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RISK PREMIUM: INSIGHTS OVER THE THRESHOLD

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Abstract

The aim of this paper is twofold: First to test the adequacy of Pareto distributions to describe the tail of financial returns in emerging and developed markets, and second to study the possible correlation between stock market indices observed returns and return's extreme distributional characteristics measured by Value at Risk and Expected Shortfall. We test the empirical model using daily data from 41 countries, in the period from 1995 to 2005. The findings support the adequacy of Pareto distributions and the use of a log linear regression estimation of their parameters, as an alternative for the usually employed Hill's estimator. We also report a significant relationship between extreme distributional characteristics and observed returns, especially for developed countries.

Keywords: Expected Shortfall; Tail Risk; Pareto Distribution; Value at Risk.

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RISK PREMIUM: INSIGHTS OVER THE THRESHOLD

I. Introduction

Financial risk management is closely related with extreme events and so is highly concerned with profit and loss distribution's tail quantiles (e.g., the value of x such that P(X>x)=0.05) and tail probabilities (e.g., P(X>x), for a large value of x). No matter whether we are concerned with market, credit or other financial risks, one of the main challenges is to model the unusual but really damaging events to permit measurement of their consequences. Specifically in this paper we deal with market risk, say the day-to-day determination of the losses we may incur on a trading book due to adverse market movements.

Recent literature (see for instance Embrechts et al., 1997) claims that extreme value theory (EVT) holds promise for accurate estimation of extreme quantiles and tail probabilities of financial asset returns, and hence could be a relevant tool for the management of extreme financial risks. This theory posits that methods based around assumptions of normal distributions are likely to underestimate tail risk. The key idea of EVT is that one can estimate extreme quantiles and probabilities by fitting a model to the empirical survival function (defined as 1-F*(x) where F*(x) is the empirical cumulative probability function) of a set of data using only the extreme event data rather than all the observed data available, thereby describing the tail, and just the tail, which is the main concern in the extreme events risk management. However, Diebold, Schuermann, and Stroughair (1998) describe several pitfalls in the use of extreme value theory in risk management. Also, up to now the question whether the market is compensating for extreme risk factors is still not answered.

In this paper, we address one of those pitfalls related to the estimation by log linear regression of the parameters of the Pareto distribution, and study if there is a risk-premium associated to extreme risk measures (Value-at-Risk and Expected Shortfall), considering developed and emerging markets. We test the empirical model using daily data from 41 countries obtained from Morgan Stanley Capital International (MSCI). The findings support the use of a log linear regression estimation of the parameters, as an alternative for the Hill's estimator (Maximum Likelihood Estimator). Related to risk premium determinants, we present empirical evidence suggesting significant relationship between extreme distributional characteristics and observed returns, especially for developed countries.

The remaining of this paper is as follows. In Section 2 we provide a literature review of topics concerning Value-at-Risk (VAR), Expected Shortfall (ES) and cross section of average returns' explanations. In Section 3 the theoretical framework is shown and in Section 4, we describe methodology analysing the main results. Section 5 concludes.

II. Literature Review

Already considered a stylised fact, asset prices in financial markets are characterized by presenting a higher probabilistic density on the tails of its returns' distribution than the Normal distribution, and this effect is more salient in emerging markets, in comparison with developed markets; a phenomenon known as "fat tails". Extreme value theory has extensively been applied in such context in order to adjust the measure of those assets' VAR.

McNeil (1999) presents an overview of extreme value theory as a method for modelling and measuring extreme risk, in particular showing how the Peaks-Over-Threshold (POT) method may be embedded in a stochastic volatility framework to deliver useful estimates of VAR and expected shortfall, a coherent alternative for measuring market risk[1]. Embrechts (2004) discusses the economic implication of studying EVT in its relation with risk management highlighting the need for further research on theoretical aspects of the empirical findings.

Among the empirical papers we find McNeil (1997) and McNeil (1998) that describe parametric curve-fitting methods for modelling extreme historical losses; Longin and Solnik (2001) focus on extreme correlation and find that it increases in bear markets but not in bull markets; Delfiner and Girault (2002) analyze the “fat tails” phenomena for emerging markets and posit that measures based on EVT explains returns better than the ones based on the normal hypothesis. Malevergne, Pisarenko and Sornette (2006) evaluates the use of both Pareto and Stretched-Exponential distributions to describe the “fat tails” and found mixed results related to the generalized extreme value (GEV) estimator and the Generalized Pareto distribution (GDP) estimator, with the sample size influencing the convergence toward the asymptotic theoretical distribution. LeBaron and Ritirupa (2005) analyze the significance of booms and crashes in developed and emerging Markets. Their findings are consistent with prevalent notions of crashes being more salient in emerging markets than among developed markets. Additionally, Salomon and Groetveld (2003) study risk premium in a number of international markets and suggest that investors should focus more on downside risk instead of standard deviations.

Harvey (1995) reports that international CAPM fails when applied to emerging markets. During the seventies some researchers developed the "below-target semivariance capital asset pricing model" or better known as ES-CAPM (Hogan and Warren (1972), Hogan and Warren (1974), Nantell and Price (1979)) which is suggested to be useful when the distribution of returns is nonnormal and non-symmetric as is usually the case in emerging markets. Bawa and Lindenberg (1977) generalize those results and show that their model explains the data at least as well as the CAPM. Finally, Harlow and Rao (1989) derive a mean-lower partial moment (MLPM) model for any arbitrary benchmark return, which incorporates the previous models as particular cases[2]. Faff (2004) tests the Fama and French three-factor model using Australian data, and when estimated risk premia are taken into account, the support for the model is weak.

In a paper close to ours, Harvey (2000) considered the VAR as one of his 18 risk factors for developed and emerging markets. He found evidence that an asset pricing framework that incorporate skewness has success in explaining average returns. In this line of arguments, Estrada (2000) estimated returns in emerging markets using a measure of downside risk finding significant results. More recently, Stevenson (2001) used downside risk measures to construct an optimal international portfolio while Estrada (2002) proposed the downside CAPM (D-CAPM) which considers the systematic downside risk as a way to estimate the required returns in emerging markets.

Our paper contains four new contributions. First we discuss efficient procedures for estimating parameters of Pareto extreme value distribution and use these results to estimate Expected Shortfall. Second, from a range of 21 risk measures we study their statistical relationship with observed returns. Third, we implement a factor analysis to find different components, reaching a simplified but efficient empirical model; and fourth more recent price information is used with data until the end of 2005.

III. Theoretical Framework

Extreme Value Theory and Expected Shortfall

Measuring risk, in some sense, is to try to summarize its distribution with a number known as risk measure. For instance, Value-at-Risk (VAR) is a high quantile of the distribution of losses, typically the 95th or 99th percentile. VAR provides a sort of upper bound of losses, which is expected to be exceeded in very few occasions. In EVT, we are effectively concerned about those few moments in time where this limit is exceeded, which can be very harmful for the company or portfolio. The idea is to calculate the Expected Shortfall given by the average expected loss exceeding VAR. It was first proposed by Artzner et al. (1997) who posit that it is a coherent risk measure, while VAR is not. Then expected shortfall is the conditional expectation of loss given that loss is beyond the VAR level; that is expected shortfall is defined as follows:

$$ES_{\alpha}(X) = E\left[X / X \geq VAR_{\alpha}(X)\right] \qquad (1)$$

Observe that if the profit-loss distribution is Normal, the VAR and the ES essentially offers the same information, as assets with a higher VAR would also have a higher ES. VAR and ES would be just scalar multiples of the standard deviation (Yamai and Yoshiba, 2005). The problem arises when the distribution is not Normal, as tail risk may be present in the case of fat-tails and so high potential for large losses. In this case, it’s crucial to identify the distribution function that best describes those extreme bad returns and one approach is to use Extreme Value Theory.

The modern approach to EVT is through peaks-over-threshold (POT) models; these are models for all large observations, which exceed a high threshold. McNeil (1999) posits that POT models are considered to be the most useful for practical applications, due to their more efficient use of the limited data on extreme values. Within POT class of models we will concentrate on those related to a Frechet distribution. Much of our discussion is related to the idea of tail estimation under a power law assumption, say, we assume that returns are in the maximum domain of attraction of a Frechet distribution, so that the tail of the survival function is a power law times a slowly-varying function:

$$P(X > x) = k(x)x^{-\alpha} \qquad (2)$$

As pointed out by Diebold et al. (1998), often it is assumed that *k(x)* is constant; in which case, attention is restricted to densities with tails of the form:

$$P(X > x) = kx^{-\alpha} \qquad (3)$$

, with the parameters *k* and α to be estimated, respectively representing the scale and shape parameters of a Pareto distribution. In this case, the first moment is given by:

$$\mu = \left[\frac{\alpha}{\alpha - 1}\right] k^{\frac{1}{\alpha}} \qquad (4)$$

There are several ways of estimating these parameters. One of the most popular is the Hill's estimator, which is based directly on the extreme values and proceeds as follows. Order the observations $x_{(1)} > x_{(2)} > \ldots > x_{(m)}$ and form an estimator based on the difference between the *m*-th largest observation and the average of the *m* largest observations.

$$\hat{\alpha} = \left[\left(\frac{1}{m}\sum_{i=1}^{m}\ln(x_i)\right) - \ln(x_m)\right]^{-1} \qquad (5)$$

The Hill's estimator is the maximum likelihood estimator, and so has good theoretical properties: it can be shown that it is consistent and asymptotically normal, assuming iid data and that *m* grows at a suitable rate with sample size[3]. A crucial problem in using the previous estimator is to define the size of *m*, as there is an important bias-variance trade-off when varying *m* for fixed sample size: increasing *m*, we are using more data (moving toward the centre of the distribution), which reduces the variance but increases the bias, as the power law is assumed to hold just in the extreme tail. In this paper, as our main purpose is to investigate the existence of a relative risk premium among country index returns, we adopt a procedure suggested by Yamai and Yoshiba (2002) and select *m* as the one day $VAR_{95\%}$ and $VAR_{99\%}$ for each country index.

In fact we use Hill's estimator as a reference point, but will privilege the estimation by linear regression. Simply note that

$$P(X > x) = kx^{-\alpha} \qquad (6)$$

Implies,

$$\ln P(X > x) = \ln(k) - \alpha \ln(x) \qquad (7)$$

, so that α is the slope coefficient in a simple linear relationship between the log tail of the empirical survival function and the log extreme values, and then we are able to estimate both k and α by a linear regression over the survival *m* observations.

The basic insight in this approach is that the essence of the tail estimation problem is fitting a log-linear function to a set of data, and so easy to handle and good for practical purposes. In order to verify the goodness of fit, we use MSE (mean square error), R^2, comparing the results with the ones obtained using the Hill's estimator.

Moreover, in order to calculate the expected shortfall (ES), we know that it is related to VAR by:

$$ES_\alpha = VAR_\alpha + E\left[X - VAR_\alpha / X > VAR_\alpha\right] \qquad (8)$$

, where the second term is the mean of the excess distribution over the threshold VAR_α. This has the probability distribution given by the power law assumption (Pareto distribution).

Risk Premium

The framework proposed in this paper to infer whether there is a risk premium associated to the VAR and to ES, follows APT's basic insight. Any required return can be thought of as having two components: a risk-free rate, and a set of risk premium. The first component is compensation required for the expected loss of purchasing power, which is demanded even for a riskless asset. The second component is extra compensation for bearing risk, which depends on the asset considered, say, expected returns should include rewards for accepting risk. APT provides an approximate relation for asset returns with an unknown number of unidentified factors

We take the perspective of a U.S.-based, internationally diversified investor. Thus the risk-free rate should compensate this investor for the dollar's expected loss of purchasing power, and the risk premium should compensate the investor for bearing different risks (world market portfolio, extreme events, etc.). Our empirical model may be seen as an extension of Estrada (2000). We define observed return for country index *i* as:

$$RR_i - R_f = a_0 + a_{1i}(RM_{1i}) + \ldots + a_{Ki}(RM_{Ki}) + \text{error} \qquad (9)$$

, where RR_i is the observed dollar return, R_f is the U.S. risk free rate and RM_{ji} are risk factors $j = 1,..K$, and i indexes the markets. In the empirical application we use average daily observed returns in U.S. dollars for country index i, which we express as RR_i.

From the several risk measures available in the literature (see for instance Harvey, 2000) we choose 21 different factors, focusing on extreme risk measures like VAR and ES. The measures are:

1. **SR** (systematic risk) measured by standard market's model beta and is estimated through the following "world version" of a single factor model with R_{mt} denoting the return on the MSCI world index. We estimate the following regression[4]:

$$R_{it} - r_{ft} = \alpha_i + \beta_i \lfloor R_{mt} - r_{ft} \rfloor + e_{it} \qquad (10)$$

,where r_{ft} is the U.S. 3-month Treasury bill rate and e_{it} is the residual.

2. **Down-βiw** is the β coefficient from market model (10) using observations when country returns and world returns are simultaneously negative.

3. **Down-βw** is the β coefficient from market model (10) using observations when world returns are negative.

4. **IR** (idiosyncratic risk) is the standard deviation of residuals e_{it} , in model (10)

5. **TR** (total risk) is the standard deviation of the country's index return.

6. **σ-garch(1,1)** is the volatility forecast considering a GARCH (1,1) model to describe country index return's volatility.

7. **VAR95t** is the parametric VAR at 95% confidence level. The volatility considered is the one-day-ahead GARCH(1,1) forecast. Expected daily return is

set equal to zero. One day $VAR_{95\%}$ is calculated for each index, using the expression (see Jorion, 1995) given by:

$$VAR_{95\%} = z * PV * \sigma * \sqrt{\Delta t} \qquad (11)$$

, which in our case, considering a unit monetary investment (present value (PV) = 1), one day investment horizon[5] and a confidence level of 95% (z = 1.6448, assuming a Normal Distribution), is equivalent to:

$$VAR_{95\%} = 1.6448 * \sigma$$

8. **VAR95d** is the empirical VAR, say the value of the observed return representing the 5th lowest percentile.

9. **VAR99t** is the parametric VAR similar to VAR95t but considering a confidence interval of 99%.

$$VAR_{99\%} = 2.3263 * \sigma$$

10. **VAR99d** is the empirical VAR, say the value of the observed return representing the 1st lowest percentile.

11. **ES95t** is the parametric Expected Shortfall using as the threshold VAR95d.

12. **ES95d** is the sample average of returns below the 5th percentile level (VAR95d).

It's important to point out that we are assuming ES and VAR as measures of losses and so are represented by positive numbers. In order to consider risk factors related to semi-standard deviation, we used the following expression:

$$Semi - B = \sqrt{(1/T)\sum_{t=1}^{T}(R_t - B)^2} \text{ , for all } R_t < B, \qquad (12)$$

13. **Semi-Mean** is the semi standard deviation with respect to the average return of the market. (B = average return of the market)

14. **Semi-0** is the semi standard deviation with respect to 0. (B = 0)

15. **Semi-rf** is the semi standard with respect to the risk free rate. (B = U.S. risk free rate)

16. **kurt** is the kurtosis of the return distribution.

17. **skew** is the unconditional skewness of returns. It's calculated by:

$$skew = \frac{\text{Mean}(e_i^3)}{\text{StandardDev}(e_i)^3} \qquad (13)$$

18. **skew5%** is given by:

{(return at the 95th percentile level – mean return) - (return at the 5th percentile level – mean return)} – 1 (14)

19. **coskew1** represents coskewness definition 1 (Harvey and Siddique, 2000).

$$\cos kew1 = \frac{\frac{\sum e_i e_m^2}{T}}{\sqrt{\frac{\sum e_i^2}{T} * \frac{\sum e_m^2}{T}}} \qquad (15)$$

$$\text{where}$$

$$e_{mt} = R_{mt} - Avg(R_{mt}) \qquad (16)$$

20. **coskew2** represents coskewness definition 2 (Harvey and Siddique, 2000).

$$\cos kew2 = \frac{\frac{\sum e_i e_m^2}{T}}{(\sigma_{e_m})^3} \qquad (17)$$

21. **size** which is given by the natural logarithm of the market capitalization related to each country participation in the MSCI world index[6].

For further reference, in this paper, when we say beta related risk factors, we are considering the factors: SR, Down-βiw and Down-βw; when we refer to distributional risk factors, we are considering: TR, σ-garch(1,1), VAR95t, VAR95d, VAR99t, VAR99d, ES95t, ES95d, ES99d, Semi-Mean and Semi-0; and when we refer to skewness factors we are considering skew, skew5%, coskew1 and coskew2.

So, firstly we use the time series of each country index to calculate their individual different risk factors (for instance, the beta is estimated using eq. (10)) and then we implement a cross-sectional analysis (based on eq. (9)) to identify the risk premium associated to each risk factor.

IV. Methodology and Results

In order to verify if there is a risk premium associated to the measures of risk previously discussed, we examined a sample of daily index returns (MSCI database) for the period from April 4^{th} 1995 to December 30^{th}, 2005. We considered the following developed markets: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States; and the following emerging markets: Argentina, Brazil, Chile, Colombia, Hong Kong, India, Indonesia, Israel, Jordan, Malaysia, Mexico, New Zealand, Pakistan, Peru, Philippines, Poland, Singapore, South Africa, South Korea, Taiwan, Thailand, Turkey and Venezuela. The use of country indices is to avoids the thin trading effect - the returns data impacted by censoring, when using daily data (Brooks et al., 2005).

So in the end we used 41 country indices daily dollar returns time series, the risk free rate time series and the time series of the world index also given by MSCI, representing 123,376 observations. We considered the 3-month Treasury Bill yield as the risk free rate. Once we had the time series of the observed daily returns it was possible to calculate the risk factors defined in the previous section for each country. Tables 1 and 2 list observed returns and risk factors for each market.

Surprisingly, observed average daily dollar return for emerging markets (0.015%) is lower than for developed countries (0.034%). This is due to the negative result offered by most Asian emerging markets (Hong Kong, India, Indonesia, Israel, Jordan, Malaysia, Pakistan, Philippines, Singapore, Taiwan and Thailand). If we exclude those countries, average observed return rises to (0.038%). A similar result happens with beta related risk factors as they were in general higher for developed markets. It's important to notice that despite the fact that the average betas (SR) for both markets were lower than 1, if we weight each country's beta by its size, we would reach an overall beta close to 1. Observe that the U.S. market represents almost 50% of the world market and has a beta of 1.141.

About the distributional risk factors, usual results are found as emerging markets present in general higher values for extreme risk measures. As usually found (Delfiner and Girault, 2002), kurtosis for both markets were higher than 3 pointing out to fat tails in return's distribution, and this coefficient was higher for emerging markets. The skewness factors were negative for both markets, except for skew5%.

In order to highlight the fat tail phenomena, in Figure 1 we may observe the QQ plot comparing observed daily returns for the world index to the normal distribution. It can be seen that for negative extreme tail, observed returns appear more often than what is predicted by the normal distribution, especially when returns are lower than -1.4% (aprox. equals to the VAR95d). So the normal approximation seems to perform well for the central part of the distribution but offers bad results for the tails. Recall that the kurtosis is usually higher than 3 (for the world index, kurtosis is equals to 5.747). This fact raises the need for a better understanding of the tail distribution in order to predict extreme losses. Chung, Johnson and Schill (2004) using US equity data from 1930 to 1998 rejected normality of returns for daily, weekly, monthly, quarterly and semi-annual intervals. In the absence of normality, we expect investors to be concerned about the shape of the tails of the distribution of portfolio returns.

We then implemented the EVT analysis. We considered *m* as the number of failures in the VAR prediction, say when the observed return loss was greater than the empirical VAR measures (VAR95d, VAR99d). Then, we adjusted a power function to each of the instrument's excess loss time series. At this point we estimated the parameters by the two procedures described above: Hill (Maximum Likelihood Estimator) (eq. 5) and LS (eq. 7). The average results for the parameters of the Pareto distribution considering all the individual stock indices are given in the Tables 3 and 4.

Figure 2 shows the graph of the Pareto Distribution considering the average (LS) parameters estimation for developed and emerging markets. It can be seen that the slope parameter (alfa) is higher for the developed countries' sample while the scale parameter (K) is lower. This supports the evidence that extreme losses should be higher for emerging markets. For instance, the probability that in the emerging market's sample we have a daily loss over 5% is around 40%, while this probability is just 14.5% for the developed countries' sample.

Table 7 provides the result of a t-test over the difference between the alfa parameters estimated by LS and Hill procedures. Observe that the difference is statistically different from zero[7], indicating that the estimation procedures are offering different results. In order to compare the estimation procedures we calculated the coefficient of determination (R^2) for the LS estimation and the MSE (mean square error) for both methodologies. Average results are presented in Tables 5 and 6.

If we consider as the threshold the VAR95d, we may observe that least squares estimator offered a better fit to the extreme losses data with a mean square error (MSE) much lower than the one provided by the Hill estimator. Also the R^2 for the regression used in the LS analysis was on average 0.96 again supporting the good adjustment of the Pareto distribution.

Taking VAR99d as the threshold, LS provides again better results than Hill. The R^2 greater than 0.90 supports the LS estimation of the Pareto distribution parameters. It's good to highlight that when we use the VAR99d as the threshold, we reduce the number of extreme losses of each return time series. In our case, each extreme loss series had around 28 observations instead of 150 when we used the VAR95d as the threshold.

As the LS methodology offered satisfactory results, we used the LS estimators of α and k to define the survivor function of the extreme losses (Pareto Distribution). We calculate the parametric expected shortfall (ES95t) for each country following the discussed model (eq. 4). The summary results for developed and emerging markets are given in Tables 1 and 2. On average, the parametric expected shortfall measure (ES95t) for emerging markets was higher than the one for developed countries. Also, we may observe that the ES of the index is much lower than the one obtained just by averaging the ES for each market. For instance, ES95t is equal to 2.208 for the world index (Table 1) and equals to 4.202 if we average this measure for all countries. This empirical evidence shows that the diversification effect holds for the expected shortfall.

Once we have all the risk measures defined and calculated over each market's time series, we begin the cross section regression analysis and the starting point is to compute the correlation matrix among the considered risk variables (Tables 8 and 9). These tables show the correlation matrix for developed and emerging countries. A correlation coefficient higher, in absolute terms, to 0.560 is statistically significant at a 1% confidence level; and if its value is between 0.430 and 0.560, it is significant at a 5% confidence level.

Taking these ranges into account we can observe some differences among both samples. Considering the correlations with the return we can see that the beta risk factors' coefficients were positive but not significant for developed and emerging countries, except for Down-β_w which was significant for emerging markets. This initially indicates that CAPM models will not perform well in both samples. About the distributional risk factors, most of them offered positive and significant coefficients for developed countries, while they were negative and not significant for emerging markets. Coskewness was negative and significant for emerging markets, supporting the results found in Harvey (2000) and skew5% was positive and significant for developed countries.

All previous results are in line with the general intuition that a higher risk should imply a higher return, however when we compare both samples we observe that emerging markets present higher risk and lower return when compared to the developed countries sample. At the same time the numbers suggest that the two groups of risk factors (beta-related and distributional factors) are positively (and in general significantly) correlated within each group for both samples. This empirical fact could be considered as an evidence of "home-bias"[8] as market participants are investing and so requiring reward for the risk faced just within an asset group, say developed countries or emerging markets.

It's important to highlight the correlations between VAR and ES risk measures. The correlations among them in both samples were in general positive and significant, however far from 1 which would be the case if returns were following a Normal Distribution.

Regression Analysis

We first implemented regressions considering each of the risk factors acting individually. These regressions examine the bivariate relation between the average returns and the average risk measures. Throughout all this section, we considered to be significant a p-value lower than 5% and we used White (1980) heteroskedasticity consistent covariance matrix. The results related to the all markets sample are summarized in Table 10.

We can observe that the beta-related risk factors were positive and significant, explaining around 22% of the average returns. The first regression is the classic CAPM and has an R^2 equals to 20.17%. Downside betas work well for the world sample explaining around 23% and this result comes especially from emerging markets. The intercepts are not significantly different from zero. This seems to support the CAPM. The distributional risk factors came out to be in general negative but not significant. The results for the skewness risk factors were negative but not significant. Size provided a positive but not significant result. The previous results give support to the use of the CAPM model in a world sample analysis. We performed the same analysis segregating developed and emerging markets (Tables 11 and 12).

From those tables, we have evidence that CAPM doesn't perform well in both samples taken apart. As pointed out in Estrada (2000), the lack of explanatory power of beta can be explained by: the markets are not fully integrated; the world-market portfolio is not mean-variance efficient; the model is mispecified; and finally, returns and betas may be uncorrelated if these two magnitudes are summarized by long-term averages but their true values change over time. In the multivariate regressions we could see that adding another risk variable (like VAR95t for the case of developed markets) the systematic risk (beta) became significant.

Considering the developed countries sample, distributional risk measures were in general positive and significant explaining up to 33.18% (in the case of ES95t) of the returns variation, indicating the relevance of downside extreme risk for sophisticated markets. VAR, ES and Semi-deviation measures are important and priced in developed countries. Related to skewness risk factors, just skew5%, which is correlated to distributional risk measures, offered a positive and significant coefficient.

In the case of emerging markets, downside betas and coskewness' measures were significant explaining in the case of coskew1 30.52%. Estrada

(2000) already has shown the relevance of down-side beta for emerging markets. In terms of coskewness, as suggested in Harvey and Siddique (2000), if asset returns have systematic skewness, expected returns should include rewards for accepting this risk. Emerging markets exhibit a more skewed return distribution and so we found significant coefficients for this risk measure. Also note that in a world where investors care about skewness of their portfolios, coskewness should count and skewness itself should not (this is similar to the roles played by beta and volatility in the classic CAPM). This is what we found for emerging markets.

On the other hand, idiosyncratic risk (IR) and total risk (TR) were not significant for emerging markets, however were priced in developed countries. Asset pricing theory predicts that only systematic variance should be priced but 31.75% of the developed countries' return was explained by total variance.

As the usual beta came out to be significant in the bivariate regression, considering the whole sample, we decided to investigate multivariate possibilities. The following regressions use two risk factors (SR + another risk measure). Tables 13, 14 and 15 provide the results for the whole sample, developed markets and emerging markets respectively.

When we considered all countries in the sample, the multivariate regressions don't seem to improve the CAPM analysis as we remain with an explanation (R^2) of around 27%, due to the beta, and together with it, just coskew1 risk factor was negative and statistically significant. Kurtosis, skewness, skew5% and size were negative but not significant. However when we segregate the sample we find better results for developed countries, with the R^2 raising up to around 34.67%. The distributional risk factors were positive and significant.

Beta factor became significant when considered together with σ-garch(1,1), VAR95t, VAR99t, coskew1, coskew2 and size.

In emerging markets, beta became positive and significant in most regressions, explaining around 20%, however, Down-βw, coskew1 and coskew2 offered better results and in these cases beta was not significant[9].

As a last analysis we addressed the several risk measures presented before and, using a principal components approach, tried to identify if they load in different factors or are offering the same information. We can observe from the Tables 8 and 9 that there's a high correlation among groups of risk factors, which would lead us to initially suppose the existence of two risk components, one related to the market risk and the other dealing with the distributional characteristics. An important result to highlight is the positive and significant correlation between the observed ES and the one using the model presented in this paper, specially for developed countries (0.984), again supporting the use of the Pareto approximation to describe the tail distribution. When we run a principal components' analysis over the 21 risk factors we find the results presented in Table 16.

The principal components analysis offered 4 factors for the all countries sample; one related to beta, another related to a distributional characteristic and the remaining 2 linked to kurtosis and skewness measures. The results for developed and emerging markets are mixed reaching 3 and 4 factors respectively. Recall that distributional risk factors in general were not significant for the emerging market's sample.

Summing up, the empirical evidence suggests that returns extreme distributional characteristics have a risk-premium associated with them in developed markets. In the case of emerging markets, it is coskewness measures and, to a lower degree, downside betas which are correlated with observed returns.

V. Conclusions

In this paper, we had four main goals: first discuss efficient procedures for estimating parameters of Pareto extreme value distribution and use these results to estimate Expected Shortfall; second, from a range of 21 risk measures we studied their statistical relationship with observed returns; third, we implemented a factor analysis to find different components, reaching a simplified but efficient empirical model; and fourth more recent price information was used with data until the end of 2005, and the results were compared with previous works (Estrada, 2000; Harvey, 2000). We used daily data from 41 emerging and developed countries, in the period from 1995 to 2005.

Surprisingly, over a ten years period, observed average daily dollar return for emerging markets (0.015%) is lower than the one for developed countries (0.034%) whereas total risk is higher in emerging (1.833%) than in developed (1.227%) countries. This fact challenges conventional wisdom. The findings support the use of a Pareto distribution to describe the tails and the log linear regression estimation of its parameters, as a better procedure if compared to the Hill's estimator (Maximum Likelihood Estimator). Related to the risk premium, we observed a significant relationship between observed returns and extreme distributional characteristics for developed countries, but not for emerging markets. CAPM seems to be supported when we consider the whole sample but fails when we segregate the markets. The results also support the use of coskewness as a risk measure in line of the Harvey and Siddique (2000) model, especially for emerging markets. The factor analysis for all countries indicated four components: one related to beta, another related to distributional risk factors and the others mixed among the kurtosis and coskewness variables.

It's important to highlight that the explanation of the cross section of average returns remains a puzzle, as we were able to explain in the multivariate situation no more than 35% of the variation in returns. This level of explanation is usually what one gets when uses CAPM or APT based models like us. So, an avenue of opportunities is open by extending these models to accept for instance behavioural aspects, relaxing the assumption about completely rational (risk-averse) market participants as suggested in Barberis and Thaler (2003).

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Footnotes:

[1] Extreme value theory provides a natural approach to VAR and ES estimation, and there is already a considerable literature on the subject. See Danielsson and de Vries (1997), Embrechts et. al (1997), and Diebold et. al (1998).

[2] Nawrocki (1999) presents a review of such literature.

[3] See, e.g., Embrechts, Klüppelberg and Mikosch (1997, p. 348).

[4] We used Newey-West procedure in order to generate a covariance matrix that is consistent in the presence of both heteroskedasticity and autocorrelation of unknown form. Some recent papers like Cho and Engle (1999), Andersen, Bollerslev, Dieboldc and Wu (2003) and Hwang and Salmon (2006) use this procedure.

[5] "The Application of Basel II to Trading Activities and the Treatment of Double Default Effects" (BIS, 2005) suggests the use of a one-to-ten days horizon in the measurement of VAR.

[6] Barry, Goldreyer, Lockwood and Rodriguez (2002) found that mean returns for small firms exceed mean returns for large firms. Fama and French (1995) proposed size as a factor for their pricing model. This effect has been reversed in recent periods (Al-Rjoub et al. 2005)

[7] Considering a significance level of 0.10.

[8] See French and Poterba (1991) or Shapiro (1999) for a review of the house bias effect.

[9] We also tried to improve the model introducing in the regression the size of each country's market. A 3-factor model was fitted where two of them are beta and the natural logarithm of the market capitalization. The coefficient for size was negative in almost all regressions suggesting that small markets get a

premium. However, due to colinearity problems results are of dubious statistical significance. Full results are available on request.

Appendix

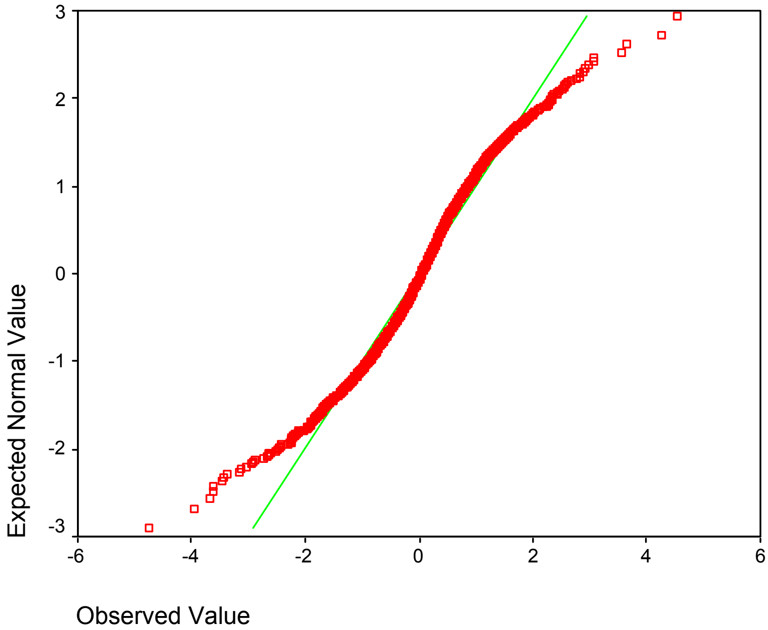

Figure 1. Quantile-Quantile graph

This figure presents QQ plot comparing observed daily returns for the world index with the normal distribution.

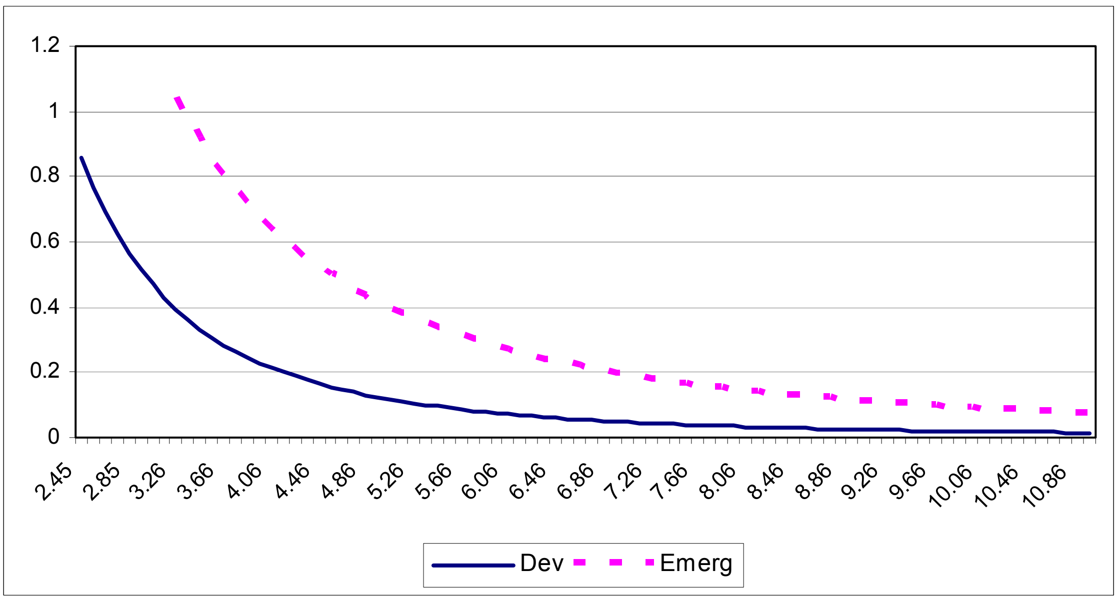

Figure 2. Pareto Distribution for Developed and Emerging Markets

This figure represents the Pareto Distribution considering the average parameters of Alfa and K, for the developed and emerging markets samples.

Table 1: Summary Statistics – Developed Markets (MSCI Data)

Market	E[R]	SR	Down-βiw	Down-βw	IR	TR	σ-garch(1,1)	VAR95t	VAR95d	VAR99t	VAR99d	ES95t	ES95d	Semi-Mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	size
Australia	0,030	0,407	0,419	0,483	1,043	1,096	0,834	1,316	1,713	1,884	2,861	2,885	2,455	0,789	0,773	0,779	6,622	-0,188	2,407	-0,153	-0,191	10,258
Austria	0,035	0,371	0,425	0,377	1,011	1,057	0,807	1,268	1,634	1,818	3,068	2,897	2,489	0,764	0,747	0,752	5,212	-0,305	2,388	-0,130	-0,158	7,809
Belgium	0,030	0,766	0,675	0,729	0,999	1,185	0,751	1,158	1,897	1,670	3,280	3,157	2,784	0,846	0,832	0,836	7,551	0,079	2,700	0,105	0,126	8,776
Canada	0,046	0,953	0,883	1,105	0,861	1,171	0,687	1,041	1,866	1,509	3,280	3,587	2,803	0,868	0,846	0,852	8,805	-0,672	2,658	-0,168	-0,173	10,642
Denmark	0,045	0,561	0,508	0,591	1,036	1,136	0,803	1,252	1,816	1,799	3,233	3,046	2,704	0,827	0,804	0,810	5,294	-0,315	2,649	-0,069	-0,086	8,197
Finland	0,054	1,503	1,140	1,505	2,020	2,376	1,028	1,590	3,765	2,291	6,352	7,006	5,668	1,733	1,707	1,712	9,581	-0,475	6,505	0,042	0,102	8,975
France	0,031	1,057	0,919	1,082	0,955	1,299	0,767	1,205	2,114	1,728	3,759	3,377	3,013	0,938	0,922	0,927	5,198	-0,153	3,150	0,012	0,014	10,866
Germany	0,026	1,227	0,990	1,202	1,038	1,457	0,752	1,175	2,438	1,687	4,173	3,847	3,489	1,054	1,042	1,046	5,521	-0,172	3,756	0,085	0,106	10,561
Ireland	0,025	0,541	0,471	0,581	1,080	1,170	0,885	1,399	1,853	2,003	3,074	3,350	2,764	0,855	0,843	0,848	6,983	-0,440	2,634	-0,038	-0,049	8,391
Italy	0,029	0,913	0,722	0,955	1,055	1,300	0,755	1,196	2,114	1,711	3,391	3,286	2,953	0,930	0,916	0,921	5,205	-0,105	3,199	-0,011	-0,014	9,946
Netherlands	0,025	1,040	0,896	1,037	0,997	1,320	0,742	1,155	2,072	1,660	3,910	3,720	3,127	0,951	0,938	0,943	6,396	-0,187	3,123	0,068	0,082	9,862
Norway	0,033	0,657	0,633	0,781	1,130	1,256	0,948	1,479	1,902	2,125	3,695	3,760	2,993	0,922	0,906	0,912	7,360	-0,430	2,844	-0,161	-0,218	8,266
Portugal	0,024	0,538	0,539	0,606	0,993	1,090	0,756	1,197	1,741	1,711	2,988	2,878	2,536	0,783	0,771	0,776	5,671	-0,190	2,505	-0,137	-0,163	7,429
Spain	0,048	0,993	0,856	1,019	1,078	1,359	0,741	1,144	2,229	1,649	3,588	3,435	3,106	0,975	0,951	0,956	5,324	-0,066	3,352	0,002	0,002	9,946
Sweden	0,044	1,200	0,976	1,270	1,391	1,713	0,839	1,281	2,714	1,853	4,778	4,445	3,986	1,220	1,198	1,203	6,095	0,006	4,335	0,032	0,053	9,470
Switzerland	0,035	0,788	0,642	0,797	0,921	1,131	0,746	1,168	1,787	1,677	3,055	2,983	2,605	0,809	0,792	0,797	6,548	-0,064	2,523	-0,008	-0,009	10,551
United Kingdom	0,022	0,820	0,675	0,805	0,849	1,090	0,612	0,962	1,774	1,378	2,972	2,829	2,538	0,786	0,775	0,780	5,211	-0,156	2,468	0,043	0,044	11,794
United States	0,032	1,141	1,026	1,161	0,559	1,102	0,565	0,874	1,786	1,259	2,858	2,875	2,514	0,788	0,772	0,777	6,440	-0,115	2,519	0,034	0,023	13,325
Average	0,034	0,814	0,705	0,847	1,001	1,227	0,738	1,151	1,959	1,653	3,385	3,335	2,870	0,886	0,870	0,875	6,054	-0,208	2,932	-0,024	-0,027	9,214
World Index	0,024	1,000	1,000	1,000	0,000	0,833	0,461	0,712	1,364	1,027	2,241	2,208	1,965	0,604	0,592	0,597	5,747	-0,196	1,6536	-0,16725	-1,48E-16	14,075

E[R] : Mean returns; **SR** : Systemic risk; **Down-βiw** : Downside beta calculated using observations when index and world index fall together; **Down-βw** : Downside beta calculated using observations when world index falls; **IR** : Idiosyncratic risk; **TR** : total risk; **σ-garch(1,1)** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR** : Value at risk; **VAR95t** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99t** is the theoretical VAR similar to VAR95t but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d; **ES99t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99d; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-Mean** : semideviation with respect to mean; **Semi-0** : semideviation with respect to 0; **Semi-rf** : semideviation with respect to Risk free rate; **kurt** : kurtosis; **skew** : skewness; **skew5%** : skewness calculated using formula 14; **coskew1:** coskewness definition 1; **coskew2:** coskewness definition 2; **size** : natural logarithm of the market capitalization.

Table 2: Summary Statistics – Emerging Markets (MSCI Data)

Market	E[R]	SR	Down-βiw	Down-βw	IR	TR	σ-garch(1,1)	VAR95t	VAR95d	VAR99t	VAR99d	ES95t	ES95d	Semi-Mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	size
Argentina	0.021	0.957	0.872	1.212	2.262	2.398	1.537	2.436	3.341	3.483	6.275	9.135	5.521	1.792	1.783	1.788	56.665	-2.413	5.633	-0.160	-0.435	6.251
Brazil	0.040	1.265	1.228	1.522	1.929	2.198	1.469	2.291	3.534	3.292	6.225	6.284	5.262	1.597	1.578	1.583	8.301	-0.185	5.753	-0.226	-0.524	9.246
Chile	0.009	0.623	0.487	0.713	1.052	1.173	0.899	1.455	1.809	2.068	3.162	3.154	2.699	0.838	0.833	0.839	6.607	-0.147	2.672	-0.159	-0.201	7.262
Colombia	0.046	0.188	0.112	0.249	1.417	1.426	1.269	2.041	2.131	2.906	3.979	3.951	3.312	0.991	0.969	0.975	9.391	0.253	3.408	-0.131	-0.222	5.963
Hong Kong	0.011	0.664	0.674	0.776	1.536	1.633	0.683	1.076	2.469	1.542	4.432	4.980	3.847	1.156	1.151	1.156	13.792	0.112	3.841	-0.090	-0.166	9.113
India	0.026	0.258	0.324	0.438	1.572	1.587	1.249	1.967	2.432	2.818	4.621	4.556	3.787	1.153	1.140	1.146	6.961	-0.294	3.890	-0.111	-0.209	8.626
Indonesia	-0.018	0.492	0.523	0.418	3.250	3.275	1.265	1.996	4.184	2.858	9.114	17.798	7.910	2.385	2.392	2.396	32.719	-1.030	7.212	-0.014	-0.055	7.167
Israel	0.034	0.855	0.639	0.933	1.389	1.561	1.000	1.569	2.482	2.251	4.708	4.642	3.816	1.128	1.112	1.117	7.896	-0.291	3.930	0.014	0.023	8.000
Jordan	0.042	0.017	0.097	0.090	0.958	0.957	1.620	2.664	1.337	3.767	2.584	3.281	2.205	0.651	0.634	0.639	13.920	0.207	1.919	-0.093	-0.107	5.558
Malaysia	-0.016	0.308	0.331	0.322	1.923	1.940	0.493	0.786	2.536	1.122	5.627	7.294	4.590	1.312	1.319	1.324	40.239	1.184	3.898	-0.085	-0.195	7.860
Mexico	0.061	1.231	1.145	1.477	1.422	1.753	1.258	1.948	2.656	2.805	4.446	4.902	3.850	1.243	1.213	1.219	10.312	-0.095	4.468	-0.175	-0.299	8.626
New Zealand	0.008	0.294	0.345	0.315	1.311	1.333	1.153	1.842	2.143	2.627	3.507	3.768	3.077	0.970	0.966	0.971	16.151	-0.552	3.109	-0.006	-0.010	6.810
Pakistan	0.006	0.056	0.052	0.135	2.060	2.061	1.215	1.972	3.297	2.800	5.825	6.309	5.121	1.509	1.506	1.511	9.302	-0.396	5.314	-0.047	-0.117	5.558
Peru	0.035	0.441	0.560	0.499	1.363	1.412	1.265	2.019	2.069	2.881	3.978	4.234	3.233	0.997	0.980	0.985	9.002	-0.029	3.363	-0.191	-0.313	6.251
Philippines	-0.040	0.307	0.375	0.382	1.747	1.765	1.246	2.053	2.676	2.903	5.011	4.878	4.138	1.198	1.218	1.223	19.291	1.097	4.180	-0.070	-0.146	5.963
Poland	0.037	0.714	0.683	0.764	1.797	1.893	1.368	2.180	2.982	3.112	5.119	5.017	4.331	1.353	1.335	1.340	5.613	-0.158	5.059	-0.093	-0.200	7.349
Singapore	-0.002	0.550	0.558	0.546	1.344	1.420	0.546	0.879	2.213	1.252	3.776	4.023	3.262	0.996	0.997	1.002	10.644	0.185	3.373	0.059	0.095	8.448
South Africa	0.018	0.760	0.723	0.899	1.377	1.515	1.247	1.976	2.349	2.826	4.445	4.562	3.691	1.111	1.103	1.108	7.635	-0.514	3.729	-0.179	-0.296	9.141
South Korea	0.020	0.734	0.577	0.819	2.617	2.687	1.312	2.074	3.983	2.968	7.304	8.134	6.264	1.867	1.858	1.863	18.057	0.570	6.797	-0.086	-0.269	9.701
Taiwan	-0.008	0.375	0.440	0.483	1.746	1.774	1.179	1.906	2.880	2.709	4.560	4.538	4.008	1.238	1.242	1.248	5.498	0.016	4.955	-0.084	-0.176	9.459
Thailand	-0.036	0.543	0.274	0.508	2.234	2.279	1.014	1.654	3.533	2.345	6.369	5.843	5.184	1.528	1.546	1.552	10.753	0.784	5.980	0.048	0.128	7.429
Turkey	0.042	0.681	1.134	0.896	3.202	3.251	1.706	2.712	4.994	3.874	9.251	9.354	7.703	2.317	2.297	2.302	8.011	-0.092	8.729	-0.105	-0.403	7.637
Venezuela	0.012	0.525	0.688	0.800	2.662	2.698	2.479	4.015	3.309	5.704	6.671	22.017	5.891	2.098	2.094	2.098	182.880	-6.362	5.754	-0.107	-0.343	4.864
Average	0.015	0.535	0.535	0.633	1.757	1.833	1.186	1.896	2.722	2.705	5.041	6.361	4.279	1.310	1.303	1.308	21.235	-0.340	4.457	-0.087	-0.185	7.178

E[R] : Mean returns; **SR** : Systemic risk; **Down-βiw** : Downside beta calculated using observations when index and world index fall together; **Down-βw** : Downside beta calculated using observations when world index falls; **IR** : Idiosyncratic risk; **TR** : total risk; **σ-garch(1,1)** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR** : Value at risk; **VAR95t** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99t** is the theoretical VAR similar to VAR95t but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d; **ES99t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99d; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-Mean** : semideviation with respect to mean; **Semi-0** : semideviation with respect to 0; **Semi-rf** : semideviation with respect to Risk free rate; **kurt** : kurtosis; **skew** : skewness; **skew5%** : skewness calculated using formula 14; **coskew1:** coskewness definition 1; **coskew2:** coskewness definition 2; **size** : natural logarithm of the market capitalization.

Table 3: Summary Statistics - Pareto Distribution Parameters (Developed Markets)

Threshold	Parameter	Mean	Min.	Max.	Std. Dev.
VAR95d	Alfa (Hill)	3.007	2.574	3.393	0.234
	Alfa (LS)	2.735	2.121	3.203	0.341
	K (LS)	9.919	3.881	27.609	7.183
VAR99d	Alfa (Hill)	4.441	2.977	6.531	0.947
	Alfa (LS)	3.886	2.396	5.315	0.994
	K (LS)	362.450	16.202	2009.700	520.450

Table 4: Summary Statistics - Pareto Distribution Parameters (Emerging Markets)

Threshold	Parameter	Mean	Min.	Max.	Std. Dev.
VAR95d	Alfa (Hill)	2.712	1.977	3.443	0.363
	Alfa (LS)	2.193	1.167	2.911	0.450
	K (LS)	13.963	1.754	51.001	13.219
VAR99d	Alfa (Hill)	3.581	2.274	6.217	0.841
	Alfa (LS)	2.784	0.846	4.435	0.940
	K (LS)	352.340	4.197	3328.100	723.450

Table 5: Pareto Distribution Parameter Estimation (Developed Markets)

Threshold	Criteria	Mean	Min.	Max.	Std. Dev.
VAR95d	R2 (LS)	0.956	0.922	0.991	0.026
	MSE (LS)	0.006	0.001	0.015	0.004
	MSE (Hill)	0.014	0.010	0.018	0.002
VAR99d	R2 (LS)	0.953	0.875	0.989	0.031
	MSE (LS)	0.004	0.001	0.012	0.003
	MSE (Hill)	0.007	0.003	0.012	0.003

Table 6: Pareto Distribution Parameter Estimation (Emerging Markets)

Threshold	Criteria	Mean	Min.	Max.	Std. Dev.
VAR 95d	R2 (LS)	0.960	0.920	0.995	0.019
	MSE (LS)	0.003	0.000	0.011	0.003
	MSE (Hill)	0.016	0.009	0.028	0.005
VAR99d	R2 (LS)	0.919	0.778	0.977	0.057
	MSE (LS)	0.005	0.001	0.011	0.003
	MSE (Hill)	0.010	0.002	0.020	0.005

Table 7: Pareto Distribution Parameter Estimation Alfa (LS) - Alfa (Hill)

	Threshold	Mean	Correlation	t	p-value
Developed Markets	VAR95d	-0.272	0.552	-2.795	0.008
	VAR99d	-0.555	0.812	-1.715	0.095
Emerging Markets	VAR95d	-0.519	0.716	-4.302	0.000
	VAR99d	-0.797	0.730	-3.029	0.004

Table 8: Correlation Matrix – Developed Markets

	E[R]	SR	Down-βiw	Down-βw	IR	TR	σ-garch(1,1)	VAR95t	VAR95d	VAR99t	VAR99d	ES95t	ES95d	Semi-Mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	size
E[R]	1.000	0.371	0.372	0.420	0.535	0.564	0.352	0.289	0.548	0.309	0.540	0.576	0.561	0.571	0.558	0.558	0.420	-0.302	0.556	-0.106	-0.010	-0.104
SR	0.371	1.000	0.982	0.987	0.391	0.747	-0.014	-0.063	0.782	-0.048	0.742	0.688	0.740	0.740	0.741	0.740	0.296	0.081	0.759	0.650	0.711	0.468
Down-βiw	0.372	0.982	1.000	0.981	0.289	0.670	-0.090	-0.144	0.703	-0.128	0.677	0.623	0.665	0.664	0.664	0.664	0.297	0.052	0.681	0.582	0.637	0.506
Down-βw	0.420	0.987	0.981	1.000	0.395	0.744	0.016	-0.037	0.774	-0.021	0.740	0.699	0.738	0.740	0.740	0.740	0.350	-0.006	0.751	0.537	0.604	0.455
IR	0.535	0.391	0.289	0.395	1.000	0.900	0.833	0.808	0.873	0.816	0.885	0.903	0.902	0.903	0.903	0.903	0.468	-0.226	0.889	0.116	0.218	-0.448
TR	0.564	0.747	0.670	0.744	0.900	1.000	0.597	0.556	0.995	0.569	0.984	0.976	0.997	0.999	0.999	0.999	0.484	-0.138	0.997	0.375	0.476	-0.092
σ-garch(1,1)	0.352	-0.014	-0.090	0.016	0.833	0.597	1.000	0.996	0.544	0.998	0.611	0.654	0.608	0.609	0.609	0.609	0.427	-0.372	0.571	-0.217	-0.156	-0.675
VAR95t	0.289	-0.063	-0.144	-0.037	0.808	0.556	0.996	1.000	0.503	1.000	0.568	0.611	0.566	0.568	0.568	0.569	0.380	-0.362	0.531	-0.235	-0.180	-0.687
VAR95d	0.548	0.782	0.703	0.774	0.873	0.995	0.544	0.503	1.000	0.516	0.976	0.959	0.992	0.993	0.993	0.993	0.442	-0.102	0.998	0.424	0.524	-0.047
VAR99t	0.309	-0.048	-0.128	-0.021	0.816	0.569	0.998	1.000	0.516	1.000	0.581	0.624	0.579	0.581	0.581	0.581	0.395	-0.366	0.544	-0.230	-0.173	-0.684
VAR99d	0.540	0.742	0.677	0.740	0.885	0.984	0.611	0.568	0.976	0.581	1.000	0.969	0.989	0.984	0.985	0.985	0.457	-0.157	0.980	0.368	0.465	-0.136
ES95t	0.576	0.688	0.623	0.699	0.903	0.976	0.654	0.611	0.959	0.624	0.969	1.000	0.984	0.984	0.983	0.983	0.624	-0.319	0.966	0.261	0.369	-0.148
ES95d	0.561	0.740	0.665	0.738	0.902	0.997	0.608	0.566	0.992	0.579	0.989	0.984	1.000	0.999	0.999	0.999	0.506	-0.180	0.995	0.366	0.469	-0.117
Semi-Mean	0.571	0.740	0.664	0.740	0.903	0.999	0.609	0.568	0.993	0.581	0.984	0.984	0.999	1.000	1.000	1.000	0.506	-0.179	0.996	0.352	0.454	-0.100
Semi-0	0.558	0.741	0.664	0.740	0.903	0.999	0.609	0.568	0.993	0.581	0.985	0.983	0.999	1.000	1.000	1.000	0.504	-0.176	0.996	0.357	0.460	-0.099
Semi-rf	0.558	0.740	0.664	0.740	0.903	0.999	0.609	0.569	0.993	0.581	0.985	0.983	0.999	1.000	1.000	1.000	0.504	-0.176	0.996	0.357	0.459	-0.100
kurt	0.420	0.296	0.297	0.350	0.468	0.484	0.427	0.380	0.442	0.395	0.457	0.624	0.506	0.506	0.504	0.504	1.000	-0.543	0.444	-0.088	0.001	-0.074
skew	-0.302	0.081	0.052	-0.006	-0.226	-0.138	-0.372	-0.362	-0.102	-0.366	-0.157	-0.319	-0.180	-0.179	-0.176	-0.176	-0.543	1.000	-0.124	0.577	0.506	0.232
skew5%	0.556	0.759	0.681	0.751	0.889	0.997	0.571	0.531	0.998	0.544	0.980	0.966	0.995	0.996	0.996	0.996	0.444	-0.124	1.000	0.392	0.493	-0.081
coskew1	-0.106	0.650	0.582	0.537	0.116	0.375	-0.217	-0.235	0.424	-0.230	0.368	0.261	0.366	0.352	0.357	0.357	-0.088	0.577	0.392	1.000	0.989	0.385
coskew2	-0.010	0.711	0.637	0.604	0.218	0.476	-0.156	-0.180	0.524	-0.173	0.465	0.369	0.469	0.454	0.460	0.459	0.001	0.506	0.493	0.989	1.000	0.354
size	-0.104	0.468	0.506	0.455	-0.448	-0.092	-0.675	-0.687	-0.047	-0.684	-0.136	-0.148	-0.117	-0.100	-0.099	-0.100	-0.074	0.232	-0.081	0.385	0.354	1.000

A correlation coefficient higher, in absolute terms, to 0.560 indicates that it is significant at a 1% confidence level; and if its value is between 0.430 and 0.560, it is significant at a 5% confidence level.

E[R] : Mean returns; **SR** : Systemic risk; **Down-βiw** : Downside beta calculated using observations when index and world index fall together; **Down-βw** : Downside beta calculated using observations when world index falls; **IR** : Idiosyncratic risk; **TR** : total risk; **σ-garch(1,1)** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR** : Value at risk; **VAR95t** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99t** is the theoretical VAR similar to VAR95t but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d; **ES99t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99d; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-Mean** : semideviation with respect to mean; **Semi-0** : semideviation with respect to 0; **Semi-rf** : semideviation with respect to Risk free rate; **kurt** : kurtosis; **skew** : skewness; **skew5%** : skewness calculated using formula 14; **coskew1:** coskewness definition 1; **coskew2:** coskewness definition 2; **size** : natural logarithm of the market capitalization.

Table 9: Correlation Matrix – Emerging Markets

	E[R]	SR	Down-βiw	Down-βw	IR	TR	σ-garch(1,1)	VAR95t	VAR95d	VAR99t	VAR99d	ES95t	ES95d	Semi-Mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	size
E[R]	1.000	0.356	0.420	0.435	-0.230	-0.167	0.336	0.309	-0.118	0.317	-0.176	-0.179	-0.173	-0.130	-0.156	-0.157	-0.120	-0.118	-0.090	-0.552	-0.513	0.063
SR	0.356	1.000	0.901	0.977	0.141	0.269	0.071	0.032	0.329	0.044	0.249	0.080	0.250	0.281	0.270	0.270	-0.004	-0.127	0.333	-0.369	-0.450	0.506
Down-βiw	0.420	0.901	1.000	0.925	0.286	0.397	0.253	0.215	0.447	0.226	0.375	0.210	0.376	0.418	0.405	0.405	0.091	-0.227	0.455	-0.452	-0.616	0.418
Down-βw	0.435	0.977	0.925	1.000	0.144	0.271	0.208	0.169	0.327	0.181	0.241	0.116	0.245	0.296	0.283	0.283	0.087	-0.233	0.335	-0.480	-0.581	0.446
IR	-0.230	0.141	0.286	0.144	1.000	0.991	0.409	0.398	0.942	0.402	0.980	0.777	0.987	0.982	0.984	0.984	0.390	-0.327	0.944	0.132	-0.231	-0.040
TR	-0.167	0.269	0.397	0.271	0.991	1.000	0.412	0.396	0.960	0.401	0.988	0.768	0.994	0.993	0.994	0.994	0.376	-0.334	0.963	0.074	-0.286	0.022
σ-garch(1,1)	0.336	0.071	0.253	0.208	0.409	0.412	1.000	0.999	0.345	0.999	0.343	0.591	0.368	0.472	0.462	0.462	0.617	-0.724	0.386	-0.349	-0.523	-0.449
VAR95t	0.309	0.032	0.215	0.169	0.398	0.396	0.999	1.000	0.326	1.000	0.325	0.586	0.351	0.455	0.446	0.446	0.625	-0.722	0.367	-0.329	-0.499	-0.478
VAR95d	-0.118	0.329	0.447	0.327	0.942	0.960	0.345	0.326	1.000	0.332	0.964	0.586	0.963	0.941	0.941	0.941	0.179	-0.182	0.996	0.079	-0.284	0.147
VAR99t	0.317	0.044	0.226	0.181	0.402	0.401	0.999	1.000	0.332	1.000	0.331	0.588	0.356	0.460	0.451	0.451	0.623	-0.723	0.373	-0.335	-0.506	-0.470
VAR99d	-0.176	0.249	0.375	0.241	0.980	0.988	0.343	0.325	0.964	0.331	1.000	0.705	0.996	0.973	0.974	0.974	0.278	-0.232	0.960	0.089	-0.261	0.055
ES95t	-0.179	0.080	0.210	0.116	0.777	0.768	0.591	0.586	0.586	0.588	0.705	1.000	0.737	0.814	0.816	0.816	0.817	-0.757	0.604	0.054	-0.217	-0.296
ES95d	-0.173	0.250	0.376	0.245	0.987	0.994	0.368	0.351	0.963	0.356	0.996	0.737	1.000	0.986	0.986	0.986	0.314	-0.282	0.960	0.085	-0.270	0.040
Semi-Mean	-0.130	0.281	0.418	0.296	0.982	0.993	0.472	0.455	0.941	0.460	0.973	0.814	0.986	1.000	1.000	1.000	0.443	-0.431	0.946	0.042	-0.322	-0.017
Semi-0	-0.156	0.270	0.405	0.283	0.984	0.994	0.462	0.446	0.941	0.451	0.974	0.816	0.986	1.000	1.000	1.000	0.446	-0.426	0.945	0.057	-0.307	-0.020
Semi-rf	-0.157	0.270	0.405	0.283	0.984	0.994	0.462	0.446	0.941	0.451	0.974	0.816	0.986	1.000	1.000	1.000	0.445	-0.426	0.945	0.057	-0.307	-0.019
kurt	-0.120	-0.004	0.091	0.087	0.390	0.376	0.617	0.625	0.179	0.623	0.278	0.817	0.314	0.443	0.446	0.445	1.000	-0.886	0.202	-0.042	-0.237	-0.454
skew	-0.118	-0.127	-0.227	-0.233	-0.327	-0.334	-0.724	-0.722	-0.182	-0.723	-0.232	-0.757	-0.282	-0.431	-0.426	-0.426	-0.886	1.000	-0.214	0.144	0.314	0.411
skew5%	-0.090	0.333	0.455	0.335	0.944	0.963	0.386	0.367	0.996	0.373	0.960	0.604	0.960	0.946	0.945	0.945	0.202	-0.214	1.000	0.057	-0.300	0.136
coskew1	-0.552	-0.369	-0.452	-0.480	0.132	0.074	-0.349	-0.329	0.079	-0.335	0.089	0.054	0.085	0.042	0.057	0.057	-0.042	0.144	0.057	1.000	0.895	-0.065
coskew2	-0.513	-0.450	-0.616	-0.581	-0.231	-0.286	-0.523	-0.499	-0.284	-0.506	-0.261	-0.217	-0.270	-0.322	-0.307	-0.307	-0.237	0.314	-0.300	0.895	1.000	-0.062
size	0.063	0.506	0.418	0.446	-0.040	0.022	-0.449	-0.478	0.147	-0.470	0.055	-0.296	0.040	-0.017	-0.020	-0.019	-0.454	0.411	0.136	-0.065	-0.062	1.000

A correlation coefficient higher, in absolute terms, to 0.560 indicates that it is significant at a 1% confidence level; and if its value is between 0.430 and 0.560, it is significant at a 5% confidence level.

E[R] : Mean returns; **SR** : Systemic risk; **Down-βiw** : Downside beta calculated using observations when index and world index fall together; **Down-βw** : Downside beta calculated using observations when world index falls; **IR** : Idiosyncratic risk; **TR** : total risk; **σ-garch(1,1)** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR** : Value at risk; **VAR95t** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99t** is the theoretical VAR similar to VAR95t but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d; **ES99t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99d; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-Mean** : semideviation with respect to mean; **Semi-0** : semideviation with respect to 0; **Semi-rf** : semideviation with respect to Risk free rate; **kurt** : kurtosis; **skew** : skewness; **skew5%** : skewness calculated using formula 14; **coskew1:** coskewness definition 1; **coskew2:** coskewness definition 2; **size** : natural logarithm of the market capitalization.

Table 10: Bivariate Regressions – All Markets

$RRi = c0 + c1*Risk_i + ui$

Risk variable	c0	p-value	c1	p-value	R2
SR	0,0035	0,6914	0,0286	0,0041	0,2017
Down-βiw	0,0006	0,9524	0,0354	0,0058	0,2202
Down-βw	0,0006	0,9480	0,0298	0,0024	0,2372
IR	0,0399	0,0000	-0,0113	0,0434	0,1144
TR	0,0401	0,0000	-0,0102	0,1038	0,0748
σ-garch(1,1)	0,0239	0,0022	-0,0003	0,9603	0,0000
VAR95t	0,0255	0,0008	-0,0012	0,7777	0,0012
VAR95d	0,0381	0,0009	-0,0059	0,2357	0,0465
VAR99t	0,0250	0,0011	-0,0006	0,8305	0,0007
VAR99d	0,0405	0,0001	-0,0038	0,1134	0,0788
ES95t	0,0323	0,0000	-0,0017	0,0279	0,0808
ES95d	0,0399	0,0000	-0,0043	0,1072	0,0744
Semi-Mean	0,0379	0,0000	-0,0123	0,1283	0,0579
Semi-0	0,0390	0,0000	-0,0134	0,0966	0,0695
Semi-rf	0,0391	0,0000	-0,0134	0,0965	0,0696
kurt	0,0260	0,0000	-0,0002	0,0860	0,0450
skew	0,0230	0,0000	-0,0017	0,6408	0,0072
skew5%	0,0346	0,0002	-0,0028	0,2630	0,0403
coskew1	0,0209	0,0000	-0,0411	0,2295	0,0261
coskew2	0,0217	0,0000	-0,0157	0,4215	0,0131
size	-0,0004	0,9811	0,0028	0,0708	0,0617

Table 11: Bivariate Regressions – Developed Markets

$RRi = c0 + c1*Risk_i + ui$

Risk variable	c0	p-value	c1	p-value	R2
SR	0.0244	0.0008	0.0113	0.1356	0.1374
Down-βiw	0.0224	0.0060	0.0157	0.1341	0.1381
Down-βw	0.0224	0.0028	0.0131	0.0878	0.1764
IR	0.0156	0.0113	0.0175	0.0007	0.2860
TR	0.0122	0.0193	0.0169	0.0000	0.3175
σ-garch(1,1)	0.0102	0.5372	0.0307	0.1603	0.1236
VAR95t	0.0145	0.4062	0.0161	0.2726	0.0838
VAR95d	0.0129	0.0225	0.0103	0.0002	0.3008
VAR99t	0.0132	0.4438	0.0120	0.2354	0.0952
VAR99d	0.0128	0.0230	0.0059	0.0003	0.2916
ES95t	0.0144	0.0012	0.0056	0.0000	0.3318
ES95d	0.0131	0.0076	0.0069	0.0000	0.3150
Semi-Mean	0.0121	0.0134	0.0235	0.0000	0.3262
Semi-0	0.0128	0.0121	0.0232	0.0000	0.3108
Semi-rf	0.0127	0.0132	0.0232	0.0000	0.3111
kurt	0.0144	0.2011	0.0031	0.0739	0.1762
skew	0.0308	0.0000	-0.0151	0.2070	0.0912
skew5%	0.0178	0.0003	0.0053	0.0001	0.3094
coskew1	0.0338	0.0000	-0.0112	0.6189	0.0113
coskew2	0.0341	0.0000	-0.0009	0.9648	0.0001
size	0.0405	0.0051	-0.0007	0.5947	0.0107

Table 12: Bivariate Regressions – Emerging Markets

$RRi = c0 + c1*Risk_i + ui$

Risk variable	c0	p-value	c1	p-value	R2
SR	-0.0007	0.9518	0.0287	0.0836	0.1266
Down-βiw	-0.0040	0.7565	0.0345	0.0469	0.1765
Down-βw	-0.0044	0.7152	0.0298	0.0284	0.1896
IR	0.0329	0.0452	-0.0096	0.2684	0.0527
TR	0.0289	0.0826	-0.0071	0.4010	0.0280
σ-garch(1,1)	-0.0116	0.4324	0.0217	0.0764	0.1127
VAR95t	-0.0091	0.5215	0.0123	0.0922	0.0952
VAR95d	0.0258	0.1816	-0.0037	0.5874	0.0140
VAR99t	-0.0099	0.4932	0.0089	0.0871	0.1003
VAR99d	0.0295	0.0934	-0.0027	0.4115	0.0309
ES95t	0.0221	0.0156	-0.0010	0.2317	0.0322
ES95d	0.0291	0.0837	-0.0031	0.3950	0.0299
Semi-Mean	0.0254	0.1122	-0.0074	0.4988	0.0168
Semi-0	0.0274	0.0817	-0.0089	0.4127	0.0245
Semi-rf	0.0275	0.0820	-0.0089	0.4120	0.0245
kurt	0.0171	0.0097	-0.0001	0.2635	0.0144
skew	0.0145	0.0221	-0.0021	0.4870	0.0139
skew5%	0.0221	0.1830	-0.0015	0.6756	0.0080
coskew1	-0.0024	0.7568	-0.1941	0.0020	0.3052
coskew2	-0.0008	0.9180	-0.0831	0.0024	0.2629
size	0.0065	0.8066	0.0012	0.7268	0.0039

Table 13: Multivariate Regressions – All Markets

$RRi = c0 + c1*SR_i + c2*Risk_i + ui$

SR / Risk	c0	p-value	c1	p-value	c2	p-value	R2
Down-βiw	0,0007	0,9478	0,0068	0,7509	0,0279	0,3169	0,2219
Down-βw	0,0001	0,9949	-0,0286	0,3876	0,0565	0,0915	0,2483
IR	0,0187	0,1021	0,0258	0,0075	-0,0090	0,0866	0,2732
TR	0,0207	0,0763	0,0293	0,0022	-0,0108	0,0645	0,2859
σ-garch(1,1)	-0,0041	0,7040	0,0304	0,0016	0,0063	0,3520	0,2132
VAR95t	-0,0031	0,7722	0,0303	0,0016	0,0034	0,4197	0,2103
VAR95d	0,0205	0,1109	0,0307	0,0017	-0,0074	0,1270	0,2748
VAR99t	-0,0034	0,7515	0,0303	0,0016	0,0025	0,3986	0,2111
VAR99d	0,0209	0,0806	0,0291	0,0022	-0,0040	0,0796	0,2870
ES95t	0,0121	0,2175	0,0274	0,0047	-0,0015	0,0506	0,2639
ES95d	0,0204	0,0803	0,0293	0,0022	-0,0046	0,0657	0,2847
Semi-Mean	0,0188	0,1022	0,0296	0,0021	-0,0136	0,0712	0,2723
Semi-0	0,0196	0,0836	0,0294	0,0021	-0,0144	0,0563	0,2819
Semi-rf	0,0197	0,0830	0,0294	0,0021	-0,0144	0,0563	0,2820
kurt	0,0063	0,4875	0,0274	0,0054	-0,0001	0,1373	0,2266
skew	0,0033	0,7151	0,0284	0,0038	-0,0012	0,6529	0,2054
skew5%	0,0162	0,1494	0,0303	0,0021	-0,0035	0,1451	0,2637
coskew1	-0,0037	0,7027	0,0325	0,0017	-0,0699	0,0230	0,2736
coskew2	-0,0015	0,8824	0,0311	0,0033	-0,0282	0,1134	0,2424
size	0,0062	0,6683	0,0301	0,0020	-0,0004	0,7561	0,2026

Table 14: Multivariate Regressions – Developed Markets

$RRi = c0 + c1*SR_i + c2*Risk_i + ui$

SR / Risk	c0	p-value	c1	p-value	c2	p-value	R2
Down-βiw	0.0232	0.0120	0.0049	0.8917	0.0091	0.8500	0.1390
Down-βw	0.0201	0.0102	-0.0539	0.0578	0.0675	0.0199	0.2546
IR	0.0132	0.0103	0.0058	0.3309	0.0151	0.0045	0.3168
TR	0.0119	0.0202	-0.0035	0.6679	0.0194	0.0036	0.3233
σ-garch(1,1)	0.0000	0.9991	0.0114	0.0491	0.0312	0.0259	0.2646
VAR95t	0.0027	0.8249	0.0119	0.0483	0.0175	0.0656	0.2356
VAR95d	0.0123	0.0236	-0.0046	0.6028	0.0125	0.0075	0.3097
VAR99t	0.0018	0.8748	0.0118	0.0480	0.0127	0.0502	0.2441
VAR99d	0.0126	0.0245	-0.0020	0.7950	0.0065	0.0087	0.2936
ES95t	0.0145	0.0020	-0.0015	0.8335	0.0059	0.0005	0.3331
ES95d	0.0129	0.0072	-0.0030	0.7011	0.0078	0.0024	0.3195
Semi-Mean	0.0119	0.0131	-0.0035	0.6555	0.0270	0.0016	0.3322
Semi-0	0.0126	0.0113	-0.0029	0.7167	0.0261	0.0025	0.3148
Semi-rf	0.0125	0.0123	-0.0028	0.7172	0.0261	0.0025	0.3150
kurt	0.0110	0.3041	0.0082	0.1997	0.0025	0.1026	0.2428
skew	0.0200	0.0016	0.0121	0.0628	-0.0168	0.0532	0.2483
skew5%	0.0182	0.0005	-0.0037	0.6598	0.0061	0.0061	0.3157
coskew1	0.0126	0.0785	0.0232	0.0050	-0.0634	0.0041	0.3467
coskew2	0.0128	0.1012	0.0233	0.0106	-0.0470	0.0124	0.2892
size	0.0419	0.0016	0.0163	0.0208	-0.0022	0.0637	0.2355

Table 15: Multivariate Regressions – Emerging Markets

$RRi = c0 + c1*SR_i + c2*Risk_i + ui$

SR / Risk	c0	p-value	c1	p-value	c2	p-value	R2
Down-βiw	-0.0036	0.7838	-0.0097	0.7532	0.0434	0.1680	0.1792
Down-βw	-0.0032	0.7685	-0.1217	0.0659	0.1307	0.0188	0.2945
IR	0.0194	0.3136	0.0319	0.0225	-0.0120	0.1646	0.2065
TR	0.0189	0.3274	0.0348	0.0146	-0.0121	0.1735	0.2013
σ-garch(1,1)	-0.0247	0.0981	0.0269	0.0842	0.0202	0.0816	0.2234
VAR95t	-0.0238	0.1074	0.0279	0.0726	0.0119	0.0915	0.2149
VAR95d	0.0188	0.4015	0.0357	0.0160	-0.0083	0.2926	0.1888
VAR99t	-0.0241	0.1045	0.0276	0.0758	0.0085	0.0884	0.2174
VAR99d	0.0190	0.3422	0.0343	0.0156	-0.0043	0.2052	0.2012
ES95t	0.0065	0.6413	0.0300	0.0507	-0.0012	0.1744	0.1701
ES95d	0.0185	0.3388	0.0343	0.0158	-0.0050	0.1802	0.1997
Semi-Mean	0.0156	0.4030	0.0343	0.0168	-0.0143	0.2035	0.1840
Semi-0	0.0171	0.3550	0.0346	0.0145	-0.0155	0.1656	0.1954
Semi-rf	0.0172	0.3533	0.0346	0.0144	-0.0156	0.1653	0.1956
kurt	0.0011	0.9269	0.0286	0.0771	-0.0001	0.2707	0.1406
skew	-0.0008	0.9493	0.0279	0.0901	-0.0013	0.5846	0.1320
skew5%	0.0138	0.4868	0.0349	0.0197	-0.0039	0.3369	0.1753
coskew1	-0.0082	0.4139	0.0142	0.4063	-0.1713	0.0151	0.3319
coskew2	-0.0056	0.6153	0.0126	0.5240	-0.0717	0.0316	0.2825
size	0.0177	0.4961	0.0351	0.0285	-0.0029	0.3319	0.1451

Table 16: Principal Components

Risk Factor	All	Dev.	Emerg.
SR	2	2	2
Down-βiw	2	2	2
Down-βw	2	2	2
IR	1	1	1
TR	1	1	1
σ-garch(1,1)	1	1	1 or 3
VAR95t	1	1	1 or 3
VAR95d	1	1	1
VAR99t	1	1	3
VAR99d	1	1	1
ES95t	1	1	1
ES95d	1	1	1
Semi-Mean	1	1	1
Semi-0	1	1	1
Semi-rf	2	2	2
kurt	1, 2 or 3	1	2 or 4
skew	1, 2 or 4	2 or 3	2 or 4
skew5%	1	1	1
size	1 or 2	3	2 or 3