Working Paper 91-07
February 1991

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UNCERTAINTY AND TOBIN'S Q IN A MONOPOLISTIC COMPETITION FRAMEWORK

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Abstract

This paper combines the adjustment cost hypothesis of Tobin's q models with Malinvaud's proposition that demand uncertainty matters in explaining investment. Demand uncertainty allows for ex-post excess capacity and leads firms to look at the expected excess capacity in deciding about investment. Marginal q is shown to be smaller than average q, the difference being explained by the degree of capacity utilization (DUC).

Key Words
Tobin's q; Investment; Monopolistic Competition; Quantity Rationing Model

* Departamento de Economía, Universidad Carlos III de Madrid. I thank Ch. Arnsperger, D. de la Croix, Ph. De Villé, J. Drèze, J.P. Lambert, Pierre Malgrange and H. Sneessens for helpful conversations. They should not be held reponsible for any mistakes. Financial support of the Fonds de Développement Scientifique (FDS) of the Catholic University of Louvain is gratefully acknowledged.

Uncertainty and Tobin's q in a Monopolistic Competition Framework *

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February 1991

1 Introduction

In modern macroeconomic theory Tobin's q models have become the most frequent explanation for investment behavior. The original idea, proposed by Tobin (1969), says that investment depends on "average q" defined as the ratio of firm's value evaluated in the stock market to its replacement cost. In the tradition of Jorgenson's (1963) neoclassical model, the introduction of adjustment costs allows for investment functions obeying Tobin's principle. Nevertheless, the investment rate depends on "marginal q", i.e., the ratio of the marginal value of capital to the replacement cost, rather than on average q. Hayashi (1982) has given sufficient conditions, concerning technology, for the equality between average and marginal q. This has reopened the debate about the relevance of average q and the stock market valuation of the firm in investment decisions.

From a theoretical point of view, the main concern is the relation between average and marginal q. One way to examine this relation is to analyze the investment behavior of firms with adjustment costs in non-Walrasian economies; this has already received some analysis using fixed-price frameworks.[1] Precious (1987) shows that marginal q is smaller than

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[1]See Blanchard and Sachs (1982), Malgrange and Villa (1984), Michel (1986), and Precious (1987).

average q when the firm knows that it will face demand constraints in future periods, even if Hayashi's (1982) assumptions hold. The difference is explained by the marginal value of all future demand constraints. Empirical estimations have been carried out for this model by Chan-Lee and Torres (1987), using the degree of capacity utilization (henceforth DUC) as a proxy for the marginal value of all future demand constraints. They conclude that DUC plays a significant role in explaining investment. The role of DUC in empirical investment functions is also stressed by the European Unemployment Programme (EUP).[2]

Another way to understand Tobin's (1969) proposition was suggested by Malinvaud (1987) (1989), who shifts the emphasis from adjustment costs to demand uncertainty. In Malinvaud's view, the firm is essentially concerned with forecasting future demand when it must decide about capacities. The fact that demand is not known with certainty implies that capacities can be underutilized. This leads the firm to take into account expected capacity utilization when she must decide about investment. Empirical estimation of Malinvaud's q model was carried out by Lambert and Mulkay (1990).

Our main concern is to combine the adjustment cost assumption with Malinvaud's proposition that demand uncertainty matters, and to show that both approaches are complementary. This should allow for a more general result, where Malinvaud's solution holds if there are no adjustment costs on investment and where the neoclassical model verifies if capacities are fully employed. From an empirical point of view, we try to derive formally an investment equation depending on both average q and the degree of capacity utilization.

In this paper, the investment behavior of the firm is analyzed in the tradition of q investment models. Moreover, it is assumed that only demand is random and that all other state variables are known by the firm. As in rationing models, conditions are imposed so that the firm's production is equal to the minimum of supply and demand. To avoid the coexistence of fixed prices and perfectly competitive markets, firms are assumed to be in a monopolistically competitive economy and to be setting prices before knowing the stochastic demand. When the random term materializes the firm's price is already set and demand becomes an upper bound on production. Nominal price rigidities are thus imposed in the model, being justified by the existence of "menu costs" or information costs as in the New Keynesian setup.[3]

In addition to this nominal rigidity, firms are assumed to be facing technological rigidities: factors are complements -at least in the short run- and

[2]The EUP has coordinated the estimation of an "aggregation over micro-markets in disequilibrium" macromodel across 10 countries. The main conclusions are presented by Drèze and Bean (1990).

[3]See Blanchard and Fischer (1989), Chapter 8, for a survey of this literature.

the capital stock is given at each period -investment takes one period of "time to build" or alternatively is subject to a one-period "delivery lag." Potential output, or capacity, is defined as the capital stock times the technical coefficient for capital. Since in the short run the capital stock is given and factor substitutability is impossible, the firm cannot produce more than total capacities. Potential output, like demand, is an upper bound on production.

Since production is equal to the minimum of capacities and a stochastic demand, when deciding about prices and investment the firm must forecast future production. The stochastic demand shock is assumed multiplicative, implying that expected production is a linear homogeneous function of expected demand and capacities. In particular, if the random shock is lognormally distributed, expected production can be approximated by a CES function of expected demand and capacities, as in Lambert (1988). The only fundamental difference with respect to standard investment models comes from expected production. In the accelerator model production is always determined by demand and in the neoclassical model production is never demand-constrained. Our model specifies expected production as a function of expected demand and capacities.

With respect to the neoclassical model, demand uncertainty changes the marginal effect of capital on profits. An increase in capital raises expected production less than marginal productivity, since the new unit of capital has a positive probability of nonutilization. In the neoclassical model this probability is equal to zero since production is never demand-constrained. This property can also be expresed in terms of elasticities: in our model the elasticity of expected production to capacities is smaller than one.

Hayashi (1982) shows that in the neoclassical model, with the adjustment cost function being linear homogeneous in capital and investment, marginal q is equal to average q. Under demand uncertainty and factor complementarity, even if the adjustment cost function is linear homogeneous in capital and investment, marginal q is no longer equal to average q. Marginal q is in fact smaller than average q, the difference being explained by future profits weighted by the elasticity of expected production to expected demand. This result is close to the one obtained in fix-price investment models, where the difference between marginal and average q is explained by the marginal value of future demand constraints.

A simpler result is also obtained in Section 3. Since the firm is assumed not to produce at full capacities, the degree of capacity utilization appears as an intuitive component of the adjustment cost function. In particular this function is supposed separable and increasing in DUC, implying that the more capacities are employed the more costly it is for the firm to implement new equipment. Under this assumption, marginal q is still smaller than average q and the investment ratio can be approximated as a loglinear function of average q and DUC. This result turns out to be equivalent to Hayashi's when DUC is unity and equivalent to Malinvaud's when marginal q is equal to one.

In Section 2 the point of departure is the monopolistic competition setup, in the particular version of Sneessens (1987).[4] We show that marginal and average q diverge by a multiplicative factor which depends on expectations about the degree of capacity utilization. Section 3 develops a discrete dynamic model for investment. Under general assumptions about the adjustment cost function, average q is shown to diverge from marginal q as a consequence of expected excess capacities.

2 The Static Problem

We start with a static model similar to that of Sneessens (1987). As we will show later, the most important characteristics of the dynamic model are already contained in this static version.

2.1 Uncertainty and Irreversibility

With a Leontief production technology and two production factors capital and labor, capacity constraints are

$$YP = BK$$

where YP is potential output, B is the technical coefficient for capital and K is the capital stock. The firm is assumed to be unconstrained in the labor market at the given wage rate. Demand is specified in a monopolistic competition framework:

$$YD = \left(\frac{p}{P}\right)^{-\epsilon}\left(\frac{\overline{YD}}{n}\right)u.$$

The demand YD to the firm depends on its own price p, relative to the aggregate price level P, on aggregate demand $\overline{YD}$ and on a stochastic term u, which is assumed lognormal with unit mean and given variance. The number of firms, denoted by n, is assumed given and large enough.

The main ideas of Malinvaud, irreversibility and demand uncertainty, lie behind these assumptions. Malinvaud's definition of "irreversibility" implies that both capital and capital intensity are fixed before the realization of the random demand shock. In this paper irreversibility takes the particular form

[4] Malinvaud (1987) also studies the behavior of a monopolistic firm. Investment decisions in monopolistic competition are analyzed by Schiantarelli and Georgoutsos (1990) in the Tobin's q tradition.

of a Leontief production function, with the capital stock decided upon before the stochastic term is known.

The model imposes a specific sequence in the decision process. Investment and prices are decided before the stochastic term is known. But production and labor demand are determined after its realization. This implies, in addition to the Leontief production function hypothesis, that production will be the minimum of supply and demand. The assumption that the firm fixes its price before knowing the stochastic demand term is a sufficient condition for non-market-clearing.[5] In addition, if capacities are scarce, demand will be rationed. Moreover, the model assumes that consumers facing a demand constraint in a market do not modify their demands on other markets.[6]

2.2 Expected Production

Under these assumptions, production is equal to the minimum of demand and potential output. Since u is lognormally distributed with unit mean, the expected value of production can be written as[7]

$$E(Y) = [E(YD)^{\rho} + YP^{\rho}]^{-\frac{1}{\rho}},$$

where

$$E(YD) = \left(\frac{p}{P}\right)^{-\epsilon}\left(\frac{\overline{YD}}{n}\right).$$

In this expression ρ depends on the variance of the demand shock. The minimum condition is the limit case when $\rho \to \infty$. Figure 1 represents the relation between price and expected values when expected production is represented by a CES function.

Lambert (1988) puts forward the CES function as a representation of aggregate production when, at the micro-level, markets are in disequilibrium. In his book, the stochastic terms describe the distribution of the constraints across the micromarkets. Sneessens uses Lambert's result to analyze the behavior of a representative firm. While the CES is a particular expression for $E(Y)$ under the assumption of a lognormal distribution, its merit is to allow for explicit results.[8]

One important characteristic of the CES function is that the probability of being in a particular regime, i.e., that a particular constraint prevails, has

[5]As in Akerlof and Yellen (1985) Mankiw (1985) and Blanchard and Kiyotaki (1987), it can be justified by the existence of "menu costs".

[6]Benassy (1990) provides the appropriate set-up to build the objective demand function faced by monopolistic firms when such spillover effects are allowed.

[7]For a proof of the CES representation of the expected value of production, see Lambert (1988).

[8]For a discussion of the implications of lognormality, see Lambert (1988).

a simple expression. For example, the weighted probability that potential output will be the main constraint in the determination of production is

$$Pr_w(YP < YD) = \Phi_p = \left(\frac{E(Y)}{YP}\right)^{\rho}.$$

Moreover, the elasticity of expected production with respect to a given constraint is equal to the weighted probability of that constraint. In particular, the elasticity of expected production with respect to potential output is equal to Φ_p. Because Φ_p becomes too small when YP becomes very large, the positive effect of potential output on expected production is decreasing and always smaller than one.

2.3 The Optimal Problem and the Adjustment Cost Function

In an uncertain world, and under the previous assumptions, the monopolistic competitive firm's optimization problem can be written as

$$V = \underset{\alpha, p}{Max}\ E[pY - wL - P\Omega(\alpha)K^*]$$

$$subject\ to$$

$$Y = min\{YD, YP\}$$

$$YD = \left(\frac{p}{P}\right)^{-\epsilon}\left(\frac{\overline{YD}}{n}\right)u$$

$$YP = BK$$

$$K = K^*(1+\alpha),$$

where w is the wage rate, L is employment, K^* is the initial capital stock before investment is decided and α is the investment rate, that is, investment divided by the initial capital stock. The price of investment goods is assumed equal to the aggregate price level. Total investment costs are represented by $P\Omega(\alpha)K^*$. The $\Omega(.)$ function is increasing and convex, and verifies $\Omega(0) = 0$ and $\Omega'(0) = 1$.

Because it is assumed that labor demand is determined after the realization of the stochastic terms, optimally $L = A^{-1}Y$, where A is the Leontief coefficient for labor. Thus

$$pY - wL = (p - MLC)Y,$$

where $MLC = wA^{-1}$ is the marginal labor cost.

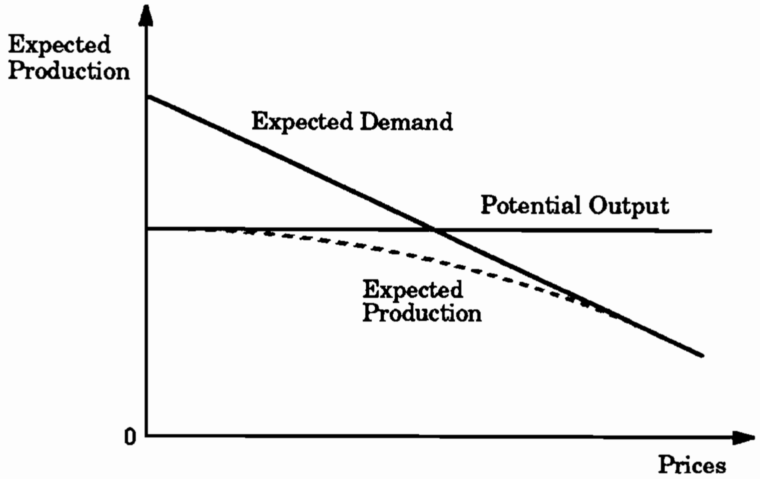

Figure 1: The CES representation of Expected Production

In addition, as long as P, w, A, B, $\overline{YD}$ and n are assumed to be known by the firm, we can write the optimization problem as

$$V = \underset{\alpha, p}{Max} \ \{(p - MLC)E(Y) - P\Omega(\alpha)K^*\} \tag{1}$$

$$subject\ to$$

$$E(Y) = \left[E(YD)^{\rho} + YP^{\rho}\right]^{-\frac{1}{\rho}}$$

$$E(YD) = \left(\frac{p}{P}\right)^{-\epsilon}\left(\frac{\overline{YD}}{n}\right)$$

$$YP = BK$$

$$K = K^*(1+\alpha).$$

2.4 Optimal Investment and Prices

The first-order conditions for this problem are

$$\frac{\partial V}{\partial p} = (p - MLC)\frac{\partial E(Y)}{\partial p} + E(Y) = 0, \tag{2}$$

$$\frac{\partial V}{\partial \alpha} = (p - MLC)\frac{\partial E(Y)}{\partial \alpha} - P\Omega'(\alpha)K^* = 0. \tag{3}$$

The derivatives of $E(Y)$ with respect to p and α are

$$\frac{\partial E(Y)}{\partial p} = \frac{\partial E(Y)}{\partial E(YD)}\frac{\partial E(YD)}{\partial p} = \Phi_k\frac{E(Y)}{E(YD)}\frac{\partial E(YD)}{\partial p} \tag{4}$$

$$where\, \Phi_k = \left(\frac{E(Y)}{E(YD)}\right)^\rho,$$

and

$$\frac{\partial E(Y)}{\partial \alpha} = \frac{\partial E(Y)}{\partial YP}\frac{\partial YP}{\partial \alpha} = \Phi_p \frac{E(Y)}{K} K^* \tag{5}$$

$$where\, \Phi_p = \left(\frac{E(Y)}{YP}\right)^\rho.$$

The ratio of $E(Y)$ to YP is the expected degree of capacity utilization $E(DUC)$. We must always remember that the elasticities Φ_p and Φ_k represent expected values.

Condition (2), together with the derivative (4), implies the well-known condition for a monopoly:[9]

$$p = \left(1 - \eta^{-1}\right)^{-1} MLC \tag{6}$$

$$where\, \eta = \epsilon \Phi_k.$$

As shown by Sneessens, the price elasticity η of expected production depends on the price elasticity ϵ multiplied by the weighted probability Φ_k of excess demand.

If we take (3) and (5), the first-order condition for investment becomes

$$\Omega'(\alpha) = \Phi_p \frac{(p - MLC)E(Y)}{PK}. \tag{7}$$

Let us assume that each unit of real capital is financed by one share and define average q, denoted by Q, as

$$Q = \frac{(p - MLC)E(Y)}{PK}. \tag{8}$$

Then the marginal condition (7) becomes

$$\Omega'(\alpha) = Q\Phi_p. \tag{9}$$

The main characteristic of this solution is that average q diverges from marginal q by a multiplicative factor: the weighted probability Φ_p of a capacity constraint. This result can be interpreted as follows. If capacities are larger than demand, the marginal value of capital is zero. But if there is excess demand, as long as the production function is linearly homogeneous, the marginal value of capital is equal to the average value. When there is

[9]This result is in the tradition of Dixit and Stiglitz (1977). The main difference is that the price elasticity of production depends on the probability Φ_k. The firms are supposed identical up to product differentiation, so that individual prices are equal to the aggregate price level for all firms.

demand uncertainty, the marginal value of investment is equal to the average value weighted by the probability Φ_p of a capacity constraint.

Implicit in the price equation (6) and in the investment equation (9) is the steady state value for expected *DUC*, which is the solution of

$$\frac{E\,(DUC)^{\rho+1}}{1 - E\,(DUC)^{\rho+1}} = \frac{\epsilon}{B}. \tag{10}$$

It can be easily verified that there is a unique positive solution for expected *DUC*, which is strictly smaller than one for finite values of ϵ and ρ. It suggests that the firm optimally expects to partially employ its capacities. Nevertheless, when ϵ goes to infinity prices tend to the perfect competitive ones and capacities become fully employed.

2.5 Example

It is interesting to look at the particular case where

$$\Omega'\,(\alpha) = exp\,\{a + b\alpha\}\,,$$

which satisfies the conditions imposed on $\Omega\,(\alpha)$ when $b > 0$. The optimality condition for investment (9) becomes

$$\alpha = a_0 + a_1 log\,(Q) + a_2 log\,(E\,\{DUC\})\,. \tag{11}$$

Figure 2 shows the relation between the investment rate α and average q, for values of expected *DUC* between 1 and .70, with $a = 0$. The larger the excess capacity, the lower the level of investment associated with a given value of Q. In particular, since expected *DUC* is smaller than 1, a unit Q is no longer associated with zero investment, but rather with negative investment. In other words, zero investment requires a Q above unity if expected *DUC* is smaller than one.[10]

The specification of the investment function in equation (11) is very similar to that estimated by Chan-Lee and Torres (1987). Looking at the disequilibrium investment models proposed by Malgrange and Villa (1984) and Michel (1986), these authors specify investment as a function of average q adjusted for the degree of capacity utilization, relying on the theoretical argument that the degree of capacity utilization is correlated with the demand constraint. In our framework, *DUC* appears not as an instrumental variable for the marginal value of demand constraints, but rather as an approximation

[10] Equation (11) is drawn without taking into account that changes in α modify expected *DUC* and Q, for given values of the exogenous variables. The solution for α and p must be obtained from equation (6) and (7) using the definitions of Φ_p, Φ_k and $E(Y)$.

of the weighted probability of a capacity constraint. In this sense this paper can be seen as providing a theoretical foundation for this type of empirical approach to investment.

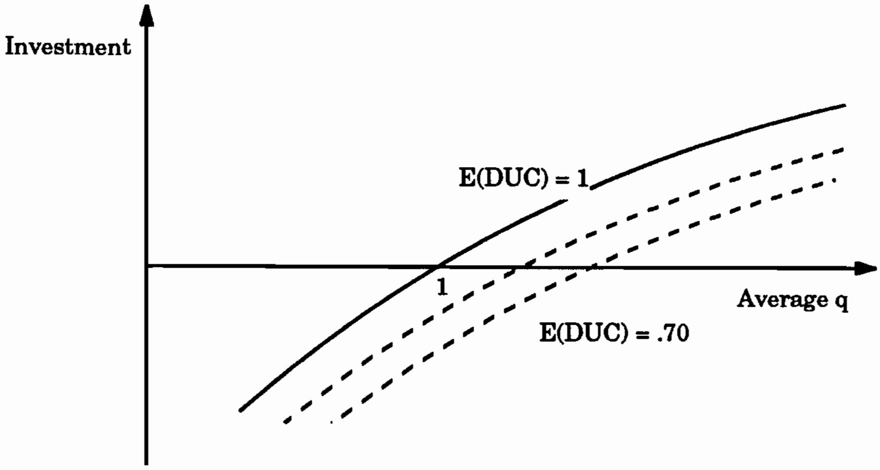

Figure 2: Investment and Tobin's q

3 The Dynamic Model

The static version of the model presented in the previous section enables us to link investment to both average q and the degree of capacity utilization in a very simple way. We shall now analyze a dynamic problem and look for conditions that reproduce the static result.

Let us start by taking a more general functional form for the investment cost function. Since the firm is assumed not necessarily to produce at full capacity, the degree of capacity utilization appears as an intuitive component of the adjustment cost function; total investment costs are thus defined as $\Psi(I, K, DUC)$. For simplicity, we assume that the function $\Psi(.)$ is separable in DUC and linearly homogeneous in capital and investment, i.e., it can be written as

$$\Psi(I, K, DUC) = \Omega\left(\frac{I}{K}\right) K\, \Gamma(DUC), \tag{12}$$

where the $\Gamma(.)$ and $\Omega(.)$ are increasing and convex functions. This implies that, for given values of I and K, investment costs rise with capacity utilization. It is as if the nearer the firm is to the full utilization rate, the more difficulties it faces in devoting resources to new projects. Conversely, underemployed resources can be devoted to install new equipment, lowering the installation costs.

It is instructive to distinguish inside Ψ between adjustment costs and purchasing costs. We thus decompose total investment costs into

$$p^I\Psi(I,K,DUC) = p^I I + p^I \left\{\Omega(\alpha)\Gamma(DUC) - \alpha\right\} K, \qquad (13)$$

where p^I represents the price of investment goods and the second term on the right-hand side represents adjustment costs. According with the normalization assumptions proposed in the previous section for the $\Omega(.)$ function, $\Gamma(DUC)$ must be at least equal to one to ensure that adjustment costs are everywhere non-negative.

3.1 Optimization Problem and Optimality Conditions

The stochastic demand term is assumed i.i.d. and lognormally distributed. All other exogenous variables are assumed known by the firm. Under the previous assumptions the optimization problem for the firm can be written as[11]

$$V_t = \underset{\{\alpha_s, p_s\}}{Max} \sum_{s=t}^{\infty} \left[(p_s - MLC_s)E_t(Y_s) - p_s^I \Omega\left(\frac{I_s}{K_{s-1}}\right) K_{s-1} E_t(\Gamma(DUC_s)) \right] (1+r)^{-(s-t)} \qquad (14)$$

subject to

$$E_t(Y_s) = \left[E_t(YD_s)^\rho + (B_s K_{s-1})^\rho\right]^{-\frac{1}{\rho}}, \qquad (15)$$

$$E_t(YD_s) = \left(\frac{p_s}{P_s}\right)^{-\epsilon} \overline{YD}_s, \qquad (16)$$

$$K_s = I_s + (1-\delta) K_{s-1}, \qquad (17)$$

with given initial value K_{t-1}. V_t is the firm's value. The discount rate r, the demand elasticity ϵ the depreciation rate δ and the parameter ρ are assumed constant. The paths of the exogenous variables P, MLC, p^I and B are taken as known by the firm. Time-t investment and prices are decided at period t-1, so that E_t are expectations taken with respect to the information set at t-1 (note that this information set does not contain the stochastic demand shock u_t).

[11]The non-autocorrelation assumption on the stochastic terms allows for a non-stochastic path in the control variable, which implies that profits are a linear function of two random variables, production and the Γ function.

The Lagrangean for this problem is

$$\mathcal{L}_t = \sum_{s=t}^{\infty} \left[(p_s - MLC_s) E_t(Y_s) - p_s^I \Omega\left(\frac{I_s}{K_{s-1}}\right) K_{s-1} E_t(\Gamma(DUC_s)) \right] (1+\mathrm{r})^{-(s-t)}$$

$$-\sum_{s=t}^{\infty} \mu_s \left(K_s - I_s - (1-\delta)K_{s-1}\right)(1+r)^{-(s-t)},$$

where μ represents the marginal value of capital. The first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial p_t} = E_t(Y_t) + (p_t - MLC_t)\frac{\partial E_t(Y_t)}{\partial p_t} - p_t^I \Omega_t K_{t-1} \frac{\partial E_t(\Gamma_t)}{\partial p_t} \quad (18)$$

$$\frac{\partial \mathcal{L}_t}{\partial I_t} = -p_t^I \Omega_t' E_t(\Gamma_t) + \mu_t = 0, \quad (19)$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = 0. \quad (20)$$

The optimality condition for prices can be reduced to

$$p_t = \left(1 - (\epsilon \Phi_{k,t})^{-1}\right)^{-1} \left(MLC_t + p_t^I \frac{\xi_t E_t(\Psi_t)}{E_t(Y_t)}\right), \quad (21)$$

where ξ_t is the elasticity of $E_t(\Gamma_t)$ with respect to $E_t(DUC_t)$. In the particular case of $\xi = 1$, the price condition becomes

$$p_t = \left(1 - (\epsilon \Phi_{k,t})^{-1}\right)^{-1} \left(MLC_t + \frac{p_t^I \Omega_t K_t}{YP_t}\right). \quad (22)$$

As is standard in monopoly theory, the firm fixes a markup rate over marginal costs. As in the static model, the demand elasticity ϵ is multiplied by the probability Φ_k of a demand constraint. This product represents the price elasticity of expected production. However, the firm now also considers investment spending (in addition to labor costs) in the determination of total marginal cost. Because the investment cost function depends negatively on demand, the firm knows that all modifications in its price must have an effect on profits through the costs of investment. If the price rises, expectations about demand, production and capacity utilization are revised downwards. The reduction in expected *DUC* decreases the installation costs. Thus firms are induced to set a higher price than in the standard monopolistic case, where the $\Psi(.)$ function is independent of *DUC* and the markup is applied only to marginal labor costs.

Condition (19) allows for a first characterization of the optimality condition for investment:

$$\Omega'(\alpha_t) E_t(\Gamma_t) = \frac{\mu_t}{p_t^I} = q_t. \quad (23)$$

Just as in the neoclassical investment model, optimal investment is a function of marginal q, which is represented by the ratio of the marginal value of capital to the replacement cost. The identity of this result with Hayashi's depends on the specification of the investment cost function; if $\Gamma(DUC)$ is equal to one, equation (23) is equivalent to Hayashi's result.

The Euler condition for capital can be derived from equations (19) and (20). Solving it recursively forward we obtain

$$p_t^I K_t \Omega_t' E_t(\Gamma_t) = \sum_{s=t+1}^{\infty} (p_s - MLC_s) E_t(Y_s) \Phi_{p,s} (1+r)^{-(s-t)} - \sum_{s=t+1}^{\infty} p_s^I E_t(\Psi_s)(1 - \Phi_{k,s}\xi_s)(1+r)^{-(s-t)}. \tag{24}$$

This intermediate result will be necessary below to compare marginal q and average q. In equation (24), on the left-hand side, the marginal cost of investment is multiplied by the capital stock. It differs from the standard condition because adjustment costs depend on expected DUC, which is not necessarily equal to one. On the right-hand side, we have the marginal value of total capital. In the particular case where the firm expects to fully use its capacities, equation (24) reduces to the standard condition. But it is an extreme case in which capacities for all future periods are so small or expected demand is so large that the probability of excess capacity is zero.

3.2 Average Q and Marginal Q

Let us define average q as

$$Q_t = \frac{E_t(V_{t+1})}{p_t^I K_t} (1+r)^{-1}. \tag{25}$$

Dividing both sides of equation (24) by $p_t^I K_t$, using the optimality condition (23) and the definition of average q, we can deduce a general relation between marginal and average q

$$q_t = Q_t \hat{\Phi}_{p,t} - \frac{1}{p_t^I K_t} \sum_{s=t+1}^{\infty} p_s^I E_t(\Psi_s) \Phi_{k,s} (1 - \xi_s)(1+r)^{-(s-t)} \tag{26}$$

where

$$\hat{\Phi}_{p,t} = \sum_{s=t+1}^{\infty} \Phi_{p,s} \frac{\pi_s (1+r)^{-(s-t-1)}}{E_t(V_{t+1})}. \tag{27}$$

π_s represents time-s profits, as in equation (14). Note that in each period s, the elasticity Φ_p is weighted by time-s profits divided by the expected value of the firm. This weight has the property of a probability measure, since the value of the firm is the sum of all future profits. Thus $\hat{\Phi}_p$ is a mean value of all future elasticities of expected production to capacities and it will be called the "mean potential output elasticity."

Equation (26) represents the relation between marginal and average q without any particular assumption about the $\Gamma(.)$ function. In this general case the solution of the simple dynamic problem is augmented by a new term which depends on investment costs weighted by $\Phi_k(1-\xi)$. This will become clearer in the next section.

3.3 Hayashi's Investment Cost Function Hypothesis

Hayashi (1982) shows that, together with the linear homogeneity condition for the production function, the linear homogeneity of the investment cost function with respect to investment and capital is a sufficient condition for the equality between marginal q and average q. In our model this proposition no longer holds. Following Hayashi, assume that $\Gamma(DUC) = 1, \forall\, DUC$. In this case the ξ elasticity will be zero and it can be easily verified that

$$q_t = Q_t - \frac{1}{p_t^I K_t} \sum_{s=t+1}^{\infty} \Phi_{k,s}(p_s - MLC_s) E_t(Y_s)\,(1+r)^{-(s-t)}. \qquad (28)$$

Since new equipments can be partially utilized, marginal q is smaller than average q. The difference is represented by the second term on the right-hand side, where the value of expected production is weighted by the probability Φ_k of an excess demand. In a certainty model, when demand is known by the firm, the difference between marginal and average q depends on the marginal value of the future demand constraints. As shown by Precious (1987), under certainty we would have

$$q_t = Q_t - \frac{1}{p_t^I K_t} \sum_{s=t+1}^{\infty} \lambda_s E_t(Y_s)(1+r)^{-(s-t)},$$

where λ is the Lagrangean multiplier associated with the demand constraint. The marginal value λ of the certain demand constraint in this equation is equivalent to the marginal value the expected demand constraint $\Phi_k(p - MLC)$ obtained in (28).

Finally, assume that $\Gamma(.)$ is linear, or it can be linearized in such a way that ξ is equal to unity, so that the second term in the right-hand side of equation (26) vanishes. This implies a simple relation between marginal and average q

$$q_t = Q_t \hat{\Phi}_{p,t}. \qquad (29)$$

Under these conditions the solution for the dynamic model is very similar to the solution for the static model: average q is multiplied by a mean value of future potential output elasticities. When expectations about Φ_p are identical for all future periods, the relation between marginal and average q is the same as in the myopic static problem.

4 Conclusions

The important role of capital constraints in macroeconomic fluctuations was explained by disequilibrium models in the seventies. In this paper, we point to the role of capacity constraints in explaining investment in an economy with monopolistic competition. In such a context, capacity constraints do not depend on an assumption of exogenously fixed prices since firms themselves set prices in an optimal way. The static model developed in Section 2 enables us to produce a very simple investment function, in which marginal q diverges from average q by the weighted probability of a capacity constraint (or the elasticity of expected production to capacities). The crucial assumptions for this result are irreversibility in the sense of Malinvaud (1987) and demand uncertainty. The irreversibility assumption implies that capacities are decided before the realization of the stochastic demand. When technology is defined in such a way, excess capacity is possible and firms must look at the expected excess capacity when deciding about investment. When the stochastic demand shock is given a lognormal specification, as in Sneessens (1987), the weighted probability of a capacity constraint is a power function of the expected degree of capacity utilization (DUC). The merit of lognormality is to allow for explicit results, i.e., the rate of investment depends on average q and expected DUC.

In Section 3 a dynamic investment problem is solved. It is shown that marginal and average q diverge by a mean value of all future probabilities of a capacity constraint. To obtain this result, a more general condition is imposed: adjustment costs are assumed to depend positively on DUC in addition to capital and investment. This assumption does not comply with Hayashi's (1982) linear homogeneity hypothesis. Moreover, it presumes that the closer the firm is to its full utilization rate, the larger are the costs associated with investment, reflecting the fact that the firm is less and less able to devote resources to new projects. Finally, because prices affect expected DUC, the firm charges a markup rate over marginal investment costs in addition to marginal labor costs.

Appendix

Full Derivation Equation (24)

The first-order condition (20) can be written as

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = (p_{t+1} - MLC_{t+1})\frac{\partial E_t(Y_{t+1})}{\partial K_t} - p^I_{t+1}\left[\Omega_{t+1} - \Omega'_{t+1}\frac{I_{t+1}}{K_t}\right]E_t(\Gamma_{t+1})$$
$$-p^I_{t+1}\Omega_{t+1}K_t\frac{\partial E_t(\Gamma_{t+1})}{\partial K_t} - \mu_t(1+r) + (1-\delta)\mu_{t+1} = 0.$$

The partial derivative of $E(\Gamma)$ with respect to K is

$$\frac{\partial E_t(\Gamma_{t+1})}{\partial K_t} = \xi_{t+1}\frac{E_t(\Gamma_{t+1})}{E_t(DUC_{t+1})}\frac{\partial E_t(DUC_{t+1})}{\partial K_t}$$

and

$$\frac{\partial E_t(DUC_{t+1})}{\partial K_t} = \frac{\partial E_t(DUC_{t+1})}{\partial YP_{t+1}}B_t = -\Phi_{k,t+1}\frac{E_t(DUC_{t+1})}{K_t},$$

where ξ is the elasticity of $E(\Gamma)$ with respect to $E(DUC)$ and Φ_k is the elasticity of $E(Y)$ with respect to $E(YD)$, which verifies $\Phi_k + \Phi_p = 1$. Using that Φ_p is the elasticity of $E(Y)$ with respect to YP, substituting μ by its expression in equation (19) and multiplying both sides by K_t, the optimality condition for capital becomes

$$p^I_t K_t \Omega'_t E_t(\Gamma_t) = (p_{t+1} - MLC_{t+1})E_t(Y_{t+1})\Phi_{p,t+1}(1+r)^{-1}$$
$$p^I_{t+1}E_t(\Psi_{t+1})(1 - \Phi_{k,t+1}\xi_{t+1})(1+r)^{-1}$$
$$+p^I_{t+1}K_{t+1}\Omega'_{t+1}E_t(\Gamma_{t+1})(1+r)^{-1}.$$

Solving this equation recursively forward we obtain equation (24)

$$p^I_t K_t \Omega'_t E_t(\Gamma_t) = \sum_{s=t+1}^{\infty}(p_s - MLC_s)E_t(Y_s)\Phi_{p,s}(1+r)^{-(s-t)} \quad (24)$$
$$-\sum_{s=t+1}^{\infty} p^I_s E_t(\Psi_s)(1 - \Phi_{k,s}\xi_s)(1+r)^{-(s-t)}.QED$$

References

AKERLOF George and Janet YELLEN (1985), "A Near-Rational Model of the Business Cycle with Wage and Price Inertia." *Quarterly Journal of Economics*, supplement, **100**, 823-838.

BENASSY Jean-Pascal (1990), "Monopolistic Competition." In Hildenbrand and Sonnenschein, ed, *Handbook of Mathematical Economics*, Volume 4, (forthcoming).

BLANCHARD Olivier and Stanley FISCHER (1989), *Lectures on Macroeconomics.* The MIT Press.

BLANCHARD Olivier and Nobu KIYOTAKI (1987), "Monopolistic Competition and the Effects of Aggregate Demand". *American Economic Review*, **77**, 4, 647-666.

BLANCHARD Olivier and Jeffrey SACHS (1982), "Anticipations, Recessions, and Policy: An Intertemporal Disequilibrium Model." National Bureau of Economic Research, W.P. 971.

CHAN-LEE James and Raymond TORRES (1987), "q deTobin et taux d'accumulation en France." *Annales d'Economie et de Statistique*, **5**, 37-48.

DIXIT Avinash and Joseph STIGLITZ (1977). "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, **67**, 3, 297-308.

DREZE Jacques and Charles BEAN (1990), "European Unemployment Lessons from a Multicountry Econometric Study." *Scandinavian Journal of Economics*, **92**, 136-165.

HAYASHI Fumio (1982), "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica*, **50**, 213-224.

JORGENSON Dale (1963), "Capital Theory and Investment Behavior." *American Economic Review*, **53**, 247-256.

LAMBERT Jean-Paul (1988), *Disequilibrium Macroeconomic Models: Theory and Estimation of Rationing Models Using Business Survey Data.* Cambridge University Press.

LAMBERT Jean-Paul and Benoît MULKAY (1990), "Investment in a Disequilibrium Context or Does Profitability Really Matter?" In Gabszewicz and al, ed, *Economic Decision-Making: Games, Econometrics and Optimization*, North Holland.

MALGRANGE Pierre and Pierre VILLA (1984), "Comportement d'investissement avec coûts d'ajustement et contraintes quantitatives." *Annales de l'INSEE*, **53**, 31-70.

MALINVAUD Edmond (1987), "Capital Productif, Incertitudes et Profitabilité." *Annales d'Economie et de Statistique*, **5**, 1-36.

MALINVAUD Edmond (1989), "Profitability and Factor Demands under Uncertainty." *De Economist*, **137**, 2-15.

MANKIW Gregory (1985), "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *Quarterly Journal of Economics*, **100**, 529-539.

MICHEL Philippe (1986), "Dynamique de l'accumulation de capital en présence de contraintes de débouchés." *Annales d'Economie et de Statistique*, **2**, 117-145.

PRECIOUS Mark (1987), *Rational Expectations, Non-Market Clearing and Investment Theory.* Claredon Press, Oxford.

SCHIANTARELLI Fabio and D. GEORGOUTSOS (1990), "Monopolistic Competition an the Q Theory of Investment." *European Economic Review*, **34**, 1061-1078.

SNEESSENS Henri (1987), "Investment and the Inflation-Unemployment Trade off in a Macroeconomic Rationing Model with Monopolistic Competition." *European Economic Review*, **31**, 781-815.

TOBIN James (1969), "A General Equilibrium Approach to Monetary Theory." *Journal of Money, Credit and Banking*, **1**, 15-29.