of equations (4.1) and (4.2) to be related. Unfortunately, no closed form solution can be obtained for the asymmetric first order conditions (4.3). To link the notion of asymmetric speeds of adjustment (λ_1,λ_2) with the notion of

2) More general nonlinear adjustments will be considered in section 6.

asymmetry in adjustment costs (δ), we proceed proposing a piecewise closed form solution of (4.3) depending on the direction of the adjustment. Linearizing (4.2) using a piecewise second order Taylor series expansion gives

$$AAC((1-B)Q_t) = \begin{cases} \gamma_1((1-B)Q_t)^2 & \text{iff } (1-B)Q_t > 0 \\ \gamma_2((1-B)Q_t)^2 & \text{iff } (1-B)Q_t < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (4.7)$$

where γ_1 and γ_2 are constant positive cost parameters of respectively rising and declining adjustment. Expression (4.7) implies that

$$\begin{aligned} \gamma_1 &> \gamma_2, && \text{iff } \delta > 0 \\ \gamma_1 &< \gamma_2, && \text{iff } \delta < 0, \text{ and} \\ \gamma_1 &= \gamma_2, && \text{iff } \delta = 0. \end{aligned} \qquad (4.8)$$

Thus, in the quest for a closed form solution of the nonlinear second order difference equation (4.3) the continuously differentiable asymmetric specification (4.2) has been approximated by a piecewise linear quadratic expansion. From the previous section the closed form solution for each piecewise linear-quadratic approximation is known. The two linearized necessary conditions are

$$E[(1-B)Q_{t+1}|\Omega_t] = \beta^{-1}(1-B)Q_t + (\beta\gamma_1)^{-1}(Q_t-Q_t^*), \qquad \text{iff } (1-B)Q_t > 0 \qquad (4.9a)$$

and

$$E[(1-B)Q_{t+1}|\Omega_t] = \beta^{-1}(1-B)Q_t + (\beta\gamma_2)^{-1}(Q_t-Q_t^*), \qquad \text{iff } (1-B)Q_t < 0. \qquad (4.9b)$$

When $Q_t-Q_t^*$ is negative, we expect Q_t to rise in the next period.
Thus $(1-B)Q_t > 0$ corresponds with $(Q_t-\alpha P_t)^+$. Vice versa, when $Q_t-Q_t^*$ is positive, we expect Q_t to fall in the next period. So $(1-B)Q_t < 0$ corresponds with $(Q_t-\alpha P_t)^-$. The closed form solution of (4.9) is therefore the NEC model (4.6). The relationships between the adjustment speed parameters (λ_1,λ_2) of (4.6) and the parameters of the piecewise linearly approximated asymmetric adjustment costs model are as follows

$$\lambda_1 = \tfrac{1}{2}(1+\beta^{-1}+(\beta\gamma_1)^{-1}) - \tfrac{1}{2}((1+\beta^{-1}+(\beta\gamma_1)^{-1})^2 - 4\beta^{-1})^{\frac{1}{2}} \quad (4.10a)$$

$$\lambda_2 = \tfrac{1}{2}(1+\beta^{-1}+(\beta\gamma_2)^{-1}) - \tfrac{1}{2}((1+\beta^{-1}+(\beta\gamma_2)^{-1})^2 - 4\beta^{-1})^{\frac{1}{2}}. \quad (4.10b)$$

This completes the formal derivation of the relationship between asymmetric error correction models and asymmetric adjustment models.

5. NONLINEAR ERROR CORRECTION MODELS AND THE IMPLICATIONS OF HAVING TRENDS IN THE MEAN

In the case of Q_t and P_t having trends in the means the NEC representation specified in terms of P_t and Q_t according to equation (2.10) is as follows

$$\phi_\alpha(B)(1-B)Q_t + \theta_\alpha(B)(1-B)P_t = \phi_\alpha(B)(1-B)\mu_{q_t} - \theta_\alpha(B)(1-B)\mu_{p_t} + \Gamma_1(\mu_{q_{t-1}} - \alpha\mu_{p_{t-1}}) - \Gamma_1(Q_{t-1}-\alpha P_{t-1}) - g_\alpha(-\mu_{q_{t-1}} + \alpha\mu_{p_{t-1}} + Q_{t-1} - \alpha P_{t-1}) + \epsilon_{\alpha t} \quad (5.1)$$

However, equation (5.1) is usually written as

$$\phi_\alpha(B)(1-B)Q_t + \theta_\alpha(B)(1-B)P_t = C_1 - \Gamma_1(Q_{t-1} - \alpha P_{t-1}) - g_\alpha(Q_{t-1} - \alpha P_{t-1}) + \epsilon_{\alpha t}. \quad (5.2)$$

For (5.2) to be a well specified model several conditions need to be satisfied. Differencing once should be a good detrending procedure for the means so that $(1-B)\mu_{q_t}$ and $(1-B)\mu_{p_t}$ are not trending. The trends in mean of Q_t and P_t should be proportional (co-trending in mean), such that $\mu_{q_t}-\alpha\mu_{p_t}$ is no longer trending. Notice also that the asymmetric terms of $g_\alpha(.)$ are forced to satisfy also the requirements as well. The cointegrating vector $(1,-\alpha)$ is also the vector that is making the trends in the mean to be co-trending.

To obtain the structural counterpart of the asymmetric error correction model with P_t and Q_t having trends in the means we have to redefine some of the characterizations presented in section 4.

The error correction components now becomes

$$(Q_t-\alpha P_t)^+ = \begin{cases} (Q_t-\alpha P_t) - (\mu_{q_t}-\alpha\mu_{p_t}) & \text{iff } (1-B)((Q_t-\mu_{q_t})-\alpha(P_t-\mu_{p_t})) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5.3.1)$$

and

$$Q_t-\alpha P_t)^- = \begin{cases} (Q_t-\alpha P_t) - (\mu_{q_t}-\alpha\mu_{p_t}) & \text{iff } (1-B)((Q_t-\mu_{q_t})-\alpha(P_t-\mu_{p_t})) < 0 \\ 0 & \text{otherwise} \end{cases} \qquad (5.3.2)$$

The piecewise second order Taylor series expansion of the asymmetric adjustment costs function is then as follows

$$AAC((1-B)Q_t) = \begin{cases} \alpha_1[(1-B)(Q_t-\mu_{q_t})]^2 & \text{iff } (1-B)(Q_t-\mu_{q_t}) > 0 \\ \alpha_2[(1-B)(Q_t-\mu_{q_t})]^2 & \text{iff } (1-B)(Q_t-\mu_{q_t}) < 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (5.4)$$

After these redefinitions equations (4.5), (4.6) and (4.8) to (4.10) applies.

To clarify the specification error when trends in the means are not correctly accounted for, a simple example is given in figures 5A and 5B. Here equations (4.4.1) and (4.4.2) are no longer true, because the area A', that corresponds to $(1-B)(Q_t-\alpha P_t) = (1-B)u_t$ is positive, and $Q_t-\alpha P_t = u_t$ is positive as well. However, in area A $(1-B)(Q_t-\alpha P_t)$ is negative, whereas $Q_t-\alpha P_t$ is positive. The same argument holds for the condition of equation (4.4.2) which is related to the areas B' and B of figure 5A and the corresponding area of figure 5B.

Moreover, asymmetries may occur between situations where the growth rate of the observed decision variable (Q_t) exceeds the growth rate of the target Q_t^* on the one hand, and situations where the growth rate of Q_t is lower than the growth rate of Q_t^* on the other hand. This asymmetry may even be observed during periods where the decision variable is above the target (see figures 5A and 5B). Similarly, we can account for asymmetry in growth rates between areas B and B'.

In order to implement the notion of trending asymmetry in an error correction framework, the adjustment towards the equilibrium should be a correspondence, instead of a function (see figure A2). We are currently investigating the implications of trending asymmetries in dynamic time series models. This analysis, however, is considered to be beyond the scope of this paper.

6. TYPES OF ASYMMETRIES

In this section we consider more general forms of the nonlinear function $g_\alpha(.)$ that can be found in the still young literature on nonlinear error corrections and asymmetric adjustment. All the dynamic representations are in deviations from the mean, $\bar{Q}_t = Q_t - \mu_{q_t}$ and $\bar{P}_t = P_t - \mu_{P_t}$.
First, we consider a piecewise linear adjustment type of function (see figure A.3)

$$\Gamma_1(\bar{Q}_t-\alpha\bar{P}_t) + g_\alpha(\bar{Q}_t-\alpha\bar{P}_t) = \mu_1 D_{1t}(\bar{Q}_t-\alpha\bar{P}_t) - \mu_2 D_{2t}(\bar{Q}_t-\alpha\bar{P}_t) - \mu_3 D_{3t}(\bar{Q}_t-\alpha\bar{P}_t), \quad (6.1)$$

$$D_{1t} = \begin{cases} 1 \text{ iff } \bar{Q}_t - \alpha\bar{P}_t \leq C^- \\ 0 \text{ otherwise} \end{cases}$$

$$D_{2t} = \begin{cases} 1 \text{ iff } C^- \leq \bar{Q}_t - \alpha\bar{P}_t \leq C^+ \\ 0 \text{ otherwise} \end{cases}$$

$$D_{3t} = \begin{cases} 1 \text{ iff } \bar{Q}_t - \alpha\bar{P}_t \geq C^+ \\ 0 \text{ iff } \bar{Q}_t - \alpha\bar{P}_t < C^-. \end{cases}$$

From figure 6C it is clear that the equilibrium is unique, although the adjustment is slower in a small interval (C^-,C^+) around the equilibrium. However, to impose uniqueness of the equilibrium may be too restrictive in general. If, in the interval (C^-,C^+) close to the equilibrium there is no adjustment, (see figure A4), a <u>continuum of equilibria</u> exists. Particular cases of interest that are nested in this formulation are obtained if $C^- = 0$ and $C^+ > 0$, or if $C^- < 0$ and $C^+ = 0$.
Next, we consider a second type of functions, namely the more general cubic poynomials (see figure A5)

$$\Gamma_1(\bar{Q}_t-\alpha\bar{P}_t) + g_\alpha(\bar{Q}_t-\alpha\bar{P}_t) = \mu_1(\bar{Q}_t-\alpha\bar{P}_t) + \mu_2(\bar{Q}_t-\alpha\bar{P}_t)^2 + \mu_{3t}(\bar{Q}_t-\alpha\bar{P}_t)^3, \quad (6.2)$$

where μ_{3t} is time dependent in order to guarantee the asymptotic stability conditions (see Escribano (1986), (1991b), and Hendry and Ericsson (1991)). Equation (6.2) is only an approximation to more general adjustment mechanisms, which can be obtained by nonparametric techniques (smoothing splines). Figure A6 represents the adjustment mechanism observed for UK money demand during the period 1878-1970. Equation (6.2) has the nice property that the adjustment is faster when the distance between the decision variable and the target becomes larger.

7. AN EMPIRICAL APPLICATION

The asymmetric adjustment error correction approach may proof useful in structurally analyzing any economic time series that is assumed to be endogenously generated by the optimizing behavior of (representative) agents. Examples are investment : Gould (1968), consumption : Hall (1978), employment : Sargent (1978), and so forth. The empirical application presented in this section will be limited to the theory and practice of dynamic labor demand, and is founded on the research described in Pfann (1990).

The following notations will be used. L_t = the number of white collar workers employed in the U.K. manufacturing sector at time t; W_t = the real U.K. manufacturing sector white collar wage costs at time t; K_t = the U.K. manufacturing sector capital stock at time t. The annual U.K. data run from 1955 to 1986 (see appendix 1 for the sources and the definitions).

The characteristics of the series are as follows (see appendix 2)

1: L_t, K_t and W_t have a unit root

2: L_t, K_t and W_t have one cointegrating vector.

In this example the decision variable is white collar employment, and the set of forcing variables consists of real white collar wage costs and capital. Hence forth, in correspondence with the preceding section, the following relation hold

$$Q_t = L_t, \text{ and } P_t = (K_t, W_t).$$

The equilibrium errors from the cointegration relationship are as follows

$$\hat{u}_t = \underset{}{L_t} + \underset{(4.41)}{5.35} + \underset{(8.20)}{0.28}\ OC74 + \underset{(9.32)}{1.89}\ W_t - \underset{(10.66)}{2.59}\ K_t \qquad (7.1)$$

$R^2 = 0.83 \qquad \sigma = 0.056 \qquad ADF = -4.56$

In all equations absolute t-values are given within parentheses. OC74 is a step-dummy for the oil crisis of fall 1973 being one from 1974 on. In addition to the correctly specified test of the cointegrating vectors of the system (see Phillips (1991)), as presented in appendix 2, we also present the Augmented Dickey Fuller statistic (ADF) in (7.1).
Judging from table IIb of Phillips and Ouliaris (1990) the hypothesis of no cointegration is rejected in equation (7.1). Leaving out OC74 reduced the Augmented Dickey Fuller statistic (ADF) to -1.49. Consequently, structural breaks may blur cointegration relations if they are not adequately dealt with (see also Palm and Pfann (1991), Perron (1989), Escribano (1991a)). Figure 7.1 shows $\hat{u}_t$ of equation (7.1), where the horizontal line plays the role of the longrun equilibrium between L_t, K_t and W_t.
The estimated linear error correction model (3.12) is as follows

$$\underset{}{(1-B)L_t} = \underset{(4.16)}{-0.006} - \underset{(0.32)}{0.007}\,(1-B)W_{t-1} + \underset{(4.36)}{2.28}\,(1-B)K_{t-1} - \underset{(1.90)}{0.24}\,\hat{u}_{t-1} \qquad (7.2)$$

Sample = 1957-1986

$R^2 = 0.46 \quad \sigma = 0.033 \quad \chi^2_{AR}(2) = 0.70 \quad \chi^2_{NORM}(2) = 1.85 \quad \chi^2_{ARCH}(2) = 0.78$

The reported statistics are the residual based Ljung-Box test for residual autocorrelation (χ^2_{AR}), the residual based normality test (χ^2_{NORM}), and the residual based ARCH test (χ^2_{ARCH}). All tests have two degrees of feedom. None of the tests are significant. Thus the model would be an acceptable econometric model. Next, we report the estimated asymmetric error correction model (4.5)

$$(1-B)L_t = \underset{(3.37)}{-0.06} - \underset{(0.25)}{0.05}\,(1-B)W_{t-1} + \underset{(4.13)}{2.17}\,(1-B)K_{t-1} - \underset{(2.19)}{0.42}\,\hat{u}^+_{t-1} - \underset{(0.41)}{0.07}\,\hat{u}^-_{t-1} \qquad (7.3)$$

Sample = 1957-1986

$R^2 = 0.49 \quad \sigma = 0.032 \quad \chi^2_{AR}(2) = 0.24 \quad \chi^2_{NORM}(2) = 1.95 \quad \chi^2_{ARCH}(2) = 0.51$

The estimated error correction parameters in (7.3) provide us with useful additional information with respect to the asymmetry between underequilibrium adjustment and overequilibrium adjustment towards the longrun cointegration relation. The finding that $\hat{u}^-_{t-1}$ and $\hat{u}^+_{t-1}$ both have negative signs is in line with the expected error corrections for procyclical variables. However, the two models are not statistically distinct : the F-statistic testing the statistical significance of the included asymmetry (7.3) versus the linear symmetric model (7.2) yields $F(1,25) = 1.58$. The adjustment speed towards a higher target level of white collar employment ($1-\gamma_1 = 0.42$) exceeds the adjustment speed towards a lower target level ($1-\gamma_2 = 0.07$). The characteristic roots $\lambda_1 = 0.58$ and $\lambda_2 = 0.93$ lie within the unit circle. Using (5.9a) and (5.9b) the piecewise linear asymmetric adjustment costs parameters yield $\gamma_1 = 3.07$ and $\gamma_2 = 114.04$, assuming $\tau = 0.95$. Thus, we find that $\gamma_1 < \gamma_2$, implying $\delta < 0$ (see (4.6)), which is in accordance with the finding of Pfann and Palm (1988) for U.K. manufacturing white collar workers : white collar workers are more easily hired in times of economic growth than fired in times of economic recession.

8. CONCLUSIONS

In this paper we showed that nonlinear error correction mechanisms that are found to exist in time series data may be endogenously generated resulting from the optimizing behavior of (representative) agents that face asymmetric costs of adjustment. The rationale for asymmetric costs is equivalent to the notion of nonlinear error correction mechanisms: the adjustment path to a higher target level should not necessarily be symmetric with the adjustment path to a lower target level. Several types of asymmetry are discussed and we explained how trends should be included in the nonlinear error correction model. In a numerical example we estimated the adjustment speeds in different phases of the economic cycle for U.K. manufacturing white collar workers, finding that white collar workers are more easily hired in times of economic growth than fired in times of economic recessions.

Figure A : Several Types of Asymmetries

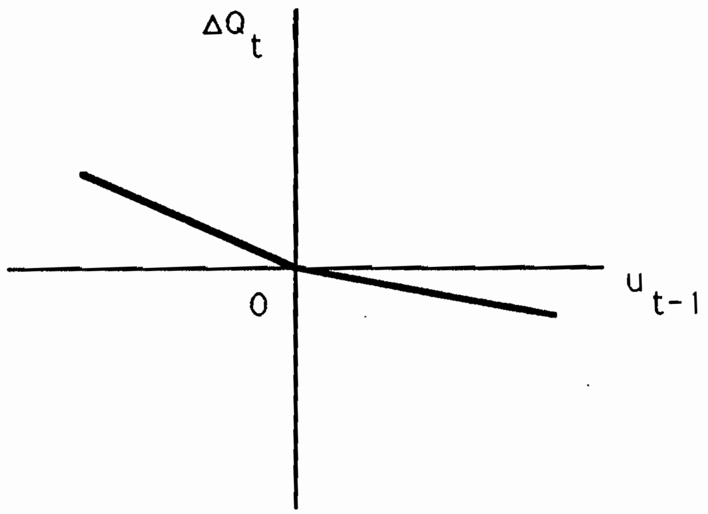


FIGURE A1

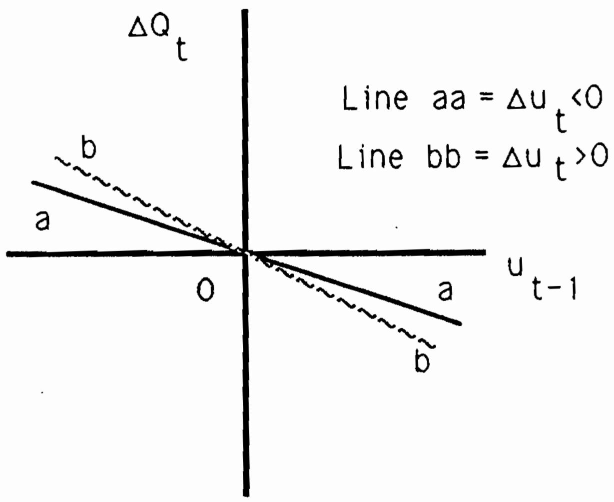


FIGURE A2

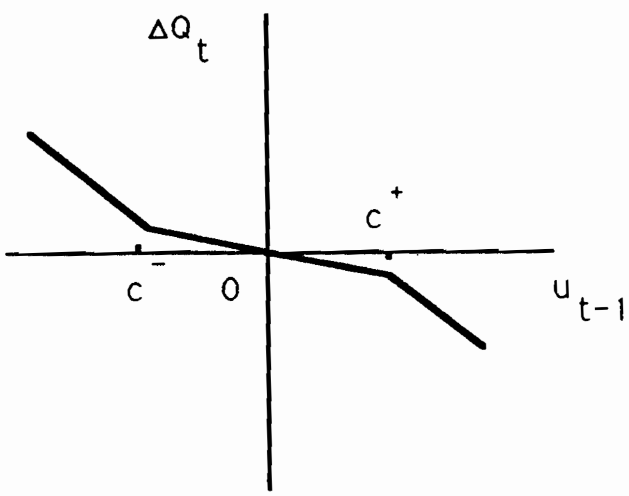


FIGURE A3

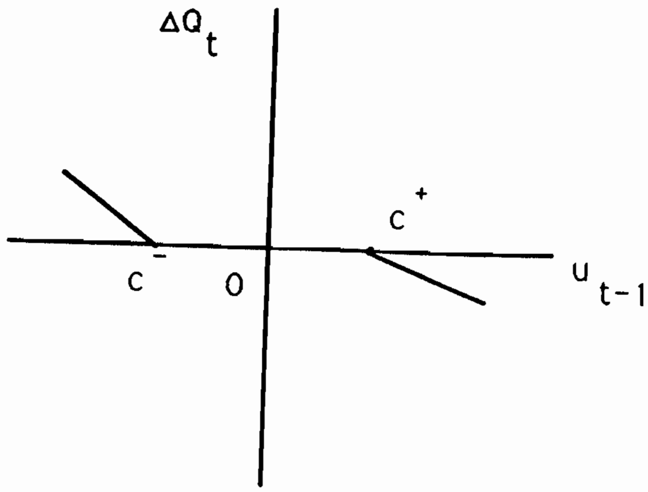


FIGURE A4

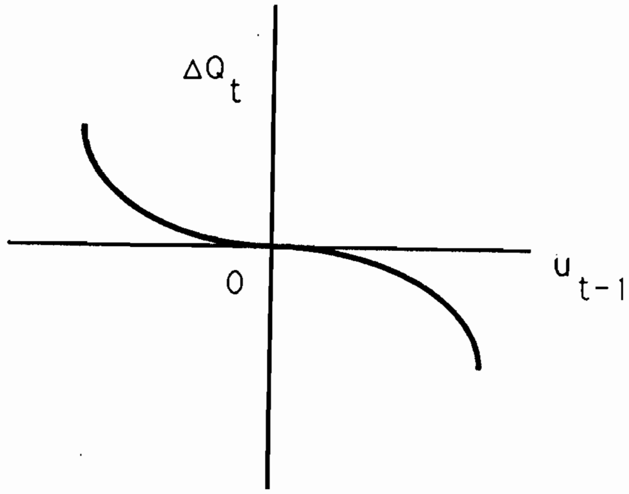


FIGURE A5

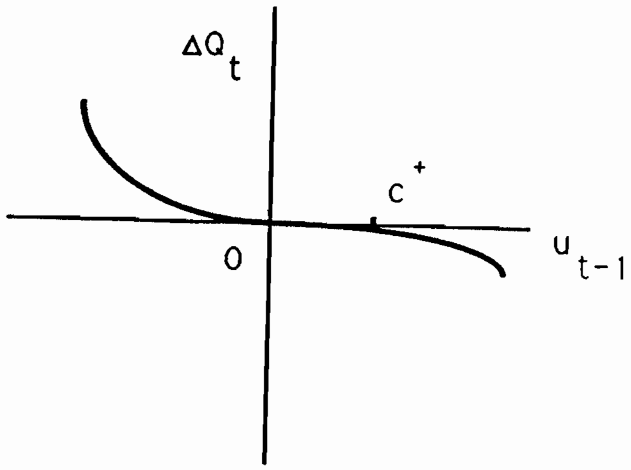


FIGURE A6

Figure 5.1 : Nonlinear Error Correction with Trend in Mean.

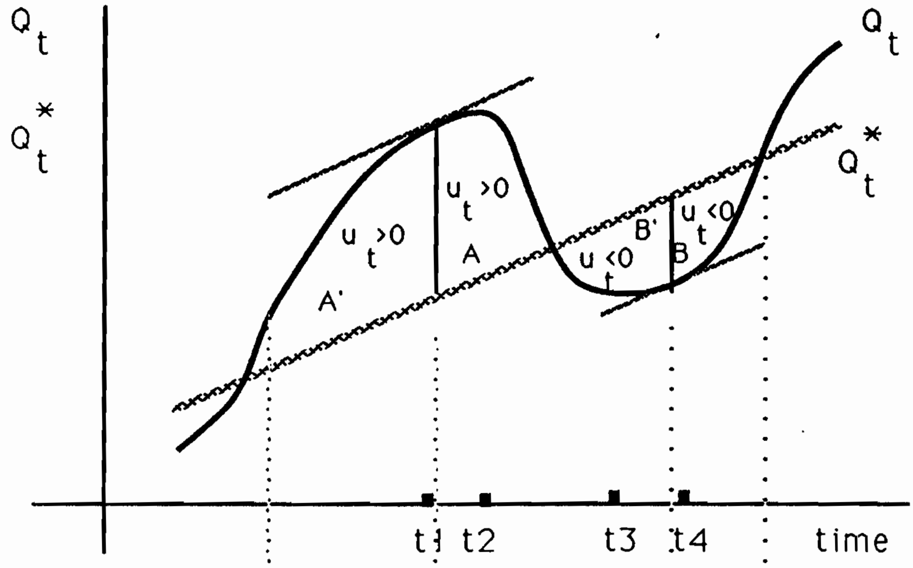


Figure 5a

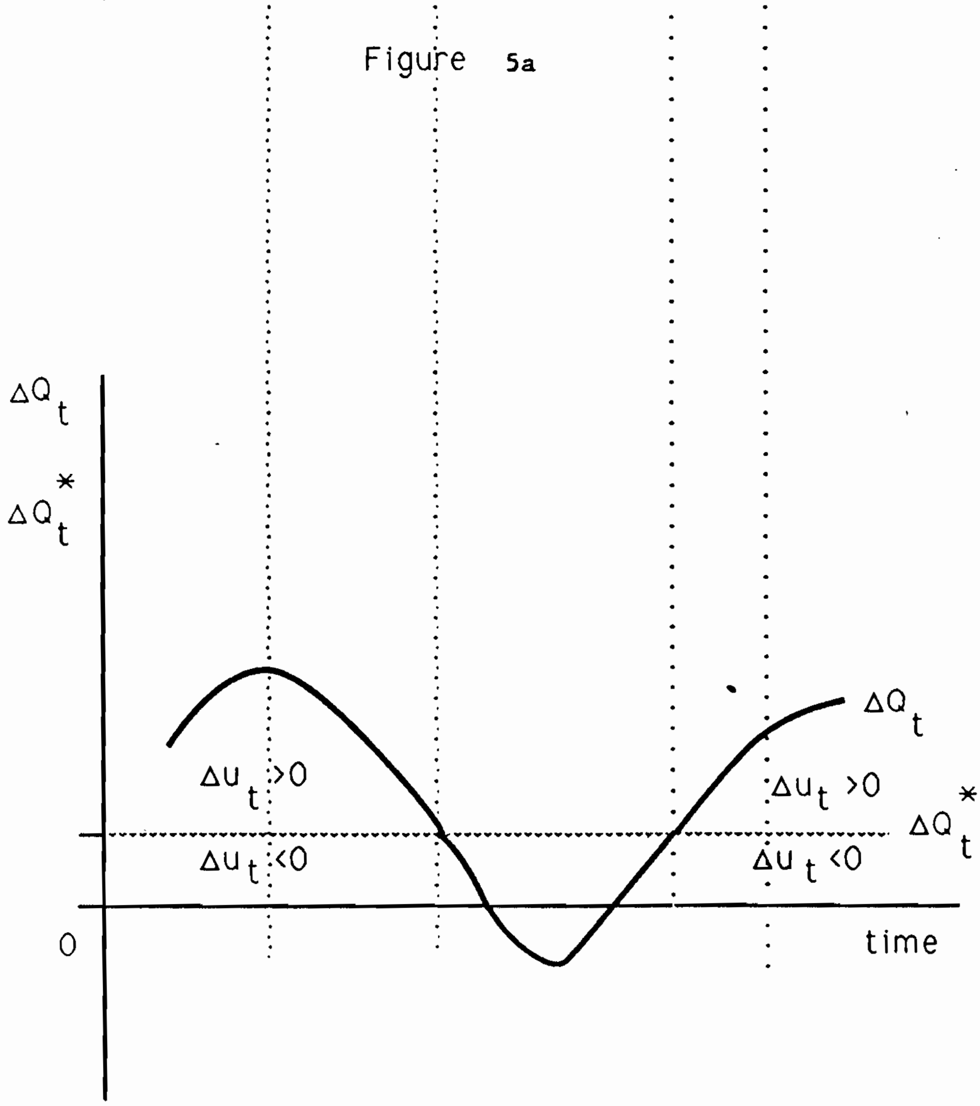
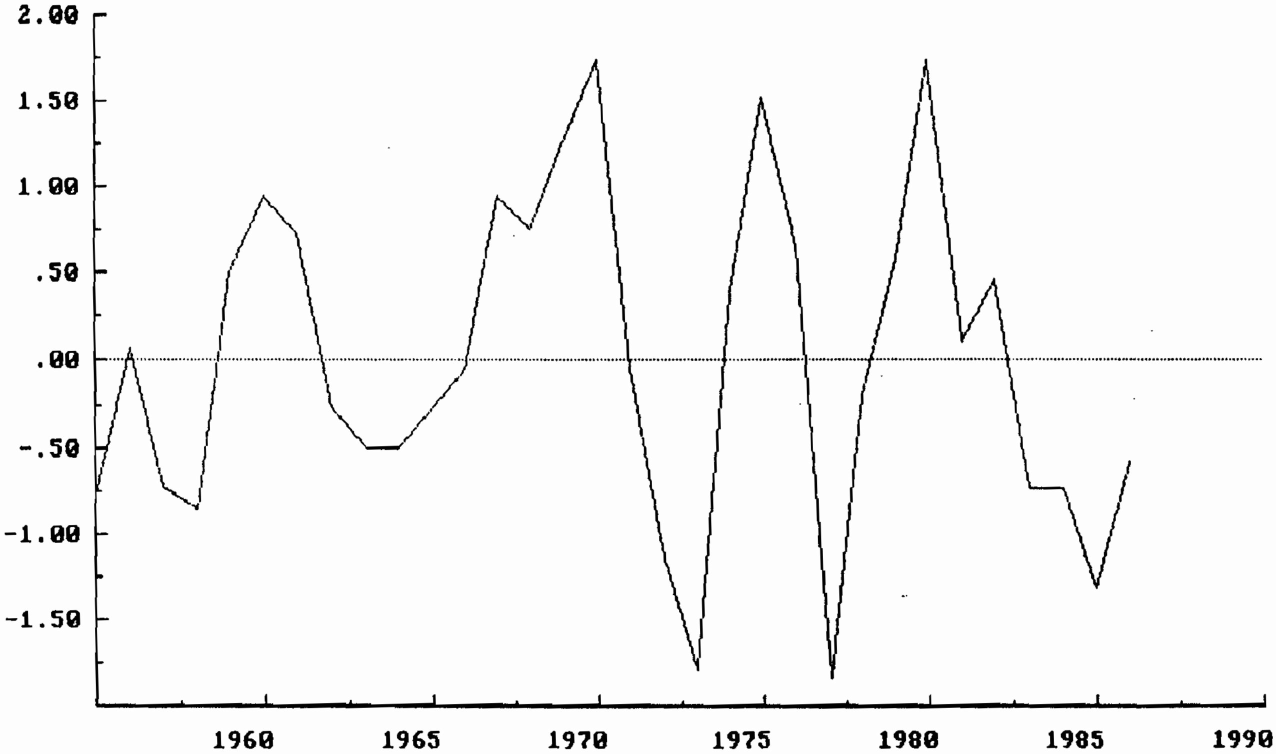


Figure 5b

Figure 7.1

LONGRUN ERRORS OF L, K AND W

Cointegration relation



APPENDIX 1 : SOURCES OF THE ANNUAL U.K. MANUFACTURING DATA.

The base year of all prices and indices is 1980.
Sample : 1955-1986.
The following main data sources were used :

BB	:	Blue Book
DEG	:	Department of Employment Gazette
ETAS	:	Economic Trend Annual Suplement
HABLS	:	Historical Abstract of British Labour Statistic
MM	:	Mendis L. and J. Muellbauer (1984), British Manufacturing Productivity 1955-1983 : Measurement Problems, Oil Shocks, and Thatcher Effects, CEPR Discussion Paper No. 34.

The variables are defined as follows :

L	:	The natural log of the total numbers of employees in U.K. manufacturing, have been obtained from ETAS.
W	:	The natural log of the real weakly earnings index have been obtained by deflating gross weekly earnings of manual and nonmanual workers (pre-1970 data : HABLS; from 1970 on data : New Earnings Survey in DEG) by Py.
K	:	The natural log of the gross capital stock at constant prices (K) have been obtained from BB for data from 1963 and from MM for pre-1963 data.

APPENDIX 2 : UNIT ROOTS AND COINTEGRATION.

UNIT ROOTS TESTS

Model : $(1-B)V_t = a'X_t + \alpha_1 V_{t-1} + \alpha_2(1-B)V_{t-1} + \epsilon_t$

$V_t \in \{L_t, K_t, W_t\}$

$X_t + (CONST, OC74)^*$

$H_0 : \alpha_1 = 0$

Sample : 1957-1986

	L_t	K_t	W_t
Fuller's $\hat{\tau}_\tau$	-1.37	-2.53	-0.92
Adjusted R^2	0.33	0.85	0.50
DW-statistic	2.01	1.42	1.96

* OC74 is a step dummy equal to 1 1974 and zero elsewhere.

According to Fuller's $\hat{\tau}_\tau$ statistic (Fuller 91976), table 8.5.2) we do not reject the hypothesis that the univariate time series have a unit root. Also, if we take account of the fact that one dummy variable (OC74) is included in the model and therefore use the distribution given by Perron (1989), we reach the same conclusion.

JOHANSEN'S COINTEGRATION TESTS

Exaplanatory Variable : L_t

Forcing Variables : W_t, K_t

Sample : 1957-1986

	$H_0 : r$
CV = 1 (r = 0)	33.46
CV = 2 (r = 1)	4.01

CV : number of cointegration vectors.

The critical values for a three variate cointegration system are given in table 1 of Johansen (1988) : 23.8 and 26.1 for 5 percent and 2.5 percent significant levels respectively. We find that the hypothesis of no cointegration is rejected in favor of one cointegration vector.

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