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THE EFFECT OF REALISED VOLATILITY ON STOCK RETURNS RISK ESTIMATES*

Aurea Grané [1] and Helena Veiga [2]

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Keywords: Asymmetry, High-Frequency Data, Minimum Capital Risk Requirements, Realised Volatility
JEL Classification: C14, C15, G13.

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The effect of realised volatility on stock returns risk estimates

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Abstract

In this paper, we estimate minimum capital risk requirements for short, long positions and three investment horizons, using the traditional GARCH model and two other GARCH-type models that incorporate the possibility of asymmetric responses of volatility to price changes; and, most importantly, we analyse the models performance when realised volatility is included as an explanatory variable into the models' variance equations. The results suggest that the inclusion of realised volatility improves the models forecastability and their capacity to calculate accurate measures of minimum capital risk requirements.

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1 Introduction

In recent years, attention has been drawn to the increase of volatility in financial markets across the world because financial institutions such as banks can incur into substantial trading losses. This worry has started moving research into the direction of developing models able to forecast volatility accurately and creating quantitative techniques that aim at specifying the potential losses. With respect to the direction of developing new models, the papers of Engle (1982) and Bollerslev (1986) were the starting points of a new literature that continues to grow due to new extensions that cope the main empirical features of financial data. One of these empirical facts is the high "sensitiveness" of volatility to negative shocks (see Pagan and Schwert, 1990; Nelson, 1991; Campbell and Hentschel, 1992; Engle and Ng, 1993; Glosten et al., 1993, among others). The reasons for this to occur are, first, the "leverage effect" explained in Black (1976) and Christie (1982), according to which lower equity values lead to a higher debt-to-equity ratio that, in turn, increases the risk of equity holder's positions, and second, the "volatility feedback" that permits changes in volatility to affect returns through changes in future expected returns (see Brooks and Persand, 2003).

Concerning the direction of quantifying losses, value-at-risk (VaR) is a very popular technique that provides an estimate of the probability of likely losses to occur over a given time horizon due to changes in market prices. A very related concept is the minimum capital risk requirement (MCRR) defined as the minimum sufficient capital to absorb all except a pre-specified percentage of unforeseen losses (see Brooks et al., 2000). Several methods have been proposed to calculate the VaRs, among them

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we include the "delta-normal" method, the historical simulation that involves the estimation of the quantile of the portfolio returns, and the structured Monte Carlo simulation (see Dowd, 1998; Jorion, 2001). Although the Monte Carlo approach is powerful and flexible for generating VaR estimates because it can be specified any stochastic process for the asset price, it is not free of important drawbacks. The first and most important one is related to the process that has been assumed for the price of the asset, because if this assumption is not correct, the calculated VaRs can be inaccurate. The second drawback is related to the computational time required to compute the VaRs. It may be very high for a large portfolio. An alternative approach that could overcome the first drawback, is to use bootstrap rather than Monte Carlo simulation.

In this paper, we address an approach to the calculation of the MCRRs similar to the works of Hsieh (1993) and Brooks et al. (2000). We calculate MCRRs for four stock prices series defined on long and short positions for 1, 10 and 30 days horizons, using the traditional GARCH model and two extensions of it that allow for asymmetric responses of volatility to negative and positive shocks as in Brooks and Persand (2003). Moreover, we include into the models' variance equations realised volatility as an explanatory variable as in Koopman et al. (2005), and we do compare the models performance (with and/or without realised volatility) in calculating accurate MCRR's. The results report that the inclusion of realised volatility is of extreme importance, at least for the considered GARCH-type models, since it works as a volatility persistence absorbing mechanism, i.e, it reduces the estimated volatility persistence, and consequently, leads the models to generate more accurate volatility forecasts and estimates of minimum capital risk requirements. In fact, the differences observed in the MCRRs with and without including realised volatility are quite substantial that would have impact upon the costs of holding positions in these stocks. Consequently, the most important findings in this paper are: first, GARCH models including realised volatility (denoted GARCH-RV) perform better in terms of providing accurate MCRRs; second, the MCRRs based upon the traditional GARCH specification (without including realised volatility) are generally larger for short investment horizons and smaller for long investment horizons than the ones obtained with the GARCH-RV models, which is due to the decreasing volatility forecastability registered by the former models when the forecasting horizon increases (see Christoffersen and Diebold, 2000); and finally, models that allow for asymmetric responses of volatility to price changes (with and without including realised volatility) perform better in out-of-sample tests.

The remainder of the paper proceeds as follows: In Section 2 we present a description of data and its main statistical properties. In Section 3 we focus on the realised volatility estimator used in the paper. We estimate several conditional heteroscedastic models and we present the forecasting and the MCRRs methodologies in Section 4. Section 5 reports the main empirical results and we conclude in Section 6.

2 Data Analysis

In this study we calculate the MCRRs for stock price data on the American Express company, the Coca-Cola company, the Disney (Walt) company and on the Pfizer Company. These companies are included in the daily Dow Jones Stock Index and represent four different industries. The data was collected from Yahoo Finance and spans the period 22 October 1997 - 22 January 2007, summing up 2324 observations.

Figure 1 and Figure 2 show graphs of the financial returns and their volatility evolutions. In Table 1, where we report some summary statistics, we observe that the four returns series are negatively skewed and have a kurtosis between 6.159 and 10.068.

Next, we proceed by applying some statistical tests to the data to highlight its underlying generator process. We start by testing whether the returns are independently and identically distributed (iid) with the BDS test of Brock et al. (1996). Table 2 shows the results of the BDS test. The null hypothesis

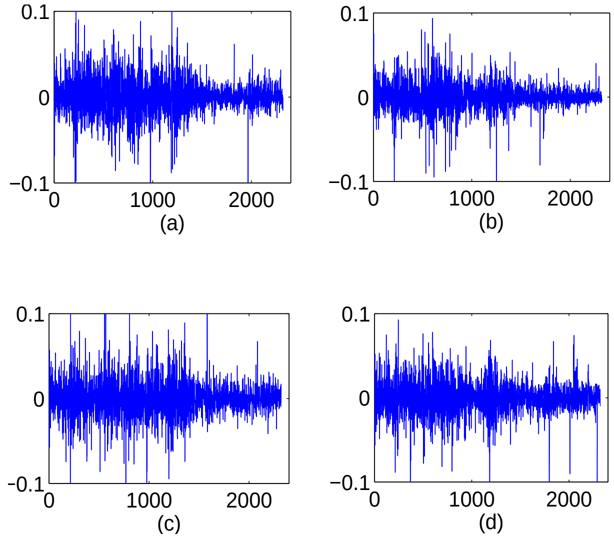

Figure 1: Financial returns: (a) American Express, (b) Coca-Cola, (c) Disney (Walt) and (d) Pfizer.

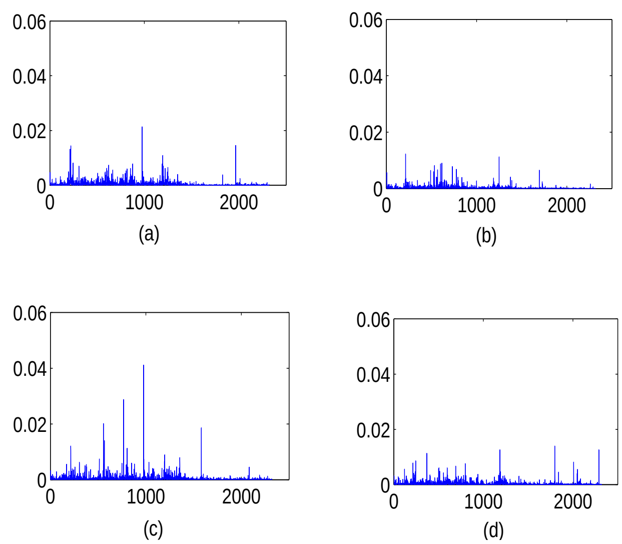

Figure 2: Squared returns: (a) American Express, (b) Coca-Cola, (c) Disney (Walt) and (d) Pfizer.

Stock Price Data	American Express	Coca-Cola	Disney (Walt)	Pfizer
Mean	0.0002	-0.0001	0.0001	0.0000
Variance	0.0005	0.0003	0.0005	0.0004
Skewness	-0.1478	-0.2429	-0.1961	-0.2729
Kurtosis	6.7037	8.2439	10.0677	6.1590

Table 1: Summary statistics of returns.

of iid is rejected for all returns series at a 5% significance level as in Hsieh (1993) and Brooks et al. (2000).

Hsieh (1991) showed that the BDS test can detect many types of non iid causes including linear dependence, non-stationarity, chaos and non-linear stochastic processes. In order to understand the underlying reason for the rejection of the null hypothesis, we calculate the autocorrelation functions of the returns and squared returns up to order 15 and we test if they are statistical significant. The Ljung-Box Q statistic is also computed and works here as a mere indicator since, with the violation of the iid assumption, it does not converge anymore to a χ^2 distribution.

Table 3 shows that the autocorrelations and the Ljung-Box Q statistic of squared returns are larger

ϵ/σ	Contracts	Embedding dimensions			
		2	3	4	5
0.5	American Express	13.7	21.2	28.7	40.5
	Coca-Cola	12.9	18.7	25.5	33.8
	Disney (Walt)	11.4	17.3	22.6	28.6
	Pfizer	11.9	14.8	18.6	23.1
1.0	American Express	13.5	18.5	22.9	28.1
	Coca-Cola	12.2	16.5	20.2	24.4
	Disney (Walt)	10.9	14.8	18.0	20.9
	Pfizer	12.0	14.4	16.9	19.4
1.5	American Express	12.9	16.5	18.7	20.9
	Coca-Cola	10.9	13.8	15.7	17.7
	Disney (Walt)	10.6	13.6	15.5	17.1
	Pfizer	11.5	13.4	14.9	16.1
2.0	American Express	11.7	14.9	16.1	17.3
	Coca-Cola	9.9	12.3	13.4	14.4
	Disney (Walt)	10.0	12.0	13.1	13.9
	Pfizer	14.4	11.8	12.6	13.3

Table 2: BDS test statistic for financial returns. The critical values of the statistic for a two-tailed test are: 1.645 (10%), 1.960 (5%), 2.326 (2%), and 2.576 (1%).

Lag length	A. Express Returns	Sq. Returns	Coca-Cola Returns	Sq. Returns	Disney Returns	Sq. Returns	Pfizer Returns	Sq. Returns
1	0.007	0.244*	0.039	0.162*	−0.006	0.096*	0.029	0.153*
2	−0.053*	0.195*	−0.055*	0.210*	−0.026	0.066*	−0.102*	0.097*
3	−0.008	0.182*	-0.003	0.082*	-0.005	0.109*	-0.039	0.113*
4	−0.006	0.151*	-0.009	0.110*	-0.005	0.073*	-0.006	0.098*
5	−0.041*	0.252*	0.001	0.084*	−0.036*	0.031	-0.014	0.102*
6	−0.052*	0.086*	0.023	0.080*	-0.020	0.054*	-0.033	0.098*
7	-0.018	0.182*	-0.002	0.078*	−0.012	0.048*	-0.017	0.104*
8	-0.017	0.182*	−0.072*	0.082*	-0.027	0.056*	0.043*	0.076*
9	0.010	0.133*	-0.035	0.069*	-0.020	0.055*	-0.007	0.106*
10	-0.012	0.134*	0.055*	0.125*	-0.005	0.083*	0.002	0.053*
11	0.009	0.070*	−0.017	0.081*	-0.025	0.053*	-0.034	0.045*
12	0.023	0.120*	0.036	0.068*	−0.013	0.031*	-0.013	0.063*
13	0.018	0.083*	-0.011	0.087*	-0.016	0.035*	-0.030	0.062*
14	-0.024	0.106*	0.007	0.124*	0.029	0.017*	-0.002	0.046*
15	-0.04	0.096*	-0.030	0.122*	0.014	0.030*	-0.009	0.078*
Q(15)	22.6	870.5	40.2	431.7	13.6	131.4	43.5	289.0

Table 3: Autocorrelations of returns and squared returns. The last line contains the values of the Ljung-Box Q statistic. * means that the correlation of order m with $m = 1, ..., 15$ is significant at a 5% significance level.

than the ones of returns. Moreover, the individual significance tests show evidence (at a 5% significance level) that both returns and squared observations are autocorrelated, although the autocorrelation is much stronger for the series of squared returns.

So far, we have found a non-linear dependence in the series. In order to check if this non-linearity is in mean or in variance, we test the null of zero conditional mean with the proposal of Hsieh (1989, 1991). If the null hypothesis is true, the bicorrelation coefficients, $\rho(i,j) = E(y_t\, y_{t-i}\, y_{t-j})/[\mathrm{Var}(y_t)]^{3/2}$, are zero for all $i, j \geq 1$. These coefficients are asymptotically normal distributed with zero mean and variance $[(1/T)\sum y_t^2\, y_{t-i}^2\, y_{t-j}^2]/[(1/T)\sum y_t^2]^3$. The bicorrelation coefficients are reported in Table 4 and we observe that none of them are statistically significant, which leads us to conclude that the non-linear dependence is in variance.

$\rho(i,j)$	A. Express	Coca-Cola	Disney	Pfizer
$\rho(1,1)$	-0.05	-0.06	0.06	0.08
$\rho(1,2)$	0.05	0.02	0.00	0.01
$\rho(2,2)$	0.02	0.07	-0.19	0.04
$\rho(1,3)$	0.02	0.01	0.04	-0.01
$\rho(2,3)$	0.01	0.06	-0.00	0.03
$\rho(3,3)$	0.04	0.00	-0.14	-0.02
$\rho(1,4)$	0.00	-0.00	0.04	-0.00
$\rho(2,4)$	-0.05	-0.08	0.05	0.01
$\rho(3,4)$	-0.03	0.05	-0.01	0.06
$\rho(4,4)$	0.02	0.03	0.14	0.02
$\rho(1,5)$	0.01	-0.05	-0.05	-0.03
$\rho(2,5)$	0.11	0.07	-0.01	0.01
$\rho(3,5)$	-0.01	-0.08	-0.05	0.04
$\rho(4,5)$	0.08	-0.13	0.06	-0.02
$\rho(5,5)$	0.24	-0.05	0.04	0.02

Table 4: Bicorrelation coefficients of the stock returns.

3 Realised Volatility

It can be shown, under innocuous regularity conditions, that realised volatility, measured as the sum of squared overnight returns and cumulative squared intraday returns, provides a much better estimator of daily volatility than squared daily returns. Some authors such as, for instance, Andersen and Bollerslev (1998); Andersen et al. (2001); Barndorff-Nielsen and Shephard (2001, 2002a,b, 2004); Comte and Renault (1998); Andersen et al. (2003, 2005) showed that realised volatility is a consistent estimator of the integrated volatility, that is the time integral of the instantaneous volatility when the asset price follows a diffusion process.
In this paper, we use the 5-min squared returns (from www.price-data.com) to calculate the measure of realised volatility proposed by Martens (2002) and used in Koopman et al. (2005), that consists on scaling the sum of 5-min returns by

$$\tilde{\sigma}_t^2 = \frac{\hat{\sigma}_{oc}^2 + \hat{\sigma}_{co}^2}{\hat{\sigma}_{co}^2} \sum_{d=1}^{D} R_{t,d}^2, \tag{1}$$

where t corresponds to a specific day ($t = 1, ..., T$), D is the total number of 5-min intraday returns, in our case $D = 78$, and the open-to-close sample variance, $\hat{\sigma}_{oc}^2$, and the close-to-open sample variance, $\hat{\sigma}_{co}^2$, are calculated in the following way:

$$\begin{aligned}
\hat{\sigma}_{oc}^2 &= \frac{10000}{T} \sum_{t=1}^{T} (\log P_{t,D} - \log P_{t,0})^2, \\
\hat{\sigma}_{co}^2 &= \frac{10000}{T} \sum_{t=1}^{T} (\log P_{t,0} - \log P_{t-1,D})^2,
\end{aligned}$$

where $\log P_{t,0}$ is the logarithm of the open market price and $\log P_{t,D}$ is the logarithm of the close market price. The need for this scaling is justified by the fact that the market is not open 24 hours a day, and consequently, the overnight return is more volatile than the intraday 5-min returns, which introduces extra "noise" into the realised volatility estimator. Hansen and Lunde (2002) proposed a different estimate of the open-to-close variance that is based on the average of $\sum_{d=1}^{D} R_{t,d}^2$ and Areal and Taylor (2002) proposed a different scaling where the weights depend on the proportions of variance.
The summary statistics of daily squared returns and realised volatility for the Coca-Cola data are reported in Table 5 and Figure 3. We observe various changes in realised volatility that correspond more or less to the ones of squared returns, but high volatility periods seem to be more amplified in daily squared returns than in realised volatility.

	Squared returns	Realised Volatility
Mean	2.77	3.98
Variance	55.76	27.97
Skewness	7.94	5.50
Kurtosis	90.31	56.56

Table 5: Coca-Cola summary statistics comparison.

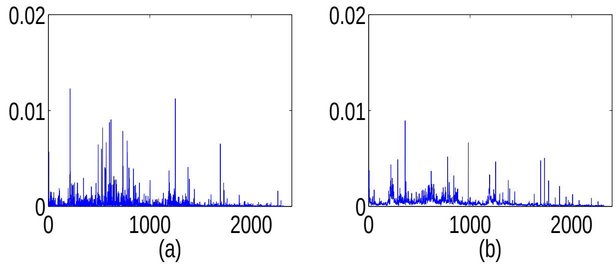

Figure 3: (a) Daily squared returns and (b) Realised volatility.

4 Conditional Heteroscedastic Models

4.1 Model Selection and Estimation

Since we have found in Section 2 that the non-linear dependence is in variance and financial econometrics research has suggested that GARCH-type models may be useful in modelling and forecasting the conditional variance of returns series, we start our analysis by presenting the GARCH(1,1) model of Bollerslev (1986) and some extensions of it that allow for asymmetric responses of volatility to price changes. Let y_t be the return at time t given by

$$y_t = \mu + \varepsilon_t = \mu + \sigma_t \, \epsilon_t, \tag{2}$$

where μ is the expected value of y_t, ε_t is the prediction error, σ_t^2 is the variance of y_t given information at time $t-1$ such as

$$\sigma_t^2 = \gamma + \alpha \, \varepsilon_{t-1}^2 + \beta \, \sigma_{t-1}^2, \tag{3}$$

$\sigma_t > 0$ and $\epsilon_t \sim NID(0,1)$.

As in Brooks and Persand (2003), we have carried out the test of Engle and Ng (1993) to infer about the need of an asymmetric conditional volatility model. The test is given as follows

$$\frac{\varepsilon_t^2}{\sigma_t^2} = \Phi_0 + \Phi_1 \, Z_{t-1} + \Phi_2 \, Z_{t-1} \, \frac{\varepsilon_t}{\sigma_t} + \Phi_3 \, Y_{t-1} \, \frac{\varepsilon_t}{\sigma_t} + w_t, \tag{4}$$

where w_t is a white noise error, Z_{t-1} is a dummy variable that takes value 1 if $\varepsilon_t/\sigma_t < 0$ and 0 otherwise, and Y_{t-1} is equal to $1 - Z_{t-1}$. The significance of Φ_1 and Φ_2 or Φ_3 suggest that there are sign and size biases, respectively. In order to perform a joint test for sign and size biases, we estimate equation (4) by OLS and we use a Lagranger Multiplier Principle TR^2 that follows a χ^2 distribution. The results of the test are reported in Table 6. They evidence for all returns series that the conditional volatilities are affected by the sign and size of shocks to volatility, which suggest additional modelling structure that incorporates the possibility of asymmetry into the variance equation (3).

For this purpose we consider two GARCH-type models. The first is the Threshold GARCH model proposed by Glosten et al. (1993) and denoted GJR, in which the conditional variance is given by

$$\sigma_t^2 = \gamma + \alpha \, \varepsilon_{t-1}^2 + \beta \, \sigma_{t-1}^2 + \lambda \, S_{t-1}^- \varepsilon_{t-1}^2, \tag{5}$$

	Φ_0	Φ_1	Φ_2	Φ_3	TR^2
American Express	-0.898	-1.816	-5.133	2.483	1370.168*
	(0.106)	(0.144)	(0.097)	(0.106)	
Coca-Cola	-0.977	-0.930	-3.948	2.650	1611.124*
	(0.067)	(0.094)	(0.065)	(0.067)	
Disney (Walt)	-1.488	-0.413	-3.927	3.360	1468.145*
	(0.084)	(0.116)	(0.081)	(0.084)	
Pfizer	-0.842	-0.966	-3.789	2.405	1670.663*
	(0.061)	(0.085)	(0.057)	(0.063)	

Table 6: Engle and Ng (1993) test for the GARCH model. * means that we reject the null hypothesis at any relevant significance level.

where $S^-_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and 0 otherwise. In this model the impact of ε^2_{t-1} on the conditional variance σ^2_t is different when ε_{t-1} is positive than when it is negative, reflecting that negative shocks are associated with increases in variance and positive shocks are associated with a small decrease in variance.

Another GARCH-type model that is able to take into account the asymmetric response of volatility to positive and negative price changes is the model proposed by Nelson (1991) and denoted EGARCH, where the conditional variance is expressed in the following way

$$\log\left(\sigma^2_t\right) = \gamma + \beta \log\left(\sigma^2_{t-1}\right) + \alpha\, \epsilon_{t-1} + w\left(|\epsilon_{t-1}| - \sqrt{2/\pi}\right). \tag{6}$$

The parameter α in this specification is the responsible for the asymmetric feature. In fact, if $-1 < \alpha < 0$, a positive shock increases volatility less than a negative one, and if $\alpha < -1$ a negative shock increases volatility while a positive shock reduces it.

We estimate all models by quasi-maximum likelihood (QML) with the Ox GARCH 4.2 package of Laurent and Peters (2006). In order to check if the models are correctly specified we apply the BDS test to the standardised residuals. Note that, in this case we need to calculate new critical values because the test favors the null of iid when we apply it to the standardised residuals of GARCH-type models. For this purpose, we have simulated 2000 data series from each model with a sample size similar to the original one, fitted each model on the simulated data and run the BDS test on the residuals.

		Embedding dimensions			
American Express	ϵ/σ	2	3	4	5
GARCH	0.5	-0.054	1.205	1.042	1.269
	1.0	0.057	0.898	0.944	1.109
	1.5	0.162	0.863	1.012	1.070
	2.0	0.309	1.108	1.370	1.113
GJR	0.5	0.570	0.900	1.112	1.223
	1.0	2.231	2.030*	1.865	1.748
	1.5	2.293	2.311*	2.199*	2.022
	2.0	2.530	2.740*	2.544*	2.039
EGARCH	0.5	-1.921	-1.844	-1.615	-1.308
	1.0	-1.242	-1.666	-1.706	-1.477
	1.5	-1.117	-1.762	-1.717	-1.552
	2.0	-.219	-1.463	-1.432	-1.429

Table 7: BDS test statistic for the standardized residuals (* significant at a 5% significance level). The critical values can be obtained from the authors upon request.

Table 7 presents the results of the BDS test for the standardised residuals obtained from fitting the selected models to the American Express data. We observe that we do not reject the null hypothesis of iid standardised residuals for the GARCH and EGARCH models.[1]

[1]The results for the other series are similar to the ones presented here, except for the GARCH model that performs

It is well known that the GARCH estimated persistence is too high to generate observed volatility pattern. Some authors such as Brooks et al. (2000) addressed this problem by including a proxy for overnight volatility into the variance equation (3). According to these authors this helps in explaining the level of volatility persistence. Others such as Hamilton and Susmel (1994) and Lamoureux and Lastrapes (1990, 1994) tried to solve this problem by using a Markov Switching ARCH model and the total volume of stock traded within a day, respectively.
In this paper, as in Koopman et al. (2005), we extend the presented models by including into the variance equations an extra explanatory variable that takes into account information from high-frequency data, realised volatility. We have included this variable due to the good forecasting results obtained in Koopman et al. (2005). In this case, the variance equation is the following

$$\sigma_t^2 = \gamma + \alpha\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2 + \delta\, \tilde{\sigma}_{t-1}^2,$$

where $\tilde{\sigma}_t^2$ is the estimate of realised at time t. The GJR and EGARCH variance equations, formulas (5) and (6), are extended in the same way and we denote these models GJR-RV and EGARCH-RV, respectively.

	Parameters					
	μ	γ	α	β	λ	w
GARCH						
American Express	0.0006	0.055	0.074	0.914		
	(0.0003)	(0.040)	(0.020)	(0.022)		
Coca-Cola		0.007	0.053	0.946		
		(0.007)	(0.027)	(0.026)		
Disney (Walt)	0.0007	0.039	0.088	0.913		
	(0.0004)	(0.032)	(0.049)	(0.044)		
Pfizer		0.142	0.119	0.850		
		(0.077)	(0.040)	(0.048)		
GJR						
American Express		0.009		0.938	0.110	
		(0.020)		(0.012)	(0.024)	
Coca-Cola		0.009		0.956	0.086	
		(0.005)		(0.011)	(0.025)	
Disney (Walt)		0.018		0.961	0.077	
		(0.013)		(0.018)	(0.041)	
EGARCH						
American Express	0.003		-0.076	0.999		0.248
	(0.001)		(0.032)	(0.0006)		(0.025)
Coca-Cola			-0.059	0.998		0.282
			(0.026)	(0.0008)		(0.040)
Disney (Walt)			-0.061	0.997		0.306
			(0.024)	(0.001)		(0.035)

Table 8: Final estimates and standard errors (in parenthesis) of the GARCH-type models.

From Table 8 we observe that the persistence estimated by the GARCH-type models, that depends on the sum of α and β for the GARCH model, is quite high. As an example and for the American Express and Coca-Cola data, we obtain values of the estimated persistences of 0.988 and 0.999, respectively (see Table 8). On the contrary, considering the GARCH-RV model, the estimated persistences for these series are 0.839 and 0.521, respectively (see Table 9). Note that in Table 8 and Table 9 we only present the models that are statistical significant.

slightly worse for the Coca-Cola and Disney data. All test results are available upon request from the authors.

	Parameters			
	α	β	w	δ
GARCH-RV				
American Express		0.839		0.142
		(0.029)		(0.026)
Coca-Cola		0.521		0.345
		(0.100)		(0.080)
Disney (Walt)		0.728		0.181
		(0.174)		(0.122)
Pfizer	0.104	0.658		0.197
	(0.046)	(0.141)		(0.110)
EGARCH-RV				
Coca-Cola	-0.050	0.998	0.235	501.31
	(0.023)	(0.0006)	(0.030)	(106.360)

Table 9: Final estimates and standard errors (in parenthesis) of the GARCH-RV-type models.

4.2 Forecasting

The main aim of this subsection is to highlight the volatility forecasting methodology. Concerning the GARCH model, the estimated one-day-ahead conditional variance at time $t-1$ is

$$\hat{\sigma}_t^2 = \hat{\gamma} + \hat{\alpha}\,\varepsilon_{t-1}^2 + \hat{\beta}\,\hat{\sigma}_{t-1}^2,$$

and given that $y_t = \mu + \varepsilon_t$, the previous expression can be written in the following way

$$\hat{\sigma}_t^2 = \hat{\gamma} + \hat{\alpha}(y_{t-1} - \hat{\mu})^2 + \hat{\beta}\,\hat{\sigma}_{t-1}^2.$$

Finally, it follows directly that the one-day-ahead variance forecast at time T can be calculated as

$$\hat{\sigma}_{T+1}^2 = \hat{\gamma} + \hat{\alpha}(y_T - \hat{\mu})^2 + \hat{\beta}\,\hat{\sigma}_T^2.$$

If the variance equation includes realised volatility, the one-day-ahead variance forecast at T can be computed as

$$\hat{\sigma}_{T+1}^2 = \hat{\gamma} + \hat{\alpha}(y_T - \hat{\mu})^2 + \hat{\beta}\,\hat{\sigma}_T^2 + \hat{\delta}\,\tilde{\sigma}_T^2,$$

where $\tilde{\sigma}_T^2$ is the value of realised volatility at T and $\hat{\delta}$ is the quasi-maximum likelihood estimate of δ (see Koopman et al., 2005). Similar expressions of the one-day-ahead variance forecasts for the other models can be deduced.

4.3 MCRR Methodology

Capital risk requirements, given by the percentage of the initial value of the position for a 95% coverage, are estimated for 1, 10 and 30 days investment horizons. To this end, we proceed as in Grané and Veiga (2007) by generating 20000 paths of future values of the price series with the help of the parameter estimates, the disturbances obtained by sampling with replacement from the iid standardised residuals (iid bootstrap), and the multi-step ahead volatility forecasts. The maximum loss over a given holding period is then obtained by computing

$$Q = (P_0 - P_1)\,n,$$

where n is the number of contracts, P_0 is the initial value of the position and P_1 is the lowest simulated price (for a long position) or the highest simulated price (for a short position) over the period. If the number of contracts is one, without loss of generality, we can write $\frac{Q}{P_0} = \left(1 - \frac{P_1}{P_0}\right)$ for a long position,

and $\frac{Q}{P_0} = \left(\frac{P_1}{P_0} - 1\right)$ for a short position. Note that, since P_0 is constant, the distribution of Q only depends on the distribution of P_1.
In this paper, we proceed as in Hsieh (1993) assuming that simulated prices are lognormal distributed, which it is frequent in the finance literature. Consequently, the maximum loss for a long position over the simulated days is given by $Q/P_0 = 1 - \exp(c_\alpha\, s + m)$, where c_α is the $\alpha \times 100\%$ percentile of the standard normal distribution and s and m are the standard deviation and mean of the $\ln(P_1/P_0)$, respectively. The analogous for a short position is given by $Q/P_0 = \exp(c_{1-\alpha}\, s + m) - 1$, where $c_{1-\alpha}$ is the $(1-\alpha) \times 100\%$ percentile of the standard normal distribution (see Brooks, 2002).
The confidence intervals for the MCRRs are obtained as the 95% percentile intervals estimated by iid bootstrap. For each model we estimate the parameters, we forecast the volatility and we keep the standardised residuals. Each value of the MCRR is obtained from 200 re-samples of the standardised residuals, proceeding as described above, and the confidence intervals are computed from 1000 estimated MCRR values. We choose the percentile intervals because it is possible to obtain a better balance in the left and right sides using the empirical distribution of the MCRRs (Efron and Tibshirani, 1993, chapter 13). The confidence intervals give us an idea about the sample dispersion in the MCRR estimates.

5 Results

The series show larger MCRRs for short positions than for long positions. As an example, for the American Express stock returns and according to the GARCH model, 1.79%, 5.16% and 8.02% of the value of a long position (as a percentage of the initial value of the position) will be enough to cover 95% of the expected losses if the position is held for 1, 10 and 30 days, respectively. The MCRRs for a short position are 1.89%, 5.58% and 9.68%, respectively. This finding could be explained by the existence of a positive drift in the returns over the sample period, indicating that series are not symmetric about zero. In fact, the mean for all series, except for the Coca-Cola returns, is positive over the sample period (see Table 1).

Long Position				
Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	1.79	2.03	1.87	1.71
10	5.16	6.32	5.73	5.18
30	8.09	8.65	9.55	8.95
Short Position				
Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	1.89	2.14	1.93	1.94
10	5.58	6.90	6.20	6.84
30	9.68	10.01	10.90	13.93

Table 10: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the American Express quotes.

From Tables 10–13 we observe that the MCCRs derived from the GARCH model are, in general, larger than those obtained with the other GARCH-type models. The reason for this to happen is the excessive volatility persistence implied by this model that leads to high values of volatility forecasts, and consequently, to high values of MCRRs. The inclusion of realised volatility into the variance equations usually improves the performance of models in forecasting volatility since the forecasts become more "sensitive" to changes in volatility (see Koopman et al., 2005). Consequently, and also because the level of volatility at the beginning of the MCRRs calculation period is low relatively to its historical level for half of the series, we obtain values of MCRRs for one day investment horizon smaller than those obtained without including realised volatility as an explanatory variable. The opposite may be observed when the volatility at the calculation period is high (see Table 13).

Long Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH	EGARCH-RV
1	1.03	0.93	0.98	0.81	0.79
10	3.21	3.34	3.08	2.66	2.67
30	5.58	5.80	5.38	5.19	5.32

Short Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH	EGARCH-RV
1	1.03	0.94	0.99	0.81	0.79
10	3.25	3.37	3.12	2.68	2.69
30	5.74	6.08	5.55	5.30	5.28

Table 11: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Coca Cola quotes.

Long Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	1.76	1.44	1.68	1.74
10	5.28	5.11	5.18	5.62
30	8.92	9.37	9.01	10.91

Short Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	1.86	1.44	1.72	1.78
10	6.24	5.11	5.62	6.12
30	11.55	9.37	10.04	12.40

Table 12: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Disney quotes.

Long Position

horizon	GARCH	GARCH-RV
1	1.86	2.10
10	5.61	5.38
30	8.64	16.49

Short Position

Horizon	GARCH	GARCH-RV
1	1.89	2.12
10	5.64	5.42
30	8.83	18.50

Table 13: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Pfizer quotes.

Tables 14–17 show the 95% confidence intervals for the MCRRs based upon the considered specifications. The results show that the amplitude of the intervals increase with the investment horizon, which makes the MCRR estimates for longer horizons less reliable, and the inclusion of realised volatility, in general, decreases their amplitude, specially, for very short investment periods.

For a full evaluation of the results, we perform an out-of-sample test of the MCRRs calculated with the selected models. By definition, the failure rate of a model is the number of times the estimated MCRRs are inferior to the returns (in absolute value). If the MCRR model is correctly specified, the failure rate should be equal to the pre-specified MCRR level (in our case, 5%). Therefore, we calculate the MCRRs for one day horizon for both long and short positions, and then, we check if

Long Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	[1.42,2.36]	[1.42,2.32]	[1.55,2.54]	[1.38,2.72]
10	[4.36,6.05]	[4.45,6.03]	[4.95,6.73]	[4.22,6.56]
30	[7.05,9.12]	[7.54,9.58]	[8.47,10.64]	[7.47,10.42]

Short Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	[1.53,2.36]	[1.48,2.32]	[1.62,2.49]	[1.59,2.83]
10	[5.26,6.58]	[5.05,6.36]	[5.46,6.88]	[6.06,7.52]
30	[9.61,11.84]	[8.80,10.95]	[9.81,12.21]	[12.66,15.34]

Table 14: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the American Express quotes.

Long Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH	EGARCH-RV
1	[0.84,1.28]	[0.78,1.09]	[0.80,1.20]	[0.82,1.18]	[0.66,0.97]
10	[2.85,3.66]	[2.90,3.77]	[2.73,3.50]	[2.74,3.48]	[2.34,3.02]
30	[4.98,6.25]	[5.76,7.19]	[4.80,6.02]	[4.98,6.21]	[4.75,5.85]

Short Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH	EGARCH-RV
1	[0.85,1.25]	[0.79,1.10]	[0.82,1.18]	[1.59,2.83]	[0.66,0.95]
10	[2.85,3.62]	[2.96,3.76]	[2.74,3.48]	[6.06,7.52]	[2.35,2.98]
30	[5.15,6.40]	[5.77,7.25]	[4.98,6.21]	[12.66,15.34]	[4.76,5.92]

Table 15: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Coca Cola quotes.

Long Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	[1.43,2.23]	[1.21,1.74]	[1.37,2.12]	[1.44,2.14]
10	[4.60,6.03]	[4.40,5.73]	[4.57,5.85]	[4.96,6.32]
30	[7.85,10.06]	[8.23,10.22]	[8.02,10.03]	[9.79,12.06]

Short Position

Horizon	GARCH	GARCH-RV	GJR	EGARCH
1	[1.43,2.23]	[1.23,1.84]	[1.42,2.16]	[1.48,2.20]
10	[4.60,6.03]	[4.75,6.20]	[4.93,6.33]	[5.40,6.89]
30	[7.85,10.06]	[9.26,11.77]	[9.03,11.45]	[11.22,14.21]

Table 16: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Disney quotes.

Long Position

Horizon	GARCH	GARCH-RV
1	[1.54,2.26]	[1.71,2.51]
10	[4.90,6.26]	[4.74,6.04]
30	[7.65,9.48]	[14.66,18.26]

Short Position

Horizon	GARCH	GARCH-RV
1	[1.58,2.23]	[1.79,2.48]
10	[5.00,6.35]	[4.82,6.04]
30	[7.98,9.90]	[16.65,20.82]

Table 17: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Pfizer quotes.

these MCRRs have been exceeded by price movements in day $t+1$. We roll this process forward and we calculate the MCRRs for 252 days.[2] In Table 18, we present the number of violations of the MCRR estimates generated by the models. We observe, relatively to the GARCH model, that the number of violations (in percentage) never exceeds the 5% nominal value for all series, which may indicate that the model generates "slight" excessive MCRRs, but the inclusion of realised volatility seems, in all cases, to improve the performance of the model. Finally, the best performances are registered by the asymmetric models: the GJR model for the Disney returns, the EGARCH for the American Express and the EGARCH-RV for the Coca-Cola returns, with failure rates closer to the nominal 5% level.

	American Express		Coca-Cola		Disney		Pfizer	
	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position
GARCH	2.0%	4.4%	1.6%	3.2%	2.4%	4.0%	2.4%	2.4%
GARCH-RV	2.8%	6.0%	1.6%	3.6%	3.2%	5.2%	2.8%	2.8%
GJR	2.8%	3.6%	1.6%	3.6%	3.6%	4.4%	*	*
GJR-RV	4.0%	6.3%	1.6%	5.2%	2.4%	6.0%	*	*
EGARCH	*	*	2.0%	4.8%	*	*	*	*
EGARCH-RV								

Table 18: Results of the out of sample test. Estimates of the failure rate. The MCRR's are computed to cover the 95% of expected losses. * means that we do not calculate the failure rate for these models.

Since the calculation of the empirical failure rate defines a sequence of ones (MCRR violation) and zeros (no MCRR violation), we can use the well known likelihood ratio test for a proportion in order to test $H_0 : f = 5\%$ vs. $H_1 : f \neq 5\%$, where f is the theoretical failure rate. We apply this test to the failure rates for long and short positions. Table 19 reports its p-values. The results evidence that there is not "the best" model, for which we never reject the null hypothesis that the theoretical failure rate is equal to the nominal level, but we have clear indications that models including realised volatility and/or asymmetries perform better in computing accurate estimates of minimum capital risk requirements.

	American Express		Coca-Cola		Disney		Pfizer	
	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position
GARCH	0.000	0.321	0.000	0.052	0.004	0.209	0.004	0.004
GARCH-RV	0.017	0.252	0.000	0.116	0.052	0.443	0.017	0.017
GJR	0.017	0.116	0.000	0.116	0.116	0.321	*	*
GJR-RV	0.209	0.198	0.000	0.443	0.004	0.252	*	*
EGARCH	*	*	0.000	0.441	*	*	*	*
EGARCH-RV								

Table 19: p-values of the out of sample test. The MCRR's are computed to cover the 95% of expected losses. * means that we do not calculate the failure rate for these models.

6 Conclusions

In this paper, we examine the performance of three conditional heteroscedastic models in the calculation of minimum capital risk requirements for long and short positions, using 1, 10 and 30 days investment horizons. Some of the models take into account the possibility of asymmetric responses of volatility to positive and negative price changes and they are extended by including into their variance equations realised volatility as an explanatory variable. The results show that the inclusion of realised volatility decreases the estimated persistence and leads to more accurate estimates of minimum capital risk requirements. Moreover, models' volatility forecastability decreases with the increase of the

[2] For a long position the failure rate is obtained as the percentage of negative returns smaller than one day ahead MCRRs for long positions. Analogously, for a short position the failure rate is estimated as the percentage of positive returns larger than one day ahead calculated MCRRs for short positions (see Giot and Laurent, 2003, 2004).

investment horizon, which is reflected by the range of the MCRRs confidence intervals. Finally, the results also stress that asymmetric models (with and without including realised volatility) perform better in out-of-sample tests. This paper may be of some reference to those financial institutions that use or plan to use these conditional heteroscedastic models to calculate minimum capital risk requirements, since it stresses the important role of realised volatility and asymmetries on stock returns risk estimates.

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