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Three essays on economic dynamics

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THREE ESSAYS ON ECONOMIC DYNAMICS

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Abstract

This Ph.D. dissertation is composed of three chapters, in which several dynamical aspects about several economic problems are analyzed.

In the first chapter, entitled “Dynamical Stability in Repeated Games”, we propose a definition of dynamic stability that can be applied to repeated games. For this purpose, we extend the traditional theory by extending the infinite repetition of the stage game not only towards the future, but also towards the past. This allows the existence of stationary strategies and hence the possibility of dynamic stability.

In the second chapter, entitled “On the Role of Educational Subsidies” we develop an overlapping generations model, and we analyze its properties concerning the steady state and the transitional dynamics. We also calibrate the model with data on several European countries, and we find that the optimal distribution of public expenses between research and development and educational subsidies should be changed towards the former in a majority of countries.

Finally, in the chapter entitled “Inflationary Effects of a Monetary Union” we develop a model that formalizes the Balassa-Samuelson effect, and we analyze its existence both in economies with independent monetary policy and in economies belonging to a monetary union. We find that this effect should only appear in countries inside a monetary union. For countries with independent monetary policy, the sign of the effect is the opposite. We also analyze the effects of an enlargement concerning fast growing countries. We find that both the average inflation rate of the Union and the inflation rate of the old members would be reduced, whereas there are two opposite effects for the new countries: the change towards a less relaxed monetary policy and a money attraction effect. We find that for certain countries of a hypothetical enlargement of the Euro zone, the former effect dominates, so the inflation rate of the new members would also be reduced.

Dynamic Stability in Repeated Games[1]

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Abstract

A concept of dynamic stability in infinitely repeated games with discounting is presented. For this purpose, one modification of the available theory is needed: we need to relax the assumption that the game starts in a given period. Under this new framework, we propose stable strategies such that a folk theorem with an additional stability requirement still holds. Under these strategies, convergence to the long run outcome is achieved in a finite number of periods, no matter what actions or deviations have been played in the past. Hence, we suggest a way in which a player can build up his reputation after a deviation.
JEL classification codes: C70, C72

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1 Introduction

The dynamic features of repeated games have been analyzed in a number of studies. If the same players play the same game repeatedly, then they can choose their actions as a function of the history of what have been played. This implies that dynamic programming tools can be used in this context. This approach has been revealed as a very useful tool. One example of its applications is the work by Abreu, Pearce and Stachetti (1990).

A dynamic consequence of repeated games is that there can be equilibria that do not imply to play a static Nash equilibrium (NE from now on) each period. Players may choose in one period strategies which are different from the static best response in order to induce, through the history, some kind of play in the future. The natural question arises: What kind of actions (different from the static NE) can be supported in a repeated game? The 'folk' theorems provide an answer for subgame perfect equilibria (SPE).[1] Apart from some technical conditions, all action profiles with an individually rational static payoff can be supported as a SPE of an infinitely repeated game.

This result sometimes relies on the so-called trigger strategies. However, the use of this type of strategies brings additional problems. For example, if for whatever reason the history is slightly perturbed, the continuation path induced by trigger strategies changes dramatically, switching from a path of cooperation to a path of (usually) Nash reversion. Obviously, this cannot satisfy any concept of dynamic stability, as we will see later. This lack of dynamic stability is also present in other, more general, punishment schemes used in the literature. Optimal punishment schemes, like those described in Abreu (1998), are an example.

Against this background, we propose a concept of dynamic stability for repeated games. The first observation is that we need to modify the usual theory in the following way: We need a game with no beginning. In other words, it is often assumed that a repeated game has a first period in which the game has started. As a consequence, the history of the game is always of a finite and increasing dimension. It will be seen that this fact brings some problems for the question of dynamic stability. Hence, we propose a slightly different framework: the game has no beginning, i.e. history is always of the same (infinite) dimension.

This modification allows us to define stationary strategies, i.e. strategies that can be represented by only one function relating the past history with

[1] See Fudenberg and Tirole(1991), section 5.1.2.

the current action. Then, we will be able to define our concept of dynamic stability in repeated games.

Another step in the analysis is to see if the requirement of dynamic stability modifies in a substantial way the set of possible outcomes in a repeated game. Our approach here will be to show that the folk theorem also holds when we require dynamic stability. The proof of the theorem is constructive, since it yields the class of (stationary) strategies that can support a certain outcome. These strategies have the property that, after a deviation, the profile played each period converges again to the long run profile in a finite number of periods.

The following additional remark concerning the necessity of games with no beginning may be useful. As already pointed out, the strategies at any period are functions of the history, under a dynamic programming approach. One can interpret this history as the object that determines the reputation of both players, because it summarizes all cooperations and defections conducted by the players. But, if the game starts in a given period, nothing can be said about the initial reputation by looking at the history, since in the first period the history is the empty set. Indeed, it is implicitly assumed that play begins with full reputation, because strategies often recommend players to cooperate in the first period.[2] The available theory is silent about how this initial reputation is determined. Note that it is indiferent to make an implicit assumption, or to say that we assume explicitly (and exogenously) a certain degree of reputation at the beginning of the game: we are in both cases analyzing the convergence for only one initial reputation. By contrast, we make in this paper the complete exercise: we take all possible histories in a given period for a game that has no starting period, and we check if the strategies induce convergence for all these possible histories. In this way, we are studying convergence for all initial reputations, whereas the traditional analysis focuses only on one initial reputation.

Of course, this new approach is appropriate to study games with no beginning. However, it should be clear that our motivation is not (only) to study this class of games. In a game starting today, players need to fix some idea about the likelihood that their opponents will cooperate or not. Clearly, we might do this by assuming some prior distribution over a set of possible types of the opponent. The other possibility is to do what we're doing here. Take all possible histories of the game as if it would have no beginning. If the equilibrium converges to certain profile for all of them, then it is clear that this profile will be the long run outcome. This is precisely our definition of

[2] More precisely, the strategies that support a certain outcome usually start by choosing in the first period the profile to be supported.

dynamic stability. With this approach, we avoid the use of stochastic tools. Moreover, we obtain the additional result that following any deviation (or mistake), the equilibrium will converge to the long run outcome again, and in a finite number of periods. In this sense, our approach not only provides a more precise[3] answer to the question: "why are we cooperating?", but also answers the question: "if we are not in a cooperative situation (maybe because someone has deviated in the past), how can we reach it again?".

Finally, note that the main contributions of this paper are the proposed definition of stability, the proposed stable strategies, the management of initial conditions in a repeated game, and how players can build up again their reputation after a deviation. The folk theorem is presented only as a complementary result. Of course, the proof is very similar to standard proofs of folk theorems in repeated games; it is just a matter of extending it to an environment without starting period and with reversion to the cooperative outcome.

The rest of the paper is organized as follows. Section 2 presents a review of the existing theory of repeated games, and proposes the main definitions in which the rest of the work is based, including the definition of a stable SPE. Section 3 presents a theorem useful to characterize outcomes supportable as a stable SPE. Section 4 analizes the memory requirements of the proposed stable strategies and relate them to the theory on bounded recall. Finally, concluding thoughts and possible extensions are presented in section 5.

1.1 Related Literature

Abreu (1988) argues that we can have a penalty greater than Nash reversion (the penalty usually assumed in trigger strategies). In this paper we also use a penalty greater than Nash reversion. In fact, we use a penalty worse than (or equal to) the minmax, but only for a finite number of periods. On the other hand, the penalty in Abreu (1988) is for an infinite number of periods (if there are no deviations in the punishment phase), and it is not worse than the minmax.

There are some recent studies concerned with the size of the penalty. Evans and Thomas (2001) use the concept of perturbed game to study repeated games and cooperation. They criticize the previous work by Aumann and Sorin (1989) and Anderlini and Sabourian (1995), showing that the result "perturbation implies efficiency" can be achieved only if there are 'draconian' penalties in the support of the perturbation, i.e. penalties designed to min-

[3] More precise in the sense that, under our framework, cooperation does not depend on initial conditions in the long term.

max a player almost all the time. We have the same concern. Even Nash reversion is a very big penalty to be fully credible. By contrast, our stability requirement ensures that penalty is at least limited in time.

The strategy presented below is related to at least two other strategies proposed in the previous literature. Green and Porter (1984) present a strategy that has a finite number of punishment periods. However, since their model is of imperfect information, the punishment phase is defined as a function of one observable variable (the price) that imperfectly reflects the actions of other players. Therefore, even when players do not deviate, the observable variable sometimes induces the punishment phase. This occurs when it falls below a trigger level, generating fluctuations in the behavior of players. Hence, this environment is not adequate to study stability issues; the motivation is totally different. A similar strategy can be found in Piccione (2002).

Fudenberg and Tirole (1991) use in their theorem 5.4 a strategy with three phases. One is the cooperative phase. If someone deviates, the game goes to a finite punishment phase. However, at the end of this second phase the play doesn't return to the cooperative phase. Instead, it goes to a third phase with payoffs between the other two. Therefore, the strategies do not induce a convergent equilibrium path. Moreover, it is imposed exogenously that play begins in the cooperative phase, which amounts to assume that players begin the game with reputation.

Finally, Kalai and Stanford (1988) and related papers study the possible implementation of strategies by finite automata. At first glance, one can interpret this approach as a concern about stationarity of strategies. However, this is only one requirement of our definition of stability (the other is convergence). Moreover, we think that their automata do not cover stationarity issues properly, because they assume that automata start the game in some initial "state of mind" and, as already discussed, this initial assumption is not innocuous. The same can be said about Kalai, Samet and Stanford (1988), with the addition that they show that reactive equilibria can exist only by chance, i.e. under a particular combination of parameters of measure zero.

2 Preliminaries

In this section we start with some basic concepts to be used. Then, we modify the concept of a repeated game with an initial period or node to define a repeated game without initial period. Finally, we define the concept of stable subgame perfect equilibrium.

2.1 Stage Game

Let's define the following symmetric game, which will be called the stage game. There are two players. Both of them must play an action simultaneously from the same set of possible actions. This set can be discrete or continuous, finite or infinite. Let's call this set S. The particular choices of the two players will be denoted by $s_1, s_2 \in S$. Let's define the instantaneous payoff function for player i, u_i as follows:

$$u_i : S \times S \to \mathbb{R}$$

Where the first argument in both payoff functions is the action of player 1 and the second the action of player 2. Each player is trying to maximize his own payoff.

Assumption A1: The functions u_i are bounded, and the (static) best response correspondences $BR_i(s_{-i}) = \arg\max_{s_i} u_i(s_1, s_2)$ are non-empty valued.

2.2 Repeated Game

We construct a new game by the infinite iteration of the stage game defined before, in which past actions are observable. Now the objective function is the sum of the instantaneous payoff functions, discounted by the parameter δ, which is the same for both players. It is usual in the literature to assume that the repeated game starts in a given period and continues up to infinity. Later we modify this concept using a repeated game that goes from minus infinity up to infinity. To facilitate comparison between the two concepts we present here some definitions about a repeated game with starting node.

Let's call period 0 the first stage game, period 1 the second stage game, and so on.

It is useful to define the object 'history' in period t as the actions played by both players in previous periods, and period t itself: $h^t = \{h_k^t\}_{k=0}^t ; h_k^t = \left\{h_k^{1,t}, h_k^{2,t}\right\}$. Denote the set of all possible histories at t by H^t.

Now we can define a strategy for player i in this repeated game as follows:

Definition 1 *A strategy σ_i for player i in the repeated game with starting node is a sequence of functions, one for each $t \in \{1, 2, \ldots\}$ of the form $s_i^t : H^{t-1} \to S$, and an action $s_i^0 \in S$ for period 0.*

Note that the set $H^{t-1} = (S \times S)^t$ varies with t.

In each subgame h^{t-1}, the strategies σ_1 and σ_2 determine a sequence of pairs of actions, or continuation path, composed of the actions that players would play if they followed the strategies σ_1 and σ_2 after history h^{t-1}. Let $P\left(h^{t-1},\sigma_1,\sigma_2\right)=\left\{P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right\}_{k=0}^{\infty}$ be the combination of a given history and its associated continuation path given σ_1 and σ_2. It can be defined in the following recursive manner:

$$P_k\left(h^{t-1},\sigma_1,\sigma_2\right)=\begin{cases} h_k^{t-1} & \text{if } k\leq t-1 \\ \left\{s_1^k(\{P_m\left(h^{t-1},\sigma_1,\sigma_2\right)\}_{m=0}^{k-1}),s_2^k(\{P_m\left(h^{t-1},\sigma_1,\sigma_2\right)\}_{m=0}^{k-1})\right\} & \text{if } k\geq t \end{cases}$$

A subgame perfect equilibrium (SPE) can be defined in the following way:

Definition 2 *A pair of strategies* $\{\sigma_1,\sigma_2\}=\{s_1^t,s_2^t\}_{t=0}^{\infty}$ *constitute a SPE of the discounted repeated game with starting node if, for all periods* $t\in\{0,1,\ldots\}$*, and for all histories* $h^{t-1}\in H^{t-1}$ *in each period, the following two conditions are satisfied:*

$$\begin{aligned}\sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_1\left(P_k\left(h^{t-1},\tilde{\sigma}_1,\sigma_2\right)\right),\forall\tilde{\sigma}_1\neq\sigma_1\\ \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right) &\geq \sum_{k=t}^{\infty}\delta^{k-t}u_2\left(P_k\left(h^{t-1},\sigma_1,\tilde{\sigma}_2\right)\right),\forall\tilde{\sigma}_2\neq\sigma_2\end{aligned}$$

Now we can focus on the repeated game without starting node. In this case history in period t is $h^t=\{h_k^t\}_{k=-\infty}^{t}$.[4] Note that here we need to include all periods up to minus infinity, since there is no starting node. Looking at the definition of h^t, it is clear that the set of all possible histories is now the same for all periods: $h^t\in H$, where $H=(S\times S)^{\infty}$.

The definition of a strategy and of an equilibrium are very similar to the previous ones:

Definition 3 *A strategy* σ_i *for player* i *in the repeated game without starting node is a sequence of functions, one for each* $t\in\{\ldots,-1,0,1,\ldots\}$ *of the form* $s_i^t:H\rightarrow S$.

Definition 4 *A pair of strategies* $\{\sigma_1,\sigma_2\}=\{s_1^t,s_2^t\}_{t=-\infty}^{\infty}$ *constitute a SPE of the discounted repeated game without starting node if, for all periods* $t\in$

[4]The number of periods in $P\left(h^{t-1},\sigma_1,\sigma_2\right)$ also changes, so now the continuation path is $P\left(h^{t-1},\sigma_1,\sigma_2\right)=\left\{P_k\left(h^{t-1},\sigma_1,\sigma_2\right)\right\}_{k=-\infty}^{\infty}$ with the same recursive definition exposed above.

$\{\ldots,-1,0,1,\ldots\}$, *and for all histories $h^{t-1} \in H$ in each period, the following two conditions are satisfied:*

$$\begin{aligned}
\sum_{k=t}^{\infty} \delta^{k-t} u_1 \left(P_k \left(h^{t-1}, \sigma_1, \sigma_2\right)\right) &\geq \sum_{k=t}^{\infty} \delta^{k-t} u_1 \left(P_k \left(h^{t-1}, \tilde{\sigma}_1, \sigma_2\right)\right), \forall \tilde{\sigma}_1 \neq \sigma_1 \\
\sum_{k=t}^{\infty} \delta^{k-t} u_2 \left(P_k \left(h^{t-1}, \sigma_1, \sigma_2\right)\right) &\geq \sum_{k=t}^{\infty} \delta^{k-t} u_2 \left(P_k \left(h^{t-1}, \sigma_1, \tilde{\sigma}_2\right)\right), \forall \tilde{\sigma}_2 \neq \sigma_2
\end{aligned}$$

Note that in both cases we have an infinite countable set of conditions to be checked with an infinite countable set of functions that constitute the strategies.

In the traditional repeated games with initial node we have a very well defined utility function for the whole game (which is usually assumed to be additively separable over time). When we extend the model to cover a repeated game with no beginning, this is no longer true. But this is not a problem for definitions 3 and 4, as long as we keep the assumption of separability over time of utility functions. With this assumption, the payoff function of each player at each period is defined conditionally, and therefore the optimization problems are well defined, and deviations can be analysed exactly in the same way as in a game with initial period.

2.3 Stationary Strategies and Stable Equilibria

Once we have defined an equilibrium for both cases (with and without initial period), the next step is to define stationary strategies.

Definition 5 *A strategy σ_i for the repeated game is stationary if there is a function s_i such that $s_i^t = s_i, \forall t$.*

It is interesting to note that the concept of stationary strategies is not applicable to a repeated game with initial period,[5] because the functions s_i^t are defined over different sets (H^t) for different periods. Contrariwise, when there is no initial node, all functions s_i^t are defined over the same set (H), so it is possible to have stationary strategies. Here we have one advantage of our approach: it is possible to study stationary strategies, and this fact can simplify the analysis. Note that with stationary strategies, we do not need to check deviations in all periods in all histories. Instead, it is enough to fix one period t, and analyze deviations in all histories or subgames h^t,

[5] This is true unless the strategy is independent of history for all periods. If this is the case, we can only have as equilibrium one NE of the stage game every period.

but only for this period, because the only difference between two subgames is the previous history, not the period itself.

Now consider the continuation path $P(h^t, \sigma_1, \sigma_2) = \{P_k(h^t, \sigma_1, \sigma_2)\}_{k=-\infty}^{\infty}$ determined by the strategies σ_1 and σ_2 given the history h^t, defined above. The definition of a dynamically stable SPE is the following:

Definition 6 *A subgame perfect equilibrium for a repeated game with no starting node satisfies dynamic stability if the following two conditions are satisfied:*

(a) Stationarity: The strategies σ_1 and σ_2 are stationary.

(b) Convergence: $\lim_{k\to\infty} P_k(h^t, \sigma_1, \sigma_2) = \lim_{k\to\infty} P_k\left(\tilde{h}^t, \sigma_1, \sigma_2\right), \forall h^t, \tilde{h}^t \in H$

Condition (b) in the previous definition states that in a stable SPE, the continuation path converges to the same profile for all histories. Consequently, the long run payoff for player i, $u_i\left(\lim_{k\to\infty} P_k(h^t, \sigma_1, \sigma_2)\right)$, does not depend on the history.

Since the definitions of stationarity and stability cannot be applied to repeated games with initial period, the next section focuses only on repeated games with no starting period.

Now, we can investigate the existence of a stable SPE. For this purpose we cannot rely on the traditional arguments of existence of NE in each subgame, because we are focusing on pure strategy equilibria. Existence, however, can be obtained if there is at least one NE in pure strategies of the stage game. This is true because the strategies "play the static NE in each period, regardless of history" are always a stable SPE. In order to obtain a more general existence result, we should have a definition of dynamic stability also for mixed strategies, which is beyond the scope of this paper.[6]

3 Outcomes supportable as a stable subgame perfect equilibrium

In this section we present a folk theorem with an additional stability requirement. We start by defining the minmax value in pure strategies for player 1, $v = \min_{s_2}\max_{s_1} u_1(s_1, s_2)$. Since, for simplicity, we are focusing on symmetric games, this is also the minmax value in pure strategies for player 2. Let $m = \{m_1, m_2\}$ be one strategy profile in which this minmax value is attained for player 1. Again by symmetry, minmax for player 2 is attained

[6]Some comments about mixed strategies can be found in the concluding section.

at $\{m_2, m_1\}$. Suppose we want to support a certain outcome $\{c_1, c_2\}$, and for simplicity assume $c_1 = c_2 = c$. We will make two more simplifying assumptions. Later on we will suggest how they can be relaxed. The first one is:

Assumption A2: $\exists p$ such that $m_2 \in \arg\max_s u_1 (s, p)$.

Take p according to the previous assumption, and take $q = m_2$. Normalize $u_1 (p, q) = u_2(q, p) = 0$. Then it should be clear that $v \geq 0$. Furthermore, under assumption A2, q is the (static) best response to p, so $u_1 (q, p) = u_2 (p, q) \geq v \geq 0$. Another simplifying assumption is:

Assumption A3: $p \neq c$, $q \neq c$.

The three following subsections are as follows. The first presents the type of strategies that will be used in the proof of the folk theorem, which is presented in the second subsection. Finally, we study how long can be the punishment interval T that follows a deviation for a given value of δ.

3.1 Strategies

Our strategy of proof is to support a certain outcome $\{c_1, c_2\}$, by imposing a penalty of T periods in the case of a deviation, in which the deviating player receives a payoff smaller than or equal to v. As stated above, it is assumed for simplicity that this outcome is such that $c_1 = c_2 = c$.

The intuition of the strategy that is going to be used to support the outcome is very simple. In fact, the strategy can be defined very easily in the case of an initial period in the following way. Start in the cooperative phase, with the two players playing c. If player 1 deviates, switch to phase 1, and if player 2 deviates, switch to phase 2. In phase 1, the prescribed profile is $\{p, q\}$. The game remains in this phase until $\{p, q\}$ is observed for T consecutive periods, in which case the game switches again to the cooperative phase, or until a deviation occurs, in which case a penalty to the deviating player starts again. The same for phase 2, but with a prescribed profile $\{q, p\}$.

The problem, however, arises when there is no initial node, since we cannot impose a given phase at the start of the game. The alternative is to define the phase as a function of the previous history. Also, we need strategies to be independent of calendar time, by stationarity.

We will present now the strategy in a formal way. It is very similar to the one just described for games with initial period. First, we try to find a suitable starting point in history (τ), and then we use θ_j to record the phase in which the game was in period $j > \tau$. The expression $I(A)$ denotes a function that takes the value 1 if the condition A is true, and 0 otherwise.

The strategy for player 1 with T periods of punishment is as follows:[7]

Step 1: Analyze previous history h^{-1}. If $\sum_{j=-\infty}^{-1} I\left(h_j^{-1} = \{c, c\}\right) \geq 1$, then take $\tau = \max\left\{k : h_k^{-1} = \{c, c\}\right\}$, initialize $\theta_{\tau+1} = 0$, and go to step 2. If $\sum_{j=-\infty}^{-1} I\left(h_k^{-1} = \{c, c\}\right) = 0$ and $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) \geq 2$, then take $\tau = \max\left\{k < 0 : \sum_{j=k}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 2\right\}$, and initialize $\theta_{\tau+1} = 1$ if $h_\tau^{2,-1} = c$, and $\theta_{\tau+1} = 2$ if $h_\tau^{1,-1} = c$, and go to step 2. If $\sum_{j=-\infty}^{-1} I\left(h_j^{-1} = \{c, c\}\right) = 0$ and $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 1$, then take $\tau = \left\{k : I\left(h_k^{1,-1} = c\right) + I\left(h_k^{2,-1} = c\right) = 1\right\}$, and initialize $\theta_{\tau+1} = 1$ if $h_\tau^{2,-1} = c$, and $\theta_{\tau+1} = 2$ if $h_\tau^{1,-1} = c$, and go to step 2. Finally, if $\sum_{j=-\infty}^{-1}\left(I\left(h_j^{1,-1} = c\right) + I\left(h_j^{2,-1} = c\right)\right) = 0$, then initialize $\theta_0 = 0$ and go to step 3.

Step 2: If $\tau + 1 = 0$, then go to step 3. If not, compute $\theta_{\tau+2}$ as follows. If $\theta_{\tau+1} = 0$ and $h_{\tau+1}^{-1} = \{c, c\}$ or both arguments are different from c, then $\theta_{\tau+2} = 0$. If $\theta_{\tau+1} = 0$ and $h_{\tau+1}^{i,-1} \neq c, h_{\tau+1}^{-i,-1} = c$, then $\theta_{\tau+2} = i$. If $\theta_{\tau+1} = 1$, then if $h_k^{-1} = \{p, q\}, \forall k \in \{\tau + 2 - T, \ldots, \tau + 1\}$, $\theta_{\tau+2} = 0$, else $\theta_{\tau+2} = i$, where i is 1 unless $h_{\tau+1}^{1,-1} = p$ and $h_{\tau+1}^{2,-1} \neq q$, in which case $i = 2$. If $\theta_{\tau+1} = 2$, then if $h_k^{-1} = \{q, p\}, \forall k \in \{\tau + 2 - T, \ldots, \tau + 1\}$, $\theta_{\tau+2} = 0$, else $\theta_{\tau+2} = i$, where i is 2 unless $h_{\tau+1}^{2,-1} = p$ and $h_{\tau+1}^{1,-1} \neq q$ in which case $i = 1$. Iterate until θ_0 is computed.

Step 3: If $\theta_0 = 0$, then play c. If $\theta_0 = 1$, then play p. If $\theta_0 = 2$, then play q.

Note that the previous maxima always exist because, at any given point, the history finishes in the previous period.

As anticipated above, the variable θ can be interpreted as the phase in which the game is. Note that it is only used to determine the action to be played, with no other future effects. The strategy is presented in this way to avoid confusion between imposing a particular phase in a particular period, and determining the phase in one period endogenously. This is done in step 1. In step 2 we analyze deviations from cooperation, or from a punishment phase, that have been occurred in the past, to be sure that at each time we know who is the player that has deviated most recently. In step 3 we simply require the players to play according to the current phase, which has been determined in the two previous steps. It is worth noting that we need to do the three steps at each period. Formally, it is not possible to make them once

[7] We assume that period is equal to 0 without loss of generality, by stationarity of the strategy. Also, only strategy for player 1 is presented, by symmetry.

and then follow the argument at the beginning of this subsection for games with initial period (because we require stationarity), although the intuition is similar.

3.2 The folk theorem

The proof of the following folk theorem for stable equilibria is quite standard, and hence we will only focus on the steps needed to fulfill the new stability requirement (for further details see Fudenberg and Tirole (1991)).

Theorem 7 *Suppose we have a stage game in normal form satisfying assumption A1, and consider the associated repeated game with no initial period. Then (a)* $\exists\bar{\delta} \in (0,1)$ *such that, for all* $\delta \in [\bar{\delta}, 1)$*, every pure strategy profile with payoffs greater than the minmax values can be the limit of the equilibrium path of a stable SPE, for all histories; and (b) for all* $\delta \in [\bar{\delta}, 1)$*, convergence to this limit can be achieved in a finite number of periods, for all histories.*

Proof. Part (b) is done in detail in the following subsection. For simplicity, the proof will make use of assumptions (A2) and (A3).[8]

Take the strategies defined in the previous section, with a penalty length T arbitrarily chosen, and $\{c, c\}$ as the profile to be supported.[9] If $\{c, c\}$ is a Nash equilibrium of the stage game, the proof is trivial, so assume that it is not (note that this implies that $\max_s u_1(s, c) - u_1(c, c)$ is strictly greater

[8] Assumptions A1 and A2 are made for simplicity. They are not necessary to derive the result, and in this note we are going to suggest how the result could be proved if they don't hold.

Suppose players are in phase 2 for certain history. Under A1, player 1 has no incentives to deviate because he is playing his static best response. On the other hand, if A1 doesn't hold, then player 1 could deviate for a short run profit. After this deviation, player 1 will be punished for T consecutive periods with the profile (p, q). But, the profit is for only one period, so it is bounded, and the punishment is more intense the larger are δ or T. Hence, for δ and T sufficiently large the incentive disappears.

Concerning assumption A2, please note that the second part of the assumption is not restrictive, because $q = m_2$ and hence if $q = c$ then $u_i(c, c) \leq v$.

Finally, if we need the punishment profile to be such that $p = c$, it would be difficult to assess whether $\{q, c\}$ is a deviation of player 1 or the punishment phase that follows a deviation of player 2. In the second case, if player 1 deviates and plays c, the strategy in section 3.1 would identify $\{c, c\}$ as a cooperation, instead of a deviation of player 1. But note that q is the static best response to c, so this deviation is not profitable.

[9] The same proof can be replicated for asymmetric games or symmetric games with $c_1 \neq c_2$, although with a little bit more notation. Simply we would have two different $\bar{\delta}$, one for each player, and it suffices to take the bigger of the two.

than 0). We need to show three facts. First, the strategies are stationary. Second, they induce a convergent continuation path, for all histories. And third, there are no profitable deviations. Since the game is symmetric, only deviations of player 1 will be considered.

The first and second parts are straightforward, given the definition of the strategy.

Concerning deviations, the following conditions[10] are enough to avoid them:

$$u_1(q,p)\left(1+\delta+\cdots+\delta^{t-1}\right)+\tfrac{\delta^t}{1-\delta}\cdot u_1(c,c)\geq$$

$$\geq \max_{s\in S-\{q\}} u_1(s,p)+\tfrac{\delta^{T+1}}{1-\delta}\cdot u_1(c,c)\,;1\leq t\leq T \tag{1}$$

$$\frac{\delta^t}{1-\delta}\cdot u_1(c,c)\geq \max_{s\in S-\{p\}} u_1(s,q)+\frac{\delta^{T+1}\cdot u_1(c,c)}{1-\delta};1\leq t\leq T \tag{2}$$

$$\frac{u_1(c,c)}{1-\delta}\geq \max_s u_1(s,c)+\frac{\delta^{T+1}}{1-\delta}\cdot u_1(c,c) \tag{3}$$

Inequalities (1) and (2) rule out deviations in histories following a deviation of player 2 and 1, respectively, with t remaining punishment periods. Finally, inequality (3) rules out deviations when players are supposed to cooperate and play c.

It is easy to see that, under our assumptions, condition (1) is always satisfied.

Condition (2) is equivalent to

$$\delta^T\geq \frac{\max_{s\in S-\{p\}} u_1(s,q)}{u_1(c,c)} \tag{4}$$

Note that $\max_{s\in S-\{p\}} u_1(s,q)\leq \max_s u_1(s,q)=v<u_1(c,c)$. Hence, for a finite T there exists a $\delta_1\in(0,1)$ such that (4) is satisfied for all $\delta\in[\delta_1,1)$.

Finally, condition (3) can be expressed as:

$$u_1(c,c)\cdot\delta^{T+1}-\left(\max_s u_1(s,c)\right)\delta+\left(\max_s u_1(s,c)-u_1(c,c)\right)\leq 0 \tag{5}$$

[10] We are using the principle that, if the strategies constitute an equilibrium, then it is enough to check deviations as the following: deviate in one period and then follow again the strategy. See Fudenberg and Tirole (1991), section 4.2.

Note that the left hand side of (5) is a polynomial (in δ) of degree $T+1$. Let's call it $Y(\delta)$, so condition (5) is equivalent to $Y(\delta) \leq 0$. It's easy to see that $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$, $Y(1) = 0$, $Y'(\delta) = ((T+1) \cdot u_1(c,c)) \cdot \delta^T - \left(\max_s u_1(s,c)\right)$, and that $Y(\delta)$ is (strictly) convex in $(0,\infty)$, since $u_1(c,c) > 0$.

Now, two cases are possible:

Case 1: $\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} \geq 1$: In this case the polynomial $Y(\delta)$ is (strictly) decreasing in $(0,1)$. Since $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$ and $Y(1) = 0$, then $Y(\delta) > 0$ for $\delta \in (0,1)$. Therefore, there is no equilibrium in this case.

Case 2:$\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} < 1$: Here the function $Y(\delta)$ is decreasing in $\left(0,\hat{\delta}\right)$, and increasing in $\left(\hat{\delta},1\right)$, with $\hat{\delta} = \left(\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)}\right)^{\frac{1}{T}} \in (0,1)$. Again, since $Y(0) = \max_s u_1(s,c) - u_1(c,c) > 0$ and $Y(1) = 0$, then $\exists \delta_2 \in (0,1)$ such that $Y(\delta) \leq 0$ for all $\delta \in [\delta_2, 1)$.

Note that the value of T determines whether we are in case 1 or 2, and case 1 can be avoided by taking $T > \frac{\max_s u_1(s,c)}{u_1(c,c)} - 1$.

The proof is complete by considering $\overline{\delta} = \max\{\delta_1, \delta_2\}$. ■

3.3 Bounds for the punishment interval

In this subsection we make a different type of analysis. Once we have shown that a stable perfect equilibrium exists for sufficiently patient players, we can ask ourselves the following: for a given (sufficiently high) δ, what values of T constitute a stable SPE?

We have seen that a necessary condition is $\frac{\max_s u_1(s,c)}{u_1(c,c)\cdot(T+1)} < 1$, so the first constraint on T is:

$$T > \frac{\max_s u_1(s,c)}{u_1(c,c)} - 1 \tag{6}$$

Other constraints on T are conditions (4) and (5), but now for a fixed δ. With some algebra we can obtain the following from (4):

$$T \leq \frac{\log\left(\frac{\max_{s \in S-\{p\}} u_1(s,q)}{u_1(c,c)}\right)}{\log(\delta)} \tag{7}$$

The numerator and the denominator are negative, and δ is big enough so that the denominator is small enough (in absolute value) to produce a sensible bound.

Now, from (5) we get:

$$T \geq \frac{\log\left(-\frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} \cdot (1-\delta) + 1\right)}{\log \delta} - 1 \tag{8}$$

Therefore the admissible values for T are the positive integers such that (6), (7) and (8) are satisfied. The lower bound is[11]

$$\max\left\{\frac{\log\left(-\frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} \cdot (1-\delta) + 1\right)}{\log \delta} - 1, \frac{\max\limits_{s} u_1(s,c)}{u_1(c,c)} - 1, 1\right\}$$

and the upper bound is

$$\frac{\log\left(\frac{\max\limits_{s \in S-\{p\}} u_1(s,q)}{u_1(c,c)}\right)}{\log(\delta)}$$

Note that the numerator is finite under assumption A1, because $\max\limits_{s \in S-\{p\}} u_1(s,q) < u_1(c,c)$. The denominator is also finite and different from 0 for $\delta \in (0,1)$. Therefore, the upper bound is strictly finite, so convergence must be achieved in a finite number of periods.

The intuition for the upper bound is very simple. Consider the incentives for a player that is being punished. Clearly, deviation is more profitable if the punishment period is longer. Therefore, the punishment period cannot be very long, because it would induce deviations.

4 Memory Requirements

In this section we study the memory requirements of the strategies presented in section 3.1, in an attempt to compare our approach with the theory on bounded recall. First we will show that our strategies do not work under bounded recall. This could cast doubt on the complexity of our strategy and hence on its applicability in the real world, as Aumann (1997) suggests. For this reason we define a concept of memory requirements weaker than bounded

[11] Remember that condition (6) is a strict inequality, and the other two conditions are weak inequalities.

recall, and we provide upper bounds for memory requirements under this weaker definition.

We start by defining absolute memory of a strategy:

Definition 8 *A strategy σ_i has absolute memory $\Phi(\sigma_i) = \lambda$ if $\lambda \in \mathbb{N}$ is the minimum number such that $s_i^t(h^{t-1}) = s_i^t(\tilde{h}^{t-1})$, for any period $t \in \mathbb{Z}$, and for every pair of histories $h^{t-1} \in H$ and $\tilde{h}^{t-1} \in H$ such that $h_k^t = \tilde{h}_k^t, \forall k \in \{t-\lambda, \ldots, t-1\}$.*

We can say that a strategy σ_i satisfy bounded recall if and only if $\Phi(\sigma_i) < \infty$. Now, it is easy to see that the strategy defined in section 3.1 does not satisfy bounded recall, as the following proposition shows:

Proposition 9 *The strategy defined in 3.1 has infinite absolute memory.*

Proof. Consider a history $h^{t-1} \in H$ such that $h_k^{t-1} = \{q, q\}, \forall k \leq t-1$. Now, consider another history $\tilde{h}^{t-1} \in H$ such that $\tilde{h}_k^{t-1} = \{q, q\}, \forall k \leq t-1, k \neq t-\lambda$, and $\tilde{h}_{t-\lambda}^{t-1} = \{c, q\}$, for some $\lambda \geq 1$. According to the strategy defined in 3.1, player 1 should play c after history h^{t-1}, whereas he should play q after history $\tilde{h}^{t-1}$. The proof is complete by taking the limit $\lambda \to \infty$. ■

The previous proposition shows that our stable strategies cannot be defined under a bounded recall framework. But this does not mean that our strategies are infinitely complicated, because we can provide bounds for their memory requirements using a definition weaker than absolute memory. We call this weaker concept conditional memory, and it is defined as follows:

Definition 10 *A strategy σ_i has conditional memory $\phi(\sigma_i, h^{t-1}) = \lambda$ in subgame h^{t-1} if $\lambda \in \mathbb{N}$ is the minimum number such that $s_i^t(h^{t-1}) = s_i^t(\tilde{h}^{t-1})$, for every history $\tilde{h}^{t-1} \in H$ such that $h_k^t = \tilde{h}_k^t, \forall k \in \{t-\lambda, \ldots, t-1\}$.*

The definition of conditional memory calculates memory requirements, conditional on being in a particular subgame. On the other hand, absolute memory is defined without any conditioning, so it can be interpreted as the maximum conditional memory over all possible histories.

Now, we are in a position to provide bounds for conditional memory requirements of the stable strategy defined in 3.1:

Theorem 11 *Let σ_1 and σ_2 be the stable strategies defined in 3.1 for player 1 and 2, respectively. Then, $\phi(\sigma_i, P_k(h^{t-1}, \sigma_1, \sigma_2)) = 1$ for any $t \in \mathbb{Z}, h^{t-1} \in H, i \in \{1, 2\}, k \in \mathbb{Z}$ such that $k \geq t + T$.*

Proof. If players play according to the stable strategies defined in 3.1, then $P_k(h^{t-1}, \sigma_1, \sigma_2) \in \{\{c,c\}, \{p,q\}, \{q,p\}\}, \forall k \geq t$, depending on the value of θ_k computed in steps 1 and 2 in the definition of the strategy. Moreover, the profiles $\{p,q\}$ and $\{q,p\}$ can only last for at most T periods, so $P_k(h^{t-1}, \sigma_1, \sigma_2) = \{c,c\}, \forall k \geq t + T$. The proof is complete by noting that step 1 in the definition of the strategy ignores the part of the history observed before the most recent $\{c,c\}$. ■

The previous theorem states that, if players play according to the stable strategies defined in 3.1, then conditional memory requirements eventually collapse to 1, because eventually both players will be playing c. This means that conditional memory requirements are finite and small, except from histories that are very far from the equilibrium path implied by our stable strategies.

5 Concluding Comments

In this paper we have proposed a concept of dynamic stability in repeated games with discounting for which a modification of the traditional theory is needed. In particular, we need to introduce a game with no beginning, i.e. a game with an infinite history at all periods. One should keep in mind issues like reputation or robustness to initial conditions when interpreting the proposed concept of dynamic stability.

A characterization[12] of payoffs supported as a stable equilibrium is also presented, with the additional result that convergence to the long-run strategy profile can be achieved in a finite number of periods, for all previous histories. The proof is constructive, giving the strategies that support the long run profile. We have also shown that memory requirements of these strategies are bounded, but under a concept of memory requirements weaker than bounded recall.

One can look at the contributions of this work in at least three different ways. The most direct is the study of games with an infinite history, i.e. with no beginning. A more interesting interpretation is that the theory developed here provides a formal justification for the folk results often used in the literature, because all equilibrium paths converge to the cooperative outcome. In other words, we are checking the robustness of the folk results to the assumption that strategies begin with certain profile. Finally, the analysis of the convergence process is interesting by itself. If for whatever reason players are not cooperating (maybe because someone has deviated in

[12]This characterization is incomplete in the sense that we have focused on pure strategies, and on games with no uncertainty.

the past), the strategies presented here provide a way in which players can build up again their reputations.

The analysis developed here has nevertheless several limitations, and further research will be useful. The main limitation is that the theorem requires a supported outcome in pure strategies. Although the rest of the assumptions are innocuous, and are made for simplicity, the requirement of pure strategies cannot be generalized in a straightforward manner. If we want to support convex combinations of the pure strategy payoffs, we cannot apply the usual argument of a public randomizing device and a correlated distribution. The reason is that we require the equilibrium path to be convergent, not to switch at random over a set of outcomes. But one can still have a notion of dynamic stability in these cases. Perhaps the simplest way is to substitute condition (b) in the definition of a stable SPE by convergence in expected payoffs, not in the profile played. Maybe there are other possibilities, like convergence in distribution over outcomes, even though it may be difficult to develop stationary strategies with the property that future probability distribution over outcomes is invariant, irrespective of the realization of the present (stochastic) outcome.

Second, we have focused on games with no uncertainty, so a concept of stability for games with some source of uncertainty may be useful. Probably, the definition will be close to Ely and Välimäki (2002) and Green and Porter (1984), but the extension is not straightforward. One possibility would be to make the definition analogous to those used in stochastic processes, assuming that from now on all stochastic variables take a realization equal to their mean value (including variables related to imperfect information). Then, the definition of convergence could be modified accordingly.

Third, it is interesting to note that the folk result presented here is a limit result. The theorem in section 3.2 states that we can support stable equilibria for a sufficiently high discount factor. It doesn't say anything about optimal punishment schemes (satisfying stability), in the sense that there may exist other strategies that require weaker conditions for $\bar{\delta}$. This is obviously another possible extension.

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On the Role of Educational Subsidies[1]

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Abstract

We present an overlapping generations model with human capital and public R&D that has several interesting features. First, it can generate expectation cycles of period two in the time that each generation is spending on education. Second, a subsidy to education may affect positively growth by making cheaper the good used in the production of public R&D, human capital. Third, a subsidy to education plays the role of a financial system, transferring resources from old to young people, in the absence of private financial markets.

We also develop a calibration exercise for a set of european countries to obtain the recommendations of the model concerning educational and R&D policies. We also see if actual policies are close or not to the recommendations.

1 Introduction

Nowadays, it is almost common knowledge that education is a very important part of the process of sustained growth. Human capital represents a big fraction of the total accumulated factors in modern economies. Further, human capital is the main resource employed in the production of new knowledge, as well as in the adoption of the existing technology.

On the other hand, governments in OECD economies subsidize at least part of the costs of education. Then, understanding the effects of these subsidies in the process of growth turns out to be of capital importance.

The main goal of the paper is to study the optimal distribution of subsidies between R&D and education. From a theoretical point of view, R&D subsidies can be justified by their effects on growth. By contrast, justifications of educational subsidies are more varied. In this paper we are interested in two of them: First, the role of educational subsidies as a substitute for the financial markets faced by young agents. Second, the positive effect of education on growth through R&D activities. We show that, in theory, both effects can be present in the economy. This has important implications. For example, if a financial market for young people could be constructed, it would be still optimal to have educational subsidies, due to the second justification. Or, in other words, policies with the only aim of maximizing growth may find optimal to assign some resources to educational subsidies even if growth is only produced by R&D.

With this motivation in mind, we construct a very simple overlapping generations model with educational subsidies and public R&D. In this model, educational subsidies can have three main effects. First, a change in the subsidy could lead an economy to a situation of multiplicity of equilibria, and perhaps it can induce cycles in the educational effort of different generations.

Second, the subsidy encourages human capital accumulation, hence making human capital cheaper. If the production of new knowledge is intensive in human capital, as it is in our model, this effect can raise growth. Note, however, that a higher growth may not be optimal; it will depend on the existence of externalities. In our model, human capital has an indirect externality that will be discussed below.

Third, educational subsidies can act as a substitute of financial markets. Young people are often borrowing constrained, and on the other hand they expect an increase in their real income through the life cycle. As a consequence, young people have incentives to borrow in order to smooth consumption and to finance their investments in education. Borrowing constraints typically bind in this situation, and therefore an educational subsidy may increase welfare, by playing the role of the missing financial market.

The main features of the model are the following. People live for two periods. In the first period, they must allocate their time between human capital accumulation and the labor market. They also consume in the first period. The monetary cost of education is zero, but nevertheless there is an opportunity cost in the form of foregone earnings. The government subsidizes a fraction of this opportunity cost. In the second period, agents simply sell all their raw time and their human capital in the market, and consume the proceeds. There is no intergenerational altruism. The government levies an income tax on both young and old people, and uses the proceeds to finance the subsidy, and to purchase some human capital in the market. It uses this human capital to provide public R&D.

Now, the indirect externality mentioned above becomes clear. If an agent studies more time, the relative price of human capital falls. Then, the government can buy more human capital with the same resources, and therefore it provides more public R&D. The benefits of this increase not only are divided among all agents in the economy, but also the first time they appear is two generations after. Clearly, the agent does not internalize this effect in the absence of fiscal policy.

There are several topics in the literature related to this paper. One is the endogenous growth literature, both with human capital accumulation and with R&D as the engines of growth. Another is the literature about optimal fiscal policy. Finally, this paper has also implications regarding financial markets and borrowing constraints.

There is a wide variety of R&D models with endogenous growth in the literature. Examples are, among many others, Jones (1995), Howitt (1999), Segerstrom (2000) and Aghion and Howitt (1992). In many of them there are firms that produce R&D services, to be used in the same firm or to be sold in the market in the form of patents. As a consequence, it is often true that the competitive equilibrium is not optimal in the presence of R&D activities. This can be due to monopoly power generated by patents, to externalities in the process of R&D, or to the existence of imitation posibilities.

Our model has, by contrast, public R&D. This allows us to avoid issues concerning market structure and market power. Furthermore, the main results of the paper probably would not be affected by the introduction of private R&D, and some can be reinforced. Consider for example the second effect of an education subsidy described above. A lower relative price of human capital not only induces the government to increase its R&D, but also it would generate an increase in private R&D if the latter were present in the model.

Concerning the endogenous growth literature with human capital accumulation as the engine of growth, the most known model is the Lucas (1988) and Uzawa (1965) model. In this type of economies, long term growth occurs because the production function for new human capital has constant returns to scale in some set of reproducible factors, allowing unbounded accumulation of human capital. The general properties of the model have been analyzed in a number of works.[1] Other papers have studied more specifically the problem of fiscal policy in this kind of models. King and Rebelo (1990) find important growth effects of fiscal policy, and Jones, Manuelli and Rossi (1993) argue that this is because they use an endogenous growth model. Focusing on the educational subsidy, Milesi-Ferretti and Roubini (1998) show, among other results, that the long term optimal subsidy must be zero, and Alonso-Carrera (2000) analyzes in detail the theoretical effects of the presence of the subsidy.

Our model has at least two important differences with respect to the Lucas-Uzawa model. First, a model of infinitely lived agents appears to be inappropriate to study the effects of borrowing constraints in education. Therefore, we have chosen to develop an overlapping generations model. In this sense, our model is closer to Hendricks (1999, 2001) and de Gregorio (1996).

Second, in the Lucas-Uzawa model the engine of growth is human capital *accumulation*. If this were true, we would expect income *level* and human capital to be correlated. There is, however, some evidence suggesting that income *growth* is related with human capital.[2] If we identify human capital with concepts like average years of education, one would expect precisely the second kind of relation, because in a lifetime context, average years of education (or similar measures) cannot grow without bound. Moreover, with this latter interpretation, human capital *level* contributes to the growth of another variable (knowledge) that indeed can grow without bound. Therefore, the level of human capital is expected to be related with income growth.

[1] See, for example, Caballé and Santos (1993), Ortigueira and Santos (1997) and Mulligan and Sala-i-Martin (1993).

[2] See, for example, Benhabib and Spiegel (1994).

Our strategy here is to model explicitly the link between human capital and growth of knowledge, by assuming that R&D services are produced with human capital services.[3]

Note, however, that the previous discussion is only about language. We can interpret human capital in the Lucas-Uzawa model as knowledge. Or one can have a wider definition of human capital including also the level of knowledge learned. These issues become important in applied work, where one needs to identify very clearly which concept one is using.

There is extensive literature covering level and growth effects of tax reforms, and optimal taxation.[4] Our model is not very appropriate to study taxation, because in our model the income tax is non-distorting. We focus on the study of educational subsidies, and the optimal composition of public expenditure (for a given income tax) between education subsidies and R&D. This optimal composition is the main lesson in the section of optimal fiscal policy. A related work is Rustichini and Schmitz (1991). They analyze a model in which the government subsidizes both the research activity and the imitation activity. The composition of public expenditure between these two concepts is specifically studied. They find that it is optimal to subsidize both activities.

Also, in our model, there is no waste of public resources. Either they are spent on education subsidies or on public R&D. By contrast, many models of optimal taxation assume public expenditure to be non-productive. This separation between revenues and expenses can be problematic, as discussed in Jones, Manuelli and Rossi (1993), particularly in the analysis of optimal government size. We present in the conclusions an extension that would make the model appropriate to cover this question.

A close model is the one by Glomm and Ravikumar (2001). They present an OLG model with two periods in which human capital is produced with the parental human capital, the resouces spent by the government in education,

[3] An alternative way of obtaining growth effects from levels of human capital would be to assume a production function with an ad hoc *growth* external effect of human capital. But this would be something difficult to interpret in the traditional terms of imporvements in productivity when the worker is surrounded by colleagues with high human capital. Moreover, this formulation would make impossible to analyze the optimal allocation of public resources between educational and R&D subsidies, which is the main goal of the paper.

[4] See, for example, King and Rebelo (1990), Milesi-Ferretti and Roubini (1998), Jones, Manuelli and Rossi (1993), Chamley (1986), Lucas (1990), Stokey and Rebelo (1995) and Judd (1985).

and the time of young people. The model is quite similar to our model. There are, however, some differences. First, they study the policy choice problem, assuming that decissions are taken by old people, using majority voting, whereas we focus on optimal policy. Second, in their model, there is altruism between generations, and young people cannot work; they choose between studying or leisure. By contrast, we don't allow for intergenerational altruism, and young people can work, which means that they must pay the opportunity cost of studying. With this approach we are able to study issues concerning borrowing constraints. Third, we model explicitly the relationship between human capital and R&D (or growth). This allows us to study the link between subsidies to education and long term growth.

De Gregorio (1996) presents evidence about the importance of borrowing constraints in the process of development. We agree with his view that borrowing constraints can reduce growth by reducing the incentives of the young people to accumulate human capital. As already discussed, we add to his argument the fact that education subsidies can effectively act as a substitute of financial markets in the presence of borrowing constraints.

Finally, Trostel (1993, 1996) shows that, if education has both time and monetary costs, then the income tax distorts the composition between both factors because time is actually tax deductible but monetary cost is not. Then, he shows that a subsidy to monetary costs of education can offset the distortion previously mentioned. He obtains that the optimal subsidy should be roughly equal to the marginal income tax. Since he focuses on this particular consequence of subsidies to education, whereas we are focusing on other effects, his work should be seen as complementary to our study.

The rest of the paper is organized as follows. The next section presents the description of the model. In section 3, the equilibrium is analyzed. In sections 4 and 5 the stationary solution and the transitional dynamics of the model are, respectively, analyzed. Optimal policy is studied in section 6. Finally, section 7 has some concluding comments and discusses some possible extensions.

2 The Model

At each period t there are two agents, the young agent and the old agent. Let c_t^y and c_t^o be the consumption of the agent that is young in period t, and old in period t, respectively. There is an instantaneous utility function, denoted by $u(\bullet)$ and a discount factor ρ. Both agents (and in particular the young agent) are endowed with perfect foresight.

The young agent in period t is endowed with one unit of time, and he has to decide how much time he devotes to human capital accumulation, which will be denoted by v_t. The remaining time is sold in the market as labor. The cost of human capital accumulation is foregone earnings.

The old agent in period $t+1$ is endowed also with one unit of time, which he devotes entirely to the labor market, and in addition he has $f(v_t)$ units of human capital, depending on the time he devoted to human capital accumulation when he was young. The function f satisfies:

Assumption A1 The function $f(v_t)$ is increasing, concave, continuously differentiable, and satisfies $f(0)=0$, $f'(1)=0$ and $\frac{d\left(\frac{-f''(v_t)f(v_t)}{(f'(v_t))^2}\right)}{dv_t} \geq 0$.

The $f(v_t)$ units of human capital are sold in the market of human capital, which is independent from the labor market. When the old agent dies, all his human capital is lost.

There is a representative firm which produces the consumption good y_t using labor and human capital, with the following production function:

$$y_t = A_t H_t^{\beta} L_t^{1-\beta} \tag{1}$$

Where L_t and H_t are, respectively, the amount of raw labor and human capital employed by the firm, and A_t is a productivity parameter that can vary over time. The prices of labor and human capital are respectively w_t^l and w_t^h.

Finally, there is a government that levies an income tax τ. The proceeds of the tax are used to pay a subsidy to human capital accumulation, which is a fraction s of the post-tax wage $(1-\tau)\cdot w_t^l$ per unit of time invested, and to buy a quantity R_t of human capital in the market that is used to provide public R&D.

We assume that all the parameters satisfy the following *strict* inequality: $0 < s, \tau, \beta, \rho < 1$.

The representative young agent's problem is to maximize (2)

$$u\left(c_t^y\right) + \rho \cdot u\left(c_{t+1}^o\right) \tag{2}$$

Subject to

$$c_t^y = (1-\tau)\cdot w_t^l \cdot (1-v_t) + s\cdot(1-\tau)\cdot w_t^l \cdot v_t \tag{3}$$

$$c_{t+1}^o = (1-\tau)\left[w_{t+1}^l + w_{t+1}^h \cdot f(v_t)\right] \tag{4}$$

$$0 \le v_t \le 1 \tag{5}$$

Note that, since there is no bequest motive in the model, the solution to the problem of the old agent is trivial: sell all his time and human capital, and consume all his income.

Concerning the problem of the firm, profit maximizing conditions are:

$$w_t^l = A_t \cdot (1-\beta)\cdot \left[\frac{H_t}{L_t}\right]^{\beta} \tag{6}$$

$$w_t^h = A_t \cdot \beta \cdot \left[\frac{L_t}{H_t}\right]^{1-\beta} \tag{7}$$

There are three markets in this economy: consumption goods, raw labor, and human capital. Note that the supply of raw time is one unit from the old and $1-v_t$ units from the young, and the supply of human capital is simply the human capital of the old, $f(v_{t-1})$. Thus, the three market clearing conditions are, respectively:

$$c_t^y + c_t^o = y_t = A_t H_t^{\beta} L_t^{1-\beta} \tag{8}$$

$$L_t = 2 - v_t \tag{9}$$

$$H_t + R_t = f(v_{t-1}) \tag{10}$$

It's assumed that the government has a balanced budget at each period of time. Then, the following equation must be satisfied:

$$s\cdot(1-\tau)\cdot w_t^l \cdot v_t + w_t^h \cdot R_t = \tau\left[w_t^l \cdot (2-v_t) + w_t^h \cdot f(v_{t-1})\right] \tag{11}$$

Now, plug (9) and (10) into (8), (6) and (7) to obtain

$$c_t^y + c_t^o = y_t = A_t \cdot \left[f(v_{t-1}) - R_t\right]^{\beta}\left[2 - v_t\right]^{1-\beta} \tag{12}$$

$$w_t^l = A_t \cdot (1-\beta) \cdot \left[\frac{f(v_{t-1}) - R_t}{2 - v_t} \right]^{\beta} \tag{13}$$

$$w_t^h = A_t \cdot \beta \cdot \left[\frac{2 - v_t}{f(v_{t-1}) - R_t} \right]^{1-\beta} \tag{14}$$

Finally, the growth rate of the productivity parameter A_t is some increasing function A of resources spent on research by the government, R_t:

$$A_{t+1} = A(R_t) \cdot A_t \tag{15}$$

3 Equilibrium

Now we can define an equilibrium for this economy. In this section, we will focus on the positive analysis of the economy. Therefore we will define the equilibrium taken as exogenously given the two policy parameters s and τ. In section 6 we will make them endogenous by obtaining the optimal policy.

Definition 1 *An equilibrium is a sequence of variables* $\left\{c_t^y, c_t^o, v_t, H_t, L_t, R_t, w_t^l, w_t^h\right\}_{t=1}^{\infty}$ *such that, given a value for* v_0 *and given values for the set of parameters* $\{s, \tau, \beta, \rho\}$, *the following conditions are satisfied:*

(a) For each $t \in \{1, 2, \ldots\}$, *the variables* c_t^y, c_{t+1}^o, *solve the consumer's problem described above, taking as given the variables* $w_t^l, w_t^h, w_{t+1}^l, w_{t+1}^h$.

(b) The variable c_1^o *satisfies equation (4).*

(c) For each $t \in \{1, 2, \ldots\}$, *the variables* H_t, L_t, w_t^l, w_t^h *satisfy the profit-maximizing conditions (6) and (7).*

(d) The government has a balanced budget each period: (11) is satisfied for all $t \in \{1, 2, \ldots\}$.

(e) For each $t \in \{1, 2, \ldots\}$, *the market clearing conditions (8), (9) and (10) are satisfied.*

Now the first step is to obtain the solution for the representative agent. The first order condition for an interior solution in the consumer's problem is:

$$u'(c_t^y)\, w_t^l (1-s) = \rho \cdot u'\left(c_{t+1}^o\right) w_{t+1}^h \cdot f'(v_t) \tag{16}$$

Note that $u'\left(c_t^y\right)$ is increasing in v_t, and $u'\left(c_{t+1}^o\right)$ and $f'\left(v_t\right)$ are decreasing in v_t. Therefore, if the solution is interior, then it is uniquely determined by (16).

If the solution is $v_t = 0$, the following must be true:

$$u'\left(\left(1-\tau\right) w_t^l\right) w_t^l \left(1-s\right) \geq \rho \cdot u'\left(\left(1-\tau\right) w_{t+1}^l\right) w_{t+1}^h \cdot f'\left(0\right) \tag{17}$$

Analogously, if the solution is $v_t = 1$, the following must be true:

$$u'\left(s\left(1-\tau\right) w_t^l\right) w_t^l \left(1-s\right) \leq \rho \cdot u'\left(\left(1-\tau\right)\left(w_{t+1}^l + w_{t+1}^h \cdot f\left(1\right)\right)\right) w_{t+1}^h \cdot f'\left(1\right) \tag{18}$$

The following proposition establishes under which conditions the income tax τ is non-distorting.

Proposition 2 *The value of v_t that solves the young agent problem does not depend on τ if the function $u'\left(\cdot\right)$ is homogeneous.*

Proof: The factors in (16), (17) and (18) are all independent of τ, except for marginal utilities. Therefore, the optimal v_t will depend on τ if and only if the ratio $\frac{u'\left(c_t^y\right)}{u'\left(c_{t+1}^o\right)}$ depends on τ. Now, if $u'\left(\cdot\right)$ is homogeneous of degree ϑ, then:

$$\begin{aligned}\frac{u'\left(c_t^y\right)}{u'\left(c_{t+1}^o\right)} &= \left(\frac{c_t^y}{c_{t+1}^o}\right)^{\vartheta} = \left(\frac{\left(1-\tau\right)\cdot w_t^l \cdot \left(1-v_t\right) + s \cdot \left(1-\tau\right)\cdot w_t^l \cdot v_t}{\left(1-\tau\right)\left[w_{t+1}^l + w_{t+1}^h \cdot f\left(v_t\right)\right]}\right)^{\vartheta} = \\ &= \left(\frac{w_t^l \cdot \left(\left(1-v_t\right) + s \cdot v_t\right)}{w_{t+1}^l + w_{t+1}^h \cdot f\left(v_t\right)}\right)^{\vartheta}\end{aligned}$$

To finish the proof, note that the last expression is independent of τ.■

The intuition of the previous proposition is the following. Time can only be allocated either to work or to study. And, given that the income tax lowers the return of both activities in the same proportion, the tax is non distorting when marginal utility is homogeneous. As we mention in the concluding section, in order to obtain distorting taxes, we should include a non-taxed alternative to allocate time (v.gr. leisure).

Among the marginal utilities that are homogeneous, we assume for simplicity logarithmic utility, which implies that $u'\left(\cdot\right)$ is homogeneous of degree -1. Then (16) becomes

$$\frac{1}{\frac{1}{1-s} - v_t} = \frac{\rho \cdot f'(v_t)}{\frac{w_{t+1}^l}{w_{t+1}^h} + f(v_t)} \tag{19}$$

and (17) and (18) become, respectively

$$(1-s) \geq \rho \cdot f'(0) \frac{w_{t+1}^h}{w_{t+1}^l} \tag{20}$$

$$\frac{1-s}{s} \leq \rho \frac{f'(1)}{\frac{w_{t+1}^l}{w_{t+1}^h} + f(1)} \tag{21}$$

Note that (no matter the solution is interior or not), (19), (20) and (21) imply that the optimal solution depends only on the ratio $\frac{w_{t+1}^l}{w_{t+1}^h}$, and the parameters of the model:

$$v_t \left(\frac{w_{t+1}^l}{w_{t+1}^h}, \ldots \right) \tag{22}$$

Equation (22) gives us the solution of the consumer's problem. Once v_t is obtained, we can obtain consumption in both ages using (3) and (4).

Concerning the government, we can rearrange (13), (14) and (11) to obtain:

$$\frac{1-\tau}{\tau} \left[\frac{s \cdot v_t}{2 - v_t} (1-\beta) + \beta \cdot \frac{R_t}{f(v_{t-1}) - R_t} \right] = 1 \tag{23}$$

Equation (23) defines the value of R_t as a function of v_t, v_{t-1}, and the parameters of the model:

$$R_t (v_{t-1}, v_t, \ldots) \tag{24}$$

Finally, we can calculate the ratio $\frac{w_{t+1}^l}{w_{t+1}^h}$ from (13) and (14):

$$\frac{w_{t+1}^l}{w_{t+1}^h} = \frac{1-\beta}{\beta} \left(\frac{f(v_t) - R_{t+1}}{2 - v_{t+1}} \right) \tag{25}$$

Now, if we introduce (25) into (22) we can obtain the solution for v_t as a function of v_{t+1}, R_{t+1}, and the parameters of the model:

$$v_t (v_{t+1}, R_{t+1}, \ldots) \tag{26}$$

Equations (24) and (26) describe the dynamic behavior of the economy. Note that the optimal solution given by (26) is driven by expectations on future investments in education and research. It does not depend on current or past conditions like v_{t-1} or A_t. For this reason, the solution of the dynamic system cannot be characterized by an initial condition, and multiplicity of equilibria can arise.

In particular, all sequences $\{v_t, R_t\}_{t=1}^{\infty}$ satisfying equations (24) and (26) can be an equilibrium. The initial value v_0 has influence only on the variable R_1. It has no effect on the rest of research efforts, and it has no effect on any educational effort.

As a consequence, if a stationary solution exists, then the stationary outcome each period is always an equilibrium.[5] Of course, there can be other equilibria too. We need to check if these other equilibria either converge to the stationary solution or have an explosive (and inconsistent) behavior. If the latter is true, we can focus on the study of the stationary solution.

The next two sections deal with this issues. In the following one, the stationary solution is characterized, and in section 5 the dynamics of the equilibrium are analyzed.

4 Stationary solution

In this section we characterize the stationary solution of the dynamic system (24) and (26). We denote with an * the stationary values of the variables.

The strategy is to start with the conditions for an interior solution, and then rule out corner solutions.

Assuming interiority we have the following. From (25) we obtain the stationary price ratio

$$\left(\frac{w^l}{w^h}\right)^* = \frac{1-\beta}{\beta}\left(\frac{f(v^*) - R^*}{2 - v^*}\right)$$

Hence, substituting in the (interior) first order condition of the consumer (19) we obtain

[5] The variable v is equal to its steady state level from period 1 on. The variable R, however, achieves its stationary value in period 2, with the value of R_1 given by (24).

$$\frac{1}{\frac{1}{1-s}-v^*}=\frac{\rho\cdot f'(v^*)}{\frac{1-\beta}{\beta}\left(\frac{f(v^*)-R^*}{2-v^*}\right)+f(v^*)} \tag{27}$$

Now, from the government budget constraint (23) we have

$$\frac{s\cdot v^*(1-\beta)}{2-v^*}+\frac{\beta\cdot R^*}{f(v^*)-R^*}=\frac{\tau}{1-\tau} \tag{28}$$

Solving for R^* in (27) we obtain

$$R^*=f(v^*)-\frac{\beta}{1-\beta}(2-v^*)\left[\left(\frac{1}{1-s}-v^*\right)\rho\cdot f'(v^*)-f(v^*)\right] \tag{29}$$

Replacing (29) into (28) and simplifying we can obtain

$$\frac{s\cdot v^*(1-\beta)}{2-v^*}+\frac{1-\beta}{(2-v^*)\left[\left(\frac{1}{1-s}-v^*\right)\frac{f'(v^*)}{f(v^*)}\rho-1\right]}-\beta-\frac{\tau}{1-\tau}=0 \tag{30}$$

This equation determines interior stationary solutions. Note that under assumption A1, the expression $\left[\left(\frac{1}{1-s}-v^*\right)\frac{f'(v^*)}{f(v^*)}\rho-1\right]$ is decreasing in v^*, tends to infinity at $v^*=0$ and tends to -1 at $v^*=1$. Therefore, by continuity, there is one $\hat{v}$ such that $\left(\frac{1}{1-s}-\hat{v}\right)\frac{f'(\hat{v})}{f(\hat{v})}\rho-1=0$. We can rule out solutions of the form $v^*>\hat{v}$ because $\left[\left(\frac{1}{1-s}-v^*\right)\frac{f'(v^*)}{f(v^*)}\rho-1\right]$ would be negative, and then we can see from (29) that $R^*>f(v^*)$, which is obviously a contradiction because human capital in the private sector cannot be negative.

Focusing then on the interval $[0,\hat{v}]$, note that the left hand side of equation (30) is increasing in v^*, tends to $\left(-\beta-\frac{\tau}{1-\tau}\right)<0$ at $v^*=0$, and tends to infinity at $v^*=\hat{v}$. Hence, by continuity, there is only one interior stationary value v^*.

To be sure that the stationary solution is unique we need to rule out corner solutions. By contradiction, suppose that $v^*=0$. Then, from (20) we have $(1-s)\geq\rho\cdot f'(0)\left(\frac{w^h}{w^l}\right)^*$. But this is a contradiction since $\left(\frac{w^h}{w^l}\right)^*$ tends to infinite as v^* tends to 0.

Concerning the other corner solution, assume that $v^*=1$. Then (21) implies that $\frac{1-s}{s}\leq\rho\frac{f'(1)}{\left(\frac{w^l}{w^h}\right)^*+f(1)}$ which is a contradiction under A1 since $f'(1)=0$, provided that $s<1$.

Note that this argument can be applied to rule out corner solutions also in equilibria different from the steady state. The key assumptions here are the infinite marginal productivity of a factor in zero supply and the condition $f'(1) = 0$.[6]

In summary, there is only one stationary solution, it is interior, v^* is given by (30) and the associated value for R^* is given by replacing v^* into (29).

5 Dynamics

To study the dynamics of the model we need to obtain the dynamic equation that links v_t and v_{t+1}. Thus, using (18) and (25) we obtain from the consumer's problem:

$$\frac{1}{\frac{1}{1-s} - v_t} = \frac{\rho \cdot f'(v_t)}{\left(\frac{1-\beta}{\beta}\right)\left(\frac{f(v_t)-R_{t+1}}{2-v_{t+1}}\right) + f(v_t)} \tag{31}$$

Now, evaluate (23) in $t+1$ to obtain:

$$R_{t+1} = \frac{f(v_t)}{\frac{1}{\frac{\tau}{(1-\tau)\beta} - \frac{s \cdot v_{t+1}(1-\beta)}{(2-v_{t+1})\beta}} + 1} \tag{32}$$

Substituting (32) into (31), and after some algebra, we obtain the following equation:

$$\rho \cdot \frac{f'(v_t)}{f(v_t)}\left(\frac{1}{1-s} - v_t\right) - 1 = \frac{1}{(2-v_{t+1})\left(\delta - \frac{s \cdot v_{t+1}}{2-v_{t+1}}\right)} \tag{33}$$

where $\delta = \frac{\beta}{1-\beta} + \frac{\tau}{(1-\tau)(1-\beta)}$.

Equation (33) defines a function $v_{t+1}(v_t)$. This function characterizes the dynamic behavior of the variable v. In particular, it is obvious that $v_{t+1}(v^*) = v^*$. And since v_t must belong to $[0, \hat{v}]$ for all periods, a *divergent* sequence $\{v_t\}_{t=1}^{\infty}$ satisfying (33) for all t is not an equilibrium, because it will eventually violate either condition *(d)* or *(e)* in the definition of equlibrium.

[6] Also note that we can rule out corner solutions with weaker assumptions, namely a sufficiently high marginal productivity of a factor in zero supply and a sufficiently low value for $f'(1)$. In order to save functional flexibility, this is the approach used in section 6.

Note, however, that the usual interpretation of absence of equilibrium for certain initial conditions does not apply in our model, because equation (33) represents a forward looking condition, not a backward looking one. As a consequence, the initial condition v_0 does not affect future values of v. The correct interpretation is the following:

Proposition 3 *There exists at least one equilibrium. Moreover, in any equilibrium, v_1 must be such that the sequence induced by (33) is non-divergent.*

Proof: Once v_1 is chosen, the rest of the sequence $\{v_t\}_{t=2}^{\infty}$ is uniquely determined by equation (33). Therefore, the values for v_1 that induce a divergent pattern are not equilibria, because eventually equilibrium conditions *(d)* or *(e)* will be violated. Hence the first young generation will not choose such values for v_1 in equilibrium. The exsitence of at least one equilibrium is easily shown by noting that $v_t = v^* \ \forall t$ is an equilibrium.■

In other words, the first young generation chooses v_1 based on the expectations $\{v_t\}_{t=2}^{\infty}$. And perfect foresight implies that expectations must be consistent, both internally and with regards to the value v_1 chosen. Hence, the first young generation neither makes an inconsistent choice of v_1 nor bases his decision on inconsistent expectations.

Using (33) we can easily establish the following property concerning $v_{t+1}(v_t)$:

Proposition 4 *The function $v_{t+1}(v_t)$ satisfies the following: $v_t > v^* \Rightarrow v_{t+1}(v_t) < v^*$, and $v_t < v^* \Rightarrow v_{t+1}(v_t) > v^*$*

Proof: Assume $v_t > v^*$. Then, $\rho \cdot \frac{f'(v_t)}{f(v_t)} \left(\frac{1}{1-s} - v_t \right) - 1 < \rho \cdot \frac{f'(v^*)}{f(v^*)} \left(\frac{1}{1-s} - v^* \right) - 1$. Now, (33) implies that $(2 - v_{t+1}) \left(\delta - \frac{s \cdot v_{t+1}}{2 - v_{t+1}} \right) > (2 - v^*) \left(\delta - \frac{s \cdot v^*}{2 - v^*} \right)$, which implies the result: $v_{t+1} < v^*$. If $v_t < v^*$ the proof is essentially the same.■

The previous proposition establishes that, if the steady state is stable, then convergence is non-monotone. However, remember from proposition 2 that the dynamics of this model are generated by forward looking behavior, so unstable equilibria can be ruled out. Therefore, we need to study stability in order to see if non-monotonicity (and multiplicity of equilibria) is possible.

We can calculate explicitly the function $v_{t+1}(v_t)$ from (33):

$$v_{t+1}(v_t) = \frac{2\delta}{\delta + s} - \frac{1}{(\delta + s) \left(\rho \cdot \frac{f'(v_t)}{f(v_t)} \left(\frac{1}{1-s} - v_t \right) - 1 \right)} \tag{34}$$

We can see clearly that it is decreasing in $[0, \hat{v}]$. The intuition behind this result is the following: Since (34) reflects a forward looking condition, then the fact that the function is decreasing means that a generation wants to study more if the next generation is expected to study less. But if generation t believes that generation $t+1$ will study less, then generation t will have more working time to complement with his human capital in $t+1$. Therefore, the relative price of human capital will be higher in $t+1$, and it is optimal for generation t to study more.

The slope of the function $v_{t+1}(v_t)$ is what determines stability in our model. It can be obtained by taking the derivative with respect to v_t in expression (34):

$$\frac{dv_{t+1}(v_t)}{dv_t} = -\left(\frac{1}{\delta+s}\right) \frac{\frac{f'(v_t)}{f(v_t)} - \left(\frac{1}{1-s} - v_t\right)\left[\frac{f''(v_t)}{f(v_t)} - \left(\frac{f'(v_t)}{f(v_t)}\right)^2\right]}{\rho\left[\frac{f'(v_t)}{f(v_t)}\left(\frac{1}{1-s} - v_t\right) - \frac{1}{\rho}\right]^2} \tag{35}$$

We see that $\frac{dv_{t+1}(v_t)}{dv_t} < 0$ for $v_t \in (0, \hat{v})$. Therefore, if the steady state is locally stable, then there can be equilibria with a non-monotone convergence across all the transition. Other properties of the function are presented in the following proposition:

Proposition 5 *The function $v_{t+1}(v_t)$ satisfies the following:*
(a) $\lim\limits_{v_t \to 0} v_{t+1}(v_t) = \frac{2\delta}{\delta+s}$
(b) $\lim\limits_{v_t \to \hat{v}} v_{t+1}(v_t) = \lim\limits_{v_t \to \hat{v}} \frac{dv_{t+1}(v_t)}{dv_t} = -\infty$
(c) $\lim\limits_{v_t \to 0} \frac{dv_{t+1}(v_t)}{dv_t} = -\frac{1-s}{\rho(\delta+s)}$
(d) $\frac{d^2 v_{t+1}(v_t)}{(dv_t)^2} < 0$ *for* $v_t \in (0, \hat{v})$

Proof: Parts (a), (b) and (c) are just a matter of taking the corresponding limit in the appropriate function, (34) or (35). To prove part (d), note that (35) can be expressed as

$$\frac{dv_{t+1}(v_t)}{dv_t} = \left(\frac{-1}{\rho(\delta+s)}\right) \frac{\frac{1}{\frac{f'(v_t)}{f(v_t)}\left(\frac{1}{1-s} - v_t\right)} + 1 + \left(\frac{-f''(v_t) f(v_t)}{(f'(v_t))^2}\right)}{\left(1 - \frac{1}{\rho \frac{f'(v_t)}{f(v_t)}\left(\frac{1}{1-s} - v_t\right)}\right)^2 \left(\frac{1}{1-s} - v_t\right)} \tag{36}$$

The result is obtained by noting that under assumption A1 the numerator of the second fraction is increasing in v_t whereas the denominator is decreasing in v_t.$\blacksquare$

The properties concerning $\frac{dv_{t+1}(v_t)}{dv_t}$ (and in particular property (b) in proposition 4) ensure that there is no global stability, in the sense that there are sequences $\{v_t\}_{t=1}^{\infty}$ satisfying (34) and starting into the interval $(0, \hat{v})$ that are not credible, because eventually the values of the sequence will lie outside the interval $(0, \hat{v})$ if v_1 is near enough 0 or $\hat{v}$.

Thus, we need to study if the steady state is locally stable, in which case there is a continuum of equilibria starting in a neighborhood of v^* and converging non-monotonically to the steady state. Note that local stability is characterized by the condition $\left|\frac{dv_{t+1}(v^*)}{dv_t}\right| < 1$. In addition, there can be pairs $\{\tilde{v}, \bar{v}\}$ with $\tilde{v} < v^* < \bar{v}$ such that $v_{t+1}(\tilde{v}) = \bar{v}$ and $v_{t+1}(\bar{v}) = \tilde{v}$, so that the non-convergent sequence $\{\tilde{v}, \bar{v}, \tilde{v}, \bar{v}, \tilde{v}, \bar{v}, \ldots\}$ is also an equilibrium.

If we have neither local stability nor persistent oscillations of period 2, then perfect foresight implies that the sequence $\{v^*, v^*, v^*, \ldots\}$ is the *unique* equilibrium.

The next task is to obtain necessary and sufficient conditions for both local stability and persistent oscillations of period two. Now, it is worth noting that the policy parameters cannot be chosen freely in the open square $(0,1)^2$. As an example, note that taking $s \to 1$ and $\delta \to 0$ is not feasible, because if $\beta > 0$, then $\delta \to 0$ implies $\tau \to 0$, and the subsidy needs to be financed by some taxes. In general, there is a function $\hat{\tau}(s)$ that gives us the minimum feasible tax rate for a given subsidy.[7] Thus, the constraint that the two policy parameters belong to the set $\{s, \tau : \tau \geq \hat{\tau}(s)\}$ is implicitly assumed in this paper.

A necessary condition for both local stability and persistent oscillations of period two is given in the following proposition:

Proposition 6 *Under assumption (A1) the following is true:*
(a) Local stability implies $\frac{1-s}{\rho(\delta+s)} < 1$
(b) The existence of persistent cycles with period 2 *implies* $\frac{1-s}{\rho(\delta+s)} < 1$

Proof: First recall from proposition 4 that $\lim_{v_t \to 0} \frac{dv_{t+1}(v_t)}{dv_t} = -\frac{1-s}{\rho(\delta+s)}$ and $\frac{d^2 v_{t+1}(v_t)}{(dv_t)^2} < 0$ for $v_t \in (0, \hat{v})$. Local stability is equivalent to $\left|\frac{dv_{t+1}(v^*)}{dv_t}\right| < 1$.

[7] Recall that tax is non-distorting, so government revenues are always increasing in taxes.

Now, part (a) is straightforward because $\frac{dv_{t+1}(v_t)}{dv_t}$ is decreasing and hence $1 > \left|\frac{dv_{t+1}(v^*)}{dv_t}\right| > \left|\frac{dv_{t+1}(0)}{dv_t}\right| = \frac{1-s}{\rho(\delta+s)}$. To prove part (b) note that the existence of persistent oscillations of period 2 implies that there exist two points $\{\tilde{v}, \bar{v}\}$ and $\{\bar{v}, \tilde{v}\}$ different from the steady state that belongs both to the function $v_{t+1}(v_t)$ and to its inverse $v_{t+1}^{-1}(v_t)$. The point $\{v^*, v^*\}$ belongs to both functions. Now, by contradiction, suppose that $\frac{1-s}{\rho(\delta+s)} \geq 1$. By the strict concavity of $v_{t+1}(v_t)$ the absolute value of the slope of this function is increasing and therefore $\left|\frac{dv_{t+1}(v^*)}{dv_t}\right| > 1$ and $\left|\frac{dv_{t+1}^{-1}(v^*)}{dv_t}\right| < 1$. Now, since $\left|\frac{dv_{t+1}(v_t)}{dv_t}\right| > 1$ for $v_t \in (0, \hat{v})$, then $\left|\frac{dv_{t+1}^{-1}(v_t)}{dv_t}\right| < 1$ for $v_t \in (0, \hat{v})$. But this contradicts persistent oscillations because to the right of v^* the slopes of both functions are respectively greater that 1 and smaller that 1 in absolute value, so the two functions cannot cross themselves in a point different than $\{v^*, v^*\}$.■

Sufficient conditions are more difficult to obtain. In simulations performed by the author, the steady state is locally stable only under very rare parameter combinations, and typically the assumptions in A1 that ensure an interior solution do not hold. In any case, all the calibration exercises in section 6 yield parameter values that clearly imply local unstability and hence uniqueness.

It is interesting to study the role that the subsidy s can have on the existence of multiple equilibria. It can be seen that a small s may rule out both local stability and persistent oscillations. In particular, assume that $\delta\rho < 1$. Then, note that the condition $s \leq \frac{1-\delta\rho}{1+\delta}$ implies uniqueness, by proposition 4, so one can conclude that a small enough subsidy may rule out multiplicity.

Up to this point we have studied the positive features of the model. In the next section we will deal with the normative analysis.

6 Optimal policy

Having characterized the competitive equilibrium for a given policy $\{s, \tau\}$,[8] the next task is to find policies such that the associated equilibrium satisfies certain properties. Our focus will be on optimal outcomes. There are,

[8]In fact, the variable R_t should be part of the policy. Therefore when we speak about the policy $\{s, \tau\}$, what we mean is the policy composed by $\{s, \tau\}$, *and* the associated equilibrium values for R_t.

however, several issues to be discussed here.

First, remember that the model can exhibit multiplicity of equilibria. As a consequence, a number of problems arise. If, for example, for a certain pair of policies the model has multiplicity, we cannot directly compare them, because we would need to know the specific equilibrium that is going to be realized. We would need some equilibrium selection mechanism in order to make policies comparable. We will deal with this problem by selecting the stationary equilibrium for each policy with multiplicity. This makes all policies comparable, because the stationary equilibrium always exists. In addition, we will see later that in our simulations there is uniqueness, both in the initial and in the optimal policy, so the stationary assumption is not restrictive.

Second, since we are using an overlapping generations model, we cannot maximize the utility of the representative agent because, at best, we have a representative agent *each period*. As a consequence, optimality requires the existence of a planner who maximizes some social welfare function, involving utilities from all generations in the economy. Clearly, the optimal outcome will be different for different weights in the social welfare function. For example, if the planner puts all the weight in the current old generation, the optimal policy would probably be $s = \tau = R_t = 0$, since both education subsidies and public R&D yield only future benefits, and the unique current effect is that a bigger subsidy reduces current labor supply, causing current returns to human capital to decrease. Our strategy here will be to assume that the planner maximizes a weighted average of the utilities of all generations, with weights decreasing at the rate ρ.[9]

Third, as already discussed, the initial condition (i.e. v_0) has no effects on the equilibrium values of v_t, but the first value for public R&D, R_1, is indeed affected by v_0. We will not deal with this problem. Instead, we will focus on long term optimality, i.e., taking $v_0 = v^*$.[10] In addition, we will

[9]It should also be mentioned that the result that maximum steady state growth implies efficiency, present in some OLG models, does not hold in our model. This is because in our model the maximum growth rate is attained by taking the limit $\tau \to 1$ and $s \to 1$. But, as long as $\tau \to 1$, consumption is tending to 0 for all periods and generations, which shows that the result does not hold. This is the reason why we are using a social welfare function.

On the other hand, we will discuss later that the model is not best suited to deal with the optimal size of the government, so we will fix τ and maximize only over s. In this case, maximum growth is a sufficient, albeit not necessary, condition for Pareto efficiency.

[10]Note that in the model, the relative effects of initial conditions finish with the gener-

normalize $A_1 = 1$.

Fourth, we are avoiding a very serious problem, which is time inconsistency. We will assume that the planner selects a policy at the beginning and then it is committed to follow this policy forever.[11] The fact that we do not take as given initial conditions (remember from the previous paragraph that we are taking $v_0 = v^*$) mitigates in part the problem. We will later try to measure the size of this problem by introducing the initial condition and comparing the resulting equilibrium with the one obtained with $v_0 = v^*$.

Now we are able to describe the planner's problem. The planner realizes that individuals will react to its policy according to the competitive equilibrium. Therefore, we include the competitive equilibrium equations as constraints in the planner's problem. As already discussed, we take as initial conditions $v_0 = v^*$ and $A_1 = 1$.[12] Thus, the planner's problem is to choose τ, s, and R^* to maximize

$$\log\left(c_1^o\right) + \sum_{t=1}^{\infty} \rho^{t-1} \left[\log\left(c_t^y\right) + \rho \log\left(c_{t+1}^o\right)\right] \tag{37}$$

subject to

$$c_1^y = (1-\tau)(1-\beta)\left(\frac{f(v^*) - R^*}{2-v^*}\right)^{\beta} (1 - v^* + s \cdot v^*) \tag{38}$$

$$c_1^o = (1-\tau) \cdot \left[(1-\beta)\left(\frac{f(v^*) - R^*}{2-v^*}\right)^{\beta} + \beta\left(\frac{2-v^*}{f(v^*) - R^*}\right)^{1-\beta} f(v^*)\right] \tag{39}$$

$$c_t^y = c_1^y \cdot \left(A\left(R^*\right)\right)^{t-1}; t \in \{2, 3, \ldots\} \tag{40}$$

$$c_t^o = c_1^o \cdot \left(A\left(R^*\right)\right)^{t-1}; t \in \{2, 3, \ldots\} \tag{41}$$

ation born at $t = 2$, with the effect that R_1 has on A_2. However, this should not be seen as a very fast transition, since people in the model live for two periods, so one period is about 20-25 years. Also, the *scale* of consumption of all future generations is affected by A_2.

[11] Several studies deal with this problem. See, for example, Phelan and Stacchetti (2001), Benhabib and Rustichini (1997) or Chari and Kehoe (1990).

[12] Note that $v_0 = v^*$ implies that initial conditions are varying with the policy choice.

$$0 \leq R^* \leq f(v^*) \tag{42}$$

and the previous equations (28) and (30).

The first two constraints are simply the constraints in the consumer's problem, but with the equilibrium prices substituted. Notice that the variables v^* and R^* are given by equations (28) and (30).

Constraints (40) and (41) take into account the growth rate of the economy, which is $A(R^*)$. Finally, the first part of constraint (42) is to avoid the choice of non-affordable education subsidies,[13] and the second part is, as already discussed, to avoid $v^* > \hat{v}$.

We have calibrated the model for a set of european countries (Belgium, Spain, France, Netherlands, Portugal, United Kingdom, Czech Republic, Hungary and Slovak Republic). The two policy parameters are included in the calibration process. Also, we have solved the previous problem for each calibration. Since the tax in the model is non distortionary, and public expenses are only educational subsidies and R&D (which together represent a small fraction of total public spending), we have solved the maximization problem only over s, i.e. keeping the calibrated τ fixed. In other words, we are answering the question of how to allocate a given amount of resources between subsidies and R&D. But there is also another interesting question, which is: How much resources we should devote to both uses? The model (as it is now) is not able to answer it; an extension, commented in the conclusions, is required. Once the solution for s is obtained, we compare it with the previously calibrated parameter, in order to assess if actual policies are close to the optimal ones for each country.

The calibration process is described in the following lines.

The human capital production function that is going to be used is

$$f(v) = B\left(1 - e^{-n \cdot v}\right); B, n > 0 \tag{43}$$

Note that this function do not satisfy the condition $f'(1) = 0$ in assumption A1. Nevertheless, for large values of n, $f'(1)$ is very near to zero, and we indeed obtain in our calibrations large values of n, so that the solution is interior for all calibrations. It's easy to verify that the remaining requirements of assumption A1 are satisfied.

[13] Or, in our terminology, to ensure that $\tau \geq \hat{\tau}(s)$

Taking into account that a period should be interpreted as 20-25 years, we have composed 25 times the average real interest rate between 1991 and 2003[14] to calibrate ρ.

Concerning the policy parameters, the tax τ is chosen directly as the sum of public expenditures in tertiary education[15] and R&D, as a fraction of GDP. The subsidy is calibrated by dividing public spending in tertiary education over the total cost of studying. We assume that the opportunity cost of studying that comes from foregone earnings is given by the official minimum wage in the country. To estimate properly the cost of education we need to include also the monetary cost of education, which is absent in the model. We assume that the monetary cost of education is given by total expenditures in tertiary education.

The parameters β and n are calibrated in the following way. First, we take two times[16] the number of students in tertiary education divided by the number of workers as the empirical counterpart of v. Second, we obtain data on the ratio between average wage and minimum wage by dividing GDP per worker in 2000 over the official minimum wage. Note that in the model this ratio can be expressed as

$$1 + \frac{\beta}{1-\beta} \cdot \frac{1}{1 - \frac{R}{f(v)}} \tag{44}$$

Given these two sources of information, we plug the observed value for v in the equations of the model, and then we choose β and n to satisfy equations (28) and (30), and to match expression (44) with the observed ratio of average wage over minimum wage.

Concerning the function $A(R^*)$, we have chosen a concave function to reflect decreasing returns within a period in the production of knowledge:

$$A(R^*) = 1 + \phi \cdot (R^*)^{\alpha}; \phi \geq 0, 0 < \alpha < 1 \tag{45}$$

[14] The period is shorter for Czech Republic (2000-2002), Hungary (1999-2003) and Slovak Republic (2000-2003).

[15] We use data only on tertiary education because in the model the variable v is the time spent on education when agents have an alternative use of the time, which is working. Hence, we don't want to include previous stages of education.

[16] We multiply by two because v refers only to the time spent by the young generation, and data on the number of workers include both young and old people.

The data on both students and workers refers to the year 2000

We take $\alpha = 0.1$ to ensure that the long run solution is not going to be very dependant on parameter changes.

Up to this point, the remaining parameters are A_0, B and ϕ. We normalize $A_0 = 1$ and $B = 1$, and choose ϕ to replicate the long term growth rate of the economy, measured as the average real GDP growth rate between 1991 and 2003. The first normalization needs no justification, because it deals only with the units in which output is measured. But the second one deserves more reasoning, because we cannot change the scale of human capital without changing at the same time the scale of labor. We indeed can normalize $B = 1$ because it can be shown that this parameter only affects the long term growth of the economy, and this variable can be replicated using the parameter ϕ.

A summary of the results of the calibration is presented in table 1. The last column is the ratio between average wage and minimum wage.

Table 1: Results of the calibration

Country	τ	s	ρ	β	n	v	$\frac{\bar{W}}{W_{\min}}$
Belgium	0.0134	0.3623	0.3360	0.5697	8.33	0.1601	2.32
Spain	0.0111	0.3336	0.4053	0.7193	6.30	0.2082	3.56
France	0.0140	0.3222	0.3400	0.5643	8.57	0.1533	2.30
Netherlands	0.0163	0.3897	0.4328	0.4621	19.31	0.1133	1.87
Portugal	0.0125	0.3879	0.4788	0.6160	16.11	0.1406	2.61
United Kingdom	0.0100	0.2745	0.3811	0.5756	13.19	0.1294	2.36
Czech Republic	0.0111	0.4493	0.5705	0.6276	35.23	0.0940	2.69
Hungary	0.0125	0.4172	0.8131	0.7269	21.28	0.1485	3.67
Slovak Republic	0.0089	0.4309	0.8490	0.7135	27.70	0.1258	3.49

Having calibrated the model, we are now able to study optimal policy. As mentioned above, we will maintain fixed the parameter τ, and maximize over the parameter s. We have included the boundary case $s = 1$ in the maximization, in contrast with the previous sections. The reason is that when $s = 1$ the solution to the consumer's problem is the corner solution $v = 1$, and not in all cases the solution is continuous at $s = 1$. For example, it is easy to show that if $\frac{2\delta}{\delta+1} < 1$, then $\lim_{s \to 1} v(s, \ldots) < 1$. Nevertheless, enforcing $v = 1$ is always a possibility for the planner, and this is the reason why we need to include the case $s = 1$ in the maximization.

We have done two exercises. First, we have maximized (37) over s for each country using as initial condition $v_0 = v^*$ (case A). Second, we have tried to measure the importance of the time consistency problem (or the size of the incentives to deviate from a committed policy) by using as initial condition the calibrated value of v, which can be interpreted as the initial condition when the policy is changing to the optimal one (case B).

Table 2 presents the change in policy for all the countries considered, both under case A and case B. Note that once τ is fixed, policy is fully characterized by s. We present also the change in average annual growth rates.[17]

Table 2: Change to the optimal policy

Country	Obs. policy		Optimal policy			
			Case A		Case B	
	s	Gr(%)	s	Gr(%)	s	Gr(%)
Belgium	0.3623	1.55	0.299	2.11	0.226	2.22
Spain	0.3336	2.25	0.268	2.62	0.197	2.73
France	0.3222	1.28	0.332	1.26	0.279	1.35
Netherlands	0.3897	1.65	0.316	1.72	0.218	1.77
Portugal	0.3879	1.84	0.242	2.07	0.143	2.12
United Kingdom	0.2745	2.03	0.220	2.13	0.119	2.22
Czech Republic	0.4493	0.92	0.290	0.98	0.221	0.99
Hungary	0.4172	1.75	0.000	1.93	0.000	1.93
Slovak Republic	0.4309	1.11	0.000	1.38	0.000	1.38

We see that for all countries except France, the optimal subsidy is lower than the observed one. Clearly, when we switch to case B, the optimal subsidy is further reduced, because the first old agent does not benefit from any increases in s (he cannot study more), whereas in case A the first old agent starts with more human capital under a higher subsidy. Concerning growth under optimal policy, it is higher for a lower subsidy.[18] This is not a general result. For higher values of τ, it may be the case that growth is increasing in the subsidy, because a higher subsidy makes human capital cheaper, allowing the government to provide more public R&D with the same

[17] The column "observed growth" corresponds to the growth rate used in the calibration process, i.e. average real GDP growth rate between 1991 and 2003. The two growth columns under the label "optimal policy" refer to the steady state annual growth rate of the model under case A and case B, respectively.

[18] It should be noted that the extrapolation of growth rates depends crucially on the assumption about the parameter α. As a consequence, we should interpret results in a qualitative way, rather than cuantitative, until we have a more precise calibration or estimation of the parameter α. We nevertheless consider that our value (0.1) is quite conservative, and therefore is not responsible of the low optimal education subsidies obtained.

budgetary resources. If this effect is greater than the reduction in R&D expenses, the real quantity of human capital purchased by the government can be increased. But given that we have calibrated very small values for τ, the second effect dominates and we obtain a negative relationship between subsidies and growth.

Our optimality criterion is the maximization of a social welfare function involving utilities from all generations. Therefore, it is interesting to see if the change to the optimal policy is also a Pareto improvement, compared to the observed situation. If the answer is not, then we want to know what generations are worse-off under the new policy. This is done in table 3. The numbers refer to the period in which the generation is young, and the first old generation is represented by 0.

Table 3: Generations worse-off under the new policy

Country	Case A	Case B
Belgium	None	None
Spain	None	None
France	All	1
Netherlands	1	1
Portugal	None	None
United Kingdom	None	None
Czech Republic	1	1
Hungary	1	1
Slovak Republic	None	None

The welfare effects are different depending on generations. We have three main groups. First, note that a reduction in the subsidy always improves welfare of the first old generation, and viceversa. This is because the first old generation can only be affected by the factor prices, and a reduction in the subsidy increases government demand for human capital and at the same time increases the time devoted to work by the young. Both effects tend to benefit the first old generation. Second, future generations (in our case starting from the second) are also better-off under a lower subsidy, and viceversa. The reason is that, under our calibration, a lower subsidy implies a higher growth. Therefore, a generation will be better-off if it is sufficiently far in the future. For our calibration, this starts at generation 2. Finally, the first young generation is less straightforward to analyze, because there are two opposite effects. On one hand, a higher subsidy transfers resources from old to young people, when the marginal utility of consumption is much higher due to the missing financial market. The subsidy plays here the role of a substitute of the financial market. But on the other hand (under our low calibration of τ), a higher subsidy means a lower growth, and consumption of generation 1 when old is indeed affected once by the growth rate. This reduces further consumption when old, in addition to the pure intertemporal redistribution.

With these considerations in mind, we can classify countries in three main groups. First, there are countries (Belgium, Spain, Portugal, United Kingdom and Slovak Republic) in which the change to the optimal policy is a Pareto improvement both under case A and case B. This is because the change is towards a lower subsidy, and the mentioned growth effect dominates for generation 1. The second group (Netherlands, Czech Republic and Hungary) are countries in which the growth effect is dominated both under case A and case B. As a consequence, the change to the optimal policy is not a Pareto improvement, because generation 1 is worse-off. Finally, there is a country (France) in which the change in policy is towards a higher subsidy under case A. This case is particularly interesting because it appears to be a mistake: under the new optimal policy, all generations are worse-off. This is a technical problem related to the fact that under case A the initial human capital is varying with s. The optimization procedure chooses a high value of s such that the first young generation would be better-off *if* the initial human capital would be the steady state value under the new subsidy. But once we take into account that the initial human capital is lower, we obtain that all generations are worse-off. In other words, optimization under case A does not take into account transitional costs to a higher subsidy.

In summary, our model recommends educational subsidies between 0.220 and 0.332 (with the exceptions of Hungary and Slovak Republic, for which the model recommends no subsidies at all).[19] In almost all cases, this implies a reduction in subsidies to education and an increase in R&D expenses. Also, the model warns about the incentives to reduce subsidies further if initial conditions are taken into account. Concerning generational analysis, the model suggests that the unique generation that may be willing to have high subsidies is the first young generation, because the subsidy makes possible to transfer resources to the age in which the marginal utility of consumption is higher. On the other hand, future generations prefer higher growth[20] (more

[19] Given that, for our calibrated parameters, the subsidy has no growth effects, the whole recommended subsidy can be interpreted as a substitute of the missing financial market.

[20] Remember that the negative relation between subsidies and growth is not true in

R&D), and the first old generation prefers high demand of human capital (more R&D), and high supply of labor (less time devoted to study). This suggests a possible explanation for the high subsidies observed. If we had positive population growth, then the median voter would always be young. Of course, another explanation is that our calibration procedure overstates actual subsidies. This would be the case if, for example, the minimum wage used in the calibration is lower than the wage of a worker with no human capital.

7 Conclusions

We have developed an overlapping generations model as a framework to understand some effects of educational subsidies. We have shown that a change in the subsidy may lead the economy toward a situation of multiplicity of equilibria, and may generate cycles of period 2. We have also shown that these cycles can be persistent or non-persistent in the long run.

We have analyzed two effects that could explain a positive optimal subsidy. First, a higher subsidy encourages investments in human capital, making the future price of public R&D cheaper. As a consequence, it may be useful to transfer resources from public R&D to education subsidies in order to rise *real* quantity of R&D services. This is true if human capital under the current subsidy is low, and the tax is high. The second effect is due to the fact that in our model the young agent is borrowing constrained. He expects a high increment in his income along the life cycle, but he needs to pay the opportunity cost of education now, so he would want to borrow if possible. The absence of financial markets makes this impossible, and the government can enforce here a transfer of resources from old to young people using the subsidy to education, which is an imperfect subsitute of the missing financial market.

A calibration exercise for several european countries suggest that the optimal composition of expenses involve a reallocation of funds from educational subsidies to R&D expenses. The current state of our model, however, cannot say anythig about the optimal size of total expenses in both concepts.

There are some possible extensions of this work. First, as mentioned in the previous section, our calibration procedure is designed to study the

general. Under higher values of τ it is possible that a higher subsidy indeed increases long term growth, through a cheaper human capital.

optimal distribution of resources between educational subsidies and public R&D, but it does not allow us to analize if total amount of resources devoted to these two concepts should be increased. This is because the government in our model has only these two kinds of expenses and, as a consequence, if we would maximize over the tax, we would obtain unrealistic high values. Then, an interesting extension would be to include in the model pure public consumption, which would be an exogenous parameter to be calibrated, so that we can also maximize over the tax. Also, it would be positive to include leisure in the utility function, to ensure that the tax is distortionary. These two modifications would allow us to study the interesting problem of the optimal size of the government.[21]

Another important extension is to include a third period in agents' lives.[22] This would allow the existence of a financial market in which middle aged people save and young people borrow. By comparing the optimal subsidy with and without financial markets we would be able to isolate the two effects mentioned above, because in the economy with financial markets the effect related with borrowing constraints disappears.

Finally, the extension of the model to cover demographic issues may allow us to develop a political analysis of the actual policy. Remember that interests of different generations at a fixed moment in time are in general different so one would not expect the political equilibrium to satisfy any kind of optimality. As already mentioned, this may explain why actual subsidies are different from the optimal ones in democratic economies.

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[21]Or, at least, the optimal amount of resources that the government should invest in R&D and educational subsidies.

[22]See Boldrin and Montes (2002) for a similar model with three periods.

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Inflation Effects of a Monetary Union*

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Abstract

We present a very simple model useful to clarify the relation between the Balassa-Samuelson effect and the existence of a monetary union. More precisely, we obtain the usual result that countries with more productivity growth in the traded goods sector will have a bigger inflation rate. However, we find this effect only for countries belonging to a monetary union. As a consequence, a central bank of a monetary union can only target average inflation because inflation differentials are an inevitable consequence of productivity growth differentials across member countries.

We also explore the implications of the model concerning an enlargement of a monetary union. The result is that the direct effect of the accesion of high growth countries is a reduction in the average inflation of the monetary union. We also obtain the following additional effect for the candidate countries: if a candidate is growing faster than the monetary union average, then it will suffer an increase in its inflation rate when entering into the monetary union, in addition to the possible change in the monetary policy.

1 Introduction

We present a very simple model useful to clarify the relation between the Balassa-Samuelson effect and the existence of a monetary union. More pre-

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cisely, we obtain the usual result that countries with more productivity growth in the traded goods sector will have a higher inflation rate. However, we argue that this effect will only be present in countries that belong to a monetary union, whereas the partial effect of productivity growth for independent countries will be the opposite. As a consequence, a central bank of a monetary union can only target average inflation because inflation differentials are an inevitable consequence of productivity growth differentials across member countries.

Concerning the enlargement of a monetary union, one extended argument is that the accession of countries with high productivity growth will increase the average inflation rate of the monetary union due to the Balassa-Samuelson effect. Our claim here is that this argument is actually composed of two. First, the accession of rapid-growth countries will *decrease* average inflation. Second, the central bank could relax monetary policy as a response in such a way that the net effect on average inflation would be possitive. The fact that new (fast growing) countries will have more inflation than the average is compatible with a reduction in the average inflation rate because, if monetary policy does not change, the enlargement tends to reduce the inflation rates of all old members of the monetary union. And now we can see why the central bank could relax monetary policy: the reduction in inflation of old members can put inflation rates of slow growth countries very low, or even negative. In an independent paper, Sinn and Reutter (2001), using a similar argument, obtain empirically a minimum inflation rate for Europe.

The model also identifies two effects on the inflation rate for the acceding countries. The first one is the possible change in monetary policy. The second one is an increase(decrease) in the inflation rate for countries growing faster(slower) than the monetary union average. This latter effect is independent of the monetary policy effect, so the total variation in the inflation rate is the combination of both effects. The intuition for this result is the following. Monetary policy in the monetary union must be defined in terms of aggregate money supply, i.e. total money supply for the whole union. However, the way in which this money supply is distributed accross members is not homogeneus. In particular, the model predicts that a fast growing country will attract money from the slow growing countries.

Using data on money supply, inflation and growth, we assess the effect that becoming a member of EMU will have on the inflation rate of an acceding country. We also decompose this effect into the two components exposed in the previous paragraph.

In the light of the model, our interpretation of the existing inflation differentials in the European Monetary Union (EMU) is the existence of productivity growth differentials, translated into inflation through the Balassa-

Samuelson effect, whereas in other countries differences are mainly determined by different monetary policies. The Balassa-Samuelson argument for a monetary union works as follows. The prices of tradeable goods cannot differ between countries due to arbitrage. Therefore, inflation differentials must come from the prices of non-tradeable goods or services. In addition, it appears that the productivity in the sector of tradeable goods grows faster than in the services sector. Now, a country with a higher productivity growth in the tradeable goods sector will have a higher growth in wages, which implies a higher difference between both sectorial inflation rates. Finally, since prices of tradeable goods are the same across countries, a faster-growing country will have a higher inflation in the services sector and hence a higher aggregate inflation.

There are, however, other explanations for inflation differentials that we want to mention. One possible explanation is that prices are simply adjusting because the fixed exchange rates between local currencies were (more or less) wrongly fixed when the common currency was launched. In the EMU, the adjustment cannot come from a movement of the exchange rates, and therefore if a currency was priced at a depreciated value, the price level of this country will tend to raise until a new equilibrium is reached. However, this can hardly be the reason behind the inflation differentials in the EMU: This adjustment process would be very fast, due to arbitrage opportunities, and on the other hand inflation differentials remain today, several years after the fix in exchange rates.

Another possible reason is that the countries with a high inflation have structural problems, like lack of competitive markets or high labor costs. If this is true, economic institutions in these countries must work towards a solution to these problems. But in this case, one should only expect level effects in prices.[1] It is difficult to argue that these structural problems can influence inflation rates in a persistent way.

The rest of the paper is organized as follows. In the next section we present the model. In sections 3 and 4 we analyze the equilibrium for countries with independent monetary policy and countries inside a monetary union, respectively. In section 5 we study the inflation effects of a monetary union for a candidate country. In section 6 we present the macroeconomic data of the EMU, and we see if the predictions of the model are in the data. Also we make a simulation exercise to see the possible effects of the enlargement of the EMU. Finally, we present in section 7 final conclussions, and

[1] If there are inflation differentials, either between countries or between sectors, not supported by different developments in real costs or productivities, then some relative price is tending to infinity, which is clearly not sustainable in the long run.

some thoughts about european policies.

2 The Model

In this section we present a very simple model that helps to explain inflation differentials between countries of a monetary union as a function of productivity growth differentials. The model is a reduced form of a cash-in-advance model. The link between the two models is presented in appendix 1.

Assume that there is a set of countries, which is denoted by I, and $i \in I$ represents a generic country.

The population of each country has an endowment of one unit of time at each period. There is no leisure, so the consumers will sell all their labor endowment in the market. We will suppose that migration costs are prohibitive, so that we will not have migration at equilibrium. Alternatively, we could suppose that productivity differentials are embodied in people, for example in the form of human capital, and therefore the incentives for migration are small and could hardly compensate migration costs.

There are two sectors in each country. Both sectors use only one input, labor, to produce the output.

The first one is the services sector, with a per capita output denoted by S_t^i. The productivity in this sector is equal to the constant s, which is the same for all countries and periods. The output of this sector is not tradeable between countries.

The other sector, the goods sector, produces a per capita quantity of G_t^i. All countries enjoy exogenous technological progress. This productivity growth is not common to all countries, so two different countries will have in general different productivity growth rates. The productivity of the goods sector in country i at period t will be denoted by A_t^i, and its growth rate will be denoted by $\gamma_{A_t^i}$. In the sequel, we will denote the growth rate of a variable by the symbol γ. Finally, the output of this sector is tradeable between countries.

Suppose that country i devotes to the goods sector a fraction θ_t^i of its labor force at period t. Then, the per capita output of both sectors can be expressed as:

$$S_t^i = s\left(1 - \theta_t^i\right) \tag{1}$$

$$G_t^i = A_t^i \cdot \theta_t^i \tag{2}$$

Now, note that the price of goods (in terms of local currency) will be the same across countries at each period t, because goods are tradeable between countries. Specifically, let Pg_t^i be the price of goods, in local currency terms, in country i at period t. Now, let δ_t^{ij} be the price of one monetary unit of country i in terms of monetary units of country j. Then, the prices Pg_t^i and Pg_t^j satisfy the following relation:

$$Pg_t^j = Pg_t^i \cdot \delta_t^{ij} \tag{3}$$

Now, if countries belong to a monetary union, the price of goods is the same across countries: $Pg_t^i = Pg_t, \forall i \in I$.

On the other hand, the price of services may be different across countries even in a monetary union, because they are not tradeable. Therefore, we will denote by P_t^i the price of services, in local currency terms, in country i at period t, no matter if we have a monetary union or not.

Since there is no migration, wages may be different across countries. As in the price of services, let W_t^i denote the price of one unit of labor, in local currency terms, in country i at period t, no matter if we have a monetary union or not.

Now, profit maximization in both sectors implies the following conditions:

$$P_t^i = \frac{W_t^i}{s} \tag{4}$$

$$Pg_t^i = \frac{W_t^i}{A_t^i} \tag{5}$$

Using (4) and (5) we can obtain the relation between prices in one country:

$$P_t^i = \frac{Pg_t^i \cdot A_t^i}{s} \tag{6}$$

To close the model, we assume that there is a representative consumer in each country, endowed only with one unit of labor each period. The consumer sells this unit in the labor market. The preferences over goods and services are represented by a Cobb-Douglas utility function. Then, the problem is to maximize

$$\left(G_t^i\right)^{\alpha_i} \left(S_t^i\right)^{1-\alpha_i} \tag{7}$$

subject to

$$Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i = W_t^i \tag{8}$$

The solution to this problem is given by the following demand functions:

$$G_t^i = \alpha_i \frac{W_t^i}{Pg_t^i} \tag{9}$$

$$S_t^i = (1 - \alpha_i) \frac{W_t^i}{P_t^i} \tag{10}$$

Since relative prices are determined by non-arbitrage conditions, these demand functions determine only the fraction of time devoted to each sector. Consequently, using (4), (5), (9) and (10) we can conclude that

$$\theta_t^i = \alpha_i \tag{11}$$

Note that, due to the fact that there is only one tradeable commodity, trade between countries will not occur in equilibrium.

3 Countries with Independent Monetary Policies

Assume that per capita money supply in each country is given by M_t^i. Then, the monetary equilibrium[2] is given by the following equation

$$Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i = M_t^i \tag{12}$$

assuming that expenditures equate the demand for money. Alternatively, one can use income as the demand for money. In this case, the monetary equilibrium is given by

$$W_t^i = M_t^i \tag{13}$$

Both expressions are equivalent. As already pointed out, the reader can find some microfoundations of equations (12) or (13) in the appendix 1.

Monetary policy is summarized by the parameter μ_t^i, which is a measure of money supply growth rate.

Then, using equations (4), (5) and (13), the growth rate of prices in both sectors is given by

$$\gamma_{P_t^i} = \mu_t^i \tag{14}$$

$$\gamma_{Pg_t^i} = \frac{1 + \mu_t^i}{1 + \gamma_{A_t^i}} - 1 \tag{15}$$

[2] Assuming that velocity of money is one.

Using (3), we can express the exchange rates as a function of monetary policies:

$$\delta_t^{ij} = \frac{M_t^j}{M_t^i} \cdot \frac{A_t^i}{A_t^j} \tag{16}$$

Now, define inflation in country i, π_t^i, as

$$1 + \pi_t^i = \lambda^g \left(1 + \gamma_{Pg_t^i}\right) + \lambda^s \left(1 + \gamma_{P_t^i}\right) \tag{17}$$

with

$$\lambda^g = \frac{Pg_t^i \cdot G_t^i}{Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i} = \alpha_i \tag{18}$$

$$\lambda^s = \frac{P_t^i \cdot S_t^i}{Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i} = 1 - \alpha_i \tag{19}$$

Hence, we can express the inflation rate as:

$$1 + \pi_t^i = \alpha_i \frac{1 + \mu_t^i}{1 + \gamma_{A_t^i}} + (1 - \alpha_i)\left(1 + \mu_t^i\right) = \left(1 + \mu_t^i\right)\left(\frac{\alpha_i}{1 + \gamma_{A_t^i}} + (1 - \alpha_i)\right) \tag{20}$$

We can see that, in countries with independent monetary policy, the inflation rate depends negatively on the size of the goods sector (α_i), and its productivity growth ($\gamma_{A_t^i}$), and depends positively on the monetary policy itself (μ_t^i). Consequently, we need a money growth measure to be able to compare inflation rates across countries. In particular,

$$\frac{\partial \pi_t^i}{\partial \gamma_{A_t^i}} = -\frac{\left(1 + \mu_t^i\right)\alpha_i}{\left(1 + \gamma_{A_t^i}\right)^2}$$

Hence, the negative effect of $\gamma_{A_t^i}$ on π_t^i is more intense the higher is μ_t^i.

In summary, the model establishes that a higher inflation differential across sectors caused by the Balassa-Samuelson effect implies a lower inflation rate in goods rather than a higher inflation rate in services, if monetary policy does not change.

Another interesting question is how productivity growth differentials could affect the relative price between two countries, or the real exchange rate. From (3), it is clear that the real exchange rate computed only with good prices is always equal to 1. However, if we compute the quotient between aggregate price indexes of both countries, this is no longer true. Let's call this quotient ϑ. Then, from (16) and (20) we get:

$$\begin{aligned}
1+\gamma_{\vartheta} &= \frac{1+\pi_t^i}{1+\pi_t^j}\left(1+\gamma_{\delta_t^{ij}}\right) \\
&= \frac{\left(1+\mu_t^i\right)\left(\frac{\alpha_i}{1+\gamma_{A_t^i}}+(1-\alpha_i)\right)}{\left(1+\mu_t^j\right)\left(\frac{\alpha_j}{1+\gamma_{A_t^j}}+(1-\alpha_j)\right)}\frac{\left(1+\mu_t^j\right)\left(1+\gamma_{A_t^i}\right)}{\left(1+\mu_t^i\right)\left(1+\gamma_{A_t^j}\right)} \\
&= \frac{\alpha_i+(1-\alpha_i)\left(1+\gamma_{A_t^i}\right)}{\alpha_j+(1-\alpha_j)\left(1+\gamma_{A_t^j}\right)} \\
&= \frac{(1-\alpha_i)\,\gamma_{A_t^i}}{(1-\alpha_j)\,\gamma_{A_t^j}}
\end{aligned}$$

Hence, movements in the real exchange rate calculated with both goods and services are determined by relative productivity growth rates and relative service shares.

4 Countries Inside a Monetary Union

Let β_j denote the weight of country j in the monetary union.[3] Now, the money supply is M_t for the whole monetary union. Since the output of the goods sector is tradeable, the price needs to be the same in all countries inside the monetary union, say Pg_t. As before assume that

$$\gamma_{M_t} = \mu_t \tag{21}$$

Now, the monetary equilibrium is characterized by the equation:

$$\sum_{j\in I}\beta_j\left(Pg_t\cdot G_t^j+P_t^j\cdot S_t^j\right)=M_t \tag{22}$$

Or, in terms of income:

$$\sum_{j\in I}\beta_j\cdot W_t^j=M_t \tag{23}$$

[3] These weights should be proportional to population measures, not to production measures, because all variables in equations (12) and (13) are in *per capita* terms.

Now we can use (5) to substitute for W_t^j:

$$\sum_{j\in I}\beta_j\cdot Pg_t\cdot A_t^j = M_t$$

$$Pg_t = \frac{M_t}{\sum_{j\in I}\beta_j\cdot A_t^j} \tag{24}$$

The inflation rate of the goods sector depends on the (common) monetary policy (M_t) as well as on the weighted average productivity of the goods sectors across the countries inside the monetary union. As before, assume it is denoted by γ_{Pg_t}. Note that this rate is the same for all countries, and it depends negatively on the productivity growth rates of all countries.

We can use (6) to obtain the inflation rate of the services sector for country i:

$$1+\gamma_{P_t^i} = \left(1+\gamma_{Pg_t}\right)\left(1+\gamma_{A_t^i}\right) \tag{25}$$

We see that it depends on the common inflation in the goods sector, and on the productivity growth rate of country i.

The last step is to obtain the average inflation rate combining the growth rates of both prices:

$$1+\pi_t^i = \alpha_i\left(1+\gamma_{Pg_t}\right)+(1-\alpha_i)\left(1+\gamma_{P_t^i}\right) \tag{26}$$

Now, introducing (25) into (26):

$$1+\pi_t^i = 1+\gamma_{Pg_t}+(1-\alpha_i)\cdot\gamma_{A_t^i}\cdot\left(1+\gamma_{Pg_t}\right) \tag{27}$$

We see that the differences in the inflation rate between countries depend only on the product of the productivity growth in the goods sector of the country and the share of the services sector in the country, $(1-\alpha_i)\cdot\gamma_{A_t^i}$.

More precisely, we can summarize these findings in the following result:

Theorem 1 *Under the above conditions, for each pair of countries i,j we have:*

$$\pi_t^i \geq \pi_t^j \Longleftrightarrow (1-\alpha_i)\cdot\gamma_{A_t^i} \geq (1-\alpha_j)\cdot\gamma_{A_t^j}$$

Proof: Immediate from equation (27).■

5 Inflation Effects of a Monetary Union

Now suppose that a country i with an independent monetary policy is considering to enter into a monetary union. We can compare the inflation rate under the independent regime with the inflation rate under the monetary union. Clearly, the result of the comparison will depend on the differences between the independent policy and the policy adopted by the union. If the country has a more expansive monetary policy than the union, then probably inflation will be reduced when entering into the union.

To isolate other effects from the change in monetary policy, assume that the monetary growth under the independent regime is equal to the monetary growth of the union, $\mu_t^i = \mu_t$. Remember that the inflation rate under the independent regime is given by equation (20). Now we need to obtain the explicit expression of the inflation rate under the monetary union. We can rewrite (27) as

$$1 + \pi_t^i = \left(1 + \gamma_{Pg_t}\right)\left(1 + (1 - \alpha_i) \cdot \gamma_{A_t^i}\right) \tag{28}$$

Now let's call $\bar{A}_t = \sum_{j \in I} \beta_j \cdot A_t^j$. Then, equation (24) can be expressed as

$$Pg_t = \frac{M_t}{\bar{A}_t} \tag{29}$$

or, alternatively

$$1 + \gamma_{Pg_t} = \frac{1 + \mu_t}{1 + \gamma_{\bar{A}_t}} \tag{30}$$

Now, plug (30) into (28) to obtain the explicit expression of the inflation rate of a country inside the monetary union:[4]

$$1 + \pi_t^i = \frac{1 + \mu_t}{1 + \gamma_{\bar{A}_t}} \cdot \left(1 + (1 - \alpha_i) \cdot \gamma_{A_t^i}\right) \tag{31}$$

At this point we are able to characterize the relative size of both inflation rates:

Theorem 2 *Suppose that monetary growth is the same in both regimes, $\mu_t^i = \mu_t$. Then, the inflation rate under the independent regime will be higher or equal than the corresponding rate inside the monetary union if and only if $\gamma_{\bar{A}_t} \geq \gamma_{A_t^i}$.*

[4] Note that $\bar{A}_t$ includes A_t^i.

Proof: Comparing equations (20) and (31) we can obtain the following set of inequalities

$$\begin{aligned}
\left(1+\mu_t^i\right)\left(\frac{\alpha_i}{1+\gamma_{A_t^i}}+(1-\alpha_i)\right) &\geq \frac{1+\mu_t}{1+\gamma_{\bar{A}_t}}\cdot\left(1+(1-\alpha_i)\cdot\gamma_{A_t^i}\right)\\
\alpha_i+(1-\alpha_i)\left(1+\gamma_{A_t^i}\right) &\geq \frac{1+\gamma_{A_t^i}}{1+\gamma_{\bar{A}_t}}\cdot\left(1+(1-\alpha_i)\cdot\gamma_{A_t^i}\right)\\
1+(1-\alpha_i)\cdot\gamma_{A_t^i} &\geq \frac{1+\gamma_{A_t^i}}{1+\gamma_{\bar{A}_t}}\cdot\left(1+(1-\alpha_i)\cdot\gamma_{A_t^i}\right)\\
1+\gamma_{\bar{A}_t} &\geq 1+\gamma_{A_t^i}
\end{aligned}$$

which proves the result.■

The intuition of this result, as stated in the introduction, is that although aggregate money supply is the same for all countries, the distribution accross them is not homogeneus. In particular, fast growing countries are attracting money from slow growing countries. As a consequence, money supply in fast countries is indeed growing faster than the aggregate money supply and hence they will have an additional inflationary pressure.

6 Macroeconomic Data

The Euro was implemented as the official currency by 12 countries of the European Union at the beginning of year 1999, and hence we will use data from 1999 up to the present. We portray in tables 1 and 2 the evolution of inflation and GDP growth for several groups of european countries. The column Av. is the average of the corresponding variable between 1999 and 2003. The first important feature that we see in the data about inflation and GDP growth is that there is at least some heterogeneity. The highest inflation rate is more than three times bigger than the lowest, and differences are greater in GDP growth.

Table 1: Inflation and GDP growth in EMU members

	Inflation						*GDP growth*					
Country	**99**	**00**	**01**	**02**	**03**	**Av.**	**99**	**00**	**01**	**02**	**03**	**Av.**
Belgium	1.1	2.7	2.4	1.6	1.5	1.9	3.2	3.8	0.6	0.7	0.8	1.8
Germany	0.6	1.4	1.9	1.3	1.0	1.2	2.0	2.9	0.8	0.2	-0.1	1.2
Greece	2.1	2.9	3.7	3.9	3.4	3.2	3.4	4.4	4.0	3.8	4.7	4.1
Spain	2.2	3.5	2.8	3.6	3.1	3.0	4.2	4.2	2.8	2.0	2.3	3.1
France	0.6	1.8	1.8	1.9	2.2	1.7	3.2	3.8	2.1	1.2	0.1	2.1
Ireland	2.5	5.3	4.0	4.7	4.0	4.1	11.3	10.1	6.2	6.9	1.6	7.2
Italy	1.7	2.6	2.3	2.6	2.8	2.4	1.7	3.1	1.8	0.4	0.3	1.5
Luxembourg	1.0	3.8	2.4	2.1	2.5	2.4	7.8	9.1	1.2	1.3	1.2	4.1
Netherlands	2.0	2.3	5.1	3.9	2.2	3.1	4.0	3.5	1.2	0.2	-0.9	1.6
Austria	0.5	2.0	2.3	1.7	1.3	1.6	2.7	3.4	0.8	1.4	0.9	1.8
Portugal	2.2	2.8	4.4	3.7	3.3	3.3	3.8	3.4	1.7	0.4	-0.8	1.7
Finland	1.3	3.0	2.7	2.0	1.3	2.1	3.4	5.1	1.1	2.3	1.5	2.7
EURO zone	1.1	2.1	2.3	2.3	2.1	2.0	2.8	3.5	1.6	0.9	0.4	1.8

Source: Eurostat

Table 2: Inflation and GDP growth in other european countries

	Inflation						*GDP growth*					
Country	**99**	**00**	**01**	**02**	**03**	**Av.**	**99**	**00**	**01**	**02**	**03**	**Av.**
Bulgaria	2.6	10.3	7.4	5.8	2.3	5.6	2.3	5.4	4.1	4.8	4.5	4.2
Cyprus	1.1	4.9	2.0	2.8	4.0	3.0	4.7	5.0	4.0	2.0	2.0	3.5
Czech Rep.	1.8	3.9	4.5	1.4	-0.1	2.3	0.5	3.3	3.1	2.0	2.2	2.2
Denmark	2.1	2.7	2.3	2.4	2.0	2.3	2.6	2.8	1.6	1.0	0.8	1.8
Estonia	3.1	3.9	5.6	3.6	1.4	3.5	-0.6	7.3	6.5	6.0	4.4	4.7
Hungary	10.0	10.0	9.1	5.2	4.7	7.8	4.2	5.2	3.8	3.5	2.9	3.9
Iceland	2.1	4.4	6.6	5.3	1.4	3.9	5.4	6.5	3.0	-0.6	2.1	3.2
Lithuania	0.7	0.9	1.3	0.4	-1.1	0.4	-1.8	4.0	6.5	6.8	6.6	4.4
Latvia	2.1	2.6	2.5	2.0	2.9	2.4	2.8	6.8	7.9	6.1	6.0	5.9
Norway	2.1	3.0	2.7	0.8	2.0	2.1	2.1	2.8	1.9	1.0	1.0	1.8
Poland	7.2	10.1	5.3	1.9	0.7	5.0	4.1	4.0	1.0	1.4	3.3	2.8
Romania	45.8	45.7	34.5	22.5	15.3	32.2	-1.2	2.1	5.7	4.9	4.6	3.2
Slovenia	6.1	8.9	8.6	7.5	5.7	7.4	5.9	4.1	2.9	2.9	2.1	3.6
Slovak Rep.	10.4	12.2	7.0	3.3	8.8	8.3	1.5	2.0	3.8	4.4	3.8	3.1
Sweden	0.6	1.3	2.7	2.0	2.3	1.8	4.6	4.3	0.9	1.9	1.4	2.6
United Kingdom	1.3	0.8	1.2	1.3	1.4	1.2	2.8	3.8	2.1	1.7	2.0	2.5

Source: Eurostat

We have also differences in the data of european countries outside the EMU.

As we see in equations (27) and (31), our model predicts that inflation should be positively related with GDP growth inside the EMU. Further, due to differences in monetary policy, the model also predicts a very weak relation between both variables for countries outside the EMU, or for Euro countries before the existence of EMU, unless we introduce money supply as a control variable. To check these facts, we have run two regressions.[5] We have taken real GDP growth[6] as the regressor, and inflation as the dependent variable. The first regression uses the average data from countries inside the EMU from 1999 to 2003. The results are as follows (standard errors are shown below the estimated parameter):

[5]All variables in this regressions are expressed in percentage points. We have used average annual growth rates of prices, money supply and GDP. Please note that we are not using annual data, so there is only one observation per country. As a consequence, the number of observations is very low and results should be interpreted with caution.

[6]The results do not change neither qualitatively nor quantitatively if we use real GDP per capita or real GDP per worker instead. The two equations are, respectively, $HICPgr = \underset{(0.3888)}{1.6961} + \underset{(0.1491)}{0.3576} \cdot GDPgr$ and $HICPgr = \underset{(0.3089)}{2.1304} + \underset{(0.1834)}{0.3136} \cdot GDPgr$.

$$HICPgr = \underset{(0.3786)}{1.5792} + \underset{(0.1194)}{0.3333} \cdot GDPgr \tag{32}$$

where $HICPgr$ represents the inflation rate calculated using the HICP, and $GDPgr$ represents the growth rate of real GDP. Clearly, the positive relationship between inflation and GDP growth is significative. The R^2 statistic is 43.8%. This is exactly what the model predicts.

Now let's focus on countries with independent monetary policies. To avoid differences in the countries included in the sample, we have repeated the exercise for the same set of countries, but before the Euro was launched.[7] Equation (20) establishes that the marginal effect of productivity growth on inflation is negative. Also, we need to include money supply as a regressor if we want to isolate this marginal effect of productivity growth. This is studied in the second regression:

$$ICPgr = \underset{(0.6198)}{2.5470} - \underset{(0.2354)}{1.2277} \cdot GDPgr + \underset{(0.0827)}{0.5812} \cdot Mgr \tag{33}$$

Mgr is the growth rate of the money measure M2. The two marginal effects are clearly significative, and they have the expected sign. The R^2 statistic of this regression is 85.5%. Again, this is what we expect from the model.

To further explore if this is a feature of countries with independent monetary policies, we have repeated regression (33) using data from european countries outside the EMU. According to the model, we expect the same signs in this new regression because we have in both cases countries with independent monetary policies. The estimated equation confirms these signs:

$$HICPgr = \underset{(2.4838)}{5.6723} - \underset{(0.6940)}{3.2123} \cdot GDPgr + \underset{(0.0868)}{0.9146} \cdot Mgr \tag{34}$$

Here Mgr is the growth rate of the money measure M2 except for Sweden, where M3 is used instead. As before, the two marginal effects have the expected sign and are significative.[8] The R^2 statistic of this regression is 89.5%.

[7] In particular, we have considered annualized growth rates between years 1991 and 1997. The HICP's have been substituted by the national ICP's.

[8] One striking result is that the coefficient of GDP growth is indeed too negative. One possible interpretation is that a higher growth gives sustainability to fiscal policy, and hence it reduces the expectations of using money to finance fiscal deficits. Probably, this effect was less important in EMU countries during the 90's due to the reputation and independence of most of their central banks. Therefore, the inflation response to an increase in GDP growth is greater for the acceding countries. See Sargent (1982) for details about this interpretation.

Hence, we can conclude that the different sign of the coefficient is due to the different behavior of inflation in countries with different monetary policy regimes.

We have repeated the analysis using growth rates of a given year instead of average growth rates across several years. The sign of all parameters for all years is the expected, although the larger dispersion of annual data tends to reduce significance. We have also tried using GDP deflactor instead of HICP, with very similar results.

Andrés et al. (1996) provide further evidence. They analyze the effects of inflation on growth, using four years averages. Their finding is that the coefficient is more negative for countries under a floating exchange rate regime. The evidence concerning countries under a fixed exchange rate regime is less clear; the coefficient is positive or negative depending on the specification, but in all cases its magnitude is small. To the extent that a fixed exchange rate regime is a situation close to a monetary union, their evidence supports our results.

In summary, our model has predictions qualitative close to the data, particularly in the medium or long run.

Now we can use the model to study the effects of the enlargement of EMU on the inflation rates of previous members and candidate countries, as well as on the average inflation rate of the enlarged Euro zone, assuming that Euro area monetary policy does not change.[9] We will analyze specifically the accession of ten countries;[10] other scenarios can be easily analyzed.

The model predicts different effects for countries previously in the EMU and candidate countries. Assume that money supply growth in EMU does not change. Assume also that productivity growth rates for all countries remain unchanged. If we look at equation (31), we see that the unique effect for a previous member is the possible variation in $\gamma_{\bar{A}_t}$. We can expect a catching-up process in the new members, so it is sensible to expect an increase in $\gamma_{\bar{A}_t}$ following the enlargement. Therefore, the prediction of the model for all previous members is a reduction in their inflation rates.

The same reasoning can be applied to the average inflation rate of the EMU. Interpreting the monetary union as an independent country, we can use equation (20) to make the analysis. The two effects that we can indentify in the equation are the change in productivity growth and the change in sectorial shares. We have argued that we expect an increase in the productivity growth. Concerning α_i, the candidate countries have services sectors smaller

[9] In other words, the effects described below are in addition to a possible change or reaction in the monetary policy of the Euro area.

[10] Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovak Republic, Slovenia, Cyprus and Malta.

than the EMU average, so the services share will decrease following the enlargement. Both effects work in the same direction, so the model predicts a reduction in the average inflation rate of the enlarged EMU.

The last step is to study the inflation effects for candidate countries. The model here identifies two different effects. One is the change in the monetary policy and the other is the money attraction effect described in section 5. In order to compare both effects and to see which of them dominates, we have made a simulation exercise, described in Appendix 2. This procedure estimates the total effect for candidate countries and splits it into the two mentioned effects, change in monetary policy and money attraction. The results are presented in the following table.[11]

Table 3: Inflation Effects of the Enlargement for Candidate Countries[12]

Country	Prev.Inf	New Inf	Total Ef.	Mon. Policy	Money Attr.
Cyprus	4.66	5.71	**1.05**	-3.96	5.01
Czech Rep.	2.98	0.57	**-2.42**	-1.19	-1.23
Estonia	4.64	4.91	**0.28**	-4.99	5.27
Hungary	8.72	3.48	**-5.24**	-8.44	3.2
Lithuania	-0.10	2.97	**3.08**	-0.05	3.12
Latvia	3.22	6.45	**3.23**	-4.58	7.81
Malta	3.26	1.80	**-1.47**	-1.53	0.06
Poland	4.87	0.67	**-4.20**	-3.59	-0.61
Slovenia	7.04	1.65	**-5.38**	-6.50	1.12
Slovak Rep.	5.61	2.19	**-3.42**	-4.61	1.19

Table 3 should be interpreted in the following way. The column labeled "Mon. Policy" gives the variation in inflation of a country if it implements the same money growth rate than the Euro area, but without entering into it. We obtain negative values for all countries, which means that monetary policy in Euro area is more restrictive than in all the acceding countries. The last column is the change in the inflation rate of a candidate country that would occur if monetary policy were the same both in EMU and in the candidate country. The predominance of positive values reflects the fact that almost all acceding countries have real GDP growth rates higher than the EMU average. Finally, column "Total Ef." is the combination of the two mentioned effects, i.e. it gives the total variation in inflation of a candidate

[11] We use here GDP deflactor as a measure of prices because HICP is not available for Malta. The results are, nevertheless, very similar using HICP.

[12] The data are expressed in percentage points.

country when becoming a member of the EMU, given its current real GDP and money growth rates. We can see that most countries would experiment a decrease in their inflation rates entering the EMU. And this is despite of the fact that these countries are growing faster than the EMU. The reason is that the effect of a change in monetary policy is so important that it dominates the money attraction effect.

7 Concluding Comments

We have presented a model that clarifies the relation between the Balassa-Samuelson effect and the existence of a monetary union. We argue that the partial effect of productivity growth on inflation is positive *only* for countries belonging to a monetary union. This partial effect for countries with independent monetary policies becomes negative once we take into account money supply growth. As a consequence, the central bank of a monetary union can only target average inflation rate, whereas inflation differentials will be determined by differences in productivity growth. We have also seen that this implications are close to the empirical evidence, especially for medium or long run data.

An additional result of the model is that if a country enters into a monetary union, then its inflation rate will rise if the country is growing faster than the union average, and viceversa. This effect is in addition to the effect of a change in the monetary policy. The intuition is that the central bank determines the total money supply, but the distribution of this total amount is not homogeneous: A fast growing country will attract money from low growing countries and, as a result, money supply of the former is indeed growing more than the average. This money attraction effect introduces an additional inflationary pressure for fast growing countries.

We have studied the implications of the model concerning the EMU enlargement. In particular, we have analyzed the effects of an enlargement when the acceeding countries grow faster than the EMU average. The model provides very clear predicitions for the inflation rates of previous members: they will be reduced. The same prediction applies to the EMU average inflation following the enlargement. Concerning candidate countries, the prediction of the model is not so clear. As already mentioned, the total effect is a combination of a change in monetary policy and a money attraction effect. We have made a simulation exercise to analyze both effects. The result is that the monetary policy effect is negative for all countries, which means that the EMU monetary policy is less expansive than the policies of candidates. On the other hand, the money attraction effect is positive for eight of the ten countries analyzed. The combination of both effects is negative for six of ten countries, so, in general, candidate countries will have less inflation inside the EMU.

There are several implications of the model that are very important for the EMU. First, the observed inflation differentials are not a short term phenomenon. They will remain until the countries grow at the same rate. The empirical evidence shows that the process of convergence in productivity growth is slow[13], so we cannot expect homogeneus inflation rates in the short run. An important implication is that a requirement of homogeneous inflation rates for EMU members is senseless.

There is an assumption in the model that deserves special mention, namely the absence of migration across countries. We have already mentioned two possibilities here, prohibitive migration costs or productivity differentials embodied in workers in the form of human capital. The policy implications are different. In the second case the governments cannot do anything but fostering real convergence to reduce inflation differentials. But in the first case, policies aimed to increase labor mobility are appropriate to achieve more homogeneous inflation rates. It is very likely that the european reality is in the middle of the two possibilities. If this is the case, there is room for economic policy in the task of reducing inflation differentials.

Another important implication is that candidate countries should have in mind that entering the EMU has a rising effect in inflation for fast-growing countries. We have seen, however, that this effect is dominated in most countries by a more restrictive monetary policy, so the predicted net effect is in most cases a decrease in inflation. On the other hand, the Euro zone countries need to take into account the predicted reduction in inflation rates. And if this imply a danger of deflation for some countries, all of them (EMU members and candidates) must realize the posibility of a movement in the European Central Bank (ECB) towards a more expansive monetary policy.

Concerning the requirement of homogeneus inflation rates for candidate countries before they enter the EMU, we can make the following reasoning. Monetary policy for candidate countries should be such that the change in the inflation rate is not very drastic at the entering moment. However, this doesn't imply that inflation rates should be very close to the average EMU inflation. It depends on the productivity growth differentials. As a consequence, our proposal is that inflation requirements should be designed taking into account growth differentials, i.e. allowing a fast growing country to have more inflation before entering, because it will still have more inflation after entering.

[13] See for example Barro and Sala-i-Martin (1992)

One can argue that the small growth of the services sector is due to measurement problems concerning the quality of these services. If this is true, the high inflation rate of services is reflecting an increase in quality. But as long as european policies are designed as a function of measured inflation, the fact that inflation is reflecting or not quality growth is not relevant. From our point of view, it is equivalent to define inflation thresholds as a function of GDP growth, or to impose a unique threshold, but for inflation rates that take into account quality increases.

Finally, we want to stress that the unique concern of this paper is the study of inflation rates. Of course, monetary unions have other benefits (like reductions in transaction costs) and costs (like difficulties to accommodate asymmetric shocks) that need to be analyzed by implicated governments.

Appendix 1

In this appendix we show that the presented model is a reduced form of a cash-in-advance constraint model. This would help to interpret equations (12) and (22).

Suppose that both goods and services need to be paid with money. The consumer spends her current money holdings and a direct transfer (T_t^i) from the government in purchasing goods and services. On the other hand, the consumer earns a wage, which is used to accumulate money for the next year. The transfer is financed by issuing more money, so it needs to satisfy

$$T_t^i = M_t^i - M_{t-1}^i \tag{35}$$

Now we are in a position to obtain the reduced model. The consumer's problem is to maximize

$$\left(G_t^i\right)^{\alpha_i}\left(S_t^i\right)^{1-\alpha_i}$$

subject to

$$Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i = M_{t-1}^i + T_t^i \tag{36}$$

$$M_{t-1}^i = W_{t-1}^i \tag{37}$$

Combining equations (36) and (37) we get

$$Pg_t^i \cdot G_t^i + P_t^i \cdot S_t^i = W_{t-1}^i + T_t^i \tag{38}$$

The first order conditions are

$$G_t^i = \alpha_i \cdot \frac{W_{t-1}^i + T_t^i}{Pg_t^i} \tag{39}$$

$$S_t^i = (1-\alpha_i) \cdot \frac{W_{t-1}^i + T_t^i}{P_t^i} \tag{40}$$

Now plug (35) and (37) into (39) and (40):

$$G_t^i = \alpha_i \cdot \frac{M_t^i}{Pg_t^i} \tag{41}$$

$$S_t^i = (1-\alpha_i) \cdot \frac{M_t^i}{P_t^i} \tag{42}$$

Finally, use equation (37) in period t to obtain equations (9) and (10). Equation (12) comes directly by plugging (35) into (36).

Appendix 2

The inputs of the calibration procedure are: Real GDP growth rate, GDP deflactor growth rate, population and percentage of workers in the services sector for all countries (Euro countries and the ten candidates mentioned in section 6). For all variables we take averages between 1999 and 2003. The source of all these data is Eurostat.

The calibration procedure is as follows: First, we take directly α_i as the percentage of workers in the services sector. Second, using real GDP growth rate and α_i, we take the value $\gamma_{A_t^i}$ that replicates the observed GDP growth. Now, for current Euro area countries we calculate the weights β_i using population. Using them, we obtain the weighted average $\gamma_{\bar{A}_t}$ for the EMU. Finally, the parameter μ_t is chosen as the one that minimizes the (weighted) sum of the squares of the differences between the observed inflation rates and the inflation predicted by equation (31), for Euro area countries.

In order to study the enlargement, we need to obtain the new weights for the enlarged monetary union using population of the 22 countries. With these weights we obtain the new weighted average $\gamma_{\bar{A}_t}$. At this point we can use again equation (31) to obtain an estimate of the new inflation rates (and the total effect of the enlargement) for the 22 countries.

To isolate the two effects described in section 5 for candidate countries we construct an intermediate inflation estimate by plugging the calibrated parameter μ_t in equation (20) for each of the candidate countries. This intermediate inflation tries to estimate the inflation rate for the country if it remains independent but implements the monetary policy of the EMU. Therefore, the difference between the observed inflation rate and the intermediate inflation captures the effect of a change in monetary policy, and the difference between the intermediate and the new estimated inflation rate captures the money attraction effect.

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