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**Abstract:**

Understanding patterns of conflict and pathways in which political history became established is critical to understanding how large states and empires ultimately develop and come to rule given



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regions and influence subsequent events. We employ a spatiotemporal Cox regression model to investigate possible causes as to why regions were attacked by the Neo-Assyrian (912-608 BCE) state. The model helps to explain how strategic benefits and costs lead to likely pathways of conflict and imperialism based on elite strategic decision-making. We apply this model to the early 9th century BCE, a time when historical texts allow us to trace yearly campaigns in specific regions, to understand how the Neo-Assyrian state began to re-emerge as a major political player, eventually going on to dominate much of the Near East and starting a process of imperialism that shaped the wider region for many centuries even after the fall of this state. The model demonstrates why specific locations become regions of conflict in given campaigns, emphasizing a degree of consistency with which choices were made by invading forces with respect to a number of factors. We find that elevation and population density deter Assyrian invasions. Moreover, costs were found to be more of a clear motivator for Assyrian invasions, with distance constraints being a significant driver in determining where to campaign. These outputs suggest that Assyria was mainly interested in attacking its weakest, based on population and/or organization, and nearest rivals as it began to expand. Results not only help to address the emergence of this empire, but enable a generalized understanding of how benefits and costs to conflict can lead to imperialism and pathways to political outcomes that can have major social relevance.

**Supporting material:**

Modeling Strategic Decisions in the Formation of the Early Neo-Assyrian Empire

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## **Modeling Strategic Decisions in the Formation of the Early Neo-Assyrian Empire**

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Understanding patterns of conflict and pathways in which political history became established is critical to understanding how large states and empires ultimately develop and come to rule given regions and influence subsequent events. We employ a spatiotemporal Cox regression model to investigate possible causes as to why regions were attacked by the Neo-Assyrian (912-608 BCE) state. The model helps to explain how strategic benefits and costs lead to likely pathways of conflict and imperialism based on elite strategic decision-making. We apply this model to the early 9th century BCE, a time when historical texts allow us to trace yearly campaigns in specific regions, to understand how the Neo-Assyrian state began to re-emerge as a major political player, eventually going on to dominate much of the Near East and starting a process of imperialism that shaped the wider region for many centuries even after the fall of this state. The model demonstrates why specific locations become regions of conflict in given campaigns, emphasizing a degree of consistency with which choices were made by invading forces with respect to a number of factors. We find that elevation and population density deter Assyrian invasions. Moreover, costs were found to be more of a clear motivator for Assyrian invasions, with distance constraints being a significant driver in determining where to campaign. These outputs suggest that Assyria was mainly interested in attacking its weakest, based on population and/or organization, and nearest rivals as it began to expand. Results not only help to address the emergence of this empire, but enable a generalized understanding of how benefits and costs to conflict can lead to imperialism and pathways to political outcomes that can have major social relevance.

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## **Introduction**

The Neo-Assyrian period (912–608 BCE) was a time when the Assyrians politically dominated large parts of the Near East. By the early ninth century BCE, a series of campaigns by the new Assyrian king, Ashurnasirpal II (r. 883–859 BCE), began to shape the region that ultimately led to the establishment of a large-scale empire by the eighth century BCE that dominated much of the Near East until the end of the seventh century BCE (Cline and Graham 2011). The empire became the largest state in the ancient Near East and had direct influence on many cultural groups, but it also began a long process whereby empires and imperialism became the norm in the Near East as successive states and empires began to dominate even more territory. This makes the early ninth century BCE an important period to investigate if we are to understand how this long process of imperialism emerged and the strategic path dependencies in which later decisions were shaped by earlier outcomes.

Often, most ancient states' strategic decisions are difficult to evaluate and understand within contemporary contexts. Particularly, historical data are usually missing and the problem of identifying ancient toponyms makes key battles, alliances, and events difficult to place in time and space. Furthermore, multiple factors affect strategic circumstances of states at any given time, making historical contexts often unclear for researchers and important questions, such as why and how the process of imperialism in a given region began, hard to answer. In this paper, we propose a method that evaluates strategic military decision-making by elites that affected pathways in which the Neo-Assyrian Empire began to emerge. This model considers the likelihood and propensity of given states to be attacked by the Neo-Assyrians and addresses what factors could determine observed conflict. We demonstrate the utility of a spatiotemporal Cox regression model that investigates the determinants of strategic decisions and the context of relevant political dynamics and spatial scales. Our goal is to better understand why certain states were attacked by the Neo-Assyrians and uncover the underlying processes that could have shaped their strategic decision-making.

We begin this paper by providing background into the case study, specifically the early ninth century BCE when the Neo-Assyrian state began to more aggressively launch yearly campaigns against neighboring states and key decisions by its leadership shaped which states increasingly came into conflict with the Neo-Assyrians. Next, we articulate a series of six hypotheses concerning factors that might have influenced these decisions and which are testable via the proposed model. We then present our model, demonstrating its suitability for addressing questions concerning where and when given campaigns are fought. The model results are considered in the context of strategic decision-making undertaken by the elite. We see that elevation and population density appear to deter Assyrian invasion. The clearest results, however, show that deterrence in the form of

distance to the Assyrian army is the largest driver affecting Assyrian invasion. Given these results, we conclude by considering the extent to which the presented approach answers our research goal and how it might be applicable to other cases.

## **Historical Background**

Assyria's documented history stretches back to c. 2000 BCE. Originally a small state centered on the city of Ashur (modern Qal'at Sherqat) in northern Mesopotamia, it again rose to prominence during the Late Bronze Age (c. 1600–1200 BCE), when it gained independence from the neighboring kingdom of Mitanni, located in the Khabur region in upper Mesopotamia, and embarked on a program of territorial expansion (Radner 2014). The Assyrian king Ashur-uballit (r. 1365–1330 BCE) cemented Assyria's newfound status by becoming a latecomer to the so-called Great Powers Club, a group of powerful states that dominated the Near East in the Late Bronze Age (Moran 1992).

Although the collapse of the Late Bronze Age system around 1200 BCE dramatically redesigned the political landscape of the Near East (Radner 2014), leading to the disappearance of the Hittite empire and a weakening of the once-powerful Egyptian and Babylonian states, Assyria emerged less affected, putting it in a relatively good political position. During the reign of Tiglath-pileser I (r. 1114–1076 BCE), Assyrian territory continued to encompass a significant part of northern Mesopotamia, and Tiglath-pileser sought to extend Assyria's boundaries further by campaigning repeatedly to the west of the Euphrates river. His immediate successors, however, were less successful and by the start of the first millennium BCE, Assyria's territorial holdings had been pushed back to a modest strip of land bordering the Tigris River. The end of the tenth century, however, saw the start of renewed efforts to regain Assyria's former status. This marked the beginning of the Neo-Assyrian period (934–610 BCE), during which Assyria would emerge as the most powerful empire to date, controlling most of the ancient Near East (Cline and Graham 2011).

The reign of king Ashurnasirpal II (r. 883–859 BCE) was instrumental in Assyria's rise to greatness, ultimately having an impact long after the fall of Assyria, as the ninth century ushered the beginning of a millennia-long period of large-scale empires and states dominating the region. From his ascension, Ashurnasirpal pursued the policy of restoration and conquest begun by his grandfather Adad-nirari II (r. 911–891 BCE), but expanded it to a greater scale, campaigning vigorously and regularly: his inscriptions record no fewer than 14 military campaigns during his 24 years on the throne. He was particularly active in the first few years of his reign, sometimes conducting two separate campaigns in a single year (Grayson 1982:253). The primary focus of his exploits were the regions to the east, north, and west of the empire's heartland, which lay between the cities Ashur, Nineveh (modern Mosul), and Arbela (Erbil; Radner 2011). Figure 1 shows the map

of the region at the beginning of Ashurnasirpal's reign, including the principal Assyrian cities, and the states that existed during this time. Advances in military technology, including more efficient siege machines and reliance on cavalry rather than chariotry, improved the effectiveness of the Assyrian army (Fischer 1998:205). At the same time, Ashurnasirpal made contributions to the system of provincial administration, under which conquered regions were put under the control of an Assyrian governor and subjected to regular tribute (Grayson 1982:258). Mass deportations of local populations sought not only to distribute manpower where it was needed, but also to minimize the risk of future revolts (Oded 1979).

### **Ashurnasirpal's Campaigns**

The region of Mazamua, located in parts of modern Iraqi Kurdistan and parts of western Iran (Figure 1: states 0, 9, 21, 37, and 41), was the site of three Assyrian expeditions between 881 and 880 BCE. Located to the northeast of Assyria, it represented an important gateway to the Iranian plateau and its rich trade network. The region may have been under Assyrian control prior to Ashurnasirpal's reign but rebelled soon after his accession. Having put down the revolts, Ashurnasirpal stamped his authority on the region by renovating the city Atila, likely located in Figure 1:9, and giving it an Assyrian name, Dur-Ashur ("fortress of the god Ashur"). For the next two centuries, the city served as a garrison from which armed expeditions in the Zagros Mountains could be launched easily and with minimal delay (Radner 2013:442).

The middle Euphrates region engaged Ashurnasirpal intermittently from 878 BCE. In that year, the new ruler of Suhu (Figure 1:35), aided by the Babylonians, rebelled against Assyria. The revolt was eventually joined by the neighboring states of Laqu (Figure 1:24) and Hindanu (Figure 1:16), and despite swift and harsh retaliation from Ashurnasirpal, rebellions continued to break out in the region (Grayson 1991).

Ashurnasirpal's expeditions to the west took him from Bit-Bahiani (Figure 1:6) and Hatti (Figure 1:15) in Syria as far as the Levantine coast, where the king performed the traditional ritual of washing his weapons in the Mediterranean Sea (Grayson 1991:298). He received tribute from local rulers and cut down trees in the cedar groves of Lebanon, whose wood had been highly prized by Mesopotamian kings since the third millennium BCE (Klein and Abraham 2000). Although no direct Assyrian control was established in this region, the expedition served an important political and ideological purpose, raising Assyria's visibility and status and extending the empire's symbolic presence to the Mediterranean Sea, a representation of one of the traditional boundaries of the "officially existing" world (Liverani 1990:59).

As a result of Ashurnasirpal's skilled leadership and military exploits, Assyria regained territories that had been lost centuries earlier and established itself as one of the leading powers of the ancient Near East. Ashurnasirpal's successors would capitalize on this momentum to further extend Assyria's territorial gains and political influence. At its height in the seventh century BCE, the Assyrian empire controlled a vast territory, which stretched from Egypt in the southwest to the mountainous regions of the Taurus and Zagros ranges in the north and east, and the Persian Gulf in the south. Even after its defeat at the hands of the Babylonians and the Medes by 610 BCE, Assyrian influence continued to be felt. Its imperial dominance set the tone for its successors, from the Neo-Babylonian, Achaemenid, Seleucid empires, and perhaps even to the Abbasid Caliphate, which ruled the Middle East from its Mesopotamian capital into the thirteenth century CE (Cline and Graham 2011). The Achaemenids, for example, replicated or were influenced by innovations made by the Assyrians in governing and military affairs.

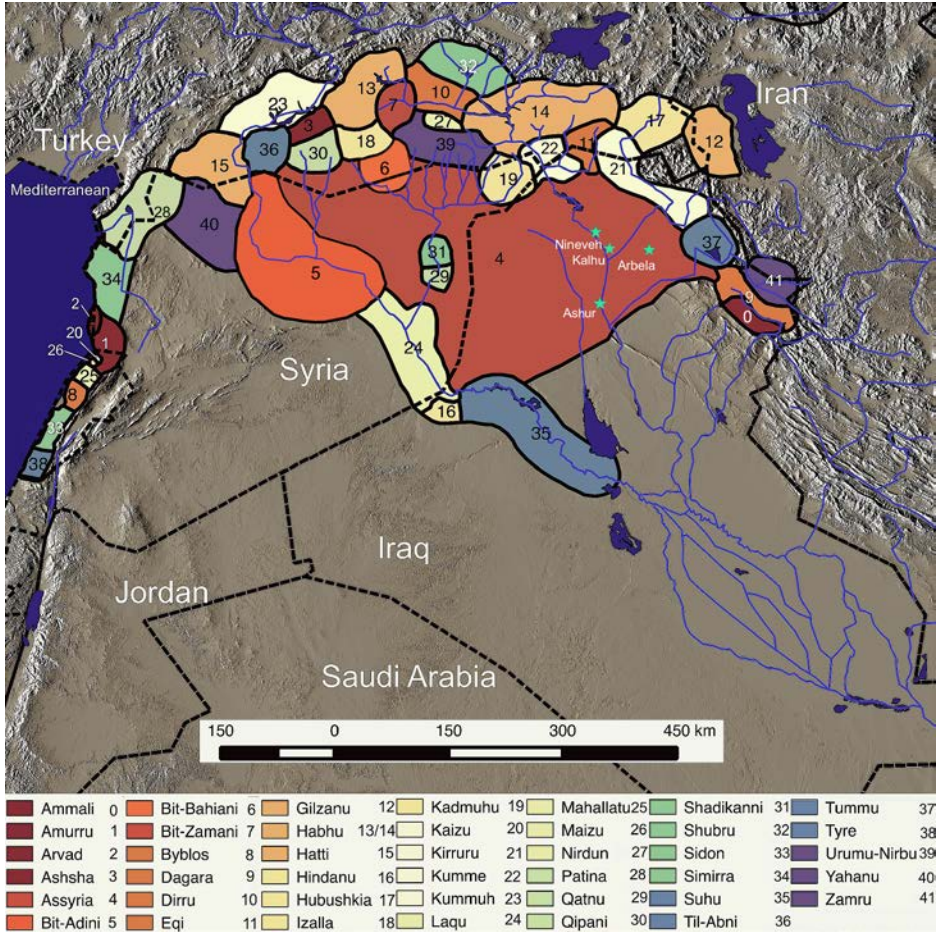
### **Historical Sources**

The reign of Ashurnasirpal is relatively well-documented. Like his predecessors, Ashurnasirpal commemorated his achievements in official inscriptions recorded on a variety of media, from palace walls and free-standing steles to clay prisms buried in the foundations of important buildings (Grayson 1991:189-393). A large number of such compositions survive from Ashurnasirpal's new royal residence, Kalhu (modern Nimrud; Figure 1), whose buildings and public areas were lavishly decorated with images and texts celebrating the king's deeds. Royal inscriptions invariably take the form of an autobiographical account and, in accordance with the demands of Assyrian royal ideology, tend to focus on the king's role as military leader. The result is a series of detailed accounts of military campaigns led by the king, narrated in chronological order or grouped thematically (or even a combination of the two). Campaign narratives may include information about the route followed by the army, enemy casualties, and tribute or loot (Tadmor 1997).

Despite their obvious appeal as historical records, royal inscriptions remain a problematic source whose very genre defies modern classification. The information included in them was carefully selected by the royal scribes in order to portray their royal masters in the best possible light, and edited as the need arose to accommodate additional material or constraints of space. The scribes who composed royal inscriptions availed themselves of a range of source materials, including contemporary records and itineraries, but also literary narratives and mythological accounts designed to convey an ideological message about the supremacy of the Assyrian king (Tadmor 1997).

One way of strengthening this message was to maintain the fiction of a single army led by the king, who alone is responsible for all victories; for rare exceptions to this convention, see Yamada (2000:26, 221–222) and Niederreiter (2005). In

reality, the Assyrian empire must have relied on multiple armies with some military leadership delegated to generals, but this is almost never reflected in official texts. Although inscriptions often record quite detailed information about the army's itinerary, it is not always possible to reconstruct this as accurately as we might like, due to the difficulties of correlating ancient and modern toponyms.



**Figure 1.** Map of Assyria (4) and surrounding states in c. 883 BCE. Colors and numbers indicate states in the region in the early ninth century BC; dashed lines indicate modern borders, and solid lines indicate ancient borders.



Our knowledge of the reality of Assyrian state governance in the eighth and seventh centuries BCE is enhanced immeasurably through surviving corpora of letters exchanged between the king and his officials (Parpola 1987; Lanfranchi and Parpola 1990; Fuchs and Parpola 2001; Luukko and Van Buylaere 2002; Dietrich 2003; Reynolds 2003; Luukko 2012). This state correspondence allows us to counterbalance the elevated rhetoric of royal inscriptions with more down-to-earth and mundane communiqués and to fill in important information gaps that facilitated Assyria's wars and expansion. However, no such archives survive from the reign of Ashurnasirpal, and we do not have complementary sources from outside Assyria to corroborate royal inscriptions or provide an alternative point of view.

## **Hypotheses**

We investigate the factors that might have influenced the Assyrian army's decision to invade a particular state and compare this choice with other states that might have been chosen but were not. In this section, we derive a number of testable hypotheses concerning attributes of the states invaded that may have played a role in these decisions. Two types of attributes are considered: the costs that the invading Assyrian army would have to endure should they select that state and the potential opportunities associated with each state should their invasion be successful. Categorizing the attributes of each state into 'push' (i.e., costs such as elevation, organized defense, etc.) and 'pull' (e.g., benefits such as metals, distance to trade, etc.) factors from the perspective of the Assyrian army enables us to determine the balance between, on the one hand, whether the Assyrian army chose states to invade by minimizing the effort expended in invading new territory and expanding the empire in accordance with the principle of least effort (Zipf 1949) and, on the other, whether they sought to maximize the potential opportunities associated with those choices. In what follows, we derive six hypotheses that are used in the construction of the model, each of which can be considered as either a cost or a benefit associated with the target selection of the Assyrians.

The re-emergence of the Assyrian empire during the period under consideration occurred over a relatively short time-scale. As a consequence, there may have been time constraints on the decisions made by the army, with invasions taking place in certain states simply because they were *en route* to more desirable locations. In this way, some states that were a long distance from the location of the Assyrian army may have been seen as undesirable, given the time and effort it would have taken to travel there. Travel might have been viewed as a significant cost associated with selecting states to invade. As a consequence, our first hypothesis asserts:

**Hypothesis 1.** The Assyrian army was more likely to invade states that were near to their previously recorded location.

As well as the distance to potential invasion sites, there are other factors that may have influenced the Assyrians' cost of travelling. We hypothesize that a significant cost may have arisen if the terrain led to difficulty in travelling through the state. More mountainous regions, for instance, would have likely led to higher perceived costs by the army. In addition, uneven or high terrain might have favored the population of that state, who may have turned to insurgent tactics to counter the threat posed by the Assyrians. There is evidence to suggest that mountainous regions are more likely to provide favorable conditions for insurgencies during modern conflicts (Fearon and Laitin 2003). Our second hypothesis states:

**Hypothesis 2.** The Assyrian army was more likely to favor invading states with low mean elevation.

There is evidence to suggest that the invaded states had some resistance to the Assyrian army (Tadmor 1997) and it is likely that the Assyrian army encountered a number of militant groups of varying strengths during invasions. Our next hypothesis asserts that if the states were able to organize effective defense via these militant groups, the Assyrians would have been deterred from invading. The Assyrian army might have considered effective defense of a state to be a significant cost due to the potential for damaging the strength of the army via, for instance, Assyrian loss of life. In order to estimate the military potential of each state, we use a measure of population density, the operationalization of which is described in the sections that follow. In particular, we suppose that states with high population density will have had the organizational capacity to enable more effective defense. Thus, our third hypothesis states:

**Hypothesis 3.** States with organizational capacity to enable effective defense, for which high population density serves as a proxy, were less likely to be invaded by the Assyrian army.

Certain states may have also made alliances with contiguous states in order to counter the threat posed by the Assyrian army. Indeed, the empirical record mentions three instances of alliance formation between the smaller states. Although the available data do not allow us to test a formal hypothesis regarding whether these alliances were successful in deterring the Assyrians, we consider whether other such alliances might have been made after Assyrian invasion. The specification of the model used to detect such effects is discussed in the results section.

As well as invading states in order to expand the empire, the Assyrians may have invaded some states because control over those states offered more tangible benefits and opportunities. In particular, the attributes of each of the states may have had a significant influence in selecting the states to invade. We hypothesize that states with more desirable attributes were more at risk to invasion. We

consider three attributes of each state, which represent the associated opportunities that may have attracted invasion by the Assyrians.

First, we hypothesize that states with better conditions for agriculture are likely to be more desirable to the Assyrians. Although current rainfall conditions are likely to differ from those of the past, we consider the modern level of precipitation of each state (NOAA 2014) as a proxy for past agricultural conditions, leading to:

**Hypothesis 4.** States with higher levels of precipitation were more likely to be invaded.

Next, we hypothesize that the Assyrians were attracted to a particular area for its level of natural resources, specifically iron deposits, which became increasingly desired for creating iron weapons and tools during the Iron Age (Maxwell-Hyslop 1974), leading to:

**Hypothesis 5.** States with higher levels of metal resources were more likely to be invaded.

Our final hypothesis asserts that the Assyrians sought to seek out new trading opportunities to the west, which were likely to have been more prevalent towards the coast of the Near East (Sherratt and Sherratt 1993), leading to:

**Hypothesis 6.** States that were closer to the Mediterranean coast were more likely to be invaded.

In what follows, we describe our analytical approach, which includes an overview of the dependent and independent data used in our analysis. We then present the results of our analysis, which enable us to evaluate each of the hypotheses stated above.

## **Methodology**

The onset and evolution of conflict, particularly with regards to historical conflict, is traditionally discussed using anecdotal perspectives, rather than by employing mathematical or statistical models to seek out underlying mechanisms or patterns that might be exploited to obtain insights. More recently, there has been a dramatic increase in the quantity and quality of such models exploring the location and timings of various examples of conflict (Turchin 2003; Weidmann and Ward 2010; Zammit-Mangion et al. 2012; Turchin et al. 2013; Bhavnani et al. 2014). This is partly due to increased data availability, which is crucial for modeling because it enables the development of models that are empirically consistent, and partly due to an increased range of sophisticated modeling techniques, some of which are well-suited to scenarios involving only partial data. Indeed, historical conflicts pose an additional complication because available data are typically scarce, biased, or often interpreted from other sources. In many cases, the availability of data constrains the sophistication with which models can be constructed.

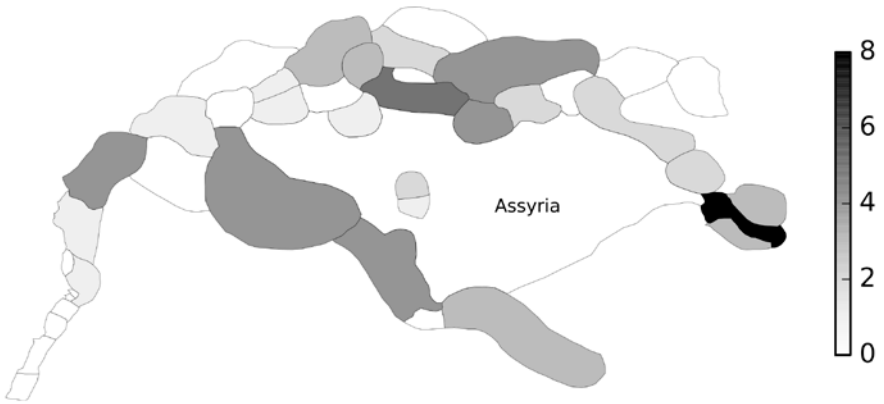
The dependent variable used in this study is given by the states that are invaded by the Assyrian army, as detailed through inscriptions made during the reign of Ashurnasirpal (Grayson 1991). The descriptions of the activities of the Assyrian army were collated and, where possible, georeferenced according to the state in which each activity occurred. The geographic data were obtained from using a geographic study (Liverani 1992) of the Assyrian state at the time of Ashurnasirpal's rise. The data are made available as supplementary data to this work. In total, 65 separate invasion activities were identified between 883–865 BC and used in the analysis. Figure 2 shows a thematic map of the geographic area under consideration, with counts for the number of times each state was invaded as the dependent variable.

We propose a spatiotemporal Cox regression model to investigate the relationship between the invasions of the Assyrian army and the costs and benefits associated with those decisions. The model estimates the likelihood that each state will be selected as a target and has two components. The first captures inherent variation in the times at which Assyrian invasions took place, variation which is not dependent on the states themselves but is due to other factors that influence campaign times. Variation in attack frequency might, for example, arise due to external factors such as the weather or army weariness. We do not explicitly model this natural variation, but retain the term in the model to demonstrate that the states themselves are not the only factors influencing the timings of the attacks. The second component in the model considers the differences in each of the states at the times at which these campaigns took place, and supposes that the decisions made by the Assyrians with regards to where to invade depended on these differences. In an attempt to capture the variation across different spatial regions, a number of independent variables are incorporated into the model. We now outline our model before explaining how each of the independent variables is operationalized.

We suppose that the Assyrian army invades one of  $N$  surrounding states at times given by the random variables  $T_1, T_2, \dots, T_j$  and that, at each time  $T_j$ , the random variable  $S_j \in \{1, 2, \dots, N\}$  denotes the specific state that is invaded. Thus, the actions of the Assyrian army are summarized by the sequence of tuples given by  $(T_1, S_1), (T_2, S_2), \dots, (T_j, S_j)$ .

Following previous studies that consider the time until events occur within a geographic area (e.g., Myers 1997; Raleigh and Hegre, 2009), we model the hazard function  $\lambda_i(t)$  for state  $i$ , taken to be the instantaneous risk at time  $t$  that state  $i$  will be invaded. This is formally defined as

$$(1) \quad \lambda_i(t) = \lim_{\delta t \rightarrow 0} \frac{\Pr(T_j \leq t + \delta t, S_j = i | T_j \geq t)}{\delta t},$$



**Figure 2:** Choropleth map of the study area (Assyria and surrounding states), detailing the number of times each state was invaded by the Assyrian army between 883–865 BC.

where  $T_j$  is a random variable specifying the time of the next invasion, assuming that invasions at times  $T_1, T_2, \dots, T_{j-1}$  have already taken place.

The Cox regression model, first described in Cox (1972) and further elaborated upon in several subsequent papers (Cox 1975; Anderson and Gill 1982; Gill 1984), assumes that pure temporal variation in the frequency of event occurrence can be separated from dependencies specific to each of the different areas  $i$ . Specifically, it supposes that the hazard function can be written as:

$$(2) \quad \lambda_i(t|z_i(t)) = \alpha(t) \exp(z_i(t)^T \beta),$$

for each area  $i$  for a temporally dependent baseline hazard  $\alpha(t)$  and temporally varying covariates  $z_i(t)$  with a vector of parameters  $\beta$ .

An advantage of this form of the model is that to estimate the parameters  $\beta$ , representing the effect from the independent variables on the risk of invasion, an explicit form of the baseline hazard  $\alpha(t)$  is not required, and purely time-varying actions of the Assyrian army can be neglected. As a consequence, the times  $T_1, T_2, \dots, T_j$  can be treated as given and the task is reduced to modelling the associated random variables  $S_1, S_2, \dots, S_j$ , which detail the invaded state. This highlights a particular strength of the Cox regression approach: the precise event times are, in fact, irrelevant and only the order in which states are attacked is required to estimate the parameters  $\beta$ . For the case of the Neo-Assyrian campaigns, this is particularly salient because accurate dates are not recorded by inscriptions, but

the order in which the inscriptions are presented give an indication of the ordering of invasions during any given campaign.

The conditional probability that state  $i$  is invaded at time  $T_j$ , given that we know that one state is invaded at that time, is given by

$$(3) \quad \Pr(S_j = i | T_j) = \frac{\exp(z_i(T_j)^T \beta)}{\sum_k \exp(z_k(T_j)^T \beta)},$$

and the parameters  $\beta$  are found via maximization of the partial likelihood function given by

$$(4) \quad \mathcal{L}_p(\beta) = \prod_{j=1}^J \prod_{i=1}^N \left( \frac{\exp(z_i(T_j)^T \beta)}{\sum_k \exp(z_k(T_j)^T \beta)} \right)^{\mathbf{1}(S_j=i)},$$

where  $\mathbf{1}(S_j = i)$  is an indicator function, equal to one if  $S_j = i$  and equal to zero otherwise. Note that the model estimates a counting process rather than a survival process because it is possible that each state can be invaded more than once (see Anderson and Gill, 1982; Gill, 1984).

In what follows, the independent variables denoted by  $z_i(t)$  that are used to populate the model are described. The first variable to be incorporated is required to evaluate hypothesis 1 and measures the distance between the Assyrian army's previous location and each state for each time period during the study. This is calculated as the geographic distance between the centroid of the state of the last invasion, and the centroid of the state in question (measured in units of 100 km).

The second variable is a measure of the mean elevation of each state, as obtained from a Level 1 digital elevation model (DEM; USGS 2014). The mean elevations are included in the model using units of 100 m for ease of interpretation of the parameter values.

Determining the potential defense within each state, and the danger to the Assyrian army being invaded, is given by the population density of each state, which is calculated by relative size of settlements found in given regions modeled (Liverani 1992; Wilkinson et al. 2005; Wilkinson et al. 2007; Radner 2014). Although these estimates are not precise, they allow us to determine areas where we expect relatively more or less people being concentrated based on historical and archaeological data in the region. Values are assigned 1–4, with 4 indicating the highest population; values indicate the relative degree to which given regions were more populated than others. These values are then divided by the area of each state, as measured in units of 100 km<sup>2</sup>.

Average precipitation, obtained from NOAA (2014), is used to determine whether a state was suitable for agriculture. Although these are modern data,

paleoclimate studies show the current pattern is similar to the Iron Age precipitation ranges (Issar and Zohar 2007), or at least indicate relatively which regions are wetter or drier (e.g., along the Mediterranean coast). The average precipitation is measured in units of 100 mm for ease of interpretation.

The amount of metal resources available in each state is estimated by relative distance from iron deposits known in the region (Maxwell-Hyslop 1974), and in Anatolia in particular. States that are closer to known deposits score higher (i.e., a value of 2 or 3), whereas states that are farther away score lower (i.e., 1). Figure 3 shows some of the relevant variables utilized in the model.

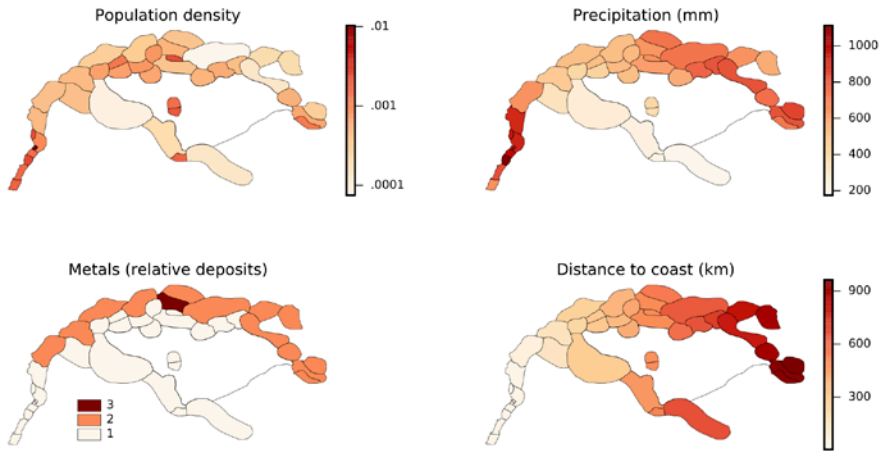
A further variable is added to incorporate a measure of the distance of each state to the coast. This is given by calculating the geographic distance between the centroid of each state and the nearest coastal location; it is taken in units of 100 km for ease of interpreting the corresponding parameter estimates. This variable acts as a proxy for access to beneficial trade routes that emerged along the coast (Sherratt and Sherratt 1993).

Finally, two control variables are also included in order to alleviate problems associated with unobserved heterogeneity and the varying geography of the study area. The first control, indicating the number of previous times that each state had been invaded by the Assyrians at each point in time, reduces potential bias resulting from unobserved heterogeneity. This arises when factors that are largely responsible for influencing the choices made are not included in the model. Unobserved heterogeneity may also influence parameter estimates of those variables that are included in the model. Incorporating the number of prior invasions for each state goes some way to incorporate some of the factors that influenced the Assyrians' choices that we have not (or could not) incorporate into the model. The area of each state is also included as a control variable because the states vary considerably in size. As a consequence, even if the invasions were made completely at random, it is likely that some states would be more invaded than others if they are larger. Including this variable reduces size as a source of bias.

The final expression for the hazard function  $\lambda_i(t)$  is given by

$$(5) \lambda_i(t|z_i(t)) = \alpha(t) \exp(\beta_1 D_i(t) + \beta_2 E_i + \beta_3 P_i + \beta_4 R_i + \beta_5 M_i + \beta_6 C_i + \beta_7 A_i + \beta_8 I_i(t)),$$

where  $D_i(t)$  is the distance between state  $i$  and the previous recorded location of the Assyrian army;  $E_i$  is the elevation of state  $i$ ;  $P_i$  is the population density of state  $i$ ;  $R_i$  is the precipitation in state  $i$ ;  $M_i$  is the amount of metal resources in state  $i$ ;  $C_i$  is the distance from state  $i$  to the coast;  $A_i$  is the area of state  $i$ ; and  $I_i(t)$  is the number of prior invasions in state  $i$ .



**Figure 3:** A selection of independent predictors of invasion frequency mapped on the study area of Assyria and surrounding states.

## Results

The `coxph` function in the R survival package (Therneau 2013) was employed to estimate the parameters  $\beta$ . Fox (2002) provides an overview of its implementation with regards to time-varying covariates. Note that even though the dependent variables are time-dependent, the estimated parameters are not: the regression is performed over multiple time-steps and leads to one estimate for the relative effect of each variable.

Figure 4 summarizes the parameter estimates for two models: one containing only those variables which represent costs to the Assyrian army (Model 1) and one containing both costs and opportunities (Model 2). The exponential value of the parameter is plotted (the odds ratio), together with a 95 percent confidence interval for that estimate. This value is chosen for ease of interpretation and gives the estimated change in odds that a state will be invaded from a one-unit increase in the associated independent variable. To explain, if the exponential of the coefficient in  $e^\beta$  is equal to one, then any change in the associated variable has little effect on the risk of invasion. If  $e^\beta$  is greater than one, then a one-unit increase in the value of the associated variable increases the odds of invasion by a factor of  $e^\beta$ . Similarly, if  $e^\beta$  is less than one, then a one-unit increase in the associated variable decreases the odds of invasion multiplicatively by the same amount.

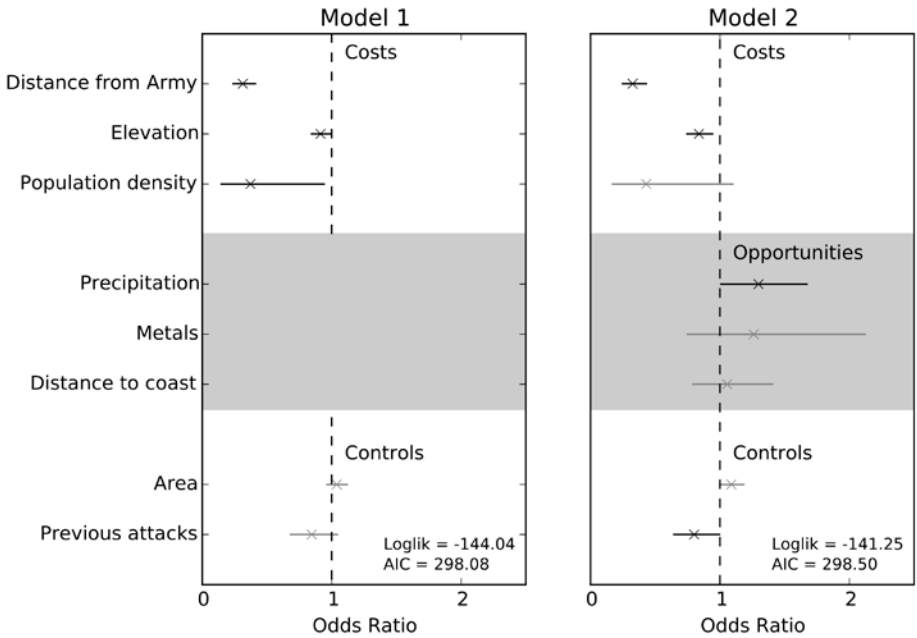
In keeping with Hypothesis 1, distance between the state and the Assyrian army is negatively associated with the risk of invasion in both models. Given that the values of the exponential of each parameter estimate is around 0.3, if all else is



equal, every 100 km between the state and the Assyrian army reduces the probability of invasion to around a third of what it otherwise would be.

An increase in elevation negatively impacts the likelihood of invasion, a finding in agreement with Hypothesis 2. This effect was significant in both models. With all other things being equal, every 100 m increase in the mean elevation of a state reduced the likelihood of invasion by 10–15 percent.

Population density, a proxy for the extent to which each state could coordinate effective defense, was significantly negatively associated with the probability of invasion in Model 1, in agreement with Hypothesis 3. In Model 2, in which both cost and opportunity variables were included, the effect was not significant at the 95 percent level.



**Figure 4.** Parameter estimates and associated 95 percent confidence intervals for each of the independent variables incorporated in the model. The left hand side presents a model containing costs to the Assyrian army and the right hand side presents a model with both costs and opportunities. If a confidence interval crosses the value 1, then the associated parameter is not significant at the 95 percent level and the confidence interval is shaded grey.

Only precipitation levels were found to be statistically significant opportunity for Neo-Assyrians that was predictive of invasion, supporting Hypothesis 4.

Specifically, states with relatively higher levels of precipitation, and therefore states which are more likely to have conditions suitable for agriculture, were more likely to be invaded by the Assyrians, suggesting that the benefits associated with potential for agriculture were a significant driver in their decision-making. The presence of metals (Hypothesis 5) and the distance to the coast (Hypothesis 6) were not significant predictors of invasion at the 95 percent level.

Finally, although the control variables incorporated did not test a specific hypothesis articulated in this section, it is interesting to note their direction. The point estimate for the area of the state was positively (but not significantly) associated with the probability of invasion, implying that larger states were more likely to be invaded.

The point estimate for the number of previous attacks was negatively associated with the probability of invasion and was significant for Model 2. This negative association indicates that the Assyrians might have favored invasion of states that they had not invaded previously, suggesting that expansion was one of their principal objectives.

Model 2 led to a slightly higher Akaike's Information Criterion (AIC; Akaike 1974; Anderson 2008) score than Model 1 (298.08 vs. 298.50) despite the added variables. This suggests that the three opportunity variables (precipitation, metals, and distance to coast) do little to improve the model. Omitting the variables for metals and distance to coast but retaining the variable for precipitation resulted in an AIC value of 295.36, meaning that the inclusion of just the opportunity variable associated with better agricultural conditions leads to a better model. Parameter estimates and significance levels of this model were consistent with those shown in Model 1.

Table 1 presents an analysis of deviance table for Model 2. This table shows the results of a sequence of likelihood ratio tests to establish the extent to which the inclusion of each variable improves the model fit. The table is constructed as follows: beginning with a null model containing none of the independent variables, the log-likelihood is calculated. Eight models are then specified, each of which includes just one of the eight variables in Model 2. For each of these models, a likelihood ratio test is performed against the null model to determine whether the inclusion of each variable significantly improves model fit. The variable that increases the log-likelihood by the largest amount is selected as the next variable in the table (which, in this case, is distance from the army). This process is then repeated with the remaining variables, but instead of adding each variable to a null model, we add each remaining variable to a model containing just those variables that have already been shown to cause the largest improvement in model fit and which have, therefore, already been added to the table. Thus, the improvement in model fit that is due to each variable is determined, whilst, at each stage, controlling for the variables that are found to have the largest influence. The order

of the variables in the table provides some indication of the amount of explanatory power associated with each of them.

**Table 1.** A series of likelihood ratio tests to determine whether the inclusion of each variable improved the log-likelihood value of Model 2.

	Odds ratio in final model (standard error)	Log-likelihood of model containing each variable with those variables above	<i>p</i> -value
Null model (with no variables)		-241.38	
Distance from army	0.3252 (0.1424)	-154.08	.0000
Population density	0.4305 (0.4780)	-148.66	.0010
Elevation	0.8376 (0.0606)	-145.70	.0151
Previous attacks	0.8006 (0.1114)	-144.49	.1189
Precipitation	1.2975 (0.1292)	-143.44	.1468
Area	1.0881 (0.0435)	-141.68	.0609
Metals	1.2603 (0.2654)	-141.32	.3951
Distance to coast	1.0551 (0.1465)	-141.25	.7163

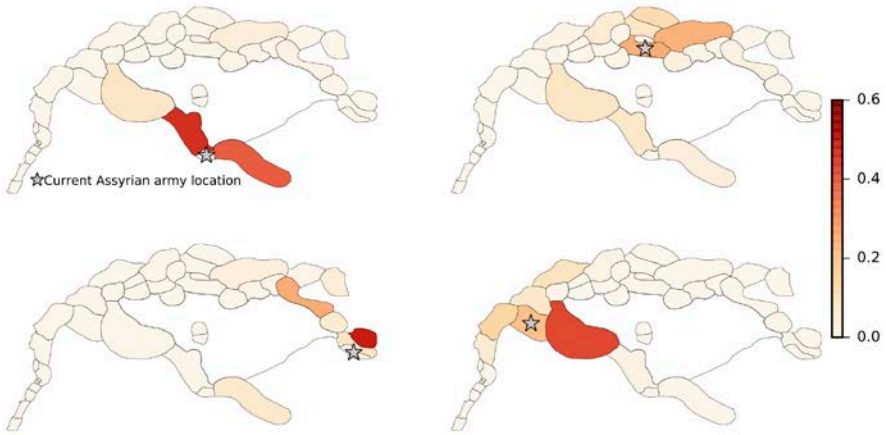
To explain, the first variable is the distance from the Neo-Assyrian army; this variable increased the log-likelihood of a null model with no covariates from -241.38 to -154.08, which was the maximum available increase of all possible covariates. The model improvement as a result of including this variable was highly significant ( $p < 10^{-5}$ ).

Considering the remaining variables, the next largest increase in the log-likelihood from the model that included only “distance from army” was found by including the population density variable. The null hypothesis, that the model is not improved by the addition of this variable, can be rejected because  $p = 0.0010$ .

The next largest increase comes from the elevation variable, which was also the final variable that significantly improved the model. This finding supports the hypothesis that elevation played a significant role in deterring the Neo-Assyrian army. The inclusion of the precipitation variable did not appear to significantly improve model fit, suggesting that its significance for the AIC statistic (Figure 4) may arise from a confounding factor.

In summary, our results suggest that regions closer to the last recorded army location, with lower population density, and which had lower mean elevation were more likely to be invaded by the Neo-Assyrians. There is little evidence to suggest that the opportunity variables tested were capable of capturing these decisions.

Figure 5 maps the relative hazards for each state in the region for four different starting locations of the Neo-Assyrian army, as specified by Equation 3. A large amount of the modeled hazard depends on the location of the Neo-Assyrian army; however, the effect from other variables can also be observed because the risk does simply decay relative to the army's location.



**Figure 5.** The probability that each state will be invaded next, as calculated by Equation 3, for four starting points of the Neo-Assyrian Empire as indicated by the star. The location of the star indicates the location of the Assyrian army and the color of each state indicates the probability that the state will be the next one to be invaded. These probabilities span from 0 to 0.6. The state of Assyria is shaded white.

A model was also constructed in order to test whether there was any evidence of alliance formation amongst the invaded states and whether such alliances might then have had either a deterrent or an incentive effect for the Neo-Assyrians. In other words, were they more or less likely to invade states which had recently been

attacked and were those recently attacked states, therefore, more likely to have formed alliances? To construct this model, a dummy variable was included equal to one if a state or any of its contiguous neighbors had been invaded previously in the same campaign. It is supposed that these states were the ones most likely to form alliances.

For this model, an additional control variable was incorporated to alleviate potential errors from confounding variables. This control was another dummy variable that indicated whether each state was contiguous with the previously invaded state. If alliances between neighbors did indeed affect the decision-making of the Neo-Assyrians, then an effect would be expected for the remainder of the campaign, and not just for the next attack. Once this was controlled for, no significant effects of neighbor alliances on likelihood of invasion were observed.

## **Discussion and Conclusion**

We have presented a spatiotemporal Cox regression model to determine the attributes of states that made them more susceptible to invasion during the Neo-Assyrian campaigns of the early ninth century BCE. The case study of the Neo-Assyrian state is generally an ideal example of early empires because it is relatively well-documented, despite some limitations. Moreover, it is an important empire to understanding the long succession of empires that continued in the Near East long after the Neo-Assyrian Empire. In essence, our results provide an idea of what the impetus may have been that began this long process of empire formation in the region.

Our results suggest that the Neo-Assyrians were under constraints with regards to how far they could travel during any one campaign. Strategic decision-making by the king would, therefore, have played a key role in deciding where the campaigns were largely fought. We have investigated a series of hypotheses to determine whether, even within each of these campaigns, the decision-making of the Neo-Assyrians were informed by factors associated with costs and opportunities of the choices that might have been made. We conclude that consideration of the costs associated with invasion had a larger impact on decision-making than opportunities. We find that distance of the army from potential states that could be attacked, elevation (where elevation would be a deterrent to invasion), and population density (which could be a proxy for the extent to which different regions were able to organize effective defense) were significant factors affecting invasion. Including the opportunity variables of precipitation, distance to coast, and the availability of metal deposits only marginally improved the fit of the model. Overall, it appears that human and geographic factors likely affected the onset of conflict during the reign of Ashurnasirpal, which subsequently shaped conflict in the region later in the ninth century BCE (Yamada 2000) and likely later periods. In effect, it appears that Ashurnasirpal's campaigns did have a practical

goal of attacking and defeating Assyria's nearest potential rivals, particularly those which appeared to be less populated or organized.

Our study is limited mainly by the lack of data associated with this time period. Nevertheless, we have demonstrated how systematically applying mathematical models can help to understand and discuss some of the key drivers during such historical periods, even when there are data constraints. The model employed in this study is particularly well-suited to understanding the features of locations that affect their risk of invasion because it can incorporate time-varying covariates without requiring detailed information on the specific times at which the invasions occurred.

The method we employed presents a simple and easy-to-use approach for studying different possibilities as to why early empires, such as the Neo-Assyrian state, undertook expeditions and warfare. Whereas most cases of past empire formation are speculative or based on purely qualitative understanding of states' motivations and their expansion, the approach we provide allows a quantitative-based assessment of impetus for expansion and pathways in which states formed. Furthermore, this method is applicable to other states and empires because the data requirements are not a great burden to obtain, particularly for empires and states occurring after the Neo-Assyrian period. Well-dated archaeological data that signify attacks on states, for example, can be used in the place of historical records of invasions. The order and temporal circumstances of attacks could be checked by the modeling approach to determine motivation plausibility. A comparative approach among these states' invasions may allow the approach presented here to be used to determine if similar patterns were also in effect. Overall, this approach is expandable to other cases, potentially even those with less data resolution.

## **Supplementary Data**

See uploaded file.

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