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VOLATILITY MODELLING AND ACCURATE MINIMUM CAPITAL RISK REQUIREMENTS: A COMPARISON AMONG SEVERAL APPROACHES

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Abstract

In this paper we estimate, for several investment horizons, minimum capital risk requirements for short and long positions, using the unconditional distribution of three daily indexes futures returns and a set of GARCH-type and stochastic volatility models. We consider the possibility that errors follow a *t*-Student distribution in order to capture the kurtosis of the returns distributions. The results suggest that an accurate modeling of extreme returns obtained for long and short trading investment positions is possible with a simple autoregressive stochastic volatility model. Moreover, modeling volatility as a fractional integrated process produces, in general, excessive volatility persistence and consequently leads to large minimum capital risk requirement estimates. The performance of models is assessed with the help of out-of-sample tests and *p*-values of them are reported.

***Keywords**:* Minimum Capital Risk Requirement, Moving Block Bootstrap, Stochastic Volatility, Volatility Persistence.

***JEL Classification**:* C14, C15, G13.

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Volatility modeling and accurate minimum capital risk requirements: a comparison among several approaches.

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Abstract

In this paper we estimate, for several investment horizons, minimum capital risk requirements for short and long positions, using the unconditional distribution of three daily indexes futures returns and a set of GARCH-type and stochastic volatility models. We consider the possibility that errors follow a t-Student distribution in order to capture the kurtosis of the returns distributions. The results suggest that an accurate modeling of extreme returns obtained for long and short trading investment positions is possible with a simple autoregressive stochastic volatility model. Moreover, modeling volatility as a fractional integrated process produces, in general, excessive volatility persistence and consequently leads to large minimum capital risk requirement estimates. The performance of models is assessed with the help of out-of-sample tests and p-values of them are reported.

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1 Introduction

In recent years, financial markets across the world have reported an increase in volatility that has started to concern financial institutions such as banks since they could incur in large trading losses. Consequently, this has created a need for quantitative techniques that aim at specifying the possible losses that these institutions can suffer. Concerned with the first goal, financial econometrics has developed models to account for the empirical facts of financial data and to provide accurate estimators of volatility. One of the most well known models for its good performance in dealing with financial data is the generalized autoregressive conditional heteroscedasticity model (GARCH) of Bollerslev (1986). Later, other GARCH-type models have appeared

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to capture important characteristics within financial time series: the asymmetric response of volatility to positive and negative returns and the volatility persistence. Some examples are the exponential GARCH (EGARCH) of Nelson (1991), the fractional integrated GARCH (FIGARCH) of Baillie (1996) and the fractional integrated EGARCH (FIEGARCH) of Baillie et al. (1996). The latter models specify volatility as a fractional integrated process with the purpose to capture the slow decay of the autocorrelation functions of non-linear transformations of returns like squares and absolute values. Relatively to the FIGARCH model, Davidson (2004) and Zaffaroni (2004) showed, respectively, that it has the unpleasant property that the persistence of shocks to volatility decreases as the long-memory parameter increases and that it cannot generate squared returns autocorrelations with long-memory. Therefore, Davidson (2004) presented an alternative to the previous model, denoted hyperbolic GARCH (HYGARCH), that re-establishes the traditional relation between the long-memory parameter and volatility. Alternatively to the GARCH-type models, Taylor (1986) proposed an autoregressive stochastic volatility model, denoted ARSV, that specifies volatility as a latent variable modeled as an autoregressive process. Carnero et al. (2004) favored the ARSV model because it explains the relationship between the kurtosis of returns distribution and the persistence of volatility better than the GARCH(1,1). Later but still in the context of stochastic volatility, Breidt et al. (1998) and Harvey (1998) proposed a stochastic volatility model whose volatility process follows a fractional integrated process and can be regarded as an alternative to the FIGARCH and the HYGARCH models.

Concerning the second goal of quantifying losses, value-at-risk (VaR) is a very popular technique providing an estimate of the probability of likely losses to occur over a given time horizon due to changes in market prices. A very related concept is the minimum capital risk requirement (MCRR) defined as the minimum sufficient capital to absorb all except a pre-specified percentage of unforeseen losses (see Brooks et al., 2000). Several methods have been proposed to calculate the VaRs, among them we include the "delta-normal" method, the historical simulation that involves the estimation of the quantile of the portfolio returns, and the structured Monte Carlo simulation (see Dowd, 1998; Jorion, 2001). Although the Monte Carlo approach is powerful and flexible for generating VaR estimates because it can be specified any stochastic process for the asset price, it is not free of important drawbacks. The first and more important one is related to the stochastic process that has been assumed for the price of the asset, because if this assumption is not correct, the calculated VaRs can be inaccurate. The second drawback is related to the computational time required to compute the VaRs. It may be very high for a large portfolio. An alternative approach that could overcome the first drawback, is to use bootstrap rather than Monte Carlo simulation.

In this paper, we address an approach to the calculation of the MCRRs similar to the works of Hsieh (1993) and Brooks et al. (2000). We calculate the MCRRs for three indexes futures (the FTSE-100 Index Futures, the Russell 2000 Index Futures and the S&P 500 Index Futures) defined on long and short positions for 1, 5, 10, 30, 90 and 180 days horizons, using the unconditional and conditional approaches. We use the moving block bootstrap of Künsch (1989) and Liu and Singh (1992) for computing the unconditional distribution of returns since, contrary to previous papers, we have found that the returns of the considered financial series are not iid, not only due to

the existence of non-linear dependence, but also due to a weak linear dependence structure of the own returns detected by the rejection of the null hypothesis of the Ljung-Box test. Moreover, we have calculated (for the FTSE-100 Index Futures) the MCRRs both with the iid bootstrap described in Efron and Tibshirani (1993) and the moving block bootstrap described in Lahiri (2003) and we have found huge differences in the MCRRs estimates, specially for long positions and larger investment horizons. Thus, the motivation of this paper is to add new evidence from the futures market to the modeling of financial data by calculating appropriate MCRRs for these three indexes futures, to highlight some volatility forecasting features of well known specifications, since accurate volatility estimators for futures positions are essential to impose optimal capital deposits, and to compare different approaches in order to understand better the risks associated to derivative positions.
As in Hsieh (1993) and Brooks et al. (2000) we have fitted several GARCH-type models and we have found that the BDS test (Brock et al., 1996) applied to the residuals rejects models that introduce asymmetries between the conditional variance and the returns, such as the EGARCH, the GJR of Glosten et al. (1993) and the FIEGARCH models, which is consistent with previous works. The "best" models according to the BDS test take into account the volatility clustering, the fat tails of the returns distributions and the volatility persistence. The performance of the models is assessed by computing the failure rates in an out-sample period.
The most important findings in this paper are: first, the simple autoregressive stochastic volatility model of Taylor (1986) with errors following a normal distribution performs better in terms of volatility forecasting and provides accurate MCRRs estimates; second, the MCRRs based upon GARCH models are generally larger for short investment horizons and smaller for long investment horizons than the ones obtained with the alternative specifications due to the decreasing volatility forecastability registered by GARCH models when the forecasting horizon increases (see Christoffersen and Diebold, 2000); and finally, the fractional stochastic volatility model, in general, generates excessive MCRRs due to the extreme volatility persistence that is produced by this model.
The remainder of the paper proceeds as follows: In Section 2, we present a description of data and its main statistical properties. In Section 3, we estimate several conditional heteroscedastic and stochastic volatility models and we present the forecasting and the MCRRs methodologies. We present the moving block bootstrap in Section 4. Section 5 reports the main empirical results and we conclude in Section 6.

2 Data Analysis

In this study, we calculate the MCRRs for three indexes futures: the FTSE-100 Index Futures, the S&P 500 Index Futures, and the Russell 2000 Index Futures. The data was collected from EconWin and spans the period 2 August 1989 - 18 May 2005 for the FTSE-100 Index Futures, the period 4 August 1989 - 16 October 2006 for the S&P 500 Index Futures and the period 5 February 1993 - 15 December 2006 for the Russell 2000 Index Futures. We have deleted from the data set observations corresponding to non trading days to avoid the incorporations of spurious zero returns, leaving 3980, 4366 and 3421 observations for the FTSE-100, S&P 500 and Russell indexes futures, respectively.

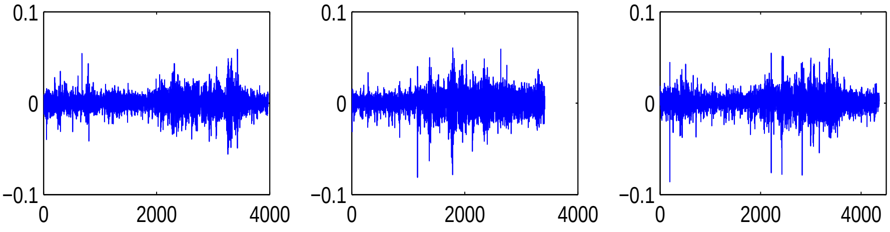


Figure 1: Series of financial returns: (a) FTSE-100 Index Futures, (b) Russell Index Futures and (c) S&P 500 Index Futures.

Figure 1 and Figure 2 show graphs of the financial returns and their volatility evolutions. In Table 1, where we report some summary statistics, we observe that the three returns series are negatively skewed and have a kurtosis between 5.7349 and 8.3197.

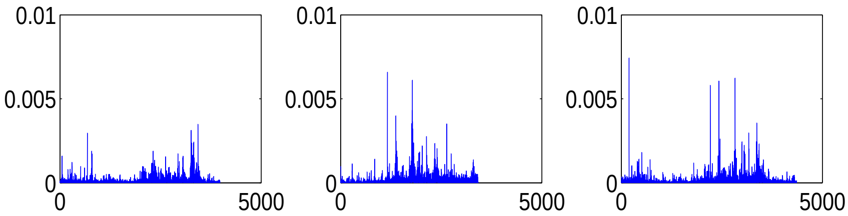


Figure 2: Squared returns: (a) FTSE-100 Index Futures, (b) Russell Index Futures and (c) S&P 500 Index Futures.

Futures Contracts	FTSE-100	Russell	S&P 500
Mean	0.0002	0.0004	0.0004
Variance	0.0001	0.0001	0.0002
Skewness	-0.0841	-0.2588	-0.2961
Kurtosis	5.8592	8.3197	5.7349

Table 1: Summary statistics of returns.

Next, we test whether the returns are independently and identically distributed (iid) because a rejection of this hypothesis leads to a difference how conditional and unconditional densities describe short term dynamics of prices (see Hsieh, 1993). To this end, we apply the BDS test of Brock et al. (1996) to the returns series. Under the null hypothesis the BDS test statistic is asymptotically normal distributed (see Brock et al., 1991). Table 2 shows the results of the BDS test. The null hypothesis of iid is rejected for all the three returns series at a 5% level of significance, which is consistent with the results of Hsieh (1993) and Brooks et al. (2000).
Hsieh (1991) showed that the BDS test can detect many types of non iid causes including linear dependence, non-stationarity, chaos and non-linear stochastic processes. In

ϵ/σ	Contracts	Embedding dimensions			
		2	3	4	5
0.5	FTSE-100	8.9	13.0	17.0	20.6
	Russell	15.4	22.9	30.3	40.9
	SPF	10.9	14.8	18.3	23.2
1.0	FTSE-100	10.7	15.0	18.6	21.7
	Russell	14.4	21.3	25.9	31.4
	SPF	11.0	15.3	18.5	22.4
1.5	FTSE-100	12.7	17.4	20.6	23.3
	Russell	14.0	20.7	24.1	27.1
	SPF	11.1	15.5	18.1	21.0
2.0	FTSE-100	13.8	18.7	21.7	23.9
	Russell	13.8	20.7	23.7	25.9
	SPF	11.9	15.9	17.9	20.2

Table 2: BDS test statistic for financial returns. The critical values of the statistic for a two-tailed test are: 1.645 (10%), 1.960 (5%), 2.326 (2%), and 2.576 (1%).

order to understand the underlying reason, we calculate the autocorrelation function of the returns and squared returns up to order 15 and we test for the joint signification of the autocorrelations with the Ljung-Box Q statistic.

Lag length	FTSE-100 Returns	Sq. Returns	Russell Returns	Sq. Returns	S&P 500 Returns	Sq. Returns
1	0.015	0.214	-0.002	0.188	-0.038	0.193
2	-0.028	0.304	-0.045	0.326	-0.055	0.168
3	-0.075	0.251	0.012	0.206	-0.017	0.149
4	0.037	0.230	0.010	0.269	0.011	0.118
5	-0.026	0.249	-0.018	0.214	-0.039	0.158
6	-0.041	0.258	-0.028	0.251	-0.018	0.113
7	-0.024	0.201	-0.010	0.186	-0.037	0.139
8	0.045	0.282	-0.003	0.189	-0.002	0.122
9	0.033	0.164	-0.004	0.232	0.014	0.101
10	-0.025	0.234	-0.033	0.205	0.002	0.116
11	0.003	0.242	0.043	0.190	-0.001	0.110
12	0.003	0.191	0.055	0.183	0.047	0.102
13	0.037	0.227	0.017	0.193	0.023	0.081
14	-0.019	0.146	-0.030	0.144	0.011	0.088
15	0.028	0.166	0.009	0.240	-0.022	0.089
Q(15)	68.631*	3113.5*	36.758*	2458.6*	50.913*	1058.7*

Table 3: Autocorrelations of returns and squared returns. The last line contains the values of the Ljung-Box Q statistic. The critical values are: 22.31 (10%), 25.00 (5%), and 27.49 (1%), and * means significant at a 5% level.

Table 3 shows that the autocorrelations and the Ljung-Box Q statistic of the squared returns are larger than the ones of the returns. Moreover, the Ljung-Box Q statistic shows evidence (at a 5% significance level) that both the returns and the squared observations are autocorrelated, although the autocorrelation is much stronger for the series of the squared returns.

So far, we have found a non-linear dependence of the series, we check next whether this non-linearity is in mean or in variance. We test the null of zero conditional mean with the proposal of Hsieh (1989, 1991). If the null hypothesis is true, the bicorrelation coefficients, $\rho(i,j) = E(y_t\, y_{t-i}\, y_{t-j})/[Var(y_t)]^{3/2}$, are zero for all $i, j \geq 1$. These coefficients are asymptotically normal distributed with zero mean and variance $[(1/T)\sum y_t^2\, y_{t-i}^2\, y_{t-j}^2]/[(1/T)\sum y_t^2]^3$. We have implemented the individual significance test of the bicorrelation coefficients in Ox, available upon request. The bicorrelation coefficients are reported in Table 4. None of the bicorrelation coefficients are statistically significant, which leads us to conclude that the non-linear dependence is in variance.

$\rho(i,j)$	FTSE-100	Russell	S&P 500
$\rho(1,1)$	0.08	0.04	0.24
$\rho(1,2)$	-0.06	-0.03	0.03
$\rho(2,2)$	0.14	0.10	0.05
$\rho(1,3)$	0.00	0.06	0.07
$\rho(2,3)$	-0.02	-0.02	0.02
$\rho(3,3)$	0.07	0.10	0.12
$\rho(1,4)$	0.04	0.04	0.03
$\rho(2,4)$	0.13	0.05	0.03
$\rho(3,4)$	0.03	-0.10	-0.07
$\rho(4,4)$	0.01	0.08	0.13
$\rho(1,5)$	-0.08	-0.04	-0.01
$\rho(2,5)$	-0.03	0.07	0.05
$\rho(3,5)$	0.02	0.01	-0.01
$\rho(4,5)$	-0.01	-0.01	0.00
$\rho(5,5)$	-0.01	-0.04	0.12

Table 4: Bicorrelation coefficients of the futures returns.

3 Conditional Approach: GARCH and Stochastic Volatility Modeling

3.1 Model selection and estimation

Given the conclusions of Section 2, we need to model carefully the conditional variance of the returns series to obtain accurate MCRR estimates. Rather than choosing a model a priori, we estimate several models in an attempt to choose the best specifications for each series. In the conditional heteroscedasticity context, we first estimate the GARCH(1,1), the FIGARCH(1,1) and the HYGARCH(1,1) with normal and t-Student errors. We estimate the FIGARCH model, despite its drawbacks, because we would like to compare its forecasting performance to the one of its alternative, the HYGARCH model. Finally, we consider also the GJR(1,1), the EGARCH(1,1) and the FIEGARCH(1,1) since Brooks and Persand (2003) found that allowing for asymmetric responses of volatility to positive and negative returns can improve the VaR estimates. The criterium chosen to select the models is based on their capacity to capture the non-linear dependence in returns. To this end, we apply the BDS test to the standardized residuals. If they are iid, the models are correctly specified. In this case, we need to calculate new critical values because the test favors the null of iid, when we apply it to the standardized residuals of GARCH-type models. Therefore, we have simulated 2000 data series from each model with a sample size similar to the original one (imposing the same coefficient estimates), fitted each model on the simulated data and run the BDS test on the residuals. The models are estimated by quasi-maximum likelihood (QML) with the Ox GARCH 4.2 package of Laurent and Peters (2006).

		Embedding dimensions			
Russell	ϵ/σ	2	3	4	5
GARCH-Gauss	0.5	0.183	1.357	1.509	1.841
	1.0	-0.371	1.028	1.231	1.541
	1.5	-0.961	0.254	0.514	0.799
	2.0	-1.539	-0.609	-0.191	0.038
GARCH-Stud	0.5	0.550	1.847	2.096	2.397*
	1.0	-0.057	1.441	1.700	2.045
	1.5	-0.580	0.710	1.042	1.353
	2.0	-1.090	-0.088	0.400	0.646
FIGARCH-Stud	0.5	1.029	1.534	1.412	1.461
	1.0	0.672	1.386	1.069	1.038
	1.5	0.251	0.786	0.566	0.556
	2.0	-0.188	0.195	0.141	0.110

Table 5: BDS test statistic for the standardized residuals (* significant at a 5% significance level). The critical values can be obtained from the authors upon request.

The selected models are the ones for which we do not reject the null hypothesis of iid standardized residuals, which are: the GARCH-Gauss (normal errors), the GARCH-Stud (t-Student errors), the FIGARCH-Stud and the HYGARCH-Gauss (this model is only significant for the FTSE-100 Index Futures). Table 5 presents the results of the BDS test for the standardized residuals obtained from fitting the selected models to the Russell Index Futures.[1]

The HYGARH model proposed by Davidson (2004) is given by:

$$y_t = \mu + \varepsilon_t = \mu + \sigma_t \, \epsilon_t,$$

where ε_t is the prediction error, σ_t^2 is the variance of y_t given information at time $t-1$, $\sigma_t > 0$, $\epsilon_t \sim NID(0,1)$ or a t-Student distribution and

$$\sigma_t^2 = \omega + \theta(L) \, \varepsilon_t^2,$$

where

$$\theta(L) = 1 - \frac{\alpha^*(L)}{\beta(L)}(1 + \psi((1-L)^d - 1)),$$

$\alpha^*(L) = 1 - \sum_{i=1}^{q} \alpha_i^* L^i$, $\beta(L) = 1 - \sum_{i=1}^{p} \beta_i L^i$, $\omega > 0$, $\psi \geq 0$ and $d \geq 0$. For values of $d \in [0, 1/2)$ the conditional variance is stationary. The model simplifies to a GARCH(p,q) and to a FIGARCH(p,d,q) for $\psi = 0$ and $\psi = 1$, respectively. For $0 < \psi < 1$, we have a nested model that is able to generate long-memory as d increases.

From Table 6 we observe that the volatility implied by the GARCH-type models, that depends on the sum of α and β for the GARCH models and also on the parameter d for the fractional integrated GARCHs, is quite high. As an example and for the FTSE-100 Index Futures, we obtain values of implied volatility of 0.988 and 0.991 for the GARCH-Gauss and GARCH-Stud, respectively.

[1] The results for the other series are similar to the ones presented here, except for the GARCH model that performs slightly worse for the FTSE-100 Index Futures. All test results are available on request from the authors.

	Parameters						
	μ	γ	α	β	DF	d	$ln(\psi)$
GARCH-Gauss							
FTSE-100	0.0003	0.013	0.076	0.912			
	(0.0001)	(0.003)	(0.009)	(0.011)			
Russell	0.0007	0.019	0.115	0.875			
	(0.0002)	(0.005)	(0.013)	(0.013)			
S&P 500	0.0005	0.008	0.057	0.936			
	(0.0001)	(0.002)	(0.008)	(0.009)			
GARCH-Stud							
FTSE-100	0.0004	0.010	0.070	0.921	14.030		
	(0.0001)	(0.003)	(0.009)	(0.010)	(2.539)		
Russell	0.0008	0.014	0.104	0.890	17.009		
	(0.0002)	(0.004)	(0.013)	(0.013)	(4.061)		
S&P 500	0.0006	0.005	0.054	0.943	6.271		
	(0.0001)	(0.002)	(0.008)	(0.009)	(0.576)		
FIGARCH-Stud							
FTSE-100	0.0004	1.197	0.142	0.587	13.575	0.4801	
	(0.0001)	(0.393)	(0.041)	(0.068)	(2.325)	(0.050)	
Russell	0.0008	0.737	n.s.	0.266	18.640	0.3374	
	(0.0002)	(0.158)		(0.039)	(4.287)	(0.033)	
S&P 500	0.0006	0.752	0.227	0.608	6.576	0.420	
	(0.0001)	(0.239)	(0.041)	(0.058)	(0.541)	(0.045)	
HYGARCH-Gauss							
FTSE-100	0.0003	0.039	0.145	0.658		0.594	-0.041
	(0.0001)	(0.012)	(0.040)	(0.076)		(0.093)	(0.020)

Table 6: Estimates and standard errors (in parenthesis) of the ARCH-type models. n.s. stands for non significant at any relevant significance level.

In the context of stochastic volatility, natural competitors to the GARCH, FIGARCH and HYGARCH models are the autoregressive stochastic volatility model (denoted ARSV) of Taylor (1986) and the autoregressive fractional integrated stochastic volatility model (denoted ARLMSV) that extends the models of Breidt et al. (1998) and Harvey (1998). The first is a short-memory model while the second has as a short-memory component and a long-memory component (a fractional integrated process is specified for the volatility). These models are estimated with the Whittle estimation method. Following the same procedure as before, we apply the BDS test to the residuals of the ARSV and the ARLMSV models and observe that the null hypothesis of iid is not rejected for the ARSV residuals in all series except for the Russell Index Futures. Relatively to the ARLMSV model, it seems that the model fits very well the FTSE-100 Index Futures returns.
The ARLMSV model is given by the following expressions:

$$y_t = \sigma\, \epsilon_t \exp\left(\frac{h_t}{2}\right) \quad (1)$$

$$(1-\phi L)(1-L)^d\, h_t = \eta_t. \quad (2)$$

In equation (1), σ denotes a scale parameter, $\sigma_t = \exp(h_t/2)$ is the volatility of y_t (the returns at time t), ϵ_t is $NID(0,1)$ and η_t is $NID(0,\sigma_\eta^2)$, where σ_η^2 is the variance of η_t. The ARSV model is obtained from equations 1 and 2 by imposing the restriction $d = 0$.

		Parameters			
		ϕ	σ	σ_η^2	d
ARSV	FTSE-100	0.994	0.003	0.007	
	Russell	0.995	0.003	0.010	
	S&P 500	0.996	0.002	0.007	
ARLMSV	FTSE-100	0.968	0.003	0.001	0.467
	Russell	0.810	0.003	0.004	0.660
	S&P 500	-0.795	0.002	0.098	0.874

Table 7: Estimates of the stochastic volatility models.

Table 7 reports the stochastic volatility models parameter estimates. The estimate of σ is $\hat{\sigma}$=exp$\{0.5\,\hat{\mu} + 0.5\ E(\log\,\epsilon_t^2)\}$ where $\hat{\mu}$ is the sample mean of $\log(y_t^2)$, and assuming the normality of errors we have that $E(\log\,\epsilon_t^2) = -1.27$ (see Zaffaroni, 2005). We also observe that the volatility persistence implied is very high, inducing that the effects of shocks to the conditional variance take time to dissipate. Remind that in the case of the ARSV, the persistence is only given by ϕ, and in the case of the ARLMSV it is given by ϕ and d.

3.2 Forecasting

The main aim of this subsection is to highlight the volatility forecasting methodology. Thus, and concerning the GARCH-type models, we start by eliciting the dynamics of the one-step-ahead conditional variance. With the idea that a GARCH(1,1) model can be written by recursive substitution as a ARCH(∞), the multi-step forecast of the conditional variance based upon the available information at t is

$$\sigma^2_{t+k|t} = \sigma^2 + (\alpha+\beta)^{k-1}\left(\sigma^2_{t+1|t} - \sigma^2\right),$$

where σ^2 (the unconditional variance) is equal to $\sigma^2 = \gamma\,(1-\alpha-\beta)^{-1}$ and it is assumed that $(\alpha+\beta) < 1$ in order to guarantee that σ^2 exists and the multi-step forecast of the conditional variance converges to the unconditional variance at an exponential rate fixed by $\alpha+\beta$ (see Andersen et al., 2005). If instead of a GARCH(1,1) we have a FIGARCH(1,d,1), the actual conditional variance forecasts are given by

$$\sigma^2_{t+k|t+k-1} = \gamma\,(1-\beta)^{-1} + \lambda\,(L)\ \sigma^2_{t+k-1|t+k-2},$$

with $\sigma^2_{t+k|t+k-1} \equiv \varepsilon_t^2$ for $k < 0$, $\lambda\,(L) \equiv 1-(1-\beta L)^{-1}\,(1-\alpha L-\beta L)\,(1-L)^d$, whose coefficients are computed from the following expressions:

$$\lambda_1 = \alpha + d, \quad \lambda_j = \beta\,\lambda_{j-1} + \left[\frac{j-1-d}{j} - (\alpha+\beta)\right]\delta_{j-1}, \quad j = 2,3,\ldots$$

and $\delta_j \equiv \delta_{j-1}\,(j-1-d)\,/j$. Note that the δ_j's are the coefficients in the Maclaurin series expansion of $(1-L)^d$ (see Andersen et al., 2005).
With respect to stochastic volatility, first, we have to estimate h_t based on the full sample. To this end, we use a state-space smoothing algorithm (Kalman filter) that leads to the minimum mean square linear estimator (MMSLE) of h_t, (see Harvey and

Shephard, 1993; Harvey, 1998). The method is based on transforming equation (1) to obtain

$$\mathbf{w} = k\,\mathbf{1} + \mathbf{h} + \boldsymbol{\xi}, \tag{3}$$

where $\mathbf{w}$ is a $T \times 1$-vector that contains the observations of log y_t^2, $t = 1\ldots,T$, $\mathbf{1}$ is a $T \times 1$-vector of ones, $\boldsymbol{\xi}$ is a $T \times 1$-vector containing log $\epsilon_t^2 - E(\log\,\epsilon_t^2)$, $t = 1,\ldots,T$, and $k = \log\,\sigma^2 + E(\log\,\epsilon_t^2)$. Under the assumptions that h_t is stationary and h_t and ξ_t are uncorrelated, the covariance matrix of $\mathbf{w}$ is $\mathbf{V} = \mathbf{V}_h + \mathbf{V}_\xi$, where $\mathbf{V}_h$ and $\mathbf{V}_\xi$ are the covariance matrices of h_t and ξ_t, respectively. Hence, the MMSLE of h_t, in matrix notation, is given by

$$\tilde{\mathbf{h}} = \mathbf{V}_h\,\mathbf{V}^{-1}\mathbf{w} + k\,(\mathbf{I} - \mathbf{V}_h\,\mathbf{V}^{-1})\mathbf{1}, \tag{4}$$

where $\mathbf{I}$ is the identity matrix $T \times T$. Moreover, since the ξ_t's are serially uncorrelated, $\mathbf{V}_\xi = \sigma_\xi^2\,\mathbf{I}$, where σ_ξ^2 is the variance of ξ_t. Then, equation (4) can be written as

$$\tilde{\mathbf{h}} = \left(\mathbf{I} - \sigma_\xi^2\,\mathbf{V}^{-1}\right)\,\mathbf{w} + k\,\sigma_\xi^2\,\mathbf{V}^{-1}\,\mathbf{1},$$

and k can be estimated by the sample mean of log y_t^2 (see Harvey, 1998).[2] Yajima (1988) showed that there is only a slight loss of efficiency if the mean is used instead of the GLS estimator. Since the matrix $\mathbf{V}$ is a Toeplitz matrix, we have implemented the Trench algorithm described in Zohar (1969) to invert it.
Forecasting the log y_t^2, for $t = T+1,...,T+l$, implies for the stationary case that

$$\tilde{\mathbf{w}}_l = \mathbf{R}\,\mathbf{V}^{-1}\mathbf{w} + k(\tilde{\mathbf{I}} - \mathbf{R}\,\mathbf{V}^{-1})\mathbf{1},$$

where $\tilde{\mathbf{I}}$ is an $l \times T$-matrix defined in blocks in the following way:

$$\tilde{\mathbf{I}} = \left(\begin{array}{ccccc|ccc} 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & & \ddots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{array}\right),$$

$\tilde{\mathbf{w}}_l$ is a $l \times 1$-vector containing the forecasts of log y_t^2, for $t = T+1,...,T+l$, and $\mathbf{R}$ is a $l \times T$-matrix of covariances between $\mathbf{w}_l$ and $\mathbf{w}$. The forecasts of σ_{T+j}^2, for $j = 1,...,l$, are obtained by taking the exponential of the elements of $\mathbf{w}_l$ and multiplying them by $\tilde{\sigma}^2 = T^{-1}\sum_{t=1}^{T}\tilde{y}_t^2$, where $\tilde{y}_t = y_t\,\exp(-\tilde{h}_t/2)$.

3.3 MCRR Methodology

Capital risk requirements, given by the percentage of the initial value of the position for 95% coverage, are estimated for 1, 5, 10, 30, 90 and 180 days investment horizons. To this end, we generate 20000 paths of future values of the price series with the help of the parameter estimates, the disturbances obtained by sampling with replacement from the iid standardized residuals (iid bootstrap), and the multi-step ahead volatility forecasts. The maximum loss over a given holding period is then obtained by computing

$$Q = (P_0 - P_1)\,n,$$

[2] Harvey (1998) showed that if h_t is not stationary, we should differentiate equation (3) and then estimators of the first differences of h_t can be calculated from an analogous equation to equation (4).

where n is the number of contracts, P_0 is the initial value of the position and P_1 is the lowest simulated price (for a long position) or the highest simulated price (for a short position) over the period. We assume that the futures position is opened on the final day of the sample (see Brooks et al., 2000; Brooks, 2002). If the number of contracts is one, without loss of generality, we can write $\frac{Q}{P_0} = \left(1 - \frac{P_1}{P_0}\right)$ for a long position, and $\frac{Q}{P_0} = \left(\frac{P_1}{P_0} - 1\right)$ for a short position. Note that, since that P_0 is constant, the distribution of Q only depends on the distribution of P_1.

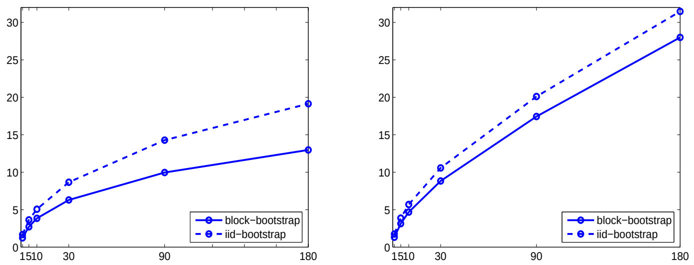
In this paper we proceed as in Hsieh (1993) assuming that simulated prices are lognormal distributed since this hypothesis is frequent in the finance literature. Consequently, the maximum loss for a long position over the simulated days is given by $Q/P_0 = 1 - \exp(c_\alpha\, s + m)$, where c_α is the $\alpha \times 100\%$ percentile of the standard normal distribution and s and m are the standard deviation and mean of the $\ln(P_1/P_0)$, respectively. The analogous for a short position is given by $Q/P_0 = \exp(c_{1-\alpha}\, s + m) - 1$, where $c_{1-\alpha}$ is the $(1-\alpha) \times 100\%$ percentile of the standard normal distribution (see Brooks, 2002).

The confidence intervals for the MCRRs are obtained as the 95% percentile intervals estimated by iid bootstrap. For each model we estimate the parameters, we forecast the volatility and we keep the standardized residuals. Each value of the MCRR is obtained from 200 re-samples of the standardized residuals, proceeding as described above, and the confidence intervals are computed from 1000 estimated MCRR values. We choose the percentile intervals because it is possible to obtain a better balance in the left and right sides using the empirical distribution of the MCRRs instead of the underlying normal distribution (Efron and Tibshirani, 1993, chapter 13). The confidence intervals not only allow us to determine if the differences in the MCRRs are significant for the conditional and unconditional approaches, but they also give us an idea about the sample dispersion in the MCRR estimates.

4 Unconditional Approach: Moving Block Bootstrap

We now proceed to compute the unconditional density of the returns series. Instead of using the iid bootstrap technique of Efron and Tibshirani (1993), as it was done by Hsieh (1993) and Brooks et al. (2000), we apply the moving block bootstrap (see Lahiri, 2003) on the observed price changes directly. We have seen in Section 2 that the return series are not iid mainly due to the existence of non-linear dependence. In fact, the autocorrelation functions of the squared returns are strongly significant. On the other hand, we also find that the returns of the three series present a weak dependence structure confirmed by the rejection of the null hypothesis of the Ljung-Box test. These two findings lead to the rejection of the iid hypothesis and to the inadequacy of the iid bootstrap.

In order to select the block size we have run a pilot experiment, following the algorithm described below. First, we simulate a series of size T from a GARCH model (we have seen in Section 3 that this model generates residuals that are iid for the three series, and consequently, it is a good specification for the financial returns) and we obtain the "true values" of the 2.5 and 97.5 percentiles (estimators of the VaR for long and short positions, respectively) as the mean values of 10000 realizations of the simulated series. Second, we perform a moving block bootstrap of size b. For this, we select M realizations of the simulated series and a block size b. For each realization, we split it in blocks of size b and reconstruct it B times to obtain the percentiles of the realization. Using the M computed values of the 2.5 and 97.5 percentiles we obtain a confidence interval for each one of them. Then, we evaluate the coverage of the confidence intervals obtained in the second step, using the "true values" of the percentiles computed previously. Finally, we repeat this procedure for different values of the block size b and we select the value of b for which the coverage of the confidence intervals is optimal. We have used values of $T = 2049$, $M = 1000$, $B = 200$ and $b = 2^k$, for $k = 0, \ldots, 11$ and the best results have been obtained for $b = 2$, which is a common value for the block size when the inference problem involves higher-level parameters (see Lahiri, 2003). Once the block size has been fixed to $b = 2$, the estimation of the MCRRs (see Table 8–Table 10) has been carried out over 20000 block bootstrap replicates of each returns series and the confidence intervals shown in Table 11–Table 13 have been obtained as explained in Section 3.3.



(a) (b)

Figure 3: Comparison of the moving block bootstrap and the iid bootstrap methods in computing the capital requirement for 95% coverage probability as a percent of the initial value of the FTSE-100 Index Futures for (a) Long Position and (b) Short Position.

Figure 3 shows the difference in the estimates of the MCRRs obtained with the iid bootstrap and the moving block bootstrap, specially for a long position. This difference reenforces the adequacy of the moving block bootstrap in our case.

5 Results

All series show larger MCRRs for short positions than for long positions, specially, as the investment horizon increases.

As an example, for the FTSE-100 Index Futures and according to the Gaussian GARCH(1,1), approximately 1%, 2.18% and 3.01% of the value of a long position (as a percentage of the initial value of the position) will be enough to cover 95% of the expected losses if the position is held for 1, 5 and 10 days, respectively. The MCRRs for a short position are approximately 1.06%, 2.38% and 3.41%, respectively. This finding could be explained by the existence of a positive drift in the returns over the sample period, indicating that series are not symmetric about zero. In fact, the mean for all series is positive over the sample period. The FTSE-100 Index Futures MCRRs are smaller than the ones obtained by Brooks et al. (2000) for the same

Long Position							
No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	HYGARCH-Gauss	ARSV	ARLMSV	Uncond
1	1.00	0.98	0.88	0.89	0.90	0.77	1.26
5	2.18	2.16	2.05	1.95	2.02	1.73	2.74
10	3.01	2.99	2.96	2.71	2.88	2.46	3.82
30	4.77	4.77	5.27	4.40	5.09	4.34	6.41
90	6.46	6.62	9.28	6.70	9.15	7.74	10.08
180	6.80	7.10	12.98	8.11	13.32	11.09	12.94
Short Position							
No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	HYGARCH-Gauss	ARSV	ARLMSV	Uncond
1	1.06	1.05	0.95	0.96	0.94	0.80	1.35
5	2.38	2.37	2.28	2.17	2.14	1.82	3.14
10	3.41	3.42	3.42	3.14	3.10	2.64	4.62
30	6.06	6.16	6.75	5.79	5.79	4.90	8.78
90	10.37	10.89	14.03	11.06	11.63	9.69	17.55
180	14.16	15.39	22.80	16.92	19.12	15.48	28.07

Table 8: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the FTSE-100 Index Futures.

series possibly because our sample period does not include such extreme events like the stock market crash of October 1987.

Long Position						
No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	1.21	1.23	1.29	1.09	2.00	1.57
5	2.56	2.60	2.89	2.39	4.50	3.56
10	3.55	3.62	4.24	3.44	6.34	5.00
30	5.41	5.60	7.42	5.93	11.02	8.22
90	7.11	7.67	12.45	10.44	18.52	12.62
180	7.50	8.34	16.42	15.08	25.41	16.17
Short Position						
No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	1.32	1.34	1.40	1.13	2.06	1.66
5	2.93	2.99	3.24	2.50	4.61	3.74
10	4.32	4.42	4.96	3.65	6.70	5.57
30	8.14	8.46	10.10	6.95	12.21	10.78
90	15.37	16.51	20.74	13.95	22.70	21.93
180	23.82	26.27	33.51	23.20	33.96	35.68

Table 9: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Russell Index Futures.

Moreover, the MCCRs derived from block bootstrap are in general larger than those obtained from the conditional approach. This may occur because the level of volatility at the beginning of the MCRRs calculation period is low relatively to its historical level (see Figure 2). Therefore, the conditional approach gives us lower volatility forecasts than the historical average. As the holding period increases from 1 to 180 days, the MCRR estimates converge to those of the unconditional approach, except the ones obtained with the ARLMSV model for the returns of the Russell and S&P 500 indexes futures. Those seem to diverge from the unconditionally estimated

MCRRs as the horizon increases (see Table 8–Table 10). The reason for this to happen is the excessive volatility persistence implied in the ARLMSV model for these two returns series. Note that the estimates of d (the long-memory parameter) in these two cases lead to a non-stationary model.

Long Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	0.89	0.85	0.92	0.61	3.85	1.31
5	1.89	1.82	1.96	1.35	11.89	2.82
10	2.64	2.53	2.81	1.96	16.81	3.96
30	4.08	3.93	4.87	3.49	27.77	6.32
90	5.54	5.49	8.37	6.51	43.63	9.49
180	5.96	6.11	11.20	10.11	55.31	11.68

Short Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	0.96	0.93	1.00	0.62	4.08	1.45
5	2.20	2.15	2.29	1.39	14.08	3.23
10	3.27	3.23	3.50	2.05	20.98	4.79
30	6.20	6.27	7.23	3.85	41.00	9.39
90	12.08	12.80	15.79	7.86	85.86	19.79
180	18.83	20.88	26.74	13.45	148.95	32.84

Table 10: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the S&P 500 Index Futures.

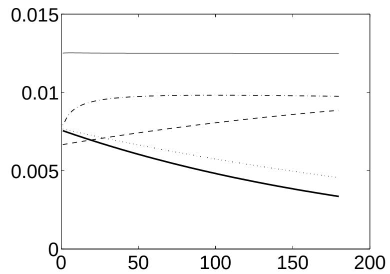


Figure 4: Russell Index Futures volatility forecasts: GARCH-Gauss (thick solid line), GARCH-Stud (dotted), FIGARCH (dashdot), ARSV (dashed) and ARLMSV (thin solid line).

We also observe that the MCRRs calculated with the Gaussian GARCH are in general higher for short investment horizons and smaller for larger investment horizons in comparison to the ones calculated with other specifications. Moreover, the MCRRs based upon the FIGARCH model (for the Russell and the S&P 500 indexes futures) are larger than the ones calculated based upon the alternative models. This is due to the low volatility forecastability of the GARCH model in larger forecasting horizons and the high volatility forecasted by the FIGARCH model. In fact, from Figure 4 we observe that GARCH models forecast high values for the volatility at the beginning of the out-of-sample period that decrease exponentially with the forecasting horizon. Table 11-Table 13 show the 95% confidence intervals for the MCRRs based upon the unconditional and the conditional approaches. The results show that the amplitude of the intervals increase with the investment horizon, which makes the MCRR estimates for longer horizons less reliable. Except for the FTSE-100 Index Futures series, the confidence intervals for 5 or more investment days for both the GARCH and the HYGARCH models never overlap with the ones obtained with the unconditional density (see Brooks et al., 2000). This indicates that there is a huge difference between the MCRRs obtained using these models and the ones obtained with the unconditional density. This is not the case for the other conditional specifications.

Long Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	HYGARCH-Gauss	ARSV	ARLMSV	Uncond
1	[0.85, 1.14]	[0.86, 1.15]	[0.76, 1.02]	[0.77, 1.04]	[0.79, 1.03]	[0.67, 0.88]	[1.07, 1.48]
5	[1.89, 2.40]	[1.91, 2.43]	[1.78, 2.29]	[1.70, 2.17]	[1.78, 2.23]	[1.52, 1.90]	[2.41, 3.07]
10	[2.64, 3.34]	[2.66, 3.37]	[2.59, 3.31]	[2.39, 3.04]	[2.53, 3.19]	[2.16, 2.73]	[3.35, 4.26]
30	[4.22, 5.30]	[4.22, 5.29]	[4.65, 5.86]	[3.90, 4.90]	[4.54, 5.62]	[3.87, 4.80]	[5.63, 7.09]
90	[5.89, 7.34]	[5.74, 7.12]	[8.21, 10.31]	[5.91, 7.46]	[8.19, 10.10]	[6.94, 8.56]	[8.96, 11.26]
180	[6.27, 7.93]	[5.99, 7.51]	[11.49, 14.39]	[7.10, 9.03]	[12.00, 14.64]	[9.98, 12.20]	[11.41, 14.37]

Short Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	HYGARCH-Gauss	ARSV	ARLMSV	Uncond
1	[0.90, 1.20]	[0.91, 1.21]	[0.81, 1.08]	[0.82, 1.10]	[0.81, 1.07]	[0.69, 0.91]	[1.14, 1.59]
5	[2.11, 2.60]	[2.12, 2.61]	[2.03, 2.51]	[1.93, 2.38]	[1.90, 2.36]	[1.62, 2.02]	[2.73, 3.66]
10	[3.05, 3.77]	[3.04, 3.77]	[3.05, 3.77]	[2.80, 3.46]	[2.75, 3.42]	[2.34, 2.91]	[4.08, 5.31]
30	[5.57, 6.73]	[5.47, 6.62]	[6.11, 7.42]	[5.24, 6.33]	[5.22, 6.40]	[4.43, 5.41]	[7.86, 9.86]
90	[9.79, 11.86]	[9.90, 11.27]	[12.62, 15.38]	[9.93, 12.02]	[10.43, 12.82]	[8.69, 10.68]	[15.72, 19.34]
180	[14.04, 16.74]	[12.88, 15.34]	[20.54, 25.00]	[15.37, 18.39]	[17.15, 21.10]	[13.90, 17.07]	[25.47, 30.91]

Table 11: Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the FTSE-100 Index Futures.

Long Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	[0.99, 1.40]	[1.00, 1.42]	[1.06, 1.50]	[0.91, 1.26]	[1.68, 2.41]	[1.24, 1.97]
5	[2.25, 2.88]	[2.29, 2.93]	[2.55, 3.26]	[2.12, 2.69]	[3.92, 5.09]	[2.97, 4.05]
10	[3.10, 3.96]	[3.17, 4.04]	[3.72, 4.75]	[3.03, 3.84]	[5.65, 7.14]	[4.28, 5.71]
30	[4.79, 6.04]	[4.96, 6.25]	[6.62, 8.29]	[5.30, 6.57]	[9.94, 12.21]	[7.14, 9.18]
90	[6.23, 8.02]	[6.75, 8.72]	[11.11, 14.00]	[9.36, 11.65]	[16.72, 20.40]	[11.20, 14.29]
180	[6.53, 8.31]	[7.30, 9.35]	[14.55, 18.32]	[13.44, 16.63]	[23.22, 27.61]	[14.27, 18.05]

Short Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	[1.14, 1.46]	[1.16, 1.49]	[1.22, 1.56]	[0.97, 1.27]	[1.77, 2.40]	[1.39, 2.01]
5	[2.64, 3.21]	[2.69, 3.27]	[2.92, 3.56]	[2.24, 2.78]	[4.04, 5.16]	[3.28, 4.21]
10	[3.89, 4.70]	[3.98, 4.82]	[4.46, 5.43]	[3.27, 4.04]	[5.89, 7.37]	[4.95, 6.22]
30	[7.35, 8.81]	[7.65, 9.16]	[9.11, 10.97]	[6.21, 7.62]	[10.89, 13.61]	[9.71, 11.84]
90	[13.92, 16.69]	[14.99, 17.99]	[18.70, 22.80]	[12.47, 15.40]	[20.06, 25.26]	[19.9, 24.34]
180	[22.02, 25.51]	[24.33, 28.32]	[30.38, 36.94]	[20.76, 25.41]	[30.14, 37.59]	[32.42, 39.46]

Table 12: Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Russell Index Futures.

For a full evaluation of the results, we perform an out-of-sample test of the MCRRs calculated with the selected models. By definition, the failure rate of a model is

Long Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	[0.72, 1.21]	[0.69, 1.17]	[0.74, 1.27]	[0.51, 0.78]	[2.93, 5.19]	[1.06, 1.73]
5	[1.60, 2.26]	[1.54, 2.18]	[1.66, 2.36]	[1.16, 1.57]	[9.71, 14.11]	[2.38, 3.40]
10	[2.26, 3.03]	[2.17, 2.93]	[2.41, 3.26]	[1.72, 2.21]	[14.40, 19.39]	[3.38, 4.56]
30	[3.54, 4.66]	[3.40, 4.51]	[4.23, 5.55]	[3.10, 3.90]	[24.95, 30.53]	[5.55, 7.16]
90	[4.76, 6.21]	[4.73, 6.21]	[7.19, 9.33]	[5.79, 7.20]	[39.95, 47.41]	[8.33, 10.73]
180	[5.06, 6.68]	[5.22, 6.92]	[9.70, 12.50]	[9.03, 11.08]	[50.86, 58.76]	[10.19, 13.19]

Short Position

No. days	GARCH-Gauss	GARCH-t-Stud	FIGARCH-t-Stud	ARSV	ARLMSV	Uncond
1	[0.83, 1.22]	[0.80, 1.19]	[0.86, 1.30]	[0.53, 0.76]	[3.06, 5.59]	[1.12, 1.77]
5	[1.94, 2.43]	[1.91, 2.38]	[2.03, 2.54]	[1.22, 1.57]	[11.14, 17.54]	[2.80, 3.58]
10	[2.91, 3.58]	[2.88, 3.53]	[3.12, 3.83]	[1.81, 2.26]	[17.64, 25.13]	[4.23, 5.36]
30	[5.64, 6.77]	[5.72, 6.84]	[6.57, 7.92]	[3.46, 4.24]	[35.18, 47.01]	[8.40, 10.33]
90	[11.08, 13.04]	[11.83, 13.83]	[14.53, 17.28]	[7.09, 8.67]	[73.68, 98.27]	[17.88, 21.42]
180	[17.42, 19.98]	[19.42, 22.28]	[24.47, 28.78]	[12.11, 14.70]	[123.87, 169.07]	[29.85, 35.62]

Table 13: Approximate 95% central confidence intervals for the minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the S&P 500 Index Futures.

the number of times the estimated MCRRs are inferior to the returns (in absolute value). If the MCRR model is correctly specified, the failure rate should be equal to the pre-specified MCRR level (in our case, 5%). Therefore, we calculate the MCRRs for one day horizon for both long and short positions and then check if these MCRRs have been exceeded by price movements in day $t + 1$. We roll this process forward and we calculate the MCRRs for 252 days.[3] In Table 14 we present the number of violations of the MCRR estimates generated by the models and by sampling with the moving block bootstrap from the unconditional distribution of returns. For both the FTSE-100 and the S&P 500 indexes futures the number of violations (in percentage) never exceeds the 5% nominal value. This indicates that the models generate "slight" excessive MCRRs. The best performance is for the ARSV model that registers failure rates closer to the nominal 5% level. Contrarily, if the models underperform in the failure rate (reject less than the nominal level) for the previous series, they overperform (reject more than the nominal level) with the returns of the Russell Index Futures. In the case of the ARLMSV model and for the Russell and S&P 500 indexes futures, we have not calculated the failure rate due to its bad performance in calculating the MCRR estimates.
Since the calculation of the empirical failure rate defines a sequence of ones (MCRR violation) and zeros (no MCRR violation), we can use the well known likelihood ratio test for a proportion in order to test $H_0 : f = 5\%$ vs. $H_1 : f \neq 5\%$, where f is the theoretical failure rate. We apply this test to the failure rates for long and short positions. Table 15 reports the p-values of this test. The results evidence that the ARSV model is the only model for which we never reject the null hypothesis that the theoretical failure rate is equal to the nominal level. We also observe that among

[3]For a long position the failure rate is obtained as the percentage of negative returns smaller than one day ahead MCRRs for long positions. Analogously, for a short position the failure rate is estimated as the percentage of positive returns larger than one day ahead calculated MCRRs for short positions (see Giot and Laurent, 2003, 2004)

	FTSE-100		Russell		S&P 500	
	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position
Unconditional	1.6%	0.8%	8.3%	7.1%	2.8%	2.8%
GARCH-Gauss	4.0%	2.4%	6.3%	5.6%	3.2%	4.0%
GARCH t-Stud	4.0%	2.4%	6.3%	5.6%	4.0%	4.0%
FIGARCH t-Stud	4.0%	2.0%	6.7%	5.6%	3.2%	3.6%
HYGARCH-Gauss	4.0%	2.4%	*	*	*	*
ARSV	4.0%	3.2%	7.1 %	7.1%	3.2%	4.8%
ARLMSV	1.6%	0.4%	*	*	*	*

Table 14: Results of the out of sample test. Estimates of the failure rate. The MCRR's are computed to cover the 95% of expected losses. The * means that we have not calculated the failure rate for these models.

the GARCH-type models, the FIGARCH has the worst performance, the GARCH models perform quite well for the Russell Index Futures returns, and the GARCH with errors following a t-Student distribution improves upon the Gaussian GARCH for the S&P 500 Index Futures returns.

	FTSE-100		Russell		S&P 500	
	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position
Unconditional	0.000	0.000	0.029	0.091	0.017	0.017
GARCH-Gauss	0.209	0.004	0.198	0.339	0.052	0.209
GARCH t-Stud	0.209	0.004	0.198	0.339	0.209	0.209
FIGARCH t-Stud	0.209	0.000	0.140	0.339	0.052	0.116
HYGARCH-Gauss	0.209	0.004	*	*	*	*
ARSV	0.209	0.052	0.091	0.091	0.052	0.441
ARLMSV	0.000	0.000	*	*	*	*

Table 15: p-values for the null hypothesis $f = \alpha$, with α =5%. The * means that we have not calculated the failure rate for these models.

6 Conclusions

This paper compares three different approaches (unconditional density, conditional heteroscedastic and stochastic volatility models) to calculate minimum capital risk requirements for long and short positions for three indexes futures. We calculate the MCRRs for 1, 5, 10, 30, 90 and 180 days investment horizons and we find that the volatility forecastability decreases with the increase of the investment horizons, which is reflected by the range of the MCRRs confidence intervals (see Christoffersen and Diebold, 2000, for similar conclusions). The results show that MCRRs based upon GARCH-type models with errors that follow a normal and/or a t-Student distribution underperform in terms of failure rate. On the other hand, the autoregressive stochastic volatility model is able to produce more accurate estimates. This paper also shows that fractional integrated stochastic volatility models produce extreme volatility persistence that conduces to very large values of the MCRRs and, consequently, to a wasting of valuable resources to those financial institutions that use or plan to use these models to calculate minimum capital risk requirements.

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