working papers

UNIVERSIDAD CARLOS III DE MADRID

CONTAGION VERSUS FLIGHT TO QUALITY IN FINANCIAL MARKETS *

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Abstract

None doubts that financial markets are related (interdependent). What is not so clear is whether there exists contagion among them or not, its intensity, and its causal direction. The aim of this paper is to define properly the term contagion (different from interdependence) and to present a formal test for its existence, the magnitude of its intensity, and for its direction. Our definition of contagion lies on tail dependence measures and it is made operational through its equivalence with some copula properties. In order to do that, we define a NEW copula, a variant of the Gumbel type, that is sufficiently flexible to describe different patterns of dependence, as well as being able to model asymmetric effects of the analyzed variables (something not allowed with the standard copula models). Finally, we estimate our copula model to test the intensity and the direction of the extreme causality between bonds and stocks markets (in particular, the flight to quality phenomenon) during crises periods. We find evidence of a substitution effect between Dow Jones Corporate Bonds Index with 2 years maturity and Dow Jones Stock Price Index when one of them is through distress periods. On the contrary, if both are going through crises periods a contagion effect is observed. The analysis of the corresponding 30 years maturity bonds with the stock market reflects independent effects of the shocks.

Keywords: Contagion, Copula functions, Causality in the Extremes, Flight to quality, Interdependence, Multivariate extreme value theory

JEL code: C12, C13, C15, C51, C52, G1

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Contagion versus Flight to Quality in Financial Markets

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Abstract

None doubts that financial markets are related (interdependent). What is not so clear is whether there exists contagion among them or not, its intensity, and its causal direction. The aim of this paper is to define properly the term contagion (different from interdependence) and to present a formal test for its existence, the magnitude of its intensity, and for its direction. Our definition of contagion lies on tail dependence measures and it is made operational through its equivalence with some copula properties. In order to do that, we define a NEW copula, a variant of the Gumbel type, that is sufficiently flexible to describe different patterns of dependence, as well as being able to model asymmetric effects of the analyzed variables (something not allowed with the standard copula models). Finally, we estimate our copula model to test the intensity and the direction of the extreme causality between bonds and stocks markets (in particular, the flight to quality phenomenon) during crises periods. We find evidence of a substitution effect between Dow Jones Corporate Bonds Index with 2 years maturity and Dow Jones Stock Price Index when one of them is through distress periods. On the contrary, if both are going through crises periods a contagion effect is observed. The analysis of the corresponding 30 years maturity bonds with the stock market reflects independent effects of the shocks.

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1 Introduction

There is common consensus about the concept of crisis, that is, everyone detects a crisis when she is going through one. However, the definitions for this phenomenon are different depending on the features of the economy under study. For example a firm manager concerned about the levels of output may consider that the firm is going through a crisis period if she detects a loss of productivity for certain levels of labor and capital. On the other hand, if most of the firm's business is based on exports abroad the manager will be concerned with sharp appreciations of the local currency against the foreign currency.

These examples raise the issue of finding a general definition of crisis that gathers the different types of crisis regardless the cause. In this way, a naïve and very general definition of crisis in an economy may be given by a threshold that represents a tolerance level. The questions that arise here are how determining this tolerance level and how an exceedance of this threshold affects the tolerance level of other economies or related markets worldwide.

The latter question clearly points out that a crisis is something more than an isolated phenomenon affecting independent markets (financial, credit, currency markets). A crisis in one market is characterized by the collapse not only of that market but by the negative effects produced on other markets. Therefore it seems natural to think of the transmission channels that connect the markets. From an economic viewpoint this involves the analysis of different mechanisms that affect the system: economic fundamentals, market specific shocks, the impact of bad news, or phycological effects (herd behavior). The discussion surges here in the direction and intensity of the dependence between the markets in turmoil periods. There is a large amount of literature concerning these features of dependence. For example Forbes and Rigobon (2001), or Corsetti, Pericoli, Sbracia (2002) where the concepts of interdependence and contagion are analyzed in detail. Regarding the intensity of the dependence, contagion implies that cross market linkages are stronger after a shock to one market, while interdependence implies no significant change in cross market relationships. Regarding the direction, contagion implies that the collapse in one market produces the fall of the other market, whilst interdependence implies that both markets collapse because both are influenced by the same factors.

From an statistical viewpoint, the linkages between markets are usually measured by Pearson correlation. Baig and Goldfajn (1998) compare the correlation between two markets for a pre-crisis and a post-crisis period determined by a shock. They find that there is an increase in cross market correlation after a crisis and therefore there exists a contagion effect. This conditional correlation, however, does not carry the adequate information about an increase in dependence. Forbes and Rigobon (2001) propose an adjusted correlation measure that corrects

the problem of conditioning to turmoil periods where cross market correlation is biased upwards because stock market volatility of the conditioning variable (market in crisis) is higher, even if the linkages between the markets remain constant. They find that the cross dependence between the markets is hardly altered after a shock, so there is interdependence but not contagion. Corsetti, Pericoli and Sbracia (2002) find something in the middle, sometimes contagion, sometimes interdependence. They consider that the absence of contagion found in Forbes and Rigobon (2001) can be attributed to pitfalls in their testing procedure.

Correlation, therefore, can lead to misleading results or at least to different interpretations depending on the way of using it. This measure only presents a complete picture of the dependence structure between the markets when their corresponding random variables are jointly gaussian. Under this assumption cross correlations are sufficient to fully describe the dependence structure between the random variables. In this setting multivariate GARCH models are sufficient to describe the dynamics (co-movements) of the vector of random variables. There are many specifications of these models, however a natural specification is given by the extension of the univariate GARCH, that is, the covariances and variances are linear functions of the squares and cross products of the data. Engle and Kroner (1995) propose the vec model that in the first order case is,

$$vec(\Sigma_t) = vec(\Omega) + A vec(X_{t-1} X'_{t-1}) + B vec(\Sigma_{t-1}),$$

where A, B are $m^2 \times m^2$ matrices with some restrictions, with m the number of random variables. For $m = 2$, $vec(\Sigma_t) = (\sigma_{1t}^2, \sigma_{12t}, \sigma_{21t}, \sigma_{2t}^2)$, σ_{it}, $i = 1, 2$ are the conditional volatilities and σ_{12t}, σ_{21t} the conditional covariances at time t. Engle and Kroner (1995) also introduce BEKK models, that in the first order case can be written as

$$\Sigma_t = \Omega + A X_{t-1} X'_{t-1} A' + B \Sigma_{t-1} B',$$

where A, B are $m \times m$ matrices. These models are really complex: the number of parameters to be estimated for the vec model of order 1 is $2m^4$, and for the BEKK model is $2m^2$. In addition, unless the observations are jointly gaussian the cross correlations are not able to fully describe the pattern of multivariate dependence and therefore some dependence is misspecified. Consider for example the asymmetric linkages, corresponding to the left and the right tail, found between most of the financial assets returns. These stylized facts are far from being explained by these models.

Engle (1999) proposes dynamic conditional correlation models (DCC) that extend constant conditional correlation models (CCC) introduced by Bollerslev (1990). The vocation of DCC

is to model the structure of dependence between a vector of random variables (m=2) by means of the conditional correlation that is allowed to evolve over time. First the serial dependence of each random variable is individually modelled (GARCH, Stochastic Volatility (SV)), and then the cross dependence between the innovations is modelled by another univariate model (exponential smoothing, GARCH, etc.)

$$\left.\begin{array}{l} X_{i,t} = \epsilon_{it}\sigma_{it}, \quad i = 1, 2, \\ \sigma_{i,t}^2 = \omega_i + \alpha_i X_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2, \end{array}\right\} \quad \text{and} \quad \rho_t^2 = \omega_o + \alpha_o \epsilon_{1,t-1}\epsilon_{2,t-1} + \beta_o \rho_{t-1}^2,$$

with ρ_t the conditional correlation, and ω_i, α_i, β_i, $i = o, 1, 2$ the corresponding parameters of the GARCH processes.

The martingale property is imposed on the vector of innovations, *i.e.* $E[\varepsilon_{it}|\Im_{i,t-1}] = 0$, $i = 1, 2$, with $\Im_{i,t-1}$ the set of information available at $t-1$ for each random variable.

These assumptions do not preclude the case $E[\varepsilon_{it}|\Im_{1,t-1} \bigcup \Im_{2,t-1}] \neq 0$ (Granger causality, Granger (1969)) and the type of specifications for the conditional correlation considered in Engle (1999) are not sufficient to explain the cross linkages between the random variables. Therefore more complex models are called for such that the innovations satisfy $E[\varepsilon_{it}|\Im_{1,t-1} \cup \Im_{2,t-1}] = 0$, $i = 1, 2$. However this assumption does not delivers us from different forms of serial dependence in the innovation vector $(\varepsilon_{1t}, \varepsilon_{2t})$. Instead, we should analyze the whole structure of dependence between the innovations. This is given by the copula function derived from the bivariate distribution $H_t(\varepsilon_{1t}, \varepsilon_{2t})$, and by their conditional counterpart obtained from $H_t(\varepsilon_{1t}, \varepsilon_{2t}|\Im_{1,t-1} \cup \Im_{2,t-1})$, see Patton (2001) or Granger, Terasvirta and Patton (2002).

The definition of copula is due to Sklar (1959). This function provides the complete structure of dependence between the random variables after taking into account the corresponding marginal distributions. In particular, the model introduced in Engle (1999) may be considered a gaussian copula where the dynamics of the dependence are given by the conditional correlation. The set of available copulas is endless providing different alternatives suiting to the problem at hand. Some examples are given by the gaussian copula used by Longin and Solnik (2001) to describe the dependence in financial asset returns, Student's t copulas (Mashal and Zeevi, (2002)) that suit better to the tails of these sequences, Joe-Clayton copula in Joe (1997) or its variation, the symmetrized Joe-Clayton copula introduced in Patton (2001) for the dependence between exchange rates series.

It is also interesting to analyze the links between the vector of random variables in the tail regions. Its joint distribution function in the tail region is derived from the multivariate extreme value theory, see Resnick (1987). Applications of multivariate extreme value distributions to examples concerning the tail regions appear in Ledford and Tawn (1996) or in de Haan and de Ronde (1998). The analysis of the dependence in the extremes provides an inter-

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esting alternative to correlation for measuring the strength of the linkages between the random variables as they become more extreme (differences between interdependence and contagion).

The vocation of this paper is modelling the dependence found between the random variables that represent different economic and financial markets. This dependence is divided in two classes regarding the origin. First, the links due to economic fundamentals (rational dependence) and second, the co-movements of the corresponding innovations (irrational dependence). Our focus lies on the latter form of dependence and the concepts of interdependence and contagion. In order to model this form of dependence (cross dependence in the innovations sequences) we introduce an innovative copula function derived from the extreme value theory that incorporates sufficient flexibility to describe different patterns of dependence, in particular asymmetric effects between the variables not reflected by standard copulas. Furthermore, the concepts of interdependence and contagion are revisited and the definitions proposed in the literature are adapted to be expressed as tail dependence measures, and in turn properties of the copula functions involving the tails of the marginal distributions. Finally, the intention of the authors is to apply this methodology to test the flight to quality phenomenon, that is, outflows of capital from the stocks markets to the bonds markets when the first ones are facing crises periods.

This paper is structured as follows. Section 2 introduces the copula function derived from the multivariate extreme value theory. The next section proposes tail dependence measures as an alternative to correlation; these measures are used to define contagion and interdependence. The cases of asymptotic dependence and independence are also studied. Finally the section studies the statistical aspects of the model, and provides a test for the existence of these effects. In Section 4, this innovative copula function as well as the new dependence measures are applied to analyze the dependence structure between bonds and assets (flight to quality phenomenon). Section 5 presents the conclusions.

2 The model

Consider the model

$$
\left.
\begin{aligned}
X_{1,t} &= g_1(X_{1,t-1}, \ldots, X_{m,t-1}) + \varepsilon_{1,t}, \\
\ldots \quad \ldots \quad \ldots \\
X_{m,t} &= g_m(X_{1,t-1}, \ldots, X_{m,t-1}) + \varepsilon_{m,t},
\end{aligned}
\right\}
\tag{1}
$$

and assume that $(\varepsilon_{1,1}, \ldots, \varepsilon_{m,1}), \ldots, (\varepsilon_{1,t}, \ldots, \varepsilon_{m,t})$ are independent vectors, that is, the multivariate dependence between the innovations is given by $H_t((\varepsilon_1, \ldots, \varepsilon_m)|\Im_{t-1})$, with $\Im_{t-1} = \Im_{X_1,t-1} \cup \ldots \cup \Im_{X_m,t-1}$. Note that the structure of dependence is time varying though the marginal distributions of the observations are independent of time. Otherwise if the innova-

tions satisfied the martingale property, the marginal distributions would not be free from the time index, that is, the joint distribution function would take the form

$$H_t((\varepsilon_{1,t}|\Im_{t-1},\ldots,\varepsilon_{m,t}|\Im_{t-1})|\Im_{t-1}),$$

with $\varepsilon_{i,t}|\Im_{t-1}$, $i = 1,\ldots,m$ the conditional random variables.

Both distribution functions, however, give rise to the type of conditional copulas introduced in Patton (2001) where the dynamics of the joint distribution function is driven by a parameter that is time varying. Instead, for appropriate functions $g_1,\ldots,g_m$ we propose a multivariate distribution function $H(\varepsilon_1,\ldots,\varepsilon_m)$ time invariant motivated by the dependence found between the vector of maxima of the corresponding random variables.

2.1 The structure of dependence: The copula function

This section studies the dependence structure between m random variables via the copula function. The concept of copula is due to Sklar (1959) and refers to the class of multivariate distribution functions supported in the unit cube with uniform margins.

Definition 2.1. *A function $C : [0,1]^m \to [0,1]$ is a m-dimensional copula if it satisfies the following properties:*

- *For all $u_i \in [0,1]$, $C(1,\ldots,1,u_i,1,\ldots,1) = u_i$.*

- *For all $u \in [0,1]^m$, $C(u_1,\ldots,u_m) = 0$ if at least one of the coordinates is zero.*

- *The volume of every box contained in $[0,1]^m$ is non-negative, i.e., $V_C([u_1,\ldots,u_m] \times [v_1,\ldots,v_m])$ is non-negative. For $m = 2$, $V_C([u_1,u_2] \times [v_1,v_2]) = C(u_2,v_2) - C(u_1,v_2) - C(u_2,v_1) + C(u_1,v_1) \geq 0$ for $0 \leq u_i, v_i \leq 1$.*

The copula $C(u_1,\ldots,u_m)$ is the joint distribution function of the probability integral transforms of each of the variables $X_1,\ldots,X_m$ with respect to the marginal distributions $F_1,\ldots,F_m$. It may be seen as the component of the multivariate distribution function of a vector of random variables that captures the dependence structure.

Theorem 2.1. *(Sklar's theorem): Given a m-dimensional distribution function H with continuous marginal distributions $F_1,\ldots,F_m$, then there exists a unique copula $C : [0,1]^m \to [0,1]$ such that*

$$H(x_1,\ldots,x_m) = C(F_1(x_1),\ldots,F_m(x_m)), \quad \forall x_1,\ldots,x_m \in \mathbb{R} \cup \{\infty\}. \tag{2}$$

Conversely, if $C(u_1,\ldots,u_m)$ is a m-dimensional distribution function with uniform margins, and $F_1,\ldots,F_m$ are continuous univariate distribution functions for the random variables

$X_1, \ldots, X_m$, *then the function H defined in* (2) *is a m-dimensional distribution function with margins* $F_1, \ldots, F_m$.

It is immediate to see that if we have a model for the joint distribution of the m random variables and the marginal distributions of the X_i are continuous, the complete dependence structure of the corresponding variables is known,

$$C(u_1, \ldots, u_m) = H(F_1^{-1}(u_1), \ldots, F_m^{-1}(u_m)), \tag{3}$$

with $F_i^{-1}(u) = inf\{x \in \mathbb{R} | F_i(x) \geq u\}$, for all $0 \leq u \leq 1$.

This measure of dependence extends the notions of linear correlation (Pearson) and rank correlation (Spearman). More important, it overcomes the typical problems of these scalar measures. Embrechts, McNeil and Straumann (1999) provides an excellent review about the properties and problems of these dependence measures.

It is shown that under very general conditions on the marginal distribution functions the dependence structure of any multivariate distribution is described by the copula function. In particular this interesting result is found for the joint distribution of the maxima of a vector of random variables. Moreover, there exists a copula function that drives the dependence in the extremes whose expression is derived from the extreme value theory.

Consider $M_n = (M_{n1}, \ldots, M_{nm})$ the vector of componentwise maxima, with components $M_{ni} = \max\{X_{1i}, \ldots, X_{ni}\}$, and the vector of sequences $a_n = (a_{n1}, \ldots, a_{nm})$ with each $a_{ni} > 0$, and $b_n = (b_{n1}, \ldots, b_{nm})$. Under some smoothness condition in the tail of F_i, Leadbetter, Lindgren and Rootzén (1983) show that

$$\lim_{n \to \infty} F_i^n(a_{ni} x_i + b_{ni}) = G_i(x_i), \quad i = 1, \ldots, m, \tag{4}$$

where $G_i(x_i)$ is an extreme value distribution of one of the three possible types, Gumbel, Weibull and Fréchet. The distribution F_i is said to belong to the maximum domain of attraction of G_i, see also Embrechts, Klüppelberg and Mikosch (1997). Denote the distribution of the multivariate maximum by

$$H^n(a_{n1} x_1 + b_{n1}, \ldots, a_{nm} x_m + b_{nm}) = P\{a_{ni}^{-1}(M_{ni} - b_{ni}) \leq x_i, i = 1, \ldots, m\}, \tag{5}$$

where $H(x_1, \ldots, x_m) = P\{X_1 \leq x_1, \ldots, X_m \leq x_m\}$. The core result of the multivariate extreme value theory is that (4) may be extended to

$$\lim_{n \to \infty} H^n(a_{n1} x_1 + b_{n1}, \ldots, a_{nm} x_m + b_{nm}) = G(x_1, \ldots, x_m), \tag{6}$$

with G a non degenerate multivariate extreme value distribution (*mevdf*). The class of these particular distributions is precisely the class of max-stable distributions (Resnick (1987), proposition 5.9). These distributions are defined by this property

$$G^t(tx_1, \ldots, tx_m) = G(\alpha_1 x_1 + \beta_1, \ldots, \alpha_m x_m + \beta_m), \tag{7}$$

for every $t > 0$, and some constants $\alpha_i > 0$ and β_i.

The marginal distribution functions of G are the univariate extreme value distributions $G_i(x_i)$. By Sklar's theorem, (6) may be written as

$$\lim_{n \to \infty} H^n(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm}) = C(G_1(x_1), \ldots, G_m(x_m)), \tag{8}$$

with C a copula function.

It can be seen under some simple algebra that C also describes the dependence structure of the largest observations. Our aim in the following lines is to derive a suitable analytical expression for this copula function. In order to do this, the marginal distributions are transformed to obtain identical and parameter free versions of these univariate distributions, in particular Fréchet distributions of the type $\Psi_\alpha(z) = \exp(-z^{-\alpha})$ with $\alpha = 1$.

Let $Z_i = 1/\log \frac{1}{F_i(X)}$ be such transformation, and denote $P\{Z_i \leq z\} = F_i^*(z)$ with $z = 1/\log \frac{1}{F_i(a_{ni}x + b_{ni})}$. This distribution satisfies these interesting properties: $F_i^*(z) = \Psi_1(z)$, $F_i^*(z) = F_i(a_{ni}x + b_{ni})$ and $F_i^{*n}(nz) = \Psi_1(z)$. Note that these conditions imply $F_i^{*n}(nz) = F_i^*(z)$ and

$$\lim_{n \to \infty} H^{*n}(nz_1, \ldots, nz_m) = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)), \tag{9}$$

with $H^*(z_1, \ldots, z_m) = H(a_{n1}x_1 + b_{n1}, \ldots, a_{nm}x_m + b_{nm})$. This condition holds for any vector $(z_1, \ldots, z_m)$ in $[z_{o1}, \infty) \times \ldots \times [z_{om}, \infty)$, with $(z_{o1}, \ldots, z_{om})$ a threshold vector. The function C is called extreme copula because satisfies this property,

$$C^t(\Psi_1(z_1), \ldots, \Psi_1(z_m)) = C(\Psi_1^t(z_1), \ldots, \Psi_1^t(z_m)), \quad t > 0, \tag{10}$$

where the margins are extreme value distributions. The proof immediately follows from (7). This condition entails an invariance property given by the logs of the corresponding distributions, that is,

$$t \, \log C(\Psi_1(tz_1), \ldots, \Psi_1(tz_m)) = \log C(\Psi_1(z_1), \ldots, \Psi_1(z_m)).$$

Then, for n and $(z_1, \ldots, z_m)$ sufficiently high,

$$\lim_{n \to \infty} n \left(1 - H^*(nz_1, \ldots, nz_m)\right) = -log\, C(\Psi_1(z_1), \ldots, \Psi_1(z_m)), \tag{11}$$

and

$$\lim_{n \to \infty} \frac{H^*(nz_1, \ldots, nz_m)}{1 + log\, C(\Psi_1(nz_1), \ldots, \Psi_1(nz_m))} = 1. \tag{12}$$

Other interesting result derived from the invariance property is

$$P\left\{Z_1 \leq nz_1, \ldots, Z_m \leq nz_m \,\Big|\, \bigcup_{i=1}^m Z_i > nz_{0i}\right\} = P\left\{Z_1 \leq z_1, \ldots, Z_m \leq z_m \,\Big|\, \bigcup_{i=1}^m Z_i > z_{0i}\right\}.$$

The left term in (11) may be considered as a sequence of measures that converge to a constant given the vector $(z_1, \ldots, z_m)$, see Resnick (1987) or de Haan and de Ronde (1998) for different transformations of the marginal distributions. Expression (12) provides the joint distribution function of the largest observations, that is, for n sufficiently high the denominator may be approximated by the copula function C. Therefore

$$P\left\{Z_1 \leq z_1, \ldots, Z_m \leq z_m\right\} = C(\Psi_1(z_1), \ldots, \Psi_1(z_m)), \tag{13}$$

for the vector $(z_1, \ldots, z_m)$ sufficiently high.

The latter expression is promising in the sense that C is a good approximation of the dependence structure in the largest observations. However, the challenge of choosing a suitable threshold vector that determines the region satisfying condition (11) still remains.

On the other hand the invariance property implies that the copula C must be of exponential type. There are different characterizations of this distribution. A general expression for $m = 2$ is given in the form of the Pickands representation (Pickands, 1981), that is,

$$C(u_1, u_2) = \exp^{D(t)log(u_1 u_2)}, \tag{14}$$

where $u_1 = \Psi_1(z_1)$, $u_2 = \Psi_1(z_2)$, $t = \frac{log(u_1)}{log(u_1 u_2)}$, and $D(t)$ is a convex function on $[0, 1]$ such that $\max(t, 1 - t) \leq D(t) \leq 1$ for all $0 \leq t \leq 1$. This family of distributions is included in the class of Archimedean copulas (Nelsen 1999, chapter 4). The dependence in these copulas is driven by a single variable t for $m = 2$. The Gumbel-Hougaard family is within this class of distributions and satisfies the invariance property. It is represented by

$$C_G(u_1, u_2; \theta) = \exp^{-[(-log\, u_1)^\theta + (-log\, u_2)^\theta]^{1/\theta}}, \quad \theta \geq 1. \tag{15}$$

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This distribution function is usually known as Gumbel bivariate logistic copula. The main problem that arises if we assume that C is modelled by a Gumbel distribution C_G in (13) is the choice of the threshold. Condition (11) may be violated for low thresholds where the extreme value theory is not a reliable technique. Other drawback of modelling C as C_G is the asymmetry, the random variables modelled by the Gumbel copula are exchangeable and hence it is not possible to quantify different contributions of the corresponding random variables. In order to account for this asymmetric dependence we propose a version of C_G able to describe these effects. This function is denoted by $\widetilde{C}_G(u_1, u_2; \Theta)$, with $\Theta = \{\theta, \gamma, \eta\}$, and takes the following expression

$$\widetilde{C}_G(u_1, u_2; \Theta) = \exp^{-D(u_1, u_2; \gamma, \eta)[(-log\ u_1)^\theta + (-log\ u_2)^\theta]^{1/\theta}}, \tag{16}$$

with

$$D(u_1, u_2; \gamma, \eta) = \exp^{\gamma(1-u_1)(1-u_2)^\eta}, \quad \gamma \geq 0, \quad \eta > 0. \tag{17}$$

Theorem 2.2. *The function $\widetilde{C}_G : [0, 1] \times [0, 1] \to [0, 1]$ defined in (16) and (17) is a copula function if the parameters in Θ satisfy that $\widetilde{c}_G(u_1, u_2; \Theta) > 0$, $\forall (u_1, u_2) \in [0, 1] \times [0, 1]$, with $\widetilde{c}_G(u_1, u_2; \Theta) = \frac{\delta^2 \widetilde{C}_G(u_1, u_2; \Theta)}{du_1 du_2}$ the density function of the copula $\widetilde{C}_G$.*

Proof.- Let us denote $A(u_1, u_2; \theta) = [(-log\ u_1)^\theta + (-log\ u_2)^\theta]^{1/\theta}$. The conditions related to the contour of $\widetilde{C}_G$ immediately follow from the contour properties of the functions $D(u_1, u_2; \gamma, \eta)$ and $A(u_1, u_2; \theta)$. The proof that $\widetilde{C}_G$ is 2-increasing involves more algebra. Consider $V_{\widetilde{c}_G}([u_{o1}, u_{11}] \times [u_{o2}, u_{12}]) = \widetilde{C}_G(u_{11}, u_{12}; \Theta) - \widetilde{C}_G(u_{11}, u_{o2}; \Theta) - \widetilde{C}_G(u_{o1}, u_{12}; \Theta) + \widetilde{C}_G(u_{o1}, u_{o2}; \Theta)$, and define $V'(u_1) = \widetilde{C}_G(u_1, u_{12}; \Theta) - \widetilde{C}_G(u_1, u_{o2}; \Theta)$. Then, $V_{\widetilde{c}_G}([u_{o1}, u_{11}] \times [u_{o2}, u_{12}]) = V'(u_{11}) - V'(u_{o1})$. Note that $V'(u_1) \geq 0$, $\forall u_1 \in [0, 1]$, with $u_{o2} < u_{12}$. This function can be written as

$$V'(u_1) = \exp^{-D(u_1, u_{o2}; \gamma, \eta) A(u_1, u_{o2}; \theta)} \left[\exp^{-(D(u_1, u_{12}; \gamma, \eta) A(u_1, u_{12}; \theta) - D(u_1, u_{o2}; \gamma, \eta) A(u_1, u_{o2}; \theta))} - 1 \right],$$

that is greater than 0 if and only if $D(u_1, u_2; \gamma, \eta) A(u_1, u_2; \theta)$ is decreasing in u_2. The only condition that remains to see is that $V'(u_1)$ is nondecreasing. This condition will hold if the function $\frac{\delta \widetilde{C}_G(u_1, u_2; \Theta)}{du_1}$ is nondecreasing in u_2, that amounts to see if $\widetilde{c}_G(u_1, u_2; \Theta) > 0$, $\forall (u_1, u_2) \in [0, 1] \times [0, 1]$. $\square$

The choice of the threshold in (13) is overcome by adding the function $D(u_1, u_2; \gamma, \eta)$. This function by means of the pair (u_1, u_2) and the parameter γ measures the sensitivity of the dependence structure to departures from the invariance property. In other words, either the margins are further in the right tail $(u_1, u_2 \to 1)$ or $\gamma \cong 0$ the copula function $\widetilde{C}_G$ is closer to C_G and the invariance property holds. In this way the joint distribution function for the

entire range of the random variables Z_1, Z_2 is

$$P\{Z_1 \leq z_1, Z_2 \leq z_2\} = \widetilde{C}_G(\Psi_1(z_1), \Psi_1(z_2)), \tag{18}$$

where $z_i = 1/log \frac{1}{F_i(x_i)}$ in this case. This distribution function is driven by the parameters θ, γ, η. The constant γ assesses the extent of the invariance property. The parameter θ describes the level of asymptotic tail dependence between the random variables. The case of perfect independence is covered by $\theta = 1$, $\gamma = 0$. Finally η measures the level of asymmetry or exchangeability of the variables.

The following list enumerates the most outstanding advantages of our copula function $\widetilde{C}_G$.

1. This copula function is derived from the multivariate extreme value theory, in contrast to ad-hoc choices to model the dependence structure.

2. The function $D(u_1, u_{12}; \gamma, \eta)$ and in particular the parameter γ extend the results of the multivariate extreme value theory about the distribution of the largest observations to the entire range of the random variables.

3. $\widetilde{C}_G$ is able to explain asymmetric effects of the variables for $\eta \neq 1$. It may be considered as an alternative to the asymmetric logistic model in Tawn (1988).

4. This copula function is sufficiently flexible to describe different forms of dependence and asymptotic dependence, as will be shown below.

3   Contagion: types and definitions

Linear measures of dependence are not sufficient to describe the dependence patterns between a vector of random variables. The popular Pearson correlation has a number of pitfalls, see Embrechts, McNeil and Straumann (1999). Some of them are that a a zero correlation does not imply independence if the marginal distributions are not elliptical, and second, correlation is not invariant under transformations of the random variables. Spearman correlation (rank correlation) for example solves the latter, however, it also fails to give a measure of independence far from the elliptical world.

In the bivariate setting natural measures of dependence different from the traditional correlation are given by the dependence in the tails. Ledford and Tawn (1997) and Coles, Heffernan and Tawn (1999) define the asymptotic tail dependence measure $\aleph$,

$$\aleph = \lim_{t \to \infty} P\{Z_2 > t | Z_1 > t\}. \tag{19}$$

This measure takes the zero value if the random variables are asymptotically independent.

There are two classes of extreme value dependence, asymptotic dependence and asymptotic independence. Both forms of dependence permit dependence for moderately large values of the variables, however the likelihood of joint extreme events under asymptotic independence converges to 0 as the events become more extreme. Loosely speaking, the probability of one variable being extreme given the other is extreme is 0 in the limit. The copula $\widetilde{C}_G$ supports both types of asymptotic dependence. It can be seen that $\aleph_{\widetilde{C}_G} = 2 - 2^{1/\theta}$, that reflects asymptotic independence for $\theta = 1$ and asymptotic dependence otherwise.

The definition in (19) can be extended to the entire range of the random variables. Lehman (1966) defined two random variables Z_1, Z_2 as positively quadrant dependent (PQD) if for all $(z_1, z_2) \in \mathbb{R}^2$,

$$P\{Z_1 > z_1, Z_2 > z_2\} \geq P\{Z_1 > z_1\}P\{Z_2 > z_2\}, \tag{20}$$

or equivalently if

$$P\{Z_1 \leq z_1, Z_2 \leq z_2\} \geq P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}. \tag{21}$$

In the same way negative quadrant dependence (NQD) is defined reversing the inequalities in both expressions.

Definition 3.1. *We say that two random variables are interdependent if they are PQD. In consequence interdependence is characterized by joint movements in the same direction (co-movements) of the corresponding random variables.*

If Z_1 and Z_2 are NQD a large value in one random variable is corresponded by a value of the same magnitude in the opposite direction for the other variable. Economically, interdependence means that links in turmoil periods (tails of the distributions) are only consequence of the same linkages between the markets found in still periods.

In the case that the random variables are continuous these definitions are a property of the copula. From elementary probability theory

$$P\{Z_1 > z_1, Z_2 > z_2\} = \widetilde{C}_G(u_1, u_2) - (u_1 + u_2) + 1, \tag{22}$$

with $u_i = \Psi_1(z_i)$, $i = 1, 2$.

Define the function $g(u_1, u_2)$ as the difference between the probabilities in (20) in terms of the copula function,

$$g(u_1, u_2) = \widetilde{C}_G(u_1, u_2) - u_1 u_2. \tag{23}$$

If this function is positive for all $(u_1, u_2) \in [0, 1] \times [0, 1]$ the former definitions for cross dependence apply, that is, Z_1 and Z_2 are PQD.

The function $g(u_1, u_2)$ itself is not sufficient to determine the strength of the links between

the variables. A stronger condition is required to measure the amount of dependence for different values of the random variables. This condition is tail monotonicity, that is, the function (23) is either nonincreasing or nondecreasing in its arguments. In particular, increasing tail monotonicity for the function $P\{Z_1 > z_1, Z_2 > z_2\} - P\{Z_1 > z_1\}P\{Z_2 > z_2\}$ characterizes the existence of contagion in the upper tails between the random variables. Thus, contagion in this context can be defined as a significant increase in the intensity of the dependence between the variables Z_1, Z_2 when these take on extreme values.

Definition 3.2. *Suppose Z_1, Z_2 with common Fréchet distribution Ψ_1 and consider z a threshold that determines the extremes in the right tail of both random variables. Then, there exists a contagion effect between Z_1 and Z_2 if $g(u_1, u_2)$ is an increasing function for both random variables, and for $u_1, u_2 \geq u$ with $u = \Psi_1(z)$.*

On the other hand contagion in intensity for the lower tails is characterized by decreasing tail monotonicity for the function $P\{Z_1 \leq z_1, Z_2 \leq z_2\} - P\{Z_1 \leq z_1\}P\{Z_2 \leq z_2\}$. In terms of copulas the conditions in definition 3.2 for contagion amount to these properties,

$$h_1(u_1, u_2) = \frac{\delta \widetilde{C}_G(u_1, u_2)}{du_1} - u_2 > 0, \qquad h_2(u_1, u_2) = \frac{\delta \widetilde{C}_G(u_1, u_2)}{du_2} - u_1 > 0. \qquad (24)$$

The presence of tail monotonicity for the whole range of the random variables indicates something stronger than contagion. These properties, called Right Tail Increasing (RTI) and Left Tail Decreasing (LTD) in Esary and Proschan (1972), imply that $P\{Z_2 > z_2|Z_1 > z_1\} > P\{Z_2 > z_2\}$ and $P\{Z_2 \leq z_2|Z_1 \leq z_1\} > P\{Z_2 \leq z_2\}$ respectively, for any pair (z_1, z_2), and therefore are synonymous of contagion and interdependence. Note however that these phenomena do not necessarily appear together. There can exist contagion in the extremes between two random variables without being interdependent, and on the other hand, two interdependent random variables can show weaker links (though stronger than being independent) in distress periods than in calm periods.

The concepts of contagion and interdependence introduced so far regard the intensity of the dependence, that is, the strength of the links between the variables as these go further into the tails. However, other forms of contagion regarding the direction of the dependence are found, in this setup the conditional probability of (19) is interpreted as a causality relationship. Contagion in this context occurs when one variable is influencing the other, that is, a large value in one variable is raising the likelihood of a large value in the other variable. Then the relation between the variables must be asymmetric, otherwise there is only an increase in the intensity of the dependence (contagion as defined in 3.2). Note however that a condition of the type $P\{Z_2 > z_2|Z_1 > z_1\} > P\{Z_2 > z_2\}$ is equivalent to (20). Moreover, the only

difference of the former with a condition as $P\{Z_1 > z_1 | Z_2 > z_2\} > P\{Z_1 > z_1\}$ is given by the marginal distributions. In the case of $H^*(z_1, z_2)$ where the margins are identical Fréchet, both conditional probabilities are identical.

Let us focus instead in the following conditions for contagion spill-over,

$$P\{Z_2 > z_2 | Z_1 > z_1\} \geq P\{Z_1 > z_2 | Z_2 > z_1\}, \tag{25}$$

for the upper tails, with $z_2 \geq z_1$, and

$$P\{Z_2 \leq z_2 | Z_1 \leq z_1\} \geq P\{Z_1 \leq z_2 | Z_2 \leq z_1\}, \tag{26}$$

for the lower tails, where $z_2 \leq z_1$. These conditions boil down to see if $\widetilde{C}_G(u_1, u_2; \Theta) > \widetilde{C}_G(u_2, u_1; \Theta)$. Consider z_1 a threshold value that determines the extreme events, hence this inequality implies that the likelihood of Z_2 being extreme given that Z_1 is extreme is larger than the likelihood of Z_1 being extreme with Z_2 extreme. In other words, Z_1 is causing Z_2 reaches extreme values. The particular case of equality in the latter expressions represents symmetry of the variables and economically concerns directional interdependence (both economies are affected by the external factors in the same way).

To formalize directional contagion define $gd_v(u) = \widetilde{C}_G(u, v) - \widetilde{C}_G(v, u)$ and introduce the following definition,

Definition 3.3. *Suppose Z_1, Z_2 with common Fréchet distribution Ψ_1 and consider z a threshold that determines the extremes of both random variables. Then, Z_1 is influencing Z_2 in the extreme values (contagion effect) if $gd_v(u)$ is strictly positive for all $v > u$ for the upper tail, and for all $v < u$ for the lower tail, with $u = \Psi_1(z)$.*

This definition is analog for Z_2 influencing Z_1 but reversing the signs of the inequality. In terms of the parameters of the copula $\widetilde{C}_G$, there is contagion from Z_1 towards Z_2 if $\eta > 1$, and from Z_2 towards Z_1 if $\eta < 1$.

The definition may be strengthened by imposing a monotonicity condition on $gd_v(u)$. The intensity of this type of contagion can be measured by means of this monotonicity condition. In particular,

Definition 3.4. *Suppose the conditions of definition 3.3, and Z_1, Z_2 such that there is positive contagion from Z_1 to Z_2. Then, Z_1 is strongly influencing Z_2 in the extreme values (strong contagion effect) if $gd_v(u)$ is an increasing function in v for all $v > u$.*

A characterization of this definition is

$$sc(u, v) = \frac{\delta \widetilde{C}_G(u, v; \Theta)}{dv} - \frac{\delta \widetilde{C}_G(v, u; \Theta)}{dv} > 0. \qquad (27)$$

The economic interpretation behind lies on irrational increases in the probability that Z_2 becomes extreme given that Z_1 has reached extreme observations (remind that the variables represent innovations).

3.1 Estimation of the Copula: Testing Contagion

In general, to estimate the set of dependence parameters Θ of any multivariate distribution function two strategies may be employed. If the marginal distributions are known or can be estimated by valid parametric models the likelihood function for the data is easily derived. If the multivariate distribution function is $H(x_1, x_2) = C(F_1(x_1), F_2(x_2); \Theta)$ the likelihood function is

$$\pounds(\Theta; x_1, x_2) = \sum_{i=1}^{n} log\, f_1(x_{i,1}) + \sum_{i=1}^{n} log\, f_2(x_{i,2}) + \sum_{i=1}^{n} log\, c(F_1(x_{i,1}), F_2(x_{i,2}); \Theta), \qquad (28)$$

with f_i the marginal density function of F_i, and $c(F_1(x_1), F_2(x_2); \Theta)$ the bivariate density of the copula. The resulting estimates of the dependence parameters are margin-dependent, as well as the parameters of the corresponding marginal distributions. On the other hand the estimates of Θ are free of these effects for nonparametric estimates of the margins. Genest, Ghoudi, and Rivest (1995) show that the estimates derived from a pseudo-likelihood estimation are consistent and asymptotically normal. This method is implemented in two steps. First, the estimates of the marginal distributions are estimated by the respective nonparametric empirical distribution functions. In this way u_i is obtained as $u_i = \hat{F}_{i,n}(x)$, with $\hat{F}_{i,n}(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{X_i \leq x\}}$, and the log-likelihood for C is

$$\pounds(\Theta; u_1, u_2) = \sum_{i=1}^{n} log\, c(u_{i,1}, u_{i,2}; \Theta). \qquad (29)$$

In our case, $H^*(z_1, z_2) = \widetilde{C}_G(\Psi_1(z_1), \Psi_1(z_2); \Theta)$, though the marginal distribution functions are known, standard Fréchet, the underlying marginals F_1, F_2 are not, so it is preferable to consider the nonparametric case. Note that $u_i = \Psi_1(z_i)$ boils down to $u_i = F_{i,n}(x_i)$ by construction of Z_i. The log-likelihood is calculated as in (29). This function, however, does not take an easy-to-handle expression in logs, and the score function does not adopt a closed form. Instead numerical optimization methods are employed to maximize the likelihood.

An appealing property of the copula $\widetilde{C}_G$ is its nested character. It is immediate to see

that $\gamma = 0$ is the standard Gumbel distribution, that represents the class of bivariate extreme value distributions. The case of asymptotic independence for the right tail is given by $\theta = 1$ whilst perfect independence is described by $\theta = 1$ and $\gamma = 0$. Finally, η measures the level of asymmetry (exchangeability) of the variables. As a result, it is straightforward to implement tests for the corresponding hypotheses about dependence by means of likelihood ratio tests. The test statistic is

$$\Lambda_n = 2log\frac{\underset{\Theta}{sup}\overset{n}{\underset{i=1}{\Pi}}\widetilde{c}_G(u_{i,1}, u_{i,2}; \Theta)}{\overset{n}{\underset{i=1}{\Pi}}\widetilde{c}_G(u_{i,1}, u_{i,2}; \Theta_0)}, \tag{30}$$

with Θ_0 the set of parameters under the null hypothesis. The asymptotic distribution of Λ_n is chi squared-distributed with degrees of freedom equal to the difference of the dimensions between Θ and Θ_0.

The nested character of the copula makes immediate testing dependence as well as testing the existence of contagion effects in the data. The corresponding hypotheses tests are $H_0 : \theta = 1$, $\gamma = 0$ vs $H_1 : \theta > 1$ or $\gamma > 0$, and $H_0 : \eta = 1$ vs $H_1 : \eta \neq 1$ for $\gamma > 0$. Meanwhile the intensity contagion and strong directional contagion boil down to study conditions (24) and (27) respectively plugging the estimated parameters.

4   Application: Flight to quality versus Contagion

Financial crises are characterized by dramatic falls in the prices of asset returns for reference markets. The fall in prices of these returns trigger a sequence of negative effects on the prices of the rest of the assets traded in the market by different reasons: a remarkable weight of the asset in the composition of the portfolios, bilateral trade, or a psychological or contagion effect.

It seems logical to think that investors in order to avoid the pernicious effects of the crisis flee towards safe markets: the bonds market. However, sometimes it is not clear the type of market failing and originating the crisis since the overall economic structure collapses. In this situation the refuge in the bonds market may not provide with the desired coverage against losses. The phenomenon of fleeing from the stocks market to the bonds market is known as flight to quality. Measuring this effect is useful in a number of ways: it reflects the links between these markets, in cases of crisis it is useful to identify its sources (financial vs other types), or the causality of the relationship, that is, if bear stock markets imply bull bond markets, or there is some common economic factor producing the co-movements (*e.g.* low interest rates).

In this section this phenomenon is tested for two different pairs of financial indexes: the

Dow Jones Corporate 02 Years Bond Index (DJBI02) vs the Dow Jones Industrial Average: Dow 30 Industrial Stock Price Index (DJSI), and the the Dow Jones Corporate 30 Years Bond (DJBI30) Index vs the Dow 30 Industrial Stock Price Index. These series are studied for the period $02/01/1997 - 24/09/2004$. The Corporate Bonds Indexes data are taken from the official Dow Jones Index website and the Stock Price Index from *www.freelunch.com*. Sample observations corresponding to public holidays and missing data in either of the series are deleted from both data sets to avoid the incorporation of spurious zero returns and aberrant dependencies, leaving n=1942 observations. The observations considered for the analysis are the logarithmic returns measured in percentage terms and denoted as r_t,

$$r_t = 100 \ (logP_t - logP_{t-1}),$$

with P_t the original prices at time t.

The methodology followed in this empirical work starts by filtering the data by univariate models as sketched in (1) and analyzing the dependence patterns between the resultant innovation vector $(\varepsilon_1, \varepsilon_2)$ by means of the copula $\widetilde{C}_G$. This copula is sufficient for testing the existence of contagion effects, co-movements, or opposite effects in the tails that are reflected in the set of parameters Θ of the copula $\widetilde{C}_G$.

Tables 6.1, 6.2 show that *DJBI02* index is well modelled by an AR(1)-GARCH(1,1) model as follows,

$X_{1,t} = 0.00025 + 0.089X_{1,t-1} + \sigma_{1,t}\varepsilon_{1,t}$, with $\varepsilon_{1,t}$ *i.i.d.* $(0,1)$, and
$\sigma_{1,t}^2 = 6.194 \cdot 10^{-8} + 0.071\varepsilon_{1,t-1}^2 + 0.903\sigma_{1,t-1}^2$.

The *DJSI* Index is modelled by the following pure GARCH(1,1) model (tables 6.3, 6.4),

$X_{2,t} = \sigma_{2,t}\varepsilon_{2,t}$, with $\varepsilon_{2,t}$ *i.i.d.* $(0,1)$, and $\sigma_{2,t}^2 = 3.0012 \cdot 10^{-6} + 0.096\varepsilon_{2,t-1}^2 + 0.887\sigma_{2,t-1}^2$.

The bivariate sequence of innovations $(\varepsilon_{1,t}, \varepsilon_{2,t})$ is represented in figure 6.1. A first glance to the picture provides some guidance towards the existence of a flight to quality effect between the innovations of *DJSI* and the innovations corresponding to *DJBI02*. The analysis of cross correlation (figure 6.2) confirms the existence of opposite shocks in the innovation sequence as well as validates the univariate models proposed to satisfy the assumptions in (1).

The copula function $\widetilde{C}_G$ introduced in this paper is estimated numerically. The parameter estimates for this example are $\hat{\theta}_n = 1.031$, $\hat{\eta}_n = 1$ and $\hat{\gamma}_n = 0.175$. This model fits well the data for different sections of the copula for both margins as can be seen (see figure 6.3). The following pictures are derived from $\widetilde{C}_G$ estimated from the data. In this way, figures 6.4 and 6.5 show negative interdependence between the random variables in the left tail, that becomes stronger in the middle of the bivariate distribution and turns positive in the right tail. Both plots are identical indicating the absence of directional contagion, that is, asymmetric effects

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between the variables. This is also described in figure 6.6. On the other hand it is remarkable the presence of intensity contagion in the left tail (figure 6.5). A deeper analysis of figures 6.4 and 6.5 shows opposite movements in the middle of their domain, that decrease when the variables take larger absolute values. This phenomenon is more pronounced for the extreme negative values that tend to move together, or at least not in opposite directions (contagion without interdependence).

It is convenient not confusing the contagion phenomenon just illustrated that appears when both variables simultaneously take on extreme events in the same tail with the flight to quality. This phenomenon occurs when the extreme values occur in the opposite tails, in particular when *DJBI02* takes positive extreme values and *DJSI* negative extreme values. Figure 6.7 clearly describes the existence of this phenomenon in both tails, that may be interpreted as a substitution effect between these financial sequences when either of the sequences are in crises periods.

The analysis for the pair Dow Jones Corporate 30 Years Bond Index (*DJBI30*) and the Dow Jones Stock Price Index (*DJSI*) yields different results. *DJBI30* is modelled by an AR(1)-GARCH(1,1) model where *DJSI* also enters in the equation. The parameter estimates are displayed in tables 6.5 and 6.6 and can be summarized as follows,

$$X_{1,t} = 0.00037 + 0.063X_{1,t-1} + 0.048X_{2,t-1} + 0.028X_{2,t-2} + \sigma_{1,t}\varepsilon_{1,t}, \text{ with } \varepsilon_{1,t} \ i.i.d. \ (0,1),$$
$$\text{and } \sigma_{1,t}^2 = 1.375 \cdot 10^{-6} + 0.056\varepsilon_{1,t-1}^2 + 0.905\sigma_{1,t-1}^2.$$

The pair $(\varepsilon_{1,t}, \varepsilon_{2,t})$ is represented in figure 6.8. The cross correlation function (figure 6.9) indicates the absence of linear correlation between any lag combination. This graph also assesses the univariate model proposed to describe the dynamics of *DJBI30*. The parameter estimates for $\widetilde{C}_G$ in this case are $\hat{\theta}_n = 1.01$, $\hat{\eta}_n = 1$ and $\hat{\gamma}_n = 0.0003$. This model suits very well to the data (figure 6.10). The fitted copula shows that the random variables are weakly interdependent (figure 6.11), that is, both innovation sequences, though close to independence, move in the same direction. More formally, the likelihood ratio test introduced in (30) calculated for $H_0 : \theta = 1, \gamma = 0$ vs $H_1 : \theta > 1$ or $\gamma > 0$ does not reject the hypothesis of independence. Note from the value of γ the random variables are symmetric and therefore there is not directional contagion. Furthermore, figure (6.12) describes absence of intensity contagion in either of the tails reflecting a weakening in the links between the variables in the tails. In the limit these random variables are asymptotically independent in both tails. Finally the flight to quality phenomenon is not present for these two series as shown in figure 6.13.

5 Conclusions

Contagion and interdependence are different concepts. In this paper contagion is related to extreme or tail events. Via the theory of copulas, we are able to analyze and test the existence of contagion, its intensity, as well as its causal direction. This is done by creating a new copula, derived from the multivariate extreme theory, that is sufficiently flexible both to describe different patterns of dependence, and model asymmetric effects between markets.

This copula has been applied to study the links between safe and risky markets represented by the Dow Jones Corporate Bond Index (*DJBI*) and the Dow Jones Stock Price Index (*DJSI*). From the point of view of economic fundamentals, the latter index is independent of *DJBI*, while the bonds indexes, *DJBI02* and *DJBI30*, have different behaviors depending on their maturity. The price of *DJBI02* is independent of the evolution of risky markets, actually the conditional mean price is only driven by its own past price, while the conditional variance is well modelled by a GARCH(1,1) model. On the other hand, *DJBI30* is positively influenced by the evolution of *DJSI* reflecting the health of the overall economy.

Regarding the irrational links between the markets reflected in the innovations sequences and modelled by the copula function introduced in this paper, the conclusions are also different for the corresponding pairs of financial series. The shocks between *DJBI02* and *DJSI* are negatively related. In particular, the flight to quality effect is present indicating a substitution effect between both financial instruments when either of them are through distress periods. It is also remarkable the existence of a contagion effect in the intensity of the dependence in situations of crises in both markets, common negative shocks. On the other hand *DJSI* and *DJBI30* innovation sequences are almost independent. There is no contagion or flight to quality effect.

The conclusion regarding the dependence of these financial series is that while *DJBI02* can serve as refuge for investors fleeing from crises attributed to the stocks markets, *DJBI30* reflects the health of the overall economy, including the stocks markets, and are used by a type of investors not concerned with sharp fluctuations of prices in the stocks markets.

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6 Tables and Figures

Parameter	Value	Standard Error	T Statistic
C	0.00023981	$3.2242e-005$	7.4378
AR(1)	0.075112	0.023976	3.1327
AR(2)	-0.0060864	0.023504	-0.2590
Regress(1)	0.0033575	0.0023594	1.4231
Regress(2)	0.001902	0.0023953	0.7940
Regress(3)	-0.0032545	0.0027541	-1.1817
K	$4.6857e-008$	$1.0825e-008$	4.3286
GARCH(1)	0.92472	0.0088449	104.5485
ARCH(1)	0.055603	0.0053162	10.4591

Table 6.1. *Parameter estimates for DJCB02 Index for the period* $02/01/1997 - 24/09/2004$.

Parameter	Value	Standard Error	T Statistic
C	0.000254010	$3.1119e-005$	8.1626
AR(1)	0.089148	0.024137	3.6934
K	$6.1945e-008$	$1.4083e-008$	4.3985
GARCH(1)	0.90361	0.011627	77.7188
ARCH(1)	0.071044	0.0066707	10.6502

Table 6.2. *Parameter estimates for DJCB02 Index for the period* $02/01/1997 - 24/09/2004$.

Parameter	Value	Standard Error	T Statistic
C	0.00049681	0.00024775	2.0053
AR(1)	-0.0078747	0.026044	-0.3024
AR(2)	-0.011078	0.023067	-0.4803
Regress(1)	0.0060823	0.0409134	0.1487
Regress(2)	0.035009	0.043498	0.8048
Regress(3)	-0.056755	0.038002	-1.4935
K	$2.9658e - 006$	$8.5241e - 007$	3.4793
GARCH(1)	0.88763	0.011124	79.7956
ARCH(1)	0.095891	0.0092913	10.3205

Table 6.3. *Parameter estimates for DJSP Index for the period* $02/01/1997 - 24/09/2004$.

Parameter	Value	Standard Error	T Statistic
C	0.00049118	0.00024454	2.0086
K	$3.0012e - 006$	$8.5464e - 007$	3.5116
GARCH(1)	0.88719	0.010683	83.0474
ARCH(1)	0.096116	0.0084832	11.3302

Table 6.4. *Parameter estimates for DJSP Index for the period* $02/01/1997 - 24/09/2004$.

Parameter	Value	Standard Error	T Statistic
C	0.0003741	0.000129	2.9001
AR(1)	0.063049	0.023987	2.6284
AR(2)	-0.0075111	0.023612	-0.3181
Regress(1)	0.04884	0.0097296	5.0197
Regress(2)	0.028364	0.01017	2.7891
Regress(3)	0.0058778	0.011312	0.5196
K	$1.3042e-006$	$4.3895e-007$	2.9712
GARCH(1)	0.90783	0.019353	46.9090
ARCH(1)	0.055566	0.0098548	5.6385

Table 6.5. *Parameter estimates for DJCB30 Index for the period* $02/01/1997 - 24/09/2004$.

Parameter	Value	Standard Error	T Statistic
C	0.00037386	0.00012836	2.9127
AR(1)	0.063416	0.023811	2.6633
Regress(1)	0.048314	0.0097464	4.9572
Regress(2)	0.028526	0.010177	2.8031
K	$1.3752e-006$	$4.5783e-007$	3.0038
GARCH(1)	0.90551	0.020033	45.2007
ARCH(1)	0.055866	0.010029	5.5704

Table 6.6. *Parameter estimates for DJCB30 Index for the period* $02/01/1997 - 24/09/2004$.
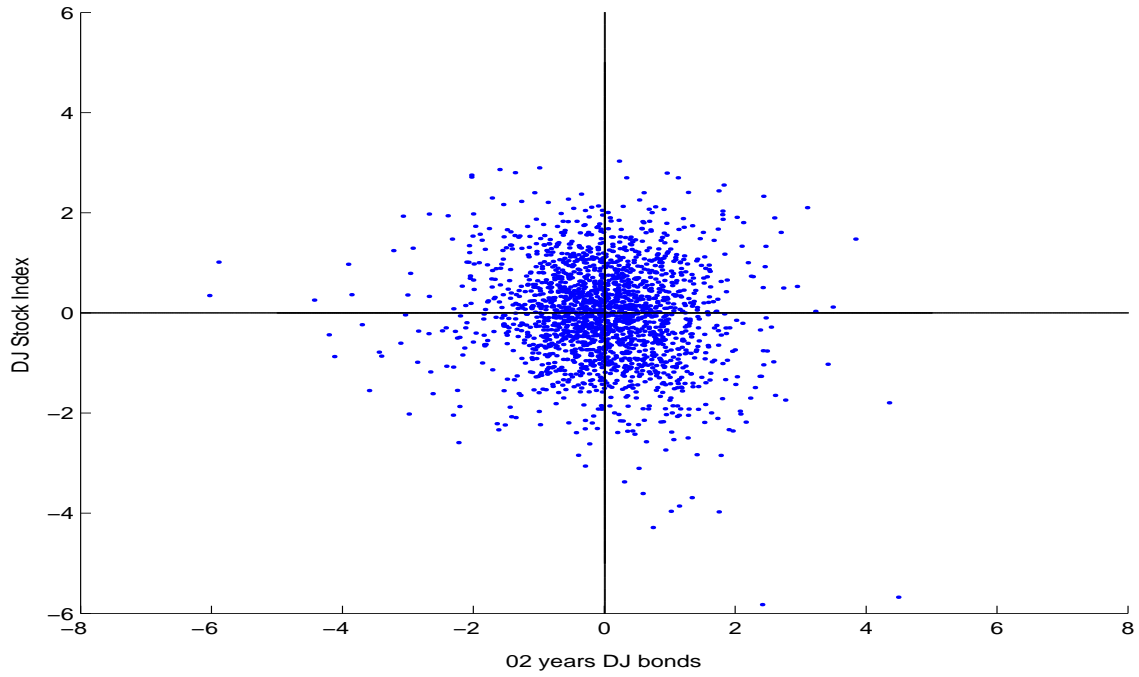
Figure 6.1. *Bivariate plot of the innovations sequences of the Dow Jones Corporate 02 Years Bonds and the Dow Jones Stock Index. The observations span the period* $02/01/1997 - 24/09/2004$, $n = 1942$ *observations.*
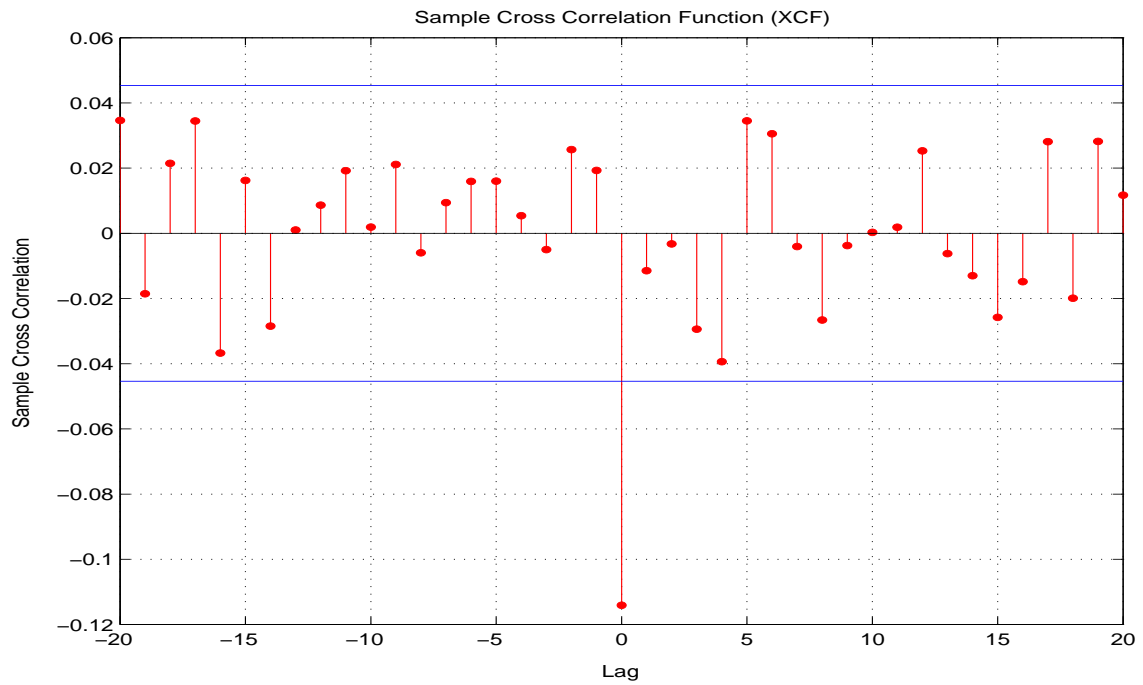
Figure 6.2. *Cross correlation for different lags of the bivariate innovation sequence, Dow Jones Corporate 02 Years Bonds and Dow Jones Stock Index, spanning the period* $02/01/1997 - 24/09/2004$, $n = 1942$ *observations.*
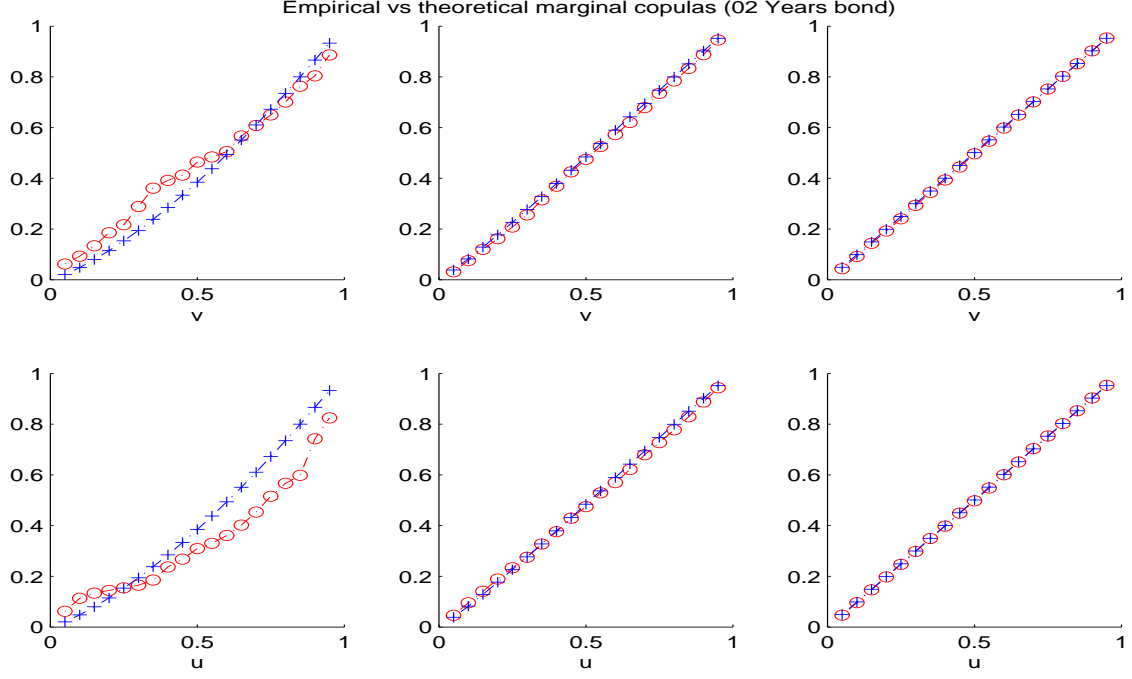
Figure 6.3. *Empirical ($o-$) and theoretical ($+-$) margins of the cumulative bivariate distribution function. The upper panel describes the vertical sections and the lower panel the horizontal section. The left panels represent 0.05 quantile, the middle panels 0.50 quantile and the right panels the $0.0.95$ quantile.*
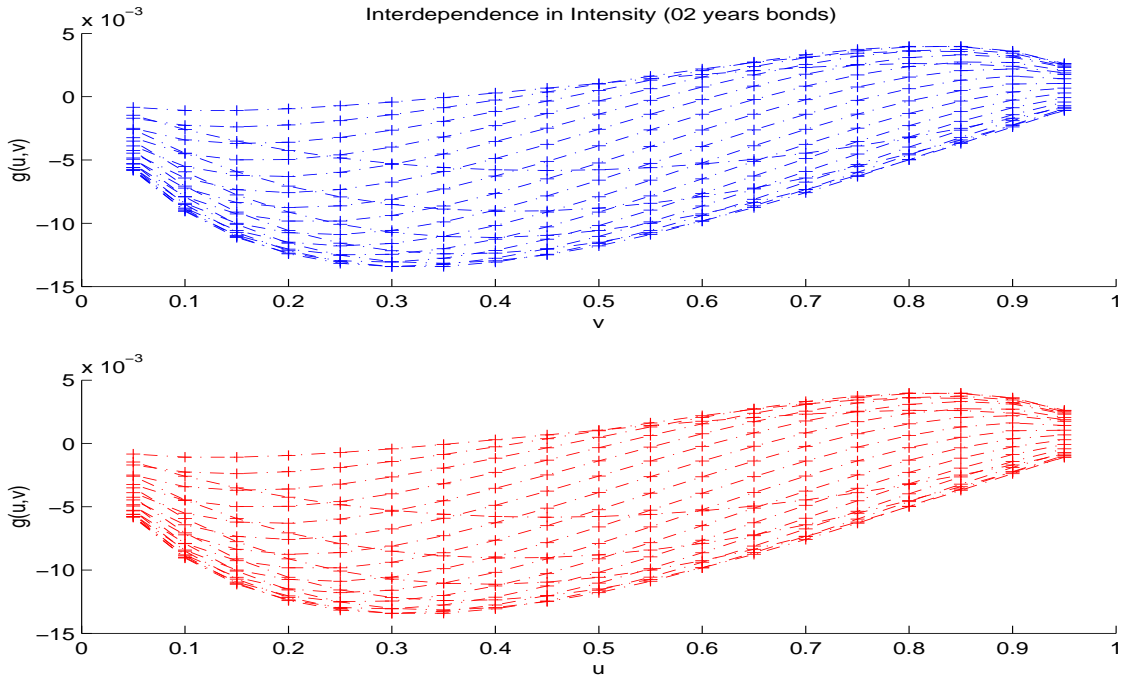
Figure 6.4. *The upper panel depicts the function $g(u,v)$ as defined in (23) plotted against the innovations of DJSI. The lower panel $g(u,v)$ plotted against the innovations of DJBI02.*
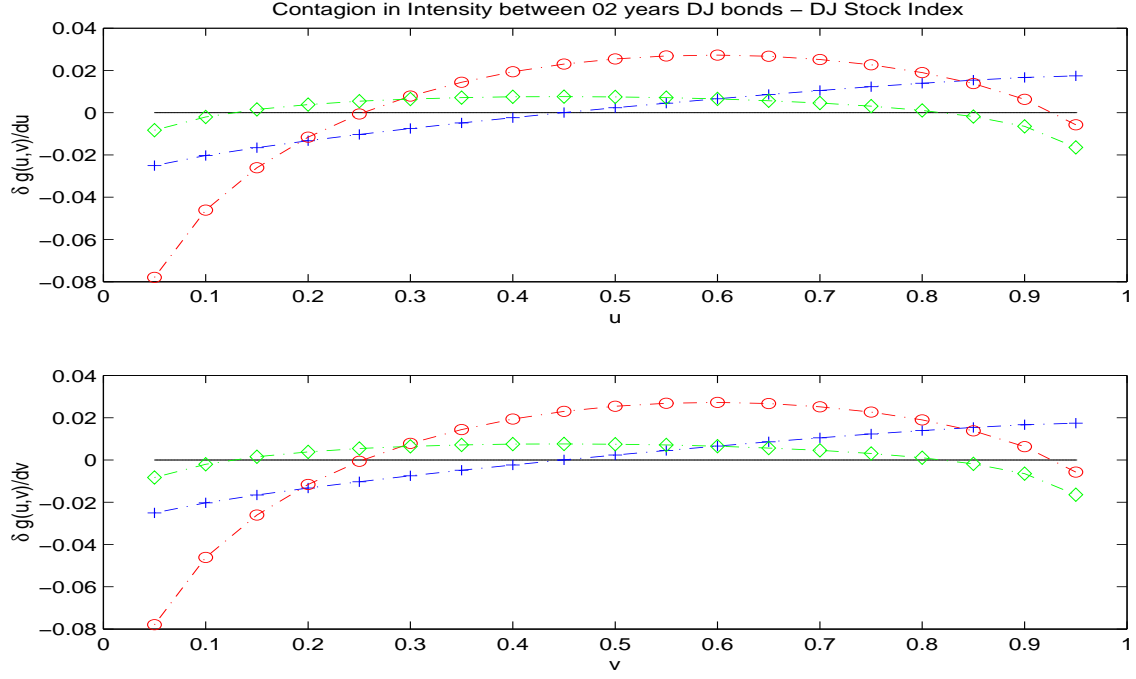
Figure 6.5. *The upper panel depicts the function $h_1(u, v)$ as defined in (24) plotted against the innovations of DJBI02 and the lower panel depicts $h_2(u, v)$ against the innovations of DJSI. $(+-)$ represents the 0.05 quantile, $(o-)$ the 0.50 quantile and $(\diamond-)$ the 0.95 quantile.*
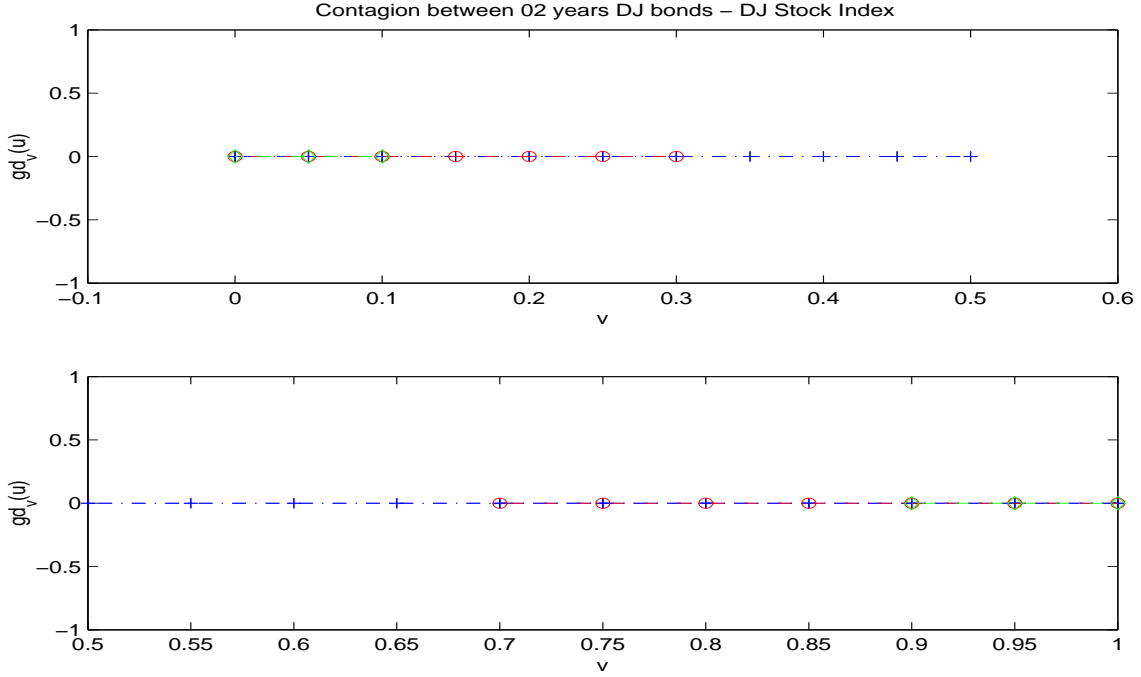
Figure 6.6. *The upper panel depicts $gd_v(u) = \widetilde{C}_G(u, v) - \widetilde{C}_G(v, u)$ for the lower tail $(v \leq u)$. $(+-)$ represents $u = 0.50$, $(o-)$ represents $u = 0.30$ and $(\diamond-)$ for $u = 0.10$. The lower panel depicts $gd_v(u)$ for the upper tail $(v > u)$. $(+-)$ represents the $u = 0.50$, $(o-)$ represents*

$u = 0.70$ *and* $(\diamond-)$ *for* $u = 0.90$.
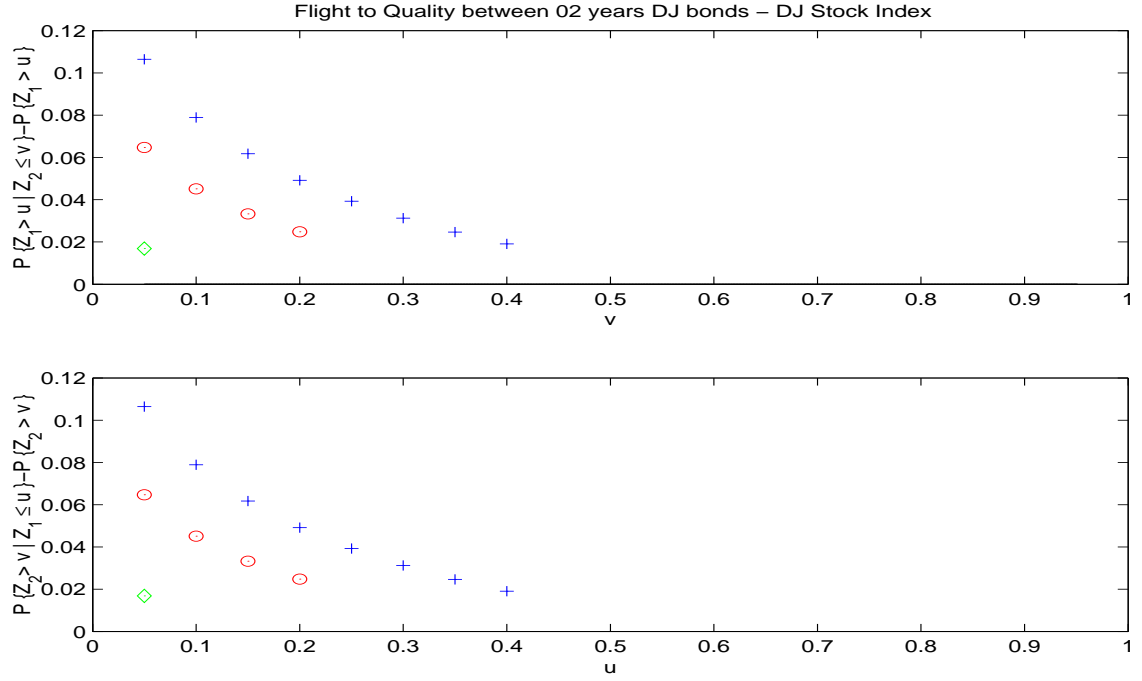
Figure 6.7. *The upper panel depicts the flight to quality from DJSI towards DJBI02.* $(+-)$ *represents* $u = 0.60$, $(o-)$ *represents* $u = 0.80$ *and* $(\diamond-)$ *for* $u = 0.95$. *The lower panel depicts the flight to quality from DJBI02 towards DJSI.* $(+-)$ *represents* $v = 0.60$, $(o-)$ *represents* $v = 0.80$ *and* $(\diamond-)$ *for* $v = 0.95$.
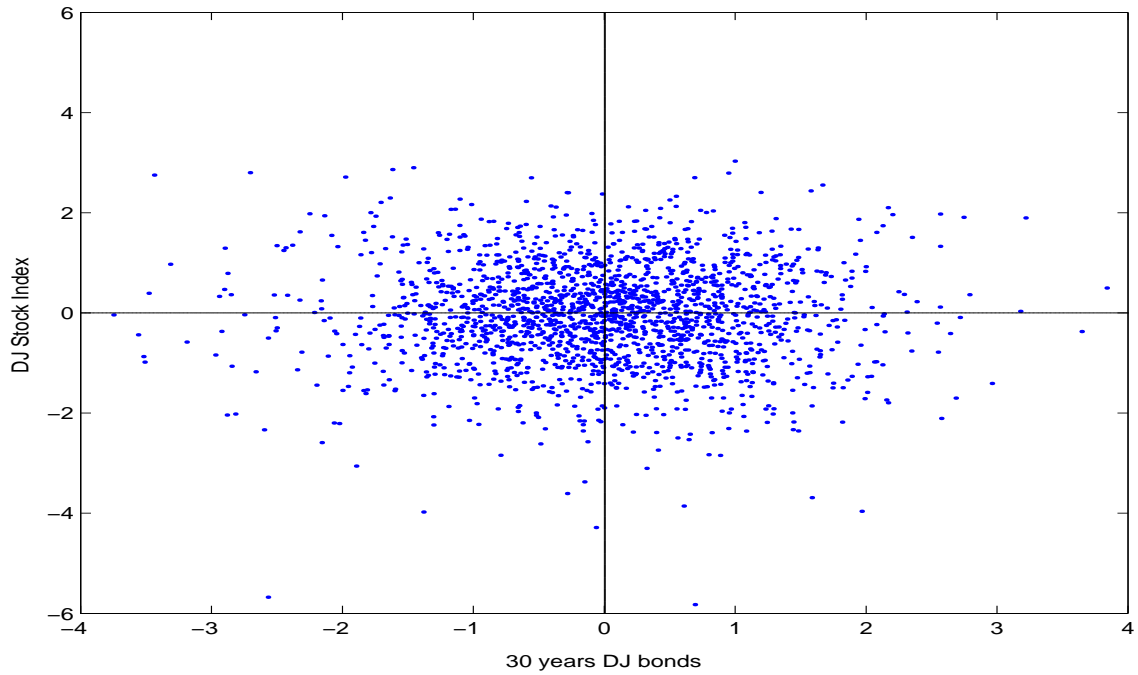
Figure 6.8. *Bivariate plot of the innovations sequences of the Dow Jones Corporate 30 Years Bonds and the Dow Jones Stock Index. The observations span the period* $02/01/1997 - 24/09/2004$, $n = 1942$ *observations.*
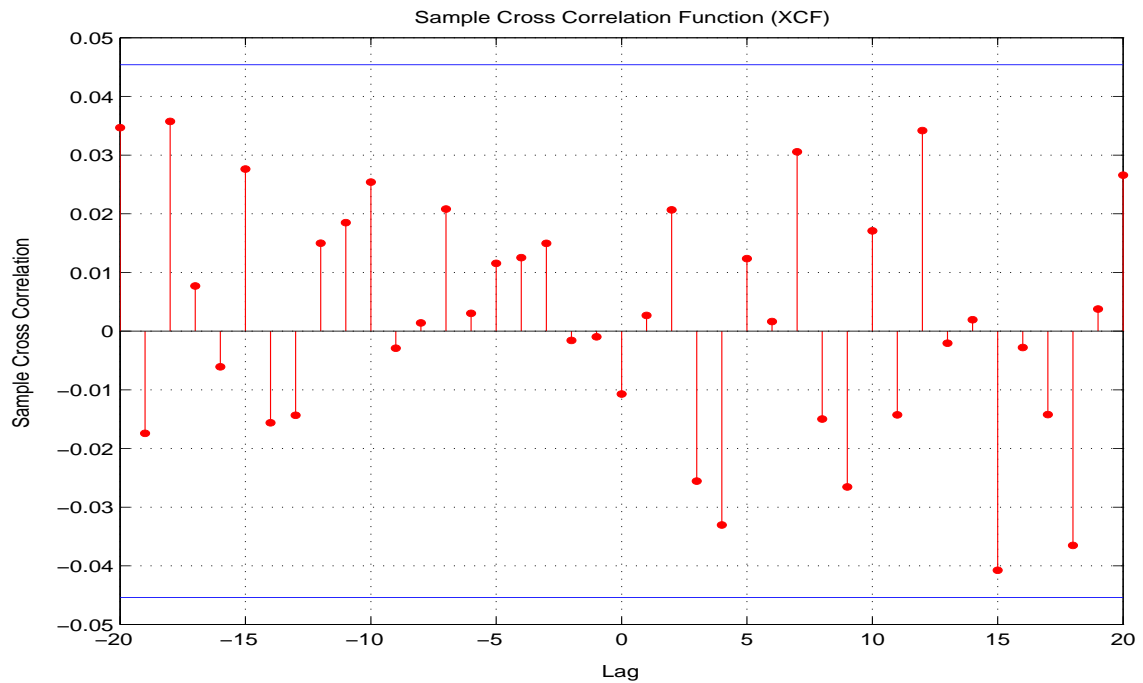
Figure 6.9. *Cross correlation for different lags of the bivariate innovation sequence, Dow Jones Corporate 30 Years Bonds and Dow Jones Stock Index, spanning the period* $02/01/1997 - 24/09/2004$, $n = 1942$ *observations.*
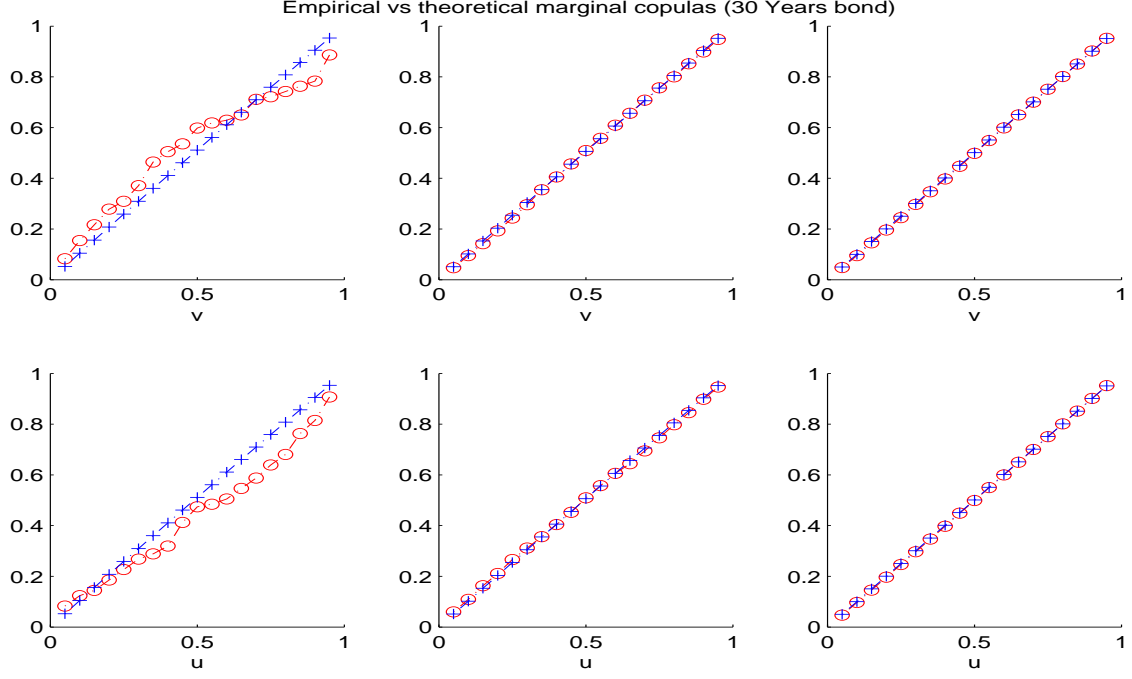
Figure 6.10. *Empirical* $(o-)$ *and theoretical* $(+-)$ *margins of the cumulative bivariate distribution function. The upper panel describes the vertical sections and the lower panel the horizontal section. The left panels represent* 0.05 *quantile, the middle panels* 0.50 *quantile and the right panels the* $0.0.95$ *quantile.*
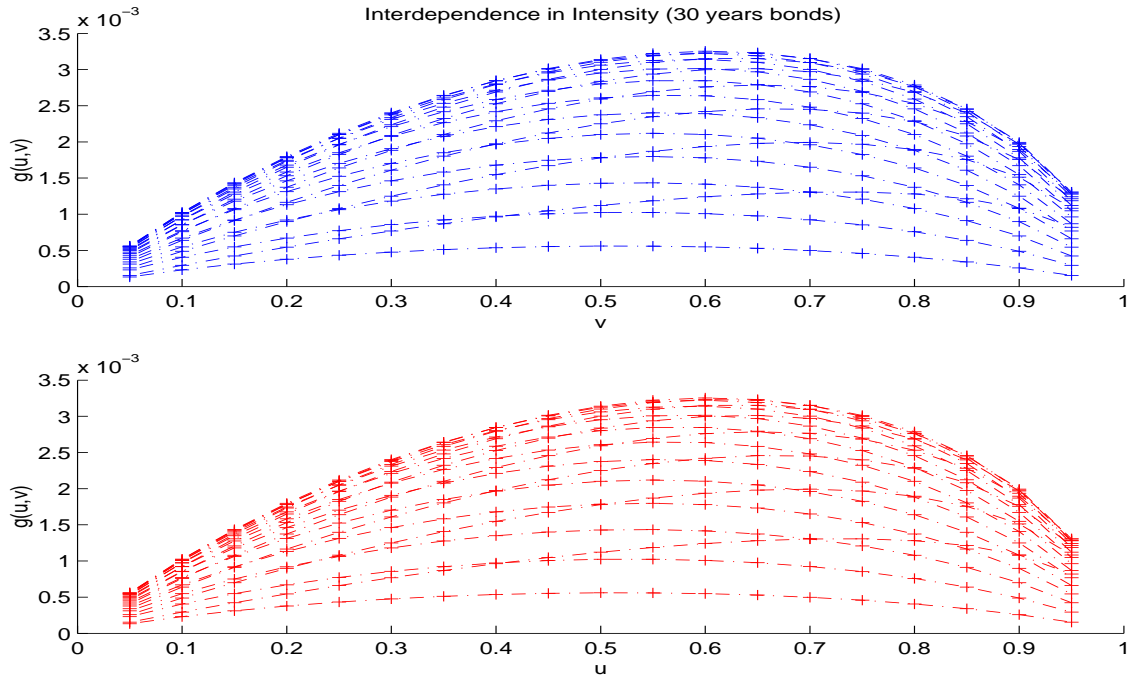

Figure 6.11. *The upper panel depicts the function* $g(u, v)$ *as defined in (23) plotted against the innovations of DJSI. The lower panel* $g(u, v)$ *plotted against the innovations of DJBI30.*
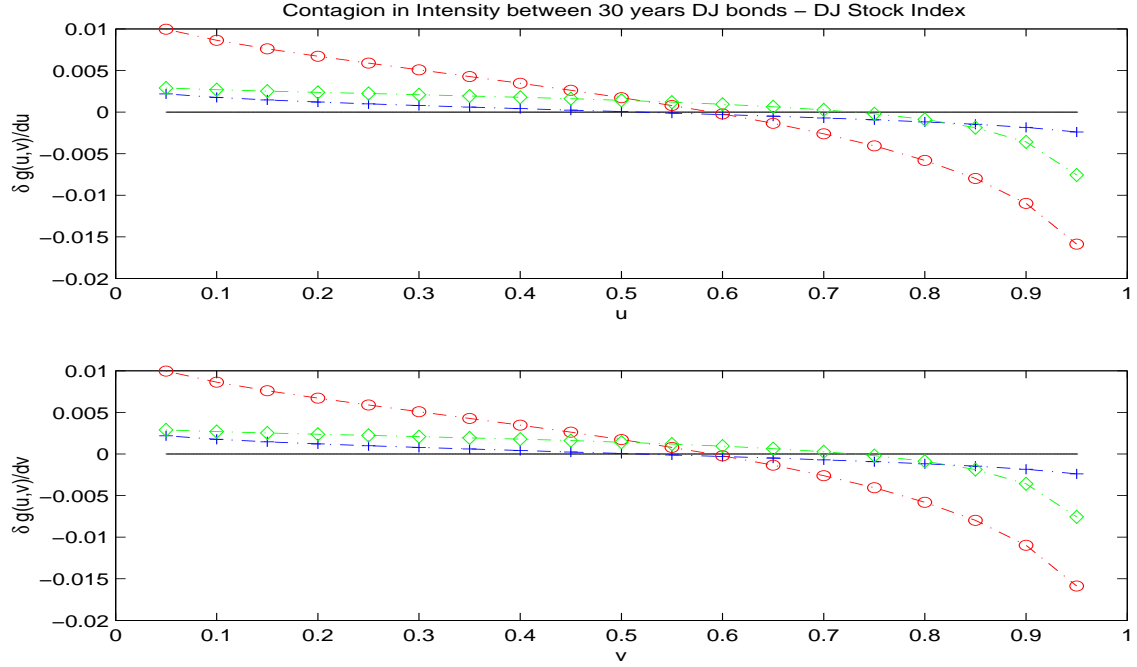
Figure 6.12. *The upper panel depicts the function $h_1(u,v)$ as defined in (24) plotted against the innovations of DJBI30 and the lower panel depicts $h_2(u,v)$ against the innovations of DJSI. $(+-)$ represents the 0.05 quantile, $(o-)$ the 0.50 quantile and $(\diamond-)$ the 0.95 quantile.*
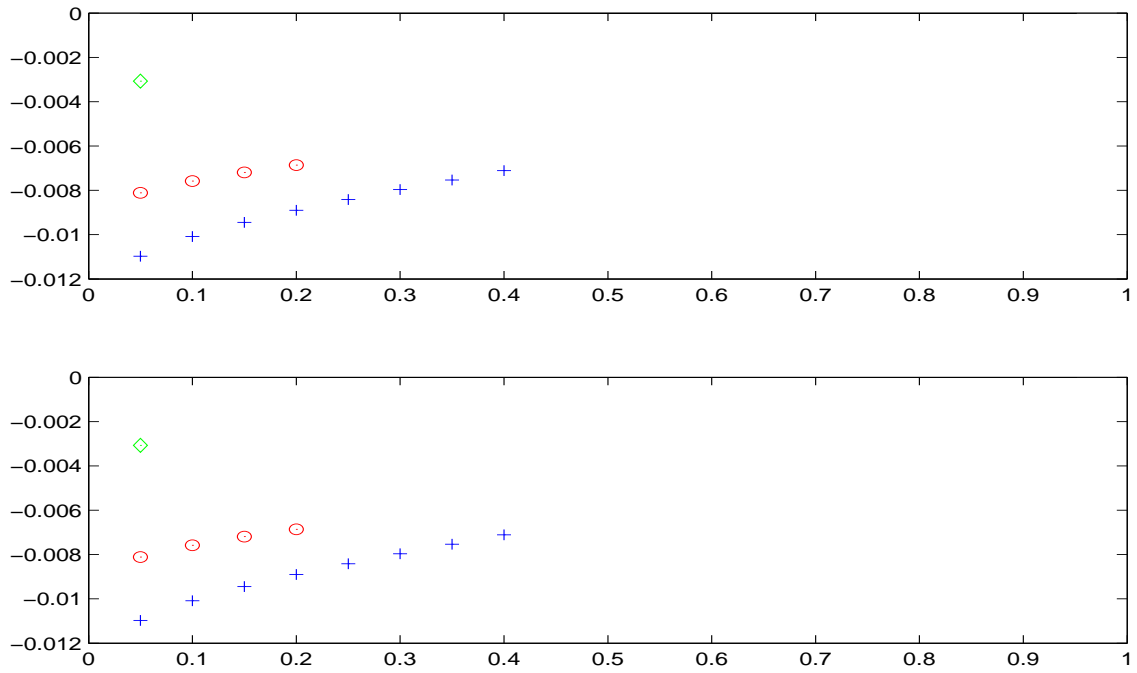

Figure 6.13. *The upper panel depicts the flight to quality from DJSI towards DJBI30. $(+-)$ represents $u = 0.60$, $(o-)$ for $u = 0.80$ and $(\diamond-)$ for $u = 0.95$. The lower panel depicts the flight to quality from DJBI02 towards DJSI. $(+-)$ represents $v = 0.60$, $(o-)$ for $v = 0.80$ and $(\diamond-)$ for $v = 0.95$.*