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THE DYNAMICS OF DURABLE GOODS MARKETS: RATIONAL EXPECTATIONS AND STICKY PRICES

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Abstract

This paper studies price dynamics in a durable good market under the assumption that consumers have rational expectations on future prices. For a wide variety of expectations, optimal consumption plans result in sticky-price demand functions. Market dynamics are characterized by intertemporal price discrimination which provides a possible explanation for the declining path of price observed in many "young" industries. Unexpected shocks on demand result in price overshooting, while unexpected supply shocks have the opposite effect on price.

Key words:
Durable Goods; Rational Expectations; Sticky Prices; Intertemporal Price Discrimination; Overshooting.

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1. INTRODUCTION

The purpose of this paper is to study price dynamics in a durable good market under the assumption that consumers have rational expectations on future prices. We find that, for a wide variety of expectations, the price of a durable good will not adjust instantaneously to its long-run equilibrium level. This provides a foundation for sticky-price demand functions similar to those studied in Fershtman and Kamien (1987), Miller (1979), Roos (1925), and Evans (1924), among others[1]. Market equilibria are analized under two alternative assumptions on the structure of the market: perfect competition and oligopolistic competition.

In addition to current prices and income, the demand for a durable good is likely to depend on some other factors. First, the stock of the good held at any particular date may influence consumers' willingness to buy new units. Thus, it is posible for demand functions (and prices) to change over time. Second, if prices are expected to change in the future, consumers may benefit from either anticipating or delaying some of their purchases; in consequence, the demand of the good will depend on consumers'expectations on future prices.

We develop a model of consumer's choice to characterize dynamic demand equations for durable goods[2] which are intertemporally consistent with consumers' expectations and preferences. This is done for sets of expectations satisfying certain smoothness requirements. We then show that these expectations are fulfilled in equilibrium .

The literature on market dynamics with sticky prices is based on the assumption that prices do not adjust instantaneously to long-run equilibrium levels. It is unclear, however, the economic motivation for the existence of adjustment lags. Fershtman and Kamien (1987) propose utility functions that depend on both current consumption and past consumption of a good. Static demand functions, resulting from myopic utility maximization, show gradual price adjustment in this case.

Our model extends the analysis of Fershtman and Kamien (1987) to the case in which consumers have rational expectations on future prices. We have chosen to develop the model in terms of a good of some (possibly limited) durability. An alternative formulation, based on preferences that depend on both current consumption and past consumption of a good, can also be developed. Indeed, it is possible to show that formally the analysis of both models is identical[3].

Myopic and anticipatory behavior result in different demands. In periods of declining prices, for instance, consumers demand less if they anticipate future prices than if they behave myopically. Thus, sophisticated consumers will delay some of their purchases in anticipation of future lower prices. The inequality is reversed in periods of increasing prices.

The implication is that models built on the asumption of a static demand will tend to make biased predictions as long as: i) the good has some durability; and ii) its price changes through time. In periods of declining prices, for example, prices will be below the static equilibrium price (for a given stock level).

The existence and properties of equilibria with rational expectations is examined under two different assumptions on the structure of the market. First, we consider the case of a competitive market with general cost and utility functions. Second, we adopt a differential game framework to analize a particular example of an oligopolistic market.

Market dynamics are similar in both cases. First, price shows a tendency to change through time in a gradual fashion. Second, price and stock of the durable good converge to a steady-state. Third, along the equilibrium path, price and stock show an inverse relation. This last result replicates the common observation that new (durable) goods are introduced at a relatively high price which declines through time, as the stock of the good in the market increases.

An inverse relation between price and stock is also predicted by models of learning-by-doing, Spence(1981). The force driving price down in models of learning-by-doing is the sequential reduction of (marginal) costs. In our model price falls for a different reason. Assume that the marginal utility of the durable good is decreasing. Then, consumers' willingness to pay will decrease as their stock of the good increases. In equilibrium, the price path is adapted to the changing willingness to pay of the consumers. Thus, gradual price reduction in our model occurs as a form of intertemporal price discrimination; see Stokey (1979) for a discusion of intertemporal price discrimination in a monopolistic market.

Even if all of the consumers are identical, price discrimination is imperfect; in intervals of declining price, for instance, the price will be below the maximum price a consumer would be willing to pay should price remain constant. In this way, buyers are induced to increase their holdings of the good.

Finally, we analize some comparative dynamic properties of the model. Suppose an unexpected shock changes any of the structural parameters of the model. How will price react to this shock? We find that the answer depends on whether the supply side or the demand side of the market is affected by the shock. Assume that the market was in a steady state equilibrium before the shock. The adjustment of price occurs in two stages. First, there is an instantaneous adjustment: the price jumps to the equilibrium level required by the current stock of the good . Second, the adjustment is completed by a gradual change, as price continuously adjusts to the changing stock level.

Supply shocks result in the price jump and the gradual adjustment running in the same direction. Thus, in the short-run price under-reacts to unexpected changes in costs or in the number of firms in the market. Demand shocks, on the other hand, result in the price jump and the gradual adjustment running in opposite directions. Thus, in the short-run price over-reacts to unexpected demand changes. We refer to these two cases as price undershooting and price overshooting, respectively. In both cases, short-run price changes are not proportionate to the long-run effect of the shock on the equilibrium price.

The rest of the paper is organized as follows. Section 2 presents the demand side of the model and compares myopic and anticipatory consumer's behavior. Section 3 studies equilibrium market dynamics under perfect competition. In section 4, a differential game framework is adopted to analize the case of an oligopolistic market. Section 5 discuss some comparative dynamics of the model. Finally, section 6 contains some concluding remarks.

2. THE DEMAND SIDE

Consider a durable good X which depretiates at a constant rate g. Let $t \in [0,\infty)$ denote time. Consumers of the good are identical[4] and infinitely lived; and there is a total of N consumers. Let $X(t) \geq 0$ be the stock of the good owned by a consumer at time t. The rate of change of the stock is:

$$(1) \qquad X'(t)=x(t)-gX(t) \qquad g\geq 0$$

where $x(t) \geq 0$ is the consumer's demand of the durable good at time t.

Let p(t) be the price of X at t. The utility rate is assumed to depend on the stock of the good and on the expenditures incurred in new purchases[5]:

$$(2) \qquad U(X, px)=u(X)-px$$

where u( ) is twice continuously differentiable, strictly increasing and strictly concave. Utility is discounted at rate r>0 over the infinite time horizon.

Suppose a consumer expects price to follow a piecewise continuous path $\{p(t):t\in[0,\infty)\}$. Then, an optimal purchase plan $x^*(t)$ will be chosen to maximize:

$$(3) \qquad \int_{[0,\infty)} e^{-rt} U(X(t),\ p(t)x(t))\ dt$$

subject to (1) and the non-negativity constraint $x(t)\geq 0$.

Optimal purchasing plans are characterized in the following:

<u>LEMMA 1</u>: Suppose that the expected price path is twice continuously differentiable. Then, individual demand satisfies:

$$(4) \quad x(t)=\max\{0,gX(t)+[(r+g)p'(t)-p''(t)]/u''(X(t))\}$$

<u>PROOF</u>: See Appendix A.///.

When the lower bound on x(t) is not binding, i.e., when consumers are active in the market, integration of (4) yields:

$$(5) \qquad p'(t)=(r+g)p(t)-u'(X(t))$$

which can be regarded as the dynamic demand equation for an active consumer. Notice that, for the set of expectations under consideration, demand depends on "local" information only. The stock of the good that the consumer wants to held is determined by the current price of the good and its current rate of change.

Myopic demand levels will be used as a benchmark. A myopic consumer is defined as one who does not anticipate future changes in the price of the good. Thus, a myopic consumer will demand units of the good up to the point in which (total discounted) marginal utility equals the current price p(t) of the good:

$$u'(X(t))/(r+g)=p(t) \tag{6}$$

Given a price p, the solution X(t) of (6) will be referred to as the myopic stock demand and denoted $X_s(p)$.

The behavioral consequences of consumers expectations are summarized as follows:

<u>PROPOSITION 1</u>: Suppose that consumers are active at time t and price p. Then, the optimal amount of stock X^* satisfies: i) $X^*>X_s(p)$ if price is expected to rise; ii) $X^*<X_s(p)$ if price is expected to decline; and iii) $X^*=X_s(p)$ if price is expected to remain constant.

<u>PROOF</u>: Simply notice that equations (5) and (6) are identical for $p'(t)=0$. Hence, iii) follows. If $p'(t)>0$, then $u'(X^*)<u'(X_m(p))$, which implies i) since u( ) is concave. ii) follows similarly.///.

Intuitively, proposition 1 indicates that if the price of the good is expected to decline, then it is optimal to delay the purchase of some units of the good to take advantage of future lower prices; and the converse holds true if the price of the good is expected to raise.

In summary, we have found that demand functions showing gradual price adjustment can be rationalized as demand functions for a durable good[6]. Furthermore, the static optimization rule, setting the stock level so that marginal utility equals price, is not optimal in this context; utility maximizing behavior requires to adapt current demand to expected future prices.

Finally, market demand can be obtained from equation (4). Notice that, as long as consumers' expectations are rational, the same price path is anticipated by every consumer. Therefore, market demand is simply $x_m(t) = Nx(t)$.

3. PERFECT COMPETITION

The results of the previous section are valid only if the set of expectations considered is consistent with the actual equilibrium behavior of price. We consider in this section the case of a perfectly competitive market to show that the dynamic demand equation (5) is indeed consistent with equilibrium dynamics.

Let c(q) be the cost of producing q units of the good, and suppose that c( ) is twice differentiable, strictly increasing and concave, and $\lim_{q\to\infty} c'(q)=\infty$. Furthermore, $c'(0)<u'(0)$, since otherwise the market will never be active.

There are n identical firms. Price-taking behavior implies that each firm's supply will satisfy:

(7) $$p(t)=c'(q(t))$$

Let's define a function b( ) as the inverse of the marginal cost, i.e., $c'(b(z))=z$ for every $z\geq 0$. Then, market supply at time t is $q_m(p(t))=nb(p(t))$. Let Q(t) be the total stock of the good in the hands of the consumers at time t. The rate of change of Q(t) satisfies:

(8) $$Q'(t)= q_m(p(t))-gQ(t)$$

A competitive equilibrium with rational expectations is a path of prices and stock (p(t),Q(t)), and a pair of demand and supply functions ($x_m(t)$, $q_m(t)$), satisfying the following conditions:

i) Each consumer correctly anticipates the price path.

ii) Supply equals demand for every t, i.e., $x_m(t)=q_m(p(t))$.

iii) The rates of change of price and stock satisfy the dynamic demand equation (5) and equation (8) simultaneously.

The dynamic properties of the equilibrium are characterized in the following:

PROPOSITION 2: i) There exists a unique steady state equilibrium (p_s,Q_s); ii) given $Q_0=Q(0)\geq 0$, there exists a price p_0 such that the resulting equilibrium path of price and stock $(p^*(t),Q^*(t))$ converges to the steady state; and iii) along the equilibrium path, price and stock are inversely related, i.e., price declines as the stock level increases and viceversa.

PROOF: i) The solution (p_s,Q_s) of $b(p_s)=gQ_s$ and $(r+g)p_s=u'(Q_s/N)$ defines a steady-state; hence p_s solves $p_s=u'(b(p_s)/N)/(r+g)$. This equation has a solution since for $p=0$, $u'(b(p)/(gN))/(r+g)-p=u'(0)/(r+g)>0$, while for $p\to\infty$, $u'(b(p)/(gN))/(r+g)-p\to-\infty<0$; continuity then requires $u'(b(p)/(gN))/(r+g)-p=0$ for some p in between. The solution is unique since $u'(b(p)/(gN))/(r+g)$ is (strictly) decreasing.

ii) First notice that, as long as $p>0$, supply will be strictly positive; hence, demand which equals supply, must also be positive; therefore the lower bound on $x(t)$ is not binding for positive prices; this implies that the dynamic demand equation is consistent with utility maximizing behavior. Suppose w.l.o.g. that $Q(0)<Q_s$. We have to show that for at least one initial price level $p(0)$, the resulting path of price and stock $L(p(0))$ converges to (p_s,Q_s). Notice that any initial condition can be continued along a unique solution path. Suppose, to obtain a contradiction, that no path $L(p(0))$ converges to the steady-state. Let $A=\{(p,Q): p'(t)=0, Q'(t)>0\}$ and $B=\{(p,Q): p'(t)<0, Q'(t)=0\}$. Then, there exists a price p_c such that: i) if $p(0)>p_c$, then $L(p(0))$ crosses A; ii) if $p(0)<p_c$, then $L(p(0))$ crosses B. Furthermore, $L(p_c)$ must cross either A or B at some

point (P_n, Q_n). In the first case, the set of points $A'=A\cap\{(p,Q): p_s<p<p_n\}$ is isolated, i.e., an initial condition in A'cannot be continued, which is a contradiction. A similar argument applies in the second case.

iii) Notice that, along the equilibrium path, convergence to the steady state requires $Q'(t)>0$ and $p'(t)<0$ if $Q(0)<Q_s$. The inequalities are reversed if $Q(0)>Q_s$.///.

Figure 1 shows the four posible dynamics of price and stock. The only stable path lies in the set $\{(p,Q): p'(t)\leq 0$ and $q'(t)\geq 0\}$ for stock levels below Q_s, and in the set $\{(p,Q): p'(t)\geq 0$ and $q'(t)\leq 0\}$ for stock levels above Q_s. Proposition 2 suggests that declining prices will be observed in young (durable good) industries under perfect competition; a long run equilibrium price will be approached as the industry reaches maturity. Furthermore, as price declines and stock increases, a larger fraction of total sales is directed to the replacement of old units of the good, and a smaller fraction corresponds to net increases in the stock held by the consumers. Total sales and profits decrease as the price of the good falls.

4. OLIGOPOLY

We will consider a dynamic version of a Cournot oligopoly in which firms choose output and recognize the influence of production decisions on price. Firms are assumed to be identical and are indexed by i; there is a total of n firms in the market. The production rate of firm i at time t is denoted by $q_i(t)$. The cost rate satisfies:

(9) $$c_i(q_i(t))=c[q_i(t)]^s$$

where c>0 and s>1. Hence, there are decreasing returns to scale in the production technology. The profit rate is:

(10) $$f^i(p(t),q_i(t))= p(t)q_i(t)-c_i(q_i(t))$$

Profits are discounted at rate r. Firm's i total discounted profits starting at t_0 are:

(11) $$P^i(t_0,p(t_0),Q(t_0))= \int_{[t_0,\infty]}e^{-rt}f^i(p(t),q_i(t))dt$$

At every date t, each firm must choose an output level after observing the current levels of price and stock. Each firm recognizes the influence of current output decisions on future prices; the current price, however, is taken as given. An strategy for firm i is a piece-wise continuous function $\mu^i:R_+^3 \longrightarrow R_+$ specifying the output rate, i.e., $q_i(t)=\mu^i(p(t),Q(t),t)$.

An equilibrium with rational expectations is a set of n strategies μ_i, one for each firm, together with a price path $p^*(t)$ such that:

i) Each consumer correctly anticipates the price path.

ii) firm's i strategy maximizes (12) starting at 0, given the other firms'strategies.

iii) Supply equals demand for every t, i.e., $x_m(t)=q_m(p(t))$.

iv) The rates of change of price and stock satisfy the dynamic demand equation (5) and equation (1) simultaneously.

An equilibrium is subgame perfect if firm's i strategy maximizes discounted profits starting at t_0 for any $t_0 \in [0,\infty)$, given the other firm's strategies.

We will be concerned with subgame perfect equilibria resulting in a continuous and differentiable price path; furthermore, we restrict our attention to equilibria in which consumers (and firms) are always active and the utility function takes the special form u(x)=lnX. For this set of equilibria, the change of the price through time is governed by:

(12) $$p'(t)=(r+g)p(t)-N/Q(t)$$

and the change in the total stock obeys:

(13) $$Q'(t)=-gQ(t)+\Sigma_{i=1,\ldots,n}q_i(t)$$

Given a set of inicial conditions $(p(t_0),Q(t_0),t_0)$, and a set of n-1 strategies, the current value of the game for firm i is:

(14) $$v^{*i}(p(t_0),Q(t_0))=\max_{\mu^i}\{P^i(p(t_0),Q(t_0),t_0)\}$$

where p(t) and Q(t) satisfy (13) and (14). The value function $v^{*i}(p(t),Q(t))$ and the equilibrium strategy $\mu_i(p(t),Q(t),t)$, must satisfy the Bellman-Jacobi equation:

$$(15) \qquad rv^*_i(p,Q)=\max_{\mu_i}\{p(t)\mu_i-c_i(\mu_i) + v^{*i}_Q(p,Q)(-gQ(t)+\Sigma_{k=1,\ldots n}\mu_k(p(t),Q(t),t)) + v^{*i}_p(p,Q)((r+g)p(t)-N/Q(t))\} \qquad i=1,\ldots,n$$

Consider the right hand side of (15). An interior optimal strategy must satisfy $\mu_i=[(p+v^{*i}_Q)/(cs)]^{1/(s-1)}$. Substituting this condition back into (15), and solving the resulting system of n partial differential equations for a symmetric solution, the following characterization of equilibrium strategies and profits is obtained:

$$(16) \qquad \mu_i(p,Q)=(k_0p)^{1/(s-1)} \qquad i=1,\ldots,n$$

where $k_0=((n-1)(sn-1)^{-1}c^{-1}$, and

$$(17) \qquad v^{*i}(p,Q)=k_1(N/r -pQ) \qquad \text{where } k_1=(s-1)/(sn-1).$$

It can be checked by direct substitution that (16) and (17) are indeed a solution of (15), and that no other symmetric solution exists. A complete characterization of this equilibrium is given in the following:

Proposition 3: Let $Q>0$. Then, the set of strategies (17) defines the unique symmetric closed-loop equilibrium of the game. For some initial price p_0 this equilibrium is stable and converges to an steady state:

$$Q_s = (n/g)^{1-1/s} \frac{[(n-1)N]^{1/s}}{[(sn-1)(r+g)c]^{1/s}} \quad \text{and} \quad p_s = \frac{N}{(r+g)Q_s}$$

Proof: Our previous considerations have shown that the proposed set of strategies defines a closed-loop equilibrium. To show that the induced price path p(t) and stock path Q(t) are consistent with the consumers' maximization problem notice that market demand must coincide with $(n\mu_i)$; hence, as long as $p(t)>0$, consumers will be active and demand must satisfy the dynamic demand equation. It only rests to show that price and stock are non-negative along the equilibrium path and converge to (p_s,Q_s); but this was demonstrated in Proposition 1 for a class of supply functions b( ) which includes (16) as a particular case.///.

As an inmediate consequence of the proposition, market dynamics share the properties of the perfectly competitive case:

COROLLARY: Along the equilibrium path, price and stock are inversely related.

According to the corollary, Proposition 2 also applies to the model of this section. Thus, intertemporal price discrimination may also be a reason for declining prices in young oligopolistic markets.

Finally, it is interesting to note that some forms of consumers heterogeneity can be easily incorporated into this framework. Suppose, for instance, that consumers can be indexed by an integer i so that $u(X,i)=h(i)\ln X$, where $h(i)>0$. Suppose that there are $H(i)$ consumers of type i, and let's define $N^*=\Sigma_i h(i)H(i)$. Then, the dynamic demand equation for the market is $p'(t)=(r+g)p(t)-N^*/Q(t)$, which is similar to equation (13). Market dynamics are thus as before. Differences among consumers result in consumers with a stronger preference $h(i)$ for the good, holding larger quantities of it.

5.-COMPARATIVE DYNAMICS

In this section we shift our phocus to markets for stablished products, to study how the equilibrium price should react to changes in the structural parameters of the model. Because of the similarities already noted, the analysis that follows applies to any of the market structures considered in the two previous sections.

Suppose that, initially, the market is in its steady state equilibrium. A structural shock may affect the number of firms or consumers in the market, or may result in a (monotonous) shift of the marginal cost or marginal utility curves. In response to an unexpected change in the structure of the model, there will be an instantaneous price jump to accommodate price to the current stock level. The jump will be followed by a process of gradual adjustment along the equilibrium path, as the stock level approaches its new long-run equilibrium level.

It seems natural to label the instantaneous adjustment as the short-run reaction of price. If the short-run adjustment is larger than the required long-run adjustment, then there is price overshoothing. Otherwise, there is price undershooting. As it turns out, whether there will be price overshooting or price undershooting depends only on the side of the market affected by the shock.

<u>PROPOSITION 4</u>: Suppose $(p(0),Q(0))=(p_s,Q_s)$. Then:

i) Non-anticipated shocks in either the number of consumers or their preferences will result in price overshooting.

ii) Non-anticipated shocks in either the number of firms or the cost function will result in price undershooting.

<u>PROOF</u>: Let (p_n,Q_n) be the new steady state after the shock.

i) Suppose that $(p_n,Q_n)>(p_s,Q_s)$ (i.e., N increases or u'( ) moves to the right). To reach Q_n it is necessary to increase Q; since p and Q are inversely related, price must be decreasing along the equilibrium path. This requires to set a price $p>p_n$. Thus, there is price overshooting. If $(p_n,Q_n)<(p_s,Q_s)$, a similar argument applies.

ii) If $p_s<p_n$ and $Q_s>Q_n$, in order to decrease Q, p must increase along the equilibrium path. Thus, price must be set to a level $p<p_n$. Furthermore, $p\leq p_s$ implies that p will be decreasing. Hence, $p_s<p<p_n$, i.e., there is price undershooting. If $p_s>p_n$ and $Q_s<Q_n$, a similar argument applies./// .

The scope of the results of proposition 4 is limited. The occurrence of zero-probability events violates the rationality of the expectations of firms and consumers. The suggestion, however, still deserves some consideration. In response to an exogenous shock, the short-run change of price may not be proportionated to the magnitude of the change required to restore the equilibrium in the long-run. There are two factors that influence the particular dynamics of price adjustment. First, in the short-run, the stock of the good owned by the consumers cannot be changed. This forces price to absorb all of the short-run impact of the shock. This short-term adjustment is similar to the kind of adjustment one would expect to observe in a non-durable market with instantaneous adjustment. Second, in order to bring the stock level to a new steady-state equilibrium, price must gradually change. These two factors work independently which explains why, in case of a demand shock, the short-run and long-run adjustment of price run in opposite directions.

6. CONCLUDING REMARKS

In summary, we have analized optimal consumer behavior with rational expectations in a durable good market; sticky price demand functions naturally emerge in this context.

The fact that consumers, in our model, anticipate future prices, does not force price to adjust inmediately to its long run equilibrium level. Firms are able to extract some consumers' surplus through time, thus taking advantage of the gradual change in the marginal utility of the durable good. On the other hand, consumers' perfect foresight has an influence on the price path; in order to induce consumers to increase their holdings of the good, price has to be kept below the current marginal utility of the good. This sets a limit to the ability of firms to extract consumers'surplus.

Durable good markets are often studied under the assumption of a perfectly inelastic (zero-one) demand. In this context, Stokey (1979) finds that consumers' expectations impose no restriction on the set of equilibria. Our analysis indicates that this result does not extend to the case of an elastic demand. Many durable goods (bulbs, compact-disks or golf balls, for instance) may be demanded in quantities greater than unity and show a limited durability (which allows for replacement sales). In all of these examples, individual demand may show some responsiveness to price and, in consequence, expectations may play a role in determining the nature of equilibria.

REFERENCES

Bulow, J. (1982): "Durable Goods Monopolists". Journal of Political Economy 90:314-332.

Fershtman, C. and M. Kamien. (1987): "Dynamic Duopolistic Competition with Sticky Prices". Econometrica 55:1151-1164.

Jorgensen, S. (1985): "Optimal Dynamic Pricing in an Oligopolistic Market: a Survey". Dynamic Games and Applications in Economics. Edited by T. Basar. Springer-Verlag. Berlin. pp.:179-237.

Kamien, M. and N. Schwartz (1981): "Dynamic Optimization". Elsevier North Holland. New York.

Miller, R. (1979):"Dynamic Optimization and Economic Applications". Mc Graw-Hill, New York.

Roos, C. (1925): "A Mathematical Theory of Competition". Journal of American Mathematics 47:163-175.

Spence, M. (1981): "The Learning Curve and Competition". Bell Journal of Economics 12:49-70.

Starr, A. and Y. Ho (1969): "Nonzero-sum Differential Games". Journal of Optimization Theory and Application 3:184-208.

Stokey, N. (1979): "Intertemporal Price Discrimination". Quaterly Journal of Economics 94:355-371.

APPENDIX A

To characterize the solution of the consumer's problem, let's associate a current value multiplier $\beta(t)$ with the differential equation (1), and define the current value Hamiltonian:

$$H(X(t),x(t),\beta(t))= u(x(t))-p(t)x(t)+\beta(t)(-gX(t)+x(t))$$

Necessary conditions for an optimum are then:

$$dH(X,x,\beta)/dx=-p(t)+\beta(t)=0$$

and

$$r\beta(t)-\beta'(t)=u'(X(t))-g\beta(t)$$

whenever the constraint $s(t)\geq 0$ is not binding, and $s(t)=0$ otherwise. Totally differentiating the two conditions above and rearranging terms it follows that, for $x(t)>0$, $x(t)=gX(t)+[p''(t)-(r+g)p'(t)]/u''(X(t))$. (The reader is referred to Kamien and Schwartz (1981) for a detailed discussion of the necessary and sufficient optimality conditions for "singular optimal control problems"./// .

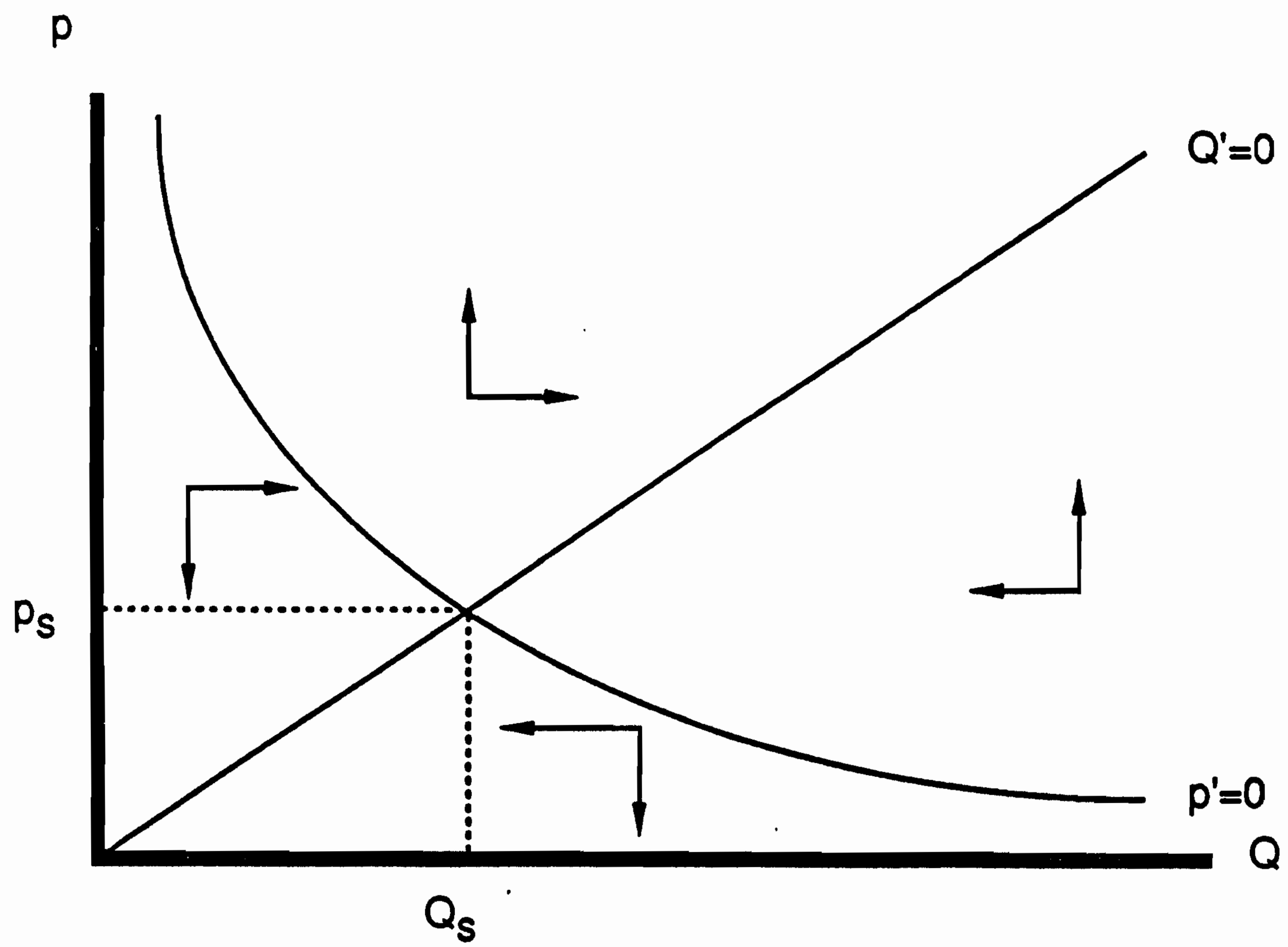

FIGURE 1: PHASE DIAGRAM OF PRICE AND STOCK

ENDNOTES

1. See Jorgensen (1985) for a survey of dynamic pricing models.

2. Durable goods are often studied in the context of an inelastic "zero-one" demand. Our model allows for consumption of an arbitrary amount of a divisible good which depretiates at a constant rate.

3. Bulow (1982) gives some plausible examples of preferences that depend on both, current and past consumption; he also notes the formal equivalence between the two alternative interpretations of our model.

4. In some cases, heterogeneous consumers can be incorporated into the model. An example is given in section 4.

5. This formulation can be slightly generalized. Suppose there is a second, non-durable, good z(t); let $p_z=1$ be the price of z and let M be income. Then, assumming that $U(X,z)=u(X)+z$ and $p(t)x(t)+z(t)<=M$, it follows that $U(X,z)=u(X)-p(t)x(t)+M$.

6. Ferhstman and Kamien (1987) consider preferences of the form $U(X,x)=u(X)x-px$ where $u(X)=A-BX$ and $X'(t)=-gX(t)+x(t)$; they analize the myopic demand $u(X)=p$. The corresponding dynamic demand is $p'(t)=(r+g)p(t)-w(X(t))$, where $w(X(t))=gX(t)u'(X(t))+(r+g)u(X(t))$ is a decreasing function of X. Hence, both models yield similar dynamics.