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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-75

# THE ROLE OF OBSERVABILITY IN FUTURES MARKETS **

José Luis Ferreira*

**Abstract**

Allaz and Vila (1993) show that oligopolistic industries may become more competitive if a futures market is added previous to the spot market. Later, Hughes and Kao (1997) show that this result occurs only if positions in the futures market are observed, and that without this condition the result is again the Cournot equilibrium. In this work we study different explicit formulations of observability and argue that the lack of it may induce a result very different from the one anticipated in Hughes and Kao (1997). By comparing the game forms of the different models, one can discuss about the suitability of either of them. In particular, the one we find most reasonable fit better some of the stylized facts of an industry like the power market in the U.K.

**Keywords:** Futures markets; Observability; Arbitrage; Cournot competition.

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* Departamento de Economía, Universidad Carlos III de Madrid, Getafe 28903, Madrid, Spain. Tel. 34-91 624 9580. jlferr@eco.uc3m.es.

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# 1 Introduction

Futures markets have been most commonly justified as a consequence of firms' attempt to hedge against fluctuations in the price or to take advantage of arbitrage opportunities. More recently, the strategic interaction between futures and spot markets has been analyzed. Williams (1987) develops a model in which positions in one market affect the costs in the other, thus altering the circumstances of competition. Allaz and Vila (1993) (from now on, A&V) show, in an oligopolistic context, that firms may use the futures markets as a way to commit to a quantity, and increase their market share. The result is that all firms have an incentive to do so, with an increase in the total quantity supplied by the industry. In a later work, Hughes and Kao (1997) (H&K), following Backwell (1995), argue that this pro-competitive effect holds only if there is perfect observability. In their analysis, if futures positions are not observed, the equilibrium is again the Cournot behavior.

In the present work we study the role of observability in more detail. First we notice that the result in H&K is based on two implicit assumptions. One is a strong version of the no-arbitrage condition: the price in the futures market must be the same as in the spot market in every contingency. This means that the two prices have to coincide, not only along the equilibrium path, but also along deviations from the equilibrium. The lack of observability is made explicit by specifying that firms do not observe futures positions by the rival, but the authors make the second implicit assumption by adding that firms cannot observe prices in the futures market. Firms realize deviations only when they sell in the spot market. The first implicit assumption is made in the two cases they study, namely total presence and total absence of observability. The second implicit assumption is made in the case with no observability. Once these assumptions are made explicit, other possibilities can be considered and compared. The following cases are worth studying.

(1) Future and spot market prices coincide in every contingency, and firms observe quantities (or prices) in the futures market.

(2) Future and spot market prices coincide in every contingency, and firms cannot observe prices or quantities in the futures market, and

(3) Future and spot market prices coincide along the equilibrium path and firms observe prices, but not quantities, in the futures market.

Case 1 is the model in A&V. Case 2 illustrates the point made by H&K, namely, that if positions are not observed we return to the Cournot outcome. Case 3 deals with a weaker version of the no-arbitrage condition and opens new possibilities, depending of how deviations from the equilibrium are observed and affect the price in the futures market. In this work we present explicit game forms for all the cases (1, 2 and subcases of 3).

The differences in the game form highlights the merits of each of the cases. In particular, cases 1 and 2 do not explicitly model the demand side of the futures market. A natural assumption is that firms can make inferences about the quantities sold in the futures market by observing the futures price. If this price does not carry full information about futures quantities, some kind of market imperfection must be called for. To this end, in case 3 we introduce a set of arbitrators whose role is to buy in the futures market and sell in the spot market. Competition among arbitrators ensures that, in equilibrium, future and spot prices coincide. However, deviations from an equilibrium may change the futures and spot prices in different ways, because of the imperfect information about quantities.

A remarkable result is that case 3 contains 1 as a subcase, but does not contain 2, as the introduction of total absence of observability in case 3 results in the competitive outcome, not the Cournot. In fact, the only equilibria that may arise in models that fit in case 3 imply prices between the competitive and the A&V prices. Thus our work presents an argument that undermines the result in H&K. Observability is important, but it may work in a very different way than thought previously. In a last section we argue that our model explains better some stylized facts observed in the futures markets of the power industry in England and Wales.

In section 2 we present the basic model. In section 3 we present the models for different assumptions on observability. Section 4 offers the discussion for the power industry in the U.K.. Section 5 concludes.

# 2 The basic model

Allaz and Vila (1993) formulate the following case: Consider a duopoly producing an homogeneous good competing a la Cournot with zero costs and facing a demand given by $p = A - q$. Suppose that before producing and selling in the spot market, firms may sell in advance part of their production in a futures market. Denote by $s_i$ and $f_i$ the quantities sold by firm i in the spot and futures market respectively ($q_i = s_i + f_i$). If positions in the futures market are observable, given $f_1$ and $f_2$, in the spot market firm i solves:

$$\max_{s_i} \; ps_i$$
$$\text{s.t } p = A - f_1 - f_2 - s_1 - s_2$$

The solution gives the reaction function $s_i = \frac{A - f_1 - f_2 - s_j}{2}$ or, in terms of total quantities, $q_i = \frac{A + f_i - q_j}{2}$. Solving for $s_1$ and $s_2$, one finds $s_i = \frac{A - f_1 - f_2}{3}$, $q_i = \frac{A + 2f_i - f_j}{3}$, and $p = \frac{A - f_1 - f_2}{3}$.

Anticipating this reaction, firms' position in the futures market is calculated as follows:

$$\max_{f_i} \; p\,(f_i + s_i) \tag{1}$$

$$\text{s.t } s_i \;=\; p = \frac{A - f_1 - f_2}{3}$$

The solution to this problem is $f_i = \frac{A - f_j}{4}$. Solving for $f_1$ and $f_2$, and substituting in the expressions for the other variables, the result is $f_i = p = s_i = \frac{A}{5}$, $q_i = \frac{2}{5}A$, with profits given by $\Pi_i = \frac{2}{25}A^2$. The equilibrium is thus showing a pro-competitive effect of the futures market as the Cournot equilibrium without it is $q_i = p = \frac{A}{3}$, with $\Pi_i = \frac{A^2}{9}$. To understand why this occurs, notice that if, for whatever reason, it is known that firm 2 does not use the futures market, then firm 1 chooses $f_2 = \frac{A}{4}$ and then $p = s_i = \frac{A}{4}$, $q_1 = \frac{1}{2}A$, $q_2 = \frac{1}{4}A$. Firms are behaving as if firm 1 were the Stackelberg leader. The pro-competitive effect is caused as firms compete to be the "leader".

Without observability, Hughes and Kao (1997) conclude a totally different story. If futures positions are not observable, the reaction functions derived from the first of the previous problems require that non observed variables be conjectural:

$$q_i = \frac{A + f_i - q_j'}{2}$$

Where $q_j'$ is anticipated to follow the same reaction:

$$q_j' = \frac{A + f_j' - q_i'}{2}$$

Solving this second system, and taking into account that in equilibrium conjectures have to be correct, one finds:

$$q_i' = \frac{A + 2f_i' - f_j'}{3}$$

Finally, reaction functions become:

$$q_i = \frac{A + \frac{2}{3}f_i + \frac{1}{3}f_i' - f_j'}{3}$$

Then, the problem of deciding the positions in the forward market is:

$$\max_{f_i} \; pq_i$$

$$
\begin{aligned}
\text{s.t } q_i &= \frac{A + \frac{2}{3}f_i + \frac{1}{3}f_i' - f_j'}{3} \\
q_j &= \frac{A + \frac{2}{3}f_j + \frac{1}{3}f_j' - f_i'}{3} \\
p &= A - q_1 - q_2
\end{aligned}
$$

The solution of this problem gives $f_i = \frac{f_j - f_j'}{2}$. In equilibrium $f_j = f_j'$, leading to the Cournot solution with $f_1 = f_2 = 0$. Hence observability is a necessary condition to obtain the pro-competitive effect in the presence of futures markets.

# 3 The role of observability

In this section we study the role of observability in more detail. According to H&K, the pro-competitive effect in A&V takes place only if positions in the futures market are observed. If they are not, the equilibrium reverts to Cournot. However, their result is based on two implicit assumptions. The first one is a result of a strong version of the no-arbitrage condition: the price in the futures market must be the same as in the spot market in every contingency. This means that the two prices have to coincide, not only along the equilibrium path, but in deviations from the equilibrium as well. The second assumption is that firms cannot observe prices in the futures market. They realize deviations only when they produce and sell in the spot market. Let us discuss these assumptions in more detail.

(i) The strong no-arbitrage condition. It is most natural to require that, in equilibrium, futures and spot prices be the same. If not, some agent can profit by buying in one marker and selling in the other. Once these arbitrage opportunities are exhausted, prices are identical. Take, then, an equilibrium situation in which the no-arbitrage condition is satisfied, and think of the consequences of a firm deviating by selling a larger quantity in the futures market. This implies an increase in the total quantity sold in the markets, and a lower price in the spot market. Should this imply also a lower (and identical) price in the futures market? If agents have perfect information of what is going on in the futures market, this seems the only reasonable consequence. On the other hand, if agents in the futures market do not have this perfect information, they may not recognize deviations (or, at least, the exact size of them), and the price in the futures market may not reflect the price that will prevail in the spot market, where Cournot competition in the residual demand (after discounting futures sales) reveals the spot price. The existence of an agent that can foresee this difference in prices may not be the only natural assumption. Only if the deviation is anticipated the change on price will re‡ect the new quantity. However, there are two reasons why this may not be the case. First, once a strategy pro…le is considered as an equilibrium candidate, there may be more than one possible deviation to anticipate. How do agents know which one is actually taking place to adjust the price? Second, the pro…tability of a deviation may depend on the reaction of other agents, and, then, on the price induced by the deviation. These considerations call for a very detailed model on how to link deviations with prices.

(ii) Observability of prices in the futures market. In the case agents have perfect information about quantities sold in the futures market it is hard to justify that some …rm may not know about this. After all, by selling a small quantity in the futures market, any …rm may enter the market and know the price, and a fortiori, the quantities sold. In the presence of perfect competition on the demand side of the futures market, only when agents in this market are not fully aware of the quantities sold (out of the equilibrium path), then one can also assume similar things for …rms. Again this calls for a very precise modeling of how agents and …rms gather information.

From the discussion above, at least three possibilities arise:

(1) Future and spot market prices coincide in every contingency, and …rms observe quantities (or prices) in the futures market.

(2) Future and spot market prices coincide in every contingency, and …rms cannot observe prices or quantities in the futures market, and

(3) Future and spot market prices coincide along the equilibrium path and …rms observe prices, but not quantities in the futures market.

Next we present explicit game forms representing all of the three cases (1, 2 and subcases of 3). The …rst case corresponds to the equilibrium outcome in A&V, case 2 corresponds to the model in H&K with no observability that leads to the Cournot outcome. Case 3 deals with a weaker version of the no-arbitrage condition and opens new possibilities, depending of how deviations from the equilibrium are observed and a¤ect the price in the futures market. The di¤erences in the game form highlights the merits of each of the cases. In particular, cases 1 and 2 do not explicitly model de demand side of the futures market, while cases in 3 do model this side of the market. Furthermore, in case 3, the arbitrage condition is a consequence of the equilibrium itself, not an assumption, like in cases 1 and 2. Finally, it should be mentioned that case 3 has as a particular case the situation in which …rms observe futures prices that are not informative. This is equivalent to the case in which …rms do not observe futures prices at all. We'll look for sequential equilibria (SE) in pure strategies.

## 3.1 Case 1. The game form of the model in A&V

We present this game form corresponding to the model in A&V for the shake of completeness. Recall that the strong version of no-arbitrage must be satisfied, and that $(f_1; f_2)$ are observed. The game is depicted in figure 1.

Insert figure 1 here

The subgame perfect equilibrium (SPE) of this game is easily calculated backwards as shown in section 2. Since it is unique, its also a sequential equilibrium (SE).

## 3.2 Case 2. The result of H&K

Again, the strong version of the no-arbitrage condition is assumed. Quantity $f_i$ is not observed by firm $j \neq i$ and there is no other information to be known form the futures market (in particular the futures market price $p_f$). The game tree is depicted in figure 2.

Insert figure 2 here

Next we show that this is, in fact, the same case, with the same solution, as the model in H&K presented in section 2. This is done in the next proposition.

**Proposition 1** Consider the game in figure 2, with payoffs as described before. Then, the only SE is the following: firm i chooses $f_i = 0$; and $s_i = \frac{A - f_i}{3}$, and firm i believes with probability one that it is in node after $f_j = 0$.

**Proof.** First show that the strategy is indeed a SE. In the equilibrium firms are playing the Cournot outcome, with profits given by $\Pi_i = \frac{A^2}{9}$: Standard Cournot analysis show that there is no profitable deviation from $s_i$. To check that there is no profitable deviation from $f_i = 0$ consider $f_i' > 0$, while $f_j = 0$. After this deviation $s_i$ changes to $s_i' = \frac{A - f_i'}{3}$, p to $p' = \frac{A - 2f_i'}{3}$, and $\Pi_i$ to $\Pi_i' = \frac{A - 2f_i'}{3}\left(\frac{A - 2f_i'}{3} + f_i'\right) = \frac{A^2 - (2f_i')^2}{9} < \frac{A^2}{9}$. To finish this part of the proof notice that beliefs are consistent.

To show that the SE is unique consider any other strategy $(f_1; f_2)$ in which one firm, say i, chooses $f_i > 0$. Again, Cournot analysis in the second stage indicates that, in equilibrium, $s_1 = s_2 = \frac{A - f_1 - f_2}{3}$: Suppose now that, instead of $f_i$, firm i plays $f_i'$. After positions $(f_i'; f_j)$, the other variables take values $s_i' = \frac{A - f_i' - f_j}{3}$, $p = A - s_i' - s_j - f_i' - f_j = \frac{A - 2f_i' + f_i - f_j}{3}$, and

$\Pi_i' = \frac{A - 2f_i' + f_i - f_j}{3}\frac{A + 2f_i' - f_j}{3}$. In the last expression observe that, for every initial $f_i$ profits achieve a unique maximum at $f_i' = \frac{f_i}{4}$. This means that the only case when there is no profitable deviation is $f_i = 0$. ■

## 3.3 Case 3.1: Weak version of no-arbitrage, $f_1 + f_2$ observed by arbitrators.

In the previous cases prices in the futures market were automatically set equal to spot market prices. To introduce prices in the futures market as an independent variable, we add new players that select these prices. The new game is as follows. In a first stage firms simultaneously decide positions in the futures market ($f_1$ and $f_2$). In the second stage, and after observing these actions, $n$ arbitrators offer simultaneously a price at which to buy these quantities. Finally, in the third stage, after observing prices in the futures market, firms sell in the spot market. The payoffs for the firms are calculated as usual. If arbitrator $j$ offers the highest price (say $p_j$) she buys all future quantities and makes profits given by $\Pi^j = (p_j - p_s)(f_1 + f_2)$, where $p_s$ is the price in the spot market. If the highest price $p$ is offered by $m$ arbitrators, each buys $\frac{1}{m}$ of the futures quantities and has profits given by $\Pi^j = (p - p_s)\frac{f_1+f_2}{m}$. The game is depicted in figure 3. For simplicity, only 2 arbitrators are shown. Nodes $a^1; a^2; a^3$ and $a^4$ belong to different information sets of arbitrator 1. Similarly nodes $b^k_k$ (alt. $c^k_k$, $d^k_k$) belong to different information sets of arbitrator 2 (alt. firm 1, firm 2).

Insert figure 3 here

**Proposition 2** Consider the game in figure 3 with payoffs as described above. Then, in all sequential equilibria firms sell $f_1 = f_2 = \frac{A}{5}$ in their first move, arbitrators set prices $p_1 = \ldots = p_n = \frac{A - f_1 - f_2}{3}$, and finally, firms sell $s_1 = s_2 = \max\{p_1; \ldots; p_n\}$ in the spot market.

**Proof.** In the third stage firms are playing the Cournot outcome, $s_1 = s_2 = p_j = \frac{A - f_1 - f_2}{3}$, and therefore, will not deviate from it. This implies $p_j = p_s = \frac{A - f_1 - f_2}{3}$, and no profitable deviations by arbitrators. In the first stage, firms must solve (1), as in case 1, to find their best reply at this stage. The solution gives $f_1 = f_2 = \frac{A}{5}$. To see that the equilibrium is unique notice that, in the third stage firms are playing the unique equilibrium strategy, that arbitrators will deviate from strategies that do not imply $p_1 = p_2 = \frac{A - f_1 - f_2}{3}$, and that the solution to firms' maximization problem in the first stage is unique. ■

## 3.4 Case 3.2: Weak version of the no-arbitrage condition, $f_1 + f_2$ not observed by arbitrators.

As long as prices in the futures market anticipate the price in the spot market, it is not sensible to assume that firms cannot deduce total quantities. Therefore, in order to have non informed firms, it must be the case that arbitrators are themselves not well informed. There may be many ways to model this situation. We start by choosing a simple and radical one. Namely, that buyers in the futures market do not know total positions in this market. I.e., we have a game as before, except that arbitrators cannot condition their actions $(p_1; ...; p_n)$ on quantities $(f_1; f_2)$. Later, we relax this extreme assumption. The game form is the same as in figure 3, except that now nodes $\{a^k\}_k$ ($\{b^k\}_k$) belong to the same information set of arbitrator 1 (2). Finally, $\{c^1; c^2\} \in u_1$, $\{c^3; c^4\} \in v_1$, $\{d^1; d^3\} \in u_2$ and $\{d^2; d^4\} \in v_2$, where $u_i$ and $v_i$ are different information sets of firm $i$.

An interesting feature of this game is that the demand in the futures market does not react to deviations made by the firms. This opens the possibility for different prices in the two markets as a result of deviations. The lack of information in the model does not allow arbitrage between them. However, in equilibrium, both prices have to coincide since the equilibrium must be anticipated by all players in the game.

**Proposition 3** Consider the game defined before; then, in all sequential equilibria, firms choose $f_1$ and $f_2$ such that $f_1 \geq A$; $f_2 \geq A$, and $s_1 = s_2 = 0$.

**Proof.** In any equilibrium it must be that $p_s = p_j = p$ for any arbitrator $j$ and that $s_1 = s_2 = p_s$ (because of Cournot behavior in the spot market). Now see that any equilibrium requires $p_s = 0$. Suppose, to the contrary, that $p_s = A - s_1 - s_2 - f_1 - f_2 > 0$. In this situation, if firm $i$ increases its future positions by $\Delta f_i$ its profits increase by $\Delta f_i \times p$. If $f_j < A$; but still $f_1 + f_2 \geq A$ and, hence, $p = 0$, firm $i$ can deviate to $f_i^0 = 0$. This deviation does not change the futures price ($p = 0$), as is not observed by arbitrators. Now firm $i$ can sell $s_i = \frac{A - f_j}{2}$ with the consequence of $p_s = \frac{A - f_j}{2}$, and profits $\pi_i = \left(\frac{A - f_j}{2}\right)^2 > 0$. Hence, the equilibrium requires $f_1 \geq A$; $f_2 \geq A$, $p_s = p_j = s_i = 0$. It is straightforward to check that this is indeed an equilibrium as profits are zero regardless of unilateral deviations. ■

If read literally, this case makes little sense, but we can provide a more reasonable interpretation. Consider again the strategy consisting of not selling in the futures market and selling the Cournot quantity in the spot market. Even if there are many agents in the futures market, some of them should detect the deviation consisting of a …rm selling a positive amount in this market. Knowing this, they have to anticipate a lower price in the spot market and, consequently, negotiate a lower price in the futures market as well. However, according to the structure of the game, the agents in the futures market cannot make this observation (they just set a price and receive payo¤s at the end of the game.) There are at least two possible justi…cations. If there are many arbitrators, the deviating …rm may sell only a little to each of them, who observes then a small quantity and anticipates a small change in the price. If this amount is small enough, the analysis may ignore it. Alternatively, the action of selling $f_i = 0$ may be interpreted as an ideal description of a reality that is closer to $f_i =$ ", where " is arbitrarily small. In this case, the deviation consisting of selling small quantities to di¤erent arbitrators may not be observed. Next we see that, in fact, this is a particular case of a more general one.

## 3.5 A general case: $f_1 + f_2$ observed imperfectly by arbitrators.

Cases 3.1 and 3.2 represent two extremes of reactions in the futures market towards changes in the quantities. In case 3.1, arbitrators observed precisely these quantities, whereas in case 3.2 they observed nothing. Next we model an intermediate situation of partial observability. Arbitrators can only observe whether total quantities in the futures market belong to a certain information set.

In the …rst stage …rms choose simultaneously quantities $f_1$ and $f_2$ within the interval $\left[0; \frac{B}{2}\right]$, where $\frac{B}{2} > A$. Setting an upper bound to the quantities …rms may sell only prevents us from considering subgames in which futures positions are in…nite. Consider the set of partitions on the interval $[0; B]$ in which sets are either intervals of length at least a pre-…xed ± > 0 or real numbers. Denote this set of partitions by $U$. Given $F = f_1 + f_2$, in the second stage arbitrator $j$'s information partition is a set $U_j$ ½ $U$. I.e., arbitrators observe $F$ or, at least, are able to determine that $F$ is in a certain interval. All arbitrators have the same information partition on futures quantities; i.e., for all $j$, $U_j = U$. Arbitrators choose prices contingent on information sets. In the third stage …rms observe prices set by arbitrators and decide $s_1$ and $s_2$.

The game form is the same as in …gure 3 except that now any nodes belong to the same information set of a given player according to the stated condition (recall that a …rm always knows its own past actions). Case 3.1 is the limit of this general case when arbitrators' information partition gets …ner, and case

3.2 corresponds to the situation in which arbitrators' information partition have only one information set containing all nodes such that $f_1 + f_2 \in [0; B]$.

**Proposition 4** Consider the game described before. Then, the only prices sustained by a SE in pure strategies are $p = 0$ and $\frac{A}{6} \leq p \leq \frac{A}{5}$.

**Proof.** Propositions 2 and 3 already show that $p = 0$ and $p = \frac{A}{5}$ can be sustained by a SE; only notice that arbitrators' information sets are in $U$. The rest of the proof is dedicated to show that the only other prices that can be sustained in equilibrium are $\frac{A}{6} \leq p < \frac{A}{5}$. Recall that to sustain a price $p > 0$ by a SE, the required total quantity must be $q = A - p$, and also that the spot quantities must satisfy $s_1 = s_2 = p$. Total futures positions compatible with these conditions require $f_1 + f_2 = F = q - s_1 - s_2 = A - 3p$ or $p = \frac{A-F}{3}$. To sustain $p$ take $f_1 > 0$ and $f_2 > 0$ such that $F = f_1 + f_2 = A - 3p$. In a SE arbitrators must assign probability 1 to the total quantity $F$, and offer prices $p_j = p$. If firms choose $f_1$ and $f_2$, firm one's profits are $\pi_1 = p(f_1 + s_1) = \frac{A-f_1-f_2}{3}\frac{A+2f_1-f_2}{3}$. To consider possible deviations from the proposed scenario we need to distinguish several cases. Given the structure of information sets, it is straightforward to see that there are only three possibilities for $F$:

(i) $F \in u$ for some interval $u \in U$. ($F$ cannot be known with certainty.)

(ii) $F \in I$, where $I$ is an interval such that for all $u \in I$, $u \in U$. ($F$ and a neighborhood around it can be known with certainty.)

(iii) $F \in U$ and is the frontier of an interval $u \in U$ such that $F \notin u$. ($F$ can be known with certainty, but is in the frontier of an interval of uncertainty.)

(i) Consider firm one's deviation to $(f_1'; s_1')$ with $f_1' + f_2$ belonging to the same information set as $F$, and with $s_1' = \frac{A-f_1'-f_2-s_2}{2}$. Because arbitrators do not observe the deviation, futures price does not change: i.e., for all $j \in N$, $p_j = \frac{A-f_1-f_2}{3}$. Because the price in the futures market does not change, firm 2 does not change its spot quantity, $s_2 = \frac{A-f_1-f_2}{3}$, and then $s_1' = \frac{2A-3f_1'-2f_2+f_1}{6}$. The spot price, however, changes to $p' = A - f_1' - f_2 - s_1' - s_2 = \frac{2A-3f_1'-2f_2+f_1}{6} = s_1'$. Then firm one's profits are given by $\pi_1' = \frac{A-f_1-f_2}{3} f_1' + \left(\frac{2A-3f_1'-2f_2+f_1}{6}\right)^2$. The difference in profits is $\pi_1' - \pi_1 = \frac{1}{4}(f_1' - f_1)^2$. This difference is positive for any $f_1' \neq f_1$, which means that no equilibrium may exist in this case.

(ii) A deviation to $f_1'$ within a neighborhood of $f_1$ gives profits $\pi_1' = \frac{A-f_1'-f_2}{3}\frac{A+2f_1'-f_2}{3}$. The difference in profits is now given by the expression $\pi_1' - \pi_1 = \frac{1}{9}(f_1' - f_1)(A - 2f_1' - 2f_1 - f_2)$. If $A - 4f_1 - f_2 > 0$, take $f_1' = f_1 + \varepsilon$, with $\varepsilon > 0$ small enough, to get $\pi_1' - \pi_1 > 0$. If $A - 4f_1 - f_2 < 0$, take $f_1' = f_1 - \varepsilon$.

(iii) There are four relevant cases. (iii.a) $A - 4f_1 - f_2 < 0$ and $f_1 + f_2$ is located at the left end of an interval of uncertainty (open to the left), (iii.b) $A - 4f_1 - f_2 > 0$ and $f_1 + f_2$ is located at the right end of an interval of uncertainty (open to the right), (iii.c) $A - 4f_1 - f_2 > 0$ and $f_1 + f_2$ is located at the left end of an interval of uncertainty, (iii.d) $A - 4f_1 - f_2 < 0$ and $f_1 + f_2$ is located at the right end of an interval of uncertainty. The cases when $A - 4f_1 - f_2 = 0$ and $A - 4f_2 - f_1 = 0$ are solved like in proposition 2 and give $p = \frac{A}{5}$ in equilibrium. In the first two cases, the deviation is perfectly observed, in case (iii.a) repeat case (ii) with $f_1' = f_1 + \varepsilon$, and in (iii.b) repeat (ii) with $f_1' = f_1 - \varepsilon$ to conclude that there are no equilibria.

For the other two cases, notice that if futures positions are not perfectly observed after the deviation, arbitrators will offer a price $p'$, their contingent price for the information set induced by firm 1. The new profits for firm 1 are given by $\Pi' = p'f_1' + \left(\frac{A - f_1' - f_2 - p'}{2}\right)^2$. In case (iii.c) $p' < \frac{A - f_1 - f_2}{3}$ and the maximum for this expression restricted to $f_1' \geq f_1$ corresponds to $f_1' = f_1$. This implies that if firm 1 deviates from $f_1$ to induce $p'$, it better do it with a very small deviation. I.e., $f_1' = f_1 + \varepsilon$ for a small $\varepsilon > 0$. The gain in profits are (except for terms in $\varepsilon$) $\Pi' - \Pi = \frac{1}{36}(5A - 17f_1 - 5f_2 - 3p')(A - f_1 - f_2 - 3p')$. The expression in the second parenthesis is positive. The expression in the first parenthesis attains its infimum with respect to $p'$, and restricted to $p' < \frac{A - f_1 - f_2}{3}$ at $p' = \frac{A - f_1 - f_2}{3}$. This infimum is $4(A - 3f_1 - f_2) > 0$ if $A - 4f_1 - f_2 > 0$, as required in this case. In case (iii.d) $p' > \frac{A - f_1 - f_2}{3}$, and the maximum of $\Pi'$ also corresponds to $f_1' = f_1$; which implies that firm one's deviation should be $f_1' = f_1 - \varepsilon$. The expression for $\Pi' - \Pi$ is the same as in (iii.c). The second parenthesis is negative, and the first parenthesis is always non negative if $A - 3f_1 - f_2 \geq 0$. By symmetry, to get that no profitable deviations exist for the other firm we have that $A - 4f_2 - f_1 \geq 0$ and $A - 3f_2 - f_1 \geq 0$. I.e., there are equilibria as long as $2A - 5f_1 - 5f_2 \geq 0$ and $2A - 4f_1 - 4f_2 \geq 0$, which implies $\frac{4}{5}A \leq f_1 + f_2 \leq A$ and $\frac{A}{6} \leq p \leq \frac{A}{5}$. ■

# 4 Discussion

According to H&K firms perform better in a situation in which futures markets are opaque (they get the Cournot profits) rather than transparent (where they get the more competitive A&V outcome). This means that, given the choice, they prefer opaque markets. However, if both markets are present, the firms face a prisoners' dilemma, as they have an incentive to use the transparent market to try to get a higher market share. Our model, however, indicates that, if the demand side of the futures market is sensitive enough in the sense of setting prices that re‡ect actual quantities, observability of futures quantities by …rms does not make a di¤erence as in any case the outcome is that in A&V. Therefore, …rms will be indi¤erent if given the choice. On the other hand if prices in the futures market do not react with enough sensibility to a change of quantities, the result may change to more competitive outcome. In this case they prefer the transparent market and face no prisoners' dilemma.

The liberalization of the power market in England and Wales provides a case of an oligopolistic industry with two futures markets, Contracts for Di¤erences (CfD) and Electricity Forward Agreements (EFA), the CfD being much more opaque. According to estimates in Power UK (1998), around 1998 the coverture of the CdF's was near 90% of the market, while the EFA's accounted for less than 30%. This contradicts H&K's model, but not ours if prices are informative. Of course, our model does not explain that …rms prefer the opaque market, but is compatible with other forces that may induce …rms to do so. For example, …rms may show a collusive behavior that may be better implemented in real life in the more opaque market. This possibility has been mentioned in OXERA (1994) and is explored in the context of a repeated situation of the model by A&V in Ferreira (2000).

# 5 Conclusion

Allaz and Vila (1993) show that if a futures market is added to the spot market in an oligopolistic industry, …rms show a more competitive behavior. Hughs and Kao (1997) argue that …rms must have perfect information about futures quantities for this result to hold. Furthermore, according to them, without this information …rms behave like in Cournot. We claimed that, while it is true that perfect information leads to the result in A&V, the lack of information may not lead to Cournot. In fact, our model shows that the result may be even more competitive if agents in the futures markets are not well informed about quantities o¤ered by …rms. If the demand side in the futures market are informed, the fact that …rms are not informed is irrelevant. This later case provides a theoretical model for the observed behavior in the futures markets of the UK power industry, which contradicts H&K's results.

Our model suggests that a closer look at the structure of the markets may provide di¤erent results that those obtained in a reduced form (in our work, case 3 versus 2). There is a large literature on Market Microstructure Theory (see O'Hara, 1995, and references within), but we are not aware of a work in which intermediaries have and information structure comparable to the one we developed here.

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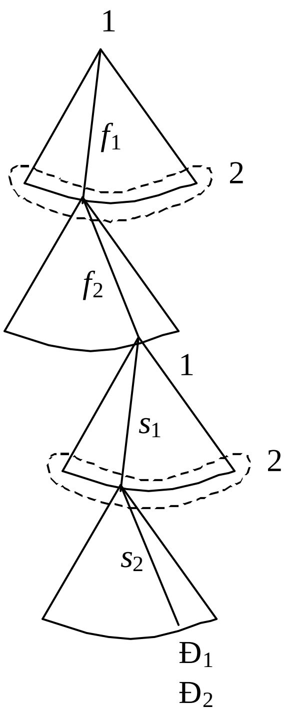


Figure 1

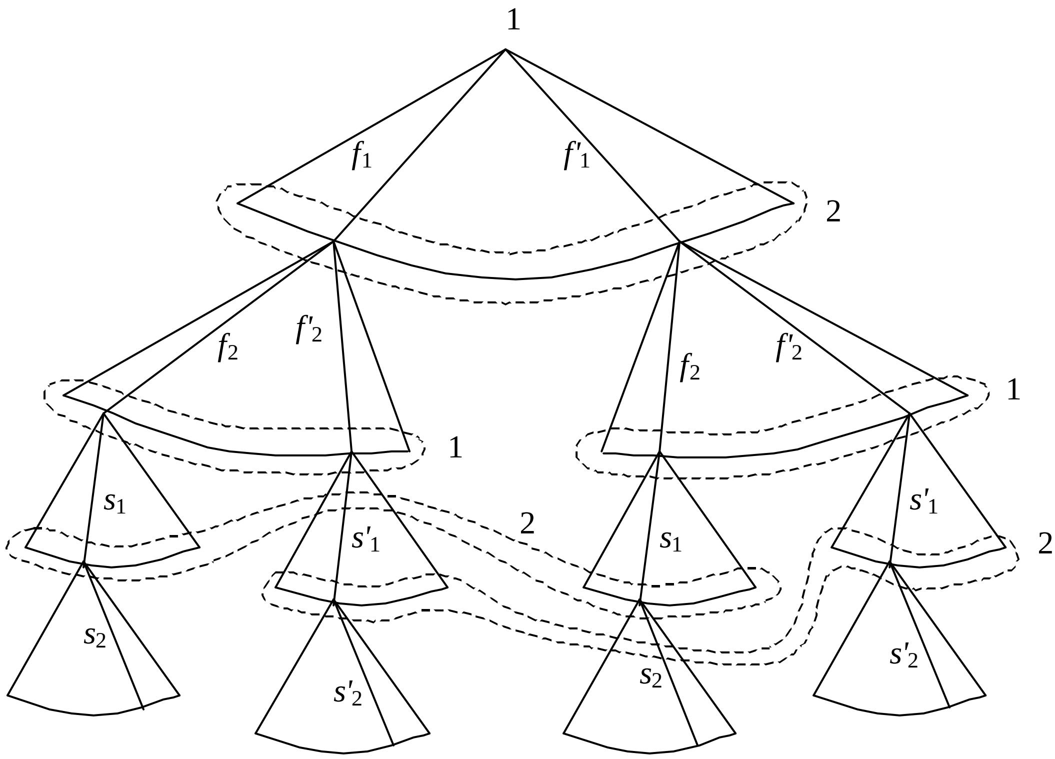


Figure 2

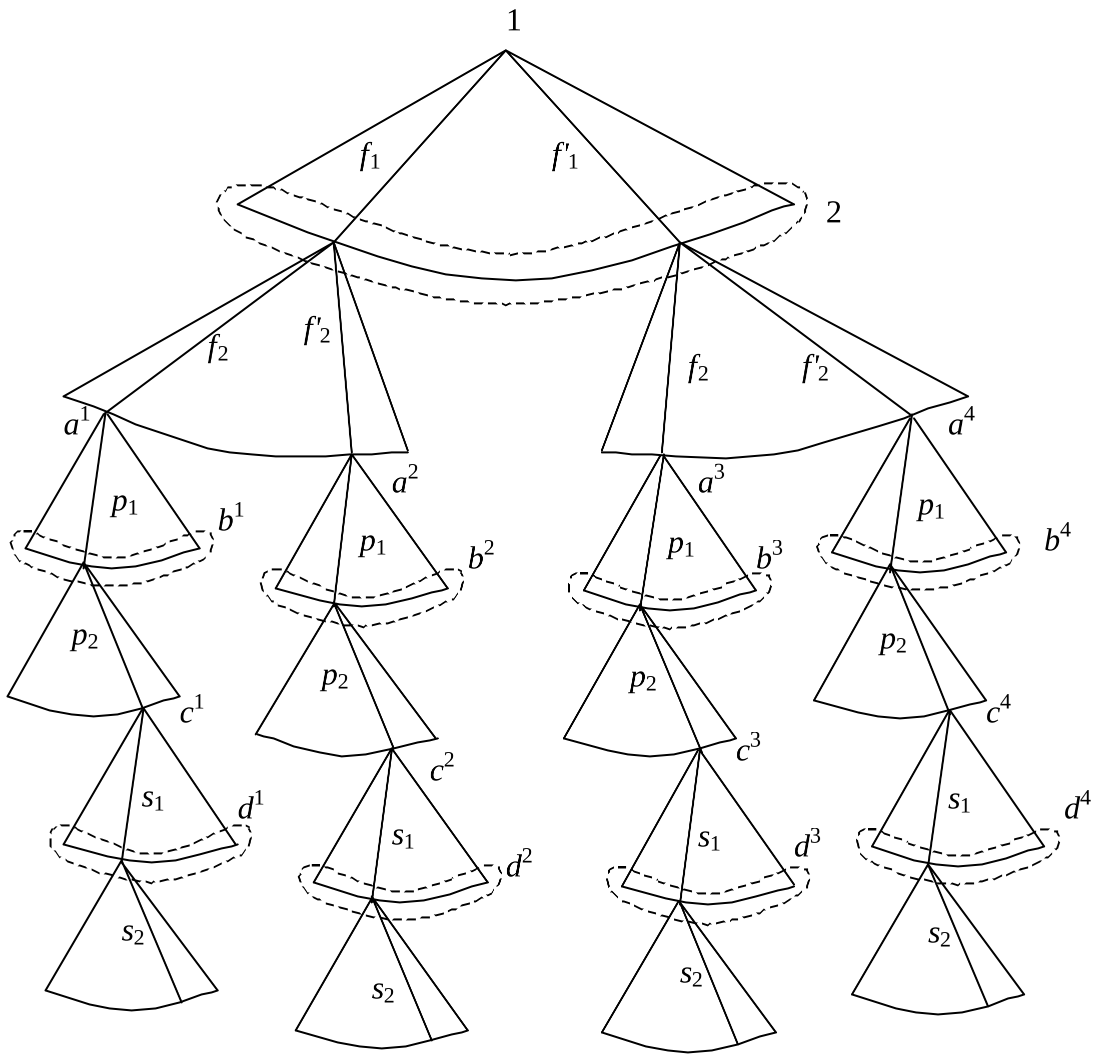
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Figure 3