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WELFARE LOSSES UNDER COURNOT COMPETITION*

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Abstract

We find that in a market for a homogeneous good where firms are identical, compete in quantities and produce with constant returns, the percentage of wel-fare losses (PWL) is small with as few as five competitors for a class of demand functions which includes linear and isoelastic cases. However with fixed costs and asymmetric firms PWL can be large. We provide exact formulae of PWL and robust constructions of markets were PWL is close to one in these two cases. We show that the market structure that maximizes PWL is either monopoly or dominant firm, depending on demand. Finally we prove that PWL is minimized when all firms are identical, a clear indication that the assumption of identical firms biases the estimation of PWL downwards.

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1. Introduction

In his classical contribution Cournot (1837, Chapter 8) established that when the number of firms in a market tends to infinity, oligopolistic equilibrium tends to perfect competition. As a corollary, Welfare Losses (WL), measured as the difference between social welfare in the optimal and the equilibrium allocation, tend to zero. But, what happens when the number of firms is finite? Is perfect competition a good approximation or, on the contrary are WL significant? (see Hotelling (1938) and Yarrow (1985) for an early treatment of this problem).

As a first cut to the problem, assume that all firms are identical and costs and demand are linear. It is easily calculated that the percentage of WL under Cournot competition, denoted by PWL, is $1/(1+n)^2$ where n is the number of firms. Thus a market composed by 7 identical firms ("the seven sisters") produces a PWL of 1.56%, not a big number.[1] This poses a serious question: were WL systematically small there would be little to be gained by considering oligopolistic behavior: A simple equilibrium concept like perfect competition may be preferable. Moreover, the motivation for public policies is weakened under small WL. Then, the dilemma is, either we find environments in which oligopoly produces WL much greater than those found in the linear model or, we abandon the oligopoly model as a leading model describing markets.

Let us first comment on papers that are relevant to our problem. McHardy (2000) studies a model with quadratic demand and presents numerical calculations. He finds that WL can be up to 30% larger than those in the linear model, which is encouraging but still does not solve the problem. Anderson and Renault (2003) calculate PWL under the assumptions made above except that they assume an inverse demand function of the form $p = A - bx^{\alpha}$, (x is aggregate output and p market price).[2] They do not study if PWL differs substantially from those in the linear model. Johari and Tsitsiklis (2005) show that if firms are identical, average costs are not increasing and the inverse demand

[1] This formula shows that once linearity is assumed, as done implicitly by Harberger (1954), WL seldom goes up to big numbers except if the number of firms is very small. A list of other empirical papers measuring WL in oligopoly can be found in Tullock (2003) p. 2.

[2] This form of demand generalizes both linear ($\alpha = 1$) and isoelastic (with elasticity of demand $1/\alpha$) forms and allows for computation of equilibria. See González-Maestre (2000) for an early application.

function is concave, PWL is bounded above by $1/(2n+1)$, which is still not very large because a market with seven firms achieves more than 93% of maximum welfare.

Our paper is a quest for markets where oligopoly produces large WL and, thus, it is a relevant model of market competition. Specifically, the purpose of our paper is twofold.

1: To provide workable formulae for PWL which depend, as far as possible, on magnitudes that are observable.[3] We regard these formulae as the main contribution of the paper from the point of view of applied economics because they show which variables to look at when dealing with WL in an actual market.

2: To use these formulae to construct markets where the Cournot equilibria yields large PWL, sometimes close to one.[4] These constructions are the main contribution of the paper from the theoretical point of view because they show that oligopoly theory is valid as a general description of markets with the perfectly competitive case as a limit.

In Section 2 we consider the baseline model, which is that of Anderson and Renault. We might expect that for suitable values of α, WL were much higher than those in the linear case. However, by using numerical methods we find that the maximum PWL obtained in this case is not very different from the one obtained in the linear case. Moreover, for some values of α, PWL is arbitrarily small. Thus, the consideration of a more general class of demand functions does not bring significant WL associated with oligopoly, but on the contrary it adds to the suspicion that WL under oligopoly may be small. We then turn our attention to fixed costs and heterogeneous firms.[5].

In Section 3 we consider free entry with a fixed (actually sunk) cost. We provide formulae for the maximal and the minimal PWL where this magnitude depends on the number of firms and α. We show that when α and the fixed cost are not observable, for any exogenously given observation on market price, output, average variable cost and number of firms, PWL can be chosen arbitrarily (Proposition 1). In particular when α

[3] The parameter α, which can be estimated but not observed, enters in the formula of PWL in Anderson-Renault (2003), so it is unavoidable in the more general set ups considered in this paper.

[4] Johari and Tsitsiklis (2005) offer an example of a market where PWL is arbitrarily close to one but in which the inverse demand function is not differentiable.

[5] Other attempts to find higher WL focus on issues ouside market competition like "X-Inefficiency" (Leibenstein (1966) and Rent-Seeking (Tullock (1967).

tends to infinite, PWL can be chosen to be arbitrarily close to one. This result implies that any given price-marginal costs margin, or elasticity of demand, is compatible with *any* PWL. When the fixed cost can be observed, the observed variables must fulfill a condition which implies that entry is blockaded. We show that any observation fulfilling this condition is compatible with many -but not all- PWL (Proposition 2). In this section WL are due to the combination of the form of the demand function -we show that PWL with linear demand is large but far from one- and there is overentry -because the optimal number of firms is one.

In Section 4 we consider heterogeneous firms. We provide a formula for PWL where this magnitude depends (positively) on the share of the largest firm, (negatively) on the Hirschman-Herfindahl concentration index, denoted by H, and on α. We find that there are market shares, number of firms and α that yield PWL close to one whereas H is close to zero (Proposition 3). In particular, the most efficient firm has a market share of $1 - PWL$, and there is an infinite number of inefficient firms in the market, so all firms have a negligible market share. We check the robustness of this construction by considering the effects on PWL when one of the above magnitudes is held fixed. In all cases, PWL is large -not necessarily close to one- and negatively correlated with H.[6] All these results points out that H is not a reliable measure of WL.[7] More importantly, they show that the concept of a "large economy" must be taken with care because seemingly innocuous departs from the model where all firms are small and identical may have serious welfare consequences. Next, we prove that the market structure that maximizes PWL is a dominant firm when $\alpha > 0$ and monopoly for $\alpha < 0$ (Proposition 4). This shows that monopoly, the target of attacks of our profession from Adam Smith on, is not the worst outcome in terms of WL. Finally we prove that PWL is minimized when firms are identical (Proposition 5). This shows that proper care of the heterogeneity of

[6] In the linear case, H is positively related to PWL but the values of H are not a reliable estimate of PWL. Thus for $n = 5$, $H = .2$, which is a value considered high for some anti-trust authorities but $PWL = 2.7\%$, not a large number.

[7] That social welfare is increasing in the marginal cost of small firms was first pointed out by Lahiri and Ono (1988). For a criticism of the idea that concentration is generally bad for social welfare see Daughety (1990) and Farrell and Shapiro (1990).

firms is essential to obtain estimates of PWL that are not biased towards small PWL.

Finally, in Section 5 we offer some thoughts about our results. Our main conclusion is twofold. On the one hand, the search for WL in actual markets should focus on economies of scale and asymmetric firms, two facts that are seldom considered in the applied literature. On the other hand the oligopoly model is still alive and well as a leading model in the study of markets. Moreover, in some cases we turn the tables: the classical vision of markets as places where (large) surplus is created may be too optimistic and markets may create little or no surplus at all, at least in several relevant cases. Other important points are the characterization of the best and the worst possible market structures from the welfare point of view when firms are different and the construction of a "large" market where PWL is arbitrarily close to one.[8] Moreover, our paper suggests that, in general, large discrepancies of oligopolistic and perfectly competitive outcomes may exist but should not be taken for granted.

It goes without saying that important causes of WL are not considered here, i.e. product differentiation, investment, R&D, location, etc. The analysis of the impact of these variables on WL requires the consideration of games that are more complicated than those considered here and, consequently, they are left for future research.

2. The Baseline Model

There is a representative consumer with a utility function $U = Ax - \frac{bx^{\alpha+1}}{\alpha+1} - px$ where x is aggregate output, p is the market price, $b\alpha > 0$ and $\alpha > -1$. The maximization of utility generates an inverse demand function $p = A - bx^{\alpha}$. Notice that if $\alpha < 0$, $b < 0$, and $A = 0$ we have an isoelastic function $p = -bx^{\alpha}$. The linear case occurs if $\alpha = 1$.

There are n identical firms each producing a single output denoted by x_i, $i = 1, ..., n$. Thus $x \equiv \sum_{i=1}^{n} x_i$. Marginal cost is constant and denoted by c. Profits for firm i are $\pi_i \equiv (p - c)x_i$. Defining $a \equiv A - c$ we have that $\pi_i \equiv (a - bx^{\alpha})x_i$. Assume $ab > 0$ and $-A\alpha < cn$. These assumptions guarantee that output and price are positive in equilibrium (see (2.1) below).

[8] Other points that have already been noticed in the literature are the importance of the functional form of demand and the failure of H and price-marginal cost margins to capture WL.

If firms compete à la Cournot, the first order condition of profit maximization yields $a - bx^{\alpha} - b\alpha x^{\alpha-1}x_i = 0$. It is easy to check that the second order condition holds and that equilibrium is symmetric. Thus Cournot equilibrium output and market price are

$$x^* = (\frac{an}{b(n+\alpha)})^{\frac{1}{\alpha}} \quad \text{and} \quad p^* = \frac{A\alpha + cn}{n+\alpha}. \tag{2.1}$$

Social welfare, denoted by W, is the sum of industry profits and the utility of the representative consumer, i.e. $W = ax - \frac{bx^{\alpha+1}}{1+\alpha}$. The optimal aggregate output is found by maximizing W, namely

$$x^o = (\frac{a}{b})^{\frac{1}{\alpha}}. \tag{2.2}$$

Social welfare in equilibrium and in the optimal allocation, are, respectively

$$W^* = \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}} \alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{1}{\alpha}}(n+\alpha)(\alpha+1)} \quad \text{and} \quad W^o = \frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)}. \tag{2.3}$$

From (2.3), the percentage of WL denoted by PWL is

$$PWL \equiv \frac{W^o - W^*}{W^o} = 1 - \frac{n^{\frac{1}{\alpha}}(n+\alpha+1)}{(n+\alpha)^{\frac{\alpha+1}{\alpha}}} \equiv L(\alpha, n), \tag{2.4}$$

see Anderson-Renault (2003) p. 262. The following properties of $L(\cdot, \cdot)$ are easily proved:

i) $\lim_{n\to\infty} L(\alpha, n) = 0$.

ii) $\lim_{\alpha\to-1,} L(\alpha, n) = 0$.

iii) $\lim_{\alpha\to\infty} L(\alpha, n) = 0$.

iv) $L(\alpha, \cdot)$ decreases with n.

v) $L(\cdot, n)$ is quasiconcave in α.

i) is the usual property of large economies, as noticed in the Introduction. The explanation of ii) is that when $\alpha \to -1$, the market produces in the limit an infinite amount of surplus, so the loss caused by oligopoly tends to zero. iii) is caused by the fact that when $\alpha \to \infty$, inverse demand is flat so firms cannot influence price and optimal and equilibrium output are identical. ii) and/or iii) imply that there are markets where, for a given n, PWL is as small as we wish, something that is impossible in the case of quadratic utility functions. iv) shows that, when there are no technological issues at stake, the more competition, the better. Finally v) follows from the fact that Anderson and Renault (2003) proved that W^o/W^* is quasi-concave on α. So W^*/W^o is quasi-convex and $-W^*/W^o$ is quasi-concave, so it is $1 - W^*/W^o$.

We now study PWL as a function of α, see Figure 1. Notice that v) guarantees that the local maximum found there is a global maximum.[9]

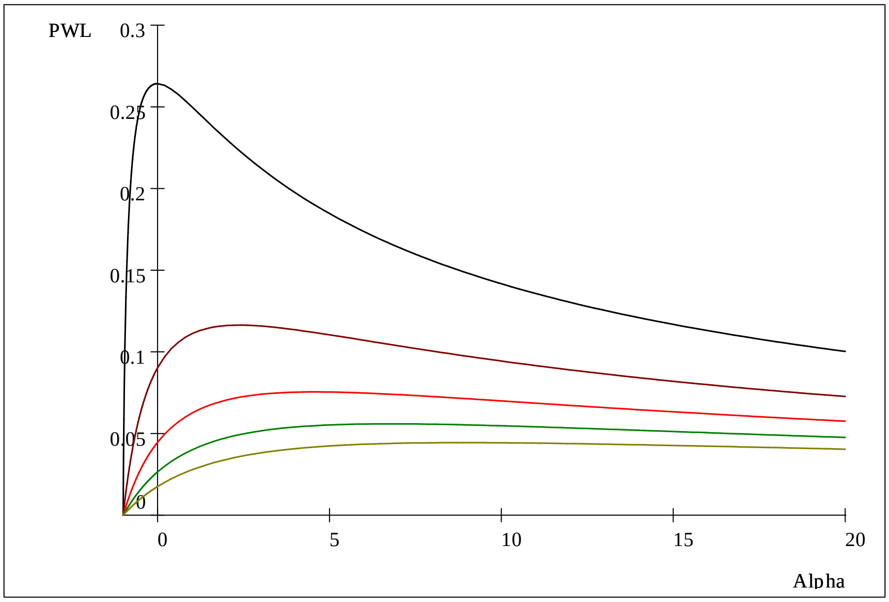

FIGURE 1: PWL for $n = 1$ (black), 2 (red), 3 (light red), 4 (green) and 5 (brown).

Table 1 below shows the maximum PWL, denoted by $\overline{PWL}$, and the corresponding values in the linear model, denoted by $PWLL$, for selected values of n. Notice that iv) above guarantees that for n larger than 10, $\overline{PWL}$ will be smaller than 2.2%.

n	1	2	3	4	5	6	7	8	9	10
$\overline{PWL}$	.27	.118	.076	.058	.044	.0357	.032	.027	.024	.022
$PWLL$	.25	.11	.0625	.04	.027	.02	.0156	.012	.01	.008

TABLE 1

[9] Figure 1 suggests that the α that maximizes $L(\cdot, n)$ increases with n.

Notice that the relative difference between $\overline{PWL}$ and $PWLL$ increases with n (from 8% for $n = 1$ to 175% for $n = 10$). However this effect is not strong enough to obtain significant WL in the cases in which the linear model yields small WL. Given this and that PWL can be much smaller than $\overline{PWL}$, we conclude that the consideration of a more general class of utility functions *alone* is not helpful to finding significant WL.

3. Fixed Costs and Free Entry

In this section we assume that in order to produce, firms must incur in a fixed cost, denoted by k, and that there is an infinite number of potential firms. The number of active firms in equilibrium is denoted by n. Given n, output is determined as in the previous section. We assume that the decision of entry is prior to the decision on output.[10] Thus, equilibrium under free entry implies that if n firms are in the market, firm n has non negative profits but firm $(n+1)$ has non positive profits, formally

$$\frac{\alpha a^{\frac{1+\alpha}{\alpha}} n^{\frac{1-\alpha}{\alpha}}}{b^{\frac{1}{\alpha}} (n+\alpha)^{\frac{1+\alpha}{\alpha}}} \geq k \geq \frac{\alpha a^{\frac{1+\alpha}{\alpha}} (n+1)^{\frac{1-\alpha}{\alpha}}}{b^{\frac{1}{\alpha}} (n+\alpha+1)^{\frac{1+\alpha}{\alpha}}}. \tag{3.1}$$

Welfare in a Cournot equilibrium with free entry is

$$W^* = \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}} \alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}} (n+\alpha)^{\frac{1}{\alpha}} (n+\alpha)(\alpha+1)} - nk, \tag{3.2}$$

where n solves (3.1). In an optimal allocation, aggregate output equals the one in (2.2). Thus, social welfare in the optimal allocation with one active firm is

$$W^o = \frac{\alpha a^{\frac{\alpha+1}{\alpha}}}{b^{\frac{1}{\alpha}} (\alpha+1)} - k. \tag{3.3}$$

Assuming $\alpha a^{\frac{\alpha+1}{\alpha}} > k b^{\frac{1}{\alpha}} (\alpha+1)$, i.e. that the fixed cost is small enough, one active firm is socially optimum because it yields more social welfare than no firms and economies

[10] Thus the fixed cost is actually sunk. Lopez-Cuñat (1999) has shown that, under conditions that are met here, the equlibrium considered in this paper is a subset of an equilibrium when both decisions are simultaneous (like in Novshek [1980] and Ushio [1983]). Thus this cost can also be considered as fixed.

of scale imply that is optimal to produce x^o in one firm. Thus PWL can be written as

$$PWL = \frac{\frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)} - \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}} \alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{1+\alpha}{\alpha}}(\alpha+1)} + (n-1)k}{\frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)} - k}. \tag{3.4}$$

In order to have a formula, in which PWL depends on observable variables, we substitute k for its upper and lower bounds in (3.1). It is clear that PWL is increasing on k. Thus, the maximal PWL, denoted by $MA(\alpha, n)$, occurs for the maximum value of k, namely

$$MA(\alpha, n) \equiv \frac{(n+\alpha)^{\frac{1+\alpha}{\alpha}} - n^{\frac{1}{\alpha}}(n+\alpha+1) + (n-1)n^{\frac{1-\alpha}{\alpha}}(\alpha+1)}{(n+\alpha)^{\frac{1+\alpha}{\alpha}} - n^{\frac{1-\alpha}{\alpha}}(\alpha+1)}. \tag{3.5}$$

Minimal PWL, denoted by $MI(\alpha, n)$, occurs for the minimum value of k, namely

$$MI(\alpha, n) \equiv \frac{(n+\alpha+1)^{\frac{1+\alpha}{\alpha}} - \frac{n^{\frac{1}{\alpha}}(n+\alpha+1)^{\frac{1+2\alpha}{\alpha}}}{(n+\alpha)^{\frac{1+\alpha}{\alpha}}} + (n-1)(n+1)^{\frac{1-\alpha}{\alpha}}(\alpha+1)}{(n+\alpha+1)^{\frac{1+\alpha}{\alpha}} - (n+1)^{\frac{1-\alpha}{\alpha}}(\alpha+1)}. \tag{3.6}$$

Figures 2 and 3 below picture $MA(\cdot, n)$ and $MI(\cdot, n)$ for selected values of n.

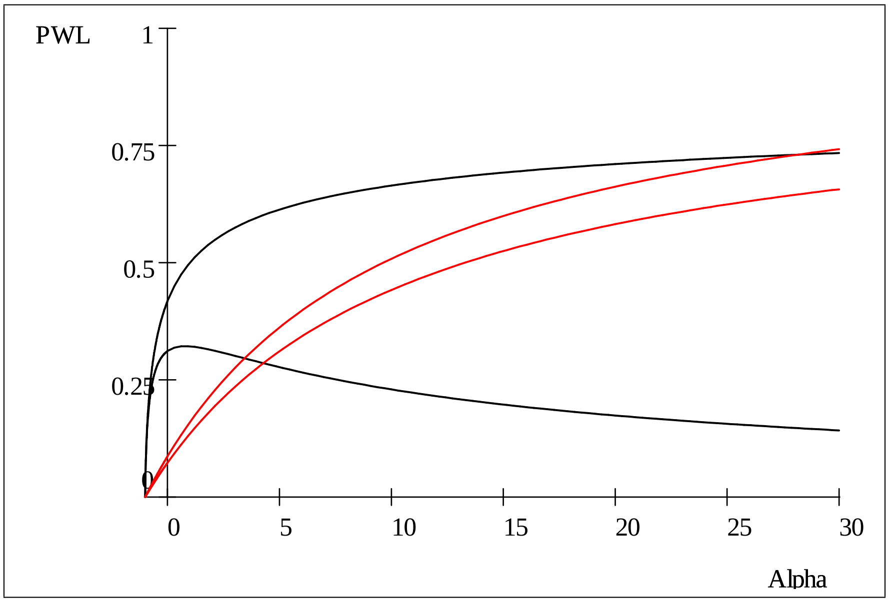

FIGURE 2: $MA(\cdot, 1)$ and $MI(\cdot, 1)$ (black) and $MA(\cdot, 10)$ and $MI(\cdot, 10)$ (red)

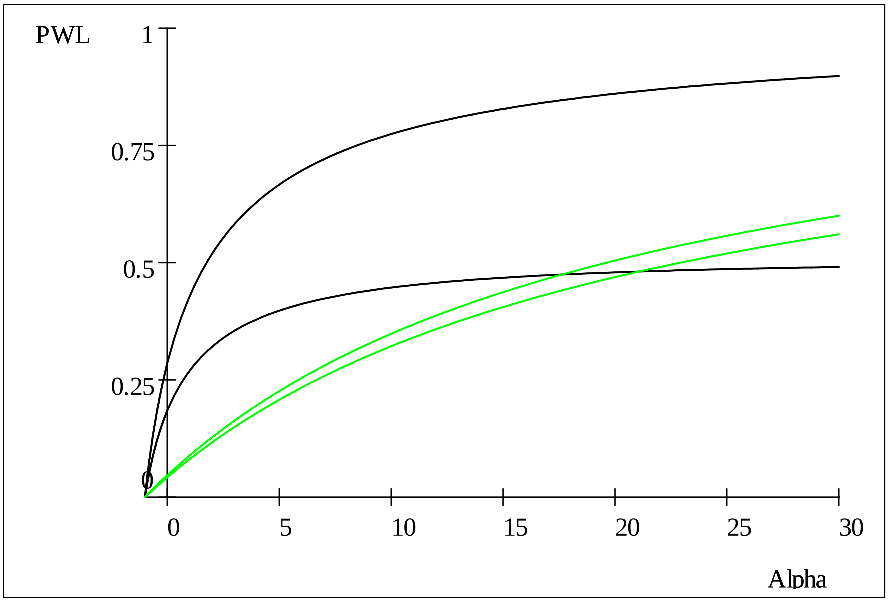

FIGURE 3: $MA(\cdot, 2)$ and $MI(\cdot, 2)$ (black) and $MA(\cdot, 20)$ and $MI(\cdot, 20)$ (green).

We now state the properties of $MA(\cdot, \cdot)$ and $MI(\cdot, \cdot)$ that correspond to i)-iv) in the previous section.

i') $\lim_{n \to \infty} MI(\alpha, n) = \lim_{n \to \infty} MA(\alpha, n) = 0$.

ii') $\lim_{\alpha \to -1} MI(\alpha, n) = \lim_{\alpha \to -1} MA(\alpha, n) = 0$.

iii') $\lim_{\alpha \to \infty} MI(\alpha, n) = \frac{n-1}{n}$, $\lim_{\alpha \to \infty} MA(\alpha, n) = 1$.

iv') Neither $MI(\alpha, \cdot)$ nor $MA(\alpha, \cdot)$ are monotonic on n.

i') implies that $\lim_{k \to 0} PWL = 0$, since (3.1) implies that when $k \to 0$, $n \to \infty$. Variations of this result have been obtained by Dasgupta and Ushio (1981), Fraysse and Moreaux (1981) and Guesnerie and Hart (1985). i') and ii') are identical to i) and ii) in the previous section. However iii') is very different from iii) because it says that markets with very large $\alpha's$ could be very inefficient. For large values of α, the contrast between monopoly and markets with a large number of firms is striking: In the former it is possible to construct examples where PWL is arbitrarily small and in the latter such examples are not possible. This is due to the fact that when n is very large, there are large WL due to the discrepancy between n and the optimal number of firms, namely one. Finally iv') is proved in Figures 2 and 3. The reason for this -apparently paradoxical- result is that k changes in order to maintain the free entry condition (3.1).

We now show that, if k and α are unknown, PWL is arbitrary even if certain variables -like price, output, marginal cost and number of firms- can be observed and we require that they correspond to the values in a Cournot Equilibrium with free entry for some parameters defining demand and costs. To formalize this, we say that a *Market* is a list of real numbers (A, c, b, α, k) such that $k > 0$, $(A - c)\alpha > 0$, $\alpha > -1$, $\alpha b > 0$, $-A\alpha < cn$ and $\alpha(A-c)^{\frac{\alpha+1}{\alpha}} > kb^{\frac{1}{\alpha}}(\alpha+1)$. An *Observation* is a list $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{c}, \mathfrak{n})$ where $\mathfrak{p}$ is market price, $\mathfrak{x}_i$ is output of firm i, $\mathfrak{c}$ $(< \mathfrak{p})$ is the marginal cost and $\mathfrak{n}$ is the number of active firms. The last variable is a positive integer and the others are positive real numbers. Under constant returns, the marginal cost equals the average variable cost so it can be observed (wages, raw materials, etc.). Now we have the following:

Proposition 1. *Given an observation* $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{c}, \mathfrak{n})$*, and a number* v *such that* $v = MA(\hat{\alpha}, \mathfrak{n})$, $\hat{\alpha} \in (-1, 0) \cup (0, \infty)$*, there is a market* $(\hat{A}, \mathfrak{c}, \hat{b}, \hat{\alpha}, \hat{k})$ *such that* $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{n})$ *is a Cournot equilibrium with free entry for this market (i.e. they fulfill (2.1) and (3.1)), and* $PWL = v$.

Proof: For k equal to the maximum value in (3.1), PWL is given by (3.5). Let v and $\hat{\alpha}$ be such that $MA(\hat{\alpha}, \mathfrak{n}) = v$. Now set

$$\hat{A} = \frac{\mathfrak{p}(\mathfrak{n} + \hat{\alpha}) - \mathfrak{c}\mathfrak{n}}{\hat{\alpha}}, \qquad \hat{k} = \frac{\hat{\alpha}(\hat{A} - \mathfrak{c})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}} \mathfrak{n}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(\mathfrak{n} + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}}, \qquad \hat{b} = \frac{(\hat{A} - \mathfrak{c})\mathfrak{n}}{\mathfrak{n}^{\hat{\alpha}} \mathfrak{x}_i^{\hat{\alpha}} (\mathfrak{n} + \hat{\alpha})}$$

This system can be solved easily because the first equation determines $\hat{A}$, the last equation determines $\hat{b}$ and with these values of $\hat{A}$ and $\hat{b}$ the remaining equation determines $\hat{k}$. By construction $\hat{A}\hat{\alpha} = \mathfrak{p}(\mathfrak{n} + \hat{\alpha}) - \mathfrak{c}\mathfrak{n}$, so $(\hat{A} - \mathfrak{c})\hat{\alpha} = \mathfrak{p}(\mathfrak{n} + \hat{\alpha}) - \mathfrak{c}(\mathfrak{n} + \hat{\alpha}) > 0$. Then, from the last equation $\hat{\alpha}\hat{b} > 0$ and the remaining equation implies $\hat{k} > 0$. Also $\hat{A}\hat{\alpha} + \mathfrak{c}\mathfrak{n} = \mathfrak{p}(\mathfrak{n} + \hat{\alpha}) > 0$. Finally we will show that $\hat{\alpha}(\hat{A} - \mathfrak{c})^{\frac{\hat{\alpha}+1}{\hat{\alpha}}} > \hat{k}\hat{b}^{\frac{1}{\hat{\alpha}}}(\hat{\alpha} + 1)$. Given the definitions of the parameters, this inequality reads $(\mathfrak{n} + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}} - \mathfrak{n}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}(\hat{\alpha} + 1) > 0$. Call $\Psi(\hat{\alpha}, \mathfrak{n})$ the left hand side of the previous inequality and extend the function to allow n to take real values. Notice that $\Psi(\hat{\alpha}, 1) = (\hat{\alpha} + 1)((\hat{\alpha} + 1)^{\frac{1}{\hat{\alpha}}} - 1) > 0$. Also $\lim_{n \to \infty} \Psi(\hat{\alpha}, \mathfrak{n}) = \infty$. Then, if $\Psi(\hat{\alpha}, \mathfrak{n}) \leq 0$ there must be a value of $\mathfrak{n}$, say $\bar{\mathfrak{n}}$ for which $\frac{\partial \Psi(\hat{\alpha}, \bar{\mathfrak{n}})}{\partial \mathfrak{n}} = 0$ and $\Psi(\hat{\alpha}, \bar{\mathfrak{n}}) \leq 0$. The former is equivalent to $(\bar{\mathfrak{n}} + \hat{\alpha})^{\frac{1}{\hat{\alpha}}} \bar{\mathfrak{n}} = \bar{\mathfrak{n}}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}(1 - \hat{\alpha})$.

If $\hat{\alpha} = 1$ this is impossible. If $\hat{\alpha} \neq 1$ plugging this equation in the definition of $\Psi(\cdot, \cdot)$ we obtain $\Psi(\hat{\alpha}, \bar{\mathfrak{n}}) = (\bar{\mathfrak{n}} + \hat{\alpha})^{\frac{1}{\hat{\alpha}}} \frac{\hat{\alpha}}{1-\hat{\alpha}}(-\hat{\alpha} + 1 - 2\bar{\mathfrak{n}}) \neq 0$. Thus $\Psi(\hat{\alpha}, \bar{\mathfrak{n}}) < 0 \Leftrightarrow \hat{\alpha} \in (0,1)$. However for $\hat{\alpha} \in (0,1)$, $(\bar{\mathfrak{n}} + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}} \geq \bar{\mathfrak{n}}^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}$ so $\Psi(\hat{\alpha}, \bar{\mathfrak{n}}) \geq \bar{\mathfrak{n}}^{\frac{1}{\hat{\alpha}}}(\bar{\mathfrak{n}} - \frac{1+\hat{\alpha}}{\bar{\mathfrak{n}}}) \geq \bar{\mathfrak{n}}^{\frac{1}{\hat{\alpha}}}(\bar{\mathfrak{n}} - \frac{2}{\bar{\mathfrak{n}}}) > 0$. Thus, $\Psi(\hat{\alpha}, \mathfrak{n}) > 0$.

Plugging the values of $\hat{A}$ and $\hat{b}$ into (2.1) we obtain

$$x^* = (\frac{(\hat{A} - \mathfrak{c})\mathfrak{n}}{\hat{b}(\mathfrak{n} + \hat{\alpha})})^{\frac{1}{\hat{\alpha}}} = \mathfrak{n}\mathfrak{x}_i \quad \text{and} \quad p^* = \frac{\hat{A}\hat{\alpha} + \mathfrak{c}\mathfrak{n}}{\mathfrak{n} + \hat{\alpha}} = \mathfrak{p}.$$

From the first inequality in (3.1) (with equality) and the definition of $\hat{k}$ it follows that

$$\frac{\hat{\alpha}(\hat{A} - \mathfrak{c})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}} n^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(n + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}} = \frac{\hat{\alpha}(\hat{A} - \mathfrak{c})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}} \mathfrak{n}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(\mathfrak{n} + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}} \quad \Leftrightarrow \quad \frac{n^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(n + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}} = \frac{\mathfrak{n}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(\mathfrak{n} + \hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}},$$

which has $n = \mathfrak{n}$ as a solution so the proof is complete. ■

There are two main implications of this result. On the one hand it points out the necessity of a good estimate of α in order to judge the efficiency of a market. Notice that first order conditions of profit maximization imply that the elasticity of demand equals $\frac{\mathsf{n}(\mathsf{p}-\mathsf{c})}{\mathsf{p}}$ so neither the elasticity of demand, nor price-marginal costs margins are related to α and/or PWL. On the other hand, together with the second part of iii'), it allows for markets yielding PWL arbitrarily close to one, the main theoretical goal of this paper. The explanation of this, is that we have constructed a market in which, in equilibrium, profits are zero and, when α tends to infinite, consumer surplus is also zero since from (2.1) we have that

$$U = \frac{\alpha}{(\alpha + 1)b^{\frac{1}{\alpha}}}(\frac{na}{n + \alpha})^{\frac{1+\alpha}{\alpha}}, \quad \text{so} \quad \lim_{\alpha \to \infty} \frac{\alpha}{(\alpha + 1)b^{\frac{1}{\alpha}}}(\frac{na}{n + \alpha})^{\frac{1+\alpha}{\alpha}} = 0.$$

The intuition of the latter equation is that large values of α make inverse demand flatter and flatter so consumer surplus goes to zero when α goes to infinite. The difference with iii) in the previous section -where $\lim_{\alpha \to \infty} L(\alpha, n) = 0$- arises from the fact that in the latter industry profits are not zero, but when α tends to infinite they tend to a.

We now consider the case where fixed costs are observable. In this case an observation is a list $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{c}, \mathfrak{n}, \mathfrak{k})$ such that $\mathfrak{k} \leq \mathfrak{x}_i (\mathfrak{p} - \mathfrak{c})$ (i.e. profits are non negative). Consider the following condition that guarantees that no firm will like to enter:

Definition 1. *Observation* $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{c}, \mathfrak{n}, \mathfrak{k})$ *and* α *fulfill condition BE (Blockaded Entry) if*

$$(\frac{\mathfrak{n}+\alpha+1}{\mathfrak{n}+\alpha})^{\frac{1+\alpha}{\alpha}}(\frac{\mathfrak{n}}{\mathfrak{n}+1})^{\frac{1-\alpha}{\alpha}} > \frac{\mathfrak{x}_i(\mathfrak{p}-\mathfrak{c})}{\mathfrak{k}}.$$

The right hand side can be interpreted as the rate of (gross) profits. BE just says that the rate of profits cannot be larger than a certain number which depends on α and $\mathfrak{n}$. The condition is more illuminating in several special cases. For instance if $\alpha \to \infty$ condition BE reads $\mathfrak{k}(\mathfrak{n}+1) > \mathfrak{n}\mathfrak{x}_i\,(\mathfrak{p}-\mathfrak{c})$. When $\alpha \to -1$ condition BE reads $\mathfrak{k}\,(\mathfrak{n}+1)^2 > \mathfrak{n}^2\mathfrak{x}_i\,(\mathfrak{p}-\mathfrak{c})$. Finally when $\alpha = 1$, BE reads, $\mathfrak{k}\,(\mathfrak{n}+2)^2 > (\mathfrak{n}+1)^2\mathfrak{x}_i\,(\mathfrak{p}-\mathfrak{c})$.

Proposition 2. *Given an observation* $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{c}, \mathfrak{n}, \mathfrak{k})$ *and a number* v *such that* $v = MI(\hat{\alpha}, \mathfrak{n})$, $\hat{\alpha} \in (-1, 0) \cup (0, \infty)$, *if BE holds, there is a market* $(\hat{A}, \mathfrak{c}, \hat{b}, \hat{\alpha}, \hat{k})$ *such that* $(\mathfrak{p}, \mathfrak{x}_i, \mathfrak{n})$ *is a Cournot equilibrium with free entry for this market (i.e. they fulfill (2.1) and (3.1)), and* $PWL \geq v$.

Proof: (Virtually identical to the proof of Proposition 1) For k equal to the minimum value in (3.1), PWL is given by (3.6). Choose $\hat{\alpha}$ such that $v = MI(\hat{\alpha}, \mathfrak{n})$. Set

$$\hat{A} = \frac{\mathfrak{p}(\mathfrak{n}+\hat{\alpha}) - \mathfrak{c}\mathfrak{n}}{\hat{\alpha}}, \qquad \hat{b} = \frac{(\hat{A}-\mathfrak{c})\mathfrak{n}}{\mathfrak{n}^{\hat{\alpha}}\mathfrak{x}_i^{\hat{\alpha}}(\mathfrak{n}+\hat{\alpha})}$$

This system can be solved, as we showed before. Plugging these values of $\hat{A}$ and $\hat{b}$ into (2.1) we obtain the required values of x^* and p^*. Finally, the left hand side of the free entry condition (3.1) holds by the definition of an observation. Notice that the right hand side of (3.1) when we plug the values of $\hat{A}$ and $\hat{b}$ obtained above reads

$$\mathfrak{k} \geq \frac{\mathfrak{x}_i(\mathfrak{p}-\mathfrak{c})(\mathfrak{n}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}(\frac{\mathfrak{n}+1}{\mathfrak{n}})^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(\mathfrak{n}+\hat{\alpha}+1)^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}},$$

which under BE holds. When the above equation holds with equality, $PWL = MI(\hat{\alpha}, \mathfrak{n}) = v$, so $PWL \geq v$. ■

Comparing these with the results obtained in the previous section we see that the consideration of fixed costs allows the possibility of finding large PWL. This is because in this case, we add the misallocation due to the wrong number of firms to the misallocation due to the wrong output.[11] The latter comes up to very large numbers because in our model the optimal number of firms is one.[12] But preferences play a role too. In the linear case, the corresponding expressions to (3.5) and (3.6) are (see Figure 4).

$$MA(1,n) = \frac{2n-1}{n^2+2n-1} \quad \text{and} \quad MI(1,n) = \frac{2n^3+3n^2+2n+2}{(n+1)^2\,(n^2+4n+2)}.$$

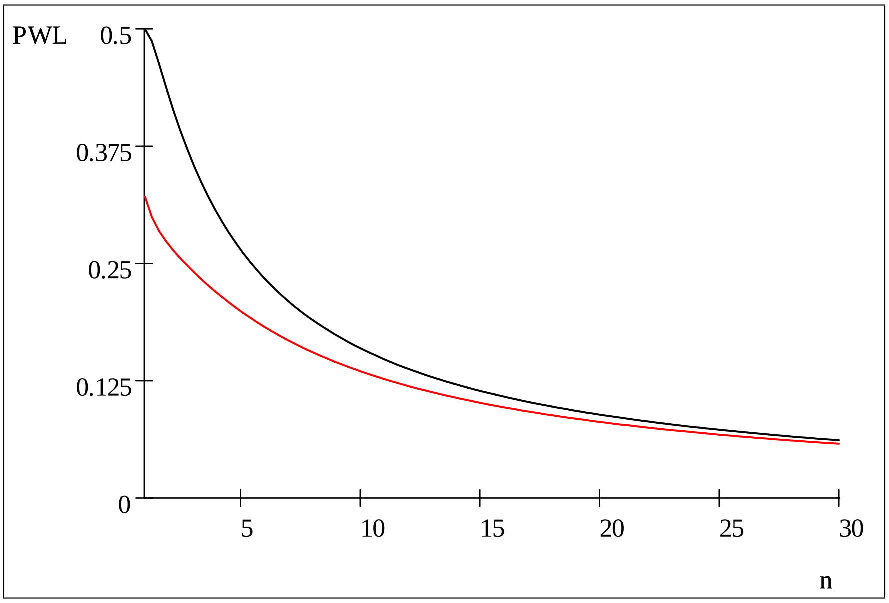

FIGURE 4: $MA(1,n)$ (black) and $MI(1,n)$ (red)

Even though for large values of n PWL is substantial (i.e. for $n = 15$, the minimum PWL is 10.14% which is just below the PWL in the case of no free entry with two firms), both $MA(1,n)$ and $MI(1,n)$ tend to zero as $n \to \infty$, which was not the case when α was allowed to vary. Moreover, in this case, values of PWL arbitrarily close to

[11] Very similar conclusions are drawn if the cost function is $cx_i + x_i^2 d/2$ with $d < 0$, i.e. under increasing returns to scale ($2c(d+b) > -da$ is required for costs to be positive in the optimum and in equilibrium). In this case PWL can be very large too because there are too many firms in equilibrium.

[12] Overentry may also occur even if the marginal cost is increasing, see von Weizsäcker (1980), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

one cannot be obtained for a given n. The reason is that the utility of the representative consumer when $\alpha = 1$ is always positive.

4. Non Identical Firms

Suppose now that firms have different productivities. Let c_i be the marginal cost of firm i. Without loss of generality let $c_1 \leq c_i$ for all i. Let $a_i \equiv A - c_i$. We will assume that for all i, $(n+\alpha-1)a_i > \sum_{j\neq i} a_j$, $b\sum_{j=1}^n a_j > 0$ and $-A\alpha < \sum_{i=1}^n c_i$. This assumption guarantees that, in equilibrium, all firms produce a positive output and market price is positive (see Equation (4.1) below). Cournot equilibrium is easily shown to be unique and given by

$$x_i^* = \frac{1}{\alpha}(\frac{\sum_{j=1}^n a_j}{b(n+\alpha)})^{\frac{1}{\alpha}}(\frac{a_i(n+\alpha)}{\sum_{j=1}^n a_j} - 1), \quad x^* = (\frac{\sum_{j=1}^n a_j}{b(n+\alpha)})^{\frac{1}{\alpha}} \quad \text{and} \quad p^* = \frac{A\alpha + \sum_{i=1}^n c_i}{n+\alpha}. \tag{4.1}$$

Social welfare is now $W = Ax - \frac{bx^{\alpha+1}}{\alpha+1} - \sum_{i=1}^n c_i x_i = \sum_{i=1}^n a_i x_i - \frac{bx^{\alpha+1}}{\alpha+1}$. In the Cournot equilibrium

$$W^* = \frac{1}{\alpha}\sum_{i=1}^n a_i(\frac{\sum_{j=1}^n a_j}{b(n+\alpha)})^{\frac{1}{\alpha}}(\frac{a_i(n+\alpha)}{\sum_{j=1}^n a_j} - 1) - \frac{b}{\alpha+1}(\frac{\sum_{i=1}^n a_i}{b(n+\alpha)})^{\frac{\alpha+1}{\alpha}}, \tag{4.2}$$

which when all a_i's are identical reduces to (2.3). In the optimal allocation only the technology in the hands of Firm 1 is used and accordingly

$$x^o = (\frac{a_1}{b})^{\frac{\alpha+1}{\alpha}} \quad \text{and} \quad W^o = \frac{\alpha a_1^{\frac{\alpha+1}{\alpha}}}{(\alpha+1)b^{\frac{1}{\alpha}}}. \tag{4.3}$$

In order to have a workable expression for PWL that depends on observable variables alone, let us define s_i as the market share of firm i. Clearly, $\sum_{i=1}^n s_i = 1$ and $s_1 \geq s_i$, $i = 2, ..., n$. Then, from (4.1),

$$s_i \equiv \frac{x_i}{x} = \frac{a_i(n+\alpha) - \sum_{j=1}^n a_j}{\alpha\sum_{j=1}^n a_j} \quad \Rightarrow \quad a_i = \frac{(\alpha s_i + 1)\sum_{j=1}^n a_j}{n+\alpha}. \tag{4.4}$$

For future reference, we will say that a list of market shares $(s_1, s_2, ..., s_n)$ is *a Market Structure*. It is clear from (4.4) that any vector $(a_1, a_2, ..., a_n)$ yields a unique market structure compatible with Cournot equilibrium and that given a market structure we can construct a vector $(a_1, a_2, ..., a_n)$ (in fact an infinite number of vectors) whose Cournot equilibrium yields this market structure. Given this equivalence, we will focus in this section on market structure that has the advantage of being observable.

Plugging the last part of (4.4) into (4.2) and after lengthy calculations we obtain PWL as a function of α and the market structure, namely

$$PWL = \frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha+1)\sum_{i=1}^{n} s_i^2 - 1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}} \equiv P(s_1, \sum_{i=1}^{n} s_i^2, \alpha). \tag{4.5}$$

When all firms are identical, (4.5) reduces to (2.4). It is noteworthy that PWL here depends only on three variables:

- α.
- The market share of the largest firm s_1.
- The Hirschman-Herfindahl index of concentration denoted by $H \equiv \sum_{i=1}^{n} s_i^2$.[13]

Equation (4.5) allows computation of PWL from s_1 and H assuming that demand is linear or isoelastic (where α is the inverse elasticity of demand). It also allows to plot PWL as a function of α for actual market structures and see what this function looks like. For instance, the numbers below represent shares of different firms in the Spanish gasoline market. Our data do not include operators with less than .013 of market share, but the consideration of these operators would hardly make any difference in H.

.41	.178	.074	.048	.034	.026	.023	.014

TABLE 2

This market has been voiced repeatedly as not very competitive. In this case, (4.5) reads like follows:

$$PWL = \frac{(1+\alpha .41)^{\frac{\alpha+1}{\alpha}} - (\alpha+1).2093 - 1}{(1+\alpha .41)^{\frac{\alpha+1}{\alpha}}}.$$

[13] In fact, s_1 and H are not independent but we prefer to write (4.5) in this way to highlight the role of H in the formula.

Looking at Figure 5 it is clear that, except for very special values of α, PWL is large. Indeed for values of α larger than $-.6$, PWL is larger than 10%. When demand is concave ($\alpha \geq 1$), PWL is always larger than 28%.

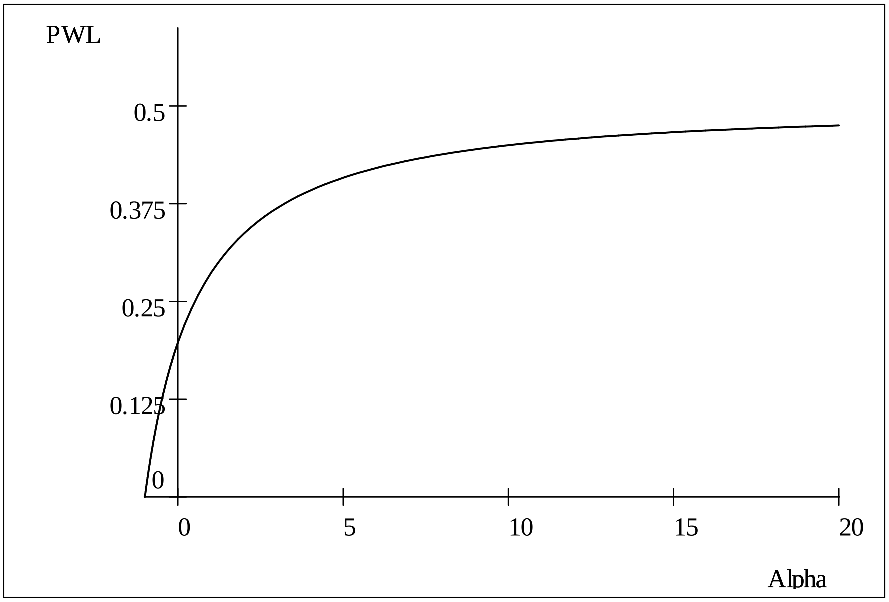

FIGURE 5

Notice the following properties of $P(\ )$ as defined by (4.5):[14]

i") $\lim_{\alpha\to-1} P(s_1, H, \alpha) = 0.$

ii") $\lim_{\alpha\to\infty} P(s_1, H, \alpha) = \frac{1}{s_1}\left(s_1 - \sum_{i=1}^{n} s_i^2\right).$

iii") $P(\cdot, H, \alpha)$ is increasing on s_1.

iv") $P(s_1, \cdot, \alpha)$ is decreasing on H.

v") $\lim_{\alpha\to 0} PWL(s_1, H, \alpha) = \frac{e^{s_1}-1-H}{e^{s_1}}.$

i") is identical to i). When firms are identical ii") reduces to ii).[15] Point iii') agrees with the received wisdom: the larger the dominant firm, the closer to monopoly, and hence the larger the PWL is. However, iv") is counterintuitive because it says the larger the concentration, the lower the WL. The reason is that when H increases, production

[14] As we mentioned before we take s_1 and H as independent when in fact they are not.

[15] If PWL is written as a function of $a_i's$, i") holds and ii") reads $\lim_{\alpha\to\infty} PWL = 1-\sum_{i=1}^{n} \frac{a_i^2}{a_1 \sum_{j=1}^{n} a_j}.$

is shifted to the less efficient firms which causes social welfare to fall. Finally v") allows us to extend $P(s_1, H, \cdot)$ to $\alpha = 0$ preserving continuity.

We now discuss why the approach followed in the previous section will not work here. An *Observation* is a list $(\mathfrak{p}, \mathfrak{x}_1, ..., \mathfrak{x}_n, \mathfrak{c}_1, ..., \mathfrak{c}_n)$ where $\mathfrak{p}$ is market price and $\mathfrak{x}_i$ and $\mathfrak{c}_i (< \mathfrak{p})$ are the output and the marginal cost of firm i and a *Market* is a list $(A, c_1, ..., c_n, b, \alpha)$ such that $(n + \alpha - 1)a_i > \sum_{j \neq i} a_j$, $\alpha > -1$, $b \sum_{j=1}^n a_j > 0$ and $-A\alpha < \sum_{i=1}^n c_i$. It is clear that not all observations are compatible with the model. In particular, the number of variables in an observation is $2n+1$ and the number of parameters defining a market is $n+3$. With $n > 2$, the number of parameters will be, in general, unable to generate the required observations. Also, first order conditions of profit maximization imply that

$$\frac{\mathfrak{x}_i}{\mathfrak{x}_j} = \frac{\mathfrak{p} - \mathfrak{c}_i}{\mathfrak{p} - \mathfrak{c}_j}$$

that may fail even for the case $n = 2$. Given this, we will study how PWL depends on α, n and the market structure focussing our attention on limiting cases, i.e. when PWL is maximal or minimal. Our first result is that when α, n and the market structure can be chosen simultaneously, PWL can be arbitrarily close to one and at the same time the concentration index H arbitrarily low.

Proposition 3. *There exists $(\alpha, n, s_1, ..., s_n)$ for which PWL is arbitrarily close to one and H is arbitrarily close to zero.*

Proof: From iv") the maximal PWL occurs when $s_2 = s_3 =, ..., = s_n$. Denoting these shares by y, we have that $s_1 + (n-1)y = 1$. Plugging this in (4.5) we have that

$$P(s_1, n, \alpha) \equiv \frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha+1)(s_1^2 + \frac{(1-s_1)^2}{n-1}) - 1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}. \tag{4.6}$$

PWL is increasing on n so the maximum PWL obtains when n is arbitrarily large, i.e.

$$\lim_{n\to\infty} P(s_1, n, \alpha) = \frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha+1)s_1^2 - 1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}. \tag{4.7}$$

We easily compute $\lim_{\alpha\to\infty} \lim_{n\to\infty} P(s_1, n, \alpha) = \lim_{n\to\infty} \lim_{\alpha\to\infty} P(s_1, n, \alpha) = 1 - s_1$. Thus when α and n are very large and s_1 very small, PWL is arbitrarily close to one (since limits are interchangeable our procedure is robust). The restriction $s_1 \geq s_i$, $i = 2, ..., n$ when firms $2, ..., n$ are identical, is equivalent to $ns_1 \geq 1$. This inequality holds when the order of magnitude at which n tends to ∞ is larger than the order of magnitude at which s_1 tends to 0.

Finally, it can be easily shown that when firms 2 to n are identical,

$$H = \frac{ns_1^2 + 1 - 2s_1}{n-1} = \frac{s_1^2 + \frac{1}{n} - 2\frac{s_1}{n}}{1 - \frac{1}{n}},$$

which when $n \to \infty$ and $s_1 \to 0$ tend to zero. ■

We now perform a robustness test on the previous result by checking what would happen to PWL and H if one of the variables in our construction is held fixed.

For given s_1, $PWL = 1 - s_1$ which for sensible values of s_1 might be large. Also, $H = s_1^2$. Thus $PWL = 1 - \sqrt{H}$ so H and PWL are negatively related.

For given n, PWL can be written as in (4.6) above. Taking limits,

$$\lim_{\alpha \to \infty} \frac{(1 + \alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha + 1)(s_1^2 + \frac{(1-s_1)^2}{n-1}) - 1}{(1 + \alpha s_1)^{\frac{\alpha+1}{\alpha}}} = \frac{s_1 n - s_1^2 n - 1 + s_1}{s_1 n - s_1}.$$

This expression achieves a maximum at $\frac{\sqrt{n}-1}{\sqrt{n}+1}$when $s_1 = \frac{1}{\sqrt{n}}$. PWL is large -the minimum value of PWL is 0.17- but not close to one. Also $H = \frac{2(\sqrt{n}-1)}{\sqrt{n}(n-1)}$ which is decreasing in n, see Figure 6. So in this case H and PWL go in opposite directions when n varies.

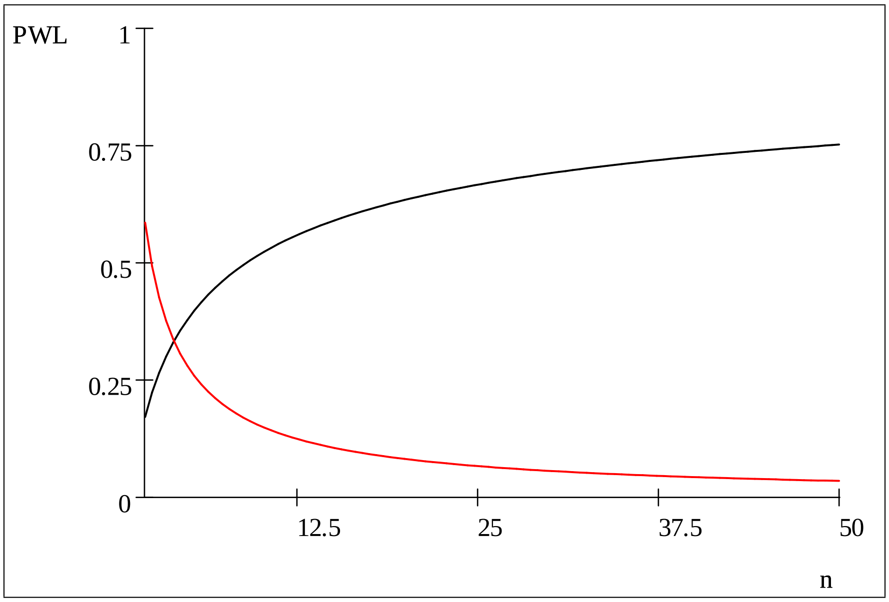

FIGURE 6: PWL (black) and H (red) for given values of n.

Finally, if α were given, for $n = \infty$,

$$PWL = 1 - \frac{(\alpha+1)s_1^2 + 1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}.$$

First order condition of PWL maximization implies $s_1^2(1-\alpha^2) - 2s_1(1+\alpha) + 1 + \alpha = 0$. If $\alpha = 1$, the maximum is achieved at $s_1 = \frac{1}{2}$ and $PWL = \frac{1}{3}$. If $\alpha \neq 1$ we have two solutions, $s_1 = \frac{-1\pm\sqrt{\alpha}}{\alpha-1}$. The root $\frac{-1-\sqrt{\alpha}}{\alpha-1}$ can be discarded because if $\alpha > 1$ it is negative, if $\alpha < 1$ it is larger than one and if $\alpha \in (-1, 0)$ it is not defined. If $\alpha \in (0, \infty)$, $\frac{-1+\sqrt{\alpha}}{\alpha-1} \in [0, 1]$ and since it can be easily shown that the maximum is interior, $s_1 = \frac{-1+\sqrt{\alpha}}{\alpha-1}$ maximizes PWL. The latter can be written as

$$PWL = 1 - \frac{\frac{1}{(\alpha-1)^2}(\alpha+1)\left(\sqrt{\alpha}-1\right)^2 + 1}{\left(\frac{\alpha}{\alpha-1}\left(\sqrt{\alpha}-1\right)+1\right)^{\frac{1}{\alpha}(\alpha+1)}} \text{ for } \alpha \in (0,\infty),$$

which increases with α, see Figure 7.[16] Again, PWL is large but not close to one. In

[16] Notice that $\lim_{\alpha\to 1} PWL = \frac{1}{3}$ and $\lim_{\alpha\to 1} \frac{-1\pm\sqrt{\alpha}}{\alpha-1} = \frac{1}{2}$ which equal the values obtained when $\alpha = 1$.

this case $H = (\frac{-1+\sqrt{\alpha}}{\alpha-1})^2$ so H and PWL go in opposite directions with respect to α. For $\alpha \in (0, -1)$ the maximum is obtained when $s_1 = 1$, i.e. monopoly.

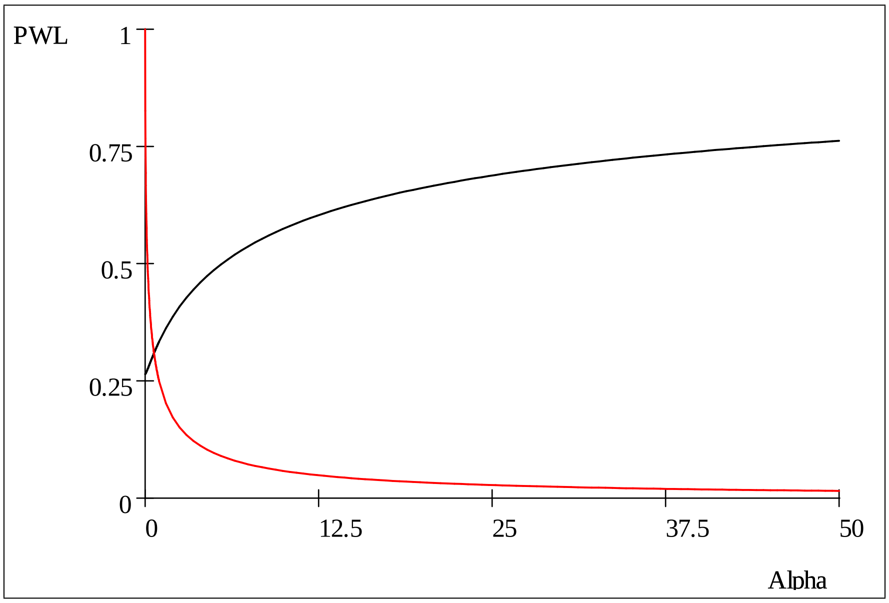

FIGURE 7: PWL (black) and H (red) for given values of α

Summing up, in the three cases considered, PWL is easily made large, but not close to one and H is far from being a reliable measure of PWL.

We now perform a more demanding exercise where PWL is studied by varying only either the market structure or α.

We first concentrate on how market shares affect PWL. A market structure such that $s_1 > s_2 =, ..., = s_n > 0$ will be called *a Dominant Firm.* A limit case of a dominant firm is *Monopoly* where only s_1 is positive.

Proposition 4. *For $\alpha > 0$, PWL is maximized when the market structure is a dominant firm with $s_1 = \frac{n+3}{2n+2}$ if $\alpha = 1$ and $s_1 = \frac{-n-1+\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n}$ if $\alpha \neq 1$. For $\alpha < 0$ the market structure that maximizes PWL is monopoly.*

Proof: The maximum of PWL in (4.5) over $\sum_{i=1}^{n} s_i = 1$ exists (by Weierestrass' theorem). As mentioned before, it occurs when $s_2 = s_3 =, ..., = s_n$. So, let us consider

PWL as given by (4.6). The extrema of this expression with respect to s_1 can be located, either when $\frac{\partial P(s_1,n,\alpha)}{\partial s_1} = 0$ or in the bounds of the interval in which s_1 must lie, namely $s_j \leq s_1 \leq 1$ for all $j > 1$. Since $(n-1)s_j \leq s_1$ the previous inequality can be written as $\frac{1}{n} \leq s_1 \leq 1$. Now, rewrite (4.6) as follows:

$$P(s_1, n, \alpha) = 1 - \frac{(\alpha+1)(ns_1^2 - 2s_1 + 1) + n - 1}{(n-1)(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}.$$

$$\frac{\partial P(s_1, n, \alpha)}{\partial s_1} = \frac{s_1^2(n - n\alpha^2) - s_1(2 + 2n + 2\alpha + 2n\alpha) + 2 + \alpha(3 + n + \alpha) + n}{(n-1)\left(\alpha s_1 + 1\right)^{\frac{1}{\alpha}+2}} \tag{4.8}$$

$$\frac{\partial P(s_1, n, \alpha)}{\partial s_1} = 0 \Leftrightarrow s_1^2(n - n\alpha^2) = 2s_1(1 + n + \alpha + n\alpha) - 2 - \alpha(3 + n + \alpha) - n \tag{4.9}$$

We have three possible cases: If $\alpha = 1$, the solution to (4.9) is $s_1^* = \frac{n+3}{2n+2} \in [\frac{1}{n}, n]$. Then, the maximum must be located either at $s_1 = \frac{1}{n}$, at $s_1 = 1$ or at $s_1 = \frac{n+3}{2n+2}$. We easily compute,

$$P(1, n, 1) = \frac{1}{4}, \quad P(\frac{1}{n}, n, 1) = \frac{1}{(n+1)^2}, \quad P(\frac{n+3}{2n+2}, n, 1) = \frac{n+1}{3n+5}.$$

From these expressions we obtain the desired result.

If $\alpha > 1$ from the first order condition we obtain two solutions,

$$s_1^* = \frac{-n - 1 \pm \sqrt{1 + \alpha n + \alpha^2 n + \alpha n^2}}{\alpha n - n}. \tag{4.10}$$

Clearly only the solution with a plus sign in front of the square root is feasible. We will show that for this solution, $s_1^* \in [\frac{1}{n}, 1]$. If $\frac{1}{n} > s_1^*$ we would have $\alpha^2(n-1) + n^2(\alpha - 1) - \alpha n + 1 < 0$ which is impossible because the left hand side achieves a minimum when $n = 2$ and $\alpha = 1$. Similarly, if $s_1^* > 1$, $\alpha n - \alpha - n + 1 < 0$, which again is impossible.[17]

Finally, notice that since there is only one value of s_1 for which $\frac{\partial P(s_1,n,\alpha)}{\partial s_1} = 0$ the shape of $P(\cdot, n, \alpha)$ is determined by the sign of $\frac{\partial P(s_1,n,\alpha)}{\partial s_1}$ at $s_1 = \frac{1}{n}$ and $s_1 = 1$. From (4.8),

$$sign\frac{\partial P(\frac{1}{n}, n, \alpha)}{\partial s_1} = sign(n + \alpha + n\alpha - \frac{1}{n} + \alpha^2 - \frac{2}{n}\alpha - \frac{1}{n}\alpha^2) \tag{4.11}$$

[17] Notice that when $\alpha \to 1$ both the numerator and the denominator in the definition of s_1^* go to zero.

which is positive because the expression on the right hand side is increasing in α and for $\alpha = -1$ equals to zero. Also from (4.8) we obtain that

$$sign \frac{\partial P(1, n, \alpha)}{\partial s_1} = sign(\alpha - n\alpha + \alpha^2 - n\alpha^2) = sign(\alpha(1+\alpha)(1-n)) \tag{4.12}$$

which is negative so the interior solution is indeed a maximum.

Finally let us consider the case $\alpha < 1$. Suppose that the negative root in (4.10) is less than one. Then

$$\frac{-n - 1 - \sqrt{1 + \alpha n + \alpha^2 n + \alpha n^2}}{\alpha n - n} < 1 \quad \Leftrightarrow \quad -\sqrt{1 + \alpha n + \alpha^2 n + \alpha n^2} > \alpha n + 1,$$

which is impossible. So there is, at most, one interior solution. Suppose first that $\alpha > 0$. From (4.11-12) we get that $sign \frac{\partial P(\frac{1}{n}, n, \alpha)}{\partial s_1}$ is positive and $sign \frac{\partial P(1, n, \alpha)}{\partial s_1}$ is negative which implies that maximum PWL is achieved at the interior solution. If $\alpha = 0$ the positive root in (4.10) equals one. Finally, if $\alpha < 0$, from (4.11-12), we have that $sign \frac{\partial P(\frac{1}{n}, n, \alpha)}{\partial s_1}$ and $sign \frac{\partial P(1, n, \alpha)}{\partial s_1}$ are both positive which given that there is, at most one value of s_1 for which $sign \frac{\partial P(\cdot, n, \alpha)}{\partial s_1}$ switches from positive to negative means that $P(\cdot, n, \alpha)$ is increasing, so it achieves the maximum when $s_1 = 1$. ■

Proposition 4 says that the most deleterious market structure is not always monopoly, the target of the wrath of economists since Adam Smith. In many cases a dominant firm structure is worse because firms other than 1 do not add much competition to the market and they are technologically inefficient. We notice that under maximal PWL,

$$H = \frac{ns_1^2 + 1 - 2s_1}{n - 1} \quad \text{and} \quad PWL = \frac{(1 + \alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha + 1)(ns_1^2 + \frac{(1-s_1)^2}{n-1}) - 1}{(1 + \alpha s_1)^{\frac{\alpha+1}{\alpha}}},$$

so H decreases with n but PWL increases with n. And H increases with s_1 but PWL not necessarily so. Thus, again, the concentration index H is a poor measure of WL.

The maximum PWL for given n and α is obtained by plugging the value of s_1 that maximizes PWL as found in Proposition 4 and denoted by $s(\alpha, n)$, into $P(s_1, n, \alpha)$.[18]

[18] Properties of $s(\alpha, n)$ when $\alpha > 0$ are: α) $\lim_{n\to\infty} s(\alpha, n) = \frac{1}{\sqrt{\alpha+1}}$. β) $\lim_{\alpha\to\infty} s(\alpha, n) = \frac{1}{\sqrt{n}}$. γ) $\frac{\partial s(\alpha,n)}{\partial \alpha} > 0 \Leftrightarrow 3\alpha n + n + \alpha n^2 + n^2 - 2 < 2\sqrt{(1 + \alpha n + \alpha^2 n + \alpha n^2)}(n+1)$ and δ) $s(\alpha, \cdot)$ is increasing in $n \Leftrightarrow (\alpha n + \alpha^2 n - 2\sqrt{(1 + \alpha n + \alpha^2 n + \alpha n^2)} + 2)(1 - \alpha) > 0$.

Let $P(s(\alpha,n),n,\alpha) \equiv F(\alpha,n)$, say. Figure 8 shows $F(\cdot,n)$ for several values of n and for $\alpha \in (0,50]$. For $\alpha \in (-1,0)$ maximal PWL is obtained under monopoly. This is why all the curves in the figure tend to the same value when $\alpha \to 0$, namely to PWL under monopoly, which by v") above is $\frac{e-2}{e} = 0.264$, see also Figure 1 above.

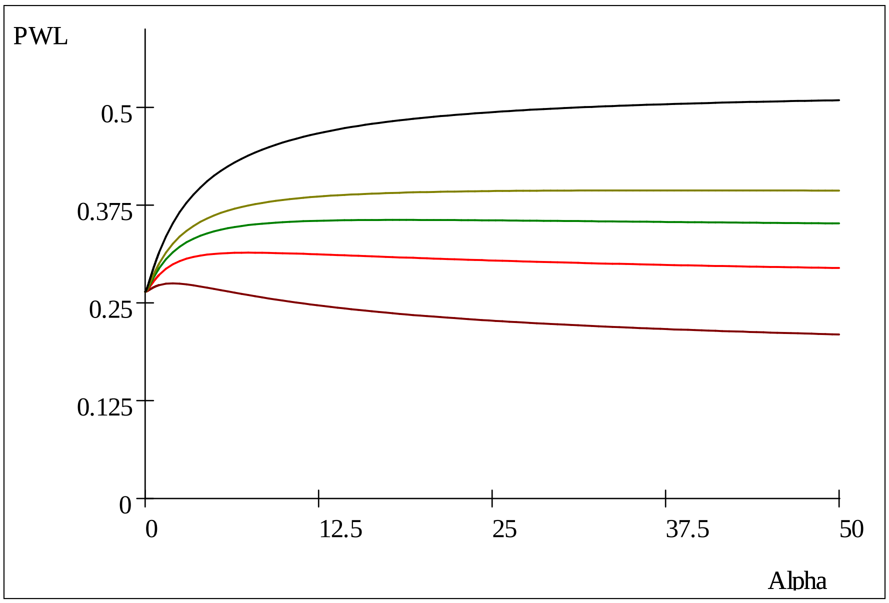

FIGURE 8: $F(\cdot,n)$ for $n=2$ (red), 3 (light red), 4 (green), 5 (brown) and 10 (black).

Now we state and prove some useful properties of $F(\cdot,\cdot)$:

I) $s_1(\cdot,n)$ and $F(\cdot,n)$ are continuous in α.

Proof: Clearly, each of the different pieces that define $s_1(\cdot,n)$ are continuous so we have only to check continuity at $\alpha=0,1$. First, when $\alpha \to 0$, $\frac{-n-1+\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n} \to 1$. The continuity of $s_1(\cdot,n)$ at $\alpha=1$ can be shown by multiplying the numerator and the denominator of $\frac{-n-1+\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n}$ by $-n-1-\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}$.[19] Finally, continuity of $F(\cdot,n)$ follows from the continuity of $s(\cdot,n)$ and $P(s_1,n,\cdot)$ ■

II) $F(\alpha,\cdot)$ is increasing in n.

[19] This is a general method to show that solutions of a quadratic equation $ax^2+bx+c=0$ are continuous in a when $a \to 0$.

Proof: Extend the functions $P(s_1, \cdot, \alpha)$ and $F(\alpha, \cdot)$ to have real values in the domain. It is clear that such functions are differentiable in n. Now compute,

$$\frac{\partial F(\alpha,n)}{\partial n} = \frac{dP(s(\alpha,n),n,\alpha)}{dn} = \frac{\partial P(s_1,n,\alpha)}{\partial s_1}\frac{\partial s_1}{\partial n} + \frac{\partial P(s_1,n,\alpha)}{\partial n} = \frac{\partial P(s_1,n,\alpha)}{\partial n},$$

where the last equality comes from the fact that $s(\alpha, n)$ maximizes PWL with respect to s_1 (this is the envelope theorem). Finally, it was established in Proposition 3 that $P(s_1, \cdot, \alpha)$ is increasing in n so the desired result follows. ■

This result implies that, for any number of firms, it is possible to find the PWL of, at least, the magnitude of $F(\alpha, 2)$ which for values of $\alpha \in (0, 50]$ never goes below $0.209\,7$. Finally, we state two limiting properties of $F(\cdot, \cdot)$:

$III)$ $\lim_{\alpha\to\infty} F(\alpha,n) = \frac{\left(\sqrt{n}\right)^3+\sqrt{n}-2n}{\left(\sqrt{n}\right)^3-\sqrt{n}}$.

$IV)$ $\lim_{n\to\infty} F(\alpha,n) = 1 - \frac{\left(\sqrt{\alpha}-1\right)^2(\alpha+1)+(\alpha-1)^2}{(\alpha-1)^{\frac{\alpha-1}{\alpha}}\left(\alpha\sqrt{\alpha}-1\right)^{\frac{\alpha+1}{\alpha}}}$.

Notice that in both cases PWL is high even for small values of α and n, see Figure 9. It is clear that $\lim_{n\to\infty,\alpha\to\infty} F(\alpha,n) = \lim_{\alpha\to\infty,n\to\infty} F(\alpha,n) = 1$. The previous properties have two interesting consequences.

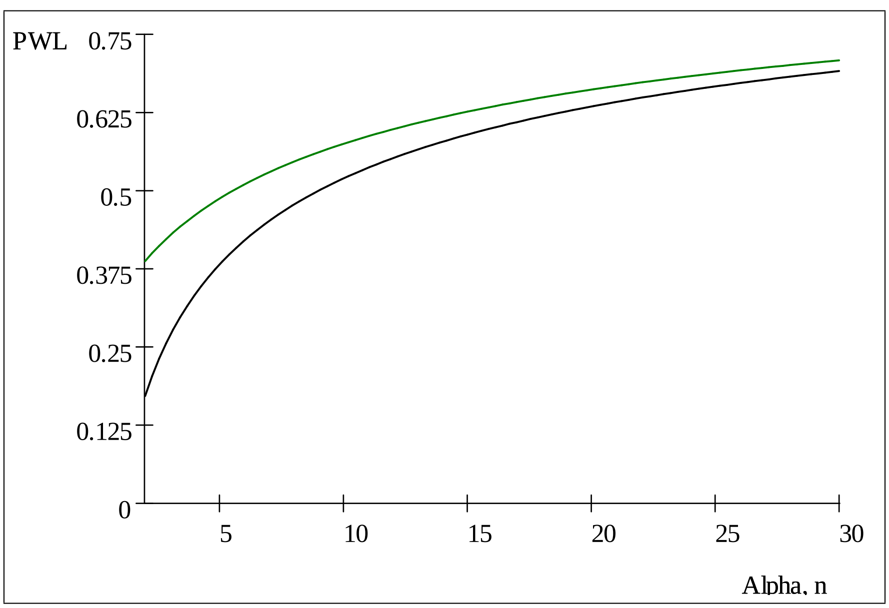

FIGURE 9: $LIM_{\alpha\to\infty}PWL$ (black) and $LIM_{n\to\infty}PWL$ (green).

Corollary 1. *1: Any* $PWL \in (0, \frac{(\sqrt{n})^3+\sqrt{n}-2n}{(\sqrt{n})^3-\sqrt{n}})$ *can be obtained for some value of* α.
2: PWL *is obtainable for some value of* n *iff* $PWL \in (0, 1 - \frac{(\sqrt{\alpha}-1)^2(\alpha+1)+(\alpha-1)^2}{(\alpha-1)^{\frac{\alpha-1}{\alpha}}(\alpha\sqrt{\alpha}-1)^{\frac{\alpha+1}{\alpha}}})$.

The first part of the Corollary follows from I) and III) and the second part from II) and IV). We now turn to the study of the market structure that minimizes PWL.

Lemma 1. *Suppose that* $(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n)$ *minimizes* $P(s_1, \sum_{i=1}^{n} s_i^2, \alpha)$. *Then* $\nexists\ \hat{s}_i, \hat{s}_j$, $j > 1$ *such that* $\hat{s}_1 > \hat{s}_i \geq \hat{s}_j > 0$.

Proof: Increasing $\hat{s}_i$ by an small amount, say dx, and decreasing $\hat{s}_j$ by dx too is feasible -i.e. $\hat{s}_i + dx$ and $\hat{s}_j - dx \in [0, s_1]$, increases H and so decreases PWL which contradicts that PWL is minimized. ■

Lemma 1 implies that only three market structures might minimize PWL.

1: All firms produce the same output. Market structure is $(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$.

2: All firms minus one, say n, produce the same output. Market structure is $(x, x, ..., y)$ with $x > y$.

3: A number of firms, say $1, ..., m$ with $m < n$ produce the same output, and the remaining firms produce zero output. Market structure is $(\frac{1}{m}, ..., \frac{1}{m}, 0, ..., 0)$.

But option 3) cannot minimize PWL since it was established that when all firms are identical, PWL decreases with the number of (active) firms (Property iv) in Section 2). So we are left with options 1 and 2.

Proposition 5. *The market structure that minimizes* PWL *is when all firms produce the same output.*

Proof: Notice that market structures 1 and 2 can be written as $(x, x, ..., 1-(n-1)x)$ with $x \in [\frac{1}{n-1}, \frac{1}{n}]$, where the lower bound of this interval comes from $1 \geq (n-1)x$. In this case $H = (n-1)x^2 + (1-(n-1)x)^2$. Plugging H into (4.5) we obtain

$$PWL = 1 - \frac{(\alpha+1)\left((n-1)x^2 + (1-(n-1)x)^2\right) + 1}{(1+\alpha x)^{\frac{\alpha+1}{\alpha}}} \equiv PW(\alpha, x, n).$$

Now, computing $\frac{\partial PW(\alpha,x,n)}{\partial x}$ this expression is found to be equal to

$$\frac{-(1+\alpha)}{(1+x\alpha)^{\frac{1+\alpha}{\alpha}}}[2n^2x-2nx-2n+2-\frac{(1+\alpha)\left((n-1)x^2+(1-(n-1)x)^2\right)+1}{1+x\alpha}]$$

Solving for $\frac{\partial PW(\alpha,x,n)}{\partial x}=0$ we obtain the following. If $\alpha=1$,

$$\frac{\partial PW(\alpha,x,n)}{\partial x}=0\Leftrightarrow 4n+4x+2-4n^2x=0\Leftrightarrow x=\frac{2n+1}{2n^2-2}<\frac{1}{n-1}.$$

So only boundary solutions are feasible and PWL is minimized when $x=\frac{1}{n}$. If $\alpha\neq 1$,

$$\frac{\partial PW(\alpha,x,n)}{\partial x}=0\Leftrightarrow x=\frac{-n^2+1\pm\sqrt{n^4+1+2\alpha n^3+\alpha^2n^2-3\alpha n^2-\alpha^2n-2n^3+\alpha n}}{(\alpha-1)(n^2-n)}.$$

Suppose that $\alpha>1$. Clearly, the negative root is not feasible, so consider the positive root, say x^*. If $x^*\leq\frac{1}{n}$, it must be that $(n-1)(\alpha^2+\alpha n-1-n)\leq 0$ which for $n>2$ and $\alpha>1$ is impossible.

Suppose that $\alpha<1$. If the negative root is less than or equal to $\frac{1}{n}$, we have that

$$-\sqrt{n^4+1+2\alpha n^3+\alpha^2n^2-3\alpha n^2-\alpha^2n-2n^3+\alpha n}\geq(n+\alpha)(n-1)$$

which is impossible. Take the positive root. If this root is larger than or equal to $\frac{1}{n-1}$, then $n(1-\alpha)\leq\alpha^2-3\alpha+2$ or $n\leq\frac{\alpha^2-3\alpha+2}{1-\alpha}$. The right hand side of this inequality has a maximum at 3 when $\alpha\to -1$. Since this value of α is never actually achieved, this inequality only may hold when $n=2$. But $\frac{\partial PW(\alpha,0.5,2)}{\partial x}=\frac{0.5\alpha+1.5}{0.5\alpha+1}>0$ which means that the minimum is achieved at the boundaries of x. Since in this case these bounds imply monopoly and duopoly, by iv) in Section 2 we achieve the desired result. ■

The implication of this result is that disregarding firms heterogeneity stacks the deck in favour of small WL because the assumption that all firms are identical implies that PWL is minimal among all market structures. Thus, minimal PWL is given by the function $L(\cdot,\cdot)$ in (2.4). Notice that since $L(\alpha,\cdot)$ is decreasing in n and $F(\alpha,\cdot)$ is increasing in n, the difference between maximal and minimal PWL increases with n for a given α, see Figure 10. Figure 10 also suggests that for a given $n\geq 5$, the distance between these two magnitudes increases with α. Finally, since $P(\cdot,n,\alpha)$ is continuous in s_1, any PWL between $L(\alpha,n)$ and $F(\alpha,n)$ is reachable by the choice of s_1.

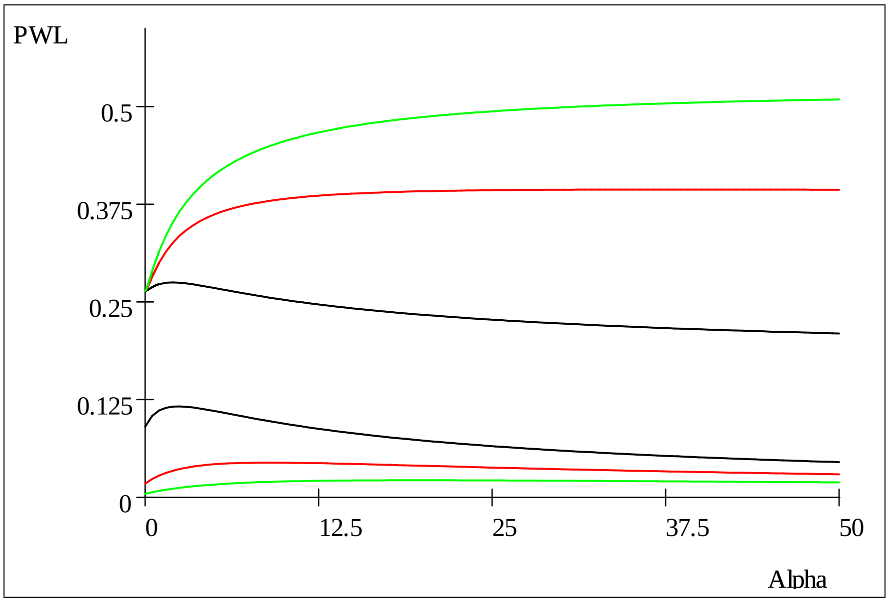

FIGURE 10: Maximal and Minimal PWL for $n = 2$ (black), 5 (red) and 10 (green).

Finally we consider the effect of α alone on PWL. We have little to say about the value of α that maximizes PWL because first order condition of maximization with respect to α is not very informative.[20] However, the continuity of $P(s_1, n, \cdot)$ has an interesting implication. Let $V \equiv \max\{\frac{s_1 - H}{s_1}, \frac{(1+s_1)^2 - 2H - 1}{(1+s_1)^2}, \frac{e^{s_1} - 1 - H}{e^{s_1}}\}$. The values in the bracket are respectively, $P(s_1, n, 0), P(s_1, n, 1)$ and $\lim_{\alpha \to \infty} P(s_1, n, \alpha)$. Then, we have:

Corollary 2. *Any $PWL \in (0, V)$ is obtainable by the choice of α.*

5. Final Comments

When one observes public policies on oligopolies one sees some concern about the number and the relative size of firms. But the question of the output set by oligopolists is cause of little or no concern at all. This paper provides some justification to this attitude: We found that WL due to the divergence between equilibrium and optimal output are small, even with as few as four firms in the market as shown in Section 2. On

[20] $\frac{\partial P(s_1, n, \alpha)}{\partial \alpha} = 0 \Leftrightarrow \left(n - 2s_1 + \alpha - 2\alpha s_1 + n s_1^2 + n s_1^2 \alpha\right) \frac{1}{\alpha} \ln(s_1 \alpha + 1) = n - 2s_1 + n s_1^2$.

the contrary WL due to the number and relative size of firms can be quite substantive as found in Sections 3 and 4. This conclusion, though, is likely to be exaggerated by our assumption that the optimal number of firms is one. Other factor that may bring down WL is the consideration of other solution concepts, e.g. Bertrand or Stackelberg equilibria, the latter particularly suited to the case of a dominant firm. However, in a dynamic framework WL can be larger than here because firms may collude. Thus, our results are just a first cut to the problem.

Our results have a number of implications for the applied literature.

1: To measure WL due to oligopolistic output setting is misguided because these are likely to be small. However WL due to overentry or to asymmetric firms can be substantial. Lack of consideration of these points biases downwards our estimates of WL.

2: Bresnahan and Reiss (1991) found markets where, as the number of firms increased beyond three, the competitive effect of additional firms on average markups was exhausted, a fact that suggests that the outcome is very close to perfect competition. A possible explanation for their findings is that they considered markets where asymmetries and economies of scale were possibly small such as doctors, dentists, druggists, plumbers and tire dealers. In contrast, Campbell and Hopenhayn (2002) find that this competitive effect persists with a large number of firms in markets were firms are asymmetric and the product is differentiated.

3: The impact of mergers and collusive agreements on social welfare depends on the characteristics of the market. For instance, with identical firms and no fixed costs our results in Section 2 suggest that anti-trust authorities should not be very concerned with mergers that do not bring the number of competing firms below, say four. However merging from duopoly to monopoly approximately doubles PWL.

4: WL depend on the parameter α that cannot be observed, *but can be estimated*. Our results point out the importance of the estimation of α for the proper account of WL.

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