HABIT FORMATION: INPLICATIONS FOR THE WEALTH DISTRIBUTION *

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Abstract

In this paper we study the role of habit formation in shaping the wealth distribution in an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty. We compare the inplications for precautionary savings and for wealth concentration between economies that only differ in the role played by habit formation. Once preferences are properly adjusted so that the Intertemporal Elasticity of Substituion is the same in all model economies studied, we find that habit formation brings a hefty increase in precautionary savings and very mild reductions in the coefficient of variation and in the Gini index of wealth. We also find that the reductions in these measures of inequality also hold when we adjust our economy so that aggregate savings are the same as in the economy without habit formation. These findings hold for both persistent and non persistent habits although for the former the quantitative size of the effects is much larger. We conclude that habit formation, while being a mechanism that increases the amount of precautionary savings generated in a model, does not change the implications for wealth inequality that arise from standard models.

Keywords: Habit Formation; Heterogeneous Agents; Precautionary Savings; Wealth Distribution.

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1   Introduction

Models with a large number of ex–ante identical agents with standard preferences subject to uninsurable, idiosyncratic shocks to income are the main tool used to answer two questions that many economists see as important: (1) what is the size of precautionary savings (savings held for the sole purpose of smoothing consumption across different contingencies)?, and (2) what accounts for the very large differences in assets holdings among American households? The accepted answer to the first question as posed, for example, by Aiyagari (1994) is that precautionary savings are small, no more than 3% of total savings. With respect to the second question, there is a relative consensus in the profession that a theory of wealth inequality based on standard and identical preferences and on uninsurable shocks to income can account for only a small part of the observed wealth inequality.[1]

In this paper we study, in the context described in the previous paragraph, the role played by habit formation in shaping the distribution of wealth, and hence, the answers to those two important questions.

Habit formation has been recently used to improve the predictions of time-separable models in different fields where savings behavior under uncertainty and the income-fluctuation problem are the chief ingredients. For instance, some authors have pursued this path and studied various formulations of habit formation to improve our understanding of the equity premium puzzle.[2] Other authors have used this class of preferences to study the observed relationship between savings and growth.[3] Finally, Fuhrer (2000) shows how the presence of habits in consumption can generate slow and hump-shaped reactions of consumption to monetary and other shocks. Despite all this work with habits,[4] its implications for the de-

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[1]For example Krusell and Smith (1998) postulate shocks to preferences to account for wealth inequality while Carroll (2000) argues that we should use models where consumers consider the accumulation of wealth as an end in itself or models where wealth yields a large unobservable flow of services. Quadrini and Ríos-Rull (1997) contains a review of the literature and its successes and failures in accounting for wealth inequality with uninsurable shocks to income. On the other hand, a recent paper, Castañeda, Díaz-Giménez, and Ríos-Rull (2000), argues that a suitably modified version of the basic model with identical and standard preferences and uninsurable shocks does account for the wealth inequality observed in the U.S. An important modification proposed by these authors is the use of a process for earnings with more volatility than those found in previous work.

[2]Abel (1990) and Constantinides (1990) show that adding habit formation to an otherwise standard exchange model economy, the equity premium puzzle, as stated by Mehra and Prescott (1985) disappears. The same result is obtained by Heaton (1995) or Boldrin, Christiano, and Fisher (1997) and others.

[3]The evidence shows that, across countries and across households, the growth rate of income has a positive and significant effect on the savings rate (see Edwards (1995), Carroll and Weil (1994), Deaton and Paxson (1994), for instance). To account for this observed pattern Carroll, Overland, and Weil (2000) modify the standard Ak model to display habit formation. They show that the model is successful to replicate the positive response of the savings rate to the growth rate of income.

[4]In this work, as in ours, households do not value leisure. The role of time non-separabilities in leisure

termination of precautionary savings and for shaping the wealth distribution have not been explored. This is precisely the target of this paper.

To study the role of habits we compare a standard economy without habits (the benchmark economy) with various habits counterparts. We proceed by first looking at economies with the same parameterization except for the habits. The two economies do not have the same Intertemporal Elasticity of Substitution (henceforth IES). Hence, households in the habits and in the benchmark economy not only differ in their attitude towards risk, but also in their willingness to intertemporally substitute consumption. To better understand the implications of habits, we compare the benchmark economy with the habits economies recalibrated to have the same IES. To further study the role of habits in shaping wealth inequality we want to isolate the effect of habits on risk aversion from the induced effects brought by the changes in the interest rate. To this end, we compare the benchmark economy with the habits economies calibrated not only to have the same IES, but also the same aggregate savings. To analyze the effect of the persistence of habits on the level of precautionary savings and wealth inequality we study two habits economies: one in which habits respond very quickly to changes in consumption (non persistent) and other in which the response of habits is very slow (persistent).

We find that the presence of habits generates a volume of precautionary savings whose size goes from two to three times the volume of precautionary savings generated by the standard model, depending on the habits persistence. With respect to inequality we find that habits do decrease wealth inequality as measured by the coefficient of variation and the Gini Index by about 10 and 18 percent also depending on habits persistence (for instance, the Gini Index is 0.404 in the benchmark model without habits and it goes down to 0.339 in one of the habits economies). These findings, although quantitatively smaller, also hold in model economies that generate Gini coefficients closer to those in the data.[5] In this case precautionary savings are between 1.5 and 2.7 its size in the non habits economy and the reduction in the inequality measures ranges between 3 8 percent both earnings processes the larger changes correspond to economies with persistent habits. Thus, our assessment is that while the effect of habits in precautionary savings can be very big, the overall effect in wealth inequality is milder.

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is dormant since its early appearance in quantitative theory in Kydland and Prescott (1982). Also we do not look at the feature opposite to habit formation, that is durability of consumption, even though in the context of our model preferences could display durability of consumption by simply setting one parameter to a negative value. The reason is that the definition of wealth that we use already includes a large fraction of the stock of consumer durables.

[5]This is achieved by calibrating the process for labor earnings as proposed by Castañeda, Díaz-Giménez, and Ríos-Rull (2000).

In this paper the vector of state variables includes both habits and assets, two variables directly controlled by the household. This feature complicates the numerical methods involved which leads us to use multidimensional splines to solve the problem of the household.

Section 2 describes the model, while Section 3 describes the calibration procedures. Section 4, describes the findings, while Section 5 explores the robustness of the findings with respect to the process for earnings. Section 6 concludes. Appendix A describes how we solved the problem of the agent and how we computed equilibria, while Appendix B describes how the intertemporal elasticity of substitution is calculated in different economies.

2 The model economy

The economy is a growth economy with production populated by a measure one of households that live forever. We only look at steady states. Section 2.1 describes preferences with habits. Sections 2.2 and 2.3 describe the technology, including the production sector, the shock process that affect households and the market arrangements. In Section 2.4 we write down the households problem while Section 2.5 presents a formal definition of steady state equilibrium.

2.1 Preferences

Households derive utility from current and past consumption. Current consumption is denoted by c. Past consumption affects the level of a stock of habits that we denote with $h \in [0, \infty)$. We write the evolution of habits as $h' = \psi(c, h)$, where we already use the recursive notation that is pervasive throughout the paper with primes denoting next period's values. We write the per period utility as $u(c, h)$, and total utility as $\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$. Notice that, since current consumption affects future per period utilities (by means of h_t), preferences over consumption are not time-separable.

The term habits refers to preferences where an increase in h lowers the utility derived from a given level of consumption: $\partial u(c, h)/\partial h < 0$. When this is positive the term used is durability, and it has very different considerations.

2.2 Technology

Each period households receive a shock to their efficiency units of labor $e \in E = \{e_1, ..., e_{n_e}\}$. This shock is Markov with transition matrix, $\pi_{e, e'}$.

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Aggregate output, Y, is produced according to an aggregate neoclassical production function that takes as inputs capital, K, and efficient units of labor, L, $Y = F(K, L)$. The aggregate labor input comes from aggregating all agents' efficiency units of labor. Aggregate capital results from aggregation of all assets. Capital depreciates at rate $\delta \in [0, 1]$.

2.3 Market arrangements

There are no state contingent markets for the household specific shock, e. Households hold assets $a \in A \equiv [\underline{a}, \infty)$ that pay interest at rate r. We assume that households are restricted by a lower bound on their assets holdings $\underline{a}$. This lower bound may arise endogenously as the quantity that ensures that the household is capable of repaying its debt in all states of the world or we can just set it exogenously as a borrowing constraint.[6] The absence of state-contingent markets and the presence of borrowing constraints are the ingredients needed to depart from the representative agent framework which is silent about distributional issues in the cross-section.[7]

2.4 The household's problem

Since we only look at steady states, the individual household's state variables are its shock, its assets and its stock of habit, $\{e, a, h\}$. The problem that the household solves is

$$v(e, a, h) = \max_{c \geq 0, a' \geq \underline{a}} u(c, h) + \beta \sum_{e'} \pi_{e, e'} v(e', a', h') \tag{1}$$

$$\text{s.t.:} \qquad a' = e\, w + (1 + r)\, a - c \tag{2}$$

$$h' = \psi(c, h) \tag{3}$$

where r and w are the return on assets and the rental rate for efficiency units of labor.

It is well known that under certain conditions problems of this type have a solution that we denote $a' = g^a(e, a, h)$, $c = g^c(e, a, h)$ with an upper bound on asset holdings, $\overline{a}$ and on the stock of habits $\overline{h}$, such that $\overline{a} \geq g^a(e, a, h) \geq \underline{a}$ and $\overline{h} > \psi(g^c(e, a, h), h) > 0$, for all $e \in E$, all $h \in \{h \mid 0 \leq h \leq \overline{h}\}$, and all $a \in \{a \mid \underline{a} \leq a \leq \overline{a}\}$. Sometimes we use the compact notation $s = \{e, a, h\}$ and $S = \{E \times [\underline{a}, \overline{a}] \times [0, \overline{h}]\}$. With respect to assets, the required

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[6]See Huggett (1993) and Aiyagari (1994) for details. Quadrini and Ríos-Rull (1997) contains a review on this topic.

[7]See Chatterjee (1994) or Alvarez and Díaz (2000).

conditions amount to have a low enough rate of return, $\beta < \frac{1}{1+r}$. Again, see Aiyagari (1994), Huggett (1993), or Quadrini and Ríos-Rull (1997) for details. With respect to habits, it suffices to have a bounded ψ.

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and from the decision rules of the agents (see Huggett (1993) or Hopenhayn and Prescott (1992) for details). Let $\mathcal{B}$ be the σ-algebra generated in S by, say, the open intervals. A probability measure x over $\mathcal{B}$ exhaustively describes the economy by stating how many households are of each type. Note that the first moment of x over e yields the aggregate labor input while the first moment over a yields aggregate capital.

Let $Q(s, B)$ denote the probability that a type $\{s\}$ has of becoming of a type in $B \subset \mathcal{B}$. Function Q naturally describes how the economy moves over time by generating a probability measure for tomorrow x' given a probability measure x today. The exact way in which this occurs is

$$x'(B) = \int_S Q(s, B) \, dx \tag{4}$$

If the process for the earnings shock is nice in the sense that it has a unique stationary distribution, then so has the economy.[8] Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.[9]

2.5 Equilibrium

We have almost all the ingredients to define a steady state equilibrium. We only need to add the condition that marginal productivities yield factor prices as functions of x. Note that to obtain a steady state, we look for a measure of households x such that given the prices implied by that measure, households actions reproduce next period the same measure x. Formally, a steady state equilibrium for this economy is a set of functions for the household problem $\{v, g^a, g^c\}$, and a measure of households, x, such that: (i), Factor inputs are obtained aggregating over households: $A = \int_S a \, dx$, and $L = \int_S e \, dx$; (ii), factor prices are factor marginal productivities, $r = F_1(K, L) - \delta$, and $w = F_2(K, L)$; (iii), given x, K, and L, the functions $\{v, g^a, g^c\}$ solve the households' decision problem described in Subsection 2.4; (iv), the goods market clears: $\int_S [g^c(s) + g^a(s)] \, dx = F(K, L) + (1 - \delta)K$, and (v), the measure of households is stationary: $x(B) = \int_S Q(s, B) \, dx$, for all $B \subset \mathcal{B}$.

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[8]For example if it satisfies the American–dream American–nightmare condition stated in Ríos-Rull (1998), then there is a unique stationary distribution of households over earning shocks, assets holdings and stock of habits.

[9]This does not mean that this will happen in equilibrium outside the steady state. The transition Q has been constructed under the assumption that the households think that prices are constant.

3 Calibration

This paper explores the role of habits in quantitatively shaping the wealth distribution. To do this, we compare economies identical in every respect but the specification of preferences: while in a benchmark economy preferences display no habit formation, in the other economies preferences display habits. We start by choosing the benchmark model economy to be essentially a version of Aiyagari (1994) that has become the standard in the literature that measures the size of precautionary savings. We describe the calibration of the benchmark model economy in Subsection 3.1 and that of the model economies with habits in Subsection 3.2.

Once we have a benchmark model economy without habits we have to calibrate the economies with habits. There is not a unique way to do this, since habits have been modeled in at least two ways. On the one side, there is a *survival consumption* branch. Past consumption piles up into a habit stock that determines a minimal consumption for today, below which utility is not defined.[10] On the other side, there is a *multiplicative habit* branch. Past consumption piles up into a habit stock that enters utility dividing today's consumption, capturing the notion that, under habit formation, it is not consumption level but relative consumption what matters.[11] Therefore, the two different approaches differ in two dimensions. First, the *survival consumption* household cares about the absolute difference between consumption and habit stock whereas the *multiplicative habit* consumer cares about the relative difference. And second, for the *survival consumption* household, consuming below the minimal level given by the habit stock is not defined (death) whereas it is well defined for the *multiplicative habit* consumer.

It is difficult to reconcile the *survival habit* approach with individual data. Even in the most conservative earnings process, any household can see its labor earnings halved between two consecutive periods. If one wants to replicate the U.S. data Gini coefficient for earnings, being unlucky may mean dividing earnings by a factor of 9 (see Section 5) in one period or even by 45 in two periods (in an extreme bad luck case). The *survival habit* utility function can hardly accommodate this variation in earnings if households do not accumulate huge precautionary savings. In contrast, variations in earnings in aggregate data are not so sharp, therefore the *survival habit* utility function can be a good representation. Since we calibrate and simulate our model economies to represent individual behaviour, we chose to work with

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[10]Pioneered by Ryder and Heal (1973) and followed for instance by Constantinides (1990), Heaton (1995), Boldrin, Christiano, and Fisher (1997) or Dynan (2000).

[11]Used for instance by Abel (1990), Carroll, Overland, and Weil (2000) or Fuhrer (2000).

the *multiplicative habit* utility function.

We choose to study two types of habits that differ in their persistence. Essentially persistent habits imply that current consumption enters negatively the per period utility function of all future periods, albeit in a decreasing manner. Non–persistent habits are those where the influence of current consumption ends next period since the per period utility function only depends on yesterday's and on today's consumption.

We have to choose not only the type of habits, but we also have to be very specific with respect to what is the habits counterpart to our benchmark model economy. We propose a sequence of economies in increasing order of appropriateness. First, we think of the habits economy as an economy like the benchmark with the addition of the term in habits but *keeping constant* all other parameters.

Habit formation breaks the link between risk aversion and the Intertemporal Elasticity of Substitution, IES. Thus, economies that keep their parameterization identical to that of the benchmark model economy except for the specification of habits differ not only in the degree of risk aversion but also in the IES. For this reason we also compare the benchmark economy with another model economy with habits but adjusted so that it has the same IES. This is achieved by changing one parameter of preferences.

This paper explores the role of habits in shaping the distribution of wealth. Part of our interest is in the size of precautionary savings, but another important part of our concerns is inequality. We want to separate the effects of habits on both characteristics and for this reason we also investigate an economy that has not only the same IES as the benchmark model economy, but also the same amount of precautionary savings, and hence of total savings.

3.1   The benchmark model economy (no habits)

In the benchmark model economy, preferences are of the CRRA form, $\sum_t \beta^t \frac{c_t^{1-\sigma}-1}{1-\sigma}$ and we set a period to be one year. Parameter β is set at .96, which places the equilibrium interest rate around 4%. We set the Intertemporal Elasticity of Substitution, $\frac{1}{\sigma}$ to be equal to 0.5, a value that is around those most preferred by economists. This is our only departure from Aiyagari (1994) in the benchmark model economy[12] since for all other choices we mimic his

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[12]Aiyagari (1994) uses values of 1, 0.33 and, 0.2. Ghez and Becker (1975) and MaCurdy (1981), both using a life cycle model and explicitly accounting for leisure postulate a low value. Mehra and Prescott (1985) and Prescott (1986) discuss other estimates in the literature and conclude that a reasonable number is not too far from 1 (notice that the models they use have quarters as periods). Cooley and Prescott (1995) point out

values. Production occurs through a standard neoclassical production function $F(K_t, L_t)$ $= K_t^\theta \, L_t^{1-\theta}$. Capital share is equal to 0.36 and the depreciation rate of capital δ is set equal to 0.08. Note that these are all standard values.

With respect to the process for earnings, Aiyagari (1994) sets an AR(1) in the logarithm of labor income. The process is fully described by two parameters: its persistence and its volatility. He chooses both values following estimates of Kydland (1984) that used PSID data and of Abowd and Card (1987) and Abowd and Card (1989) that used both PSID and NLS data. Then, he approximates the process by using a seven state Markov chain following the procedures described in Tauchen (1986). We follow the same directions although we reduce the Markov chain to three states.[13] We take our benchmark to be an autocorrelation of 0.6 and a coefficient of variation of 0.2.[14] We later provide results for an economy that has a lot more earnings dispersion, an economy capable of generating wealth dispersion more in accordance with the data.[15] The specific parameter values that we choose are summarized in Table 1.

3.2 Calibration of the Economies with Habits

As we have already stated, we use the specification of preferences used by Carroll, Overland, and Weil (2000) and Fuhrer (2000) based on that of Abel (1990) where the stock of habits h enters multiplying the level of consumption. The per period utility function is

$$u(c, h) = \frac{(c \, h^{-\gamma})^{1-\sigma} - 1}{1 - \sigma} = \frac{\left[c^{1-\gamma} \cdot \left(\frac{c}{h} \right)^\gamma \right]^{1-\sigma} - 1}{1 - \sigma}, \qquad 1 > \gamma > 0. \tag{5}$$

The second way of writing the specification highlights the fact that consumers care about a composite good which is a weighted average between the absolute value of consumption (being rich or being poor) and the relative level of consumption with respect to the past

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that this parameter is among the most difficult to pin down and settle for a value of 1. Hurd (1989) has a point estimate below one.

[13] As we describe below, habits introduce an additional choice state variable which dramatically increases the computational costs of the project. By choosing three states we reduce computer time drastically in a margin that has never proved to be important.

[14] Aiyagari (1994) provides results for autocorrelations of 0.0, 0.3, 0.6 and 0.9 and for coefficient of variations of 0.2 and 0.4.

[15] Aiyagari (1994) fails to account for the amount of wealth inequality in the U.S. The highest value of the coefficient of variation of assets in any of his model economies is 1.13 compared to 6.09 in the data. As Castañeda, Díaz-Giménez, and Ríos-Rull (2000) points out, this is in part due to the process of earnings that he chooses. According to Díaz-Gimenez, Quadrini, and Ríos-Rull (1997) the coefficient of variation for U.S. earnings from the 1992 Survey of Consumer Finances is as big as 4.19 whereas the largest value Aiyagari uses is 0.4.

Table 1: **Parameter values of the benchmark model economy**

General Parameters			
β	σ	θ	δ
0.96	2	0.36	0.08

Earnings Process			
$e \in \{e_1, e_2, e_3\} =$	{.78,	1.00,	1.27}
$\pi_{e,e'} =$	$\begin{bmatrix} 0.66 \\ 0.28 \\ 0.07 \end{bmatrix}$	$\begin{matrix} 0.27 \\ 0.44 \\ 0.27 \end{matrix}$	$\begin{bmatrix} 0.07 \\ 0.28 \\ 0.66 \end{bmatrix}$

Stationary Distribution			
$\pi^\star =$	0.337	0.326	0.337

(being better or worse than usual). For $\gamma = 0$ we are in the no-habits case: only absolute consumption matters. For $\gamma = 1$ we are in the opposite case: only relative consumption matters. Notice, hence, that the same reason that makes households willing to smooth consumption levels is going to make them willing to smooth the ratio of consumption over the habit stock (so they are better off with several small changes than with a single big change).

The evolution of the stock of habits is given by the function

$$h' = \psi(c, h) = (1 - \lambda)\, h\ +\ \lambda\, c, \qquad \lambda \in (0, 1]. \qquad (6)$$

Thus, the level of habit is a weighted average of the stream of past consumption. The parameter $(1 - \lambda)$ measures the persistence of the habit stock. The higher the value of λ the lower the duration of the influence of current consumption in future per period utilities[16]. As λ decreases the effect of c in future utilities increases and the ability of current consumption to modify the habit stock is reduced.[17]

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[16]Notice that for $\lambda = 1$ we are in the particular case that today's habit stock is only yesterday's consumption or, in other words, today's consumption only affects tomorrow's utility.

[17]Notice also that setting $\lambda = 0$ is not equal to the limit case of $\lambda \to 0$. One can rewrite the law of motion for habits as $\widetilde{h}' = (1 - \lambda)\widetilde{h} + c$ where $\widetilde{h} \equiv \frac{h}{\lambda}$ because λ is just a constant that does not affect the maximization. Under this representation we see that the role of λ is solely to control the persistence of the process and does not affect the strength of consumption in the habit stock. However, we cannot do this normalization when $\lambda = 0$ because it would imply dividing the utility function by zero, which does affect the maximization. Therefore, in this particular case λ is doing two things, namely, setting the persistence of habits equal to one and saying that consumption does not affect the habit stock.

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Calibrating the basic habits model economy requires choosing values for the parameters λ and γ. Notice that the benchmark model economy has a representation under this parameterization: the value of γ is zero (which makes irrelevant the value of λ).

There are several studies that try to estimate the parameters of habit formation in consumption. Some use individual level short panels, whereas some others use aggregate time series. Some try to find out which parameterizations are consistent with certain asset pricing regularities whereas some others try to estimate consumption demand functions or first order conditions.[18] The heterogeneity of data sets and techniques rises to a very wide range of possible values for our γ and λ. Ideally, we would be looking for estimations consistent with our model in functional forms and length of period. Unfortunately, this is hard to find.

The closest estimation to our model is the one by Fuhrer (2000) who uses quarterly data on aggregate consumption data in non-durable goods and services (from NIPA) to estimate a log-linearized consumption function where habits enter multiplicatively in the utility function as in our model. He estimates $\gamma = 0.8$ and $\lambda = 0.9985$. An estimation of $\sigma = 6.11$ is consistent with the $IES = 0.5$ we use throughout the paper.[19]

Constantinides (1990) and Boldrin, Christiano, and Fisher (1997) try to find which pairs of parameters are consistent with the risk premium and the risk premium and risk free asset rate respectively. However, they use the *survival consumption* formulation to introduce habits in the utility function. Boldrin, Christiano, and Fisher (1997) find the best fit with the equivalent to our notation of $\gamma = 0.58$ and $\lambda = 0.70$.[20] Constantindes finds several pairs that fit the risk premium, with weight of habit γ ranging from 0.09 to 0.49 and corresponding persistence parameter λ ranging from 0.10 to 0.37.[21] Although these papers solve the equity premium puzzle as stated by Mehra and Prescott (1985) they still have some counterfactual implications as the excessive unconditional variance of the risk free rate. Heaton (1995) performs a similar experiment with monthly aggregate time series in non-durable consumption and services by NIPA getting $\gamma = 0.71$ and $\lambda = 0.58$. However, he also sets a more

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[18]There are some others that just test for the presence of habit formation without estimating any closed form. A good example of these is Meghir and Weber (1996).

[19]Fuhrer (2000) also allows for a fraction of agents not to behave rationally but just to eat all their current income. He estimates this fraction to be 25% of the total population.

[20]This is an abuse of notation because their formulation is different from ours. We can somehow 'translate' parameters from one to the other seeing the *survival consumption* representation as $u(c_t, h_t) = \frac{(c_t - h_t)^{1-\sigma}}{1-\sigma}$ and $h_{t+1} = (1-\lambda)h_t + \gamma c_t$.

[21]Constantinides (1990) represents $u(c_t, h_t) = \frac{(c_t - h_t)^{1-\sigma}}{1-\sigma}$. The discrete time version of the law of motion for the stock of habits he uses is $h_t = \left(\frac{1}{1+a}\right)^t h_0 + b \sum_{i=1}^{t} \left(\frac{1}{1+a}\right)^{i-1} c_{t-i}$. Thus, we translate $\lambda = 1 - \frac{1}{1+a}$ and $\gamma = b$.

ambitious framework allowing for habit formation and consumption durability to interact (and not to offset each other as in Ferson and Constantinides (1991)) and targeting not only first moments of asset returns but also second moments. With this larger moments set he finds $\gamma = 0.00$ and $\lambda = 1.0$ if a pure habit model is used (which is evidence against habits) but $\gamma = 0.67$ and $\lambda = 0.18$ if interaction with durability is allowed for. In this last case we see a very high persistence (in monthly data) not found in other studies.

Dynan (2000) also uses the survival consumption formulation with individual level data on food consumption from PSID. Because of data restrictions[22] she imposes $\lambda = 1.0$ (i.e., only yesterday consumption matters) to find that γ cannot be said to be different from zero.[23]

We find Fuhrer estimation as the closest one to our formulation. Since he uses quarterly data and we are calibrating for a model period of one-year length we see his $\gamma = 0.8$ as an upper bound and end up choosing $\gamma = 0.75$. As for the persistence parameter, we work with a pair of values at each side of the possible range: $\lambda = 1.0$ and $\lambda = 0.25$. The former is consistent with Fuhrer estimation whereas the latter, acknowledging the diversity of empirical results, will show us what happens at the other side of the persistence range.[24,25] Our choices are described in Table 2.

Table 2: **Basic Parameters of the Habit Economies**

	γ	λ
Low Persistence Economy	0.75	1.00
High Persistence Economy	0.75	0.25
Benchmark Model Economy	0.00	–

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[22]She has the same problem as Meghir and Weber (1996). Time dimension in individual level data is very short and does not allow for estimations of persistent habits.

[23]She uses the survival consumption formulation with individual data. This also helps finding γ not different from zero since it is very difficult to accomodate the large individual variablility of consumption with the endogenous survival consumption level unless γ is very small

[24]We have obviously tried different values of γ. In none of our experiments γ has changed the qualitative results. γ behaves just as an amplifier of the habit phenomenon.

[25]We find interesting to explore a low lambda in spite of Fuhrer's findings because of the following. Meghir and Weber (1996) and Dynan (2000) reject the hypothesis of habit formation using individual level data. However, the lack of long time series forces them to equalize habit stock to yesterday's consumption. If habits are very persistent, as Heaton (1995) suggests, it might well be the case that their rejection of the habit hypothesis is driven by the fact that, under very persistent habits, yesterday's consumption ability to modify the habit stock is small.

3.3 The value of the Intertemporal Elasticity of Substitution

With CRRA preferences and no habit formation, preferences over different periods in time and over different contingencies are the same. Adding time non-separable preferences breaks this symmetry. As shown by Constantinides (1990) and Boldrin, Christiano, and Fisher (1997), habit formation breaks the link between the Intertemporal Elasticity of Substitution and the level of risk aversion. In order to disentangle the effect of habit formation in each dimension we look at model economies with habit formation exhibiting the same IES as our benchmark so that the results show the effect of habits on preferences over uncertain levels of consumption, not over intertemporally distributed consumption. In Appendix B we show that, for a version of the representative agent model without uncertainty, $IES = \frac{1}{\gamma + (1-\gamma)\sigma}$.[26] This tells us two things. First, if individuals are not financially constrained, preferences towards consumption in different moments of time do not depend on the persistence of the habit stock, $1 - \lambda$, but only on the magnitude of habits in the utility function, γ,. Second, with habit formation preferences towards consumption in different moments of time exhibit less curvature than without habit formation[27] (in other words, households desire less consumption smoothing). The reason for this being, as posed by Carroll, Overland, and Weil (2000), that "the gain or loss in utility associated with a given increase or decrease in consumption over a long horizon will be diminished by the associated movement in the habit stock". Therefore, when we want to keep IES as in the benchmark economy given γ, we will adjust σ.

4 Results

In this section we report the findings from the various model economies. The results have been computed by solving the household's problem with a two–dimensional spline tensor product (that we ensure that generates a concave function). We construct a sample of 5000 households.[28] Then, using the decision rule, the law of motion for the exogenous state and a random number generator, we simulate the decisions of these households to find a new distribution of households. We iterate until the main statistics of the samples converge. Then, we compare the statistics generated for all the economies studied by using in all of

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[26] Carroll, Overland, and Weil (2000) already show this result in a continuous time Ak growth economy.

[27] For $\sigma > 1$ and $0 < \gamma < 1$

[28] We have also tried larger samples. We see that sample sizes beyond our choice do not change aggregate results. However, for histograms and for reporting the shares of certain groups we use a larger sample size of 50000.

them the same realizations of the random numbers.[29]

In Subsection 4.1 we review the Aiyagari (1994)'s model economy, which we will refer to as the benchmark economy hereafter. We compare his findings to those of the representative agent deterministic version of his economy. Notice that the main statistics of the representative agent deterministic version of our model with habits are the same as those of the model without habits. Subsection 4.2 describes the properties of economies with habits, without any further adjustments. Sections 4.3 and 4.4 show the results for economies with habits where we recalibrate our model economy so that, respectively, IES and total savings match those of the economy without habits.

4.1 The Benchmark Model Economy

The main characteristics of the benchmark model economy, that we refer to sometimes as economy B, are described in Table 3.

Table 3: **Main statistics of the Benchmark Economy and its Deterministic Counterpart**

	D Deterministic Economy	B Benchmark Economy	Change $\frac{B-D}{D} * 100$
Aggregate Assets	2.959	3.015	1.9%
Output	1.000	1.007	0.7%
Capital Output ratio	2.959	2.994	1.2%
Interest Rate	4.17%	4.02%	-3.5%
Coeff. of Variation of Wealth	0.0	0.748	–
Gini Index of Wealth	0.0	0.404	–

The first column shows the values of the key statistics of the deterministic representative agent counterpart of the benchmark economy.[30] We denote this economy as D. As we see the interest rate is 4.17% and the capital output ratio is 2.959. For comparison purposes, we have normalized output to one in this deterministic economy. The second column includes the statistics of the benchmark model economy. We also report two measures of wealth dispersion, the coefficient of variation and the Gini Index. The last column reports the proportional variation in the main statistics of the benchmark model economy with respect to the deterministic economy. Note that both economies have different interest rates. This

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[29]For further details, see Ríos-Rull (1998).

[30]The deterministic model has been calibrated with the same parameters as those used in the benchmark economy and setting the labor endowment equal to the unconditional mean of the earning process.

means that agents are responding to different prices.[31] We want to highlight two main things from this table. First, precautionary savings, that we define as the excess in total wealth that a given economy has over its deterministic counterpart, are small, less than 2%, confirming Aiyagari's findings. Second, under this parameterization, assets holdings are very evenly distributed. The Gini coefficient, for example, is 0.40 while is 0.78 in the U.S. data.

4.2   The unadjusted habits' economies

As stated, we compare two habits economies with the benchmark model economy. Except for the existence of habits, the two habits economies have the same parameterization as the benchmark model economy. They differ from each other in the persistence of the habits. The second column of Table 4 reports the main statistics of what we refer as the non–persistent habit economy, or economy N, while the fourth column refers to the persistent habit economy, or economy P. The third and fifth columns have the rates of change between the habits economies and the benchmark model economies.[32]

Table 4: **Main statistics of the Benchmark and the Unadjusted Habits Economies**

	B Benchmark Economy	N Non–Pers. Habits $\lambda = 1$	Change $\frac{N-B}{B}$	P Pers. Habits $\lambda = 0.25$	Change $\frac{P-B}{B}$
Aggregate Assets	3.015	3.012	-0.1%	3.035	0.7%
Output	1.007	1.006	-0.0%	1.009	0.2%
Capital Output ratio	2.994	2.993	-0.0%	3.007	0.4%
Interest Rate	4.02%	4.03%	0.2%	3.97%	-1.3%
Precautionary Savings	1.9%	1.8%	-4.6%	2.6%	34.4%
Coeff. of Var. Wealth	0.748	0.721	-2.7%	0.669	-10.6%
Gini Index Wealth	0.404	0.393	-2.7%	0.367	-9.2%

Regarding total assets, we see that both economies are quite similar to the non-habits case. Aggregate savings are nearly unchanged, moving in opposite directions. Whereas in

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[31]In fact if in the benchmark model economy the interest rate were set exogenously at the level of the deterministic economy, total assets will be unbounded. See Huggett (1993), Aiyagari (1994), Ríos-Rull (1998).

[32]As in the previous Subsection, we are looking at the general equilibrium version of the economies where the interest rates adjust to ensure that aggregate asset holdings equate aggregate capital. This means that interest rates are different than in economy B. If we keep fixed the interest rate (without letting it clear markets) we see what happens in absence of this price effect. Economy N has total assets of 2.87 and economy P of 4.11, which clearly shows larger changes than their general equilibrium counterparts.

the non persistent case they fall by just a 0.09%, they increase by 0.67% in the persistent case. Movements in output and capital-output ratio follow. Even in the persistent economy, precautionary savings stay quite low, at 2.56%. However, wealth dispersion changes more. With non persistent habits, our measures of dispersion fall by about 3 percent but they fall about 10 percent with persistent habits.
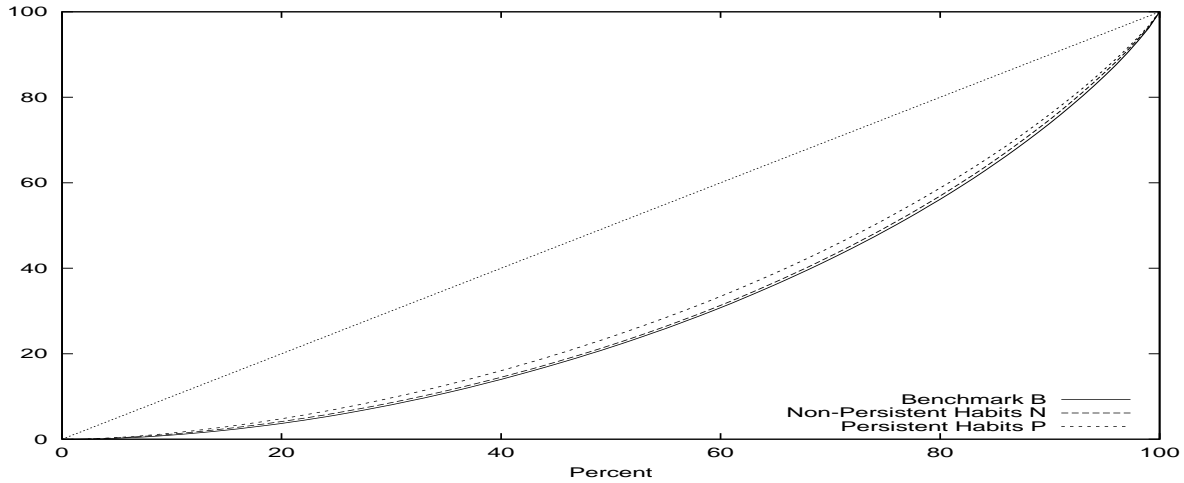

Figure 1: **Lorenz curve for assets: Unadjusted Economies, $\sigma = 2$, $\beta = 0.96$.**
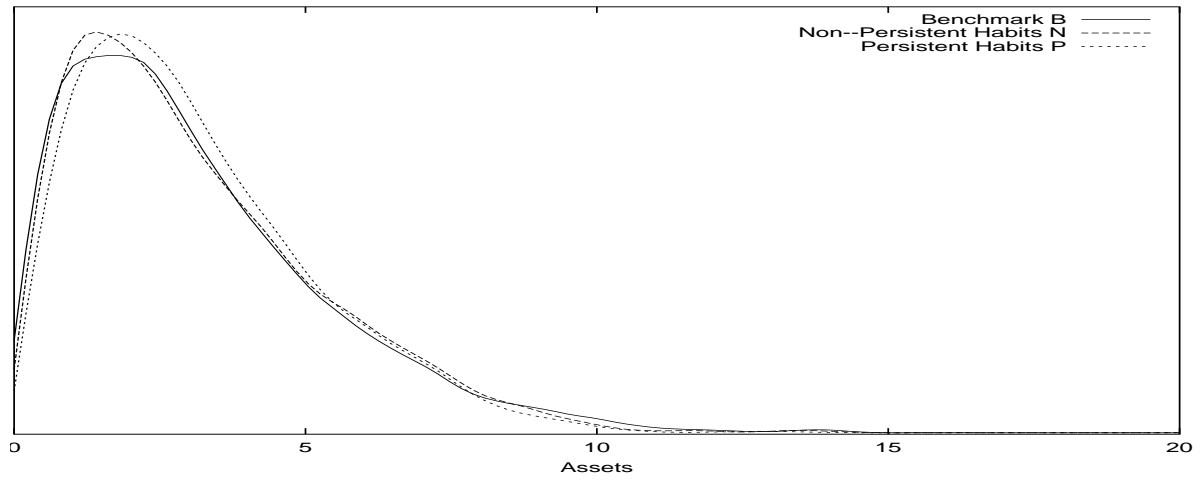

Figure 2: **Histogram for assets: Unadjusted Economies $\sigma = 2$, $\beta = 0.96$. General Equilibrium**

Figure 1 shows the Lorenz curves for assets of the three model economies. We see how similar are the benchmark and the non–persistent unadjusted habit model economies while in the persistent unadjusted model economy the distribution is a little bit more even. We

report the histograms of asset holdings in Figure 2. The histogram shows quite similar pictures for all economies.

Overall, we see that for economies that differ only in the specification of habits from the benchmark model economy, the implied differences for precautionary savings are not very big. They fall slightly for the non persistent economy and they increase for the persistent economy. This is a pattern we find throughout all experiments: precautionary savings are always larger for economies where habits are persistent than for economies where habits are non persistent. Inequality indicators fall in both economies, more in the persistent case.

However, the differences we have seen between the benchmark economy and the habits economies cannot be solely attributed to the effect of habits. In particular, the benchmark model economy has an intertemporal IES = 0.5 while that of the unadjusted economies with habits have a value of IES = 0.8. This means that households in the unadjusted habits economies have a smaller desire to smooth consumption intertemporally. In the next subsection we report the properties of economies with habits, both persistent and non persistent where the parameter σ has been adjusted to generate a $IES = 0.5$, the value of the benchmark model economy.

4.3 Adjusting habits to match the Intertemporal Elasticity of Substitution

Habits break the link between the individual's willingness to choose a contingent consumption plan, measured by the Coefficient of Relative Risk Aversion and the individual's willingness to intertemporally substitute consumption, measured by the Intertemporal Elasticity of Substitution. Each measure is the inverse of the other in the representative agent version of the benchmark economy, the economy without habits. This is no longer true in an economy with habits. Thus, to investigate the effect of habits on the level of precautionary savings we need to isolate the effect of habits on the level of risk aversion from the effect on the IES. In this subsection we recalibrate our habits economies so that the IES of the habits economies is the same as in the benchmark economy.[33]

Recall that in economies with habits $IES = \frac{1}{\gamma + (1-\gamma)\,\sigma}$, where γ is calibrated at 0.75 and IES targeted to 0.5. This implies that σ has to be increased to 5. Notice that this means that households in the unadjusted habits economies of the previous section have substantially less desire for smoothing consumption over time than they will in the economy where we adjust σ. We have then two new model economies, an economy with non-persistent habits adjusted

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[33]We are referring to the IES of the deterministic representative agent version of each model throughout the paper.

so that its $IES = 0.5$, that we denote economy M, and an economy with persistent habits also adjusted to have its $IES = 0.5$, that we denote economy Q.

Table 5: **Main statistics of the Benchmark and the Adjusted Habits Economies IES=0.5**

	B Benchmark Economy	M Non–Pers. Habits $\lambda = 1$	Change $\frac{M-B}{B}$	Q Pers. Habits $\lambda = 0.25$	Change $\frac{Q-B}{B}$
Aggregate Assets	3.015	3.066	1.7%	3.126	3.7%
Output	1.007	1.013	0.6%	1.020	1.3%
Capital Output ratio	2.994	3.027	1.1%	3.065	2.4%
Interest Rate	4.02%	3.89%	-3.2%	3.75%	-6.9%
Precautionary Savings	1.9%	3.6%	92.8%	5.7%	200.9%
Coeff. of Var. Wealth	0.748	0.676	-9.6%	0.611	-18.3%
Gini Index Wealth	0.404	0.371	-8.2%	0.339	-16.1%

The results are reported in Table 5. Notice that precautionary savings increase substantially in both economies with respect to the deterministic case. Now, they are 3.6% of total wealth in the deterministic case in the non-persistent habits economies and more than 5.6% in the persistent habits economies. So precautionary savings are between two and three times larger than in the benchmark model economy depending on the persistence. So habits indeed increase precautionary savings over the benchmark model economy, although perhaps their effect on aggregate capital is small.

With respect to wealth dispersion, we see an overall reduction of the inequality indicators, which is more evident in the economy with persistent habits where the reduction in inequality is more dramatic. Figure 3 shows the Lorenz curves for assets of the benchmark model economies and of the habits economies with $IES = 0.5$, while Figure 4 reports the histograms of asset holdings. Here we start seeing a much clearer picture than in the economies with a larger Intertemporal Elasticity of Substitution. Inequality clearly goes down, especially for persistent habits. The Lorenz curve of the habits are much closer to the diagonal than those in the benchmark model economy and the histograms seem to be much tighter. It is also more evident that a big part of the distributional differences are due to poor people: households in economies with habits, not wanting to face fluctuations on consumption, make sure they do not hold too low asset levels. In any case, the reduction of inequality as measured by the statistics that we have chosen is always less than 20 percent.
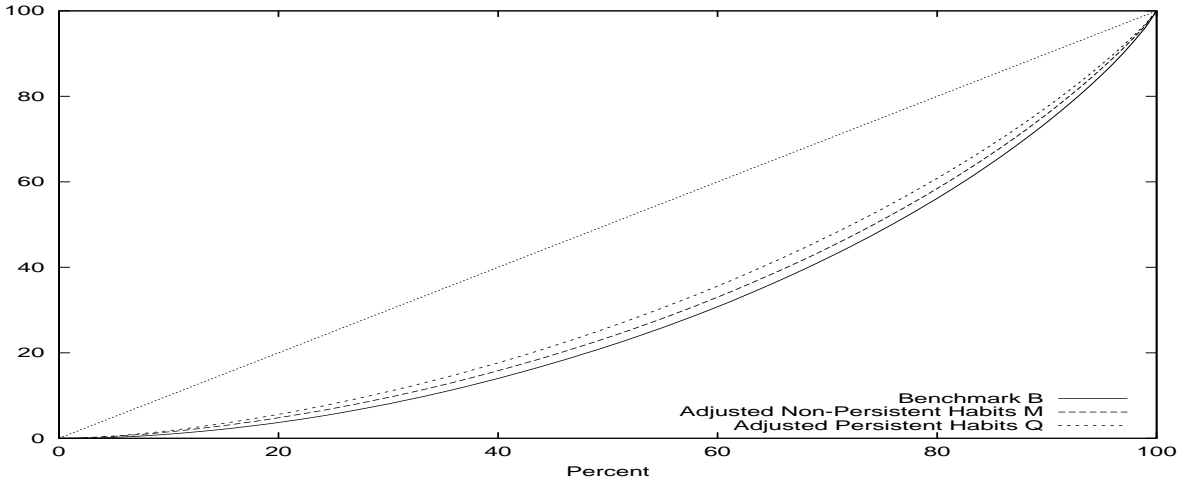
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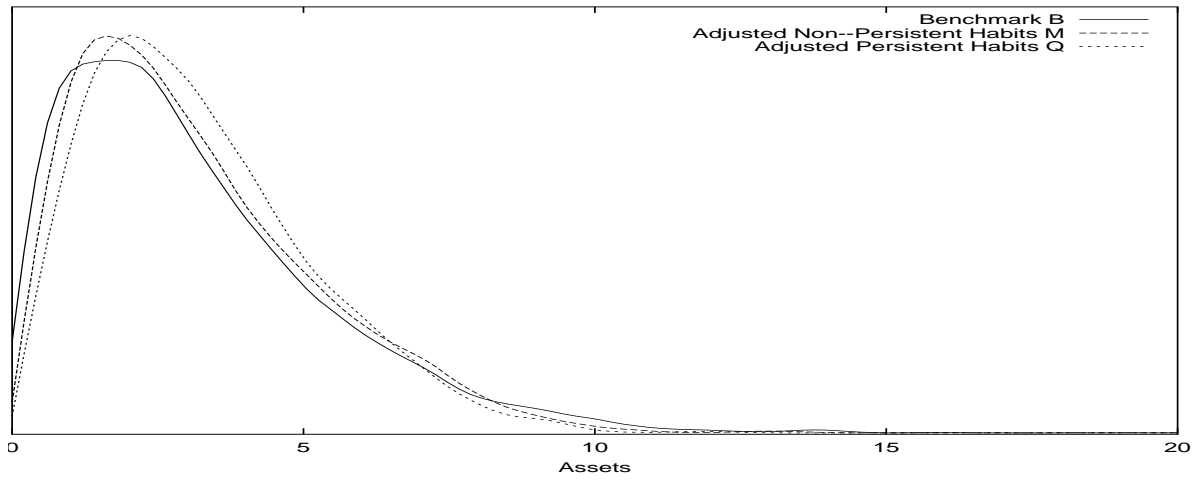Figure 3: **Lorenz curve for assets: Economies Adjusted to the IES**

Figure 4: **Histogram for assets: Economies Adjusted to the IES**

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4.4 Economies with the same savings as the benchmark model economy

To finish we want to isolate the effects of habits on inequality from those on precautionary savings. To this end we perform a second adjustment on the habit economies. We adjust the parameter β so that aggregate capital and hence the equilibrium interest rate is equal to that of the benchmark model economy.

Table 6: **Main statistics of the Benchmark and the Habits Economies Adjusted to have $IES = 0.5$ and identical precautionary savings.**

	B Benchmark Economy	M' Non–Pers. Habits $\lambda = 1$	Change $\frac{M-B}{B}$	Q' Pers. Habits $\lambda = 0.25$	Change $\frac{Q-B}{B}$
β	0.960	0.959	-0.1%	0.957	-0.3%
Coeff. of Var. Wealth	0.748	0.678	-9.4%	0.616	-17.6%
Gini Index Wealth	0.404	0.372	-7.9%	0.341	-15.6%

We label M' and Q' the non-persistent and persistent economies respectively. Results are in Table 6. As we can see the statistics for inequality are essentially identical to those for the economies of the previous subsection with the same β as the benchmark model economy. Figure 5 shows the Lorenz curves for assets of the benchmark model economy and of the habits economies adjusted to have the same IES and the same precautionary savings as the benchmark economy, while Figure 6 reports their histograms of asset holdings. Both the Lorenz curve and the histogram resemble the ones already seen in the previous section, but with the value added for the histogram that the means of the distribution for the three economies are set to be equal. This allows us to see where the differences in wealth dispersion lie. We basically see that, as already stated, the habit economies have much fewer people in low levels of assets and more people about the mean, with hardly no differences in the high values. Again, this effect is stronger for persistent habits.

4.5 Final comments

We have seen two things: first, in habits economies precautionary savings are substantially higher and the level of wealth inequality is a little lower than in their non habits counterparts and, second, these effects are stronger in economies with persistent habits. We comment each result in detail.
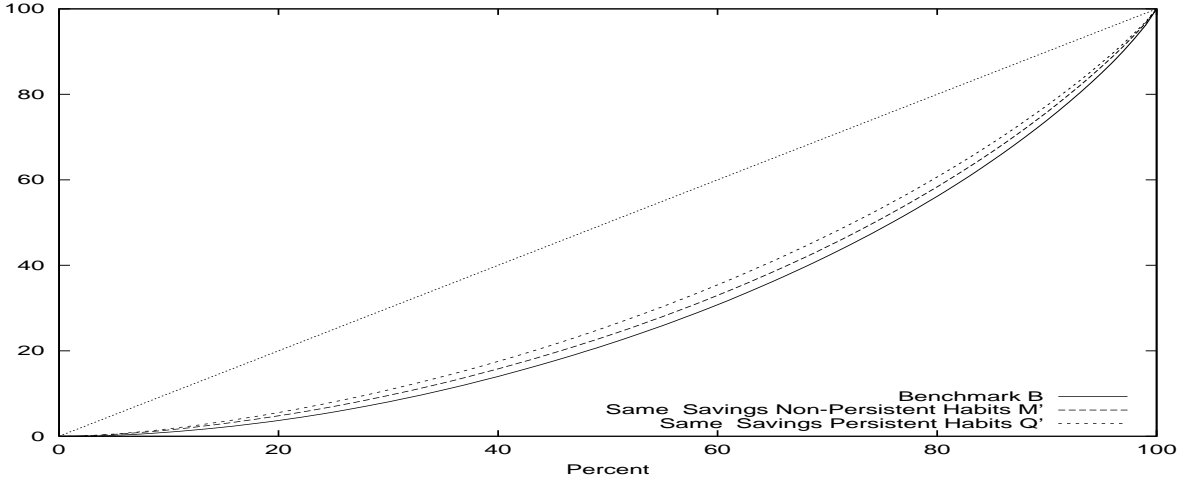
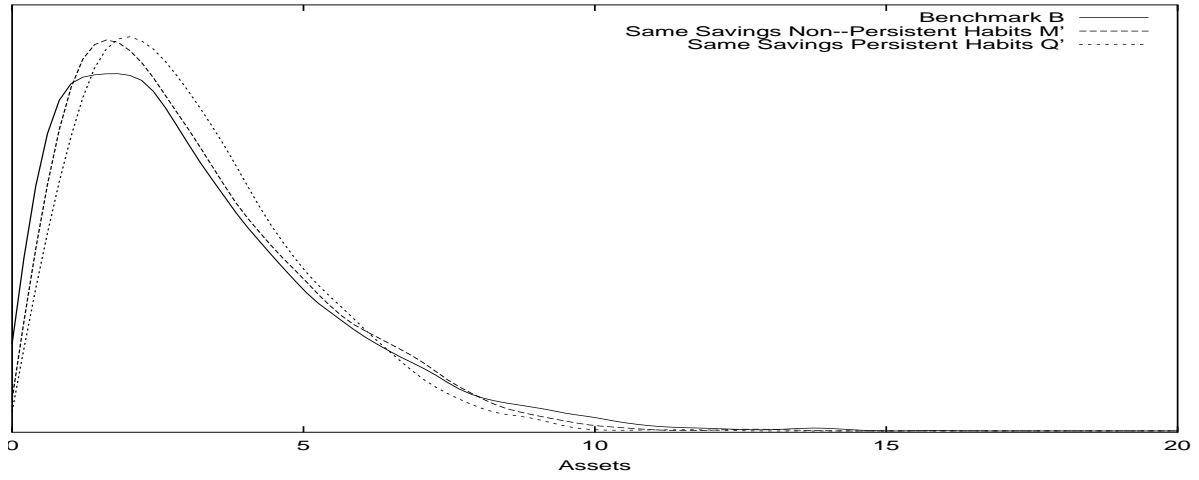Figure 5: **Lorenz curve for assets: Economies adjusted to savings, and IES**

Figure 6: **Histogram for assets: Economies Adjusted to savings and IES**

Households in habits economies are more displeased with fluctuations in consumption than their counterparts in economies without habits. Households with habits want not only a stable pattern of consumption but also a stable pattern of variations in consumption. Consequently, households in habits economies hold more assets. We see this in both the size of precautionary savings and in the shape of the lower tail of the distribution. Regarding the size of precautionary savings, we have seen that once adjusting σ to keep IES unchanged, total precautionary savings doubles or triples depending on persistence, admittedly from quite a low value (1.9%). Regarding the shape of the lower tail, we observe that it is asset-poor households who increase more their asset holdings.[34] The reason for this is twofold. On the one hand, the share of uncertain labor earnings in their total income is higher for asset-poor households than for asset-rich households. On the other hand, asset-poor households are not well self-insured, which means that income fluctuations get easily translated into consumption fluctuations. On the contrary, asset-rich households have a stock of assets large enough to buffer fluctuations. So it is for the former households that an increase on the loss of utility due to consumption fluctuations is more likely to change savings behavior. This asymmetric impact of habits on asset-poor and asset-rich households accounts for the reduction in inequality.

Both effects on precautionary savings and on inequality are stronger for persistent habits. A possible explanation for this is as follows. When habits are persistent, a fall in consumption today has a small impact in lowering the habit stock for tomorrow. In contrast, the opposite is true when habits are non persistent: a fall in consumption today is easily translated in a fall in the habit stock for tomorrow. Since not only the consumption level but also consumption relative to the habit stock matters, the fall in consumption is worse in utility terms if the habit stock stays stuck at its previous level than if it falls together with consumption. In a sense, non-persistent habits act as a safety net: being poor is not so bad because one gets easily used to it.

As we have stated, the overall effect on aggregate capital is not big. This is because the largest changes in savings behaviour are done by asset-poor people, whose share of total assets is very small.

We turn next to explore whether these findings are specific to our parameterization or also hold for a larger set of model economies.

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[34]Quantitatively, the bottom 5% of the assets distribution have an average stock of assets of 0.13 (0.2% share) in the benchmark economy whereas they have 0.27 (0.4%) and 0.31 (0.5%) in the economies labeled M' and Q'. On the other side, for the top 5% the average assets are 9.00 (15.0%) for the benchmark and 8.11 (13.5%) and 7.56 (12.6%) for the economies M' and Q'.

5 Economies with High Earnings Variability

One of the problems that Aiyagari's economy has in trying to match the U.S. wealth distribution is that the earnings distribution itself, an exogenous element, is already lacking dispersion. In his benchmark economy, he sets the coefficient of variation for the earnings distribution to be equal to 0.2, which in our experiments gives a Gini index of 0.11. The values of the Coefficient of variation and the Gini index for the U.S. economy are respectively 4.19 and 0.63.[35] One interesting robustness analysis is, hence, to see how the conclusions change if we allow for an income process generating much more earnings inequality to a level similar to the U.S. data. Castañeda, Díaz-Giménez, and Ríos-Rull (2000) calibrate the earnings process (among other features of their model economy) so that a suitably modified version of Aiyagari's model accounts for the Lorenz curve of wealth observed in the U.S. We construct a 3 point Markov process that has some of the properties of the 4 point Markov process of Castañeda, Díaz-Giménez, and Ríos-Rull (2000)[36] that we report in Table 7. To get a high Gini coefficient with just three points in the Markov chain, one needs to make each state very different, in the process that we construct the endowment of the lucky households is almost 50 times the endowment of the unlucky ones. This process for earnings has a Gini index of 0.60.[37]

Table 7: **Earnings Process of the High Earnings Variability Economies**

$e \in \{e_1, e_2, e_3\} =$	$\{1.00,$	$5.29,$	$46.55\}$
$\pi_{e,e'} =$	$\begin{bmatrix} 0.992 \\ 0.009 \\ 0.000 \end{bmatrix}$	$\begin{matrix} 0.008 \\ 0.980 \\ 0.083 \end{matrix}$	$\begin{bmatrix} 0.000 \\ 0.011 \\ 0.917 \end{bmatrix}$
$\pi^\star =$	0.481	0.456	0.063

We run the same experiments that we run for the low earnings volatility process with this new earnings process, following the same calibration procedures. The only difference

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[35]See Díaz-Gimenez, Quadrini, and Ríos-Rull (1997).

[36]There are many ways of implementing this reduction. Our choice should be seen as merely illustrative for the study of the properties that habit formation has on Economies with high Earnings variability.

[37]The process estimated by Castañeda, Díaz-Giménez, and Ríos-Rull (2000) includes retirees and it was designed for a model where households choose work effort. The process here is just intended to be in the ball park of that one. For instance, notice that even though the Gini Index is close to that in the data, its coefficient of variation is smaller than one half that in the data.

lies in the parameter β because we want the benchmark economy to have the same aggregate capital and interest rate as the benchmark economy with Aiyagari's earnings process. To do so, β must be lowered from 0.96 to 0.887. Higher variability calls for more precautionary savings.

Table 8: **Main statistics of the High Earnings Variability Economies**

	B^*	M^*		Q^*	
	No Habits	Non–Pers. Habits $\lambda = 1$	Change $\frac{N-B}{B}$	Pers. Habits $\lambda = 0.25$	Change $\frac{P-B}{B}$
Aggregate Assets	3.015	3.848	27.7%	5.948	97.3%
Output	1.007	1.099	9.2%	1.285	27.7%
Capital Output ratio	2.994	3.501	16.9%	4.626	54.5%
Interest Rate	4.02%	2.28%	-43.2%	-0.22%	-105.4%
Precautionary Savings	134.3%	199.1%	48.2%	362.3%	169.8%
Coeff. of Var. Wealth	2.491	2.405	-3.4%	2.274	-8.7%
Gini Index Wealth	0.857	0.831	-3.0%	0.805	-6.1%

Habits Economies Adjusted to have $IES = 0.5$ and identical precautionary savings.

	B^*	$M^{*'}$		$Q^{*'}$	
Precautionary Savings	134.3%	201.6%	50.2%	455.3%	239.1%
β	0.887	0.859	-3.1%	0.781	-11.9%
Coeff. of Var. Wealth	2.491	2.459	-1.3%	2.486	-0.2%
Gini Index Wealth	0.857	0.838	-2.2%	0.830	-3.2%

The header row above the first data panel reads "Habits Economies Adjusted to have $IES = 0.5$".

What we find is that qualitative results remain unchanged. In the first panel of Table 8 we can see the no habits economy B^* against what we called economies M^* and Q^*: those with non persistent and persistent habits respectively with σ adjusted to have the same IES as in the no-habits economy. In the no-habits economy precautionary savings are 134.3%, a huge number compared to the 1.9% with Aiyagari's earnings process. This is the consequence of having such a big variability in earnings. Remember that in Aiyagari's earnings process the endowment in good time is less than 50% higher than the endowment in bad times, whereas in the high earnings variability process, the endowment in good times is about 50 times

larger than in bad times. As before, the habits economies exhibit higher precautionary savings, being the increase larger for the economy Q^* with persistent habits. In the non persistent habits economy, precautionary savings are 50% larger while in the persistent habits economy, they are more than 150% larger. This time the increase is over an already very large number,[38] making the role of habits very important in shaping this variable.
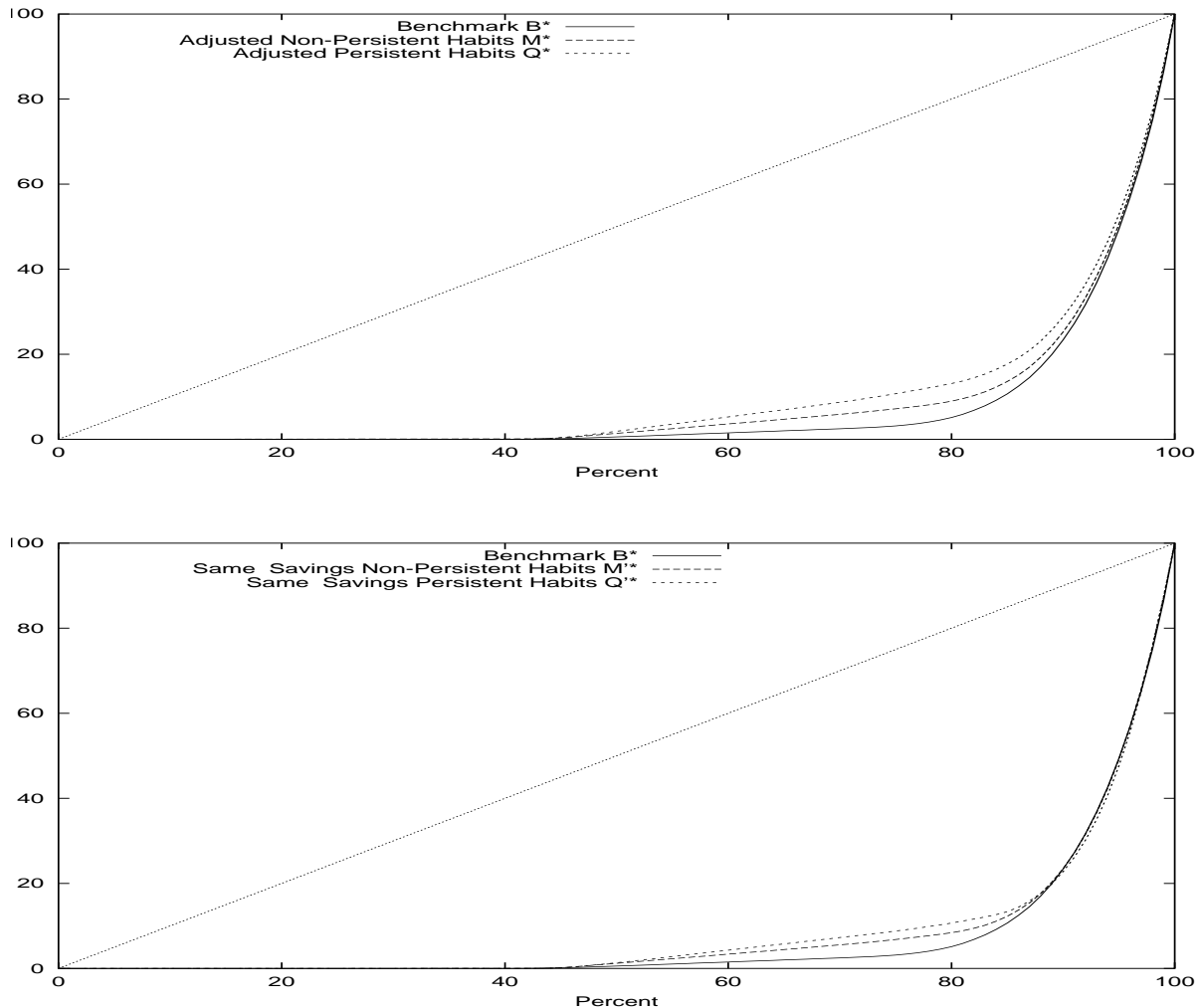

Figure 7: **Lorenz curve for assets: High Earnings Variability Economies.**

The model economies display a much higher coefficient of variation and Gini indices than the economies with Aiyagari's earnings process. Moreover, the values of the Gini index are even larger than the 0.78 of U.S. data.[39] This very high concentration of wealth can be seen both by means of the Lorenz curves plotted in Figure 7, that are much closer to the bottom

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[38]Recall that these economies are parameterized so that the no-habits economy, B^*, has the same wealth as the benchmark model economy, B, and for this the discount rates have been reduced quite dramatically.

[39]As shown in Castañeda, Díaz-Giménez, and Ríos-Rull (2000) this type of process accounts for wealth

right corner than the earlier ones, and by means of the shares of wealth held by selected groups of households reported in Table 9 where we can see that the share of wealth of the bottom 40% is zero and that of the top 10% is about 77%.

But what we really care about is the contribution of habits to shape inequality. We see that all our measures of inequality fall somewhat in the habits economies versus the no-habits economy, again more sharply for the economy with persistent habits. The main change in all economies occurs by having an increase in the share of the third and fourth quintiles at the expense of the fifth, especially of the households in the 80-95 percentiles. The fall of inequality in economies with habits adjusted to have the same precautionary savings is almost zero. However, the distribution of wealth has changed as it can be seen in Table 9, even if the total contribution of these changes to the coefficient of variation and the Gini Index is minimal.

Table 9: **The Distributions of Wealth in the High Earnings Variability Economies**

Economy	*Quintiles*					*Top Groups (%)*		
	1st	2nd	3rd	4th	5th	Top 10%	Top 5%	Top 1%
B^* (No Habits)	0.00	0.00	1.51	3.60	94.89	76.62	50.62	13.65
M^* (IES–Adj Non-Pers)	0.04	0.04	3.52	5.35	91.04	74.75	49.47	12.91
Q^* (IES–Adj. Pers)	0.01	0.02	5.23	7.85	86.89	71.21	47.21	12.05
M'^* (Same Sav Non-Per)	0.01	0.01	3.35	5.04	91.60	76.54	50.89	12.89
Q'^* (Same Sav Per)	0.00	0.00	4.28	6.39	89.32	77.23	52.43	12.59

To sum up, we just want to point out that the conclusions under this more volatile earnings process strengthen those obtained with the less volatile Aiyagari's process. We find that precautionary savings (and aggregate capital) increase dramatically whereas our measures of inequality fall slightly, these effects being stronger for persistent habits. Also as before, we see that it is asset-poor households who proportionally raise more their asset holdings.

inequality in an economy with a lot more detail built in. The actual number of the version that we use in this paper is not so important. Here, we are not after accounting for wealth inequality, but we are trying to measure the role of habit formation in changing our answers about wealth inequality and precautionary savings.

6   Conclusion

In this paper we have explored the role of habits in shaping the distribution of wealth. Our findings indicate that once we properly calibrate the economy to match the Intertemporal Elasticity of Substitution, the introduction of habits increases precautionary savings up to three times the (low) level of the benchmark model economy calibrated as in Aiyagari (1994). Its role in shaping inequality is that it reduces it: the inequality statistics go down in some cases 18%. These differences with the benchmark economy are more pronounced when habits are persistent than when they are not. When we calibrate our economies using an earnings process that better matches the observed earnings inequality in the U.S. we find the same qualitative results. Precautionary savings increase substantially from an already very large value and inequality indicators decrease a bit, with changes being bigger for persistent habits economies.

Habits affect the way households dislike consumption fluctuations. In this class of incomplete markets economies, where households are subject to idiosyncratic shocks, income fluctuations are only partially insurable through the accumulation of assets. Households with a long stream of good shocks hold big amounts of assets and reach a satisfactory degree of insurance. Households with a long stream of bad shocks are left with few assets and therefore have to bear consumption fluctuations. Not surprisingly, the comparison between an economy with habit formation and an economy without habit formation shows that it is asset-poor people behavior that differs the most. Since the presence of habits makes consumption fluctuations more painful, those households with a small level of self-insurance will try to increase it by holding higher asset stocks. This makes the wealth distribution more even by reducing the number of people holding very low levels of assets.

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A   Computational Procedures

To solve the consumer's problem described in section 2.4 we follow a successive approxima-
tions approach in the value function. Our individual state space contains two endogenous
individual variables (assets and habits) as well as the exogenous idiosyncratic shock. This
implies the need to create a two dimensional grid for the endogenous state and interpolate
for solutions of assets and habits tomorrow different from the grid points. We will do this in-
terpolation by two-dimensional splines. To our knowledge, there is no attempt done to solve
a problem of this class through bidimensional splines. We explain below how we implement
it. To solve for the steady state we proceed as follows. First, given a pair of prices $\{w, r\}$
we solve the household problem. Second, we compute the aggregate capital implied by this
solution. This aggregate capital may or may not be consistent with the given prices $\{w, r\}$.
If it is, they are the steady state prices. If it is not, we get a new pair $\{w', r'\}$ and repeat
the process. Below we describe the procedure in more detail

A.1   Solving the Household problem

The Contraction Mapping Theorem tells us that following a successive approximations strat-
egy in the functional equation (1) will guarantee finding its fixed point. Moreover, any initial
guess will do as long as it is concave in its endogenous arguments. More precisely, deriving
the FOC we get the following system:

$$
0 = -u_c(c, h) + \beta \sum_{e'} \pi_{e,e'} \left[v_a\left(e', a', \psi(c, h)\right) - v_h\left(e', a', \psi(c, h)\right) \psi_c(c, h) \right] \qquad (7)
$$
$$
c = e\,w + (1 + r)\,a - a'
$$

which defines implicitly the policy functions $a' = g^a(e, a, h)$ and $c = g^c(e, a, h) = ew + (1 + r)a - g^a(e, a, h)$. We substitute them back into equation (1) to get:

$$
v(e, a, h) = u[g^c(e, a, h), h] + \beta \sum_{e'} \pi_{e,e'} \; v\left\{e', g^a(e, a, h), \psi[g^c(e, a, h), h]\right\} \qquad (8)
$$

First, we choose a family of functions that the computer can understand. The problem
we face here is one of two endogenous choice state variables. This means that we will need
to compute the value function at any point in a bidimensional continuous support as well
as at each point of the Markov process. We choose a bispline interpolation over a grid on
a and h. Splines are very useful in this context because they guarantee continuous first
and second derivatives. We need first derivatives to write the FOC and second derivatives
to use Newton-based non-linear equation solvers. Then, we guess an initial value function
v^0, solve numerically the FOC 7 at each point (e_i, a_j, h_k) of the three-dimensional grid for
the state space, get the policy functions $g^{0,a}(e_i, a_j, h_k)$ and $g^{0,c}(e_i, a_j, h_k)$, use the bispline
interpolation to substitute them back into the functional equation 8 and get an updated v^1.

If v^1 and v^0 are close enough we reached the fixed point. If they are not we iterate on, using v^1 instead of v^0 to get certain v^2.

There are two possible problems associated with the approach just described. The first one is that the Contraction Mapping Theorem does not necessarily hold once we restrict the space of continuous and bounded functions to which v belongs to a some computer storable subspace. The second one is that the bispline approximation does not necessarily preserve concavity. Whereas we have not found any difficulty associated with the former problem, the latter deserves further comments. A spline is basically an interpolation mechanism that uses a third order polynomial in each interval between grid points. When a piece-wise linear approach is followed, what happens to a function to be approximated in one interval is totally independent of what happens to it in another one. However, this is not completely true for splines, since the requirement that first and second derivatives from the left and from the right at each grid point equal each other makes the polynomials in each interval not independent. A utility function is an object with sharp changes of slope, being first and second derivatives huge at low levels of c and h and much smaller at higher levels. This properties translate into the value function. Using few grid points means that, not only the approximation is worse than using many but also that certain properties of the function may be lost. Precisely, we observed that using a 15x15 grid[40] would lead, in the more extreme parameterizations[41], to a loss of concavity and, even worse, monotonicity. The reason of this is that the high first derivative of utility at low levels of c is translated into the adjacent intervals so that the splines approximation overshoots the function to be approximated. Only slowly the slope of the spline can go down and recover, creating a hump. To solve this, one needs to use many grid points close to zero, the area where this happens, to make sure the spline slope can fall gradually. We increased the grid to 75x20 points. Notice that this means solving the household's problem for 4500 points at each iteration

A.2 Solving for the steady state

Here we follow a standard procedure. We choose an initial guess r^1 and solve the household problem to get $g_{r^1}^a(e, a, h)$ and $g_{r^1}^c(e, a, h)$. Then, we guess an initial sample of individuals of size 5000 and apply to them $g_{r^1}^a(e, a, h)$, $g_{r^1}^c(e, a, h)$ and the law of motion for the Markov process 3000 times, which ensures in all experiments we have done that the main statistics of the sample are almost constant. This gives us the aggregate assets in the economy A_{r^1}. Then, we find r^2 such that the demand of capital by firms, $K(r)$ equals A_{r^1}. Say $r^1 < r^2$ (if not, relabel). Since capital demand is decreasing in r and aggregate assets are increasing in r, the steady state interest rate r^* belongs to the interval (r^1, r^2). From this point we start the iterative procedure. We take the middle point in the interval, call it r^3, and get the associated A_{r^3}. If $A_{r^3} < K(r^3)$ the interest rate we have tried is too small, so we set

---

[40]More dense close to the zeros than close to the upper bounds

[41]More extreme parameterizations mean high σ, high γ and high variability of the earnings process. All these three characteristics create higher differences in the marginal utility of consumption across grid points. The problems would first arise at those points with low consumption and high habits because the lower the consumption and the higher the habit, the higher the marginal utility of consumption

$r^1 = r^3$ and start again. If $A_{r^3} > K(r^3)$ the interest rate we have tried is too big, so we set $r^2 = r^3$ and start again. We stop when the distance between r^1 and r^2 is arbitrarily small. To ensure no sampling error is spoiling the convergence to r^* we use the same seed to initialize the random number generator in each iteration.

Some of our model economies (case of M and Q) are set to have the same interest rate and aggregate capital as a given one (B) by adjusting the time preference parameter β. The procedure used to get the steady state in these cases differs from the one just described in that r is fixed and in that we have to iterate in different values of β. The initialization of the procedure is not so clean because it is not possible to compute an interval (β^1, β^2) where our β^* belongs to. We proceed as follows. We guess an initial β^1. If $A_{\beta^1} < K^*$ we know we have a lower bound. If $A_{\beta^1} > K^*$ we know we have an upper bound. In the former (respectively latter) case we try higher (lower) betas until we find a β^2 for which $A_{\beta^2} > K^*$ ($A_{\beta^2} < K^*$). Then, since capital demand is invariant in beta and aggregate assets are increasing, we know that $\beta^* \in (\beta^1, \beta^2)$. From this point we can apply to β instead to r the iterative procedure described in the previous paragraph.

B   Intertemporal elasticity of substitution

To correctly compare the habits economies with the non habits economy we want to make them equal in certain dimensions. One of these dimensions is the Intertemporal Elasticity of Substitution. The following theorem gives the explicit form for the IES in the habits economies. Notice that the IES is independent of the persistence of the habit stock.

Theorem 1 *For the certainty case the Intertemporal Elasticity of Substitution in the steady state is independent of λ and equal to $\frac{1}{\gamma + (1-\gamma)\sigma}$*

Proof.   *Let's write down the Euler equation:*

$$c_t^{-\sigma} h_t^{\gamma(\sigma-1)} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1+r)]^{i-1} x_{t+i} =$$

$$= \beta(1+r) \left\{ c_{t+1}^{-\sigma} h_{t+1}^{\gamma(\sigma-1)} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1+r)]^{i-1} x_{t+1+i} \right\}$$

where $x_t = c_t^{1-\sigma} h_t^{\gamma(\sigma-1)-1}$. Then, we can write the stock of habit as a function of all past consumption:

$$h_{t+1} = (1-\lambda) h_t + \lambda c_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i c_{t-i}$$

Steady state imposes $\frac{c_{t+1}}{c_t} = \eta$. Therefore, the habit stock in the steady state is:

$$h_t = A c_t$$

with $A \equiv \frac{\lambda}{\eta - (1-\lambda)}$, which means that consumption and habit stock grow at the same rate. Finally, in the steady state x_t becomes:

$$x_t = A^{\gamma(\sigma-1)-1} c_t^{-[\gamma + (1-\gamma)\sigma]}$$

With all this, we go to the Euler equation and replace the h and x by their steady state values:

$$Ac_t^{-[\gamma+(1-\gamma)\sigma]} - \beta\gamma\lambda B\eta^{-[\gamma+(1-\gamma)\sigma]}c_t^{-[\gamma+(1-\gamma)\sigma]} =$$
$$= \beta(1+r)\left\{A\eta^{-[\gamma+(1-\gamma)\sigma]}c_t^{-[\gamma+(1-\gamma)\sigma]} - \beta\gamma\lambda B\eta^{-2[\gamma+(1-\gamma)\sigma]}c_t^{-[\gamma+(1-\gamma)\sigma]}\right\}$$

where $B = \sum_{i=0}^{\infty}[\beta(1+r)]^i\left[\eta^{-[\gamma+(1-\gamma)\sigma]}\right]^i = \frac{1}{1-\beta(1+r)\eta^{-[\gamma+(1-\gamma)\sigma]}}$

Getting $\eta^{-[\gamma+(1-\gamma)\sigma]}$ *as common factor in the rhs lets us cancel out all the lhs to finally get:*

$$1 = \beta(1+r)\eta^{-[\gamma+(1-\gamma)\sigma]}$$

or cleaner

$$\eta^{\gamma+(1-\gamma)\sigma} = \beta(1+r)$$

∎