working papers

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OPTIMAL RAILWAY INFRASTRUCTURE MAINTENANCE AND REPAIR POLICIES TO MANAGE RISK UNDER UNCERTAINTY WITH ADAPTIVE CONTROL

Javier González*, Rosario Romera*, Jesús Carretero**, José M. Pérez**

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Optimal railway infrastructure maintenance and repair policies to manage risk under uncertainty with adaptive control

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Abstract

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keywords: Infrastructure Management, Adaptative control, Risk model, RCM, railways

1 Introduction

Infrastructure management is the method through which agencies analyze data about infrastructure systems and make decisions on repair or maintenance: bridges, railways, roads or highways are examples of these activities.

The state-of-the-art in repair policies management in railway infrastructures shows a deterministic methodology, where, most of the times, the maintenance decisions are taken based on the knowledge of the system, (NAVAIR, 1996; OREDA, 1997; Schlkins, 1996; Ireson and Clyde, 1988). In 2000, the European Union founded a project named "RAIL: Reliability centered maintenance approach for the infrastructure and logistics of railway operation" aimed to study the application of RCM techniques (Anderson and Lewis, 1990; McCall, 1965; Moubray,1996) to the railway infrastructure, following the success of RCM in other industrial fields (Abdul-Nour et al.,1998; Carretero el al., 2000; Pujadas and Chen, 1996), such as aviation NAVAIR (1996), oil industry (OREDA, 1997; Pintelon et al., 1999) or ships in Schlkins (1996). The reason for the project was that railway maintenance had been traditionally planned using the knowledge and experience of each company (see Red Nacional, 32), but without any kind of reliability based methodology to support the maintenance plans and works. Some attempts to use RCM in railways, like the REMAIN (1988) project or the Norway railways (Seve et al.,2000; Vatn et al., 1996) were not adopted by the companies, probably due to their very ambitious goals. In Carretero et al. (2003) a successful experience of applying RCM to large scale railway networks is described. In the paper, a railway line is described on Figure 1.

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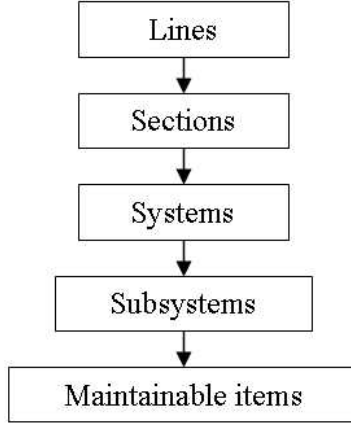
Figure 1: Railway line description

Based on this description the RCM methodology is applied to identify the failures, and to apply effective tasks to solve them.

In this paper we develop a similar approach for the maintenance of railway infrastructures, but now, we have to consider different aspects of the problem. We go a step beyond the maintenance theory shown in EN (2000) to enhance maintenance and repair policies for railway lines. The major contribution of this work is that it benefits from the RCM methodology application, but it also explicitly models uncertainty in characterizing a facility deterioration rate.

Our formulation assumes that the facility deterioration is a Markov decision process (MDP), and that the planning agency characterizes the facility deterioration rates with R Markovian models, each one determined by a deterioration level and with a matrix of transition probabilities $\pi_{ij}^r(a)$:

$$p(X_{t+1} = j | X_t = i, A_t = a, Y_t = r) = \pi_{ij}^r(a),\ a \in A\ ij \in S\ t = 0, 1, 2, ..., T-1 \text{ and } r = 1, 2, ..., R$$

where X_t is a random variable denoting the state of the infrastructure, Y_t the random variable for the environment uncertainty, T is the planning horizon and a represents the different actions, being S and A finite state-space and finite set of actions respectively. Uncertainty is captured by the random variable Y_t with mass probability Q_t in the period t. This uncertainty can be attributed to different exogenous factors as the environment, the level utilization, the design facility and the materials. In the model appears a statistical factor like the variability of the data sets used to generate the Markovian Process.

Moreover, two formulations are proposed in Durango and Madanat (2002) to solve the problem: closed loop and open loop feedback. In both, a function to minimize, that represents the minimum expected sum of discounted cost until the end of the period T, is established. The action taken at the end of each period depends on the prior sequence transition. The main difference between the two formulations is the updating rule for the mass probability Q_t. While in the pen-loop-feedback the recurrence relation does not account for the fact that the probability mass function is updated, in closed loop formulation a Bayesian rule is applied to improve the information of this variable.

In any case, agencies establish different periods, usually one year, to minimize the total optimal maintenance cost associated with a facility over a planning horizon. For example, in a road, the pavement management only depends on several deterioration states that could be discretized. Assuming that the deterioration process of the pavement is governed by different speeds, the agency only has to decide one among a group of actions to minimize the cost over a planning horizon. But in railway infrastructures the problem is different because the complexity of the problem increases with the size of the lines.

By considering that a railway line is a dynamic system, we have developed an optimal control model to establish the right maintenance policies instead of using a simple decision tree, as in RCM. But satisfying
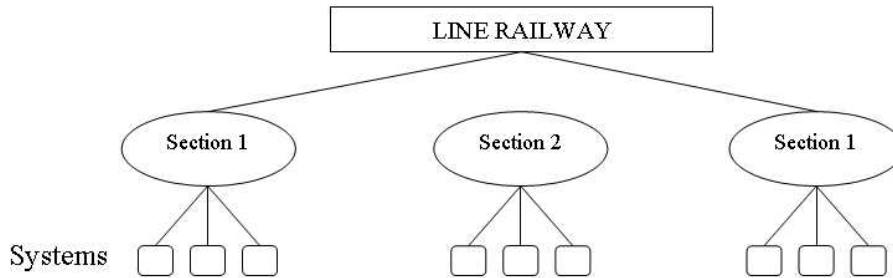
Figure 2: Scheme of a railway line

this goal with the structure of Figure 1 has an extraordinary complexity. So our model only considers that a line is divided in sections formed by systems, as shown in Figure 2.

This division transforms the simple optimal control, where we decide the maintenance policy in only one infrastructure, in a complex decision problem, which we propose a solution for in this paper.

The paper is organized as follows. Section 2 briefly presents RCM methodology. Section 3 presents the optimal control formulation for the problem of assigning a maintenance work team to a section of a railway line under uncertainty, based on the criticality and the deterioration of all sections of the infrastructure over a planning horizon. In Section 4 we present a computational study comparing the open and the closed loop formulation. In Section 5 we present conclusions and extensions of the study.

2 What is RCM?

The concepts behind RCM are not new, having their origin in the airline industry back in the 1960's. After several years of experience, in 1978, the US Department of Defense issued the MSG-3 (1978), an Airline/Manufacturers Maintenance Program Planning Document. That year, Nowlan and Heap wrote a comprehensive document on the relationships among Maintenance, Reliability and Safety, entitled Reliability Centered Maintenance (Nowlan and Heap, 1978), creating the RCM methodology. RCM spread throughout industries, specially those needing safety and reliability, during the 1980's and the 1990´s, being now extended to several industry fields. Today, RCM tools are integrated with Computerized Maintenance Management Systems (CMMS) (Hounsell, 1996), as Relex (RELEX) or ASP´s. The latest trend encompasses asset management and maintenance, supported by various methods of Condition Based Maintenance Systems (CBMS) and in-service inspection processes (Parida et al., 2000; Deshpande and Modak, 2002).

But, what is RCM? There are very good definitions of RCM in the literature (Anderson and Lewis, 1990; Nowlan and Heap, 1978; Pintelon et al., 1999; Rausand, 1998; Smith, 1993). In short, RCM can be defined as a systematic approach to systems functionality, failures of that functionality, causes and effects of failures, and infrastructure affected by failures. Once the failures are known, the consequences of them must be taken into account. Consequences are classified in: safety and environmental, operational (delays), non-operational, and hidden failure consequences. Later those categories are used as the basis of a strategic framework for maintenance decision-making. The decision-making process is used in order to select the most appropriate task to maintain a system filtering the proposed classification of consequences through a logic decision tree. In the 1970´s, and still today, RCM was a major challenge in many industries because it changed the focus of preventive maintenance from bringing back the systems to a "perfect" state to maintaining the system in a good "functional" state (within some defined operational limits) (Latino, 1999; Morsmann, 2000; Nakajima, 1988). Through this approach, it provides an understanding of how infrastructure works, what it can (or cannot) achieve, and the causes of failures.

RCM methodology (Moubray, 1996, 1997) has three major goals. First one is to enhance safety and reliability of systems by focusing on the most important functions. RCM is concerned mainly with what we want the equipment to do, not what it actually does. Second is to prevent or to mitigate the consequences of failures, not to prevent the failures themselves. The consequences of a failure differ

depending on where and how items are installed and operated. Third one is to reduce maintenance costs by avoiding or removing maintenance actions that are not strictly necessary. It is no longer assumed that all failures can be prevented by PM, or that even if they could be prevented, it would be desirable to do so. As RCM provides a ranking of maintenance tasks for a system, it can be used as a good technique for developing a preventive maintenance program (Hall, 1992; Ben-Day, 2002). A formal review of failure consequences focuses attention on maintenance tasks that are more effective, diverting energy away from those which have little or no effect (See IEC, 1982). This helps to ensure that whatever is spent on maintenance, it is spent where it will be more necessary to ensure that the inherent reliability of the equipment is enhanced.

A major problem in RCM is to establish the criticality of the infrastructure. What is criticality? It is a measure of the importance of the system from a functional point of view. Once criticality is computed for several systems, those can be classified according to their importance for the whole railway network. Criticality is the base to rank the "machines". Criticality is computed for the whole hierarchical decomposition of the infrastructure: line, section, and system level. A set of factors is defined in order to compute the criticality, which is an addition of weighted factors values.

The factors to be taken into account were defined by Carretero et al. (2002), (2003) and they were the result of the RAIL project. They concluded that the factors should be the same for lines and sections, but should be different for systems. Why? Lines and sections are classified using functional criteria specified mostly by the client (the transport companies running trains along the rails) and adding some criteria related to infrastructure and environment. According Carretero et al (2003), for railway lines criticality factors are taken from:

Table 1: Factors to compute criticality

Factor	Description
Safety	Repercussion of a failure on safety
Traffic density	Number of circulation per day
Revenues	Revenues obtained from exploitation
Availability	Number of hours that the line must be available per day
Exploitation	Number of passengers or dangerous freights
Maintainability	Maintenance process complexity
Cost (optional)	Cost associated to maintenance

Criticality is computed as its weighed sum of factors for each section of the line. After that, is classified according her importance through the values of different thresholds $\{\theta_1, \theta_2, \theta_3\}$. Each factor has a value F_i and a weight W_i. Four criticality states, that is the values of C_{it}, are defined as follows:

- A if $\sum W_i F_i > \theta_1$

- B if $\theta_1 > \sum W_i F_i > \theta_2$

- C if $\theta_2 > \sum W_i F_i > \theta_3$

- D if $\theta_3 > \sum W_i F_i$

For each railway line, there is a p-dimensional non-random vector that characterizes it:

$$C_t = \{C_{1t}, C_{2t}, ..., C_{pt}\}$$

Sections are classified in four levels from A to D depending on the criticality. Notice that criticality is not a random variable. Criticality is mostly defined by engineers, maintenance staff, or railway regulation authorities and values. We can also estimate W_i by statistical techniques (multiple regression analysis...) based on data set or/and variable information of the infrastructure. Once computed, criticality shows a ranking of the functional importance of each component of the infrastructure, including lines, sections and systems.

In the next section we present the formulation of a control problem that includes uncertainty into the RCM model defined for railway systems.

3 Control problem formulation

In an optimal stochastic control problem, we consider a dynamical system whose behavior may be influenced or regulated by a subset of the system variables, which are called control, action, or decision variables. The controls that can be applied at any given time are chosen according to rules known as control policies. In addition, we consider a function called a performance criterion defined on the set of control policies, which measures or evaluates in some sense the system's response to the control policies being used. Then the optimal control problem is to determine a control policy that optimizes (either minimizes or maximizes) the performance criterion. If the system is in the state x at time t and the control is applied, then two things happen: a cost $c(x, \overrightarrow{a})$ is incurred, and the system moves to the next state according the transition probabilities law π. (See for instance Bertsekas (1995), (2000); Henández-Lerma, 1996). We consider the discrete-time control model defined by the following elements $\{S, A, A(x) | x \in S, \pi, c\}$, where S and A are finite deterioration-levels-space and actions-space respectively. Elements $\overrightarrow{a}$ of the subset $A(x)$ are p-vectors of the form $e_i = (0, 0, ..., 1, ..., 0)$ where 1 is in the position i.

Because the railway line is divided in p different sections to maintain, our goal is to assign a work team to a section of the line. This implies that we have to decide which one is the optimum section to maintain. This is a way to represent the railway line components. It is to say, A_t represents with the value 1 the section where the work team is assigned and with zero in other case. Then, in our problem, the optimal decision is not to define an optimal maintenance policy. We only have to assign the work team to the right section. Although once this action has been taken, we must define the optimal maintenance for the systems of the chosen section.

Elements π and c represent transition probabilities and cost functional associated with Deterioration, respectively. In our problem we evaluate transitions and the costs associated with Risk instead of the Deterioration as we introduce below.

3.1 Deterioration process

Deterioration is a measure of the line degradation. Unlike Criticality, Deterioration level is captured by the random variable D_t. For each section of the line we denote deterioration by the vector:

$$D_t = \{D_{1t}, D_{2t}, ..., C_{pt}\}$$

Deterioration levels compose a finite set. Deterioration model capture the deterioration of facilities with different values of exogenous factors. We use the random variable Y_t, $t = 0, 1...T-1$, to represent the environment uncertainty. For example, this variable may assign probabilities to three levels of degradation speed: $p(Y_t = high)$, $p(Y_t = fair)$, $p(Y_t = low)$. Transitions between deterioration levels are given in terms of transition probabilities as follows:

$$p(D_{i,t+1} = q | D_{i,t} = p, A_t = \overrightarrow{a}, Y_{i,t} = r) = \pi^r_{pq}(\overrightarrow{a}) \tag{1}$$

Moreover, we consider that the deterioration process is modeled as a homogeneous Markov Chain, where the index r in Folmula 1 represents the variable status external that has influence in the degradation. Nevertheless, in a more general setting we can consider $\pi^r_{pq}(a)$ changing over time.

3.2 Risk level

Risk is a measure of the combination of the criticality and the deterioration. Then, for each section a risk level is assigned through the values of an f function, where:

$$R_{i,t} = f(C_{i,t}, D_{i,t}) \ t = 1, ..., T \ i = 1, ..., p$$

Then, the risk vector for the line at time t is given by:

$$R_t = \{R_{1t}, R_{2t}, ..., R_{pt}\}$$

Notice that the f function does not depend on the time. Criticality and deterioration are measured in a finite number of values and, consequently, we consider also a finite number of risk values. As an example, following the standard RAMS, the function f can be represented by the table in Table 2.

Table 2: (Standard RAMS) Example combination of different values for criticality and deterioration

C D	Critic. 1 (Small)	Critic. 2 (Medium)	Critic. 3 (High)	Critic. 4 (Very High)
Deterioration 1	R3	R4	R4	R4
Deterioration 2	R3	R4	R4	R4
Deterioration 3	R2	R3	R3	R4
Deterioration 4	R1	R2	R3	R4
Deterioration 5	R1	R2	R3	R3
Deterioration 6	R1	R1	R2	R3

3.3 Transition probabilities

Because the risk is the variable of the objective function of the problem, we propose to think of transitions between risks in terms of conditional probabilities. For each section $i = 1, ..., p$ we consider the following elements of the risk model: The matrix of transition probabilities among the deterioration states when the maintenance policy under the line is .

$$\Pi^r(\overrightarrow{a}) = \|\pi_{pq}^r(\overrightarrow{a})\|,\ p \text{ and } q \in S,\ r = 1, ..., R$$

Where

$$\pi_{pq}^r(\overrightarrow{a}) = p(D_{i,t+1} = q | D_{i,t} = p, A_t = \overrightarrow{a}, Y_{i,t} = r)t = 0, ..., T-1 \tag{2}$$

The Matrix of transition probabilities among the risk states when the maintenance policy under the line is $\overrightarrow{a}$, and the criticality for the section i is C_i .

$$\psi(\overrightarrow{a}, C_i) = \|\psi_{lk}(\overrightarrow{a}, C_i)\| \tag{3}$$

Where

$$\psi(\overrightarrow{a}, C_i) = p(R_{t+1} = h | R_t = l, A_t = \overrightarrow{a}, C_i) = \sum_{r=1}^{R} p(R_{t+1} = h | R_t = l, A_t = \overrightarrow{a}, C_i, Y_t = r)p(Y_t = r) \tag{4}$$

Notice that the probability distribution of Y_t is given by the mass functions Q_t probably changing over time.

We build a matrix of probabilities, depending on the criticality state of the section i, C_i, the action $\overrightarrow{a}$ and relying on the probability distribution on the deterioration rates, the maintenance policy and the environment Q_t. Thus, if we assume stationary mass functions sequence Q_t, say $Q_t = Q$ for $t = 0, ..., T-1$, then the Risk process is, in fact, a homogenous Markov Chain with matrix of transition probabilities given by $\psi(\overrightarrow{a}, C_i)$.

The following objective is to develop an expression for the probabilities $\pi_{pq}^r(\overrightarrow{a})$ in terms of the probabilities $\pi_{00}^r(\overrightarrow{a})$. For this purpose we introduce, for each Risk level R and Criticality value C_i, the auxiliary set $\alpha(R, C_i)$ containing all possible Deterioration values that join with Criticality C_i and according to the function f (see Table 2) report risk R. That is:

$$\alpha(R, C_i) = \{D_j,\ j = 1, ..., D \text{ such that } f(C_i, D_i) = R\}$$

Then

$$p(R_{t+1} = h | R_t = l, A_t = \overrightarrow{a}, C_i, Y_t = r) = \sum_{m \in \alpha(C_i, l)} \theta \sum_{n \in \alpha(C_i, h)} \pi_{mn}^r(\overrightarrow{a}) \tag{5}$$

Where $\theta = [Card(\alpha(C_i, l))]^{-1}$. From a practical point of view, when computing the transition probability $p(R_{t+1} = h | R_t = l, A_t = \overrightarrow{a}, C_i, Y_t = r)$ we also know D_t, which in fact has generated R_t. Thus, $\alpha(C_i, l) = D_t$.

3.4   The optimal function

From now on, we assign to each risk category a cost value, so we select the decision At such that it minimizes the one step-ahead line global risk. Thus, the problem can be formulated as the minimization of the expected sum of the discounted risk cost at the end of the period t. Let $d_t^{cl}(R, Q)$ be the value function at the step t, with risk R and speed deterioration distribution Q:

$$d_t^{cl}(R, Q) = min\{\sum_{k=1}^{P} \delta_k \sum_j \psi_{R_t, j}(\overrightarrow{a}, C_i) w(R_{t+1} = j)\} t = 0, ..., T - 1 \tag{6}$$

The value $0 < \delta_k < 1$ is a discount factor for the relative importance of each section and $w(R_{t+1} = j)$ represents the cost value if the section has a risk j at the step t.

CASE 1: Closed loop control formulation

In the closed loop formulation we optimize expression 6 with modified transition probabilities $\psi_{lh}(\overrightarrow{a}, C_i, Q')$ depending on the mass function Q' which is an updated probability distribution of the speed deterioration variable as it is explained below.

Because in that case we make use of the knowledge of the deterioration levels D_t and D_{t+1}, we are able to construct a new speed deterioration probability distribution Q' via the Bayesian formula after the transition probabilities $\pi_{D_t, D_{t+1}}^r(\overrightarrow{a})$ which depends on the action taken $\overrightarrow{a}$.

$$Q_r'(p, q, \overrightarrow{a}, Q) = p(Y_{t+1} = r | D_{t+1} = q, D_t = p, A_t = \overrightarrow{a}, Q) \tag{7}$$

Given by the Bayesian updating formula:

$$Q_r'(p, q, \overrightarrow{a}, Q) = \frac{\pi_{pq}^r(\overrightarrow{a})}{\sum_k \pi_{pq}^r(\overrightarrow{a}) \pi_{pq}^k(\overrightarrow{a}) Q_k} \tag{8}$$

But in the real case, our problem has p different sections. Based on the deterioration states at t and $t+1$ and the action taken at the end of the period t, the updating procedure for Q' operates as follows:

$$Q_r'(p, q, \overrightarrow{a}, Q) = E_x[\frac{\pi_{pq}^r(\overrightarrow{a})}{\sum_k \pi_{pq}^r(\overrightarrow{a}) \pi_{pq}^k(\overrightarrow{a}) Q_k}] \tag{9}$$

Where the index h takes values from 1 to R (degradation speed range). Equation 9 is a recursive evaluation from the period 1 to $T - 1$. The optimality of the Closed Loop formulation can be checked with the following experiment: under a nominal Q distribution the updating procedure giving Q' is such that we observe a clear trend approaching Q' to Q as t goes to the Horizon value T.

CASE 2: Open loop optimal feedback control formulation

This is a sub-optimal case because the mass function is not updated for each period. The value function is in this case:

$$d_t^{op}(R, Q) = min\{\sum_{k=1}^{P} \delta_k \sum_j \psi_{R_t, j}(\overrightarrow{a}, C_i) w(R_{t+1} = j)\} \tag{10}$$

In this case Q remains constant during the entire process. An upper bound for the Risk Cost of the railway line is given by the expression:

$$s(C, w) = \sum_{i=1}^{p} \delta_k w(R_k^*) \tag{11}$$

Where R^* Risk with the highest cost in section k.

4 Computational Study

In Section 2 we have presented two different Adaptive Control Formulations: Open loop, and Closed Loop. The main goal of this paper is to cover, in order to construct a model such that, working under uncertainty, provide us an optimal procedure to decide the optimal policy to maintain the railway infrastructures. The state-of-the-art in repair policies management in the railway infrastructures shows us a deterministic methodology, where most of the times, the maintenance decisions were taken based on the knowledge of the system.

In this section we present numerical results to illustrate the benefits of the optimal control formulation with uncertainty with respect to a deterministic maintenance policy. In Section 4.1 we study the performance of the two formulations and proof that the global value of the risk is decreased when we apply our modus operandi. In Section 4.2 we compare the open and the closed loop formulation under different situations of the railway line and degradation speeds and we study how the closed loop formulation performances better than open loop even when we work with incorrect deterioration models.

4.1 Optimal Stochastic Control formulation versus Deterministic Maintenance Policies

The purpose of this first study is to compare three maintenance policies for proving the benefits of the Optimal Control procedure under uncertainly. The employed matrices π_{pq}^r have been simulated according to logical deterioration probabilities among the states of degradation. We have used the following probability distribution for the environment degradation: 0.4 for slow, 0.3 for medium and 0.3 for a fast degradation in the first analysis in which we study the efficiency and the profits of the Optimal Stochastic Control strategy. We establish the same assumptions that Durango and Madanat, (2002) and Yanfeng and Madanat, (2003); say

- We have a probability distribution associated to each state of transition for each maintenance policy (to maintain or not) and for each deterioration speed.

- The maintenance decision is less effective for faster deterioration rates

- Faster deterioration has high variances in prediction.

Figure 3 presents the values of the matrices employed in the experiment. Those matrices are an example of how the environment degradation speed and the maintain policy have influence in the transition states among deterioration rates. These matrices could be different in other railway lines but they must be agree with the above assumptions. We run the example with the railway line Villalba-Cercedilla. The values of the matrix F Gives us the following criticality Table 3.

The weight of each section is 1/11 in every case in order to check the system without preferences on the section and obtaining a measure of the average cost. The differences among the sections are considered in terms of the criticality. We begin with an optimal situation in terms of deterioration; every section at time t = 0 has a deterioration rate 1 (where 1 represents that the section is in a perfect state and 7 that section is completely deteriorated). The join function of deterioration and criticality is based on the Figure 4, and the risk cost for the four levels is (1, 5, 100, 10000). That is, we consider that levels risk 1 and 2 do not have important consequences in the system but that controlling levels 3 and 4 have is very important for the system, especially Risk 4 that corresponds with a limit case, a situation absolutely undesirable. The deterministic policy 1 consists in assigning always the work team to the same section,

8

Maintain Degradation	Yes						No					
Slow	1,00	0,00	0,00	0,00	0,00	0,00	0,80	0,15	0,05	0,00	0,00	0,00
	0,99	0,01	0,00	0,00	0,00	0,00	0,00	0,80	0,15	0,05	0,00	0,00
	0,97	0,02	0,01	0,00	0,00	0,00	0,05	0,00	0,80	0,15	0,05	0,00
	0,94	0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,80	0,15	0,05
	0,90	0,04	0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,80	0,20
	0,85	0,05	0,04	0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,00	1,00
Medium	0,99	0,01	0,00	0,00	0,00	0,00	0,70	0,20	0,10	0,00	0,00	0,00
	0,97	0,02	0,01	0,00	0,00	0,00	0,00	0,70	0,20	0,10	0,00	0,00
	0,94	0,03	0,02	0,01	0,00	0,00	0,05	0,00	0,70	0,20	0,10	0,10
	0,90	0,04	0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,70	0,20	0,10
	0,85	0,05	0,04	0,03	0,02	0,01	0,00	0,00	0,00	0,00	0,70	0,30
	0,80	0,06	0,05	0,04	0,03	0,02	0,00	0,00	0,00	0,00	0,00	1,00
Fast	0,97	0,02	0,01	0,00	0,00	0,00	0,60	0,20	0,10	0,05	0,05	0,00
	0,94	0,03	0,02	0,01	0,00	0,00	0,00	0,60	0,20	0,10	0,05	0,05
	0,90	0,04	0,03	0,02	0,01	0,00	0,05	0,00	0,60	0,20	0,10	0,10
	0,85	0,05	0,04	0,03	0,02	0,01	0,00	0,00	0,00	0,60	0,30	0,10
	0,80	0,06	0,05	0,04	0,03	0,02	0,00	0,00	0,00	0,00	0,60	0,40
	0,75	0,07	0,06	0,05	0,04	0,03	0,00	0,00	0,00	0,00	0,00	1,00

Figure 3: Risk cost over four maintenance schemes

Table 3: Criticality of the line Villalba-Cercedilla

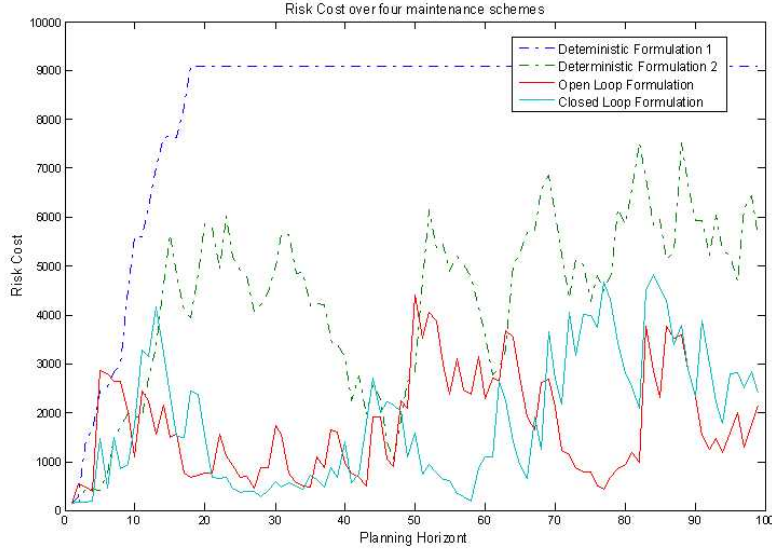| Section	Criticality
Villalba	High
Villalba- Los Negrales	Medium
Los Negrales	Médium
Los Negrales-Alpedrete	Médium
Alpedrete	Médium
Alpedrete-Collado Mediano	Médium
Collado Mediano	High
Collado Mediano-Los Molinos	Médium
Los Molinos	Médium
Los Molinos - Cercedilla	Medium
Cercedilla	High

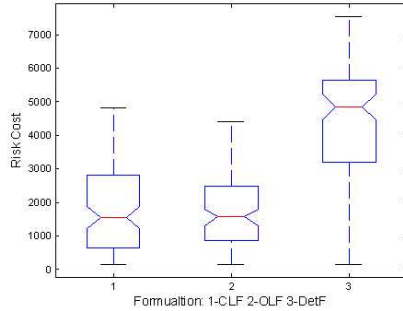Figure 4: Risk cost over four maintenance schemes

Figure 5: Risk Cost Anova analysis

in the example the section 1. The deterministic policy 2 maintains the sections of the line in a sequential order along the planning horizon, and beginning in the section 1.The total cost evaluation is denoted in this case by d_t^{det} .

Four optimization schemes are shown in Figure 4. We simulate a case of 100 iterations. It is easy to see graphically, that when applying one of the two proposed formulations the level of the risk decreases along the planning horizon.

It is clear that applying a Closed Loop Formulation the cost associated to the risk takes smaller values. It is also logical to think that even with an Open Loop formulation, the generated risk in the railway line is smaller. Deterministic policy 2 gives us values of the risk with a larger variability along the planning horizon in comparison with Close and Open loop. To proof that fact from a statistical point of view, we apply the Anova Methodology: considering methods as factor levels. Obviously, in this analysis we consider that we are working with stationary processes that take values around a mean. We obtain highly significant ($p - value \approx 0$) results among the three methods. It is obvious that the maintenance policy of assigning the work team always to the same section is undesirable but shows us the limit risk value with the employed weights. In this case both Control Formulations improve the risk value respecting to the deterministic policy. The multiple Box-Plot in Figure 5 shows graphically this result.

In terms of the Risk variability Closed and Open Loop policies behave better than the Deterministic Policy The one way Anova compares the methods considering the mean, but it is interesting to compare them with other criteria like the maximum risk cost (Max) obtained along the 100 iterations of the
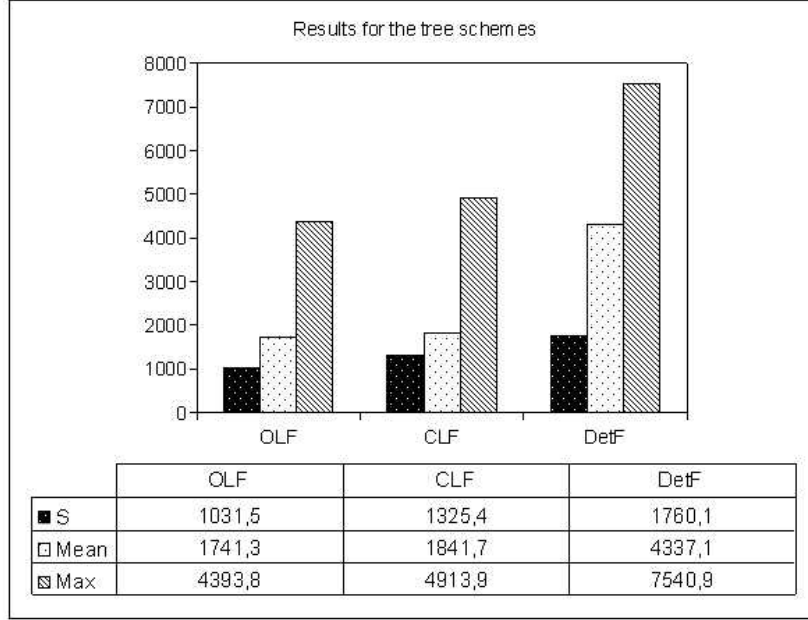
Figure 6: Absolute risk values

process, and the standard deviation of the process (S). Table 4 shows the relative improvement over the deterministic maintenance policy. The value in each cell is computed as follows:

Table 4: Relative improvement versus deterministic maintenance policy

	OLF vs DeF	CLF vs DeF
Mean	59,85%	57,54%
Max	41,73%	34,84%
S	41,40%	24,70%

$$Criteria = \frac{d_t^{det} - d_t^{cl \vee op}}{d_t^{det}} \qquad (12)$$

Where $d_t^{cl}(R,Q)$ and $d_t^{op}(R,Q)$ are shown in formulas 6 and 10 respectively. Figure 7 shows graphically the results obtained when working with the absolute risk values:

Thus, it is clear that, applying any loop formulation, the cost of the railway lines maintenance is decreased applying any of the criteria employed below.

4.2 Open Loop Formulation versus Closed Loop Formulation

In section 4.1 we have shown the existence of differences among the implemented algorithms when we compare them with a deterministic maintenance policy. In this section we study in deep those differences with several values of the probability density of the environment, and we proof that the closed loop formulation performs better that the open loop even when the deterioration model is incorrect. We simulate the Open and Closed Formulations according to tree probability mass functions: (0.05, 0.05, 0.95), (0.33, 0.34, 0.33) and (0.95, 0.05, 0.05) respectively (see Figure 7). In a first approach, the process employed for generating the deterioration transitions and Q are the same. That is to say, open loop formulation is working with a correct deterioration model. Following the same comparative procedure
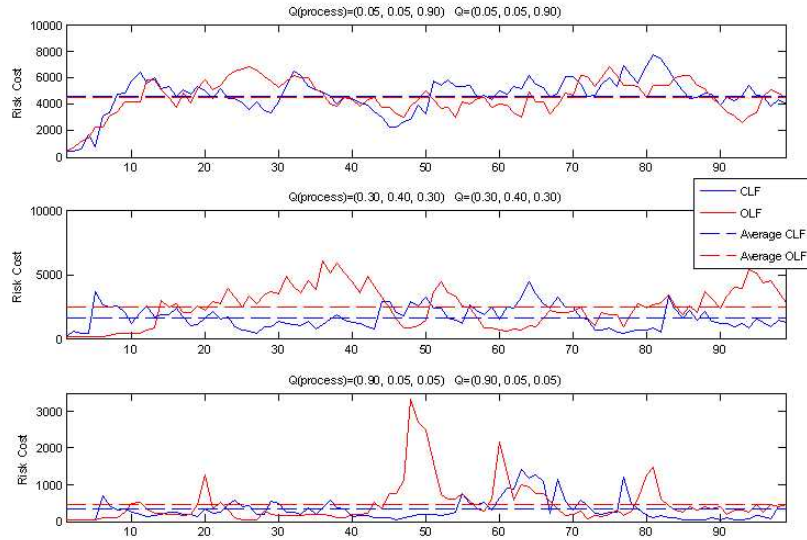
Figure 7: Three maintenance instances when the prior distribution of Q is right

that is Section 4.1, the Anova test found significant results in the second and third case and always in favor of the close loop formulation.

Once we have checked that close formulation performs better than open formulation when the deterioration model is correct, it is interesting to check the maintenance methodology when we have incorrect information about the distribution Q. We study an extreme case when we suppose that the deterioration environment follows a probability function (0.05, 0.05, 0.90), when the correct model is (0.90, 0.05, 0.05). We run the process 30 times and analyze the results (Figure 9).

Computing the mean for the two schemes in the 30 instances and plotting the results we observe the differences among the method with incorrect information. Since the closed loop employs an updating Bayesian method for the function Q', the performance is better in terms of the risk cost. Discontinued lines correspond to the average risk cost of each method.

5 Conclusions and future work

In this paper, the latent MDP model introduced by Durango and Madanat, (2002) to characterize facility deterioration has been successfully extended to evaluate Risks. We have gone a step beyond to enhance maintenance and repair policies for railway lines. For this purpose the RCM framework introduced by Carretero el al. (2003) for obtaining maintenance policies in railway infrastructures, has been provided of a mathematical tool that improves the deterministic results. Thus, our theory benefits from the RCM methodology application, but it also explicitly models uncertainty in characterizing a facility deterioration rate to decide the optimal policy to maintain the railway infrastructures. This may be the major contribution of this work.

The complexity of the railway infrastructure, specially that related with signalling, increases the complexity of the modelling problem. However, we have been able to define a formulation of the problem assuming that the facility deterioration is a Markov decision process (MDP), and that the planning agency characterizes the facility deterioration rates with R Markovian models, each one determined by a deterioration level and with a matrix of transition probabilities $\pi_{ij}^r(a)$.

By dividing the railway lines in sections, we could achieve to assign a work team to a section of the line optimally. A part of the RCM methodology has been used to compute previously the criticality of each section, so that we can use the RAMS model to relate criticality and risk. We have also developed two models: open loop and closed loop. Theoretical tests has demonstrated that the closed loop is better than open loop, but both of them performs better than the deterministic RCM methodology (which was
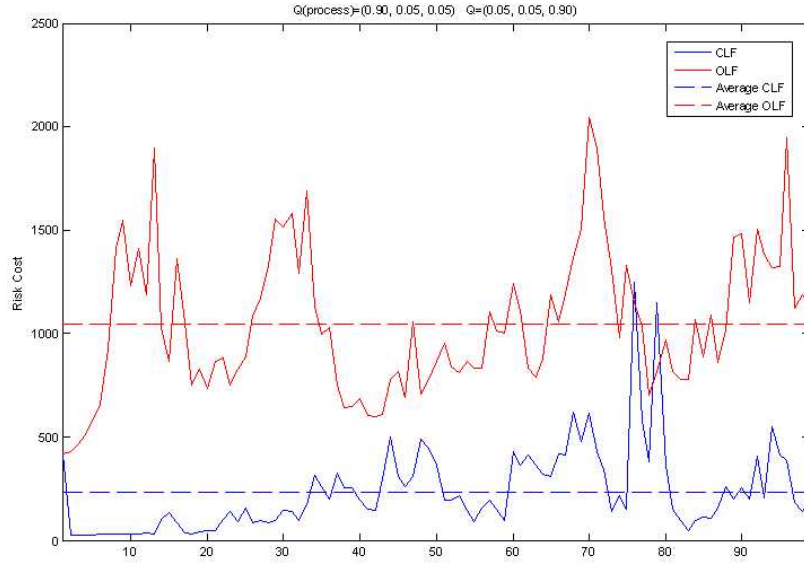
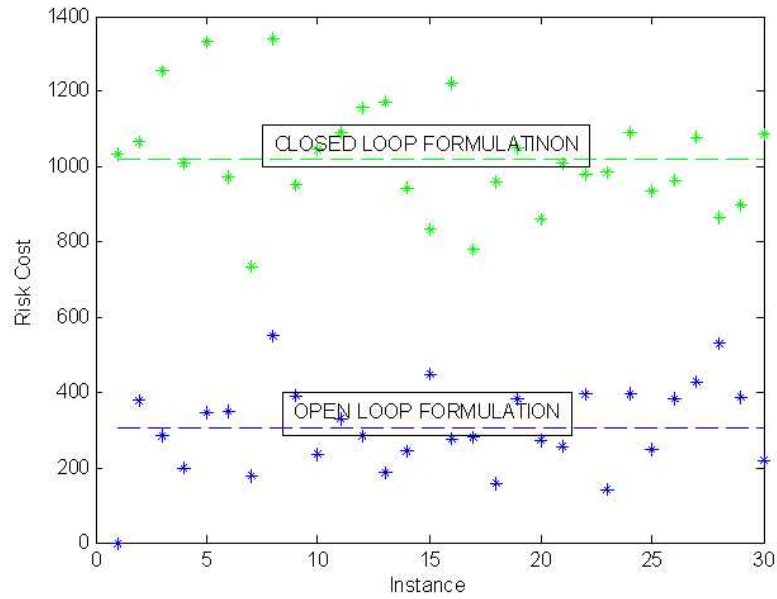Figure 8: Model results when the deterioration model is correct

Figure 9: Mean values for open and closed loop schemes

also and advance over traditional maintenance methodologies).

To verify the models presented, a computation model has also been developed and tested with a real scenario: the railway line Villalba-Cercedilla in Madrid (Spain). The test demonstrated again that applying any loop formulation, the cost of the railway lines maintenance is decreased. Moreover applying a Closed Loop Formulation the cost associated to the risk takes smaller values (40% less cost for the same risk than the deterministic approach), but with an Open Loop formulation the generated risk in the railway line is also smaller.

Work is going on to extend the model to the systems level. We are now studying how to overcome the complexity of the problem. Further research for the multicriteria cost case should be done

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