Systematic Trading: Calibration Advances through Machine Learning

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Declaration

I, Sergio Álvarez-Teleña, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

The paper with title "Construction of Emerging Markets Exchange Traded Funds Using Multiobjective Particle Swarm Optimisation" published at *Artificial Neural Networks and Machine Learning. Lecture Notes in Computer Science* by Diez, M., Alvarez-Teleña, S. and D. Gorse (2012) was originated from the technique proposed along the second experiment.

Also, a draft of the thesis was published as a book under the title "Trading 2.0: Learning-Adaptive Machines" (2012) to complete the lecturing material of COMP6006 at University College London. It was subsequently sold in Europe, Americas and Asia.

Finally, I declare that a new unit, BBVA Global Strategies & Data Science, has surged as a result of the methodologies diclosed in the thesis. During my MRes I was initially approached by the bank to run its ETF's global market making through different techniques based on my second experiment. I then used my first experiment to improve its execution capabilities based on scientific insights. My third experiment was finally used to reveal how disruptive and creative the evolution of the industry can be. By hence gradually proving the advantegeous role of science in the management of any systematic risk my unit was given a global, cross-channel (trading and commerce) and cross-asset reach.

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Sergio Álvarez-Teleña

Abstract

Systematic trading in finance uses computer models to define trade goals, risk controls and rules that can execute trade orders in a methodical way. This thesis investigates how performance in systematic trading can be crucially enhanced by both i) persistently reducing the bid-offer spread quoted by the trader through optimized and realistically backtested strategies and ii) improving the out-of-sample robustness of the strategy selected through the injection of theory into the typically data-driven calibration processes. While doing so it brings to the foreground sound scientific reasons that, for the first time to my knowledge, technically underpin popular academic observations about the recent nature of the financial markets.

The thesis conducts consecutive experiments across strategies within the three important building blocks of systematic trading: a) execution, b) quoting and c) risk-reward allowing me to progressively generate more complex and accurate backtested scenarios as recently demanded in the literature (Cahan *et al.* (2010)). The three experiments conducted are:

1. **Execution: an execution model based on support vector machines.** The first experiment is deployed to improve the realism of the other two. It analyses a popular model of execution: the volume weighted average price (VWAP). The VWAP algorithm targets to split the size of an order along the trading session according to the expected intraday volume's profile since the activity in the markets typically resembles convex seasonality – with more activity around the open and the closing auctions than along the rest of the day. In doing so, the main challenge is to provide the model with a reasonable expected profile. After proving in my data sample that two simple static approaches to the profile overcome the PCA-ARMA from Bialkowski *et al.* (2008) (a popular two-fold model composed by a dynamic component around an unsupervised learning structure) a further combination of both through an index based on supervised learning is proposed. The *Sample Sensitivity Index* hence successfully allows estimating the expected volume's profile more accurately by selecting those ranges of time where the model shall be less sensitive to past data through the identification of patterns via support vector machines. Only once the intraday execution risk has been defined can the quoting policy of a mid-frequency (in general, up to a week) hedging strategy be accurately analysed.

2. **Quoting: a quoting model built upon particle swarm optimization.** The second experiment analyses for the first time to my knowledge how to achieve the disruptive 50%

bid-offer spread discount observed in Menkveld (2013) without increasing the risk profile of a trading agent. The experiment depends crucially on a series of variables of which market impact and slippage are typically the most difficult to estimate. By adapting the market impact model in Almgren *et al.* (2005) to the VWAP developed in the previous experiment and by estimating its slippage through its errors' distribution a framework within which the bid-offer spread can be assessed is generated. First, a full-replication spread, (that set out following the strict definition of a product in order to hedge it completely) is calculated and fixed as a benchmark. Then, by allowing benefiting from a lower market impact at the cost of assuming deviation risk (tracking error and tail risk) a non-full-replication spread is calibrated through particle swarm optimization (PSO) as in Diez *et al.* (2012) and compared with the benchmark. Finally, it is shown that the latter can reach a discount of a 50% with respect to the benchmark if a certain number of trades is granted. This typically occurs on the most liquid securities. This result not only underpins Menkveld's observations but also points out that there is room for further reductions. When seeking additional performance, once the quoting policy has been defined, a further layer with a calibrated risk-reward policy shall be deployed.

3. **Risk-Reward: a calibration model defined within a Q-learning framework.** The third experiment analyses how the calibration process of a risk-reward policy can be enhanced to achieve a more robust out-of-sample performance – a cornerstone in quantitative trading. It successfully gives a response to the literature that recently focusses on the detrimental role of overfitting (Bailey *et al.* (2013a)). The experiment was motivated by the assumption that the techniques underpinned by financial theory shall show a better behaviour (a lower deviation between in-sample and out-of-sample performance) than the classical data-driven only processes. As such, both approaches are compared within a framework of active trading upon a novel indicator. The indicator, called the Expectations' Shift, is rooted on the expectations of the markets' evolution embedded in the dynamics of the prices. The crucial challenge of the experiment is the injection of theory within the calibration process. This is achieved through the usage of reinforcement learning (RL). RL is an area of ML inspired by behaviourist psychology concerned with how software agents take decisions in an specific environment incentivised by a policy of rewards. By analysing the Q-learning matrix that collects the set of state/actions learnt by the agent within the environment, defined by each combination of parameters considered within the calibration universe, the rationale that an autonomous agent would have learnt in terms of risk management can be generated. Finally, by then selecting the combination of parameters whose attached rationale is closest to that of the portfolio manager a data-driven solution that converges to the theory-driven solution can be found and this is shown to successfully outperform out-of-sample the classical approaches followed in Finance.

The thesis contributes to science by addressing what techniques could underpin recent academic findings about the nature of the trading industry for which a scientific explanation was not yet given:

- A novel agent-based approach that allows for a robust out-of-sampkle performance by crucially providing the trader with a way to inject financial insights into the generally data-driven only calibration processes. It this way benefits from surpassing the generic model limitations present in the literature (Bailey *et al.* (2013b), Schorfheid and Wolpin (2012), Van Belle and Kerr (2012) or Weiss and Kulikowski (1991)) by finding a point where theory-driven patterns (the trader's priors tend to enhance out-of-sample robustness) merge with data-driven ones (those that allow to exploit latent information).

- The provision of a technique that, to the best of my knowledge, explains for the first time how to reduce the bid-offer spread quoted by a traditional trader without modifying her risk appetite. A reduction not previously addressed in the literature in spite of the fact that the increasing regulation against the assumption of risk by market makers (e.g. Dodd–Frank Wall Street Reform and Consumer Protection Act) does yet coincide with the aggressive discounts observed by Menkveld (2013). As a result, this thesis could further contribute to science by serving as a framework to conduct future analyses in the context of systematic trading.

- The completion of a mid-frequency trading experiment with high frequency execution information. It is shown how the latter can have a significant effect on the former not only through the erosion of its performance but, more subtly, by changing its entire strategic design (both, optimal composition and parameterization). This tends to be highly disregarded by the financial literature.

More importantly, the methodologies disclosed herein have been crucial to underpin the setup of a new unit in the industry, BBVA's Global Strategies & Data Science. This disruptive, global and cross-asset team gives an enhanced role to science by successfully becoming the main responsible for the risk management of the Bank's strategies both in electronic trading and electronic commerce.

Other contributions include: the provision of a novel risk measure (*flowVaR*); the proposal of a novel trading indicator (*Expectations' Shift*); and the definition of a novel index that allows to improve the estimation of the intraday volume's profile (*Sample Sensitivity Index*).

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I would like to thank my first supervisor, Prof. Philip Treleaven, not only for his guidance during the writing of the thesis but also for having pursued the excellent idea of creating the DTC in Financial Computing & Analytics. Haven't it been for the bespoke approach that he has provided us with, across top three academic institutions, I would had never enrolled in a PhD quest. There are not enough lines in this preamble to express my gratitude.

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My industrial advisor, Mr Stefano Russo, did open his door at Renaissance Technologies to a completely unknown student that *maybe* had something interesting to say. There are many fans of RenTec out there. And me too, of course. But I am particularly a fan of Stefano.

Prof. Bob Jenkins, from London Business School, has been formally related to the thesis from inception. He gave me the self-confidence to start from scratch and *figure out* how to deliver the rest of the project. He is an inspiration to anyone in the financial industry.

On the same note, I am sincerely grateful to Laura Nuñez, from IE Business School, who became my local supervisor while I lived in Madrid and to Michal Galas (UCL), Gordon Ross (UCL), Sebastian del Bano Rolin (Queen Mary) and Abel Elizalde (Imperial College) for their valuable comments on the document. Thanks Philippe De Wilde (Kent) and Germano Guido (UCL) for your outstanding feedback and for the special care with which you scanned the thesis.

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Last but not least, thanks to all my professional colleagues for all I have learnt with them. Santander showed me how demanding it is to setup a new desk in a commercial bank and how gratifying as well. Morgan Stanley not only gave me the chance to trade through algorithms for the first time but also to witness that there are no *flow* limits in an investment bank when your trading approach delivers. And more especially, thanks to BBVA for having trusted my proposals to innovate its trading and sales systematization – another example of forward thinking in a commercial bank. Please, do note that the approaches presented herein are not necessarily being applied in any of the former institutions.

The thesis is dedicated to my wife. Marta, not only will I always be *bullish* of you but as much as you allow me, *leveraged*.

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Chapter 1

Introduction

This chapter presents an overview of the thesis and a justification of its scientific relevance. It starts by describing the motivations behind the research on techniques capable to overcome the out-of-sample robustness of standard calibration approaches in systematic trading. Second, the main objectives of the thesis are enumerated. The methodologies used throughout the different experiments are then briefly explained and followed by a mention to their major contributions to science. Finally, the chapter is concluded with the outline of the rest of the thesis.

1.1 Research motivations

Menkveld (2013) shows a recent cut in the financial markets' bid-offer spreads up to 50%. This means that there are new strategies in place which allow market makers to provide liquidity at a lower cost than that implied by a perfect risk coverage (hedge). When the coverage is not perfect there are deviations in performance between the instrument being market made and the hedge. This leads to the need of a systematic management (also automatic, at the highest frequencies of market making) to the trader. To the best of my knowledge there is no research yet focused on the analysis of the techniques that could have given rise to Menkveld's discovery hence the importance of the thesis – not only to academia but also to the financial industry[1].

Market making implies the provision of liquidity by publishing one's interest on a specific instrument (whether to buy it or sell it) to the rest of the market participants. This publication is usually referred to as the Quoting Policy. When one of these participants accepts a market maker's quote and a trade occurs the need to execute a hedge by the latter is typically triggered. As the market is dynamic the execution involves risk when the coverage is not immediate. Moreover, when the hedge is not perfect the risk is naturally increased. Both sources of risk, definition of the hedge

[1]Along the thesis, all the statements related to the "financial industry" have been extracted from conversations with different industry participants within a set of seven banks (Morgan Stanley, Citi, Barclays Capital, JP Morgan, Goldman Sachs, BBVA and Santander) and one hedge fund (Renaissance Technologies) across foreign exchange (FX), fixed income/credit (FI) and equities.

and execution, are this way required to be fully taken into account in the quoting policy hence their relevance in the thesis.

Further, backtesting seems to be the industry standard vehicle to analyse the risk involved by a strategy. However, the process of backtesting relies largely on the data sample. The over-adaption of the models' parameters to the data generates significant deviations between their performance in and out-of-sample. This core issue, known as overfitting, has been recently recalled by academia (Bailey *et al.* (2013a)) but little efforts have been yet put onto its mitigation. Hence the importance of the novel calibration technique proposed in the final experiment. It was also intended for the thesis to grow in terms of backtesting accuracy. This was attempted by gradually building each experiment upon the results of the previous. On this note, the thesis fills one of the gaps in the literature observed by Cahan *et al.* (2010) who stress the lack of research blending medium frequency trading (MFT) and high frequency trading (HFT) despite the risks involved by the latter (less known) are expected to affect the former.

As such, the core motivations of the thesis can be set out as:

1. The analysis of a scientific approach that could explain Menkveld's observations. In particular, the definition of an optimization procedure that allows for an statistically cheaper hedge in terms of liquidity provision;
2. The mitigation of the overfitting problem along such optimization process; and
3. The merge of different levels of data granularity to enhance the realism of the experiments' conclusions.

With all these features in mind follows a brief description of the further motivations behind the main domains of systematic trading included in the thesis: execution, quoting and risk-reward.

1.1.1 Execution

Execution is key to systematic trading for being the way the risk embedded in a quoting policy (market making) is eliminated. Of the several reasons why there is risk in the execution[2] I will put especial emphasis onto the placement of amounts large enough to have an impact in prices. A large part of the trading patterns that seem to be profitable in a backtesting environment turn out to be crucially eroded by their market impact and transaction costs (in especial, those designed to trade at high frequencies). The deviation between the expected execution price and the achieved execution price is called the *slippage*. The slippage is hence a significant variable to be considered in several trading strategies. However it is often disregarded due to its computational complexity.

[2]These range from the latency of the systems that affects the real availability to the trader of the liquidity present in the markets, to the information disclosure of the trader's appetite to the rest of the market participants who can consequently incorporate it into their strategies against the trader's interest.

By starting from execution I intend to include the high-frequency domain in the thesis and with it the Big Data management challenge[3]. This way, I can underpin the realism of my conclusions upon the whole range of trading frequencies.

Once the execution policy has been analysed it can be included into the optimization problem of quoting and risk/reward fine tuning.

1.1.2 Quoting

As said above, the way a market maker systematically[4] publishes her interest on a specific instrument is called the quoting policy. This also means that the trading style of the market makers is predominantly passive[5]: they wait for the rest of the agents to cross interests against them. As such, the tighter the bid-offer spread published in the markets the more transactions they can expect to cross. And the more crosses the more the opportunities to generate revenues hence the importance of being able to publish the tightest spreads.

By focusing on how to beat a standard non-scientific approach to systematic trading I have the opportunity to assess the role of applied science on the dynamics of the financial markets. Moreover, I can also bring to the foreground sound methodologies that could explain the recent academic findings about the nature of the markets[6].

Once the quoting policy has been defined it can be further fine tuned by assuming more well-defined risk dependant on the analysis of the markets dynamics.

1.1.3 Risk-Reward

The financial rationale is largely built upon the relation between the risk assumed by an investor and the reward obtained through it. In general, the higher the risk assumed by an investor the more feasible the largest rewards become. Finding the right equilibrium between the two (this is largely constrained by the investor's set of risk preferences) is the ultimate goal towards the efficiency of an investment.

By considering a chapter on risk-reward calibration I attempt to cover the problem of overfitting which is a common issue in data-driven optimization[7]. I was further motivated by the idea that out-of-sample robustness shall be favoured when the data-driven calibration process is bounded by the trader's expertise. This is, I expected that the constraints imposed by the trader's expertise

[3]After Manyika *et al.* (2011) a large part of the industry has concluded that the capacity to manage and exploit the so-called Big Data (a paradigm focused on the analysis of large volumes of data gathered from various sources of information and processed at high velocities) may lie behind the apparent take over of the markets' liquidity by traders that operate at high frequencies as observed by Jovanovic and Menkveld (2011) and Kirilenko *et al.* (2010).

[4]See Appendix for a detailed evolution of the trading industry from the manual trading to the systematic one.

[5]They can still be active in the Execution as they evacuate the risk triggered by a trade.

[6]Being it on the other hand a key aspect to the traditional industry participants who did not know how to face this disruptive change in the liquidity provision activity.

[7]Overfitting is one of the main weaknesses of backtesting when compared with the traditional heuristic fixing of the parameters.

typically favor the out-of-sample robustness of a strategy while the data-driven techniques allow exploiting hidden patterns. The challenge is to look for an intuitive model that allows the trader merging both.

Once the risk-reward approach has been properly fine tuned the trader can reasonably conclude how many alternatives are available to beat the bid-offer spread and how risky they may be out-of-sample.

1.2 Research domains

The thesis merges several disciplines ranging from financial theory and economics to computer science, statistical and stochastic analysis, and non-linear optimization – a challenge that largely confirms the aforementioned discussion started by Manyika *et al.* (2011). In particular the experiments conducted herein can be broken down into the following dimensions:

- **From a financial point of view**, I go through the core building blocks of a systematic trading model: execution, quoting and risk/reward management.
    - Execution is thoroughly explained by the study of the popular algorithm VWAP and its comparison with the rest of the algorithms currently considered in the industry. I also motivate a reason why dynamic updates of the forecasted intraday volume profile are financially undesirable when market impact is taken into account – enhancing the realism of the thesis' conclusions as advised by Cahan *et al.* (2010).
    - The hedge optimization process is defined for a set of optimal baskets that weigh off the pros and cons of not trading the full-replication hedge of an index. This requires an in-depth analysis of the transaction costs, slippage (deviation from the target) and market impact along with the tracking error (TE) implied by each strategy (hedge candidate). This approach complements and overcomes the analysis exhibited in Diez *et al.* (2012).
    - The quote optimization process is detailed for the isolated case of a market maker who needs to provide a risk quote (one whose price is commited before the hedge has concluded). I consider a risk quote on a large trade upon an ETF with the Spanish equity index IBEX 35 as underlying. This is the context in which I attempt to find the conditions that have to be met in order to reach a 50% cut on the standard bid-offer spread. This way I provide a scientific methodology that gives a sound explanation to the disruptive observations in Menkveld (2013).
    - Finally, the building blocks for an optimal risk-reward calibration are set out. With these I define a strategy which accounts for a novel trading indicator. The indicator is rooted on the analysis of the expectations embedded in the markets' quotes. Its calibration suffers from the typical overfitting of the parameters to the data sample. It is a problem still unresolved in the financial industry. Hence the proposal in the thesis

of a new technique which allows me mitigating the effects of overfitting as recently demanded by Bailey *et al.* (2013a).

- **From economics** the concept of Elasticity is borrowed and adapted to financial computing. It is used to define a first approach to the nature of the product which allows me to gradually reduce the bid-offer spread. The more the rest of the agents react to marginal changes in the trader's quotes the easier it is to passively balance the risk managed. This approach is cheaper by nature than actively managing the balance[8] as the trader does not need to pay the spread. Hence, the more elastic the activity available to the trader (the flow) the more the bid-offer spread can be reduced and the more frequently can be traded completing this way the conclusions from Menkveld (2013) and Kirilenko *et al.* (2010). By analysing each product in terms of its Elasticity instead of the nature of the asset to which it belongs economic synergies can also be exploited.

- **From computer science** I cover systems architecture, database cleaning and management as well as learning techniques:
  - Object oriented programming was used to create the systematic trading platform described in the Background. It allowed for both the efficient management of the dataset and its scientific analysis. And it combined a column-oriented open source database manager, Infobright (column oriented motivated by the fact that my data was ultimately a set of time series) with Java and R-CRAN through Python.
  - Data cleaning and management occurred at the smallest allowable price change by the exchange around a bid-offer spread (a tick) of an order book of 10 levels of depth from two different sections of the Spanish Stock Exchange (Trades and Blocks subsections). This situation largely meets the main characteristics of the so-called paradigm of Big Data – the 3 v's: volume of data, variety of channels and/or analysed at high velocity. Velocity in my case was not the main issue but the volume and the variety.
  - The machine learning techniques used herein reach both fields supervised and unsupervised learning:
    - Hidden structures in the volume's intraday profile were brought to the foreground through Principal Components Analysis (PCA). The first principal component of the matrix comprising the intraday volume profiles per stock in IBEX can be used to project the hidden seasonality of the market as a whole. Once extracted, the dynamics of the profiles can be analysed as deviations from it.
    - Reinforcement learning (RL) has also been used to allow the trader mitigate the probability of overfitting. Unlike a large part of the literature where RL is devoted to the direct training of trading systems and portfolios (Moody *et al.* (1998)) this thesis reduces its usage to the calibration of those systems. Typically, the calibration of the parameters for which the trader has no predefined prior distributions data-driven calibration tends to be the way these are fixed. The thesis proposes to

[8]Note that the market makers targets a zero end of day inventory whenever possible.

overcome this lack of a theoretical motivation behind the parameters by analysing the risk management policy embedded in each strategy. This way top raw in-sample performances traditionally selected could be disregarded if they generate a policy that is not appropriate to the trader's own experience. A standard Q-learning approach is for this purpose conducted in the attempt to extract the risk management policy (the Q-matrix) inferred by an autonomous agent that operates the strategy. This matrix is then compared with the prior defined by the trader through a simple metric of convergence. By targeting the agent that seems to mimic the most the trader's prior I build a bridge between data-driven and heuristic-driven calibrations.

◇ Support vector machines (SVM) are devoted to the improvement of the VWAP algorithm. I agree with An-Pin *et al.* (2009) in that traditional methods can hardly solve the dynamic environment issue resulting from the assumption of stationary process. In particular when referring to the dynamics of the deviations of the intra-day volume as mentioned above. Moreover, there are to the best of my knowledge no priors defined by the industry that can be exploited by researchers. As such, I consider shifting the focus from the dynamic structure of the intraday volume onto the estimation of core components in the static structure. For that I propose an SVM model which allows the trader distinguishing which slots of time along the day (bins) benefit the least from an estimation sensitive to past dynamics. This way, I largely avoid the timeframes that embed past inconsistent interest on the stock as these would negatively affect the model.

- **From statistical analysis** I use time series analysis, extreme value theory and cointegration as follows:

  - An ARMA(p,q) structure is modelled in the analysis of the dynamics around the static component of the intraday volume's profile. It is this way attempted to benefit from possible seasonality in the update of the forecast of the estimated profile along the trading session.
  - Extreme value theory is also selected to describe the statistical rationale behind the benefits derived from the analysis of the flow. In general, the trader's quoting policy shall not be based on the expected average of strategy's risk distribution unless a certain number of trades can be granted. The nature of the flow would determine the level of conditional tail risk (prudence) to be considered and subsequently priced-in hence motivating the definition of a new measure: the flow-VaR.
  - Cointegration is further proposed within execution in order to benefit from an additional layer of risk-reward optimization by selecting the hedge combination that allows for the best opportunity of mean reversion. This way the well-known Pairs Trading strategy (Elliot *et al.* (2005)) can be inherited from the active financial agents (liquidity takers) by the passive ones (liquidity providers, i.e. market makers).

- **From stochastic analysis** I borrowed Ornstein-Uhlenbeck (O-U) to model the mean-reverting (cointegrated) time series considered above and use it to propose an additional layer of risk management by taking into account both the expected margin rooted on the current deviation from equilibrium along with the expected time to reversion, i.e. the time horizon of the strategy. Both risk dimensions can help the trader better decide which mean-reverting patterns are feasible given the rest of risks managed and the risk policy followed.

- **From non-linear optimization** I took particle swarm optimization (PSO) to fix the bid-offer spread. I used it as a vehicle that allows inputting the trader's expertise along the calibration process in order to enhance the speed of finding the best candidate and, more importantly, the out-of-sample robustness as it crucially reduces the probability of spurious relationships.

1.3 Research objectives

Overall, the thesis aims at finding ways through the use of machine learning to further exploit the trader's priors along three core areas of systematic trading: execution, quoting and risk-reward. The assumption which lies behind this motivation is that along the calibration process professional experience is expected to have an advantage over raw, data-driven selection. The enhancement of the theoretical-drive of each strategy remains one of the main challenges in quantitative trading. In essence, leaving the parameters' optimization to data-driven processes typically generates over-fitting of the strategies to the sample which in turn affects their out-of-sample performance. The next chapters abound on the possibilities that machine learning render into the search for routines that allow the trader benefit from such drive enhancement. The positive externalities beyond a more robust performance can range from speed of calculations to an autonomous adaptation to changing environments. It is also intended to complete classical research on trading with these machine learning techniques and deliver more accurate results than standard literature by fusing end-of-day and intraday information. In doing so it is expected to allow me disclosing a robust approach that could be responsible for the recent decrease of the bid-offer spread in most of the financial markets.

In particular, the thesis as a whole progresses using quantitative techniques that blend finance with computational statistics in order to test a main experimental objective. As such:

1. The first target is to define a procedure that serves as a general framework for one of the most sensitive phases in the strategies' design (in terms of out-of-sample robustness): the calibration of the core parameters within a strategy – namely, take profit, stop loss and maximum time horizon. By doing so a recent demand from the literature (Bailey *et al.* (2013a)) is attempted to be fulfilled: **is it possible to find a reliable procedure that allows a trader discern what parameters' combination is more suitable out-of-sample according to her experience?**

2. Second, a part of the thesis is devoted to find the hedging features which may allow risk-averse agents quoting more aggressive prices in the markets than what has been considered a standard practice so far. Ideally converging into the discount observed by Menkveld (2013). To the best of my knowledge the literature has not provided so far a **comprehensive research on the methods that can allow for such a significant reduction of spreads**.

3. The third objective is to set out an algorithm of intraday execution that will help obtain realistic results along the rest of the experiments. Being VWAP one of the most popular execution algorithms in the industry I selected it for the thesis. I also took the chance of its setup as my execution benchmark to research on the way traders can overcome one of its most relevant challenges: the estimation of the intraday volume's profiles. For this purpose I focused on the role of machine learning. There is little that can be said about the insights on the intraday volume's profile apart from the stylized fact referring to its U-shape structure. I agree with Hobson (2006) that, as such, little relative improvements can be achieved on the field but I will still address the question: **is there any scientific methodology that could improve the execution strategy even if the enhancement is marginal?** The attempt to reach a marginal advantage upon recently popular methodologies obeys to the fact that, as long as it is robust, a marginal edge can make a large difference. And this is because execution is a business line where targeted profits per trade are measured in basis points (bps) and notional reach the billions throughout a year.

4. The fourth objective is to fuse results from intraday and end-of-day strategies as demanded by Cahan *et al.* (2010). The merge of mid frequency and high frequency strategies remains to the best of my knowledge a highly disregarded field. And that occurs even though it has a large relevance when dealing with the illiquidity issues (whether standard sizes on illiquid securities or large sizes on liquid securities) crucially present on the recent growth of the ETFs industry (Diez *et al.* (2012)). **What platform and intraday models are hence required to setup a research framework that allows for an enhanced realism in the back-testing scenarios?**

5. Fifth, I would like to define a new dimension that allows a trader distinguish the main risk management challenges that she will be facing when systematically market making a certain product. A new dimension that goes beyond the mere distinction of the nature of the asset. This is important as it would allow clustering business units that are currently dispersed across teams and, more importantly, across assets towards an efficient exploitation of synergies. **Is there any scientific motivation to change the current trading structure in the industry?** If so the scope of applied science can reach a new domain: the industry hierarchical management structure.

6. The sixth target is to find whether the flat commissions charged across liquid securities in guaranteed algorithmic execution (the so-called risk trades where the trader commits to a price level before knowing the result of her hedge) are a right practice or they should instead be a function of the complexity of the underlying' intraday dynamics. **Is the risk embedded**

in the different risk trades evenly distributed across instruments or dependent instead on a specific domain of the product's nature? An intraday analysis of the high frequency execution models may give significant information about it.

7. Seventh, I analyse the nature of the market microstructure in the Spanish market in order to understand what the **minimal level of the order book to be kept is** (as its depth is costly to store, maintain and analyse) or whether there is **hidden liquidity and, if so, whether it is randomly present or consistent**. The results from this analysis could motivate further enhancement of the algorithmic intelligence of the trading strategies.

As said, it is also important to note that the thesis covers comprehensively the main areas of systematic trading: execution trading, market making and risk-reward.

1.4 Research methodology

Follows a brief description of the experiments conducted in the thesis in the attempt to achieve the aforementioned objectives:

1. **Execution: an execution model based on support vector machines.** If a realistic backtesting of a medium frequency quoting strategy is intended to be deployed the intraday features that it would imply are required to be taken into account – allowing me to fulfill the lack of research noted by Cahan *et al.* (2010) . The execution analysis becomes this way a must.

   (a) The main issue when dealing with intraday execution is that in order not to miss valuable information it should ideally be built upon the smallest level of granularity. This leads to the so called area of Big Data. So, the first difficulty surpassed in this experiment is, as stated by Manyika *et al.* (2012) the management, cleaning and analysis of a large volume of data.

      i. The quotes document (.csv) with the tick-data was in excess of 10 times larger the size of my computer's memory (2GB). Hence Infobright, an open-source column-oriented database manager, was first used to locate the rows with the information of each stock and each day. This way I was able to extract data that could be afterwards analysed in-memory as typically, in Equities, a day of tick data of a stock is smaller than my memory allowance.

      ii. The trades document was small enough to be directly uploaded in-memory. Again, the information per stock and day was identified and fused into a single document that comprehended both quotes and trades per day for each security.

      iii. There was a third document enclosing the block trades. These are ad-hoc trades (typically large) arranged between parties that are exchanged through a different process but still officially published in the records of the order book. As neither their price nor volume are informative for the purpose of the analysis of the dynamics of the market they were matched against the previous document and extracted from the final database.

iv. Similarly, abnormal quotes and trades (such are zeroes in prices or amounts) were identified and extracted.

(b) Second, a model for execution has to be chosen and developed. The popularity across industry participants of the VWAP algorithm when large trades are considered motivated the analysis of this algorithm. Hence it was assumed as my high frequency execution strategy in the attempt to minimize the market impact of my medium frequency quoting strategy. One of the main challenges when setting up a VWAP algorithm is the estimation of the intraday volume's profile. To the best of my knowledge, there is no literature about the rationale behind the profile's dynamics. This leads to the usage of data-driven approaches in its estimation which in turn may have undesired implications out-of-sample in terms of accuracy robustness.

i. I set as benchmark a popular model referred to as PCA-ARMA (Bialkowski *et al.* (2008)) which uses a two-fold approach to the intraday profile.

A. First, through the first principal component upon the intraday volume profiles of a representative basket of stocks of the exchange (e.g. the Spanish index IBEX for the Spanish stock exchange). It sets the so-called static component that represents the main structure for the intraday volume of all the stocks.

B. Second, there is the dynamic component around that static one. As such, it fits an ARMA structure that dynamically corrects the expected profile as the observations occurred along time materialize the deviations of such expectation from the market.

ii. After developing such an execution model I motivate upon a series of criticisms the analysis of a different, static-only model.

A. The criticisms rely on the fact that the error measure used in Bialkowski *et al.* (2008) is defined through the VWAP itself. However, the only prediction is the intraday volume profile and the VWAP includes the evolution of the prices. These ultimately favour the error level of those stocks whose price evolution is less volatile along the data sample. Hence, the quality of the VWAP algorithm shall be analysed in terms of isolated volume deviations instead of VWAP deviations. This confirms the relevance of the intraday volume profile estimation. Also, in their paper, errors are net off along the trading session. That is a usual and statistically elegant approach (asymptotic theory) however, in trading though, as there are costs involved in the access to the exchanges (along with others, more subtle, like market impact) the average net off is not innocuous. As such, a different error measure is proposed: the cumulative absolute percentage errors (CAPE) upon the volume.

B. With all this in mind I first try to confirm through the autocorrelation function

(ACF) and the partial autocorrelation function (PACF) whether the ARMA(1,1) structure selected in their paper is or not robustly justified in our database. The findings exhibit the fact that it seems not to be the optimal combination of lags. Moreover, once its parameters are calibrated through linear regression and once its forecasts are included in the estimation of the intraday volume profile it can be seen how the structure acts as a multiplier of the volume outliers along time – an obvious case in Arcelor Mittal and International Consolidated Airlines across my data sample. And this challenges the assumption that the deviations are Gaussian (fat tails is a common feature instead).

C. Other candidates to the static component such are the mean and the median are hence motivated with the same result: the dynamic structure seems not to be justified. Moreover, both the median and the average yield lower CAPEs in my data compared to the PCA.

In summary, the results of the comparison, consistent across all the stocks in IBEX, exhibit the fact that in my data it is preferable to use the most basic versions of the intraday volume profile estimation, simple averages and medians, towards the static component with no dynamic component.

(c) Third, I noted that both basic approaches, mean and median, mainly differ on the sensitivity that it is given to the data sample. And as pointed out above there is not literature nor financial insights as to the length of the historical sample to be used when predicting volume bins. An indicator based on SVM is thus proposed to help identify those areas where the trader shall be less sensitive to the data sample. This is, where the information about its historical dynamics are less significant in the prediction of its future levels. If the indicator delivered positive results in my data it would allow overcoming the benchmark reaching then the **third objective listed above** (to improve the accuracy of the VWAP algorithm). Also, the average deviations from the intraday volume profiles can be used as a proxy for the risk of slippage embedded in my execution strategy. And this would allow meeting the **objective four** (to enhance the accuracy of backtesting) in subsequent experiments using this algorithm as part of its execution. As said, only once the intraday execution risk has been defined the quoting policy of a mid-frequency hedging strategy can be comprehensively analysed.

(d) Fourth, by analysing the distribution of errors from the proposed model across the stocks that belong to the IBEX35 along my data sample I can observe whether there is a profile for the errors dependent on the distribution upon market capitalization. If so, a flat commission for guaranteed algorithmic execution should not be the optimal policy as stated in the **sixth objective** (the analysis of the flat commissions as an optimal pricing policy).

(e) Finally, and following the information needs mentioned in **objective seven** (how the analysis of the order book can serve to cut costs and improve strategies), the analysis of the dynamics of the quotes within my tick database can be used to find stylized facts about the nature of the market microstructure. Not only the distribution of ticks and trades along time are relevant to understand the role of the latency on each product but further information can be inferred. As such, by simply taking the weighted average price between subsequent levels of the order book and the price of closest trade done within a certain range of ticks I can compare the distribution of errors of each weighted average price. I intend this way to assess which are the depth levels that are most relevant in terms of embedded information on the equilibrium dynamics. Also, by comparing the amount posted in a tick previous to a trade and the amount reported on that trade information about hidden interest can be brought to the foreground. By then clustering the observed hidden interest into high urgency (that between the bid-offer spread) and low urgency (that outside the bid-offer spread) further granularity about its distribution can be observed.

2. **Quoting: a quoting model built upon particle swarm optimization.** The second experiment looks for a scientific technique that could for the first time provide with a sound rationale the disruptive market facts raised by Menkveld (2013). In particular, the recent discount observed in the overall bid-offer spread of up to 50%. Two are the main possibilities considered: a general assumption by the market making participants of higher levels of risk within their strategies or the exploitation of new, scientific techniques that allow them posting more aggressive quotes at the same risk levels.

   (a) To the light of the recent changes in regulation against the assumption of risk by market makers the latter of the possibilities seems more relevant. Hence, the experiment is defined within a framework that focusses on a risk measure which can explain lower risk levels than the standard value-at-risk (VaR) with the same prudence.

      i. By noting that the nature of the market making activity crucially depends on the characteristics of the interest on the liquidity that it provides (its flow) a novel measure of risk is generated, the flow-Value-at-Risk (*flowVaR*). It is based on a novel concept, the *Elasticity of the Flow*, that serves as the new dimension reflected in **objective five** – the domain disregarded so far which may be the cornerstone for a structural change in the way the industry approaches trading[9]. The *flowVaR* allows advancing the risk analysis from a prudent perception on isolated trades to an equally prudent perception on the average outcome of a series of trades (the flow). And this gives an edge to the trader if deviations can expectedly be net off.

      ii. As such, by gradually considering the Law of the Large Numbers a fixed level of prudency can be reconcilable with different levels of tail risk as the trade is repeated. Ultimately converging to the average outcome of the distribution as the number of symmetric trades[10] grows.

   (b) This way, a medium frequency strategy for the market making of an exchange traded fund (ETF) on IBEX was created. Note again that the intraday, high frequency infor-

[9]Note that this type of assessments have finally happened at least in BBVA.
[10]Random in side and similar in size.

mation was accordingly inherited from the previous experiment. Not only from the error distribution embedded in my VWAP the slippage in execution can be estimated but the nature of the specific algorithm is also needed to design its market impact model. In the thesis, it is an adaptation of Almgren *et al.* (2005). These authors introduced a popular two-fold model for market impact:

i. Permanent market impact: component that reflects the buy/sell imbalance information that has been sent to the market participants.
ii. Temporary market impact: component that reflects the temporary price concession in order to attract interest within the execution timeframe.

Their model could be easily used to calculate the market impact of the components that I consider. However, the parameters that they calibrated were bespoke for a dataset[11] of US stocks traded by Citigroup between 2001 and 2003 that included (without any differentiation) market orders, limit orders and VWAPs. My dataset, in turn, not only refers to Spanish stocks but also to a period where electronic execution was largely developed thus I am reluctant to use the same model. More importantly, my execution will be done through VWAP only which, as said above, targets to homogenize the market impact by taking into account the intraday profile of the volume. Hence, I propose adapting Almgren *et al.*'s permanent market impact model (the one that affects the most a medium frequency strategy) to this situation in the seek for a linear model that depends on the size as a percentage of the average daily volume (ADV) traded. On IBEX, I propose an average of 2.97% of permanent market impact when the whole ADV is traded through a VWAP[12].

(c) Once the transaction costs analysis and the market impact have been appropriately considered for each of the IBEX components I can calculate the bid-offer spread to be quoted if the IBEX was completely hedged along the market making activity of its ETF. Such spread will serve as a benchmark to beat by my learning-adaptive strategy.

(d) The non-perfect hedge embeds risk and as such it becomes a strategy. As I would like its deviations to be limited the enhancement of the probability of cointegration becomes a target to the algorithm following the essence of the so-called Pairs Trading approach.

(e) Then, through the usage of particle swarm optimization as in Diez *et al.* (2012) the tracking error, TE, is optimally weighed against market impact and transaction costs along different possibilities until a hedge is decided. This hedge should hence benefit from a lower market impact and transactions costs combination at the price of entering into deviation risk – probabilistically controlled (cointegrated) but still undesirable.

(f) Interestingly, this strategic approach is not enough to beat the benchmark when a prudent level of value-at-risk, 99%, is considered in the measurement of the deviation

[11]It is important to note that these datasets are still rare to find in research.
[12]This figure turned out to be very relevant to the industry as, to the best of their knowledge, there is no other public reference in terms of the expected market impact of a VWAP. The market impact is key to the pricing of a guaranteed execution.

risk. It is important to stress that the deviation risk has an extra layer of prudency by penalizing through the usage of a Student-t – with fatter tails than the Gaussian. By attempting to beat one of the most adverse scenarios I expect my model to be more reliable.

(g) By then noting that VaR is intended to represent the case of a punctual trade and that the recurrent activity along time (the flow) has not been so far considered the *flowVaR* can be embraced in subsequent analyses.

(h) When the *flowVaR* is gradually considered as more trades are assumed it can be seen how there is a point at which the strategy can beat the benchmark. In particular, if the trader can comfortably expect at least a certain number of symmetric trades the bid-offer could be decreased by more than the aforementioned 50% I could give an answer to **objective two** (give an explanation to Menkveld (2013) findings).

Once the quoting policy has been defined for any given level of risk the optimal level of risk/reward of the strategy can be further considered.

3. **Risk-Reward: a calibration model defined within a Q-learning framework.** The third experiment completes the previous two by analysing the calibration process of a risk-reward policy. This way, I attempt to find a technique that allows for a more robust out-of-sample performance. Typically, the calibration processes are data-driven. And data-driven models are an efficient mechanism to find latent information in a data set but tend not to be designed to allow for the input of professional insights. These insights in-turn are expected to provide the strategies with out-of-sample robustness. There is hence an apparent need to find a bridge between the two opposite approaches: data-driven and theory-driven. A way to exploit the data as much as possible without missing the professional expertise.

The experiment proposes a technique that consists on setting out an autonomous agent per each vector of the possible parameters combinations within the calibration universe. Each agent is set to learn the risk policy of an ad-hoc strategy defined through one of such combinations. The ad-hoc strategy is based on an indicator, a signal upon such indicator and a set of rules that mainly define the risk and inventory management principles. A novel indicator, the *Expectations' Shift* is proposed to capture the changes in the expectations of the market making agents by scanning those shifts in the prices that, on average, do not obey to a steady market impact along the trading session. The trigger of the signal is a parameter for which the trader may not be able to motivate a particular prior (mean-reversion or momentum) hence it will be calibrated in the process along with the take-profit and stop-loss parameters. The maximum time horizon is fixed to a week (medium-frequency trading) and the trader does not build inventory upon the signal (i.e. does not trade consecutive signals on the same direction).

(a) First, the standard data-driven approaches are generated: maximum performance at the end of the sample, maximum average performance in-sample and maximum utility.

(b) Then, the use of reinforcement learning is proposed, and in particular, its Q-learning version as the process that allows to merge both approaches data and theory-driven.

i. In Q-learning, the knowledge achieved by the agent (in this case a risk management policy) is input into a so-called Q-matrix.

ii. By comparing the prior of the Q-matrix considered optimal by the trader with that learnt by the agents across the range of parameters combination within the calibration universe the agent whose trading style is closer to that of the trader can be selected.

This is, the trader can select her avatar within the calibration universe – hence the name of *Avatar Trading*. The comparison is done in the experiment through a simple metric based on rakings: the *Reasoning Convergence*.

If positive evidence of the enhancement of out-of-sample performance is found in the data sample it would allow concluding that robustness can be achieved by embracing data-driven and theory-driven approaches. This was stated as my **objective number one** (give a methodology to overcome the issue risen by Bailey *et al.* (2013)).

1.5 Major contributions

The experiments conducted along the thesis can be summarized as follows:

- The first experiment motivates the need to use an error measure, CAPE, based only on the deviations of the intraday volume profile estimation instead of based on the deviations of the VWAP. The reason is that the deviations in terms of VWAP mix the deviations of the intraday volume with the price dynamics. And this mixture generates a noise that difficults the comparison across methodologies.

  Under such measure the experiment states that the dynamic structure from Bialkowski *et al.* (2008) does not hold robustly in my data. On average across the different stocks considered, the addition of the therein proposed dynamic structure upon the first principal component triples the out-of-sample error. Moreover, the PCA itself seems not to be the best track for the static component neither within a dynamic approach nor isolated statically when compared with simple medians and averages.

  The experiment further shows that the dynamic VWAP's intraday volume analysis can be improved. Through machine learning the trader can flag whether the system has to be more or less sensitive to the sample when estimating each time slot. The *Sample Sensitivity Index* (SSI) is for such task defined. It is based on a support vector machine with inter and intraday features. And its application gives rise to a set of improvements in the original approaches of up to 3.5 basis points in my data when the Spanish market does not overlap with the US trading hours.

- On the second experiment, Particle Swarm Optimization is applied to the pricing of a risk trade on an ETF on IBEX 35. It confirms the results from Diez *et al.* (2012) and abounds on its approach through four key aspects:

    - First, it gains speed by letting the heuristics of the trader reduce the dimensions of the hedge optimization problem from the number of stocks to the number of sectors implied in the index.
    - Second, the optimization process includes a complete and detailed set of the features that affect the quotes. As such, the model for market impact is thoroughly motivated as an adaptation of Almgren *et al.* (2005) to my convex execution algorithm – note that the technique proposed in Diez *et al.* (2012) was set out for less liquid instruments (emerging markets) hence the accuracy of their model is expected to be lower. The transaction costs are also broken down into smaller granularity, in particular, the slippage of the execution algorithm is herein isolated by considering the results obtained in the previous experiment (the error distribution from the SSI) .
    - Third, a novel approach to the optimization problem, which includes a new dimension through the analysis of the flow, allows me to conclude that in the presence of elastic flow the bid-offer spread obtained following a process like Diez *et al.* (2012) can be halved without changing the risk appetite of the trader. This statement coincides with the 50% discount on bid-offer spreads observed by Menkveld (2013).
    - Fourth, the experiment is further complemented with an indicator that allows a trader consider within her hedging strategies what deviation dynamics can be expected in the mid-run. In particular, by fitting a mean-reverting signal through an Ornstein-Uhlenbeck upon the aforementioned optimal hedge. In my data, I can expect through it that the hedge itself will underperform the IBEX during the next months (106 business days) by around 14%. This result suggests that by giving different urgency to the full-replication hedge along both sides (bid and offer) the trader can expect to gain an extra profit from the mid-run dynamics of the non-full-replication hedge. The blend of this type of buy-side approaches with the market making activity can become both a competitive advantage to the market makers and a new domain of research. And it also benefits from the merge between high and medium frequencies as stated by Cahan *et al.* (2010).

- Finally, along the third experiment the typical challenges around risk-reward management are visited.

    - First, I propose a novel indicator that targets to anticipate changes in the prices through the isolation of the market makers' expectations.
    - Then, its signal is data-driven calibrated following typical approaches such are maximum end-of-sample performance, maximum average yearly return or maximum utility (that embeds the former two along with a new dimension: the maximum drawdown). In-sample results are promising as profits range between 49% and 141% for the different

approaches considered. Out-of-sample results, in turn, are largely different ranging in this case from 12% losses to 5% profits. And this suggests that the parameters selected along the aforementioned data-driven calibration process may have been overfitted.

- In order to avoid overfitting a wrapper of theory is proposed. Q-learning is this way applied to let a set of agents autonomously learn the management of the strategies implied by each combination of the calibration process. Again, the former approaches are considered within a universe now of a risk management policy that only trades signals that show some consistency in terms of performance. Even though in-sample results are again promising they are more prudent now, yielding profits that range from 41% to 85%. Out-of-sample differences are also less dramatic ranging from 4% losses to 7% benefits.
- Finally, by ranking the levels of convergence of the policies learnt by these agents with that with which the portfolio manager feels most comfortable expert criteria can be included into the calibration process. For that a measure called the Reasoning Convergence (RC) is proposed by comparing the Q-matrices of the different agents with that defined by the portfolio manager. When the parameters implied in the strategies output by the closest agents to the portfolio manager in terms of RC are considered in-sample profits range from 41% to 52% and out-of-sample from 2% to 8%. Even though deviations are still non-negligible they have been largely reduced through this approach and, equally important, losses are avoided.

From which the following core contributions can be extracted and enumerated:

1. It motivates the usage of Reinforcement Learning in the creation of a theory-wrapper upon the typically data-driven calibration process. This process allows the trader selecting those strategies that generate learning policies similar to that with which she would feel most comfortable. An approach that I called Avatar-Calibrated trading (an agent-based approach). The out-of-sample performance of this novel type of trading surpasses that of the standard data-driven calibration approaches in my data. Hence it represents a formal approach to overcome the pseudo-mathematical issues described by Bailey *et al.* (2013a).

2. It provides successful results that help disclose the disruptive techniques which can lie behind the apparent enhanced competition for the liquidity of the markets. More interestingly, the methodology proposed herein allows surpassing the 50% discount observed in Menkveld (2013).

3. It shows how the intraday volume's profile of a VWAP algorithm can be better estimated using non-linear approaches such is the pattern detection via support vector machines as in the herein defined Sample-Sensitivity Index.

4. It enhances the backtesting accuracy of a market making strategy by fusing its medium frequency with its execution high frequency strategy as proposed by Cahan *et al.* (2010) through a well-defined Big Data framework.

5. It shows that new dimensions to risk management such is the shift from isolated trades analysis to flow analysis can be significant in systematic trading and this implies a deviation from the traditional trading literature.

   (a) It defines *Elasticity of the Flow* as the main feature to determine the market making approach beyond the nature of the asset class or its trading format. This allows to research on new advances based on the concept.

   (b) It further defines a tail risk measure, the *flowVaR*, to be included into the quote optimization process that overcomes the classical VaR on market making type of trading based on the possibility for the trader to apply the Law of Large Numbers.

   (c) As mentioned above through this and other dimensions the thesis has also reached a challenging target: to be the base for significative changes in the industry. And it has achieved it within the timeframe in which it has been written – giving an idea of how rapidly the industry is evolving and how accurate the estimated relevance of this research was back in 2010. In particular, it has been crucial to underpin the setup of a new unit within BBVA: Global Strategies & Data Science. BBVA is a major bank that operates in Europe, Latin America, United States, China and Turkey. The global unit aims at exploiting strategic synergies derived from the shift from an asset class structure to a cross-asset one given the nature of the flow (elastic or inelastic).

6. It shows that the risk embedded in the volume's profile estimation is concave with the capitalization of the stocks in IBEX 35 and non-linear intraday (with larger risk during the market hours when the US markets are open).

7. Evidence is found to support that it is the second level of the order book the one that gathers most of the information responsible for the price discovery. Also, the so called hidden-liquidity tends to be consistently present in the order book for non-negligible periods of time hence motivating its estimation as a way to improve the intelligence of the execution and market making strategies.

As a whole, the thesis brings to the foreground the benefits from accounting for scientific creativity in the immediate future of the trading activity.

1.6 Thesis outline

The thesis is divided into 6 chapters including the present introduction and it is structured as follows:

Chapter 2 – *Background.* Follows a chapter with the background required to understand what I consider is the latest paradigm of trading. It starts with a literature review that asserts the role of market makers in the industry, the effects of their automation and the essence of the so-called limit order book. Especial emphasis is given to the highest frequencies of trading. Second, there is a section on the existence of a disruptive approach that differs enough from the former motivated through an evolutionary perspective and an set of economic reasons. Third, the disruptive change itself is thoroughly broken down across three main aspects: the analysis of the flow, the trading pillars involved in market making and the architecture of a systematic trading platform. Last, there is a brief overview on the mathematical foundations of the techniques that have been used along the thesis – from the classic statistical analysis to machine learning.

Chapter 3 – *Execution: an execution model based on support vector machines.* The third chapter is dedicated to the improvement of one of the most popular execution algorithms: the VWAP. After broadly disclosing the current industry-standard algorithmic trading execution models, the most popular techniques to predict the volume's profile are introduced. Once the main results from using each methodology for Telefónica (the largest stock in IBEX) are fully described and after checking that those are robust across the rest of the IBEX's members, an improvement is proposed. In particular, a novel indicator is defined in the attempt to let the trading system know whether to put more or less emphasis to past data on each bin based on SVMs. Again, results are shown to be positive for Telefónica first and then contrasted with the rest of the index's members which also turn to be positive. I exhibit this way a successful method to improve VWAP. Finally, an analysis of the market microstructure is provided to motivate a further improvement of the strategies through optimal risk-reward policies.

Chapter 4 – *Quoting: a quoting model built upon particle swarm optimization.* The fourth chapter accounts for an experiment related to the provision of synthetic exposure to an instrument when its standard (full-replication) hedge does not yield a competitive spread. First, the standard spread is calculated in detail, after an in-depth analysis of transaction costs and market impact. Then, deviations from full-replication are considered and PSO is used to approximate the best hedging candidate by weighing the deviation risk against transaction costs and market impact. A new concept, the *flowVaR*, is further introduced to let the trader exploit the nature of the flow at any given level of risk aversion. This new concept successfully allows to reach discounts of the bid-offer spread in excess of those observed by Menkveld (2013). Finally, mean-reverting strategies are briefly tested to motivate a further improvement through optimal risk-reward policies.

Chapter 5 – *Risk-Reward: a calibration model defined within a Q-learning framework.* The fifth chapter describes in detail the different core features to be considered in the risk-reward policies outlined above to improve execution trading and market making. It starts by defining a strategy as the combination of an indicator, a signal, a set of rules ranging from risk management to money management, and a calibration process around the parameters involved by the former. *Expectations' Shift* is the novel indicator upon which signals are flagged so that the trader can detect in which dates the expectations of the markets' agents changed significantly. Then, a standard set of (data-driven) calibration mechanisms is analysed and compared to the proposed "Avatar-Calibration". This novel agent-based methodology (which attempts to provide the calibration process with a theory wrapper that yields more robust patterns out-of-sample) successfully obtains positive results in my data – further motivating the need for an optimal equilibrium between data-driven calibration and theory-driven (trader's priors) instantiations.

Chapter 6 – *Conclusions and further work.* Conclusions and further work are drawn on the seventh chapter.

Chapter 2

Background

The Background is set to provide the sufficient scientific knowledge and trading acumen to reach the research objectives set out in the Introduction. As such, it starts by giving a comprehensive overview of systematic trading ranging from the definition of the terminology used in the thesis to a literature review that motivates its upcoming relevance in the research evolution. It then explains the different steps involved along the calibration processes of both fields quoting and risk-reward, and motivates the way medium and high-frequency trading are blended in the thesis. Then, the aforementioned new dimension that allows reducing the bid-offer spreads without changing the trader's risk appetite is presented. Finally, the mathematical foundations of the different research domains included in the thesis are outlined. The chapter is completed with a series of industry insights enclosed in the Appendix.

2.1 Systematic trading

This section is aimed at providing the necessary financial background to understand the trading aspects of the thesis.

2.1.1 Definition of terminology

Algorithmic trading vs trading with algorithms: given the large confusion that the term typically generates, it shall first be stated that by algorithmic trading I won't broadly refer to strategies deployed through algorithms but only to those within that domain that are devoted to execution optimization[1].

Systematic vs discretionary trading: while systematic trading is a back-testable approach discretionary trading is not; this means that traders typically cannot check ex-ante the expected performance of their strategies (just until a stop-loss is hit or a profit taken). I will refer to systematic trading as any trading process that is not dependent on discretionary decisions.

[1] I would expect that Regulation should reset its definition to its broader scope in the near future but I will still abide by its industry concept.

This is, systematic trading can be built upon strategies ranging from heuristic algorithms that rely on professional experience to quantitative algorithms that have been scientifically calibrated but in both cases they ought to be cleanly defined and consistently followed. As highlighted above, it is important to note that even though rules are not always back-tested they still could. I will refer to discretionary trading as any trading process where traders may use some indicators (quantitative or not) to guide their strategies at their discretion. Opposite to systematic trading, discretionary traders decide in what degree they follow the signals dependent on their personal expectations. This style comprehends the traditional trading and, more especially, the classical sell-side trading where traders use models developed by quants to help them decide how to hedge their positions but not systematically.

Systematic vs automated trading: throughout this thesis systematic trading comprises both trading processes constantly followed by humans (i.e. I accept the possibility for systematic manual trading) or by machines. This is, automated trading is a subset of systematic trading. As we shall see below, the higher the frequency the more the relevance of automation since there are few scenarios where intraday systematic manual trading is feasible.

Quantitative vs heuristic-rules-based trading: I will refer to quantitative trading as systematic trading that relies on optimized (quantitative) strategies. This is, rules are excluded from this group if they are not embedded into calibrated models. I consider that if the optimized model is not systematically followed the strategy cannot be categorized as quantitative. Nevertheless, given the definition above for discretionary trading there is no room for quantitative discretionary trading. A relevant question could be what the role of the heuristic rules within quantitative trading is. A further discussion hence arises at this point. First, it is important to highlight that heuristic-rules are nevertheless set by experienced professionals, specialized in localized niches, whose views should be close to the optimal strategies. They should be fine tuned using scientific approaches but there is always a trade-off between costs and benefits. Costs can range from disclosure to developers of secretive strategies and the loss of intuition behind their performance (when the trader does not account for development skills) to the inclusion of expensive delays. The latter not only refers to the development of the models itself but also to the time required to compute the parameters that may add enough delay not to be able to profit from the targeted pattern. Risk of the model is not negligible either. The large deployment of CEPs (systems for complex event processing) across the industry is a signal that a wide spectrum of patterns is being managed case-by-case; and this could be the result of using rules instead of models to quickly react to patterns especially at the highest frequencies as in the case of ultrahigh frequency trading and algorithmic trading as defined below.

HFT vs quantitative trading at high frequencies: This case is a subset of the previous one. I will refer to HFT as intraday systematic trading. It is precisely a high frequency of the transactions what requires it to be systematic. As a result, discretionary intraday trading is not considered HFT in this context while algorithmic trading is. As in systematic trading, HFT does not necessarily need quantitative intelligence: it can also be deployed using rules. This claim is easier to be understood at the highest frequencies of HFT, ultra-HFT (UHFT) also called below-second systematic trading, where the required speed of reaction restricts considerably the use of complex scientific models. These instead depend crucially on the optimization of the architecture as a whole and focus on algorithms of (largely non-statistical) arbitrage that could be directly deployed in the hardware (GPU programming). Algorithmic shortcuts such are the on-line calculation of the average or the on-line version of PCA, etc also become relevant at this stage. On the other hand, the quantitative models of HFT would require reasonable time frames in order to let their parameters be updated and as such, would be mostly deployed at the lowest frequencies within HFT[2].

At this point it should be mentioned that I will refer to as mid frequency trading those approaches that consider holding their position from two days to a week and low frequency trading when the holding time takes more than a week[3].

2.1.2 Classification of trading approaches

The main domains that account for the core differences between systematic and discretionary trading approaches are suggested to be: the type of analysis, the frequency and the back-testability. I also classify the set of the main trading approaches in the following couple of figures, Figure 2.1 and Figure 2.2, where the shade of the boxes accounts for the difficulty of back-testing[4] – the lighter the box the more reliable it is. Note that such classification is not a universal rule but an eloquent clarification instead.

Macroeconomic analysis: especially relevant on long run asset allocation, i.e. low frequency strategies. Difficult to systematize and back-test as it involves the generation of changing macroeconomic policies and expectations updates based on a global fundamental analysis. It is typically deployed by traders educated in Economics.

Technical analysis: also called charting. It is based on the repetition of figures on charts (the patterns) at different granularities. Typically these strategies are not followed systematically by the traders who tend to interpret idiosyncratically the signals generated by the identified patterns. As a result it is mostly devoted to manual trading and consequently both its accurate back-testing is not feasible and it is naturally not deployed at the highest frequencies. Its upside is its ease of pattern detection that helps its performance by herd-like application

[2]To the question, how high is HFT? I will abide by the opinion that it depends on the nature of the instrument itself. Hence, we could say that *as high as the instrument's nature allows for*. The features that fix the maximum frequency at which a strategy can be profited from range from the flow nature of the product to dynamics of the order book (resiliency is key, as we will see below), the volatility of the price and the bid-offer spread along with the costs. As such, in reality HFT should in our opinion read *highest frequency trading* instead.

[3]Not to be confused with the short, mid and long run timeframes of Economics which typically correspond to one year, five years and more than five years, respectively.

[4]Beware that the back-testing of the pattern and the back-testing of the performance are different concepts: the former is easier to back-test than the latter as it does not require assumptions as to how the rest of the agents would react to the pattern's exploitation.

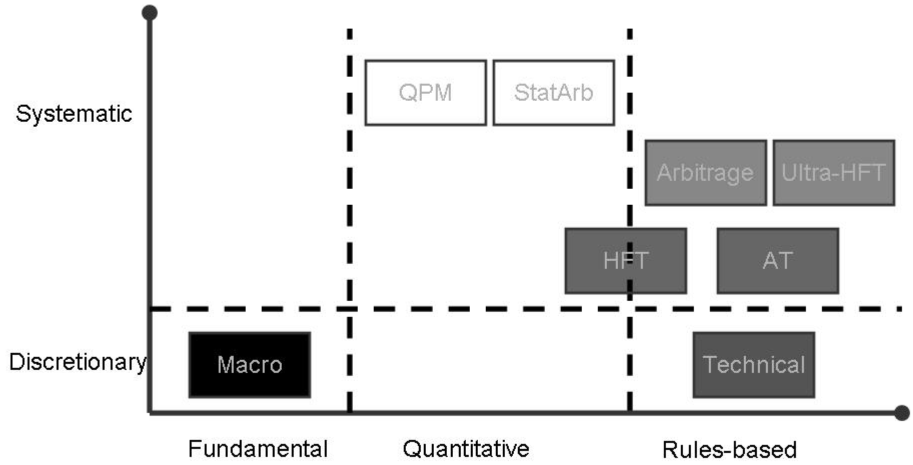

Figure 2.1: Typical classification of trading approaches by automation and type of analysis.

(the more agents use it and believe in the most popular indicators the better it works as they move the market abiding by them).

HFT: as said above, it is systematic intraday trading whether quantitative or rules-based. The inclusion of strategies within the order book makes difficult to back-test its performance as the reaction of the rest of the agents to the patterns' exploitation should be taken into account. The lower the frequency the larger the room for complex intelligence to be included in the algorithms (i.e. in the limit, ultra-HFT is defined by very simple rules with advanced programming shortcuts such is GPU programming). When it is triggered by opportunistic patterns (this typically occurs around news and corporate actions) instead of market making, positions tend to be held during longer periods as frequency is lower and volume per trade still low. As such, further access to dark pools of liquidity would enhance its performance both in levels and volatility (yielding higher levels of risk-reward indicators such is the Sharpe Ratio).

AT: traditionally rules-based but it is progressively moving onto the inclusion of intelligence making it this way face the difficulties in back-testing (same as HFT). Most of its development is done in the absence of quantitative approaches as its strategies are largely devoted to the highest frequencies within the day – heuristic rules typically fine-tuned through live tests instead of back-tests, since, as mentioned above, the reaction of the rest of the market participants to the behaviour of the algorithm is barely clear.

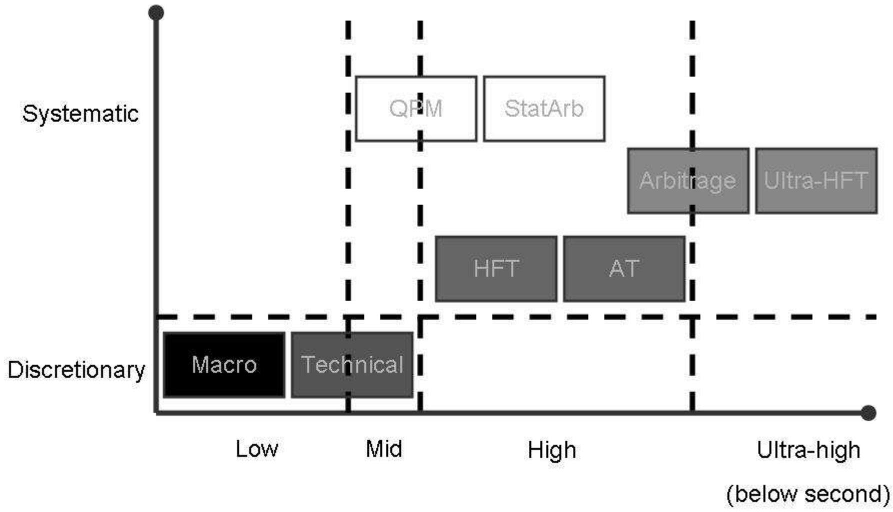

Figure 2.2: Typical classification of trading approaches by automation and frequency.

StatArb: usually referred to as Pairs Trading which consists on remaining market neutral buy buying a security or a basket of them and selling another one or another basket. This is, it targets managing TE instead of directional market risk. As explained in the section of Mathematical Foundations, it typically targets cointegrating long-short strategies whose time-to-reversion (also known as theta risk) tends to be measured through an Ornstein-Uhlenbeck model instead of a simple sample average. Its back-testing is usually accurate as it tends to target above minute time horizons (i.e. it does not necessarily requires beyond level-1 information of the order book and this eases its data management).

QPM: similar to StatArb but back-testing does not occur on individual stocks, instead it is analysed on the historic performance of the trader's book overall. The portfolio does not necessarily need to be defined across assets but can also refer to the diversification of strategies and is usually constrained by the target of zero end-of-day inventory in the market making approach. Its back-testing, although it would still be as reliable as the one of StatArb, is more difficult to implement as it requires a comprehensive platform to account for the interaction across strategies as we will see at the end of the present chapter.

Arbitrage: a mere comparison of prices typically in the presence of cross-listings such are the depositary receipts[5] (DRs) and fragmentation or corporate actions. After the trade, there

[5]A negotiable financial instrument issued by a bank to represent a foreign company's publicly traded securities within a local stock exchange.

is the clearing and settlement. These events typically take a time horizon between T+2 and T+3 (such are the German stocks) and can add variable risk to the cost of carry (e.g. overnight funding). There are also other costs often disregarded by the less experienced traders such are the taxes (sometimes the buys and sells of the stocks or the currencies are subject to changing taxes – e.g. dividends enhancement) and, more subtle risks, such are the cases where the type of the instrument cross-listed is different to the original (e.g. different voting rights of the shares, etc). The missing of those can erode the arbitrage opportunity and turn it into a loss.

UHFT: these tend to be strategies related to smart order routing (SOR) in the seek for liquidity across different pools (as in AT) but also includes some opportunistic strategies such are those that follow rules based on basic statistics to forecast the end of a FIX message[6] before it is completely read – obtaining an advantage in speed with respect to the rest of the agents at the cost of the risk taken. GPU programming and other approaches that allow direct control of the hardware in a tailored manner are also core tools utilized in this type of trading.

2.1.3 Building blocks

The main challenges of the latest market making paradigm[7], being it a cross-data-granularity (all range of frequencies), non-full replication (statistical), systematic (the result of the evolution described above), zero end-of-day inventory (a usual rule for market making) approach can be categorized as i) the data, ii) the hedge optimization (whenever it is feasible), iii) the execution, iv) the risk management and v) the quote optimization.

Data

The classical statistical analysis requires observations to be independent and identically distributed (typically Gaussian). While at the lowest frequencies of financial data this is usually achieved by simple transformations, such is the calculation of the rate of returns, in the case of the prices (where, typically, Normality is further forced by taking the logarithmic function), high frequency data analysis suffers from a series of relevant issues that the trader needs to sort out before applying classical statistics (see Engle and Russell (2004) for an in-depth analysis of most them):

- **For being data typically identified at a millisecond level** there are structural deficiencies that ought to be amended. Most of the amendments require the trader to take decisions as to the way adjustments have to be executed which is why high frequency data is delivered raw (just as it was recorded by the exchange). Typical examples of data adjustments are corporate actions (merger and acquisitions, dividends, etc) and block trades (large trades negotiated outside the market in a one-to-one basis but still, reported) but it is also usual to

[6] The Financial Information eXchange (FIX) protocol is an electronic communications protocol for international real-time exchange information for transactions occurred in the markets.

[7] See Appendix for a further overview of its platform.

find noise derived from the exchange's systems physical limitations such are zero quotes or even records of the limit order book[8] right after a trade was crossed but before its complete reset was finished (i.e. a duplication of quotes while a level was being transferred along the order book).

- **For being event-driven data** as opposed to end-of-day there is non-linearity in the time spacing. In fact, the higher the volatility of the markets the more data is generated being frequent the existence of several ticks per millisecond. Strictly speaking low frequency data is not linearly spaced either but the differences are usually negligible – they range from weekends and bank holidays that produce non-linear gaps in the dynamics of the embedded risk to movements around GMT in Summer and Winter time that can affect comparisons across stocks of different regions. However, as high frequency data arrives whenever an event occurs within the limit order book (the exchange's vehicle where appetite for the security is shown along a set of prices) the spacing is random and non-negligible. In order to overcome this issue the trader typically generates synthetic series by aggregating through different scales dependent on the needs: i) time-driven: usually targeting a time-scale that limits the impact of spurious correlations due to non-synchronous trading and completing the temporal spacing through interpolations, ii) volume-driven as in the case of flow toxicity measurement (see Easley *et al.* (2011) for a new volume-synchronized metric to estimate the Probability of Informed Trading as a measure of order toxicity), iii) volatility-driven, as Ané and Geman (2000) who increase the sample drawn from the data set the larger their measurement of stochastic volatility, in order to favour the classical analysis as mentioned above.

- **For being tick data**[9] as opposed to continuous data: a large kurtosis tends to be present in the distribution (with a sharper peak and longer, fatter tails than a Gaussian) that also seems to be discretely truncated around the best-bid and best-ask levels.

- **For accounting for persistent seasonal intraday patterns** (such is the intraday trading volume profile) **and market impact** of long-lived intraday strategies (as we will see below) there is complex temporal dependence within the time series. The field of finance that analyses the mechanism by which quotes evolve within the limit order book to reflect new information is called Market Microstructure – see O'Hara (1995) for a comprehensive review of its early literature.

It is also important to mention that from an Economics point of view, high frequency data is one of the main barriers to entry to high frequency trading as it is more expensive than end-of-day data to acquire, clean, manage and exploit.

Moreover, variables tend to be noisier the higher the frequency. Noise is expected to be larger in high frequency data than in end-of-day data as market impact typically affects more often the

[8] The order book of a security is a list of orders that a trading venue uses to record the open interest of buyers and sellers upon it.

[9] As pointed out above, ticks are multiples of the smallest allowable price change by the exchange around a bid-offer spread.

dynamics of the stock than its movements due to expectations. This implies that it is usually required to estimate their true state (filters can be used in this task). It is also a common practice to split the data dynamics into a triplet composed by: a) a common trend of the asset (sometimes calculated with PCA as shown in the first experiment), b) a gravity centre around that trend (extracted through filters such are particle filters), and c) a component with maximum entropy (the noise – which may be either discarded or exploited as it may imply trading opportunities).

Hedge optimization

The relevance of the hedge optimization has been motivated upon several dimensions. Costs-wise, an alternative basket can be traded in order to transfer risk from a set of instruments to another set where the firm has market power (e.g. naturally reduced costs such are taxes and borrow rates). Market impact-wise, the trader may decide to hedge a position with a more liquid basket than the product being market made assuming a statistical deviation (TE and tail risk) in exchange of a lower adverse impact on prices. In both examples the baskets are typically required be cointegrated within the time-horizon along which the hedge is expected to live to assure that the deviation series accounts for an stable average. Diversification-wise, the trader may include second tier components in terms of individual levels of cointegration if those fit best along the global portfolio whose open risk is running. The rationale is usually the same: to benefit from lower trading-related costs or orthogonal sources of risk by taking a controlled (back-tested) deviation risk, typically with zero mean. It is important to mention that on some instruments such are cash equities (e.g. single stocks) and foreign exchange (FX) the hedge can barely be optimized without taking non-negligible proprietary trading risks (we will refer to them as full-replication-only). This implies that the cornerstone of their market making is mostly centred on the quoting strategy and the inventory management policy.

When a hedge can be optimized the industry differentiates between two types of strategies. The hedge can be either *interim* or *long-lived* depending on whether it is used to remain market neutral while the trader enters into the full-replication one or whether the risk of the optimized hedge remains within the trading book on an on going basis. The former tends to be related to *static* hedges and the latter is usually devoted to *dynamic* ones. While static hedges remain fixed no matter their deviation along time with respect to the original instrument, dynamic hedges are devoted to reduce deviations beyond a certain threshold at the penalty of adding rebalancing costs. Alexander (1999) explains how cointegration-based hedges can reduce the danger of mispricing, over-hedging and over-rebalancing derived from correlation-based hedges. This thesis documents how to improve the performance of the optimized hedges (whether static or dynamic) through systematically accounting for adaptive-learning techniques of pattern detection.

It is also relevant to distinguish between *data-driven* and *theory-driven* approaches. While the former freely looks for the best combination of parameters along a set of back-tested scenarios, the latter further restricts the optimization process to target only patterns along a universe that the trader considers feasible. The restrictions range from the selection of the candidates' universe to usual techniques of regularization such are Akaike and Bayesian information criteria, ridge regression or lasso. This thesis, progresses gradually on the underpinning of the assumption that it is crucial for the trader to be able to exploit theory-driven approaches as much as possible in order to prevent overfitting and benefit from a longer out-of-sample performance consistency. Not only static hedges would benefit from not deviating from the expected performance path (especially if not risk-managed globally as explained below) but also dynamic hedges would benefit from a cheaper maintenance (less rebalances of the hedge in the attempt to diminish its deviation).

Last but not least, the hedge can be calculated against the raw instrument itself or against some representation of its dynamics such is the common trending approach taken by Alexander and Dimitriu (2003) or the estimation of the gravity centre of the security calculated through state-space methods such are Kalman or particle filters, as mentioned above.

Typically, once the hedge has been decided the risks and costs of its systematic execution can be estimated.

Execution

The role of execution is different in low frequency trading than in high frequency trading.

Low frequency market making (usually client-driven[10] flow of medium-to-large trades) typically involves a decision-making process rooted on standard strategies of TEmanagement as explained below and a benchmark of algorithmic trading execution (AT) usually set out by the client in order to minimize market impact (read Johnson (2010) for an excellent introduction to algorithmic trading and direct market access).

On the other hand, the execution strategy in high frequency market making (usually, electronic-driven flow of small orders as discussed later) becomes one of the main blocks of its performance. As we will see, it is typically blended within the quote optimization process. It combines automatic rules and order management strategies – usually a subset of the classical AT execution models as, by definition, it mostly deals with small quoted amounts.

Once the expected execution costs' distribution is estimated the trader can calibrate the right policy of risk management across her universe of admissible combinations.

Risk management

Systematic risk management is also different in low frequency trading than in high frequency trading. As said, at low frequencies the trader can face interim risk or long-lived one. The management of interim risk individually is proposed herein through the usage of machine learning in the improvement of the linear analysis rendered along the quote optimization process. On the other hand, it is also proposed to manage low frequency long-lived deviation risk within global books to gather orthogonal risk (this magnifies the flow) and benefit from diversification (see

[10] Also called phone trading.

Samuelson (1967) for an early research on the positive externalities from diversification). Global portfolios could then be managed using modern portfolio theory such are 'greedy' regressions on the eigenportfolios' projections (Alvarez-Teleña (2010)).

At high frequencies, risk is typically managed per security. Asymmetries of the posted quotes around the fair value are usually driven by the chosen approach for inventory management in order to affect the probability of offloading or uploading inventory. It overall focuses on finding the best risk-reward ratio when deciding whether to send limit orders (passive offers or demands with a certain probability of being activated) or market orders (active sells or buys). It hence ultimately relates to the probability of trading times the expected benefit from turning an almost degenerated variable (the execution cost and market impact of a small trade) into an stochastic one (TE or market risk depending on whether the trade is hedged or not). As said above, the rebalancing of the trader's portfolios in order to abide by the risk limits of the company can affect the symmetry of the best bid-offer spread posted in the order book by market makers.

Once the expected risk's distribution per level of aversion is properly estimated the trader can fix the price of an instrument being market made.

Quote optimization

The quote optimization is closely related to the risk attribution of each hedging strategy. In order to value the risk embedded within low frequency market making there are two main blocks to be highlighted: the analysis of the *Elasticity of the Flow* and extreme value theory. The former refers to the capacity of the market maker to manage the inventory by marginally improving the best bid and best ask within the order book, i.e. by trading passively. It has especial relevance when the trader uses an optimized hedge as it allows distinguishing between *randomly-lived* deviation risk risk (the one locked to the client's time horizon as in the presence of inelastic flow) and *flexibly-lived* TE (the one that the trader can close at her convenience, typically with another client as in the case of elastic flow).

As a result the time horizon of a market maker's optimized hedge is mostly random and usually combined with an unbiased distribution of sizes. A typical case is a client who trades-in with the market maker following a certain distribution of sizes and then trades-out at random times and following a different distribution of sizes. In this case the Law of Large Numbers may be still valid in percentages (the returns of the different trades would net off) but the market maker could face large deviations in absolute values (the notional would not net off). Hence the relevance of extreme value theory towards the application of prudence: instead of the first moment of the theoretical distribution of the deviation risk, other statistics such are value-at-risk (VaR) and conditional-VaR (cVaR) shall be considered by the trader – and this ultimately depends on the risk limits fixed by the agency where the trader works.

High frequency flow, on the other hand, tends to be flexibly-lived as it usually allows for passive inventory management (i.e. it accounts for elastic flow) and there is enough number of random trades to allow for the Law of Large Numbers to apply so that reaching the average of the TE is more feasible than in the previous case. As such, the previous blocks lose relevance in the quote optimization process. Instead, there is a further challenge that the trader needs to solve: the type of order to be sent, market order or limit order, given the expected outcome of each – basically, weighing off the cost of using an active trading approach vs the probability of the passive quote being filled times the outcome for taking that risk. This type of trading accounts for a growing literature being Lo *et al.* (1997) one of the first pioneering materials where an econometric model of limit-order execution times was developed using survival analysis that was in turn calibrated with actual limit-order data.

It is important to bear in mind that in both cases, the quoting system shall take into account the inventory of the trader whose need to balance it off along the trading session may increase or decrease the aggressiveness of her quotes around the fair value. This explains why some market makers do sometimes offer prices in a one-to-one basis (outside the venues) which are more aggressive than the best levels of the electronic limit order book (where at the same time they prefer their interest not to be seen).

Also relevant is the fact that during the quoting process the so-called winner's curse becomes a major concern. I refer to the winner's curse problem as the picking-off risk that the trader faces if we consider the limit orders to be free options to other traders who may take advantage of them if new information becomes available and those order limits have not been refreshed. Its negative impact on the markets' locative efficiency can be seen in Foucault (1999).

2.1.4 Literature review

Most of the growth in the systematic market making literature has been triggered by the iterative adaptations of the business to a changing environment as it gradually progressed towards automation[11].

The analyses of its competition scheme have been crucial to allow regulators intervene if the efficiency of the markets was at any time compromised. As such, numerous have been the papers that have covered the discussion about the reasons behind their relevance in frictional markets (Rubinstein and Wolinsky (1990)) which finally seemed to be more evident the more the returns to scale of the matching technology used (Masters (2007)). Lagos *et al.* (2007) extend the welfare of market makers from overall frictional markets to situations of market stress - a conclusion later observed by the equity market crash analysis pursued by Kirilenko *et al.* (2010) or the macroeconomic data releases analysed by Chaboud *et al.* (2009) in the presence of automated FX market making. Hollifield *et al.* (2006) further point out that competition shall be granted across market makers in order not to suffer from the perverse effects of a monopolistic liquidity supplier. This was also stressed by Jovanovic and Menkveld (2011) in the context of speed differences. Interestingly enough though, the situation analysed in the thesis is a consequence of a disequilibrium in trading

[11] See Appendix for an overview of the industry evolution.

advances reducing in general the number of competitive agents and, still, spreads have been largely reduced.

In parallel to the discussion about the competition of liquidity providers there has recently been a growing interest on the competition across the venues themselves. This has been called fragmentation (the availability of the same security across different venues). Foucault and Menkveld (2008) soon showed how the consolidated limit order book of the Dutch stock market became deeper when the London Stock Exchange entered the Dutch stock market (previously monopolized by Euronext).

On the other hand, there are the analyses of the most suitable vehicle for the cross of interests. Of the several formulae attempted it seems that order books, similar to those of the equity exchanges, are the preferred ones by FX and fixed income participants. These allow reduce searching costs by letting investors have easy access to other investors with opposite interest or to multiple market makers (Duffie *et al.* (2005)) - otherwise, markets could require intervention in order the cost to be subsidised as in Li, Y. (1996) or Duffie *et al.* (2009). Goettler *et al.* (2009), though, alerted that the limit order books can act as a volatility multiplier (prices are more volatile than the fundamental value of the asset) in the presence of informed traders (who can then alter the quotes from the rest) .

As markets evolved towards automation a new area of research was considered: the comparison between automated and non-automated trading. Johnson (2010) presents several reasons to state why the former could on average outperform the latter from a productivity perspective (Johnson (2010)). This reinforces the observations from Hendershott and Riordan (2009) who found in the Deutsche Börse's Xetra market that automated trading was better at driving prices towards efficient levels. Not only prices are apparently enhanced by automation but also liquidity (in in terms of both volumes and spreads) as referred by Barclay *et al.* (2006) upon an analysis of the auto-quoting facility introduced in NYSE back in 2003.

Automation naturally favoured the increase of the speed at which trading participants interacted with each other through the order book hence high frequency trading (HFT) on its own has recently received a large attention from academia. Bearing in mind that, as mentioned above, the analyses of Kirilenko *et al.* (2010) and Menkveld (2013) relate HFT to a market making approach there are some further analyses of HFT that should also be commented herein. First, Jovanovic and Menkveld (2011) state through a theoretical model that HFT entry can indeed increase welfare, but it might also reduce it when it trades against slow market makers. Brogaard (2010) showed that HFT activity contributes more to price discovery than non-HFT activity and further stated not only that 50% of the time their quotes are amongst the most aggressive (best bid or best ask) but also that it allows for a market impact reduction and it reduces volatility. It backed this way up most of Rosu (2006) theoretical conclusions. Brogaard (2011) used the same data set to further investigate the impact of HFT on volatility, and found evidence that HFT liquidity provision increases during times of short-term volatility. Using again the same data set, Hendershott and Riordan (2011) examined the impact of HFT on the price discovery and confirmed Brogaard (2010) results. With regards to the effect of HFT on the spreads, Menkveld (2011) studied the development of the Chi-X

European stock multilateral trading facility (MTF) in 2007 and the simultaneous entry of a large HFT participant on Chi-X. The result was that this new participant was largely responsible for the increase in market volume share of Chi-X and ultimately led to reduced spreads for the stocks that it traded[12]

The thesis contributes to the growing literature by abounding on the way automated market making can be enhanced through a more robust out-of-sample performance by injecting theory along the calibration process using learning techniques. While doing so it also covers the set of advances that could lie behind the competitive advantage of the most aggressive trading agents. And this, as seen above, could in turn serve as a way to favour the markets' efficiency by allowing those that fell behind finally catch up. It also is, to the best of our knowledge, the first research that analyses high frequency trading risks in order to appropriately include them within a mid frequency trading strategy.

2.2 The Elasticity of the Flow

As said, market makers provide liquidity by being ready to buy and sell securities at a maximum bid-offer spread typically fixed by the exchanges where they are members of or agreed through over-the-counter contracts (in a one-to-one basis) with third parties. Within this maximum range market makers setup different strategies to optimize their risk-reward performance. Traditionally, the strategy has depended on the nature of the asset being traded but this thesis proposes a subtle feature, the nature of the flow[13] as its cornerstone – not the main dimension to exploit structural synergies[14] (code, indicators, platforms, etc) but, more importantly, to benefit from a distribution analysis as opposed to an isolated, trade-by-trade risk management.

In order to analyse the broad set of market making approaches I could consider the peculiarities of two extreme types of flow:

1. The higher the frequency of the flow and the more its randomness and symmetry the more the opportunities the trader will have to net the inventory off by trading passively, in general, through limit orders (i.e. at zero cost or even at a rebate[15]). An example could be the electronic market making of a liquid future.

2. On the other hand, the less the symmetry of the flow's distribution the higher the risk of assuming deviation risk, the larger the cost of maintaining a zero-inventory and the less the robustness of the optimal approach:

    - The lower the **frequency** of the trades the more the discontinuity of the distribution (being difficult to believe in the application of the Central Limit Theory that lies behind

[12] See Kijima (2009) for a further survey on recent advances in financial engineering.

[13] Highly related to the nature of the trading venue, not in vain while this thesis was being developed the industry adopted the term of 'Equitization' to refer to situation where the rest of the assets are converging to the Equities framework in different dimensions (regulation, electronic books, etc).

[14] Their exploitation would enhance the markets' activity as noted by Masters (2007).

[15] A premium that some exchanges give to the liquidity providers in order to incentivate them.

most of the typical statistical analysis) and the larger the market risk to be assumed through the deviation risk.

- The more its **kurtosis** the larger the trades can become and as such, again the higher the risk embedded in the deviation risk.
- The more its **asymmetry** the more expensive it is to net the inventory-off as there are less passive opportunities for it.
- The more the **dynamism** of its distribution along time the less the robustness of the chosen approach.

An example of this type of flow could be a proprietary index launched by a financial institution. These usually target a subset of its clients who in turn typically enter into a large trade of such index which then they reduce gradually along time; this could hence be considered a random process to the market maker[16].

At this stage, it becomes convenient to borrow from microeconomics the concept of *elasticity*[17] and create a concept that allows us to easily cluster in terms of flow analysis the main approaches used in market making:

I will refer to Elasticity of the Flow, E, as the lowest[18] *sensitivity of the inventory (inflows and outflows) to marginal improvements of the best bid and best ask within the order book around the fair price*[19].

$$E = \min\left(\frac{\delta I}{\delta b}\frac{b}{I}, -\frac{\delta I}{\delta(-a)}\frac{a}{I}\right) \tag{2.2.1}$$

where b is a variable that represents the best bid and a the best ask.

In general, I could expect non-linearity of the sensitivity of the inventory around the market's best levels hence I should not define it with a positive a. This is, $E = \min\left(\frac{\delta I}{\delta b}\frac{b}{I}, \frac{\delta I}{\delta a}\frac{a}{I}\right)$ would be wrong since increases in the a level would generate inelastic values by definition, i.e. the inventory will not react to less aggressive levels than best ask prices as it would to more aggressive ones[20].

This is, the term elasticity accounts for the capacity that the market maker has to improve the passive inventory's net off by changing the quotes posted along the order book – i.e. using limit

[16]Similar distribution in the delta and gamma hedging of options.

[17]In microeconomics, elasticity refers to the sensitivity of the amount demanded or supplied to marginal variations in the prices. This is, a percentage change referenced to the slope of the curves of demand or supply, and to the initial level of the quantity and the price. It is often used to refer to the capacity that a firm has to change the total income by moving marginally the prices.

[18]A market maker should be equally interested in the movements along the demand and the supply given its neutral nature hence I define the overall elasticity through the elasticity of the most restrictive.

[19]Note that in reality the fair price is not necessarily the mid price.

[20]Given the essence of a trading order book I would expect: $\left|\frac{\delta I}{\delta a}\right| \leq \left|\frac{\delta I}{\delta(-a)}\right|$.

orders instead of market orders. As such, I will classify the former extreme type as the one that accounts for elastic flow being the latter a typical scenario for inelastic flow.

Follows a description of the mathematical foundations of the techniques used in the thesis which along with the details about the industry enclosed in the Appendix should be enough to understand the experiments of the next chapters.

2.3 Mathematical foundations

Four large blocks can be distinguished with regards to the techniques deployed along the thesis:

- statistical analysis, that comprehends from regression and cointegration to time series analysis,
- stochastic analysis, in especial the Ornstein-Uhlenbeck process,
- machine learning, whether unsupervised (principal components analysis and reinforcement learning) or supervised (support vector machines), and
- non-linear optimization, in particular Particle Swarm Optimization (PSO).

Follows a brief introduction to them all.

2.3.1 Statistical analysis

From statistics I have mainly borrowed three techniques: regression, cointegration and time series analysis, which are already well-known tools as detailed below.

Regression

Regression analysis is widely used in prediction and forecasting where it overlaps with other fields such is machine learning (see below). One of the reasons why it has achieved a large popularity is that it allows easily inferring what the weight of each independent variable is with respect to the dependent one; this also favours the contrast of heuristic assumptions and as such, regularization of the models based on theoretical approaches – hence its frequent use in Economics analyses.

Maybe the most popular technique within linear regression (when the independent variable is continuous) is ordinary least squares (OLS), where the overall solution is set to minimize the sum of the squares of the errors implied in the set of equations that define the model – sets that shall account for more equations than unknowns. In the case of categorical independent variables it is logistic regression instead and this usually needs to be deployed and whose parameters are typically fitted through maximum likelihood. In general, it is expressed as follows:

$$y = \beta x + u \tag{2.3.1}$$

where y represents the variable that the statisticians want to estimate by calculating its sensitivity, β, to an indepent variable x at a minimum (quadratic, as mentioned above) deviation, u.

Nonlinear regression on the other hand, is usually transformed into linear regression by taking the logarithm of the target function. Non-linear regression algorithms range from Gauss-Newton to gradient descent – see Björk (1996) and Snyman (2005), respectively for detailed descriptions of both.

Finally, kernel regression covers the non-parametric estimation of the conditional expectation of a random variable hence further typically involving the calibration of the kernels and a larger overlap with machine learning as mentioned above.

Cointegration

A large part of the strategies in quantitative trading involve the concept of cointegration. Cointegration, is a term coined in Engle and Granger (1987) to show why its possible presence shall be taken into account when deciding the technique to test hypothesis on relationships between two variables that account for so-called unit roots. It helps prevent statisticians from spurious correlation.

The Engle-Granger two step method and the Johansen test (Johansen (1991)) are probably the most popular mechanisms to test for cointegration[21]. While the former is based on the Dickey-Fuller test for unit roots in the residual, u, from a single (estimated through OLS upon the time series: $y_t = \beta x_t + u_t$) cointegrating relationship, the latter allows testing for several I(1) time series, as explained below.

StatArb, as described in Chapter 2, is a trading approach that mostly relies on cointegration relationships through the trading of pairs. In fact, the cornerstone of pairs-trading is the analysis of extreme events in the dynamics of the residual, u, being it defined as the spread of a long-short strategy on two related instruments, $y - \beta x$. Moreover, as u tends to resemble white noise-like dynamics another version of pairs-trading has also surged through the last principal components of PCA as explained below (Alvarez-Teleña (2010)).

Time series analysis

Time series analysis typically refers to the study of sequences of data points ordered in successive instants in time uniformly spaced. This feature makes time series analysis distinct from other

[21] See Blanco *et al.* (2005) for an application of cointegration on the credit market, where the price discovery is tested between the CDS and the corporate bonds markets.

common data analysis problems, in which there is no natural ordering of the observations. A time series model will generally reflect the fact that observations close together in time will be more closely related than observations further apart. And such natural temporal ordering limits the range of techniques to be used in its analysis (e.g. bootstrapping should generally not be used) but typically allows to benefit from the detection of robust patterns – I refer to them as *seasonal*.

Models for time series data can have many forms and represent different stochastic processes. When modelling the dynamics of a process, three broad classes of major concern are typically analysed: the autoregressive (AR) models, the integrated (I) models, and the moving average (MA) models whose combination produce both autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models.

The notation AR(p) indicates an autoregressive structure of order p and is defined for a process a process X_t as follows:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t \tag{2.3.2}$$

where c is a constant, φ_i is a parameter and ε_t is white noise.

The notation MA(q) refers to an integrated structure of order q:

$$X_t = b + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2.3.3}$$

where b is a constant, θ_i is a parameter and ε_t is white noise.

An ARMA(p,q) system is then defined as:

$$X_t = k + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{2.3.4}$$

where k is the result of adding c and b.

Finally, a process X_t is considered ARIMA(p,d,q) if differentiated d times, $\nabla^d X_t$, becomes an ARMA(p,q) process.

Finding appropriate values for p and q in the ARMA(p,q) process typically involves the plotting of partial autocorrelation functions (PACF) for an estimation of p and autocorrelation functions (ACF) for an estimation of q. While the latter is simply the correlation between the time series and its q^{th} lag, the former requires further explanation. In fact, it heritages from ACF as detailed below.

Given a time series z_t, the partial autocorrelation of lag k, denoted $\rho(k)$, is the autocorrelation between z_t and z_{t+k} with the linear dependence of z_{t+1} through to z_{t+k-1} removed; equivalently, it is the autocorrelation between z_t and z_{t+1}, $\varrho(z_t,\, z_{t+1})$, that is not accounted for by lags 1 to $k{-}1$, inclusive. This is:

$$\rho(1) = \varrho(z_t,\, z_{t+1}) \tag{2.3.5}$$

$$\rho(k) = \varrho(z_{t+k} - P_{t,k}(z_t + k),\, z_t - P_{t,k}(z_t)),\ \text{for } k \geq 2, \tag{2.3.6}$$

where $P_{t,k}(x)$ denotes the projection of x onto the space spanned by $z_{t+1}, ..., z_{t+k-1}$.

These functions were introduced as part of the Box-Jenkins approach to time series modelling[22]. Their rate of decay affect each other's parameter's calibration – typically, a high rate of decay in the ACF is interpreted as a more robust result in the PACF interpretation and *vice-versa*.

2.3.2 Stochastic analysis

One of the most popular models within stochastic analysis is the Ornstein-Uhlenbeck. It describes the velocity of a massive Brownian particle under the influence of friction; this is often interpreted as the continuous-time analogue of the discrete-time AR(1) process. One of its most relevant properties is its drift towards its long-term mean. In fact, it can be considered to be a modification of the random walk in continuous time, or Wiener process, in which the properties of the process have been changed so that there is a tendency of the walk to move back towards a central location, with a greater attraction when the process is further away from the centre.

An Ornstein–Uhlenbeck process, X_t, satisfies the following stochastic differential equation:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t, \tag{2.3.7}$$

where $\theta > 0$, $\mu > 0$ and $\sigma > 0$ are parameters and W_t is a Wiener process.

By interpreting that μ is the long run equilibrium of a couple of securities, σ the volatility of its dynamics and θ the rate by which the relationship reverts towards the mean, a pairs trading strategy can be defined. The half-life of the relationship, the time taken by a given amount of the substance to decay to half its mass, HL, is given by $HL = \ln(2)/\theta$ and this can help the trader decide whether the expected horizon of the mean-reverting trade allows for a profitable trade or its cost of carry may significantly erode it instead.

2.3.3 Machine learning

Depending on whether the identification of hidden structures within data uses unlabelled or labelled data we can distinguish between unsupervised and supervised learning.

[22] Box *et al.* (2008) present several techniques for an in-depth analysis of time series.

Unsupervised learning

throughout the thesis, the use of two unsupervised learning techniques can be highlighted: principal component analysis (PCA) and reinforcement learning (RL).

PCA, is a popular mathematical technique (Pearson (1901)) that deploys an orthogonal transformation of possibly correlated variables into a set of linearly uncorrelated variables, called principal components, with at most the same number of dimensions of the original data set. There are two polemic features in its application in finance worth to mention: first, the principal components are guaranteed to be independent only if the data set is jointly normally distributed and second, the whole methodology is sensitive to the relative scaling of the original variables hence transformations have to be properly taken care of (usually, normalized values are taken). However, it accounts for a key favourable characteristic: the principal components are ranked in terms of the variability of the data that they account for; and this allows traders to reduce the number of dimensions in a portfolio when looking at its dynamics – typically reducing noise at the cost of reducing also the intuition behind the strategy given the unsupervised learning nature of the hidden structures. Finally, the usual approach is the simplest of the true eigenvector-based multivariate analysis, where the eigenvectors of the covariance matrix of the data set define the directions at which most of the data's variance can be explained according to the methodology. The eigenvalues associated to such eigenvectors scale the length of each one; and this weights the relevance of each eigenvector in the data.

As mentioned above, the last principal components – those with the lowest eigenvalues associated – are expected to generate dynamics closer to white noise (in the end, the same that the trader wants to filter out). If that relationship remains out-of-sample, the trader can set bespoke baskets of the securities present in the portfolio with the weights defined by the eigenvectors, in order to benefit from mean-reverting strategies as set out above – moving from pairs trading to PCA trading. The upside from PCA trading is that it allows portfolio managers to benefit from not-publicly-eroded patterns. However, again, it does so at the price of managing an unsupervised learning strategy hence it is always recommendable to double check whether there is a theoretical rationale that backs up those structures before they are deployed in the markets.

The second core unsupervised learning technique that is used in the thesis is Reinforcement Learning (RL). Inspired by behaviourist psychology, RL is the area of unsupervised learning that focuses on the way an *agent* shall take *actions* within a certain *environment* in order to maximize some notion of cumulative *reward* – we will see an specific case for the way these parts interact in the risk-reward experiment. It is also called approximate dynamic programming for being usually formulated within a Markov decision process (MDP) framework. This is, at each time step, the process is in some known state s, and the decision maker can decide to take any available action, a. After deploying such combination, the process then responds at the next time step by moving into another state, $s*$, and giving the decision maker a corresponding reward $R_a(s,\, s*)$. RL differs from supervised learning in that pairs of correct input/output pairs are never presented. Sub-optimal actions are not explicitly corrected either.

Further, one of the most studied features of RL is its on-line performance, which involves finding a balance between exploration (of unknown/unprocessed territory) and exploitation (of current knowledge). In particular, within RL I will be using Q-learning (Watkins (1989) and Sutton and Barto (1998)) to compare the expected reward of a range of available actions at any time and state, without requiring to model the environment.

Supervised learning

One of the most popular techniques of supervised learning is support vector machines (SVM). SVM is based on the theory of structural risk minimization (Vapnik (1998)) and typically used to recognize patterns in the deployment of classification and regression analysis as follows.

Let's assume that I can represent the general prediction problem given a sample of N independent and identically distributed training instances, $\{(\mathbf{x}_i, y_i)\}_{i=1}^{N}$, in the form of the following discriminant function (a separating hyperplane):

$$\hat{y} = f(\mathbf{x}) = \langle \mathbf{w}, \phi(\mathbf{x}) \rangle + b = \sum_{i=1}^{N} \omega_i \phi(\mathbf{x}_i) + b \tag{2.3.8}$$

where $\phi(\mathbf{x}_i)$ represents a non-linear mapping of the D-dimensional input instance $\mathbf{x}_i$ into a higher dimensional feature space, i.e. $\phi: \mathbb{R}^D \longmapsto \mathbb{R}^S$, $\hat{y}$ is the corresponding prediction and ω and b are parameters learned from the N instances of training data. Finally, $\mathbf{w}$ encloses the vector of ω.

As we shall see below, it is the non-linear mapping $\mathbf{x}_i \longmapsto \phi(\mathbf{x}_i)$ the essence of SVM, whose approximation obeys mostly to the choice of an appropriate kernel[23], κ, where $\kappa(\mathbf{x}_i, \mathbf{x}_j) = \langle \phi(\mathbf{x}_i),\, \phi(\mathbf{x}_j) \rangle$ – see Cristianini and Shawe-Taylor (2000) for a reference of commonly used kernel functions.

When prediction of classification is intended, $\hat{y} = \pm 1$, ω and b can be trained using first Quadratic Programming optimization to find the levels of Lagrange multipliers, α_i, that minimize the distance between the two hyperplanes implied by $\hat{y}$:

$$\underset{\alpha}{\operatorname{argmin}} \underbrace{\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \langle \phi(\mathbf{x}_i),\, \phi(\mathbf{x}_j) \rangle}_{A} - \underbrace{\sum_{i=1}^{N} \alpha_i}_{B} \tag{2.3.9}$$

where $\alpha_i \geq 0\ \forall i$, and $\sum_{i=1}^{N} \alpha_i y_i = 0$ so that the bias is minimized. A and B represent the blocks linked to the minimization of the bias and the trade-off between model simplicity and classification error[24] respectively.

[23] Often cross-validated or combined as in Multiple Kernel Learning (Lanckriet *et al.* (2004) and Bach *et al.* (2004)).

[24] Typically expressed as $C \sum_{i=1}^{N} \xi_i$, where C represents the positive trade-off parameter and ξ, the classification error.

Once the Lagrangian multipliers have been found, the weights can be calculated through the following expression:

$$\omega = \sum_{i=1}^{N} \alpha_i y_i \phi(\mathbf{x}_i) \tag{2.3.10}$$

Only those observations with positive Lagrangians, $\alpha_i > 0$, will lie on the margin and will then be considered in the classification – being hence called the support vectors. Once the set of support vectors has been defined, SV, the bias can be calculated through the following expression:

$$b = \frac{1}{N_{SV}} \sum_{s \in SV} \left(y_s - \sum_{m \in SV} \alpha_m y_m \left\langle \phi(\mathbf{x}_m), \phi(\mathbf{x}_s) \right\rangle \right) \tag{2.3.11}$$

with which the comparison across SVM candidates can be deployed.

Literature on SVM is fairly wide and has given rise to new versions along different improvable areas as has been the case of weighted SVM or core vector machines. While the former focuses on the fact that SVM classification results are typically biased towards the class with more samples in the training (Wang *et al.* (2004)), the latter targets to improve the speed of SVM training in large data sets (Tsang *et al.* (2005)).

2.3.4 Non-linear optimization

Particle swarm optimization (PSO) is a computational method originally devoted to the simulation of social behaviour (see Kennedy and Eberhart (1995) and Kennedy (1997)). It is a non-linear technique that seeks to optimize a problem by iteratively attempting to improve a candidate solution to the light of a given measure of quality. In practice, PSO attempts iterative enhancements in the solution of a problem by having a population of candidate solutions, particles, which move around the search-space according to simple mathematical formulae over the particle's position and velocity. Each of the candidate's movement is both influenced by its local best known state and by the global best known state in the search-space, which are consecutively updated as better positions are found by other particles.

As a matter of fact, it is the update of the particle's velocity how the search dynamics are mostly determined:

$$v_{id} = \omega v_{id} + \varphi_p r_p (p_{id} - x_{id}) + \varphi_g r_g (g_d - x_{id}), \tag{2.3.12}$$

where, i is the particle, d its dimension, v is the velocity of the particle, p is the particle best known position across all the dimensions, g is the swarm's best known position and r_p and r_g are random numbers. The parameters $\omega, \varphi_p, \varphi_g$ do largely control the behaviour of the PSO method.

Given that PSO is a metaheuristic computational method, the problem being optimized can be searched along very large spaces of candidate solutions. Such feature has pros and cons. The upside is that PSO does not require the optimization problem to be differentiable as is required by classic optimization methods such as gradient descent and quasi-newton methods. PSO can therefore also be used in optimization problems that are partially irregular, noisy, that change over time, etc. The main downside though is that it can take too long time to find a solution hence its implementation in intraday trading sometimes requires the inclusion of further shortcuts through theoretically motivated structures (subsets) within the search-space as seen in Chapter 4.

See Beyer and Sendhoff (2007) for an in-depth survey on popular optimization techniques.

2.4 Summary

This chapter provides sufficient background to understand the different experiments deployed herein. It also allows to motivate the relevance of the thesis as its contribution does not relate to science only but also, and given the detailed framework that has been developed for this purpose, to systematic traders.

First, the trading terminology was defined and completed with a classification of trading approaches upon four dimensions: level of automation, type of analysis, frequency of trading and capacity to generate a realistic back-testing. Then, the main building blocks of systematic trading (data, hedge optimization, execution, risk management and quoting) were scientifically described and followed by a literature review that motivates the increasing relevance in research of the systematic trading field.

Second, the *Elasticity of the Flow* was introduced to understand how the nature of the market making activity presents a new dimension to be included into the quote optimization process. It is a novel concept that inherits from the well-known concepts of the elasticity of the demand/supply from Economic Theory. This not only allows to exploit cross-asset synergies but, more importantly, it lets to disclose the possible scientific techniques that lie behind the recent 50% discount of the bid-offer spread exhibited in Menkveld (2013).

Finally, it includes the mathematical outline of the aforementioned main research domains that have been combined along the different experiments. These include statistical analysis (regression, cointegration, time series analysis), stochastic analysis (Ornstein-Uhlenbeck), machine learning (both unsupervised and supervised) and non-linear optimization (particle swarm).

Chapter 3

Execution: an execution model based on support vector machines

This chapter analyses different models for the VWAP's core estimation of the volume's profile. A comparison of alternative approaches is first set out for the members of IBEX 35 crucially under an error measure that, unlike in most of the literature, does not depend on the prices but on the volume only. It is intended this way to avoid the effect of the low intraday dispersion of the former onto the modelling of the latter's patterns present in a popular paper that suggests the inclusion of a dynamic component. In this benchmark, such dynamic algorithm is challenged against simpler ones based on the analysis of intraday time slot as an independent time series. Evidence on the outperformance of the latter is found along the data sample and this motivates a subsequent analysis of the inclusion of inter and intraday dynamics in a non-linear manner through machine learning — fulfilling this way the third and sixth objectives of the thesis. Further proprietary improvements are finally motivated by an analysis of IBEX 35's market microstructure — objective seventh. The rest of the chapters benefit from accounting for this execution algorithm as part of the strategies deployed therein, a merge of high frequency with mid frequency trading that allows generate realistic results throughout the thesis as demanded by Cahan et. al (2010) — fourth objective.

3.1 Research challenge

If I want to generate a realistic backtesting model that allows me to find an explanation to the recent observations exhibited in Menkveld (2013) the analysis of the market impact of the hedge is crucial. The market impact depends on the execution strategy hence the model that will be used for this purpose has to be selected. I decided to take one of the most popular and challenging strategies: the VWAP. The VWAP seeks a constant market impact throughout the day by following the non-linear profile of the intraday volume. The estimation of such profile remains a challenge hence its need in the thesis represents a unique opportunity to attempt to improve it. I start taking as a benchmark a popular model that uses a combination of PCA and ARMA to approximate this curve. I challenge the model theoretically and then in practice to ultimately reach the conclusion that other, more simple and bespoke approaches beat this benchmark in my data. Finally, I provide a novel technique that combines the latter approaches through SVM which exhibits an improvement of the estimation of the curve that is robust across all the stock of my data sample.

3.1.1 VWAP within execution trading

The main algorithmic trading strategies can be clustered into three groups: benchmarking, proprietary and liquidity-seeking.

Benchmarking

- **TWAP** – *Time weighted average price.* This basic algorithm breaks the order down into equal amounts over a pre-specified time frame. Its use is particularly suitable for orders on stocks with low expected market impact and low volatility.
- **VWAP** – *Volume weighted average price.* Intraday volume's profile usually presents seasonality along each trading session in the form of a U-shape. It seems appropriate then to discount this pattern from an intraday trading execution to be more prudent (market impact-wise) in periods where crossed interest is historically lower (typically, the central hours). This is, VWAP surges as an alternative to TWAP when the size of the trade generates a non-negligible market impact. The inclusion of the volume's profile prediction adds certain complexity in its implementation and requires the removal of block trades before comparing the executed level to the benchmark as we will see below. Overall, it is typically used in orders on products with mid expected market impact and mid volatility.
- **POV** – *Percentage of volume.* It targets a certain participation in the volume traded in the market. It typically is expensive as, in order not to miss the participation rate, it sometimes needs to trade more aggressively than the former two algorithms crossing this way more often the bid-offer spread. The execution of this strategy drifts with the realized (as opposed to 'expected' as in VWAP) volume and typically no price limits nor time horizons are guaranteed. The use of a price limit could avoid undesired execution levels at the cost though of risking the fulfillment of the trade. The use of time horizons on the other hand tends to increase the aggressiveness of the trade (making it yet more expensive). It is typically used in orders on securities with high expected market impact and mid-to-high volatility. When compared to VWAP it tends to drift closer to the real volume profile both at a higher price in terms of execution but at lower costs in terms of implementation.

Proprietary

- **IS** – *Implementation Shortfall.* Attempts to balance-off the market risk assumed when waiting for the market to drift towards the trader's interest in the attempt to overcome a benchmark and the market impact incurred when executing the trade. It does so through black-box like models and as such it accounts for more model risk than POV. It is typically used in the seek for a higher target of risk-reward than VWAP and POV.

- **Peg**. It accounts for model risk (typically less than IS) and the order may not be completed either. The algorithm attempts to enhance the execution level by both not crossing the spread (sitting within the order-book instead) and allowing the client to decide how to hide the overall interest by typically letting the slice of the trade become an input.

Liquidity-seeking

- **Dark pool scanner.** It is a smart order router (SOR) that statistically seeks further liquidity to the one present in the different order books (when there is segmentation) by also scanning dark pools available for execution. It mainly accounts for the same issues than the proprietary algorithms and it is recommendable to use limit prices in order to avoid being targeted by arbitrageurs between dark pools and open markets[1]. It allows for a reduction of the role of the randomness often required to maintain anonymity in the market. Most of its popularity within equities accounts for block trading and algorithmic trading since it allows its members to benefit from lower transactions costs – a key advantage in HFT given the low margins targeted. In fact, it also allows for more accurate back-testing since it can trade at the levels of the order book without changing the quotes of most of its participants , i.e. it reduces the role of Game Theory in the calibration process.

The main experiment of this chapter abounds hence on the possible innovations within the group of Benchmarking.

3.1.2 Core features in VWAP research

The enhancement of intraday volume's profile estimation remains one of the most challenging fields in finance (Hobson (2006)). And it is the case, in our view, because there is typically neither intuition nor financial expectations behind the estimation of volume's dynamics.

Before explaining the different methodologies that shall be considered on this task there are some core features to be aware of with regards to the volume's profile:

Profile discovery. It is important to highlight the fact that the percentage profile is not defined until the very last trade has been crossed in the markets, i.e. being it referenced to the end-of-day volume it is not information available on-line. Along the day there is no accurate information about the real end-of-day profile but rather a series of probable candidates to what it could become. As it occurs with any relative distribution while time passes the marginal effect of the forthcoming trades on the participation of each bin of volume executed along the day decays. However, such

[1] A trader can shift the prices at the open market with small quotes in order to shift the best bid-offer benchmarked quotes within the dark pool with highly-probable large quotes. The probability is typically calculated after analyzing the dynamics of the dark pool through real testing.

reduction does not mean that the effect is negligible (especially in markets with an end of session auction). A large part of the literature's experiments skips this fact and focuses on the modelling of every day's volume profile as if the whole profile was available from the beginning of the session.

The effect of external stress. A usual misunderstanding around VWAP is the effect of external stress such are news or sudden increases of volume due to urgent trades. Only punctual effects that generate isolated deviations along the trading session would change the strategy of execution, usually in order to participate more than expected around the time when the stress occurred. On the contrary, those effects that remain homogeneously during the day won't change the expected relative convexity of the rest of the day – even though they can typically change the U-shape of the volume profile by flattening out the beginning of the day and adding a jump from the stress onwards (rarely the other way round). As such, if the trader cannot anticipate whether the shock will have or not a punctual effect on the volume (typically, decreasing sharply along time) during the rest of the time horizon of her execution or could not participate early the *a priori* estimated profile shall remain optimal in terms of risk/reward.

The myopia of the length of the bin. The only certainty on which the trader can count is that the bin defined for the whole trading session will be 100%. This is, the larger the length in time of the bin the lower the error of the prediction by definition. And this can mislead the decision about what length of the bins should be used to represent the profile. Moreover, if the bins are larger than the slices finally executed by the algorithm[2] the latter have to be created evenly or randomly from the former. And both approaches have issues: while the linearization of the intra-bin profile may allow the rest of the market participants spot the trader's open interest through the successive constant sizes being quoted, the randomization erodes by definition the performance of the algorithm. These features have to be taken into account by the trader before deciding the length of the bins used to predict the volume's profile.

The fuzzy effect of the price inclusion in the error measure. Most of the literature around VWAP which focuses on the estimation of the intraday volume's profile measure results upon VWAP levels. Although it seems coherent it has to be used with a dose of prudence. In fact, as prices do not have any role within the profile their inclusion simply add noise to the calculations and difficult any possible comparison, hence I propose them to be disregarded by the trader.

Absolute or quadratic errors. Finally, the misestimation of a bin should not compensate the overestimation of another since prices do not remain constant. I should hence look for the model that accounts for the best distribution of errors in absolute value – or quadratic, if I further needed the errors' distribution to be differentiable.

[2]Typically, it is not desirable to constraint the dynamics of the algorithm to the time horizon in which each bin has been defined. Note that the bins could perfectly differ in terms of length – and this can also be optimal depending on the limitations of the CEP being used. In the presence of constraints in that respect, bins could be set out more homogeneously during central hours than along the tails since the slope of the shape is flatter.

3.2 Methodology

The methodology considered for this experiment meets the following rationale:

1. The database was composed by three .csv documents, being one of them much larger than my memory allowance. As already explained in 1.4 the first entry barrier was the setup of a research framework that allowed me to deal with the Big Data issues. It was solved by setting up an open source laboratory that linked Infobright with Python and Python with R-CRAN.
2. Once the data is aligned and clean, the execution strategy has to be selected. I decided to follow the most popular algorithm in the industry.
3. I then looked for the most popular paper on the subject to reproduce it and use it to feed the rest of the experiments. The rationale I considered to follow was:
   (a) If I felt comfortable with the approach no more analyses would be required.
   (b) Otherwise, I should motivate theoretically why I doubt about its adequacy in my data sample and furthermore, propose a new technique, in line with the former, that could overcome it. From then on, such popular methodology would become the benchmark with which to compare the rest of the experiments.
      i. If these weren't used in above models, I should find a way to keep enhancing the approach to my execution algorithm exploiting machine learning techniques.
4. Take the chance of having such a rich set of data to further explain usual questions that have risen in the industry and for which there are not, to the best of my knowledge and to the knowledge of the industry participants that I consulted, answers yet.
   (a) Which level of the order book is most important in terms of price discovery? This is, off the 10 levels of depth of the order book that can be saved, how many do I have to? The cost and ease to use that data would benefit the lower the levels of depth.
   (b) Does it make sense to offer a flat rate for guaranteed execution levels across stocks? The industrial prior says no but why not? Does it depend on liquidity?
   (c) What is the nature of the hidden liquidity in the order books? Is it robustly present so that it is worth it to include it in the execution algorithm's rationale or rather random?

3.2.1 Data description

Even though most of the thesis was developed using the statistical language R-CRAN the database for this experiment was built in Python upon csv files with 1-second observations of the IBEX 35 members' order books[3]. At that frequency, block trades were scanned along a different database

[3] In the following experiment within this chapter a tick-data approach is shown.

and cleaned from the original that was subsequently aggregated into 5 minutes bars from October the 1^{st}, 2011 to January the 2^{nd}, 2012.

The third Friday of each month was also removed from the sample since IBEX 35 futures expire on those days and this has a robust effect on the intraday profile of its members' volume – as the future settles based on their average value between 16:15 and 16:44 on a minute basis.

Further, instead of the whole trading session (exchange opens at 9:00 and closes at 17:35) only intraday bins from 9:05h to 17:30h have been considered. It was attempted this way to avoid the high volatility typically present around the auctions – a usual practice in the literature in order not to compromise the results along the rest of the day.

It shall be reminded that the errors' distribution expected on the real execution of the VWAP algorithm would be typically lower than that reflected in the experiment. Not only due to the effect of the prices' dynamics (the lower the volatility of the price the lower the relevance of the errors in the volume profile estimation) but also due to the compensations in value (for a given price level the positive deviations will be compensated by the negative ones and *vice-versa*).

3.2.2 Static and dynamic methodologies

Literature regarding VWAP can be divided into two large blocks: first, static estimations that seek to forecast the seasonality of the volume's profile and second, dynamic estimations that attempt to benefit from the movements around the seasonal patterns.

Static approach

A historical average per bin has been the first approach to the seasonal structure of the volume. Its simplicity is key but it is typically affected by the outliers that may have occurred within the sample. Similarly, PCA has also been proposed to find a hidden seasonal pattern, in this case a structure that holds across the whole market to generate market-binded projections per stock and bin as we will see below. I will also discuss the pros and cons of this type of profile structures that are usually more stable along time. Finally, I will consider the median that even though it has been highly disregarded in the literature it naturally removes the outliers that negatively affect the average as mentioned above[4].

1. – Simple time average of each bin along a certain time window

The evolution of each bin can be iteratively estimated through the projection of a rolling window of the average of the bins as defined below:

[4] As noted above, patterns in volume are typically more difficult to back up by financial theory than patterns in prices.

$$A_i = \frac{1}{D}\sum_{d=0}^{D} b_i^{j,1-d},\ i = 1, \ldots, I;\ j = 1, \ldots, J \tag{3.2.1}$$

where j is the instrument; i is the specific bin intraday; b is the bin itself, the proportion of the traded volume along the time frame attached to the bin, v, over the total volume traded along the window of the trading session that I have considered, n; and$d$ is the day in the time window that ranges from 0 to D.

Once the distribution of bins has been calculated it has to be normalized so that it adds up to 100%. Apart from its simplicity its upside is that by allowing it to be sensitive to the sample it may be able to account for significant patterns that typically occur approximately at the same time of the trading session. However, if there is no further information about what can be generating a pattern robustly (about how sensitive the estimation has to be to the sample nor even about the optimal length of the window to be used in each bin) taking a simple average can also add undesired noise to the prediction.

2. – Seasonal dynamics along a latent structure

Motivated by asset management practices I could use PCA to find the profile that serves as a common guidance for a market – Darolles and Le Fol (2003). By taking the first principal component's[5] projection and forcing it not to have any negative value and to sum up to 100% I can account for an estimation of the latest latent structure of the volume's profile that allows the trader separate the common structure from the idiosyncratic dynamics around it. This technique may be a better solution than the basic approach as it should be less affected by volatile patterns i.e. it is a way to remove noise.

Overall, as in Darolles and Le Fol (2003), the problem is tackled along two temporal dimensions: the daily dimension that seeks to find the guideline for the volume's profile shape typically across market components[6], and the intraday dimension that attempts to model and progressively estimate the evolution of the volume's profile along the trading session.

$$b_i^j = v_i^j / n_i^j \tag{3.2.2}$$

Using the series of prices, P, I can define the market turnover per bin across the different stocks as a weighted sum of the bins:

[5]Note that when I outlined *PCA trading* it was stated that the eigenvectors with the lowest eigenvalues where used in mean-reverting strategies due to their white noise structure.

[6]It could be improved by looking for just the subsets that with the most cointegrated clusters following portfolio management theory like sector-wise, liquidity-wise, etc.

$$b_i^J = \frac{\sum_j P_i^j v_i^j}{\sum_k P_i^k n_i^k} = \frac{\sum_j P_i^j n^j v_i^j / n^j}{\sum_k P_i^k n_i^k} = \sum_j w_i^j b_i^j \tag{3.2.3}$$

I apply PCA to the matrix of volume profiles across stocks and generate the base of orthogonal eigenvectors that orders the observations into dimensions from larger to lower dispersion,$C_i^k = b_i'^j u_k$. It corresponds to the expectral decomposition of the variance-covariance matrix of the data.

$$\mathrm{cov}(C_i^k, C_i^l) = \lambda^k \delta^{kl} \tag{3.2.4}$$

$$\frac{b_i^j - \bar{b^j}}{\sigma^j} = \sum_k u_k^j C_i^k \tag{3.2.5}$$

Given that correlation is $\mathrm{corr}(u_k^j, C_i^k) = \sqrt{\lambda_k} u_k^j$ the former equation can be written as:

$$\begin{aligned} b_i^j - \bar{b^j} &= \sigma^j \sum_k \frac{\mathrm{corr}(b_i^j, C_i^k)}{\sqrt{\lambda_k}} C_i^k \\ &= \sigma^j \sum_k \frac{\mathrm{corr}(b_i^j, C_i^k)}{\sqrt{\mathrm{var}(C_i^k)}} C_i^k \\ &= \sum_k \frac{\mathrm{cov}(b_i^j, C_i^k)}{\mathrm{var}(C_i^k)} C_i^k \end{aligned} \tag{3.2.6}$$

Giving rise to the centred turnovers:

$$\begin{aligned} b_i^j - \bar{b^j} &= \sum_k \frac{\mathrm{cov}(b_i^j, C_i^k)}{\mathrm{var}(C_i^k)} C_i^k \\ &= \sum_k \frac{1}{\lambda_k} \mathrm{cov}(b_i^j, C_i^k) C_i^k \end{aligned} \tag{3.2.7}$$

If the first principal component's projection is isolated:

$$b_i^j - \bar{b^j} = \frac{1}{\lambda_1} \mathrm{cov}(b_i^j, C_i^1) C_i^1 + \sum_{k>1} \frac{1}{\lambda_k} \mathrm{cov}(b_i^j, C_i^k) C_i^k \tag{3.2.8}$$

$$b_i^j = \underbrace{\bar{b^j} + \frac{1}{\lambda_1} \mathrm{cov}(b_i^j, C_i^1) C_i^1}_{c_i^j} + \underbrace{\sum_{k>1} \frac{1}{\lambda_k} \mathrm{cov}(b_i^j, C_i^k) C_i^k}_{y_i^j} \tag{3.2.9}$$

$$b_i^j = c_i^j + y_i^j \tag{3.2.10}$$

This way I can split the volume profile into the sum of a so-called *market* component, c_i^j, and a *specific* component, y_i^j. The latter can be considered a residual that moves like white noise around the former (the more likely the assumption to hold the larger the first eigenvalue with respect to the rest) or further analysed as proposed below.

Note that this method is not an estimation but a decomposition of the structure driving the volatility of the profile and that, given its unsupervised nature, it is not clear the way outliers are treated by the model.

3. – Back to basics

As said above, to the light of the natural uncertainty surrounding intraday volume's profile most of the statistics around the mean could potentially be good enough candidates to the trader. As seen, a simple average tends to be the most intuitive candidate to account for some dynamics of each series of bins being its major issue that outliers would be included in the estimation. However, there is another basic statistic though that is being little used in the literature but that may be useful as well beyond the ease of its implementation: the median.

$$M_i = \acute{b}_i^{\Phi((D+1)/2)},\ \forall i = 1, \ldots, I; \tag{3.2.11}$$

where $\acute{b}_i^d$ corresponds to the ordered series of b_i^d and Φ represents a rounding function.

The median accounts for a natural way of outliers' removal while keeping part of the dynamics in the sample. This should throw similar results to the average being it worse in those bins where recent data has information about future data (i.e. where execution patterns tend to be robust) and better when the sample includes noise.

Dynamic approach

The dynamic approach seeks to benefit from the use of time-series analysis by further analysing the deviations of the bins along the day from the estimated static component (whether mean, PCA or median). The main issue of this approach is that the deviations are not materialized until the volume profile is. This is, as pointed out above, at the end of the day. As such, in order to translate the different slots of volume into estimations of the realized profile I shall consider an estimation of the day's total volume[7] first. And this becomes a new source of risk even if its reliability improves naturally along the day.

As in Bialkowski *et al.* (2008) it is usual to assume that the deviations ($\tilde{y}$) from the static component follow an ARMA(1,1) process with white noise[8] as in:

$$\tilde{y}_i^d = \psi_1 \tilde{y}_{i-1}^d + \psi_2 \epsilon_{i-1}^d + \epsilon_i^d \tag{3.2.12}$$

At the beginning of the trading session this process requires us to predict the total volume to be traded (in order to define the deviations themselves as a percentage of this very level) as well as the whole profile for the rest of the day through the ARMA(1,1) structure, a rigidity that may have the largest effect on the overall approach. However, as time passes the estimation of the consecutive bins may benefit from the intraday evolution of the volume profile as it is discovered[9].

Results from the static and dynamic approaches

After applying the previous approaches to a potential VWAP on Telefónica along the trading session described in 3.2.1, error measures non sensitive to the price dynamics of the data are calculated and compared to decide which one and why is the best technique in our data. Then, the rest of the members of IBEX are analysed to check the robustness of our first statements.

Detailed analysis for Telefónica

The experiment starts analysing the stock of Telefónica, the largest company in our data set. The errors of estimation (as said, defined as percentages of the total volume) are accounted in absolute values and accumulated both along my data sample and across bins. This is, cumulative absolute percentage errors (CAPE) will be considered in order to avoid the distortion of the price' dynamics into the error measure. Mathematically, it could be defined as:

$$C = \sum_d \sum_i |b_i^d - \hat{b_i^d}| = \sum_d \sum_i |\varepsilon_i^d|, \tag{3.2.13}$$

[7]In fact, estimations of the specific trade's period volume as typically VWAP is executed intraday, i.e. with shorter horizons than the whole trading session.

[8]They also consider a SETAR model that I won't be analysing herein.

[9]As a result, this approach has the highly disregarded issue that once a trade has started the volume profile shall be kept until its end in order not to risk over or under executing it as the profile evolves via ARMA – and this not only difficults the VWAP implementation but also smooths out the potential benefits from the gradually finer ARMA estimations.

	Dynamic methods			Static methods		
	PCA	**Average**	**Median**	**PCA**	**Average**	**Median**
Average	111.57	54.99	66.05	32.43	28.31	28.46
Standard dev.	153.94	122.97	195.75	6.13	6.49	6.54
Minimum	32.07	12.84	13.40	14.44	11.81	11.84
Maximum	925.95	759.96	1205.16	46.88	43.96	44.18

Table 3.1: Telefónica intraday volume's profile estimation: errors' statistics.

where$\hat{b_i^d}$ corresponds to the estimated percentage volume for the $i-th$ bin on day d. The absolute value of the difference is used as positive errors cannot be assumed to compensate negative ones (nor vice-versa) since the prices at which those volumes were crossed may be different. Also, I do not require this measure to be differentiable hence I can avoid the exponential penalty that is given to the extreme errors when the quadratic function was considered instead[10].

Motivated by the advances in the literature described above I first focussed on the latest technique: I started by analysing the dynamic approach and took the aforementioned market structure defined by the first principal component's projection as a reference. However, being aware of the issues related to this technique and described above (especially at a development level) I also checked the dynamic approach upon simpler structures, the average and the median. This way I expected to build up acumen with regards to the weigh-off between the pros and cons of each approach in terms of development ease and estimation accuracy. Surprisingly, as we will see below, I found that the latter leaded to more accurate approximations in my data; this triggered the comparison with the even more basic, static scenarios. And this is how I finally challenged the idea that the dynamism enhances robustly the VWAP algorithm: beyond the issues at a development level, the expected benefits of such approach could be well overcome by the new dimensions of complexity that it adds – namely, the ARMA reliability and the volume's estimation.

Results are enclosed in Table 3.1 where it can firstly be seen that, as said, the popular PCA-ARMA approach has not overcome the dynamic structures built upon the basic approaches in our data. Not only that but it has also underperformed the more basic approaches without a dynamic structure. The reason is two-fold: first, the ARMA(1,1) structure does not hold robustly in our data and second, PCA itself is not the best track for the static component within a dynamic approach nor isolated statically. Note at this point that the volume profile calculated with a dynamic method is based on the forecasts of the deviations of an ARMA(1,1) calibrated through OLS. The ARMA assumes that those deviations are Gaussian and in reality the extreme events in the volume bins are more frequent than a Gaussian would imply. The existence of the fat tails is simply another feature derived from the fact that, as said, there are little priors to be stated about the dynamics of the volume (the intraday volume is more difficult to predict than the intraday prices).

Several arguments that could possibly explain why there is such deviation with respect to Bialkowski

[10] In reality, a measure that penalized clusters along time in the errors' side/sign over swinging errors' sides could be a more appropriate one as it . This is, it is potentially worse to compensate one-sided small errors at the beginning of the day with one-sided small (and opposite) errors at the end of the day than to compensate larger errors through alternate sides consistently along the day. It is simply a consequence of assuming that the volatility of the deviation's distribution from any price level grows with the time frame.

et al. (2008) range from geographical differences across the data set used in their study and the data set used herein (different nature between the European and the American markets) to standard-practice differences given the distance in time periods covered in each experiment (the industrialization of the algorithmic execution may have had an impact in the dynamics of the volume intraday) not to mention the fact that their results were affected by the dynamics of the prices (the *fuzzy effect* mentioned above) while ours abide exclusively by the volume.

As an example, Figure 3.1 shows the autocorrelation function (ACF) and the partial autocorrelation function (PACF) for the last three days of our data set. For a direct comparison with the Bialkowski *et al.* (2008) the approach considered in the plot is the PCA-ARMA. Therein it can be seen first through the ACF subplots that the market component still resembles a similar seasonality to the one embedded in the original volume's profile leaving the specific component with no seasonality as intended. Also, observing again the ACF I can assert that given the large difference between the bar at the first lag with respect to the second along with the volatility decay of the PACF, that an MA(1) structure is possible in the sample. However, the dispersion across the largest bars of the PACF along with their similar size suggests that the possibility of an AR(1) structure shall be rejected. In fact, it would be difficult to justify the election of any AR(p) structure. Hence, the large CAPE observed in our dynamic approach over any of the proposed static components. It is also worthy to mention that, as expected, the average and the mean have similar results under the two approaches being the former marginally better.

Figure 3.2 shows some of the results in terms of CAPE per observation included in the last 2 months of our data, the estimation of the profile under the different dynamic approaches and its deviation from the real profile.

Note that the ARMA structure generates negative volume profile estimations in the PCA that should anyhow be corrected. I could easily lift those to zero and spread their weight across the rest of the bins. However, that would add more uncontrolled non-linearity to our results that would erode the expected benefits from the use of the (unrestricted) essence of the ARMA structure and would definitely bias my statements depending on the specific data that I am analysing – in fact, overfitting my statements to the sample.

Mean results across IBEX members

Figure 3.3 lets us think that, as said, the use of VWAP's itself deviations as the error measure may have hidden the real risk embedded across the different approaches considered in part of the literature, and more especially, Bialkowski *et al.* (2008). Using instead CAPE on volume-only (i.e. not including prices in the measure) the risk embedded in the dynamic approach can be distinguished throughout the peaks present on the figure. These large asymmetries that affect the three candidates to be the static component of the dynamic approach are generated on punctual dates and typically at the beginning of the trading session when estimations of the deviations are, as expected, less reliable. Note that this would have been largely diluted had I taken the prices into account or the median or average error along the day instead of the cumulative one.

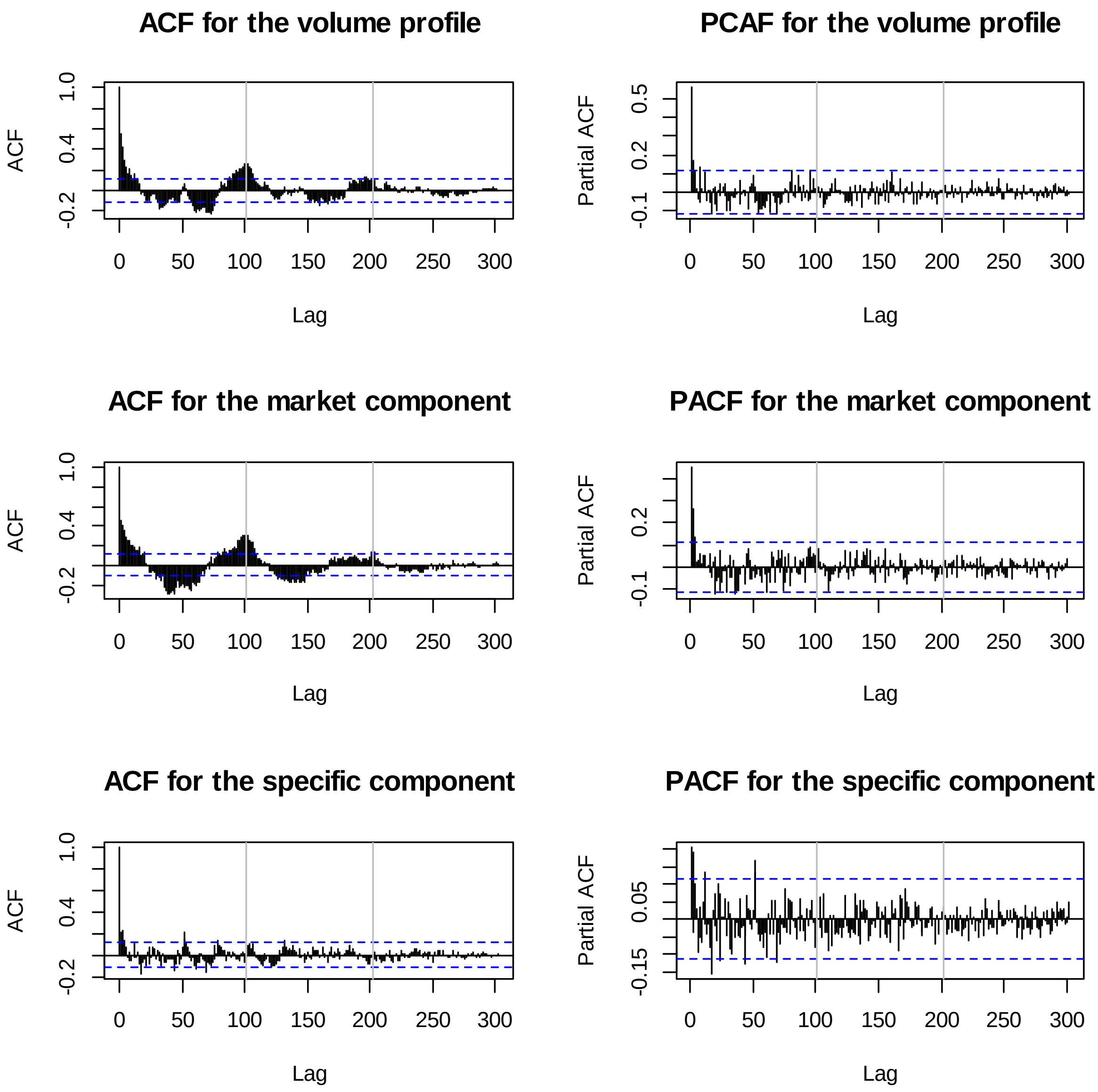

Figure 3.1: Autocorrelation and partial autocorrelation functions of the two components for the PCA-ARMA on our last three days for Telefónica.

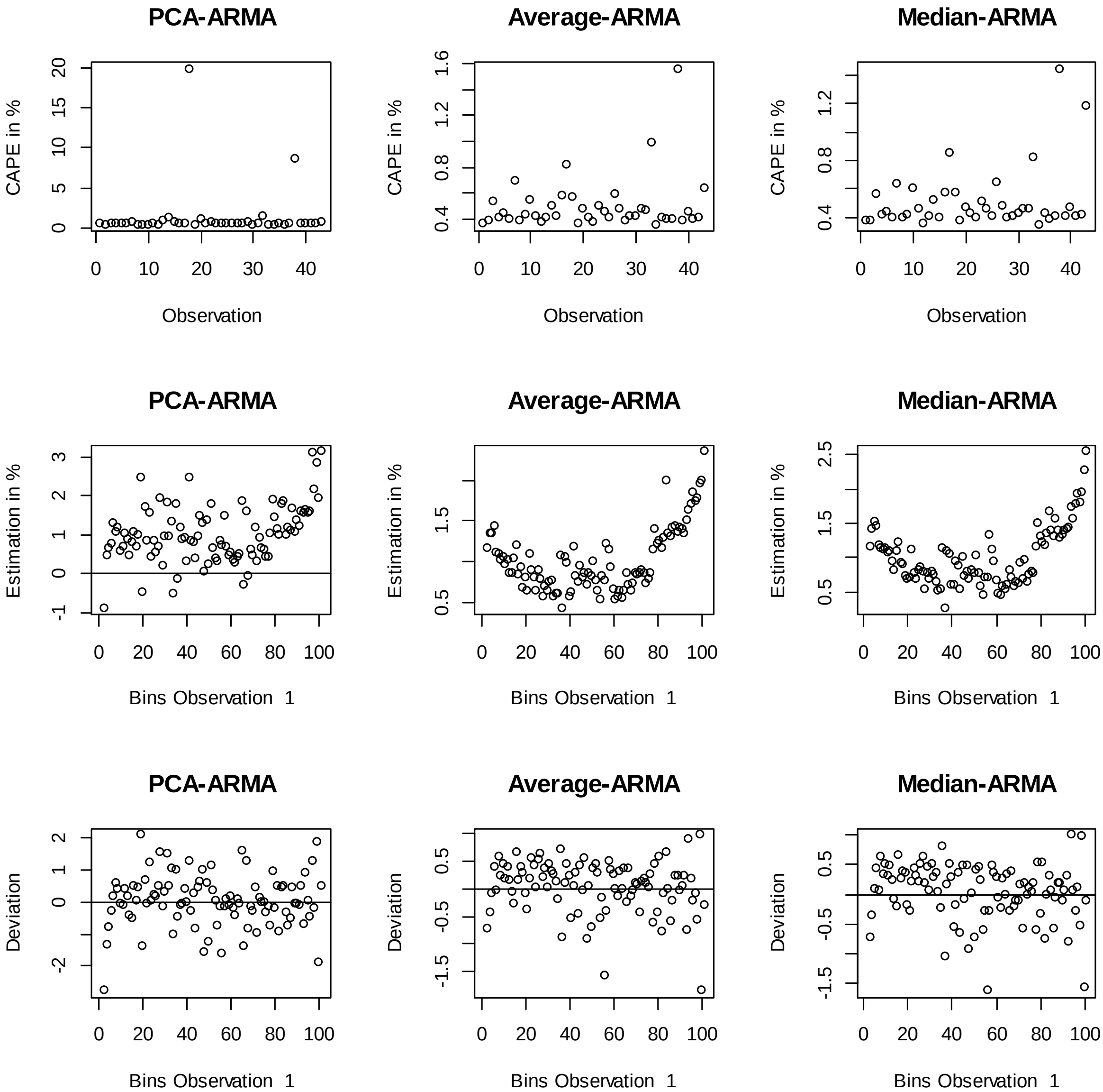

Figure 3.2: Statistics of the dynamic approaches in Telefónica with detailed information for the first observation.

Figures 3.5 and 3.6 show detailed information about the peak in Arcelor Mittal (MTS.MC) and the peak in International Consolidated Airlines (ICAG.MC) respectively, where the distribution of CAPE along the different trading sessions considered in my data sample is first shown, followed by detailed information about the estimated profile and its deviation on the dates with the highest CAPE. Note that even though a typical explanation for outlier errors in the financial predictions builds upon major news in this case it corresponds to isolated effects within the trading session. The traded volume during the morning was far from predicted; and this could largely obey to rumours or block trades that were not properly flagged in the dataset. The following corrections triggered by the ARMA structure are then propagated along the rest of the bins until the original deviation is gradually mitigated (one of the criticisms that I postulated against the election of such structure in the estimation of the volume's profile).

Finally, Figure 3.4 shows that PCA is robustly not the best candidate for the static component in our data and that the average and the mean generate close results, a statement that remains robustly across stocks[11].

Conclusion

It seems that in our data the dynamic approach adds further errors to the static one. The reasons for that range from the fact that the ARMA(1,1) structure may not hold robustly along the day to the fact that further external restrictions are typically required. And this erodes the benefit from and nature of the dynamic analysis. Nevertheless, we have seen that the PACF is not significant enough to conclude that there is any AR(p) structure in Telefónica and that the dynamic component may generate profiles that expand along the negative region. This would hence need to be amended if the trader does not want to incur into short-sale costs and restrictions for no apparent reason. Further, I expect the estimation of the deviations to become an added complexity to the model given especially at the beginning of the data when the whole volume traded has to be estimated.

It can also be inferred from the experiment that PCA is not the best candidate for the static approach when prices are not included in the errors' measure. In fact, the simple first moment of the sample distribution is in general the most suitable candidate for the estimation of the volume profile. However, it does not happen robustly being the median still better than the average in some cases[12]. This situation makes us believe that there may be a possibility to add pattern detection techniques in the attempt to use for each bin the best candidate between these two.

Last, it is important to note that the concave shape of the figures 3.3 and 3.2 may back up the industrial belief that the fee charged on each stock shall depend on its rank on the main indices it belongs to. The stocks of the figures are ordered by their participation in the index seeming hence to confirm the professionals' prior with regards to the role of the membership on a stock's demand.

[11] This is, when combined both figures I can say that the ARMA magnifies the errors of the profile structure overall.

[12] A difference that could have been magnified had not I been able to clean properly the data or had I included the third Friday of each month, when futures are due and the index rebalanced (in this case, the third Friday of December 2012).

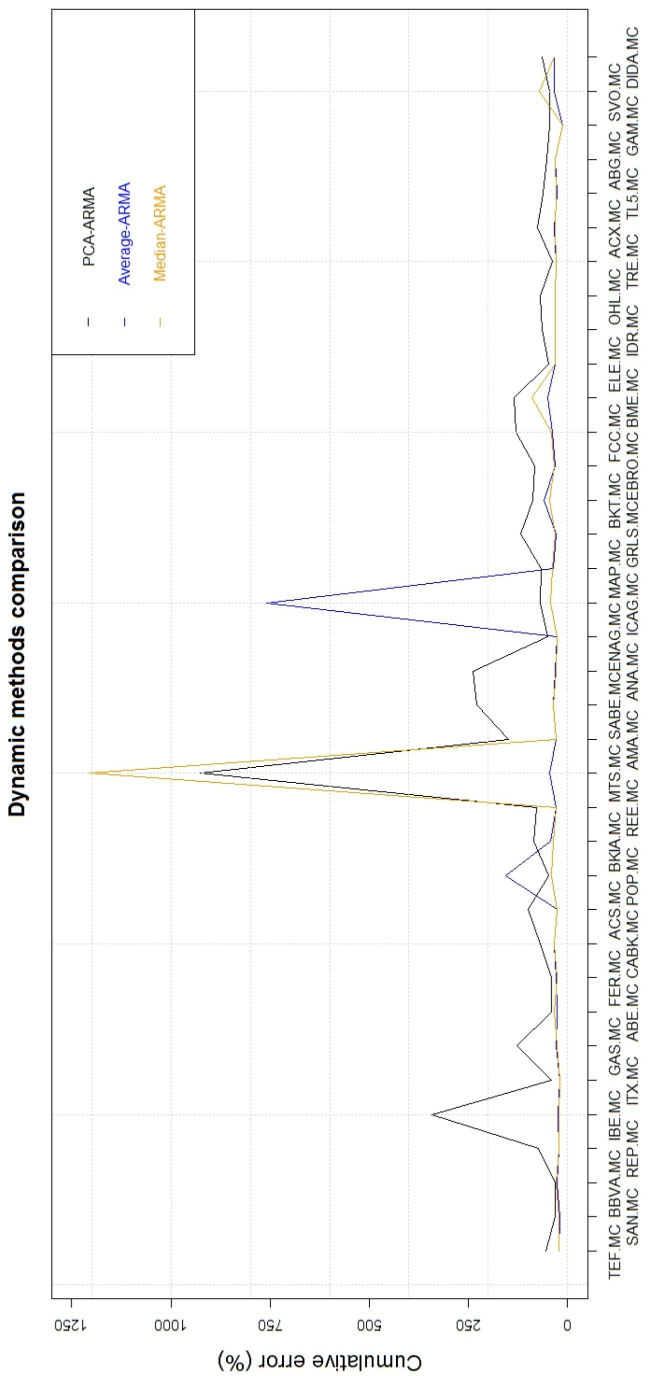

Figure 3.3: Dynamic methods comparison through the CAPE.

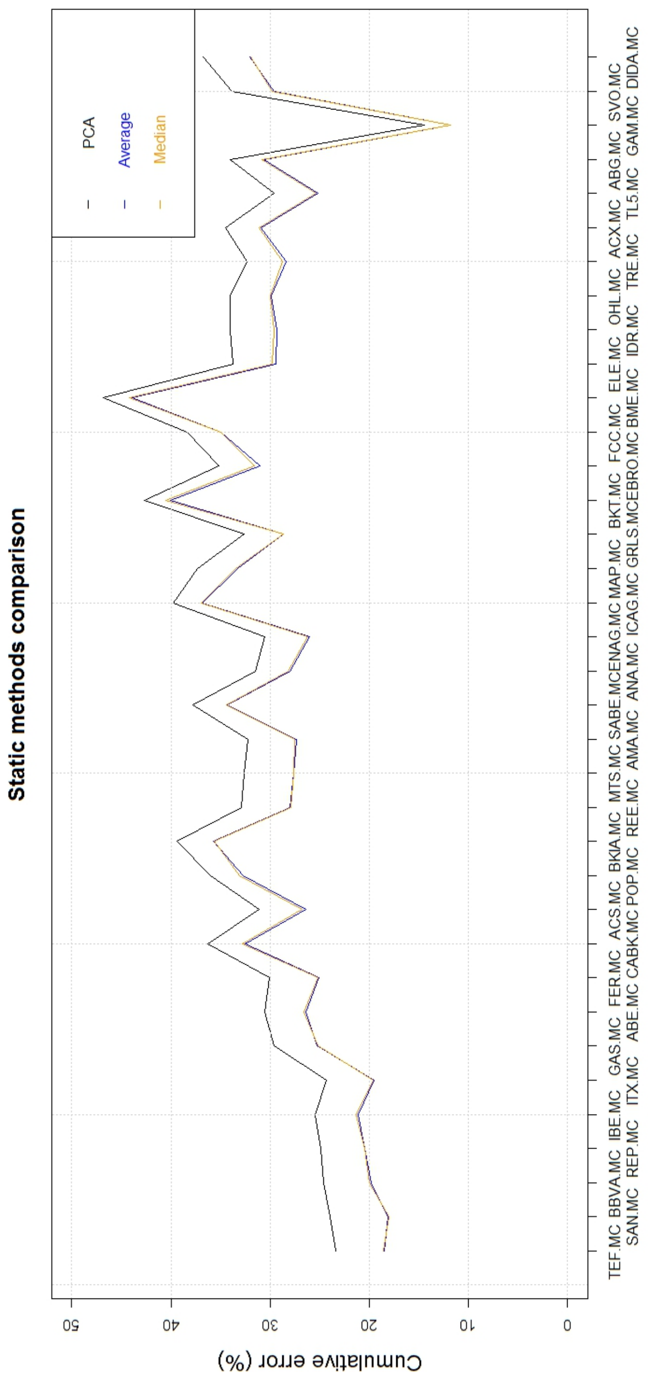

Figure 3.4: Static methods comparison through the CAPE.

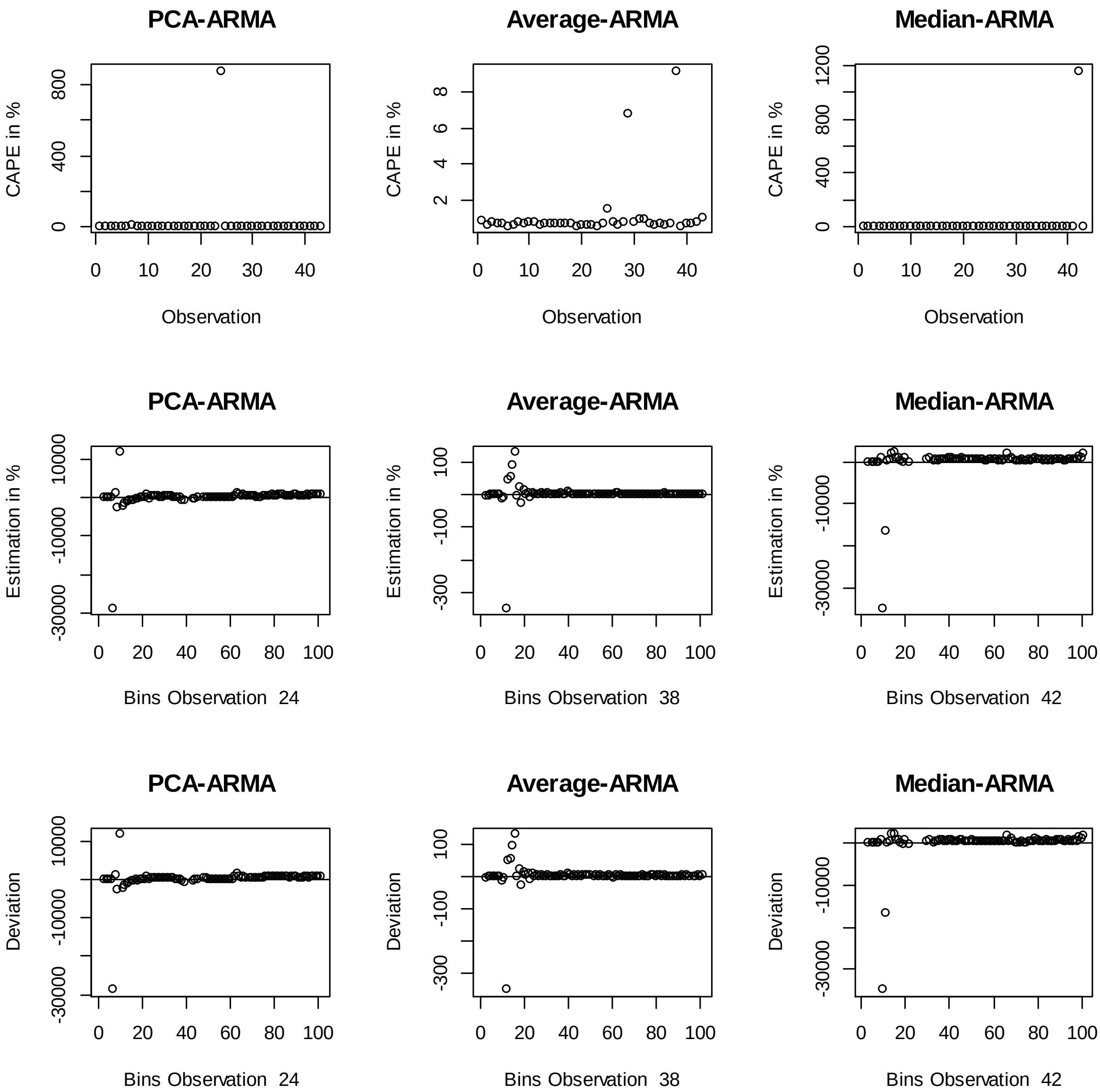

Figure 3.5: Statistics of the dynamic approaches in Arcerlor Mittal (MTS.MC) with detailed information for the observations with the largest CAPEs.

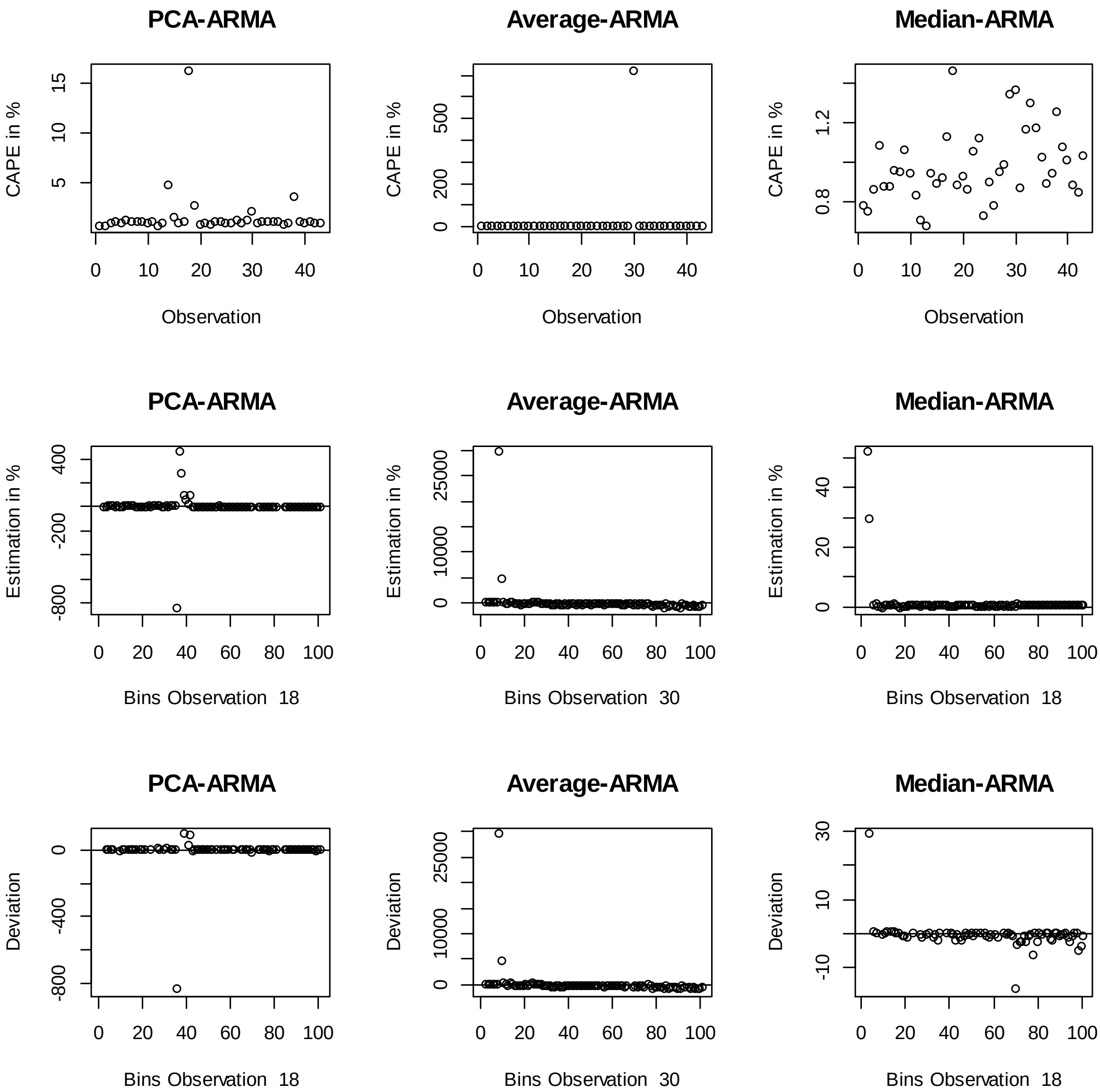

Figure 3.6: Statistics of the dynamic approaches in International Consolidated Airlines (ICAG.MC) with detailed information for the observations with the largest CAPEs.

The duality between the interest as a individual security or as part of a index generates a larger amount of noise for those in the middle of the distribution than those on the top or bottom of the list. The former because their individual demand has a large effect on the index demand and the latter because the effect of the demand of the index has a large effect on their individual one – hence, their volume should evolve more smoothly along time, on average.

3.2.3 A pattern-driven approach: the Sample-Sensitivity Index

At this stage, it seems relevant to explore the possibility to generate a signal that provides the trader with information as to how sensitive each bin should be to the sample in order to use the mean or the median in its estimation. I target to exploit stable patterns even if they account for marginal profits as inferred by the similarity of the CAPE results obtained for both candidates[13].

To the light of the previous results it will be considered the case where there is no dynamic component but only the static one which is by default to be selected as the average unless previous patterns within the trading session and from the past performance give room to the application of the median instead. The instrument that will be proposed for such task is the Sample-Sensitivity Index or SSI. The SSI allows for a dynamic component that acts though discretely along the trading session and lets the trader know whether the prediction has to be more (through the average) or less (through the median) sensitive to the data sample in the attempt to benefit from the reduction of noisy patterns present in the sample, wherever the shift is statistically significant.

In doing so, I propose to build the SSI like a support vector machine classification algorithm as follows. The chosen pattern considered in this experiment flags the bins where the deviation of the median has been smaller than the deviation of the average both in absolute values:

$$\delta_i^d = |\varepsilon_i^{A,d-1}| - |\varepsilon_i^{M,d-1}|, \tag{3.2.14}$$

where the ε components refer to the deviation of the predicted volume profile from the real one on each bin, i, on a single day, d, for the average, A, and the median, M, static components both. The further the bin from the opening auction the more patterns, n, that can be included. I will add also the robustness of such pattern intraday comparing each bin to the previous one[14]:

$$\triangle_i^d = |\varepsilon_{i-1}^{A,d}| - |\varepsilon_{i-1}^{M,d}|, \tag{3.2.15}$$

[13] It is important to note here that given the notional traded through algorithmic trading on a medium institution (in the order of billions of dollars) any marginal improvement may have a large effect on its P&L account.

[14] Note that I could have used the same notation than in the interday definition but I have decided to explicit clearly both components for the sake of clarity.

As such, the set of features can be defined as along intraday and interday information:

$$\mathbb{F} = \{\underbrace{\triangle_i^d, \triangle_{i-1}^d, ...\triangle_{i-n}^d}_{\text{intraday}}, \underbrace{\delta_i^d, \delta_i^{d-1}, ...\delta_i^{d-D}}_{\text{interday}}\} \tag{3.2.16}$$

for all $i > n-1$ and $d > D-1$. Hence,

$$\Gamma_i^d = f(\delta_i^{d+1}, \mathbb{F})$$

where f refers to the SVM function, Γ refers to the *SSI*, $\mathbb{F}$ refers to the feature space and $\delta_i^{d+1}$is the estimation for the current bin i. Then, $\Gamma_i^d = 1$ as a default case and $\Gamma_i^d = 0$ if the median has to be used instead of the average in bin i, $\delta_i^{d+1} > 0$, giving rise to a profile volume per bin defined by:

$$b_i^d = \Gamma_i^d A_i + (1 - \Gamma_i^d) M_i, \tag{3.2.17}$$

that has again to be rebalanced to meet the condition $\sum_i b_i^d = 1,\ \forall d$.

Results from the Sample-Sensitivity Index

I will consider the same information set in terms of lagged patterns along each bin of the trading session, in this case, two lags of interday patterns and one lag of intraday patterns (giving more emphasis to the former) so that the feature space will be defined as:

$$\mathbb{F} = \left\{\triangle^d, \delta^d, \delta^{d-1}\right\}$$

where each component refers to the whole distribution of bins along the trading session.

As mentioned in Subsection 2.3.3, the effectiveness of SVM does depend on the kernel selection, its parameters and the soft margin parameter, C. The kernel, κ, chosen for this experiment has been the Gaussian radial,

$$\kappa = \exp\left(-\gamma |x_l - x_m|^2\right),$$

where x_l and x_m are instances of the feature space. A 10 fold cross-validation was used to pick the best combination of parameters γ and C through a *grid search* over the sequences

$$\gamma = \left\{2^{-5}, 2^{-3}, 2^{-1}, 2\right\} \text{ and}$$

$$C = \left\{2^{-3}, 2^{-1}, 2, 2^3\right\}.$$

Two-thirds of the database were used to train the SSI and results are provided for the last one-third of out-of-sample data.

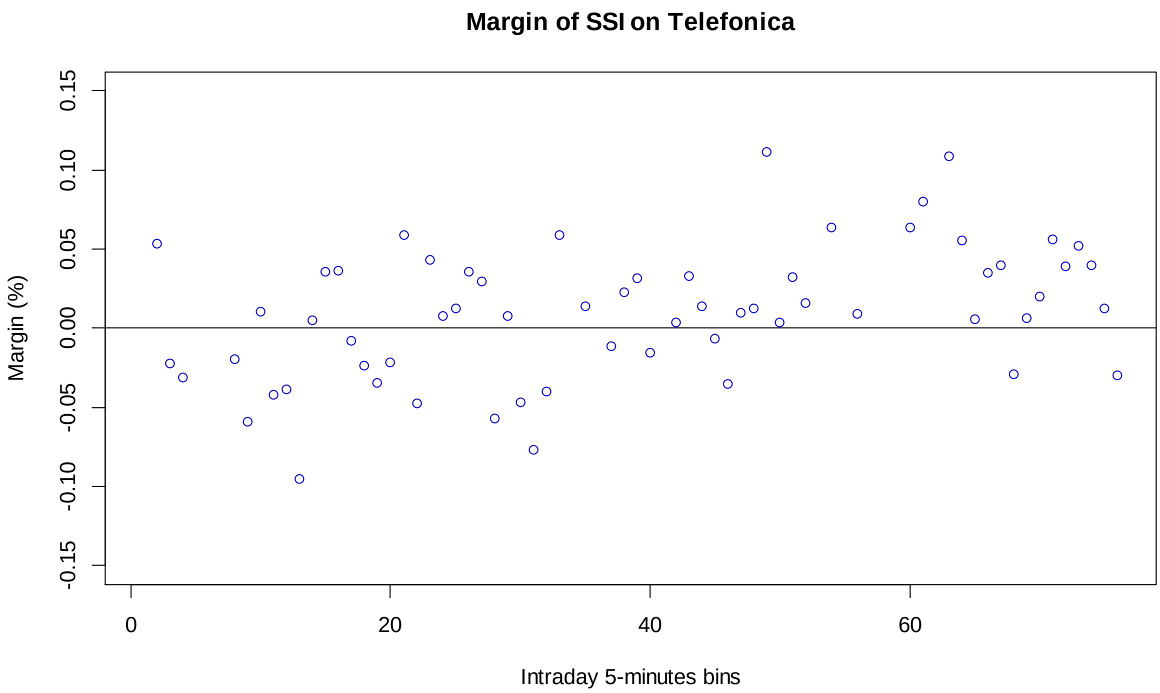

Figure 3.7: Edge for using SSI to combine the best two static components.

Detailed analysis for Telefonica

Given the U-shape of the intraday volume the beginning and the end of the trading session are expected to be the periods with the least robust patterns interday hence they shall remain outside of the analysis. Also, the opening of the US exchanges is expected to affect the dispersion of the patterns along the European trading session. This fact motivates the definition of SSI to be restricted to non-overlapping market hours.

The implementation of the SSI-profile generates an improvement of 2.1 basis points on Telefónica with respect to the use of the first moment only. As expected, SVM cannot always identify which bins require the use of the median instead as showed in Figure 3.7 where the overlapping hours with US markets have not been considered as proposed above[15]. In fact, had the US overlapping market hours been included the SSI-profile generated would have had a negative effect as seen in Figure 3.8 where the vertical line marks the beginning of the US trading session, leaving on its right side a distribution of margins largely biased towards the negative area[16].

This first approach to SSI backs up the motivation to use pattern recognition when improving the profile estimation. The wonder now is whether the mean result is held robustly across the rest of the members in IBEX or not.

[15]Where parameters chosen after the *grid-search* are: $\gamma = 2$, $C = 8$.
[16]Where parameters chosen after the *grid-search* are: $\gamma = 0.03125$, $C = 0.125$.

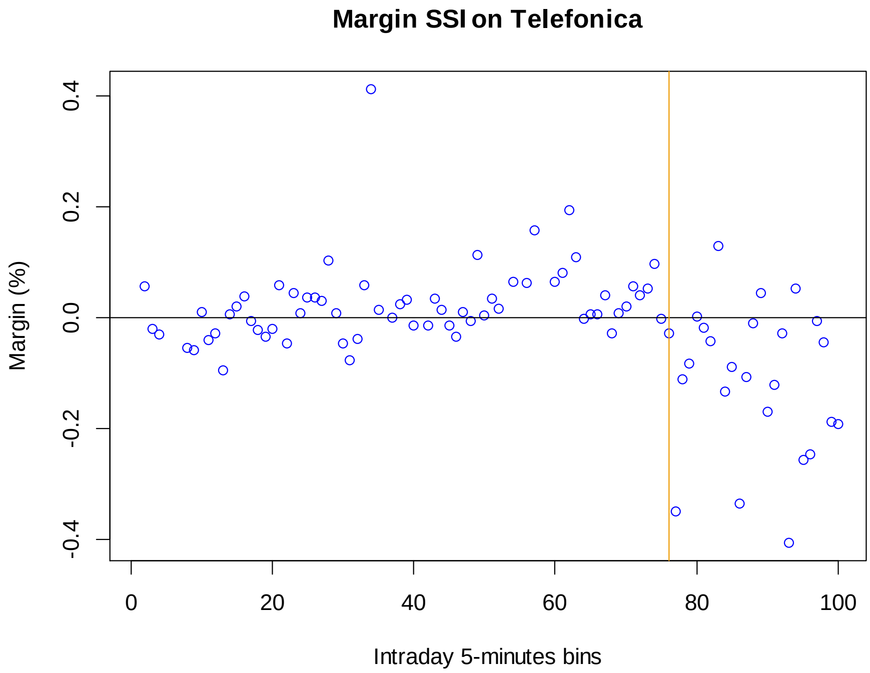

Figure 3.8: The advantage of SSI seems to suffer after the open of the US markets.

Mean results across IBEX members

Figures 3.9 and 3.10 show the margin distributions due to the use of the SSI-profile along the six largest stocks following Telefónica both when taking only care of EU trading hours and when overlapping with US market hours is considered. Not only do they back up the conclusions reached with respect the distribution generated for Telefónica but also the results are both robust across stocks and larger on average.

Moreover, Figure 3.11 shows the distribution of the mean margin from using SSI-profile along the whole set of members. It can be seen in it that none of the stocks is worst-off when the proposed pattern-detection technique is deployed during EU-only trading hours are considered. In fact, most of the time SSI would produce undesired results on average when the US overlapping is allowed, as seen in Figure 3.12.

3.2.4 Summary of results

Follows a series of bullet points that synthesize the results obtained along the experiment:

- The PCA dynamic approach approximately doubles the average error of the two simpler structures taken into account, average and median: 112 vs 55 and 66 respectively. Even though its overall distribution is less volatile than that of the median; and this could lead

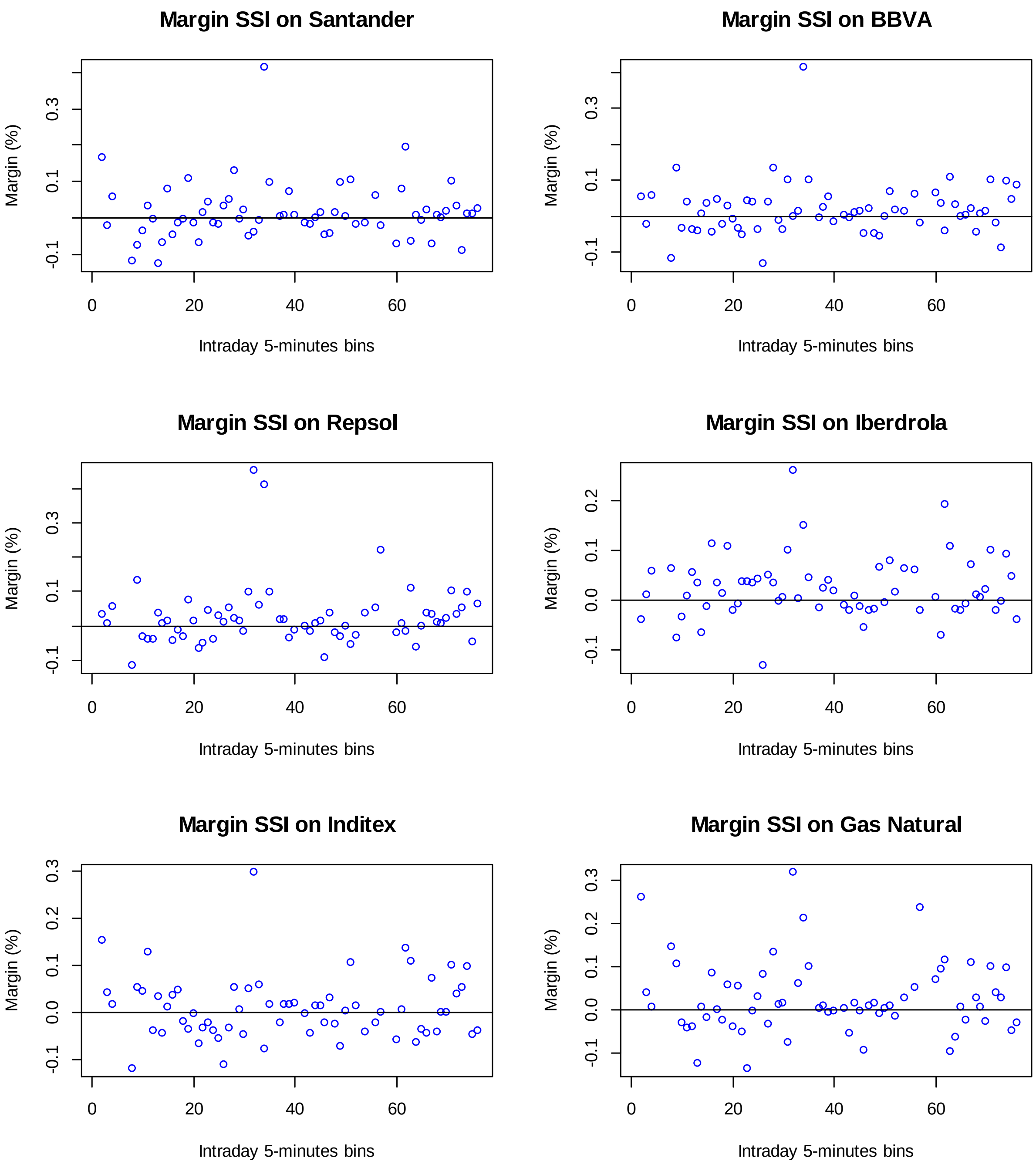

Figure 3.9: Individual results for the six largest stocks after Telefónica when overlapping hours with US are excluded.

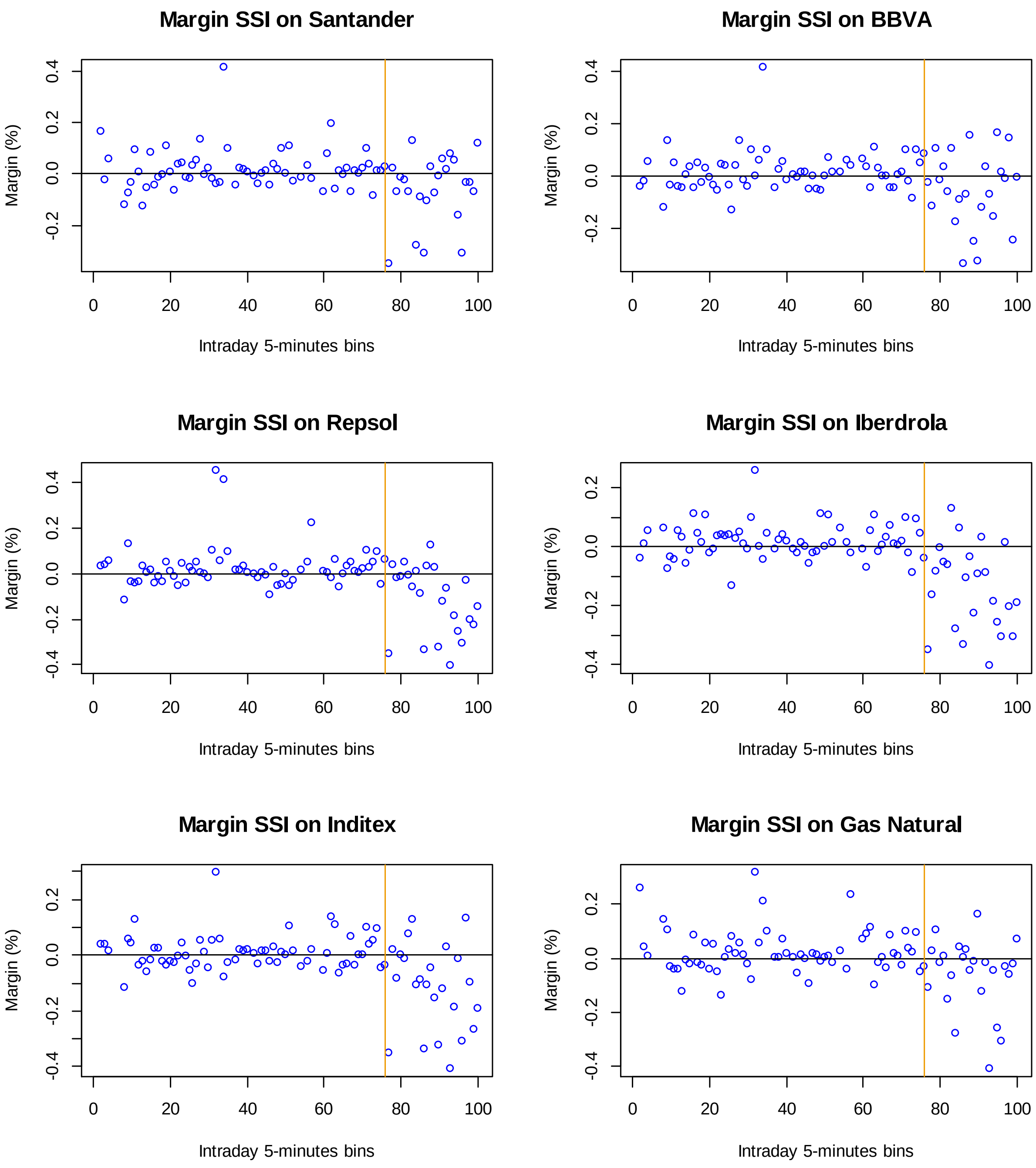

Figure 3.10: Individual results for the six largest stocks after Telefónica when overlapping hours with US are included.

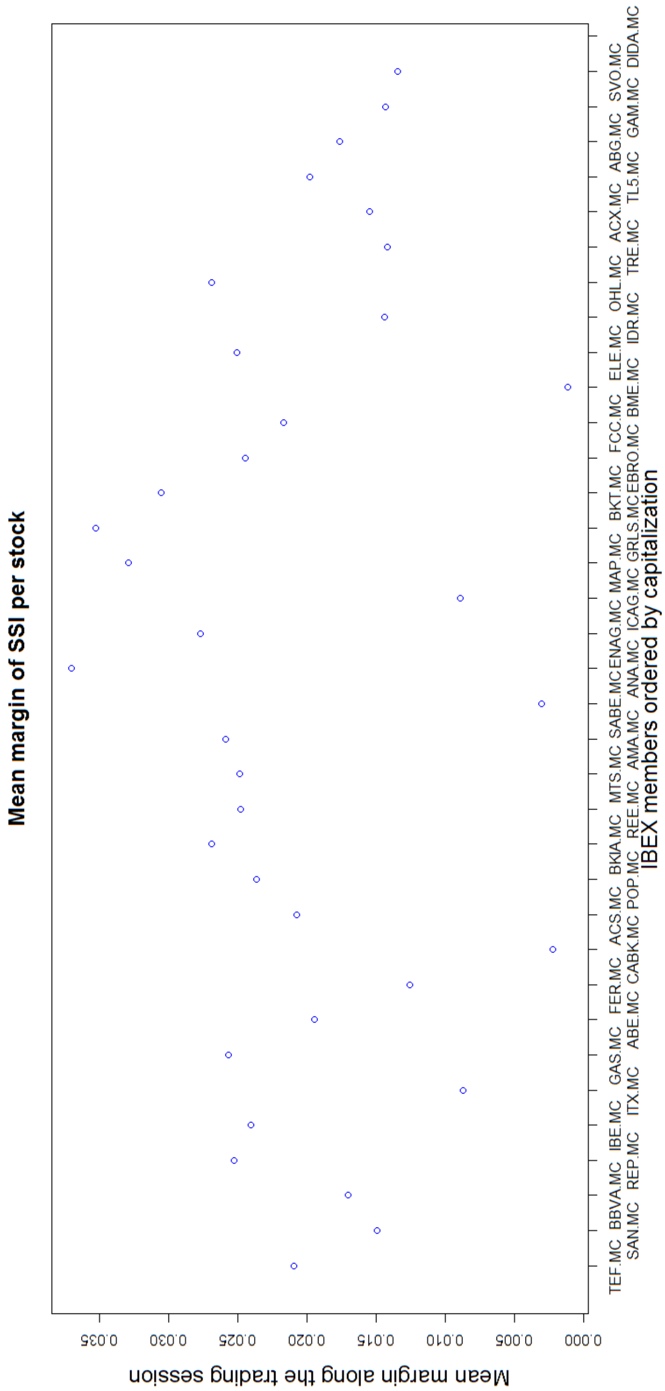

Figure 3.11: Mean margin per stock when overlapping hours with US are excluded. All positive.

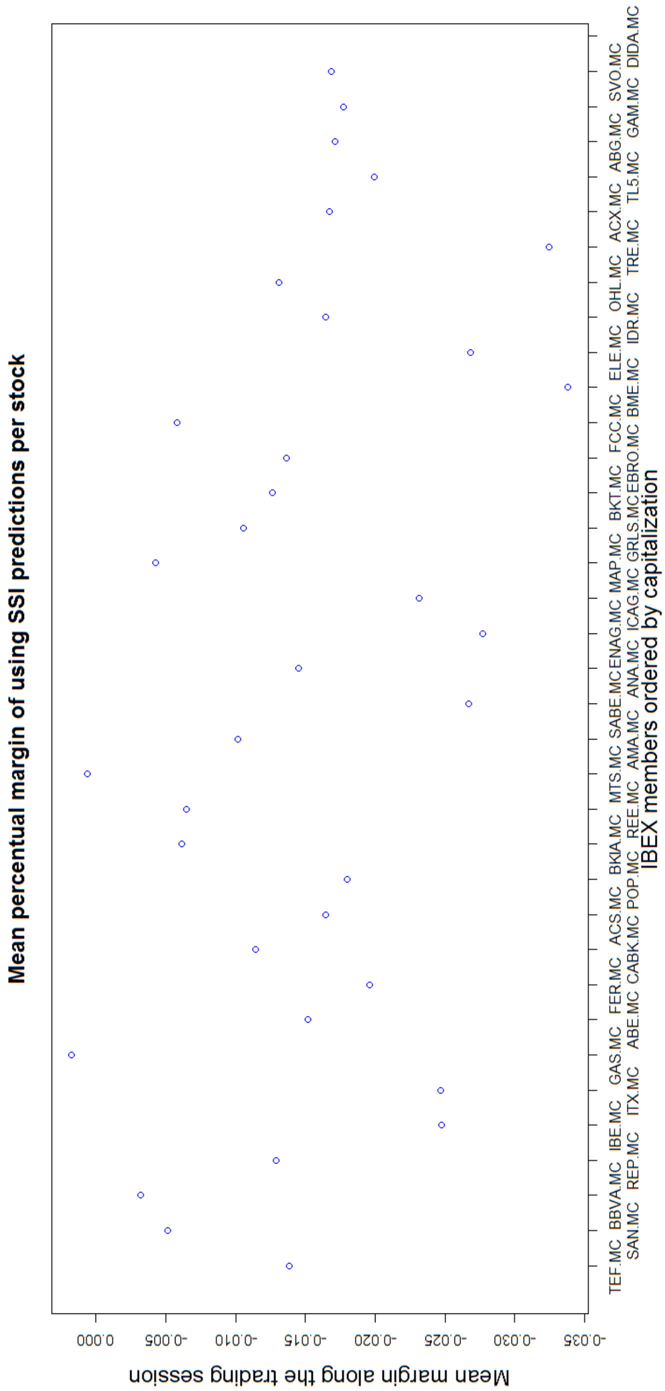

Figure 3.12: Mean margin per stock when overlapping hours with US are included. Mostly negative.

to some controversy about which one is best, the average still outstands in the rest of the domains analysed (standard deviation, minimum and maximum).

- The static approach beats the dynamic in every domain considered. This is due to the fact that it is not clear that an ARMA(1,1) structure can be forced to explain the evolution of the trading volume along the session. In my sample, while the combined analysis of the ACF, PACF set clear that an MA(1) structure is possible the appropriateness of any AR(p) is less clear. Even though the average seems to be the best static approach, its edge is now eroded by the fact that the PCA accounts for less volatility (6.13 vs 6.49 and 6.54). On the other hand, the average and the median obtain closer results than in the dynamic approach.

- The SSI, that combines through SVMs both the average and the median, is largely affected along the time span where the Spanish Stock Exchange overlaps with the US market. This is a consequence due to the fact that the latter affects the trading volume of the former. An effect that adds noise that the SSI cannot remove. In my sample, while the SSI reaches improvements of the overall error of the average static approach in excess of 3.5%. More importantly, in all the cases analysed the error was enhanced. Opposite, when the overlapping hours with the US are considered the errors during that period can be high enough to revert the positive results obtained in the previous range.

3.2.5 Summary

Even though theoretically it seemed appealing to add the dynamic approach ARMA(1,1) onto the static estimations, in reality it generates larger errors in my data when prices are not included in the errors' measure. Such process is expected[17] to render less reliable values at the beginning of the trading session when the volatility of the estimation of the total volume along the trading session is higher (fatter tails). Moreover, the MA component can generate negative bins in order to compensate from previous positive deviations. This situation, apart from affecting the freedom of the approach itself, multiplies notably the trading costs and market impact and opens the discussion as to whether there is a different way to include intraday dynamics in the profile estimation.

A movement from ARMA-PCA to SSI, an index based on SVM estimations of the best static component, is hence proposed to include dynamics through a set of features that mixes interday and intraday patterns. Results are positive and robust across stocks only along the time period where SIBE, the Spanish exchange system, does not overlap with the US trading session.

[17] In the industry, most of the execution algorithms are instantiated with an intraday volume's profile that remains constant along the life of the specific execution trade.

3.3 Further proprietary enhancements

The risk-reward chapter is devoted to the comprehensive implementation of proprietary approaches in trading hence I will try the present section not to overlap with it[18]. As such, and given the relevance of the intraday information in algorithmic trading proprietary enhancements, I will illustrate below some of the core features that have to be regarded if further improvements are considered.

Data

The data for this experiment is built upon the March future of IBEX along the stocks' trading session[19] of February the 17th, 2012. Tick data expands along 10 levels of depth within bid and ask side of the order book. Infobright was used as the column-oriented database that favoured the speed in data access.

Stylized facts

How many ticks per second may the trader expect from a specific instrument? How often are trades typically crossed? How many levels of the order book are relevant in terms of prediction? Is there any hidden liquidity and if so, is it consistently provided along time? These are the type of questions on which I will try to briefly shed light as they are essential to the trader's approach when deploying high frequency strategies in the markets.

By firstly generating the histograms of both ticks and trades per second a market participant can better understand the type of trading available on a specific security. Especially, in two core dimensions: technology suitability and business opportunities. One of the first questions that surge nowadays in the industry is how fast the trading platform does need to react on average to new ticks. The investment in technology will crucially depend on the nature of the products that the trader is planning to market make. Once the technology passes the first filter the following question typically is how much flow is generated in the market. The more the flow the more the opportunities to offset the inventory through limit-orders in the market and as such, the more the expected *Elasticity of the Flow*.

Figure 3.13 schematizes both dimensions:

1. The upper graph shows the existence of several ticks per second along the whole trading session – reaching beyond 150 after the US market opens. Advanced systems are hence required to process and react to such amount of information which is also expensive to maintain as a database (the total number of ticks in our sample for the afore mentioned date is 63,290). I will address below what could be the depth of the order book that it should be stored as a minimum in order to grant that most of the significant information is kept.

[18] There is a large literature on execution optimization from optimal control of execution costs as in Bertsimas and Lo (1998) and Almgren and Chriss (2000) to adaptive execution as in Park and Van Roy (2012).

[19] Beware that usually futures trade longer hours than stocks.

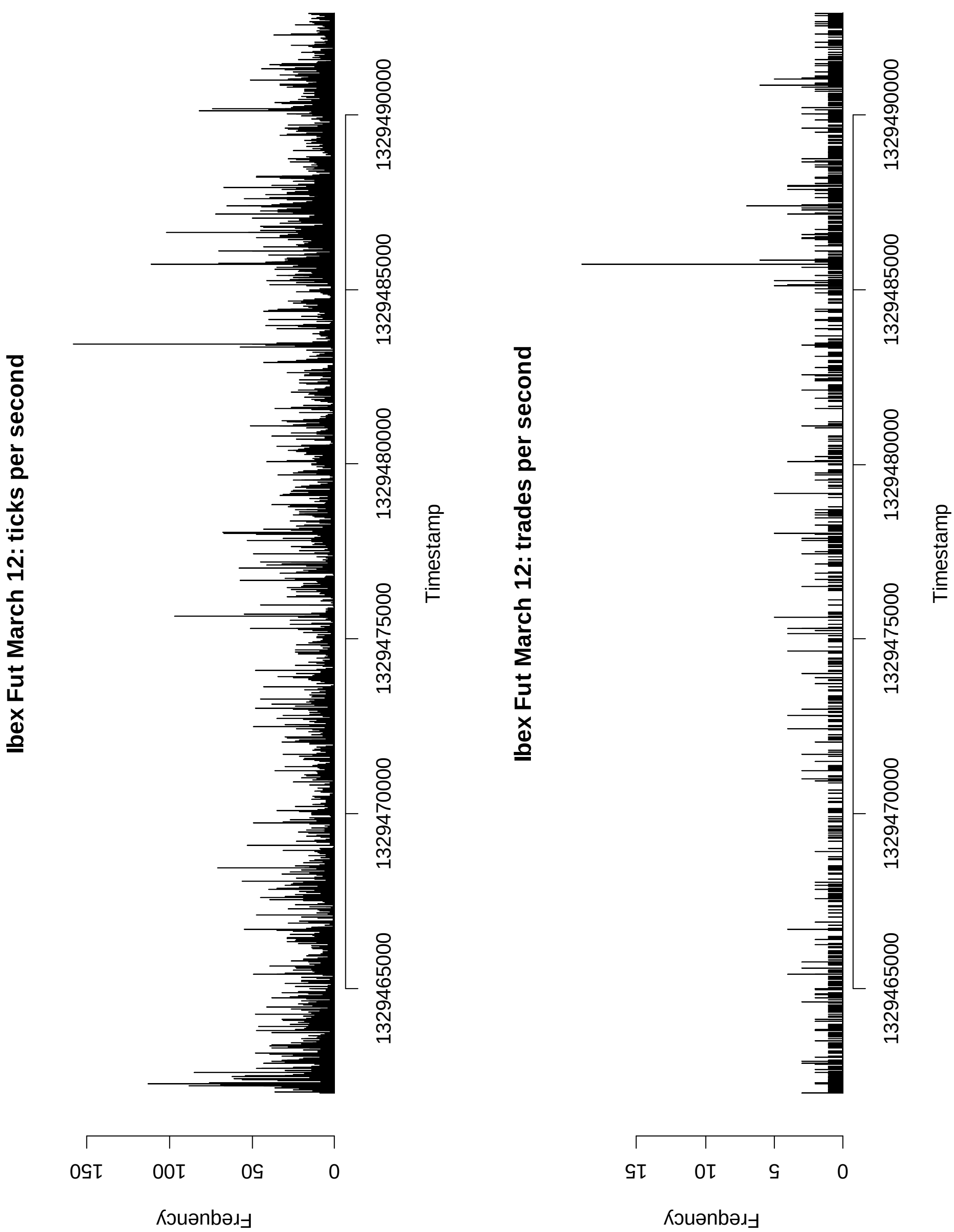

Figure 3.13: First visual approach to tick data.

2. The lower graph focuses on the number of trades per second. It can be seen this way that most of the trades occur isolated within the second where they are crossed. This gives us an idea about the nature of the flow for this specific IBEX future and helps us understand the average time horizon of the naked positions that our HFT strategy may imply. As said, during that time horizon the trader is 100% open to market risk. It also backs up our idea that sometimes below-second HFT refers to the quoting refresh instead of the actual trading. Further, the U-shape of the trading volume as explained above can be inferred through the sparseness of the data in the middle of the trading session and its accumulation in the extremes. A feature that can be more clearly seen in Figure 3.14. Finally, the total of trades observed in our data is 1,497, a figure that shall help the trader fix the target of profit per trade along with the expected average time frame pointed out above.

Depth interest

Follows the results of a simple experiment (a basic, first approach) on the quest to look for the depth of the order book whose equilibrium level closer resembles the trade levels occurred within the next 20 ticks[20]. Although volume weighted average price along both sides of the same level within the order book (not to confuse with the VWAP algorithm) is the filter used along the experiment other techniques such is particle filters could also be deployed.

Table 3.2 shows the mean quadratic error obtained through this approach in our data. Evidence is found regarding the industrial prior that it is the second level of the order book that one which embeds most of the price discovery information. Nonetheless, the second level is the one with the highest firm interest (a mean quoted size that more than doubles the size published in the first level) and, as such, also the most volatile volume-wise, i.e. the one that is updated more often. Note that the role of market impact makes us not to be able to state detailed assumptions about the order book dynamics since when the (typically shallow) first level is traded the second level becomes the first one[21].

Beyond the former stylized facts, the first and third level of the order book compete for the second role in price discovery being the first level more accurate the shorter the time required until the next trade. This is, a signal based on the information embedded in the first level of the order book would require more frequent updates than a signal based on the information of the third level – a relevant information to take into account when strategies have to be realistically adapted to the platform's constraints of the trading agent.

This analysis could ease the process of the large amount of information generated in the trading book. In fact, depending on the budget allocated to data management the trader could decide not to store data beyond the third level of the order book or, more importantly, not to take it into account in her strategy in order to speed the algorithms' reaction in the markets[22].

[20] When more than a trade occur within that time frame the closest (in time) trade is taken.
[21] And back to second in the presence of resiliency as explained in the Background.
[22] This is, taking shortcuts by processing less levels of the order book.

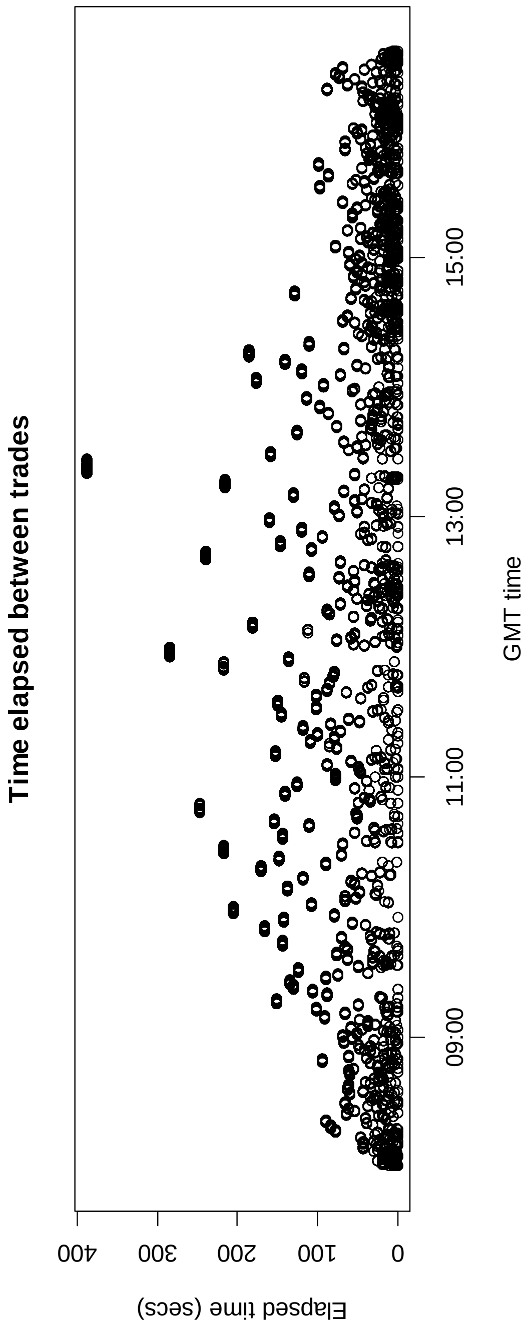

Figure 3.14: Distribution of time elapsed between trades in seconds.

	Level-1	Level-2	Level-3	Level-4	Level-5
Mean quadratic error	15.31	11.22	14.81	32.70	55.77
Standard deviation quoted size	50.20	62.03	32.26	15.82	15.22
Mean quoted size	111	231	98	56	42

Table 3.2: Comparison of candidates to the most relevant depth-level for price discovery.

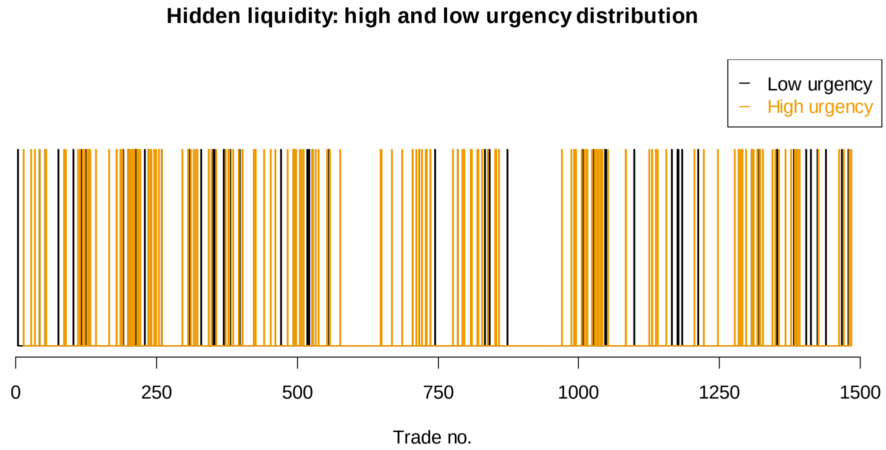

Figure 3.15: Hidden liquidity distribution along the observed trades (histogram).

Hidden liquidity

Is there further interest being hidden in the market? And if so, what is its nature? Orders of the type iceberg, fill-or-kill, etc can hide liquidity by not publishing it at the order book. However, by comparing the amount posted in a tick previous to a trade and the report of the trade being done at each level information about hidden liquidity can be brought to the foreground.

In the case where hidden liquidity is present, two are the core features of interest:

1. *Urgency*. – I calculated the level of urgency that is usually attached to it. This can be seen by identifying in which level of the order book it tends to be present. The closer to the first level the easier to benefit from it in algorithmic trading[23].

   Figure 3.15 shows how many of the trades occurred along our data sample benefited from hidden liquidity. *A priori* it can be seen that typically this type of interest is not punctual but remains instead along several trades. This persistent nature makes me question whether I can state that it is mostly used with a non-negligible dose of urgency.

   By comparing the trades with respect to the latest quotes in the order book (previous tick) and classifying the hidden liquidity discovered as high urgency if it was present within the best bid-offer spread, or low otherwise, I reach the two main conclusions – see coloured bars' distribution in figure 3.15. First, high urgency hidden liquidity is crossed more often than low

[23] Simply for being more easily tradable than those hidden at deeper levels of the order book.

	Min.	1^{st}Quartile	Median	Mean	3^{th} Quartile	Max.
High urgency	1	1	2	2.7	3	19
Low urgency	2	4	6	7.4	10	64

Table 3.3: Statistics for the number of units crossed with hidden liquidity: high and low urgency.

urgency – three times more often: while the former sums up to 169 trades the latter gathers 56. This is, most of the hidden liquidity is materialized within the bid-offer spread. And second, even though the crosses occur more often, the amount hidden within the bid-offer spread is lower than that hidden outside it, as seen in Table 3.3.

2. *Recurrence.* – I also tagged the type of appetite of the agents who hide their interest at each time. Is it random or recurrent in presence and side?

   Figure 3.15 already showed that most of trades related to hidden liquidity remain robustly along consecutive trades (i.e. hidden liquidity typically is not punctual interest but hides large amounts instead). This is, when hidden liquidity is spotted, the algorithmic trader can (statistically) count on being able to trade with a lower market impact along the next trades.

   However, is that liquidity being hidden in the same side or it does swing otherwise between bid and offer?[24] To answer this question a new benchmark is needed and this also triggers the need for further assumptions. By tagging each trade's aggressor as *buyer* or *seller* I can try to distinguish the side where the hidden-liquidity provider (if there's any) is willing to trade. I will consider that the aggressor of a trade was triggered by a buyer when it removed liquidity from the ask side of the order book. Similarly, I will consider that it was a seller when it removed liquidity from the bid side. For those cases where the trade was done within the best bid-offer spread I will consider that I don't have enough information to tag the trade and I will exclude it from the analysis.

   Figure 3.16 encloses the distribution of the hidden units traded along the day, now with the tags of Buyers and Sellers. The further existence of clusters of buy and sell side hidden liquidity can be noted. And this seems to back up in our sample the theory that it is the same agent who provides hidden-liquidity for a certain length of time. As said above, this fact is key to be able to improve the intelligence of the strategies of algorithmic trading.

Others

There is plenty of literature that targets the optimization of the execution strategy that could potentially be implemented industrially. Biais *et al.* (2002), Biais *et al.* (2000), Brunnermeir and Pedersen (2005), Carlin *et al.* (2007), Cespa and Foucault (2008), Chan and Lakonishok (1993), Chan and Lakonishok (1995), Chiyachantana *et al.* (2004), Engle and Ferstenberg (2007), Darrat *et al.* (2003), Cartea and Jaimungal (Forthcoming), Park and Van Roy (2012) and Fletcher *et al.* (2010) are just some examples of the large range of analyses from which the trader can reinforce her priors with regards to what can be more or less significant in her portfolio of strategies.

[24]There is only a pattern if the hidden liquidity's side is not randomly spread along the clusters noted above.

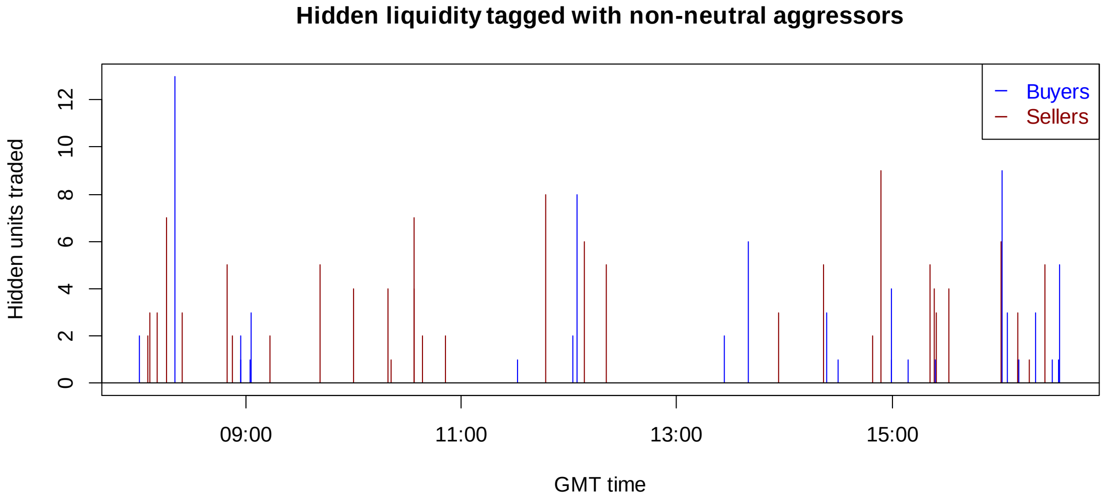

Figure 3.16: Distribution of hidden units traded across buyers and sellers.

3.4 Summary

The motivation of this chapter has been triggered by the need to find a robust estimation of the intraday volume's profile for the VWAP algorithm – its deviations will be appropriately used to approximate the slippage costs of executing through this algorithm in forthcoming experiments (meeting **objectives three, four and six**).

After justifying a challenge of the literature on popular advances of this algorithm the following conclusions are reached. ARMA-like structures are not optimal in my data set as they do not hold robustly across the sample. They are also further affected by the nature of the volume's profile, that among other specifications shall not be negative (trades-out and short-sales should be avoided). Moreover, PCA is not the best static structure either being it overcome by both the historical average and median and this motivates the use of a new estimation that mixes both through pattern detection. The Sample-Sensitivity Index (SSI) is this way proposed, an SVM index that allows the trader benefit from being less sensitive to the sample in those bins that may account for the largest noise through inter and intraday features. The application of this index generates positive results across all the members of IBEX 35 for the period considered.

Then, an analysis of tick data of the future of IBEX 35 is devoted to provide the trader with relevant information about the patterns present at a market microstructure level (**objective seven**). It is concluded that the second level of the order book seems to be the most informative in terms of price discovery. And that hidden liquidity seems to be robustly present in clusters along the trading session hence the potential benefit from including it into the intelligence of the algorithms.

Chapter 4

Quoting: a quoting model built upon particle swarm optimization

In this chapter the main inputs to be included into the initialization of the bid-offer spread in market making are addressed in a context where the classical full-replication is not a competitive strategy. In particular, it is considered the case where a client requires a sell side agent to guarantee a VWAP level on a large trade that can take several days to execute. I first take as benchmark the expected fair spread of a full-replication hedge. Then I try to beat it (objective two of the thesis) with an optimized hedge giving special emphasis to the incurred tracking error risk along with the unavoidable costs of trading into the proxy-hedge, out of the proxy and into the original instrument (from the exchange and broker fees to the execution slippage calculated in the previous chapter). In order to find the best candidate across the large universe of combinations and under the assumption that out-of-sample performance benefits from theory-driven approaches by reducing the overfitting issues, I propose Particle Swarm Optimization (PSO). It is a suitable candidate that allows directly applying regularization into the optimization process. Crucially, the heuristics upon the search-space generate an increase of the speed of optimization with respect to the non-regularized PSO (crucial for intraday trading) and allow us finding a (theoretically) robust solution. I also motivate the enhancement of the spread through the use of a different VaR measure, the flowVaR, that depends on the nature of the flow by keeping in mind that the Law of Large Numbers shall have an effect on the average tracking error priced-in by the market maker for any given level of prudence (objective four). Finally, I add StatArb information in case the trader is allowed to further enhance the bid-offer spread through proprietary strategies.

4.1 Research challenge

In this chapter I attempt to bring to the foreground what could not be explained by the literature so far: how the industry can be disruptively providing a decrease of up to 50% in the bid-offer spreads of certain products. And more especially, how can that be happening without simply assuming an increased risk appetite of the market participants. I start setting as a benchmark a full-replicating hedge, i.e. one that uses the strict definition of a product to hedge it completely. My target function is the bid-offer spread which simply is the summation of a series of variables that are often disregarded in the literature due to its complexity. First there is the market impact which requires a data sample of market impact of the strategy that the trader plans to use. I said, I will be using my enhanced version of VWAP from the previous chapter. As I do not have a data sample of its market impact I have to adapt a popular model to fulfill the idiosyncracy of my VWAP. There are the transaction costs within which I include the slippage (deviation from the target) of my execution model. Again, I will use the deviations from the previous experiment to approach this variable. And finally, there is naturally the bid-offer spread from the components of the hedge. Once the benchmark is setup I compare it with an optimized version. Being it optimized means that it weighs off the benefits from reducing market impact, transaction costs and bid-offer spreads against the assumption of further risk (the deviation produced for not following the exact composition). Such trade-off is non-differentiable as it includes discontinuous variables (such is a penalty for overnight risk). For that reason and because it is easy to add expert priors I use PSO as my optimization process. Finally, I include yet one more feature to the optimized version: the analysis of the activity of the market agent as opposed to the isolated analysis of its trades.

4.1.1 Market making strategies

Currently most of tier two market making agents (on indices, futures, ETFs...) remain providing quotes based on a full-replication basket bid-offer spread. This is, they typically make profits at a negligible risk by just passing the costs onto the clients (whether electronic or non-electronic) and charging a commission for it. However, those profits have been recently eroded with the evolution of electronic and quantitative market making. In fact, not only part of the competitors (typically, the tier one investment banks) benefit from cross venues of liquidity (the so-called dark pools) not available to the rest of the market makers but also new agents have surged and specialized in providing tighter bid-offer spreads through the use of proprietary strategies beyond full-replication.

These strategies swing between two extremes:

1. *Naked approaches:* this is, non-hedged, typically involvolving large risk and large rewards for being momentum strategies largely deployed through inventory management policies that target zero end-of-day inventory.

2. *Optimized-hedge approaches:* that typically rely on mean-reverting strategies through the use of cointegrated hedges enhanced in different dimensions as we will see below. Usually, these strategies account for less risk and less reward than the naked ones. Hence in order to build up a large figure of profits they focus on systematic trading to grow its notional performance. That is achieved through both the capacity to apply the same trading structure to a wider range of securities and the capacity to trade more frequently (giving origin to HFT). The more symmetric the distribution of the flow across both sides (buys and sells) the easier it would be for the trader not to need to trade-out of an interim hedge and into the full-replicating basket. This enhances again the role of inventory management and reduces the costs and market impact.

4.1.2 Core features in market making research

The spread quoted by the market makers has to take into account several features to be priced-in through a three-fold analysis:

- *Market impact.* – the quote of the market maker has to include the market impact implied by the size that it attaches. Sizes included in the order book tend to be both growing with the quote and more biased towards full-replication the deeper the quote is into the order book. On the other side while the most aggressive quotes tend to attach lower sizes they also tend to account for strategies that attempt to minimize the market impact. Timing and the type of the order (whether market or limit) are crucial to manage the expected market impact and with it try to improve the bid-offer spreads within the order book. Beyond-order-book sizes (large trades as in client-driven flow) also rely on market microstructure through algorithmic trading analysis (at any level of aggressiveness) in order to prudently model the market impact implied by the possible trade.

- *Transaction costs.* – they tend to be a key competitive advantage that decides which agents can account for the highest risk/reward ratios. Market makers, for instance tend to have zero commissions in the exchanges they are members of. And some exchanges do even pay them a *rebate* (part of the fee charged to the liquidity-takers) for the liquidity they post. This tends to be highly disregarded by the literature hence generating many virtually profitable strategies with no real interest in the industry. And even though at a first instance they seem easy to account for there are some grey areas that typically require the help of specialized lawyers such are the effect on the conversion of depositary receipts (DRs) of certain taxes such was the one charged by the Brazilian government (the so-called Lula's tax) to the purchases of the local currency Brazilian Real; or stamp duties like the one charged by the Irish Stock Exchange to the purchases of the stocks. Other features that can affect the transactions costs, in this case positively, and that typically are not mentioned by the literature are certain rebates such are the tax rebate granted in the *dividends arbitrage* or the returns from the equity finance management of the stocks being market made (from borrow fees if they are lent away to the funding discounts when those are used as general collateral). Also relevant is to account for the slippage as execution does not always coincide with the expected/theoretical fills.

- *Tracking error risk.* – when the trader uses an optimized hedge to reduce the market impact and transaction costs, the deviation between the optimized hedge and the instrument becomes its main source of risk. The TE, ϵ,

$$\epsilon = \sqrt{\mathrm{var}(r_t^{\mathrm{p}} - r_t^{\mathrm{h}})} = \sqrt{\mathrm{var}(u_t)} \tag{4.1.1}$$

  is the variable typically used to resume such risk. The rationale behind the usage of an optimized alternative is built upon the fact that if the performance of the residual[1], u_t,

[1] Note that I also refer to this residual as the deviation risk.

between the return of the instrument being market made, r_t^{p}, and the return of its hedge, r_t^{h}, is stationary around a mean it involves that in the limit if such mean is included in the quoted spread the risk to the market maker is zero on average. This way it is possible to benefit from decreases of market impact[2] and transaction costs.

4.2 Methodology

The methodology considered for this experiment meets the following rationale:

1. If I wanted to find a way to decrease by 50% the previous levels of bid-offer spread I had to survey across the industry the standard procedure in market making and take it as a benchmark.

   (a) If I found different ways to be standard I would select the most generic as my benchmark. This is, if, as expected, the more the market impact of the size to be market made the more the difficulty to calculate the bid-offer spread I would select an experiment based on a large trade.

2. I will consider the role of science to improve the levels of the benchmark. Usually, optimization implies entering into a hedge that has pros (expectedly less market impact and/or transactions costs) at the price of entering into more risk. That trade-off shall be optimized ideally through a flexible technique that allows me changing the model and adding discrete jumps if needed (i.e. non-differentiable target functions).

   (a) I hence have to motivate the usage of the selected optimization technique and, if possible, simplify it as much as possible based on expert insights.

   (b) The solution that beats the benchmark bid-offer spread cannot be based on the change of the trader's risk appetite. There must be a different reason that allows enhancing the bid-spread without changing the level of prudency of the trader.

   (c) If the bid-offer spread obtained this way cannot yet beat the benchmark I shall look for the logical features that would allow such scientific approach to beat the benchmark.

4.2.1 Data description

The trader has then to deploy market impact and transaction costs analyses. And contrast those with a series of hedging proxies that can provide some synthetic liquidity (based on cointegration) at the price of entering into tracking error (risk) management. Also, since this type of trades is not frequent the trader needs to decide which distribution of risk should be quoted in the book. This is done in order not to enter into highly volatile deviations but bearing in mind that that

[2]Note that there is no certainty about the market impact but also likeliness instead since, as we will see, it has to be modelled.

trade shall not be lost, i.e. the aggressiveness of the price has to be weighed-off against the risks it involves.

Follows a typical case where the trader decides to price interim deviation risk by choosing the hedge with *a priori* the most robust out-of-sample expected performance and least extreme-value risk.

Proposed scenario for the experiment

I have decided to keep my focus on IBEX-related products for to remain consistent with the results obtained from the previous chapter. Gathering its underlying such a level of capitalization and being it a basket of 35 stocks, in order to account for the benefits of an optimized hedge that reduces market impact and transactions costs by optimally increasing tracking error as explained above I have to assume a large client-driven flow, a so-called *elephant trade*. The same results would apply to smaller trades on less liquid instruments and/or shorter baskets[3].

When referring to elephant trades a frequent series of characteristics around them have to be cited. First, they typically involve risk trades. Risk trades, as opposed to agency trades, require the trader to commit to a certain execution level before knowing whether there will be slippage or not – this is, the risk of slippage is transferred from the investor to the market maker. Second, and as a consequence of the former, elephant risk trades typically are part of a cross-sales strategy across products of the trading floor, i.e. it is considered a service to tier one clients given the risk that the portfolio manager is taking. Nevertheless, risk trades' performance tends to be tracked in order to ask clients to cover the risk transferred with more agency trades (as said, seeking commissions at low risk). Third, elephant risk trades tend to be followed by sharp movements in the market against the market maker – client tends to have privileged information about the fair value of the original security with respect to the one held by the market maker. Last, elephant trades imply inelastic-flow to the market maker.

In this case it will be considered that the client wants the market maker to commit to fill an execution on IBEX, let's say through an index swap, an ETF or a future, with the mid-price of the screens on a notional of 2 billion EUR. For ease of the calculations I will assume that the client calls the trading desk early in the morning, right before the opening auction so that the first trading day is a complete one. In general it will be assumed that the market maker considers that trading up to 1/3 of the daily volume generates assumable market impact but not beyond that percentage. This is, for larger trades a typical full-replication algorithm of VWAP won't be enough to minimize the market impact and surges the need to deploy an optimized hedge – i.e. note that the algorithm will still be used for intraday execution trading. The challenge is to decide what to trade instead of the full-replicating basket and how to measure the market impact, costs and deviation risk. As said, the presence of inelastic-flow implies the need for an interim optimized hedge. This is, the trader will enter as much as possible into the full replicating hedge through

[3] Moreover, these trades tend to imply less complexity than the current experiment hence I will take the latter as an appropriate framework to analyse how to overcome most of the challenges in market making.

VWAP and the rest into an optimized interim again through VWAP to later trade out smoothly of such hedge at the same time it enters into the full-replicating one.

Finally, it will be considered that in order to define the optimized basket the trader adds to the full set of stocks of IBEX a satellite that allows releasing market impact while entering into diversified risk, a liquid instrument such is for instance, given its geographical proximity, the CAC40 ETF which also accounts for the same FX exposure. ETFs unlike futures allow the trader to avoid the expiration roll-overs and margin requirements and unlike mutual funds they provide intraday exchangeability as individual stocks. Moreover, they are convenient as they take care of other issues such are cross-currencies, corporate actions, custody of the underlying securities, etc.

Data

For this experiment I will analyse a database composed by the 1-minute evolution of the IBEX35's components along with the LYXOR's ETF for CAC40 from the October the 10th, 2007 until March the 12th, 2012. At that level of granularity, information of bids, asks and volumes will be processed towards the calculation of new (index weighted average) variables.

Also, the industry sector[4] to which each stock belongs and their distribution across indices and optimized hedge will play a relevant role in our calculations. As such, given the fact that CAC40 comprehends several sectors the ETF itself will be considered a new sector.

4.2.2 Bid-offer spread initialization following full-replication

The main challenges when trying to identify the full-replication bid-offer spread for sizes beyond those enclosed in the order book are the identification of a right market impact[5] and a complete set of transaction costs.

Market impact estimation

As explained in the previous chapter VWAP is an algorithm suited for market impact minimization. Such algorithm will be used throughout the executions estimations of this experiment leaving this way the trader with a new issue: while there is financial literature about the market impact of different proprietary trading strategies (typically more aggressive than the VWAP) there is not much information about the market impact of a VWAP algorithm itself along time – and this requires pre and post-trade analyses.

[4]BICS (Bloomberg Industry Classification System) will be considered to classify each security based on its business or economic function and characteristics.

[5]Its calculation for smaller amounts is mostly immediate in the case where I know the interest on each security at the different levels of the order book. By assuming that only market orders are sent the final execution fill would be the one resulted from wiping the order book off. See Kissell and Glantz (2003) for a market impact calculation for a portfolio.

Amongst the most popular literature on this topic, Almgren *et al.* (2005) analyse the previously proposed break down of market impact into[6]:

1. a permanent market impact: component that reflects the buy/sell imbalance information that has been sent to the market participants, to be modeled through a so-called function $g(v)$, where v indicates the trade rate in number of shares to be executed, X, over the desired time horizon, T, this is $v = X/T$. And,
2. a temporary market impact: component that reflects the temporary price concession in order to attract interest within T, to be modeled through a function $h(v)$.

I especially care about the permanent effect in the prices as that is the one that will affect our experiment the most in terms of initial deviation.

Their model postulates, that the price of the security, $S(\tau)$, follows an arithmetic Brownian motion:

$$dS = S_0 g(v) d\tau + S_0 \sigma dB, \tag{4.2.1}$$

where $B(\tau)$ is a standard Brownian motion. And set the structure of the function to be *power law* of the form:

$$g(v) = \pm\gamma|v|^{\alpha} \tag{4.2.2}$$

leading finally to the expression:

$$g(v) = \gamma\sigma\frac{X}{V}\left(\frac{\Theta}{V}\right)^{1/4} + \text{noise}, \tag{4.2.3}$$

where V is the average daily volume in shares, Θ is the total number of shares outstanding and γ was set to 0.314 as per their data set.

This expression could potentially allow us calculate the market impact's component on which I am interested. However, there are two features to be taken into account:

1. The parameters of Almgren *et al.* (2005) are calibrated to a data set of US stocks traded by Citigroup between 2001 and 2003 through a data sample where market orders, limit orders or VWAPs are not differentiated (i.e. it mixes linear and non-linear executions) and the largest

[6]See Park and Van Roy (2012) for more advanced versions of adaptive execution.

participation on the average daily volume (%ADV) accounted for was 10%. This implies that their parameters may not be universal enough to be directly applied onto our data. The model can still be used as a reference of average permanent market impact (the one I am keener on in large trades) per level of %ADV but requires some conservative parameterization.

2. On top, this model does not include the intraday seasonality observed in our previous example and I expect the permanent market impact to depend on it. However, given that our previously generated VWAP already accounts for the intraday convexity of the volume profile[7] I assume that each bin sent to the market by the algorithm does have the same market impact – allowing this way for linear projections onto the benchmark.

As said, I can still reference the market impact of our VWAP to the permanent effect model of Almgren *et al.* (2005) by projecting a benchmark of market impact per level of %ADV in each stock. Note that the time dimension will be appropriately incorporated by using in the definition of the %ADV the volume traded on average along the rest of the trading session instead of along the whole trading session. Finally, I take as representative in our IBEX data sample a company with an *inverse turnover*, $\frac{\Theta}{V}$, of 500 and a daily volatility of 4, and an extreme situation where the trader needs to trade 100% of the ADV along the trading session[8]. Under those conditions I obtain an average permanent market impact for the VWAP algorithm of:

$$g(1) = 0.314(4)^{1/2}(500)^{1/4}$$

$$g(1) = 2.97$$

As a result, weighted average market impact is calculated towards the comparison of industry baskets, index and hedges.

Transaction costs to account for

Follows the list of transaction costs that have been considered in the experiment:

- *Commissions.* – the financial instruments considered in the experiment trade in Bolsas y Mercados Españoles (SIBE) and NYSE Euronext Paris. I will assume that the commissions to be paid in order to trade in those markets through VWAP are 10bps – while it is true that the market maker would typically not need to pay for them the client would and as such these services have to still be charged in the presence of client-driven flow. I will refer to these costs in the model with a C.

- *Rebates.* – there are no rebates to profit from in neither of the considered markets hence they won't be used in the model.

[7] It was precisely defined to generate an even market impact along the trading session so that I can think of its market impact in a linear manner.

[8] By taking an extreme benchmark I enhance my prudency in terms of market making.

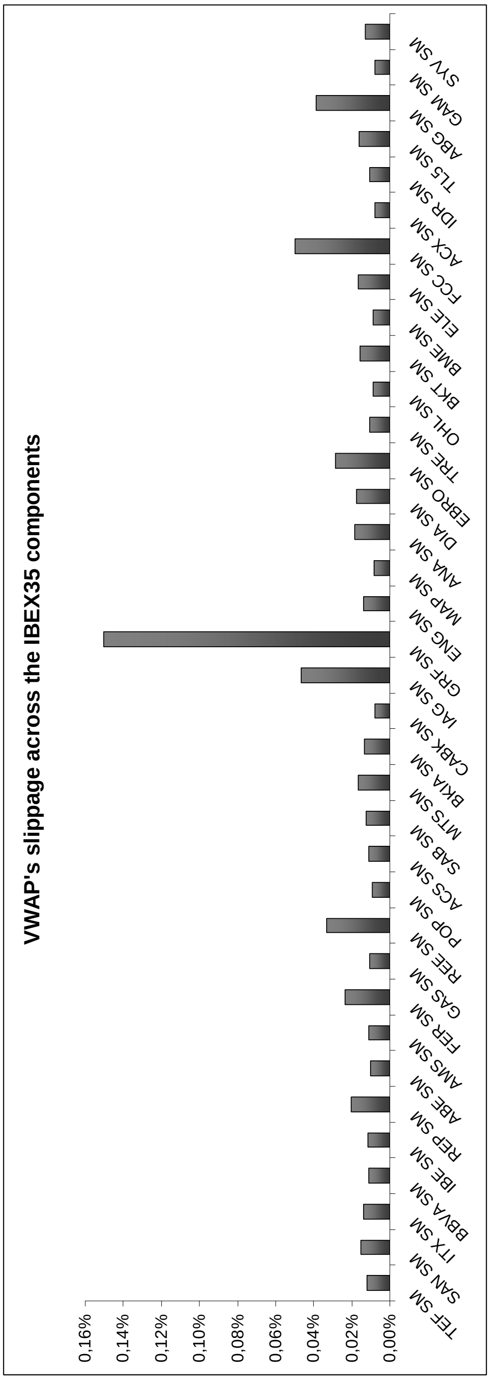

Figure 4.1: Distribution of slippage values assumed for the VWAP execution on the IBEX35 members.

- *Borrow rates.* – even though the majority of the names is considered general collateral which would have value to the Equity Finance desk these rates are negligible within the interim hedge as its expected time horizon is less than a week and this typically has little value. Hence these won't be used in the model either.
- *Slippage.* – as explained in the previous chapter it is not easy to accurately achieve the VWAP within a certain time frame through an algorithm. I will use here the deviations from the volume's profile as a prudent approximation to the VWAP's slippage. Since I already know that the effect of the prices smooth dynamics typically reduce them on average I will target to keep the same distribution by simply scaling them into basis points as seen in Figure 4.1. I will refer to the slippage in the model with an S.
- *Components' bid-offer spread.* – The first level of the order book tends to be the first charge to access an instrument (when market orders are used instead of limit ones). Moreover, it tends to be the first signal of the liquidity of a stock (and as such, its flow elasticity). For this experiment I have used the average bid-offer spread along the sample. Figure 4.2 shows the distribution of bid-offer spreads across the different members. I will refer to the bid-offer spread in the model with a ς.

Overnight risk penalty

On top of the above weighted average market impact and weighted average transaction costs, further risk has to be priced-in. Being this approach full-replication it could seem at a first instance that there are no further features involved however given the fact that it has been decided that there is a market impact limitation through the maximum %ADV to be traded within a trading session of 1/3. If more is required, the trader may consider to add 10bps more for overnight risk.

I am adding this way a discontinuity in the residuals that affects from the second trading day onwards. I will include it within the category of market impact from here on. Hence:

$$g(1) = 2.97 + 0.1\%\Theta$$

where $\Theta = \{0, 1\}$ dependent on whether the risk is held overnight risk or not.

Results

Figure 4.1 encloses an analysis of the full-replicating hedge that would involve the future of IBEX 35 under the previously defined framework for the experiment. Columns correspond to the weighted average of the cost across the components of the hedge ($\tilde{C}$), the weighted average slippage of its execution via VWAP ($\tilde{S}$), the weighted average market impact ($\tilde{g}$), the weighted average bid-offer spread of its components ($\tilde{\varsigma}$) and the proposed spread of the instrument with respect to the mid level of the order book (ϕ) where simply:

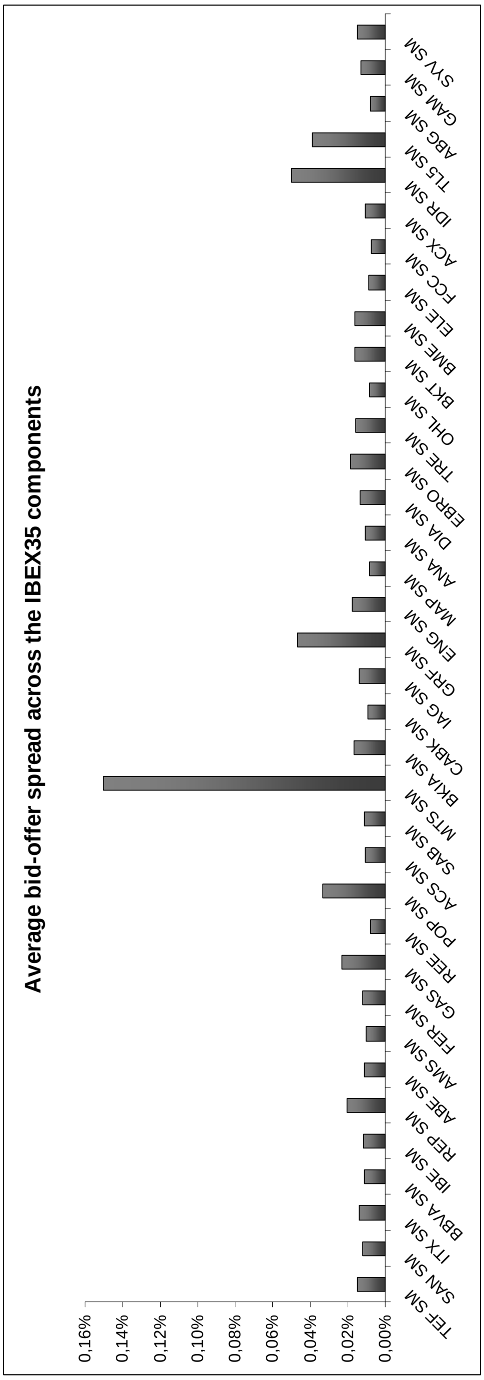

Figure 4.2: Distribution of average bid-offer spreads across the IBEX35 members.

$\tilde{C}$	$\tilde{S}$	$\tilde{g}$	$\tilde{\varsigma}$	ϕ
0.1%	0.01%	1.54%	0.07%	1.72%

Table 4.1: Analysis of the full-replicating hedge.

$$\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma}$$

Note that the curly lines represent the weighted average nature of those components. I shall now compare this figure with the ones obtained using first an optimized hedge that allows for deviation risk and then taking into account the nature of the flow.

Summary

In this section I have described in depth the way a traditional market maker would price-in the different components that may affect her full-replication hedge, exercising prudence when accurate levels are not available.

Under these criteria, the non-linear market impact model from Algram *et al.* (2005) has been used to generate a reference for the permanent effect that a 100% ADV trade would have had along a whole trading session. As VWAP already accounts for the non-linearity of the intraday volume's shape I expect a linear projection of the reference to be a fair candidate to estimate the market impact of the execution algorithm.

The transaction costs for the model have been identified and the execution risk from the previous chapter has been borrowed to account for the slippage of the algorithm.

Finally, by simply aggregating the different components thoroughly explained above and shown in Table 4.1 I have concluded that the full-replicating market maker would post a quote that would deviate from the mid quote reflected in the order book by at least:

$$\phi = 0.1\% + 0.01\% + 1.54\% + 0.07\%$$

$$\phi = 1.72$$

4.2.3 Bid-offer spread initialization following StatArb

This section considers the use of an interim hedge that cannot be immediately closed away through internalization given that, as assumed, the flow is inelastic. If that is the case the trader has two options: to keep the hedge until the client decides to trade out of her position (i.e. assuming a highly volatile time horizon) or to trade out of the hedge and into the full-replication basket and simply consider the next trade of the client as a different one. I will assume that the trader decides to take the latter option for being it the most restrictive of the two, the one that makes most difficult the quest to find a better solution than the full-replication hedge[9].

The trader has this way to enter into as much of the full-replicating hedge as possible within the first day leaving the rest for the optimal non-full replicating hedge. Then, leave smoothly the non-full replicating hedge paired with an entry onto the full replication, until the hedge is simply the full replicating one, i.e. the instrument itself. The expected market impact and transaction costs incurred throughout the hedging process shall be included into the bid-offer spread charged to the client as explained above. But now, there is a new feature: the deviation risk has to be also accounted for along with the tail risk of the deviations.

Deviations: tracking error and prudence along the extreme payoffs

Non-full replication involves by definition the assumption of deviation risk that as such shall be dealt with a certain level of prudence. The challenge is to decide how to price that risk into the bid-offer spread since the higher the prudence the larger the spread and hence the more difficult it will be for the trader to be the one finally chosen by the market to cross interests.

I will consider that the market maker is highly risk averse and wants to be protected against large losses even if they are unlikely to happen. In financial theory it means that the market maker wants to be (statistically) hedged for significantly large tail risk. Under that assumption, I can expect to shift the level of the bid-offer on each proposed hedge by its VAR^{99}, i.e. the market maker has decided to charge for the maximum loss produced along the back-tested deviations[10] with a $p = 99$ percentage of probability. In my models I will refer to this measure as $\varrho(99)$.

As Normal distributions are often criticised in the industry for not accounting accurately for tail-risk[11] I have considered the use of Student's t (Alexander (2008)). Moreover, when the risk distribution is symmetric and its tails are roughly equal, the conditional-t method to calculate VaR can be a good alternative to (the more complex) conditional extreme value theory approach as assessed by Fernandez (2003) easing this way my analysis of the tails[12].

Diez *et al.* (2012) present a Pareto front proposal to let the trader choose across baskets of candidates that optimize the relationship between tracking error risk and market impact and

[9] In fact, typically the target is to reduce as much as possible the bid-offer spread of an instrument in order to be able to apply the former approach. By doing so it is more likely that the flow resembles elasticity features being hence possible to benefit from a closer and less volatile time horizon – i.e. more independent of each individual client and built upon a portfolio of clients instead. This allows classical statistics to be used with less constraints.

[10] For its calculation, stocks with less than 4 years of history have been removed from the data sample and their weight has been spread across the rest of the names – i.e. a rebalanced IBEX has been used throughout the back-testing. The names excluded are: Amadeous IT Holding, Bankia, International Consolidated Airlines Group and Distribuidora Internacional de Alimentación.

[11] The Flash Crash is said to be one of the examples of almost impossible scenario under a Gaussian distribution that however, occur.

[12] Note that when LLN applies a Student's t converges into a Gaussian hence, in the limit both could be used indistinctly. However, the application of the LLN along the *flowVaR* following the expression defined above is more prudent than the use of the average distribution of Gaussian variables. More so, the lower the number of expected trades, η. The Gaussian approach would apply LLN by decreasing the volatility of the distribution σ, at a decaying rate of $\frac{\sigma}{\eta^2}$. Such an approach would this way render more aggressive bid-offer spreads than the one is in the experiment in those levels of η.

transaction costs using multiobjective particle swarm optimization. This experiment not only expands the financial complexity of the problem but more importantly, as opposed to the mean it focuses on the tails of the deviation risk.

Bid-offer initialization through Particle Swarm Optimization

The universe of combinations of stocks that could potentially be better than the full-replicating basket in terms of bid-offer spread, **my target function**, is large enough to consider the need for an optimizing algorithm in the calibration process. Amongst the different possibilities Particle Swarm Optimization (PSO) has been considered a good candidate for non-linear optimization not only because it allows to deal with non differentiable problems (unlike gradient descent and quasi-newton methods) but more especially because it allows to easily input heuristics into the search-space boundaries. The introduction of experts' heuristics generate a theory-driven-like calibration which, by imposing restrictions in the hedging candidates, is expected to deliver more robust results and faster than a completely data-driven calibration. In this case, I will reduce the length of the search-space by targeting only movements across industry distributions (as opposed to distributions of stocks). And then, I will reverse-engineer the hedge that could have delivered such a distribution by treating each industry as an equal-liquidity customized basket. This is, on each industry I will distribute the weight so that each stock to be traded accounts for the same %ADV.

Also, in order to grant the hedge to have more access to diversified risk with low market impact I add the already mentioned Lyxor's ETF of CAC 40. Even though it has the same FX exposure it overall adds country risk and a different industry diversification to the one of IBEX. In that sense it adds what will be called the "Diversified" industry – i.e. for the purpose of our algorithm dynamics the ETF will be treated as a different industry itself, being it expected to be transferred weight from the most illiquid stocks (more so the larger the participation of their industry in the ETF).

Follows a description of the model used towards the optimization of the hedge. As mentioned, this is just the initialization of the bid-offer spread but it should nevertheless coincide with the average bid-offer quoted along time. In other words, punctual deviations would typically occur especially in the presence of elastic-flow due to the different strategies of inventory management or sudden interest shifts on the product market made as mentioned above.

Ten particles are used, the original basket along with nine more, the latter as a result of a random change of the original's basket industry distribution as per PSO's framework. Each particle's position, x_{pi}, is initialized through the following steps:

1. *Calculate transaction costs ($\tilde{C} + \tilde{S} + \tilde{\varsigma}$).*

   Get weighted average commissions, slippage (based on Chapter 3 results) and bid-offer spread as explained above.

2. *Calculate market impact ($\tilde{g}$).*

(a) Calculate time horizon of the candidate given the least liquid industry[13]. I take the largest of the %ADV per industry and I divide it by the maximum ADV allowed per day (in this case, 1/3) in order to calculate it.

(b) Obtain the exact hedge by reverse-engineering the industry weights as mentioned above (equal liquidity customized baskets in the seek for an even market impact across the components of each sector).

(c) Project linearly our VWAP's permanent market impact reference per day of trading volume.

(d) Overnight risk is charged with 10 bps – the less liquid candidates are penalized non-linearly if they increase the time horizon.

3. *Calculate the first momentum of the deviations' distribution and the tail risk ($\varrho(99)$).*

(a) Generate the deviations of the OLS regression, u_t, of the instrument against the candidate hedge, the number of units of the hedge and the tracking error that it generates are first calculated.

(b) Fit the deviation series, u_t, into a Student's t. The estimation of its degrees of freedom is based on Venables and Ripley (2002) methodology.

(c) Calculate mean, μ and $\varrho(99)$ upon the fitted distribution for the instrument.

(d) Check stationary level through Augmented Dickey-Fuller. If the c-value for a 90% interval confidence is larger than the test statistic for the fitted distribution, i.e. if the series is non-stationary, add 1% of the deviation to the bid-offer spread[14].

(e) Fit an Ornstein-Uhlenbeck process to the deviations in order to provide the trader with further information about its mean-reverting structure for a subsequent proprietary enhancement. Following the expression explained in 2.3.2:

$$du_t = \theta(\mu - u_t)dt + \sigma dW_t, \tag{4.2.4}$$

4. *Sum the figures obtained in 1), 2) and 3) to calculate the value of the target function: the mid bid-offer spread.*

(a) $\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \varrho(99)$ where the trader aggregates the weighted average of the cost ($\tilde{C}$), of the slippage ($\tilde{S}$), of the market impact ($\tilde{g}$), and of the implied bid-offer ($\tilde{\varsigma}$); then, the risk embedded in the tracking error is priced in through the mean of the deviation (μ) and the tail's risk at a 99% confidence interval ($\varrho(99)$).

Once the particles have been initialized they would start analysing the search-space following the PSO dynamics described in subsection 2.3.4:

[13] Note that it should then be more affected by the 10 bps penalization for overnight risk than in the full-replication case.

[14] This is simply a way to give priority to cointegrated hedges. Note also that it makes the optimization problem non-differentiable on this dimension.

$$v_{pi} = \omega v_{pi} + \varphi_l r_l (p_{pi} - x_{pi}) + \varphi_g r_g (g_i - x_{pi}) \tag{4.2.5}$$

Each of the candidate's, p, movement, v_{pi}, from any position, x_{pi}, is both influenced by its local/individual best known state (minimum mid bid-offer spread), p_{pi}, and by the global/across particles best known state in the search-space, g_i, which are consecutively updated as better positions are found by other particles. As said, the dimensions to be considered are the industry weights, i, of the hedge. And this also allows me to improve the pace at which I can find an optimal solution (a more valuable feature the higher the frequency of the strategy). As said, r_l and r_g are random numbers. The vector of parameters that defines the velocity at which the particles move towards the local and global states, (ω,φ_l,φ_g), can also be changed to increase the pace of convergence of my PSO. However, that would reduce the breadth of the scan of the search-space hence it was set to (1, 2, 2) as in Kennedy *et al.* (2001).

The 10 particles will keep iterating the process until convergence is achieved. In this context I will consider that the process has converged when the bid-offer spread has not improved more than 1bp in the last ten iterations.

Results

Due to the use of a theory-driven approach and the restriction of the dimensions from the number of stocks to the number of industries, convergence is achieved within 7 iterations. The algorithm evolved as follows: it started detecting as the leader the 8^{th} particle of the first iteration which obtained a $\phi = 4.6\%$, then it kept moving randomly the particles' industry weights (dimensions) and found that the 3^{rd} particle did output a $\phi = 4.3\%$. Finally, it selected the 6^{th} particle in its third iteration that could not improve for more than 1bp during the following ten iterations. Hence the final value: $\phi = 3.85\%$.

As a result the candidate strategy for the interim hedge accounts for the features enclosed in Table 4.2, where columns correspond to the aforementioned weighted average of the cost ($\tilde{C}$), of the slippage ($\tilde{S}$), of the market impact ($\tilde{g}$), and of the implied bid-offer ($\tilde{\varsigma}$); then, there is the mean of the hedge's deviation (μ), the tail's risk at a $p = 99$ percentage confidence interval ($\varrho(99)$). Finally, in the rightmost columne there is the proposed spread respect to the mid level of the order book (ϕ). Which are the result of transferring weight to the CAC's ETF given that it accounts for a large liquidity and a diversified industry distribution that can provide the basket with low deviation risk and low market impact. This can be seen in Table 4.3, where columns correspond to: Industrial (I), Basic Materials (II), Energy (III), Utilities (IV), Consumer, Non-cyclical (V), Consumer, Cyclical (VI), Financial (VII), Technology (VIII), Communications (IX), Diversified (X) and CAC40's ETF (ETF).

So, in the end, it finally seems that our non-full replicating hedge is not enough to overcome the full-replicating basket at a 99% interval confidence for the scenario considered.

$\tilde{C}$	$\tilde{S}$	$\tilde{g}$	$\tilde{\varsigma}$	μ	$\varrho(99)$	ϕ
0.1%	0.04%	0.8%	0.01%	0.01%	2.87%	3.85%

Table 4.2: Analysis of the interim hedge.

	I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**ETF**
IBEX 35	6%	2%	8%	13%	5%	11%	33%	3%	19%	-	-
CAC 40	16%	7%	15%	5%	25%	5%	16%	1%	7%	6%	-
Hedge	12%	16%	9%	7%	2%	4%	2%	4%	1%	-	44%

Table 4.3: Weights' distribution across industries.

Summary

In this section I have thoroughly explained the strategy used by the trader to price into the bid-offer spread, 2ϕ, the different costs, market impact and risk implied in a non-full replication strategy that could in turn benefit from cointegrated, liquid instruments. Also, I have shown how the hedging candidate's components can be calibrated through a Particle Swarm Optimization process whose search-space is constrained by a series of theory-driven assumptions that allow the trader benefit from an *a priori* more robust out-of-sample results (given the fact that it follows a theory-driven process) as well as from a faster optimization process than a (raw) greedy approach.

Having stated a challenging scenario for optimization it is not surprising that for a $\varrho(99)$ I have not been able to find a better solution than the full replication as now $\phi = 3.85\%$. So, a risk-averse trader that accounts for advanced optimization models may still decide to apply a full-replication hedge.

Let's assume now that the rest of market makers do set their prudence at the same VaR p-level (99%). Would it be possible to find one of them consistently quoting more aggressive levels than the rest as it seems to happen in the markets?

I will try now to involve the analysis of the flow in a further enhancement of the spread and see whether above statement is feasible.

4.2.4 Enhanced initialization through the analysis of the flow

This section proposes a novel way to exploit the agent's insights about the flow. Not only a formal data analysis can be used to scan the nature of the flow in electronic-driven trades but the knowledge of the sales force can also be utilized in the definition of the nature of the client-driven flow.

Deviation risk: prudence, extreme risks and the LLN

A key aspect of sell-side trading that I have not seen considered in the literature so far is the fact that per every inflow there typically is a later outflow and vice-versa. This subtle difference has large effects on the deviation tail-risk that the trader faces. Typically, I would expect the trader to price in the tracking error[15] that her hedging strategy may imply. However, the lower the number of trades the trader is able to market make the more difficult it will be to assume that the Law of the Large Numbers (LLN) may apply. As such, Extreme Value Theory (EVT) shall be used instead. As said above, value-at-risk (VaR), a measure created by JP Morgan to account for tail risk could be an appropriate candidate for the trader to price in the consequences of the lack of consistent flow. The use of such measure could well generate high bid-offer spreads that could take the trader off the market. So, how can the market maker remain aggressive without violating the risk policy of her agency? There is no need to assume more risk to tighten the spread, it should be enough to try to understand better the nature of the flow.

At this point I could borrow from microeconomics the concept of elasticity through the so-called *Elasticity of the Flow* that has been previously defined in the Background. Analysing whether the trader can affect the inflows or outflows by being asymmetrically aggressive around the fair price a further improvement of the bid-offer can be granted based on the possibility for the application of the LLN.

I consider that the recurrent flow provides market makers with a similar feature to a dark pool, where they can trade away a good part of their positions through a liquidity pool to which they have a privileged access. This is, market makers do have extra-liquidity around the fair price with respect to the one estimated with full-replication only or even the previously optimized hedge. More importantly, it allows traders compensate a negative outlier by favouring more trades so that the LLN can be applied.

The *Elasticity of the Flow* and the *flowVaR*

It seems reasonable to think that the effect of the application of the LLN within market making shall not be discrete but gradually apply depending on the number of trades linked to the flow instead. There are several ways to gradually distribute the benefits from the LLN within the bid-offer spread (linearly or not) and I won't discuss herein which one is best as in the end it crucially depends on the trader's utility function.

Under this interpretation, it starts being apparent why HFT naturally accounts for the tightest bid-offer spreads in a robust manner: because the larger the *Elasticity of the Flow* (and the more popular and vanilla a product is the more elastic it becomes) the more naturally the LLN applies.

I will propose a further extension of the VaR measure to account for the *Elasticity of the Flow* through the application of the LLN by the trader[16]. Let's assume that from ϑ trades onwards I can apply the LLN. This means that the trader can safely avoid the VaR penalization in the bid-offer spread. This is, a mapping function between the VaR and the *flowVaR* can be defined:

[15] Zero, on a cointegrated series.

[16] Note that it is not the first time that VaR extensions are proposed. Almgren and Chriss (1999) derive a new measure of risk, the so called liquidity-adjusted value-at-risk, to account for both the liquidity risk on the execution of a trade and its optimal trading strategy.

$$\tilde{\varrho}(p,\vartheta,\eta) = \begin{cases} \varrho(p - \left(\frac{p-50}{\vartheta}\right)(\eta(E)-1))) & , \eta < \vartheta \\ 0 & , \eta \geq \vartheta \end{cases}, \tag{4.2.6}$$

where:

- $\tilde{\varrho}$ is the *flow VaR* function,
- ϑ is the number of trades above which the trader believes that the LLN can be applied,
- and $\eta(E)$ is the number of expected trades given the flow elasticity of the product, E.

The expression with which I linearly attempt to distribute the benefits from the application of the LLN into the bid-offer spread accounts for a couple of important features:

1. $\tilde{\varrho}(p,\vartheta,1) = \varrho(p)$, this is when the expected flow is degenerated to a single point, the *flow VaR* should not benefit from any compensation via LLN.
2. $\tilde{\varrho}(p,\vartheta,\vartheta) = \varrho(50)$, meaning that LLN is entirely applied once ϑ trades can be granted and beyond. The assumption that lies underneath is that the deviation is symmetrically distributed, so that $\varrho(50) = \sum u_t/n = \overline{u}$, where n is the number of observations.

Once the mapping function has been validated the trader can study the effect of a products' flow onto the optimal bid-offer spread[17].

Results

The final experiment takes the optimal hedge previously calculated through PSO and re-calculates the initial bid-offer spread to be quoted in the markets for successive expected number of trades. It uses a parametric approach to $\varrho(p)$: the $\tilde{\varrho}(p,\vartheta,\eta)$ that depends on the LLN that underpins the Central Limit Theorem. In order the LLN to apply so that the convergence to the *exact* mean is met requires the number of observations to be infinite. Instead I will assume that the deviation of the executed mean from the exact population value is negligible to the trader after $\vartheta = 35$ observations. As in the calculation of the PSO I have kept $p = 99$.

By generating the mapping function between the $\tilde{\varrho}(99,35,\eta)$ and the $\varrho(99)$ and running again the calculation of $\phi(\eta)$:

$$\phi(\eta) = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \tilde{\varrho}(p,\vartheta,\eta)$$

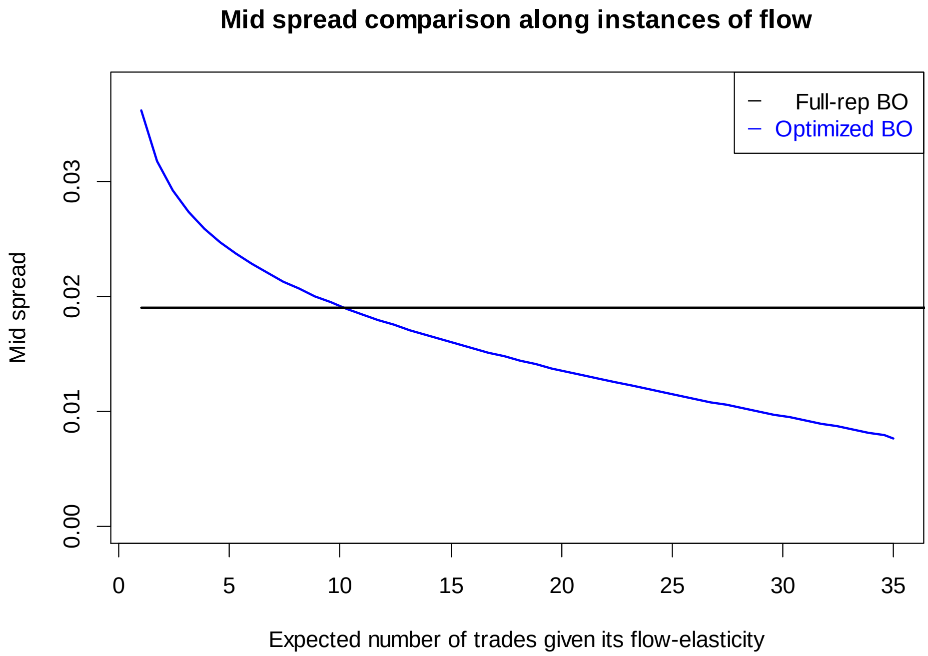

Figure 4.3: Mid spread comparison along instances of flow, for a confidence interval of 99%.

for $\eta = 1 \ldots 35$ I obtain the vector of bid-offer spreads that the trader can offer to the client based on the sales force views on the nature of the client-driven flow that they can gather. This is, based on their estimation of η.

Figure 4.3 synthesizes the different mid spreads reached along the experiment. First, it shows as a horizontal line (i.e. independent of the number of expected trades, η) the mid spread of the full-replication hedge: $\phi = 1.72\%$. Then, the first point in the convex frontier which has been built upon the successive calculations of $\tilde{\varrho}(99, 35, \eta)$ dependent on η: $\phi(1) = 3.85\%$. It obviously coincides with my first attempt to beat the full-replication spread. And finally, the rest of the points, which converge gradually into $\phi(35) = 0.79\%$.

As a result, it can be seen how beyond the point where the two curves cross I could find flow-sensitive market makers that could be consistently closing most of the trades in this product (by quoting more agressive prices) without altering their risk preferences.

Finally, it is relevant to note that lower levels of p would generate left-side shifts of the *flowVaR* frontier and *vice-versa*.

[17]Note that there are several ways to define the flowVaR. An alternative could have been defined upon a fixed 99% percentile that would instead benefit from the LLN by reducing the volatility of the distribution (e.g. to use the distribution of the average of Gaussian variables). The trader shall simply use the one with which she feels most comfortable when interpreting the model as a whole.

Summary

By being flow-sensitive a market maker can safely improve its quotes given a level of risk prudence. The rationale for that improvement is built upon the optimization of the exploitation of the nature of the flow and the gradual application of the LLN.

For the experiment analysed herein, where a demanding scenario has been considered with regards to the trader's risk aversion and to the *Elasticity of the Flow*, E, the commitment by the sales force to attract more than 10 symmetric trades (i.e. 5 inflows and 5 outflows) can allow the market maker to quote more aggressive bid-offer spreads than the full-replicating competitors.

4.2.5 Summary of results

Follows a series of bullet points that synthesize the results obtained along the experiment that can be seen in Figure 4.3:

- A full replicating hedge for the trade considered would imply a mid spread,

$$\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma}$$

$$\phi = 1.72\%.$$

- Alternatively, I consider an optimized hedge that attempts to benefit from smaller transaction costs and more importantly, smaller market impact by incurring into deviation risk. The optimization process is done through PSO restricting the particles to move the weights of each industry instead of each stock. This constraint enhances speed and reduces the probability of ultimately finding a hedge that does not cointegrate well with the market made instrument. In that case, the mid spread is:

$$\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \varrho(99)$$

$$\phi = 3.85\%$$

  This is, it seems that it is not possible to beat this way the full replicating hedge mostly because I consider an extreme prudency that penalizes the tail risk of deviation intensively.

- Last, and abiding by the extreme prudency exhibited above, I consider the activity of the market maker as a flow of trades instead of these being isolated. Following this approach and considering the $\tilde{\varrho}(p, \vartheta, \eta)$ as my penalization mechanism for the deviations I reach a distribution of mid spreads:

$$\phi(\eta) = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \tilde{\varrho}(p, \vartheta, \eta)$$

$$\phi(1) = 3.85\%$$

$$...$$

$$\phi(35) = 0.79\%$$

The distribution beats the spread implied by the full replicating hedge when the market maker can expect more than 10 trades. Moreover, when more than 30 trades can be expected the full replicating mid spread, ϕ, can be decreased by more than 50% as observed by Menkveld (2013) being it hence even surpassed above 30 trades.

4.2.6 Summary

When fixing the mid spread of a market making strategy the transaction costs and the market impact (inherited from Chapter 3) are crucial. If these are not properly considered the backtesting won't be realistic and it will lead to wrong combinations of parameters that exploit non-existing patterns. As a result, large differences between in-sample and out-of-sample performance can occur.

Of the two, the market impact is the most difficult input to consider as it has to be modelled and there is little data (and research) available that can be directly applied. I use a popular paper, Almgren *et al.* (2005), as the framework for my market impact model. Not only differences in the execution approach that generated their data but also in the activity and the market exchanges to which it refers turned into the necessity to adapt their approach. After motivating the adaptation that was required I obtained a figure, $g(1) = 2.97$, that allowed me to easily translate the average daily volume considered on a trade into market impact.

Once the transaction costs and the market impact are considered the mid spread for a full replicating hedge can be calculated and used as the reference to beat. The natural domain upon which enhancements can be analysed is on the replication of the hedge itself. If instead of replicating it fully it is replicated statistically I may be able to find more efficient combinations in terms of transaction costs or market impact. I then propose an alternative basket that benefits from lower transactions costs and market impact. However, even though it statistically replicates the product the risk assumed on that approach is not enough to be approved by a risk-averse agent.

Finally, by considering the activity of the market maker as a flow of trades instead of as isolated operations the penalization of the statistical risk can be gradually diminished as the number of trades grows. This ultimately leads to discounts larger than the 50% observed by Almgren *et al.* (2005).

Figure 4.3 visually covers how the market maker can take the decision of whether to optimize or full-replicate the index in the hedge dependent on the expected number of trades.

4.3 Further proprietary enhancements

Not only the sell-side has to be flow-aware but also pattern-aware. This is, the trader ought to have criteria as to whether the market conditions are favouring or not her non-full replicating hedge in order to account for a more informed decision with regards to the optimal timing of the hedge – from waiting naked for the market to move against the client's position to aggressively trading into the hedge.

For instance, an O-U process can be fit to the deviation, X_t, between the hedge obtained above through PSO and the instrument being quoted:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t, \tag{4.3.1}$$

where, as said in the Background, $\theta > 0$, $\mu > 0$ and $\sigma > 0$ are parameters and W_t is a Wiener process. As seen, by interpreting that μ is the long run equilibrium of a couple of securities, σ the volatility of its dynamics and θ the rate by which the relationship reverts towards the mean, a pairs trading strategy can be defined – this directly meets the nature of the relationship between the instrument and the hedge.

In the case of the experiment, the fitting yields the following information to the trader. There is a mean reverting signal, $\mu - X_t$, of a 14% margin that has been triggered after trespassing a $+2$ standard deviations of the sample's mean (a boundary fixed at 9% above and below the mean) with a half-life, $H = \ln(2)/\theta$, of 106 days (see Background for an explanation of these parameters). This is, the hedge is expected to underperform IBEX 35 during the next months (see the current overperformance in Figure 4.4) suggesting this way that the interim hedge shall be traded-out with a certain dose of urgency – the market is expected to move against the trader's strategy.

In the opposite case, if the trade is a short position (or a trade-out of an inflow), the information disclosed by the O-U process could be used in a core-and-satellite approach (i.e. multistrategy) where the market maker decides along with her risk department how much of the hedge can be allocated onto this (proprietary-like) strategy which, in this case could take up to 3 months.

The next chapter will provide us with the main framework for proprietary trading.

4.4 Summary

A complete process for an efficient bid-offer initialization has been analysed throughout this chapter. First, a series of steps towards the full-replication bid-offer spread, 2ϕ , have been proposed, being most of its difficulty concentrated in the estimation of the market impact of a VWAP algorithm as well as the estimation of the expected slippage of execution. With respect to the former, a fine tuning of the standard market impact estimation has been considered, $\tilde{g}$, given that VWAP already accounts for non-linearity. As per the latter, a prudent distribution for slippage, $\tilde{S}$, has been borrowed from the previous chapter where the error smooth caused by periods when the price

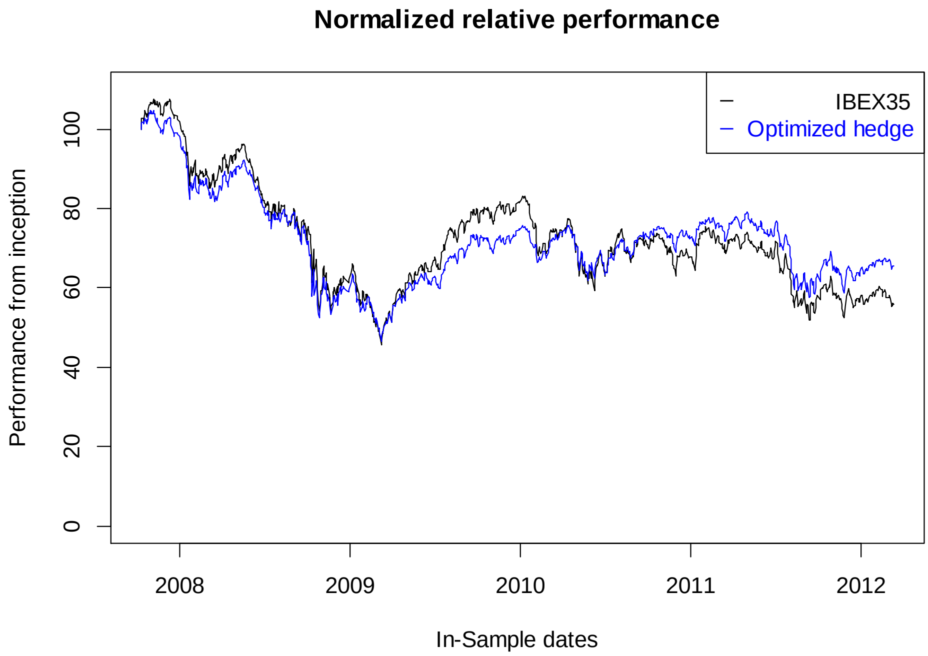

Figure 4.4: Normalized relative performance of IBEX35 and our optimized hedge.

remains stable has not been considered (i.e. there is no *myopia of the price*). It further requires to account for the transaction costs, $\tilde{C}$, and the mid spread of the components of the hedge, $\tilde{\varsigma}$. As a result, I obtained in my data:

$$\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma}$$

$$\phi = 1.72\%.$$

Then, we have seen a non-full replication approach which leverages on hedge optimization to achieve an enhanced combination of market impact and transaction costs that overcomes the unavoidable deviation risk, both mean, μ, and tail risk, $\varrho(99)$, that the trader needs to assume. Given the large number of stocks in the multidimensional optimization problem it seemed appropriate to try to benefit from the usage of a heuristic-based approach such is the restriction of the search-space within Particle Swarm Optimization. To the light of the portfolio management theory I expect cointegration to depend crucially on the industry distribution of the candidates for optimal hedge. Hence I first reduce the number of dimensions of the optimization problem from the number of stocks to the number of industries. I then generate the candidates stressing each industry's weight independently and distributing its level across the stocks that belong to it as equal-liquidity custom baskets (and this favours having similar market impact across them). Convergence is achieved after less than 10 iterations, a positive feature that is more relevant the higher the frequency of trading. However, given the challenging scenario defined for the experiment it seems apparent that this optimization process is not enough yet to beat the full-replicating bid-offer spread at a 99% interval confidence.

$$\phi = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \varrho(99)$$

$$\phi = 3.85\%$$

Finally, I introduce the added value of the sales force into the approach. By letting them define the elastic nature of the flow (along with a commitment to the number of inflows and outflows that the market maker is going to intermediate[18]), a more aggressive bid-offer than the full-replicating spread can be granted even under the strict conditions set out for the experiment (**objective two** of the thesis). The Law of Large Numbers plays a key role on this result, by assumming that I can linearly project its full application from 2 to 34 trades (**objective five**) through the $\tilde{\varrho}(p, \vartheta, \eta)$.

$$\phi(\eta) = \tilde{C} + \tilde{S} + \tilde{g} + \tilde{\varsigma} + \mu + \tilde{\varrho}(p, \vartheta, \eta)$$

$$\phi(1) = 3.85\%$$

$$...$$

$$\phi(35) = 0.79\%$$

Beyond those the trader does not charge extreme value theory but the first moment of the distribution[19] instead.

This approach could explain why the recent decrease of bid-offer spreads in the markets have coincided with the setup of highly quantitative market making agents. In fact, the approach itself shows the need for scientific expertise in the current markets. This may cause changes in the trading floors along the forthcoming years in order to give a larger role to the quantitative approaches.

Caveats

Some of the main caveats to have in mind with regards to the present chapter are the following:

[18] When trading electronically it is easier to estimate both the elasticity of the flow and the expected number of trades based on past data of the trader's activity.

[19] Typically zero, given the Efficient Market Hypothesis.

- Shall the market maker change the basket with the evolution of the patterns? The answer is "only when the systematic strategy is not compromised". In order to be able to apply the projection of the LLN the trader cannot change the basket/strategy in the case where losses are being materialized, i.e. the optimized basket has to remain fixed until the expected number of trades required to apply the LLN is reached. Otherwise, it won't be realistic to expect to achieve the average profit. In case of profits, the hedge can be changed at any time as long as the market maker bears in mind that the new expected number of trades is lower from then onwards by definition. This subtle feature can be key to HFT as even though it accounts for elastic flow, its adaptive changes shall nevertheless comply with a minimum of trades per instantiation of the model.

- Homogeneous sizes are also relevant. The application of the LLN refers to performance hence, for the trader to remain profitable, trades have to imply the same exposure – i.e. the trader wants a $p\%$ above-average performance to compensate a $p\%$ below-average performance. And this can only occur if both account for the same notional. Most of client-driven flow implies symmetric sizes being traded (at least in pairs). Also, electronic-driven flow tends to imply similar exposures given the typical distribution of sizes along the order book and the intraday volatility of the prices.

- The bid-offer spread asserted herein is the one to be fixed initially, when there is no inventory. As such, even though it shall also represent the average bid-offer spread quoted along time it does not need to hold continuously. Dynamics of the bid-offer spread largely obey to specific casuistic. The most important being the evolution of the product's appetite mostly from non-market making agents along with the evolution of the inventory of the market makers.

- Fixing the number of days to trade the hedge dependent on the least liquid industry allows the deviation risk to account for both diversification (this should favour cointegration) and an intuitive portfolio management. However, it does so at the price of increasing the notional of the deviation risk itself. The strategy could be improved by allowing for heterogeneous slices of the trade, e.g. allowing the most liquid stocks to be traded first no matter the rest of the sectors. This option though could still generate a less intuitive management of the deviation risk to be also weighed off.

Future work and further improvements

This experiment is just a first approach on how to enhance the bid-offer initialization. As such, most of the phases described herein could potentially be optimized further:

- The optimal hedge can be used to proprietary-trade intraday against the different depth levels of IBEX-linked products such are its ETFs and futures.

- Given that the presence of elastic-flow favours intraday trading, futures could be used instead of ETFs since they tend to be more liquid and, intraday, ETFs lose part of the positive features explained above.

- Instead of CAC 40 more bespoke satellite assets used to improve liquidity can be considered.
- By buying Spanish CDS and selling French CDS the deviation risk should be reduced via country-risk hedging.

Chapter 5

Risk-Reward: a calibration model defined within a Q-learning framework

This chapter addresses the typical structure required for the risk/reward enhancements proposed in the previous two. A novel indicator is first motivated, the Expectations-Shift, that seeks price movements based on market makers sentiment instead of standard market impact. Then its signals' trigger is calibrated along with two risk management parameters: stop-loss and take-profit levels. I propose the usage of Reinforcement Learning to wrap the typical data-driven calibration processes, giving rise to the so-called Avatar Calibration. This novel technique generates an agent per calibration scenario that learns a set of rules for strategy management. By then comparing each one's reasoning with a previously defined set of rules proposed by the portfolio manager it is possible to select those agents whose management approach converge the most to the trader's preferences. If the benchmark proposed by the trader is underpinned by theory the results from the agents shall in the limit be more robust than those non-theory-driven delivering hence better out-of-sample performance than the standard calibrations − fulfilling the objective number one of the thesis.

5.1 Research challenge

The previous chapters allowed me to realistically (as demanded by Cahan *et al.* (2010)) backtest a strategy that successfully achieved more than a 50% discount upon the full-replicating hedge (bringing to the foreground a technique that can explain the observations in Menkveld (2013)). However, a successful backtesting does not always imply a successful performance in the real markets. Typically, the over adaption of the parameters to the data sample generates a problem of robustness out-of-sample, known as overfitting. As such, I first show the overfitting problem on a triplet of parameters thrugh a series of greedy optimizations upon popular target functions. Then I introduce the role of agents that independently learn a policy inspired by the trader with the aforementioned target functions in order to see if that way the overfitting is mitigated. Each agent learns within a universe defined by the parameters that I want to calibrate. Finally, I include a novel approach where the target function is to find those agents that have defined a decision-matrix that mimics the most the decision-matrix that the trader would have defined. By using the parameters that defined the universe where the agent learnt such a decision-matrix I manage to successfully mitigate the overfitting problem in my data sample giving this way a mechanism to complete Bailey *et al.* (2013).

5.1.1 Proprietary trading in the sell-side

As mentioned above, during the last years regulators have been limiting the capacity of sell-side agents to take proprietary risks. As a result, most of it is hence being currently pursued in the buy-side (and more especially, in the hedge fund industry). However, even though there are strict rules for sell-side traders to manage prudently their risks it is still possible to use their expertise in the optimization of their hedging policy as long as they respect the risk limits (i.e. within a well-defined regulatory framework). This triggers the possibility to use pattern detection in risk management.

5.1.2 Core features in proprietary trading research

throughout the thesis several calls to proprietary trading have been made whenever a risk-reward had to be optimized. Being this the case it seems appropriate to dedicate a complete chapter to show a comprehensive framework for the topic. Proprietary trading typically looks for patterns both barely exploited in the markets and theoretically robust[1].

Overall, it could be said that a basic strategy is built upon the following features:

Indicator. A time series on a raw instrument (e.g. a currency rate) or on processed data (e.g. log-prices, VWAP, etc).

Signal. An event that happens on the indicator. Events tend to be defined as extreme levels such are limit-prices in technical analysis or normalized ones in StatArb. Signals tend to target momentum or mean-reversion patterns dependent on whether the events generate trend-following patterns or counter-trend-following ones.

Rules. Typically, a core rule that has been set out to the light of the signal interpretation (to buy or sell the instrument) and includes:

- *Risk management.* – Inclusion of systematic rules for take-profit and stop-loss, basically. Other rules and time horizon constraints can also be considered. In our example, I can reasonably expect that the (end-of-day) trading signals that I will witness should not last longer than a week hence I will consider a maximum time horizon of 5 days. This implies that there may be moments when there is no exposure to the markets (zero inventory).

[1] Orthogonality of returns is also important to investors in order to favour their diversification.

- *Inventory management.* – Decisions as to whether the trader shall keep building up exposure on a product or not. Most of market making strategies imply zero inventory at the end of the day. This allows for intraday inventory management building. Even though explicit signals of inventory management can be defined others, more subtle, do also affect the inventory management policy. For instance, do consecutive signals indicating a pattern to trade on a certain side have to be traded linearly (all equally weighted) or non-linearly (change the weight as more signals on the same side appear consecutively)?

- *Money management.* – The interaction of a certain strategy with other active strategies has to be clearly declared and optimized as they will all be competing for the budget allocated to the trader.

Calibration. Calibration tends to be based on performance. Performance can be stated giving emphasis to several dimensions (annualized return, maximum drawdown, volatility...) and those can be measured raw or through a utility function. The utility function has two key aspects:

- it allows for a more theoretically-driven calibration given that it weighs-off the main dimensions altogether dependent on the trader's set of preferences, but

- it adds parametric complexity to the process as not only the mean effect of each dimension is difficult to assert but also their marginal effects[2].

5.2 Methodology

The methodology considered for this experiment meets the following rationale:

1. If I want to find a way to mitigate the overfitting problem I need first to motivate a rationale on the way I foresee the solution. In this case, I will assume that the injection of the trader's expertise along the optimization process provides a sound structure to the strategy that should remain out-of-sample. This way differences between in-sample and out-of-sample performance should be reduced with respect to those from a data-driven optimization.

2. Again, the calculation of the benchmark becomes the first step.

    (a) First, I shall use a series of standard target functions in trading whose parameters are optimized through greedy maximization.

    (b) I could then take the chance of deploying a risk-reward experiment to introduce a novel indicator with which I can show that the interpretation of the signals themselves may also be a non-triviality often disregarded by the financial literature. The benchmark then will be two-fold: one series of results per possible interpretation of the signal (mean-reversion and momentum).

[2] Unlike in the goods' consumption (decreasing) marginal utilities on most of the utility parameters the marginal effect is increasing to the trader. This is the case given that better strategies can attract funds non-linearly.

i. As such, I would use each possible combination of the parameters to define a universe where an autonomous agent is trained to decide in which moment the interpretation shall be one or another.

ii. I would then compare those results with the former in order to check whether keeping flexible the interpretation of the signal adds value to the process or not.

iii. The different agents shall be trained through a methodology that allows me easily gather the knowledge surged during the learning process. A Q-learning approach embeds that knowledge in the so-called Q-matrix.

(c) Finally, I will input the trader's expertise along the optimization process through a novel approach that simply compares the theoretical Q-matrix that would have defined the trader with those output by each agent along the calibration universe. By picking the one that mimics the most that of the trader (i.e. by selecting her *avatar* along the calibration universe) I shall be able to identify a combination of parameters that in essence allows for a framework where the trader would feel most comfortable. As it takes time to change the Q-matrix the out-of-sample performance should become a mixture between the knowledge of the trader and the adaption to new scenarios. Merging this way theory-driven and data-driven approaches.

5.2.1 Data description

The data is composed by 1-minute bars of best bid, best offer and volume of IBEX 35 futures from January the 2^{nd}, 2008 to June the 15^{th}, 2012. The sample has been divided into in-sample and out-of-sample by mid-June, 2011. The minute bars have been pre-processed in the search for outliers and aggregated up to 5 seconds before the closing auction[3].

Overnight interest rates are used within the same period to account for the risk-free asset returns.

The budget that the trader is supposed to manage is 10 million euro – low enough to consider a negligible market impact in the closing auction[4].

5.2.2 A novel indicator: the Expectations' Shift

The market dynamics mainly obey to the effect of two different forces: market impact and changes in expectations. If I assume that the market impact nature of each stock does not change along time any deviation around the weighted average of the price change per share has to obey to changes in the expectations of the order book's participants[5].

I will define the *Expectations' Shift* indicator, ES, as the logarithm of today's VWAP (as a price, not the execution algorithm) up to the closing auction over yesterday's level, controlled by the

[3]In order to have time to compute the calculations proposed herein if the present strategy was set out within a real environment to trade on the close.

[4]Note that this experiment merges intraday data with mid-frequency signalling/trading.

[5]Note that intraday the nature of the market impact changes due to the seasonality observed in Chapter 3.

volume ex-block trades traded during the day (same calculation using intraday data as explained in Chapter 2). In the model, I will refer to ES indicator as ζ:

$$\zeta_t = \frac{log(P_t^{\mathrm{W}}/P_{t-1}^{\mathrm{W}})}{V_t} \tag{5.2.1}$$

where:

- P_t^{W} represents the VWAP level[6] of date t, up to but excluding the close as it is the closing auction where the execution will be filled. It implies the same calculation through intraday data as explained in the previous chapters.
- V_t represents the volume ex-block trades[7] negotiated during the day t up to the closing auction.

I target those that are extreme under the assumption that the rest of the movement does belong to market makers' shift of expectations.

5.2.3 Signal detection: when the trigger is not trivial

If instead of assuming a constant nature for the market impact as stated above I assumed that such nature may change along time with the dynamics of other features like volatility (as seen in the previous chapter) I shall be looking at just a subset of the deviations around the average – i.e. not just any deviation but the extreme.

Regrettably, traders do not have a clean criteria as to how far from the average can they define the *extreme* that will trigger the signals so I will have to leave its final definition to the calibration process. Once addressed, whenever an extreme level of the indicator is flagged I assume that the expectations have been shifted and will execute according to the trading rules.

5.2.4 Trading dimensions

The combination defined by the set of trading rules encloses the proprietary rationale which is why it tends to be the most secretive aspect of the trading industry. Once the rules are put into place it is complex to introduce new rationale about the performance of all its building blocks by just intuitively scanning the markets. That is why it is said that it is preferably to run algorithms that are built upon simple principles and few parameters rather than complex, large systems.

[6] Even though official close levels could be used I consider the VWAP since I expected it to account for less noise, providing this way more robust signals.

[7] Note that these are not considered in the VWAP calculation either.

Signal interpretation: the dual nature of the extreme events

So far I have motivated the statement that the signal provides the trader with the information that there has been an extreme reaction on the trading session's participants expectations (mainly the market makers). However, do I know anything else about how to interpret it? It seems likely that part of the time it may obey to an overreaction (when it is the result of an isolated market maker) however, it can also be a momentum's signal (if it is an overall view). In fact this is the largest challenge of most of proprietary trading strategies: it is not known whether the right strategy is to buy or to sell with the signal but only that there is an stress and with it an opportunity.

This implies that performance usually swings between profits and losses (with few periods where it remains neutral) highlighting this way the need for the inclusion of a risk management policy within the trading rules.

Risk management: take profits, stop-loss and maximum time horizon

Even though some traders have strong views as to whether there has to be a bias between the intensity level of take profit's and stop loss' rules and when so, towards what side it has to be biased, I will assume that I do not have a particularly strong rationale to define a universal rule on that respect.

I will impose though a time horizon for whichever trade that I cross in the markets as I consider that the market dislocations caused by any expectations' shift should not have a long run effect – in this case, no more than a week. This is, whether the trade is delivering a profitable performance and did not trigger the take-profits rule or it is throwing losses but has not reached any stop-loss yet, I will trade out of the position once a week has passed. I want this way to reduce the noise of non-theory driven patterns.

Inventory management: leverage on consecutive signals?

If while a position is being held there is an opposite trading signal the trade will be reverted. If the signal reinforces the side being held by the trader there are two possibilities:

1. Increasing each trades' exposure/notional in time as there are more signals backing up the former.
2. Decreasing exposure in time since the former accounts for a larger expected margin and as such it should be the one with the largest notional.

I will take the latter's view to the limit so that no inventory will be built up as a consequence of new signals on the same direction. If there is any inventory being built up it means that there are other strategies into place on the same product.

Money management: how to distribute the budget across strategies

If there are several strategies put into place on the same product I shall use Kelly Criterion to distribute the available funding across them. One of the expected upsides of systematic trading over discretionary trading is precisely that a subset of the calibrated strategies can be put into place together instead of just one. And this should favour diversification and as such out-of-sample performance – especially when no theory rationale can be considered during the optimization process (unlike in the previous chapter).

Utility function: the portfolio manager's appetite for risk

Another way to try to favour out-of-sample robustness of in-sample results is through the use of a utility function. The rationale is simple: the trader may rather look for non-corner solutions when there are several dimensions to be taken into account. The challenge though may be the fine tuning of the relationship amongst those (the so-called elasticity of substitution) that could trigger itself a further need for parametric calibration which may add more difficulties into the optimization problem.

I will assume the following benchmark for the portfolio manager's preferences:

"Increases in the performance are strictly preferred to increases in the maximum drawdown which in turn are strictly preferred to increases in the Sharpe Ratio."

As said, the inclusion of the utility function may add more complexity to the way the trader interprets the markets so even though I will add this feature here in order to account for a comprehensive experiment I won't be calibrating its parameters. Instead a materialization of the relationships that meets the proposed benchmark will be taken. In this case, I will use a linear utility function with the following characteristics:

$$U = 100(\bar{R} + 0.1D + 0.01\xi) \tag{5.2.2}$$

where:

- $\bar{R}$, represents the average of the returns, R_t, yielded by the selected strategy at the end of the sample, $t = T$.

$$\bar{R} = \frac{1}{T}\sum_{t}^{T} R_t \tag{5.2.3}$$

- D, represents the maximum drawdown along the performance of the selected strategy, its maximum historical decline[8]:

$$D = \max_{\tau \epsilon (0,T)} \left[\max_{t \epsilon (0,\tau)} R_t - R_\tau \right] \tag{5.2.4}$$

- ξ, represents the Sharpe Ratio of the selected strategy[9]. It is defined as the normalized expected difference in returns of the selected strategy, R_t, versus the risk free asset, R_t^{f}, for which the return of a 5 year Spanish bond for that period was considered. This is:

$$\xi = \frac{\mathrm{E}(R_t - R_t^{\mathrm{f}})}{\sqrt{\mathrm{var}(R_t - R_t^{\mathrm{f}})}} \tag{5.2.5}$$

By accounting for this instance of the utility I will be able to discern across Indifference Curves (i.e. Pareto fronts for the same level of utility) and compare the whole set of combinations along the calibration process.

Summary

The trading rules generate a set of distortions as to the way the markets dynamics will affect the trader's strategy – in this case, a model that seeks to find the largest shifts of the market participants' expectations is deployed through a novel signal, the Expectations-Shift.

As these distortions make the portfolio manager lose part of the intuition behind the rationale that underpins her trading policy there is the need to calibrate a series of parameters typically based on data-driven approaches. The calibration optimization is usually built upon performance analytics directly or through a utility filter that combines them by weighing-off their relative relevance according to the trader's structure of preferences.

5.2.5 Standard calibration: the data-driven approach

Throughout a calibration process the set of parameters is typically stressed so that the different combinations considered can be analysed in order to select the most suitable strategy for the trader. It is important not to confuse it with the most profitable as profitability is just one of the several dimensions included within the calibration process.

[8]Note that it cannot be simply defined as the difference between the maximum and the minimum since the maximum could occur after the minimum and that would not be a decline.

[9]Note that many others are also possible even its correlation with other strategies when the trader is running a global book and minimum variance of the overall performance is targeted.

Typically heatmaps or heatvolumes are created to visually decide what area of parameters to choose. More sophisticated optimization processes include heuristics such is the method explained in the previous chapter towards a more theory-driven process (and a shorter calculation time). As said above though I do not count on a theory-backed criteria with regards to the definition of extreme deviation or the distribution of the risk management rules (take-profit and stop-loss) so this section typically approaches calibration following data-driven techniques directly.

Calibration targets

Across the universe of parameters' combinations $\Theta = (\phi, \psi, \delta)$, where

- ϕ represents the percentage of the performance that would trigger a stop-loss. In the experiment only the universe $\phi \in [0, -0.1]$ will be considered.
- ψ represents the percentage of the performance that would trigger a take-profit. In the experiment only the universe $\psi \in [0, 0.1]$ will be considered.
- δ represents the deviation from the mean in terms of stardard deviations on the indicator that defines what is "extreme" and what not. In the experiment $\delta \in [0, 2]$, this is up to 2 standard deviations from the mean.

Three approaches will be considered, two one-dimensional and one multidimensional (utility-filtered):

1. Maximization of the performance by the end of the sample:

$$\Theta^{\mathrm{P}} = \underset{\Theta}{\operatorname{argmax}} \rho(\Theta) \tag{5.2.6}$$

   where $\rho(\Theta)$ represents the vector of end-of-sample performances across the combination of strategies.

2. Maximization of the yearly average returns throughout the sample:

$$\Theta^{\mathrm{R}} = \underset{\Theta}{\operatorname{argmax}} \bar{R}(\Theta) \tag{5.2.7}$$

   where $\bar{R}(\Theta)$ represents the vector of yearly average returns across the combination of strategies.

3. Maximization of several features related to performance analysis through the previously proposed instantiation of the utility function:

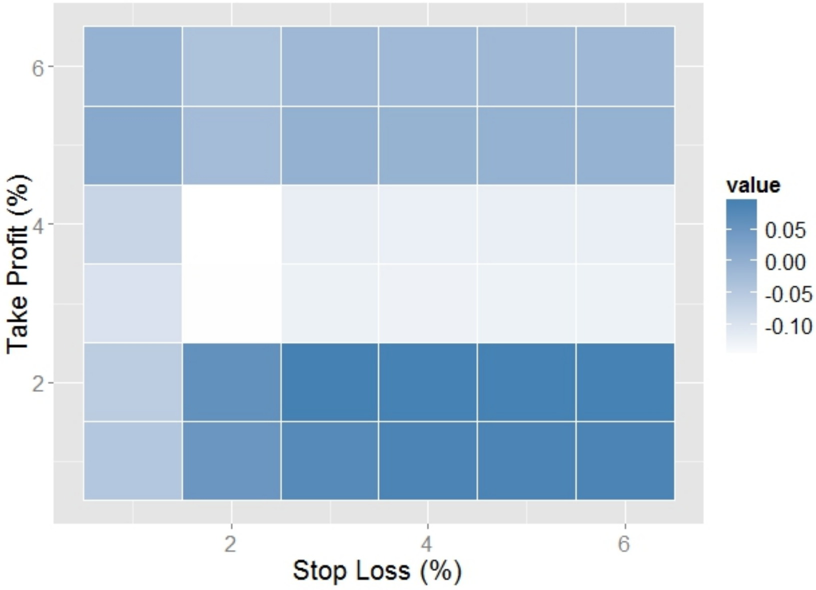

Figure 5.1: Slice of the greedy-calibration cube for the in-sample yearly returns.

$$\Theta^{\mathrm{U}} = \underset{\Theta}{\operatorname{argmax}} U(\Theta) \tag{5.2.8}$$

where $U(\Theta)$ represents the vector of utilities across the combination of strategies.

Also, as pointed out above instead of looking for a unique combination of parameters that maximizes a certain dimension of the performance or optimizes the utility I will check the set of combinations, in this case, the best 10. By combining those 10 I expect to have more diversified results and as such, more robust to the light of equity portfolio theory.

Results

Unlike in the previous chapter, given the reduced complexity considered I won't be applying any non-linear optimization algorithm. Instead, a basic greedy-search has been deployed throughout the universe of parameters' combinations Θ.

Figure 5.1 shows a slice of the heatvolume considered along this experiment for the yearly in-sample returns of the different strategies. It has been defined for different combinations of stop-loss, $|\phi|$, and take-profit, ψ, when 1.2 standard deviations are used in the definition of the signals, i.e. $\delta = 1.2$. The bluest area represents 9.2% yearly returns while the whitest one accounts for -12.6%. The presence of large blocks of the same colour indicates that even though the combination of parameters is different the way a position is traded in and out does not change. This is the typical

	Mean-reverting			Momentum		
	Θ^{P}	Θ^{R}	Θ^{U}	Θ^{P}	Θ^{R}	Θ^{U}
Performance	241%	229%	229%	149%	149%	149%
Av. yearly return	29%	33%	33%	8%	8%	8%
Utility	27.78	31.85	31.85	6.61	6.61	6.61
Max. Drawdown	-10%	-11%	-11%	-19%	-19%	-19%

Table 5.1: In-sample results for the parameters selected through the standard calibration approaches.

situation when the trade out is triggered along the data by the maximum time horizon instead of by a take-profit or a stop-loss – i.e. it is a consequence of the 5 days of maximum holding.

Tables 5.1 and 5.2 show calibrating results for both possible interpretations of the signal: mean-reverting and momentum. The results are presented in terms of the achieved performance, average yearly returns, utility and maximum drawdown. First along in-sample data and then out-of-sample data. The latter allows me to analyse the consequences of overfitting Θ.

To the light of the results it can be first noticed that, in my data, the same vector of parameters Θ optimizes both dimensions the average returns and the utility. As a result, two of the three columns for the mean-revering interpretation of the signal are exactly the same. The same applies to the momentum interpretation where the same case occurs for the three columns[10].

Table 5.1 shows how in-sample the greedy optimization the mean-reversion interpretation renders a better profile in terms of performance, average yearly return, utility and maximum drawdown. Hence, I would expect a classical trader who uses in-sample greedy maximizations to interpret the signal as mean-reverting. This is, whenever she finds an outlier in the distribution of ξ it would be interpreted as an overreaction of the market makers derived from a wrong set of expectations.

As mentioned above, it is out-of-sample when the effects of overfitting can be noticed. Table 5.2 shows how the mean-reverting interpretation suffered the most in every feature of each optimization domain. This suggests that the classical trader could have been biased by the data in her approach to the interpretation of the index and, in reality, market makers do not overreact but anticipate larger changes in the prices of the stocks.

As a result, the trader would have loss between 11% and 22% dependent on which combination of Θ would have finally selected. While, have she been able to anticipate the accuracy of the momentum interpretation, the gain would have been 5%.

Summary

By passing iteratively along the sample I can generate a universe of feasible strategies. Within it I can greedy-select those that have reached the top level along a certain performance criteria, whether one-dimensional such are the performance and the returns or a value-function of several dimensions at once such is the utility.

[10]Note that the probability for these coincidences to happen have been rised by the aformentioned selection of the top 10 combinations distributed according to the Kelly Criterion.

	Mean-reverting			Momentum		
	Θ^{P}	Θ^{R}	Θ^{U}	Θ^{P}	Θ^{R}	Θ^{U}
Performance	89%	78%	78%	105%	105%	105%
Av. yearly return	-14%	-26%	-26%	12%	12%	12%
Utility	-17.13	-30.99	-30.99	10.44	10.44	10.44
Max. Drawdown	-35%	-49%	-49%	-12%	-12%	-12%

Table 5.2: Out-of-sample results for the parameters selected through the standard calibration approaches.

It can be seen that such data-driven approach may generate high-risk for both interpretations of the signal along their out-of-sample behaviour. Moreover, while both of them largely lose their advantage out-of-sample (a usual issue in calibration), the mean-reverting interpretation does hit losses while the momentum one does still produce positive results.

5.2.6 Avatar calibration: towards a theory-driven approach

As seen above, calibration tends to be a grey area for the trader as it is not clear that the in-sample preferred performance will generate the most preferred performance out-of-sample.

An interesting feature is in fact that usually top in-sample performances generate strategy management policies that imply to keep the strategy active no matter whether it has recently performed well or wrong. This is, there is total confidence in the signal; and this among other things rises the risk of large drawdowns out-of-sample. However, I expect a portfolio manager to feel more comfortable freezing an algorithm that has recently performed poorly or randomly let alone wrong. Moreover, such rationale seems to be even more relevant when the signal informs about an unusual stress without giving more information as to whether the signal is for a momentum pattern or a reversion one as is the case in our experiment, hence especial care is needed.

I will try to target those combinations of parameters that lead to a strategy management with which the trader feels most comfortable.

The agents' strategy management policies

The main difference between the standard calibration and one that adapts within a learning process may be the robustness across them. While iterative in-sample optimization does not hold information about the result of previously optimized processes the usage of a *learning matrix* does so by acting as a brain for the autonomous agent. This also favours a finer smoothing process for the parameters' dynamics when such brain is not reset at each optimization period (episode).

Dynamic programming is a technique that *memo-izes* the optimal solutions from previously visited calibration paths increasing this way the speed of concatenated calculations. Reinforcement learning is a discipline typically built upon dynamic programming that has been developed to let robots learn without supervision how to interact with the environment. Its cornerstone is the definition of an incentives scheme to give the robot a reward when the final target is achieved or a penalty when it is not[11].

In this experiment, in order to fill in the already mentioned *learning matrix*, a Q-learning like approach has been deployed with the following features:

State. The state of the robot at each signal will be defined by a vector:

$$s_i = s(m_i \, , g_i \, , \chi_{i,j} \, , \pi_{lm} \, , a_{i-1}) \tag{5.2.9}$$

whose components are defined as follows:

1. Interpretation mode, m_i, that even though it has a default value[12] it now ranges across $mean - reversion$ (1), freeze (default mode)[13], momentum (-1) throughout the lifespan of the strategy.

2. Signal upon the indicator defined above, g_i, that ranges across buy (1), doNothing (0), sell (-1), this is $g_i \in \{1, 0, -1\}$.

3. Transition of the 2 latest trades' pattern (Λ) of the standard strategy (χ).

$$\Lambda_i^{m,g} = \text{sign}(\text{sign}(P_{i-1}^{m,g}) + \text{sign}(P_{i-2}^{m,g})) \tag{5.2.10}$$

   where $P_i^{m,g}$ is the profit of the standard strategy on the i-th signal, g_i, under the i-th selected mode, m_i. The Λ is an indicator of how the strategy is performing and can take any of the following values[14]:

   (a) allPositive(1): by trading the signal in the side controlled by the mode the 2 latest trades have been profitable.

   (b) mixture(0): when the 2 latest trades have had opposite profitability.

   (c) allNegative(-1) : when the 2 latest trades have had a negative return.

[11] The typical example is a robot that moves within a house and receives a reward when the exit is reached. The robot will optimize the flow of probable rewards from each action at each state and decide this way how to move autonomously.

[12] So that the strategy is mean-reversion or momentum biased. As said before, the default for the mode is not an innocuous decision given the non-linearity originated by the risk management rules hence even though they should now be closer in performance I will be again analysing both possibilities separately.

[13] When the model is frozen due to poor results it will still keep track of the standard strategy with the default mode, whether it is +1 or -1. I have taken this decision instead of leaving it random in order to be able to understand better the overall performance of the robots' strategy management policies.

[14] Note that unlike in Moody and Saffell (2001) it is not the performance but the profits of the trades what is considered. Also, as our intraday data is not synthetically generated through random walks with autoregressive trend process, but real instead, I do not consider the summation of the performance along a certain window but the raw pattern.

Its possible universe of transitions generates up to 7 states[15], $\chi_{i,j}$, for $j \in \{1, 7\}$, that have attached a set of transition probabilities to reach a state k from state l when action a is taken, $\pi_{lk}(a)$ where l, k are instantiations of j whose combination is restricted by the nature of the states themselves[16].

4. Latest action, a_{i-1}, as defined below that ranges across bought (1), didNothing (0) and sold (-1).

Action. To change the interpretation-mode in order to affect the execution of the signal:

$a_i = g_i m_i$ so that $a_i = \{1, 0, -1\}$ when it buys, freezes or sells the signal respectively.

This is, I allow the robot to discern whether the pattern indicates mean-reverting or momentum signals instead of sticking by one throughout the lifespan of the strategy.

Policy. Decide the mode dependent on χ, the recent performance of the strategy and a_{i-1}, the latest action[17]. There is a constraint imposed by the portfolio manager: the transition has to be gradual – i.e. the agent can't step from reversion to momentum without passing first through freeze (expected to favour robustness of a theory-driven strategy). The action will affect the second dimension of the state with a certain probability hence future simulations of the transition rule will take into account the posterior probability of upgrade and downgrade of the Λ across modes.

Reward. For each combination of the parameters within the range considered in their calibration, $\bar{\Theta} = (\bar{\phi}, \bar{\psi}, \bar{\delta})$, I back-test the standard strategy defined by it and obtain the profit linked to each signal's trade, $P_i^{m,g}$. By allocating each profit to its corresponding triplet, ($\chi_{i,j}$, a_i, a_{i-1}), and averaging out along the span of trades triggered on forward passes of that very same strategy I obtain the set of rewards, $R = R(s, a, \Theta)$.

Learning. Q-function type of approach learnt through the following value iteration update:

$$\begin{aligned} Q_i(s,a,\bar{\Theta}) = \quad & (1-\alpha)Q_{i-1}(s,a,\bar{\Theta}) \quad + \\ + \quad & \alpha \sum_{s'} \pi_{ss'}(a) \left\{ R(s',a,\bar{\Theta}) + \gamma Q_{i+1}(s',a',\bar{\Theta}) \right\} \end{aligned} \tag{5.2.11}$$

- s is the vector of states.
- a is the action of changing or not the interpretation mode to affect the strategy execution (signal multiplied by the mode).

[15] The larger the number of states the harder it becomes for the model to be able to visit all the states along the sample. If little data is available the trader could consider the use of random interpretation mode after the strategy is frozen given that keeping the default reduces the chances for the opposite to be visited. On this note, it seems reasonable to predict that this approach shall mostly not be affected by such issue with intraday strategies given the amount of data available.

[16] For example, the allPositive can give rise to another allPositive after the next trade or a mixture one. By definition of Λ it could not be followed by an allNegative one. Then, in this case the transition probability distribution for allPositive will be split into two.

[17] It resembles a Markov decision process (MDP) behaviour.

- $Q(s, a, \bar{\Theta})$ is the Q-value of taking action a in state s, for the standard strategy implied by the combination of parameters $\bar{\Theta}$. When indexed into the future $(i + 1)$ it represents the expected Q-value.
- s' is the state where I end up after action a.
- a' is the action taken in s', the one that maximizes the expected payoff.

$$a' = \underset{a}{\text{argmax}} Q_{i+1}(s', a, \bar{\Theta}) \tag{5.2.12}$$

Unlike in the case where the robot moves in a room looking for the exit, the environment in finance is changing and I do not know whether taking an action that previously was successful will also generate positive returns in the near future or not. By analysing the previous cases I can at least weight each case through the transition probabilities and estimate an expected payoff.

- α is the parameter that controls the dynamics of the Q-values matrix of states and actions. I will set it to 1/3 since I want our robot to be more sensitive to the latest patterns than to previous ones without losing sight of the rationale learnt so far.
- γ is the learning parameter. It values the forthcoming benefits as opposed to the immediate reward. Let us set a value of 0.8.

This is, at every signal[18] the state is defined. This allows me to compare the values of the Q matrix across actions taking into account the transition probabilities stated above to simulate the future expected payoff of the feasible actions and take the optimal as a policy for the next trade (exploration and exploitation per trading signal, i.e. each signal becomes an episode).

When convergence of the Q matrix is not achieved the Q matrix that has remained robustly within the convergence limits for the longest time along its iterative calculation is selected[19]. I expect the Q matrix to be biased towards maintain when the Λ is positive, towards freeze when Λ is mixture and towards invert when it is negative.

With this framework the agent will learn from experience without a teacher hence its referral as unsupervised learning. The Q matrix, this is, the strategy management policy of each combination of parameters will be used to calculate out-of-sample results.

[18]Note again that it is the signal what triggers the learning process instead of the continuous performance of the strategy.

[19]50,000 episodes are allowed and convergence is achieved when the summation of the matrix values do change in two consecutive episodes by less than 1%.

The trader's strategy management policy

By deciding the incentives reward scheme (the so-called R matrix) the trader can define her strategy management policy for a certain combination of parameters$\bar{\Theta}$. I will consider in this experiment that the policy is vaguely defined through a rule as simple as follows:

"When the trades are consistently delivering profits keep trading the signal. When they deliver random results, freeze the strategy. When they consistently deliver losses, trade the opposite."

This is, the R matrix could perfectly be one filled by zeroes everywhere but:

- in the action buy of the χ state: $\{\text{allPositive}, \text{allPositive}\}$,
- in the action freeze of the χ state: $\{\text{mixture}, \text{mixture}\}$, and
- in the action sell of the χ state: $\{\text{allNegative}, \text{allNegative}\}$.

By then using the transition probabilities the trader's Q matrix[20], $\Upsilon = \Upsilon_{\mathrm{Q}}(\bar{\Theta})$, can be calculated[21] and compared with the agents' policies, $\Xi = \Xi_Q(\bar{\Theta})$.

I can define this way what I will call the *Reasoning-Convergence* between the trader and each agent, $\Xi = \Xi(\bar{\Theta})$, that will help the trader find those agents which resemble the most similar features to her portfolio management policy:

$$\Xi(\bar{\Theta}) = 1 - \sum \left| \widetilde{\Upsilon}_{\mathrm{Q}}(\bar{\Theta}) - \widetilde{\Xi}_{\mathrm{Q}}(\bar{\Theta}) \right| / \widetilde{\Upsilon}_{\mathrm{Q}}(\bar{\Theta}), \tag{5.2.13}$$

where the curly lines indicate that in order to avoid issues with the levels of each element in the matrices I first transform the Q matrices elements by considering only the ranking across actions on each state. Otherwise, I would have found difficulties to the rewards obtained from the data with those given arbitrarily by the trader[22].

Calibration targets

Four approaches will be considered, the three seen in the previous section within the context now where a machine learns on its own the strategy management as explained above, with the addition of the *Avatar* approach:

1. Maximization of the performance by the end of the sample:

[20]Which contains the version in detail of the vaguely defined strategy management policy by filling the gaps through the value iteration update.
[21]See Appendix B for further information.
[22]This is, I am more interested in the distribution of payoffs across actions than on their levels.

$$\Theta_{\mathrm{Q}}^{\mathrm{P}} = \underset{\Theta}{\mathrm{argmax}} \rho_{\mathrm{Q}}(\Theta) \tag{5.2.14}$$

where $\rho_{\mathrm{Q}}(\Theta)$ represents the vector of end-of-sample performances across the managed strategies.

2. Maximization of the yearly average returns throughout the sample.

$$\Theta_{\mathrm{Q}}^{\mathrm{R}} = \underset{\Theta}{\mathrm{argmax}} \bar{R}_{\mathrm{Q}}(\Theta) \tag{5.2.15}$$

where $\bar{R}_{\mathrm{Q}}(\Theta)$ represents the vector of yearly average returns across the managed strategies.

3. Maximization of several features related to performance analysis through the previously proposed instantiation of the utility function that now adds the so-called reasoning-convergence.

$$U_{\mathrm{Q},t} = 100(\bar{R}_t + 0.1D_t + 0.01\xi_t + \Xi_t), \tag{5.2.16}$$

so that

$$\Theta_{\mathrm{Q}}^{\mathrm{U}} = \underset{\Theta}{\mathrm{argmax}} U_{\mathrm{Q}}(\Theta) \tag{5.2.17}$$

where $U_{\mathrm{Q}}(\Theta)$ represents the vector of utilities across the managed strategies.

4. *Avatar*. – Maximization of the $\Xi(\bar{\Theta})$. This is, the portfolio manager selects those agents that could be considered her avatar in the non-linear world defined by the set of rules proposed above.

$$\Theta_{\mathrm{Q}}^{\Xi} = \underset{\Theta}{\mathrm{argmax}} \Xi_{\mathrm{Q}}(\Theta) \tag{5.2.18}$$

where $\Xi_{\mathrm{Q}}(\Theta)$ represents the vector of Reasoning Convergences across the managed strategies.

As mentioned, I take the top 10 strategies of each prior and distribute the budget across them as per the Kelly Criterion.

	Mean-reverting				Momentum			
	Θ^{P}_{Q}	Θ^{R}_{Q}	Q^{U}_{Q}	Θ^{Ξ}_{Q}	Θ^{P}_{Q}	Θ^{R}_{Q}	Θ^{U}_{Q}	Θ^{Ξ}_{Q}
Performance	178%	174%	143%	141%	185%	175%	170%	152%
Average return	12%	14%	12%	9%	15%	16%	16%	10%
Utility	166.8	170.3	176.1	164.2	175.5	178.1	178.7	167.4
Max. Drawdown	-16%	-15%	-28%	-16%	-15%	-15%	-16%	-14%

Table 5.3: In-sample results for the parameters selected through the reinforced calibration approaches.

	Mean-reverting				Momentum			
	Θ^{P}_{Q}	Θ^{R}_{Q}	Θ^{U}_{Q}	Θ^{Ξ}_{Q}	Θ^{P}_{Q}	Θ^{R}_{Q}	Θ^{U}_{Q}	Θ^{Ξ}_{Q}
Performance	97%	97%	96%	102%	102%	107%	106%	108%
Average return	4%	10%	8%	13%	6%	8%	9%	21%
Utility	59.50	69.07	73.72	83.01	69.16	72.77	75.30	93.64
Max. Drawdown	-20%	-18%	-22%	-19%	-20%	-18%	-17%	-14%

Table 5.4: Out-of-sample results for the parameters selected through the reinforced calibration approaches.

Results

Table 5.3 shows why the reinforced calibration approaches would bias in-sample towards the momentum interpretation of the signal. This is, each optimization domain is equal or better in each feature considered when the trader interprets that the market maker is able to anticipate larger movements in the prices.

As shown in table 5.4 the negative effects of overfitting seem not to be completely avoidable out-of-sample. Again overfitting leads to losses in the case of mean-reversion across Θ^{P}_{Q}, Θ^{R}_{Q} and Θ^{U}_{Q}. But this time they are lower: between 3% and 4%. The momentum interpretation instead still outputs gains. As expected, these gains are lower than those in-sample due to the nature of overfitting. But nevertheless now higher gains than those reached in the previous analysis are now available. Lower losses and higher returns are a signal that overfitting has been partly mitigated. This is largely due to the introduction of larger frozen periods due to the agents' strategies management (in the greedy one they would only appear in the case of risk management). Basically, the agents trade less than the raw strategy would but with a higher average return. Hence, another upside of this strategy is that it does not require constantly the trader's budget. This approach hence allows releasing budget towards other sources of opportunities (such is typically a benchmark that the portfolio manager has to overcome).

An interesting case that outstands in both tables is the *Avatar Calibration*, Θ^{Ξ}_{Q}, approach. In-sample it is the one that on average renders the poorest results whether the interpretation of the signal is mean-reversion or momentum. This is by nature the expected case for a not overfitted calibration process. Hence it should render better results than the rest of alternatives out-of-sample. And that is robustly the case for both interpretations of the signal as it can be seen in table 5.4. Note that it is the only technique that outputs gains (up to 2%) in the mean-reverting interpretation which are in any case overcome by those of the momentum interpretation (8%).

In fact, when compared to the rest of methodologies the Avatar approach outperforms in every feature of the mean-reverting interpretation and all but the maximum drawdown of the momentum interpretation.

Last, the use of several strategies at once (up to 10) instead of abiding by a unique instance of the calibration volume is expected to benefit from a larger diversification (hence, robustness) and nevertheless, costs internalization that have not been included in this experiment.

Summary

A relatively simple novel approach to calibration has proved to be better than the standard techniques in my data. By introducing the trader's strategy management priors into the calibration the out-of-sample results can be improved substantially. For doing so it is necessary to let independent agents define autonomously a strategy management policy.

The trader then simply needs to select those agents that do mimic the most her own policy. This approach has been called the *Avatar Calibration* precisely because those agents resemble the same reasoning than the trader within their specific environments derived from the combination of parameters.

5.2.7 Summary of results

Follows a series of bullet points that synthesize the results obtained along the experiment:

- As seen in table 5.1 the standard calibration approach motivates the mean-reverting interpretation in-sample. It exhibits performances in excess of 200% with respect to the 149% of the momentum interpretation. However, it suffers the most when tested out-of-sample (table 5.2) reaching losses across the three scenarios considered (between 11% and 22%) while the momentum interpretations yield a 5% return.

- The usage of a risk management policy defined through Reinforcement Learning reduces the differences across the two interpretations in-sample (see table 5.3). Interestingly enough, now the momentum interpretation is better than the mean reverting within a range from 1% to 27%. Having seen the out-of-sample results of the former bullet point it seems that the RL may have led to a valuable correction. When observed out-of-sample I confirm that again the momentum interpretation is superior in my sample, as shown in table 5.4. It is also important to note that the mean-reverting deviation is now lower (with losses between 3% and 4%) while the momentum interpretation still yields positive returns (between 2% and 7%).

- Finally, by using the *Reasoning Convergence* as a link between the data-driven and the theory-driven approaches I conclude that the *Avatar Calibration* may be a more robust out-of-sample technique than those seen above. Several reasons motivate the statement:

- First, it also favours in-sample the momentum interpretation over the mean-reverting one (141% vs 152%);
- Second, it is the only one that yields positive returns out-of-sample when the mean-reverting interpretation is considered (2%); and
- Third, it yields the largest returns out-of-sample (8%) with also the largest average return and utility, and the lowest maximum drawdown.

5.2.8 Summary

This experiment attempted to complete the previous two by showing what the main challenges behind further proprietary enhancements are.

First, a novel indictor was created: the *Sample Sensitivity Index.* It is expected to show the distribution of changes in price (volume-weighted) per unit traded. By focusing on the outliers of that distribution I expect to find dynamics that do not obey to market impact. And that could have value in predicting future movements. More subtly, I need to define the trigger that discerns between what an outlier is and what not. As *a priori* I do not have any insights I have to leave it as a parameter to my model to be set through a data-driven approach. Moreover, the interpretation of the signal, whether mean-reverting or momentum had also to be left to the data as large movements in the prices adjusted by volume can mean overreaction of the markets (hence resiliency) or the beginning of a new trend.

Second, a series of standard benchmarks were calculated for each interpretation of the signals derived from the index: the maximization of the performance at the end of the sample (i.e. the last observation is crucial), the maximization of the yearly average return throughout the sample (i.e. it mitigates the relevance of the last observation) and the maximization of the utility of the trader when performance is strictly preferred to maximum drawdown which is subsequently strictly preferred to Sharpe Ratio (i.e. the distribution of returns matters). These were calculated and used as benchmark. The results out-of-sample (table 5.2) are opposite to those obtained in-sample (table 5.1) where the mean-reverting approach seemed to be the most appropriate interpretation of the signal.

Third, a Reinforcement Learning approach was proposed to add a layer with a self-defined risk management policy. There is one self-defined policy per combination of parameters (take-profits, stop-loss and trigger for the outlier). These affect the performance of the strategy (hence results can differ from the previous analysis used as benchmark) and are enclosed within a so-called Q-matrix (hence, it is information that can be further considered). Out-of-sample results (table 5.3) are more robust with respect to the in-sample figures (table 5.4) than those embedded in the standard benchmarks. But they can still yield losses if the wrong interpretation is considered (in my sample, the mean-reversion).

And finally, a new methodology was proposed: the *Avatar Calibration.* The rationale is that those self-defined risk management policies that make sense to the trader (who has the insights) the combination of parameters that have defined them shall yield more robust strategies out-of-sample. The comparison of the aforementioned Q-matrices with the prior for an acceptable risk management policy to the trader is what defines what is acceptable and what not - an approach that was called *Reasoning Convergence*. This way, a new calibration model is defined through the combination of the freedom in the parameters selection from RL (data-driven) and the robustness of the insights from the traders (theory-driven). Under this scope, results are positive out-of-sample upon both interpretations for the first time along the experiment. This underpins the claim that overfitting can be prevented by restricting the data-driven calibration universe through the expert insights. The challenge hence becomes to account for a bespoke expertise on both domains: data-driven optimization and the field in which the experiment is defined.

As a result, the target number one of the thesis was achieved.

5.3 Further enhancements

Unlinke in the previous experiments, this Section does not refer to further 'proprietary' enhancements because the whole chapter is already dedicated to the proprietary domain. Still, there are clear dimensions to be further analysed:

- The *SSI* can be improved by considering more granularities: intraday information about price changes and market impact so that finer dynamics can be identified and flagged.
- The *Reasoning Convergence* can be improved considering more sophisticated measures of convergence between the trader's prior and the Q-matrices.
- More accuracy could be achieved if I moved from the Q-matrix onto a Q-hypercube.

5.4 Summary

A whole process for proprietary trading has been thoroughly analysed in this experiment upon the following building blocks:

Indicator. A new indicator, ζ, has been proposed to scan the dynamics of the average change in price per traded share right before the closing auction in order to allow the trader both to deploy HFT strategies up to the closing auction and invest a large amount of money.

Signal. Extreme events in the indicator's dynamics are seek by the trader in order to find shifts in the expectations of the market participants. And this could be a signal for two opposite scenarios: market makers' overreaction or momentum. The lack of financial views as to what can be considered extreme deviation and what not transfers its definition into one more dimension to be calibrated within the strategy.

Rule. On the mean-reverting interpretation of the signal I sell the positive extremes and buy the negative ones. Opposite for the momentum interpretation (that won't necessarily deliver the inverse performance of the former given the non-linear effects of the risk management policy).

- *Risk management.* – The fixing of the take-profit and stop-loss rules would also be left to the calibration process. However, holding periods are going to be restricted to one week at most.
- *Inventory management.* – Inventory is not allowed to grow when several signals of the same side are sent consequently.
- *Money management.* – The budget is allocated across strategies through the Kelly Criterion.

Calibration. Two types of calibration have been proposed herein:

- *Non theory-driven.* – The classical calibration process that optimizes a certain feature of the performance or several together through the utility function. In this experiment I have distinguished between:
    - Standard: back-testing the strategy as-is for any combination of parameters (Θ^{P}, Θ^{R}, Θ^{U}).
    - Reinforced: back-testing the strategy reinforced through a new strategy management learning approach for any combination of parameters ($\Theta^{\mathrm{P}}_{\mathrm{Q}}$, $\Theta^{\mathrm{R}}_{\mathrm{Q}}$, $\Theta^{\mathrm{U}}_{\mathrm{Q}}$).
- *Theory-driven.* – A further enhancement of the reinforced approach, the *Avatar Calibration*, where the trader selects those strategies whose autonomous agents deliver an implied strategy management that mimics the most the one with which the trader feels comfortable. This gives rise to a new set of parameters, $\Theta^{\Xi}_{\mathrm{Q}}$.

It can be seen in tables 5.1 and 5.2 that in my data the momentum interpretation of the signal behaves better than the overreaction interpretation. Further, the reinforced approach improves the results of both interpretations (tables 5.3 and 5.4). And more especially, the theory-driven version of the reinforced approach, the *Avatar Calibration*, provides a valuable feature to the trader: it allows her to affect the calibration process with her own criteria and trading style. My data sample exhibits positive results about its better behaviour out-of-sample (tables 5.3 and 5.4). This hence allow me to finally meeting **target number one** of the thesis.

Caveats

The process described herein is largely demanding in data. If there is not enough data some of the scenarios within the Q matrix won't get visited so that convergence won't be achieved and noise will be added onto the strategy management policy.

As already mentioned, in order to avoid further calibration of parameters it has been considered the R matrix to remain constant throughout the experiment along with the transition probabilities. They have been all calculated in-sample and used along the out-of-sample analysis.

Chapter 6

Conclusions and future work

This chapter concludes the thesis. It starts by describing the overall conclusions reached along the research followed by a series of particular results to be highlighted. Finally, subsequent paths to be further explored are briefly considered.

6.1 Conclusions

The thesis has successfully brought to the foreground a series of methodologies that could for the first time explain two recent puzzles of the financial literature. First, it explains a scientific technique that underpins the disruptive 50% discount observed in the markets by Menkveld (2013). Such technique has been backtested against a full-replication benchmark with robust, positive results in my data sample. Second, and given the backtesting essence of the former, the thesis gives a way to solve a major challenge in quantitative trading, the mitigation of the overfitting problem present in any backtested scenario as exposed by Bailey *et al.* (2013). It does so by changing the target function from the usual greedy calibration processes onto another one where the focus is the nature of an agent that would autonomously learn within the universe defined by each combination of parameters. Last but not least, a realistic framework for the aforementioned backtesting of the experiments (as demanded by Cahan *et al.* (2010)) has been achieved by including a thorough analysis of the execution strategy. I took such a requirement as an opportunity to attempt to improve one of the most popular and challenging execution algorithms: the VWAP. The cornerstone of the algorithm is the intraday volume profile for which little financial insights can be considered. After proposing a new index that allows discerning which areas of such profile shall be more or less sensitive to past data I obtained robust, positive results during non-US market hours. The methodologies discussed throughout the thesis are based on machine learning and crucially benefit from the possibility to merge the priors of the trading experts with the edge of data-driven techniques.

As such, the objectives set out for the thesis have been met and completed with further results.

Overall, it has been mainly concluded that:

- It is possible to find a reliable procedure that allows a trader discern which parameters' combination is suitable out-of-sample. Since the classical calibration approaches typically imply strategies whose risk management seem a corner solution (i.e. to remain trading a signal no matter the recent performance of the strategy) they tend not to be reliable out-of-sample. Out-of-sample performance can be favoured by using Reinforcement Learning along the calibration process in the selection of those agents whose autonomously learnt policies most resemble that of the portfolio manager – meeting **objective number one**.
- It is possible to quote more aggressive bid-offer spreads than those for a given level of risk assumption by properly addressing the nature of the product's flow and the trade-off between deviation risk (tracking error and tail risk) and market impact – **objective two**. The perception of the risk embedded in the nature of a product's flow instead of the isolated risk present in each trade is crucial. Metaheuristic computational methods for optimization, as is the case of Particle Swarm Optimization, do account for a convenient way to drive calculations along the search-space of deviations and market impact based on the trader's acumen – which, in turn, seems to be a good practice towards a more robust out-of-sample performance. Overall, it can hence be argued that the market may be giving liquidity to certain instruments through synthetic/statistical approximations to those what enhances the liquidity of the markets but increases the risk taken therein. In that sense, the industry may need to adapt to a disruptive trading approach and create new pivotal units such is the global, cross-asset BBVA's Global Strategies & Data Science – **objective five**.
- There are still scientific methodologies that can improve our approach to VWAP – **objective three**. By simply creating an index upon support vector machines that adds the flexibility of being more or less sensitive to the data sample a series of intraday data-driven patterns can be exploited. The inclusion of this technique (that leverages in the management of the so-called big data) seems a natural way to improve the realism of the medium-frequency experiments – **objective four**.

In particular, I have reached the following conclusions in my database:

- The optimized hedge of IBEX 35 can blend Spanish and non-Spanish securities such is the CAC 40 future.
- A highly risk-averse market maker of IBEX 35 futures would still consider to assume the deviations of an optimized hedge after a flow of more than 10 trades is granted.
- Market makers expectations' shifts seem to generate robust signals with regards to the future momentum of the markets.
- The use of Particle Swarm Optimization allows to reduce the number of dimensions of the hedge optimization problem from the number of stocks in IBEX 35 to the number of sectors instead, yielding a faster optimization process.

- The comparison of volume profile estimators should remain orthogonal to the price evolution. In fact, the typically low intraday dispersion of the prices with respect to the volume blurs away the real deviations in terms of profile. Under such volume-only scenario, static estimators overcome dynamic ones in our data, opposite to what was previously found optimal.

- The dynamics of the second level of the order book seem to account for most of its relevant information in terms of price discovery. This has relevant consequences not only at a data management level but also on the development and setup of intraday strategies – **objective seven**.

- Hidden liquidity typically remains recurrently throughout a limited time frame. This gives the possibility to, if properly spotted, improve the execution of a trade – **objective seven**.

- Most of hidden liquidity is present within the best bid-offer spread. However, when it is present within deeper levels of the order book it tends to account for larger amounts than within the bid-offer – **objective seven**.

- The overlapping with US market hours generates noise in the detection of volume patterns.

- The capitalization rank of a stock included in a main index affects the detection of its volume's patterns. It adds noise especially in the mid-ranked companies. This is, the effect seems to be non-linear across stocks as it can be seen by the concavity of the deviations hence the commissions for risk trades on these stocks should not be flat – **objective six**.

The corner stones for the previous conclusions have been a series of novel concepts whose origin blends different disciplines ranging from microeconomics to robotics:

- *Elasticity of the Flow* is a concept that attempts to synthesize the nature of the interest on a security.

- *FlowVaR* is a measure that embraces the tail risk of a strategy and the effect of the Law of Large Numbers; and this allows to include the role of the *Elasticity of the Flow* within the bid-offer spread of a security.

- *Sample-Sensitivity Index* is a classification-SVM that adds dynamics to the volume profile non-linearly.

- *Expectations' Shift* is an indicator that can be fine tuned to reflect the changes in expectations from the market participants; and it generated positive performance in my data – more so when *Avatar Calibration* was used.

- *Reasoning Convergence* is a metric for the similitude between the policy autonomously learnt by an agent and that of a portfolio manager in order to identify those that mimic the most the behaviour of the trader.

6.2 Future work

The thesis has reached a large breadth of computational finance hence research can be pursued along several dimensions. In particular, it could be said that future work could focus on:

- The increase of the depth of the approaches. Once the framework has been structurally set out it is possible to compare the different results with those from different versions of the same algorithms in order to analyse the benefits and caveats from each type. E.g. the use of Multiple Kernel Learning to improve the performance of the SSI within the algorithmic trading experiment or the use of Multiobjective PSO within the experiment of market making (as in Diez *et al.* (2012)).
- The increase of the frequencies of the last experiments. The thesis moves gradually from intraday trading to end-of-day trading hence there is room to increase the frequency of the strategies deployed in the market making and proprietary trading experiments.
- The usage of asymptotic theory (the branch of statistics that studies asymptotic expansions) to further improve the definition of the *flowVaR*.
- The use of machine learning to extract information from the dynamics of the order book and improve this way the algorithmic execution – especially through the detection of hidden liquidity as explained in Chapter 3.
- The calibration of the distribution of the rest of the agents present in the order book (e.g. whether aggressive or passive) would favour the robustness of the results achieved during the testing process.
- The improvement of ultra-high frequency trading, that could ultimately benefit from a physical/chemical improvement of the hardware's performance (Colwell (2004)).

Appendix A

Trading industry review

Figure A.1 largely classifies the set of skills that my proposed approach to advanced market making requires both in terms of the type of flow and the needs for inventory management.

Flow trading: even though any of the approaches represented have their own role within both types of flow the usual ones involved in elastic flow are high frequency trading, arbitrage and ultrahigh frequency trading (further explained below). Inelastic flow on the other hand, would typically be built upon statistical arbitrage and algorithmic trading.

Inventory management: quantitative portfolio management (QPM) is used in both types of flow in order to remain as market neutral as possible even though it is run differently on both as pointed out in the Introduction.

Follows a review of the trading industry that motivates the usage of such structure. After comprehensively analysing the parallel evolution of the industry along buy and sell sides both flow trading and inventory management are presented in detail. Last, the systematic trading platform considered along the experiments is fully disclosed.

A.1 Evolution of the trading industry

Overall, the evolution of trading could be seen as supplemental between the so-called buy and sell sides. While the former has largely developed the area of risk/reward in the attempt to optimize the return on the funds that they raise the latter has in turn evolved the quoting and execution policies in the attempt to profit from the activity of liquidity provision (mostly, fees and spreads). However, the proliferation of vendors that offer execution services to the buy side along with the current reduction of the bid-offer spreads that denotes the use of risk-reward techniques in the optimization of the quoting policies of the sell-side is eroding the supplemental relationship and transforming it into an economic model of vertical integration.

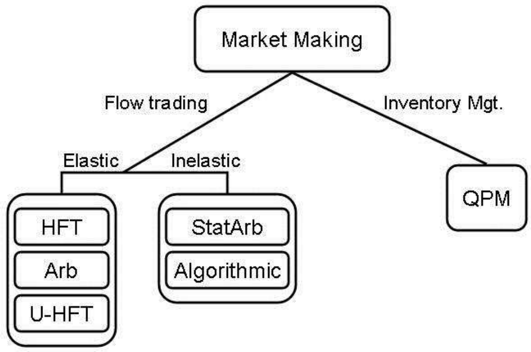

Figure A.1: Broad scope of market making

Sell-side: from discretionary to automated trading

At first, order management systems (OMS) were deployed to deal with the whole trading process, from the access to the markets to the tasks triggered by a trade[1] Later, in the early 00's the improvement of electronic trading rose the need for a broader system that also accounted for execution management. This broader platform received the name of execution management systems (EMS) and it soon embraced new (more complex) features. With the setup of algorithmic trading not only did the business become more complex in terms of both maintenance and systems' intelligence but also due to the amount of trades that ought to be booked at high frequency[2].

The rationale behind the evolution from discretionary trading to automated trading within the sell-side builds upon a discussion on several dimensions (most of them already enumerated by Johnson (2010) in reference to algorithmic trading):

- **Efficiency:** it allows to improve the trading capacity of the sell-side agent and manage a larger amount of operations per unit of time than manual trading. Also, a more frequent self-analysis and adaptation due to its enhanced speed. However, it requires a period of adaptation and a large investment in the short run – although it could be diluted given its economies of scale as we will see below.
- **Usability:** robustness of the automatic processes is key to avoid human operational risk and to release as little information as possible in the markets (anonymity can also be enhanced via random processes). Also, although a model may be the best way to setup an on going

[1]From the reporting of the trades to the associated counterparties and market authorities and the clearing/validation of both trade and settlement details to the settlement of the trade itself, whether it is physical or cash.

[2]A day of HFT on a liquid currency could easily require as many trades to be booked as the whole trading floor of discretionary traders during a complete year.

adaptation to the market dynamics as we will see below it adds *model risk* and may reduce the intuition behind the performance of the algorithm[3]. Real-time and post-trade analysis are usually deployed to check that the real performance is similar to the one expected by the model.

- **Cost:** even though at low frequencies the efficacy of manual traders (systematic or not) may be the same as the one provided by automatic agents, at high frequencies the latter has an edge. In general, the cost of that advantage is more expensive than hiring a manual trader in the short run but it is less expensive on an on going basis as, once it has been setup, it usually saves human capital (that can instead be devoted to further tasks) and it can be scaled-up at a lower cost.

- **Regulation:** requirements such is *best execution* are giving relevance to the importance of algorithmic trading as the most suited vehicle for execution. Moreover, with regulators preparing rules under the 2010 Dodd-Frank financial report towards exchange-like systems to improve transparency there have been recent changes in the industry that lead to a shift from discretionary trading to electronic trading across all assets (Goldstein (2012)).

Buy-side: from low frequency to high frequency trading

Even though Instinet's *Institutional Networks* allowed electronic block-trading for the first time in 1969 it was not until the 80's that electronic trading competed with the pitch floor for the activity (flow) in the financial industry. At its beginning it had the crucial advantage of being more efficient in crossing orders than its competitors but there was soon another positive feature that gradually became its cornerstone: the availability of reliable and easy-to-access information (in especial, historical data) allowed a new school of buy-side traders to analyse patterns in a more disciplined way.

This new approach to trading resulted first in what were called complete trading systems (CTS), a disciplined approach that was translated into rules that became progressively more sophisticated and began to include a more formal, scientific approach[4]. It rendered the seed for what was later known as quantitative trading (QT). As a result, in the 90's one of the most popular strategies was born: statistical arbitrage (StatArb), which mainly consisted on the analysis of mean-reverting patterns along medium to long run horizons, typically based on the use of daily candles (bars with information about maximum, minimum, open and close of the day). Soon, its growing

[3]The Flash Crash is an example of the risk assumed when humans do not perfectly understand the behaviour of the modeled algorithms. Apparently, it was caused by a too aggressive execution that obeyed to an algorithmic trader who did not put enough attention to the estimation of the market impact of a large trade that was being placed in the markets. As a result, one of the latest industry debates revolves around the substitution of the current traders by a new school of trading, trained in financial computing, who are capable of understanding in depth the details of the trading systems.

[4]One of the most popular CTSs was the secretive set of rules known as Turtles that were based on graphical analysis of the past history of the assets followed with precision (i.e. systematically) by a small team of Wall Street traders. Its success was said to be a proof for the advantage of discipline over intuition in trading – see Kahneman and Tversky (1979) and Barberis *et al.* (1998) for early analyses of the role of the investors' sentiments on their financial behaviour.

popularity derived onto the erosion of its margin and a quest for new profitable patterns was naturally triggered. The systematic market participants devoted their research along the '00s to a new type of trading at a shorter horizon: intraday trading. Intraday trading's barriers to entry remain active today since the analysis of its data does not only include extra costs[5] but it is also intensive in code and database management – being both demanding skills that were not always mastered by the quantitative traders of the early '00s. As a result, a new profile of traders was demanded in the markets and throughout the rest of the decade high frequency trading was born.

Vertical integration: buy-side expansion vs sell-side contraction

It is apparent that the current evolution of the financial industry is certainly moving towards the economic concept of vertical integration[6]. The irruption of the EMS off-the-shelf platforms is allowing buy-side agents to forward-vertically integrate the tasks necessary to access the markets directly with the right booking and risk analysis systems in order to reduce the payment of sell-side fees. On the other side though, there are no off-the-shelf platforms for the risk/reward strategies so the opposite integration, the sell-side backward-vertical integration (selling their proprietary strategies to final clients) is not happening so broadly yet[7].

The evolution towards more frequent patterns mentioned above seems to have reached its limit now with HFT (and, more eloquently, with Ultra-HFT). I expect the next step to be a shift on the focus of the industry onto the intelligence of the algorithms (as opposed to their speed as it has happened so far); and this shift shall bring patterns with higher rewards at the assumption of higher risk as well - a tandem that shall be optimized with techniques that favour out-of-sample stability such are those proposed herein.

A.2 Flow trading and inventory management

As seen in Figure A.1, the trading pillars of market making can be divided into two: flow trading and inventory management.

Flow trading

1. **Inelastic flow:** quote driven markets[8] would typically generate this type of flow. It is usually linked to investors with mid to long-run investment horizons who wish to access a product that has no electronic order book (e.g. bespoke indices) or who demand a size that would shift the order book if it was openly published in it (e.g. block trades). A practical

[5] Especially back in the early 00's but still relevant in the beginning of the '10s.

[6] Where firms do absorb several stages of their production process in order to reduce costs.

[7] A pure backward-vertical integration would imply sell-side agents taking themselves buy-side decisions a case that I won't consider in the thesis especially after the so-called Volcker rule – a specific section of the Dodd–Frank Wall Street Reform and Consumer Protection Act that prohibits a bank from engaging proprietary trading.

[8] Those where only the bids and asks of the market makers and other designated parties are displayed, as opposed to the more popular order driven markets where every order is published.

example within the sell-side would correspond to a scenario where a sales team pitches a proprietary index to a *set of clients* who are finally keen on it. Consequently, they trade-in *large amounts* on the *same side* along a short period of time and trade-out of their positions *progressively and independently* between the mid and the long run. As a result, the market maker of such proprietary index would be facing a low-flow, large kurtosis, asymmetric and dynamic distribution.

- **Types of exposure:** any strategy, no matter its level of complexity nor whether it has been motivated by fundamental, technical or quantitative analysis, falls into one of the thefollowing categories:
  (a) **Market exposure:** which, at the same time, is two-fold as it comprehends momentum or mean-reverting views. The former is applied when the trader wants to get full exposure to the market, whether motivated by an optimistic (bullish) or a pessimistic (bearish) view on its evolution. The latter in turn refers to the view of a market correction from an occurred movement (typically an event such are news or corporate actions) that has been opposite to the expected momentum. In market making, traders are usually not allowed to take such approach[9] as they have to remain market neutral.
  (b) **Market neutrality:** it occurs when the trader wants to remain as neutral as possible to the market. In market making the trader incurs in this type of risk when the full-replication approach leads to quotes that are not aggressive enough to remain competitive – e.g. when there are market access restrictions, when the market's competition has driven lower the spreads through optimized hedges, etc. It assumes the downside of managing tracking error risk motivated by the upside of trading a related instrument with more convenient properties – e.g. more liquid, cheaper in terms of costs and market impact, etc.
- **StatArb:** Statistical arbitrage is a strategy that typically falls into the category of market neutrality type and whose cornerstone is the exploitation of mean-reverting cointegrated time series of the deviations[10]. Those cointegrating relationships, when backed by financial theory, are seldom broken being the exceptions typically triggered by events (e.g. news) that affected asymmetrically the different instruments involved in the StatArb strategy. When not backed by financial theory, such is the case of PCA-StatArb (Álvarez-Teleña (2010)), the relationships tend to be less robust out-of-sample[11].

  It is also relevant to understand that StatArb strategies are more difficult to manage in the market making activity of the sell-side than in the buy-side since they are taken as

[9]When a sell-side trader has not hedged yet her position is eloquently said to hold a *naked position*.

[10]Not to confuse with mean-reversion market exposure as the mean-reverting patterns analysed herein are defined upon the tracking error instead of the security.

[11]The analysis of the difference between data-driven and theory-driven patterns will be present throughout the rest of the thesis.

a response to the activity of the latter – i.e. its timing is externally driven. Last but not least, given that there is TE management largely affected by exogenous variables that limits the flexibility to set stop-loss and take-profit policies, running the trader's portfolio globally (i.e. diversification across TE relationships) could likely help the trader not to suffer on average from extreme events along the year, since by bringing orthogonal risk, individual performances are easier to net-off; and this favours a larger neutrality to the market movements (hence the role of QPM).

- **Algorithmic trading:** as detailed in the next chapter even though it is delivered at high frequencies it is typically linked to mid and low frequency flow. AT was born to reduce the market impact through different strategies (e.g. linear slices as in TWAP, non-linear as in VWAP and random drifting as in Percentage-of-Volume) and soon diversified towards transaction costs minimization (e.g. Implementation Shortfall) and liquidity-seeking across venues such are exchanges, dark pools, alternative trading systems and electronic communication networks (e.g. Smart Order Routers).

  As said, currently most of the algorithms – in special those related to market impact – have been commoditized and are now offered *off-the-shelf* by different vendors[12]. The execution trader hence usually faces the challenge of how to fine tune the structure of standard strategies with open code (disclosed by the vendor) to benefit from an already-in-place framework at the price of finding restrictions in terms of flexibility given the softwares' own structure.

  TWAP and VWAP are probably the most popular algorithms for being intuitive and having perhaps the lowest model-risk. While TWAP slices the order evenly along time, VWAP does it based on a estimation of how the intraday shape of the percentage volume traded is. The latter has reached especial popularity due to the well-known fact that the seasonal pattern of the volume intraday is robust which is why it should be controlled in any trading activity. However, the intraday volume's profile becomes a core feature to determine the accuracy of the algorithm hence even when off-the-shelf packages are bought by execution traders these typically want to have ownership on its calibration. The main experiment of the next chapter is precisely devoted to this task.

2. **Elastic flow:** when the market maker faces elastic flow, the source of her margin in the markets is mostly heavily biased towards the bid-offer spread rather than tracking error management. As said above, this scenario would for example fit the one faced by a market maker who submits quotes in an electronic order book of a highly liquid instrument.

   - **Market microstructure**

     It is the limit order book's dynamics what ultimately drives the price discovery of most of the instruments. Being the elastic flow the most sensitive to marginal variations of the market price and asymmetries of the bid and the offer around the fair price, I

[12] They can also be rented out as black-boxes, however this cost usually erodes a large part of the P&L of an execution trader, given the tight margins involved in AT.

Total	Orders	Size	Bid	Ask	Size	Orders	Total
7,000	3	7,000	6.433	6.438	12,629	2	12,629
17,389	5	10,389	6.432	6.441	3,105	1	15,734
30,468	5	12,629	6.431	6.442	5,503	2	21,237
39,383	3	8,915	6.430	6.443	5,234	2	26,471
44,208	2	4,825	6.429	6.444	6,602	3	33,073

Table A.1: Snapshot of an order book

should first set the core background relative to the main features for a market maker at a microstructure level.

The limit order book represents the market interest around a the best quotes of bid and offer. The interest is published through limit orders that rest within the order book until an opposite interest is matched[13]. Usually, the number of different bids or offers published ranges up to ten, the 10^{th} depth-level, although usually beyond the 5^{th}-level onwards is typically disregarded by traders – next chapter includes an analysis of which one is the level that is expected to gather most of the information significant for price discovery.

It is relevant to highlight the fact that there may be a further, hidden interest at each level of the order book than that published – also analysed thoroughly within the next chapter. In fact, the existence of hidden liquidity clusters, usually posted through certain orders typically available at the exchanges[14], can affect the response of the order book (of its limit orders) to an exogenous stress (such is market impact) until the affected levels bounce back – being it referred to as the *resiliency* of the order book[15]. Even though it is not a usual case, it is also true that there could be instead less real interest than the one being shown: some agents alter the order book by posting limit orders with the only target of gathering information about the rest of the agents' interest – being it called *spam*.

- **Back-testing limitations**

  There are some issues to take into account when validating the performance of back-tested high frequency strategies. Not only the transaction costs and the market impact (usually negligible for the targeted sizes at this frequency) have naturally to be taken into account but also the fact that the chosen quotes may alter the order book's dynamics. This is, the quotes have an effect on the rest of the agents' strategies and, as such, the trader either develops a model that accounts for a sophisticated scheme for quotes discovery that takes into account Game Theory and Behavioural Finance or accepts that the model may need to be fine tuned once it is alive in the markets. In reality, the performance obtained in the controlled scenarios of back-testing is often taken by the

[13]This is the case of a continuous auction, the typical mechanism for crossing during trading hours. When there are open, closing or volatility auctions the order book is reduced to an equilibrium price where most of the interests would be matched – i.e. limit orders are not published at the order book during those periods.

[14] That allow hidding interest such are icebergs (iteratively show small pieces of a large trade), fill-or-kill (FoK) and inmediate-or-cancel (IOC).

[15]All terms that will be used along the rest of the thesis

trader as a benchmark. As a result, some of the back-tested strategies are first gradually tested on increasing sizes before they are definitely launched at the desired level.

- **Latency**

  The speed at which the traders' systems communicate with the exchange (FIX protocol) matters. Not only because these want to be the first to trade against an opposite limit order but also because the timestamp is key to decide what orders at a certain level are filled when there is a partial matching. This is, not only it matters when sending active orders (e.g. market orders, FoK, IoC) but also when sending passive ones (e.g. limit orders). However, not all the strategies are considered to be latency-sensitive, only those whose performance depends crucially on the latency levels such are those in the domain of ultra-HFT (UHFT) and more especially, Flash Trading[16]. As mentioned, most of quantitative HFT is not too sensitive to latency since the strategies typically target patterns that no one else is looking at and often account for some signalling mechanisms that require time before they are computed (e.g. machine learning techniques). Rules-based HFT though, being it easier to design and deploy, typically implies more latency-sensitive strategies.

- **Basics of a standard approach**

  Typically, the market maker would start placing limit orders in both sides of the order book (potentially along both depths). When those limit orders are touched (hit or lifted) the trader would start building up inventory. If the inventory's net position goes beyond a certain level (typically predefined in terms of market risk) the market maker would simply need to place asymmetric limit orders (around the fair value) to attract flow on the desired side as economically as possible[17] – i.e. instead of doing so through market orders since those require crossing the spread and do lack rebates.

  A delta exposure in the instrument or in a market neutral deviation risk is usual along the day while the market maker manages the trade-off between:

  (a) the market risk of waiting for a hit (or lift) of an opposite market order and naturally balance the inventory off, and

  (b) the cross of the spread and market impact of placing a market order that immediately achieves the inventory balance's net off.

  Typically, the longer the horizon of the investment, the larger the relevance of (a) with respect to (b) and *vice-versa*.

[16]Note that this type of trades are motivated by the fact that some exchanges allow a set of market participants view orders from others in the marketplace fractions of a second before the rest. As such, it typically generates robust performances similar to those of pure arbitrage (low volatility, not easily scalable) and there is little financial intelligence involved – it depends crucially on a continuous investment in technology systems, the optimization of the code, the hardware, the physical setup of optic fiber lines (as close to the exchange's servers as possible) and similar computer science/electrical engineering related strategies.

[17]Usually, survival analysis is used in order to balance-off the hit/miss ratio of the order type at each depth of the order book (Cox and Oakes (1984) and Kalbfleisch and Prentice (2002).

Inventory management

As said, the second trading pillar of market making is the inventory management which is highly related to the following:

1. **Risk management:** A market maker shall not target any proprietary P&L but just the fees directly charged to the client in inelastic flow and bid-offer spread (and rebates, if any) in the elastic flow. As pointed out above, the optimization processes motivated herein have as target not to suffer from the erosion of those – i.e. it shall be used as a prudential approach unlike the case of alpha generation (proprietary trading). As such, inventory management is a key part of the process that allows the trader managing the risks that could potentially erode her margin (from intraday naked positions to long-lived deviation risk).
2. **Diversification:** The trade optimization layer within systematic trading should ideally take into account the correlation between the risks that the trader is about to take and their role within the inventory already built up as well as the long-run risk profile of the institution. When there are no strategic allocation views the more the diversification of the risks assumed the better – targeting any further addition to be as orthogonal to the pool of living risks as possible seeking a zero return in terms of TE so that the pool of commissions and/or bid/offer spreads is not eroded as already mentioned.
3. **Internalization:** Not only opposite interests on a same security of different algorithms should be crossed to save exchange fees, market impact, information disclosure, etc but also the optimization of the global portfolio of inventories across assets should be performed to save costs. For example, by reducing the cost of borrow to zero on the stocks for which there are stable positions (likely related to mid and low frequency strategies) some patterns on those can be magnified and exploited.
4. **Systematization:** Not only systematization is key in terms of strategies' exploitation as it scales them across instruments and frequencies but more subtly, in terms of the enhanced risk diversification that such a cross scale generates. We will see below what the main building blocks of a systematic trading platform are.

A.3 A systematic trading platform

Narang (2009) is one of the most comprehensive (yet introductory) books on the structure that lies behind the idea of a so-called black box, a quantitative systematic trading platform[18]. Even though it is an area of finance that is progressively becoming clearer it stills remains unclear to many researchers and industry agents since the infrastructure required to manage tick data and fine tune the algorithms is still both complex (Manyika (2011)) and expensive.

I will abound below on a possible set of features for what I consider the main cornerstones of a systematic trading platform.

[18]See also Aldridge (2009) for an introduction to trading systems.

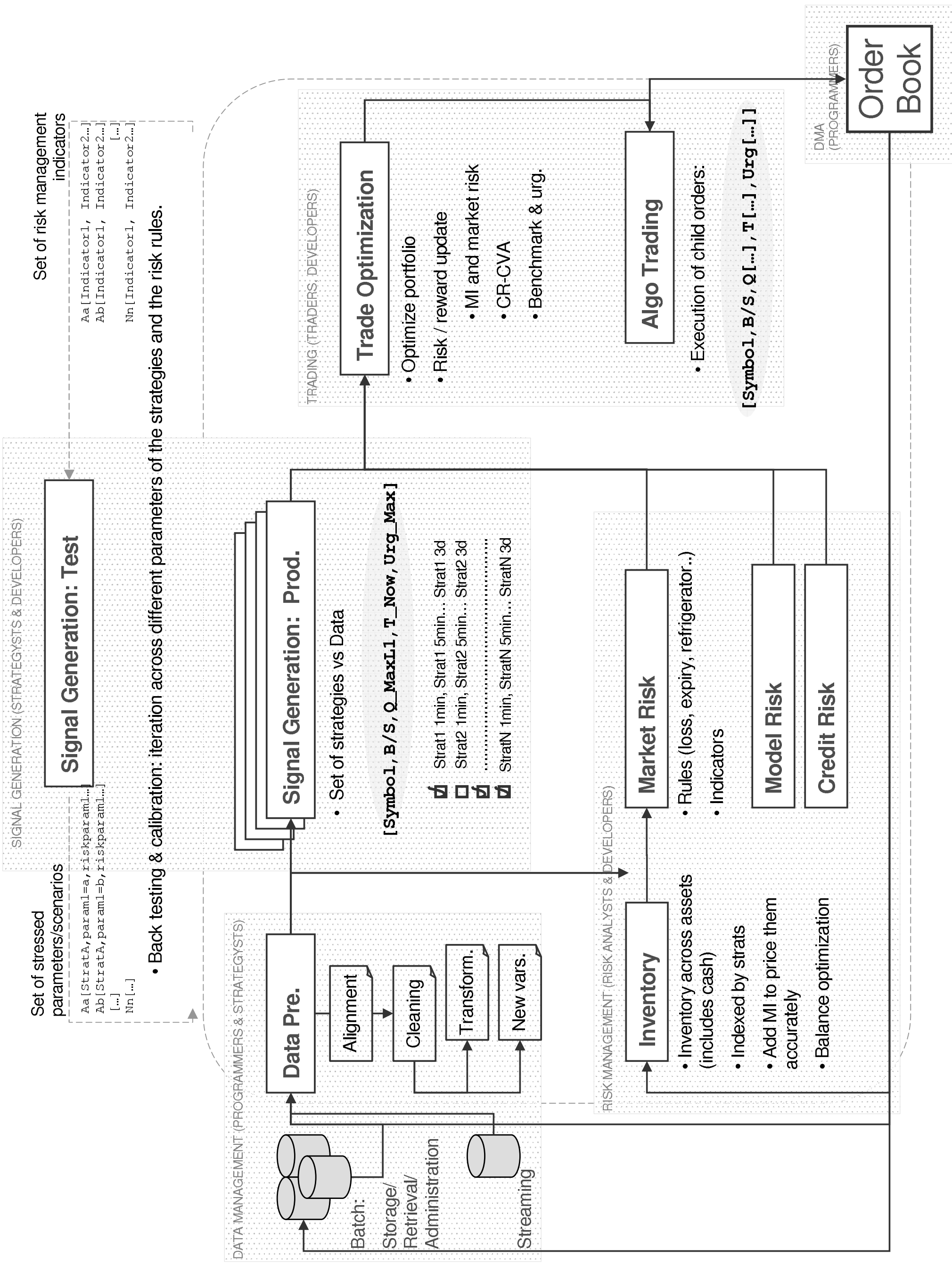

Figure A.2: A black-box inside-out.

	Buy side	Sell side
Finding alpha opportunities in increasingly competitive markets	20%	16%
Having to re-engineer models to keep them profitable	12%	7%
Managing real time risks	12%	12%
Finding capital capacity in increasingly crowded trades	9%	3%
Dealing with errors and equipment failures	7%	7%
Coping with high speed/high volume data feeds	7%	12%
Keeping up to date and compliant with regulatory changes	6%	8%
Achieving the target low latency	5%	8%
Controlling variance in latency during peak periods	3%	4%
Controlling risks of DMA or Sponsored Access	2%	6%
Coping with high throughput of orders (microbursts)	2%	6%
Other	15%	12%

Table A.2: Core differences between buy and sell-side approaches to systematic trading given their distribution of priorities.

Overview

As pointed out above, buy side and sell side have different issues and targets. Table A.2 encloses an average distribution of the market participants' concerns along the typical features related to systematic trading in both spaces[19]. It is interesting to note how the difference between buy and sell side is naturally saturated around the alpha-generation within a tight set of assets (in proprietary trading the investment universe is carefully selected) vs a fee-non-erosion approach across a wide variety of assets (in market making, the universe is increased on-demand). Figure A.2 shows where to allocate most of these features within a systematic trading platform as described below.

Data management

Most of the challenge that the step from daily to intraday systematic decision-making carries is the need for an efficient management of raw data. Its setup and maintenance is a task for developers and its scientific exploitation a task for strategists, both activities being challenging enough to be considered barriers to entry.

This block typically implies not only the management of the raw data itself but also its pre-processing.

Large raw databases: beyond its storage its efficient retrieval has to be properly addressed by the developers.

- **Storage:**

  To put it into context, we could say that while the strategies based on daily data would need around 252 data entries per year and instrument, where each entry typically embeds 5 numeric fields (such are the open price, close price, maximum along the session, minimum

[19] Figures estimated using Giffords (2011), a survey that gathered more than 500 responses from market participants around the world.

along the session and the volume) and 2 text fields (name of the instrument and date) a tick data system would easily require 75 million data entries per year and instrument[20] of 22 numeric fields (5 levels of depth on each side of the order book with two dimensions, volumes and prices posted, and traded amounts and prices) and typically, 3 text fields (name of the instrument, date and time).

This process is generally demanding enough in terms of computational resources to be treated as a "service" within the trading platform, i.e. it is usually deployed on a different server and communicates with the engine of strategies through an API in order not to affect its performance[21].

- **Retrieval**:

  Both types of access have to be considered: batch and on-line. The former would typically benefit from a column-orientation (simplifies indexing for long time series since data is stored as one-dimensional strings and sequentially read) and the use of AWK-like tools to manage and display large amounts of data efficiently – usually stored in compressed *.csv* files for a faster processing or already into column-oriented databases, e.g. Infobright or HBase. Aggregation is a core part of the retrieval and it requires several decisions to be taken by the trader. In fact, new variables are typically created in order not to lose relevant information of higher frequencies, e.g. the number of trades on each side that have been aggregated. On-line retrieval in low frequency strategies usually depends on the feeds of the standard data providers (e.g. Bloomberg, Reuters...) while the strategies related to the highest frequencies require the management of FIX messages sent directly by the exchange (DMA). There may be issues with the timestamp of the data callback service when large movements trigger enough number of ticks to collapse the exchange's information pipes so that blocks of information already aggregated are randomly sent. And this has to be properly treated.

Data pre-processing: usually, the data is first aligned at the smallest time granularity that the platform can manage (e.g. from tick data to a millisecond granularity), cleaned[22] and further transformed so that it complies with the core properties of the classical statistical analysis (independent and identically distributed data). Then, new variables can also be created at a database level in order to let the strategies' code operate with them directly[23] – e.g. slippage, internalization opportunities, etc.

As already mentioned, transaction costs and market impact are a core part of any model in market making as they can erode a substantial amount of the performance of a strategy if they are not properly accounted for. They typically need to be updated dynamically dependent on the frequency of the strategies and the nature of the costs, such are:

[20]252 days * 8.5 hours trading per day * 3,600 seconds per hour * 10 ticks per second.

[21]It also important to highlight that it has recently also benefited from cloud and parallel computing.

[22]Outlier detection is one of the main tasks at this level: even though extreme ones are easy to find (zero price, scale issues, etc) non-extreme have to be defined by the trader – i.e. is the observed data point noise or an opportunity to create alpha instead?

[23]Hence the interest on specialized database systems that allow for efficient calculation of new, simple variables and a cleaner code.

- *Negative and known ex-ante*: broker upfront fees, exchange upfront fees (removing liquidity or routing out), running funding costs, custodian running fees (clearance and settlement), stamp duty (on the sell and/or on the buy; not for members of the exchanges), taxes (typically, when selling certain currencies as it was the case for the Brazilian IOF tax until 2011).
- *Negative and unknown ex-ante*: bid-offer spread, borrow rates, margin and collateral funding fees and, most of the time, market impact (when there is resilience, i.e. when the hit or lifted levels recover after the matching).
- *Positive and known ex-ante*: exchange upfront rebate.
- *Positive and unknown ex-ante*: market impact (when some levels do not recover after the matching a low but permanent effect that increases the value of the trader's position).

Signal generation

Strategies are driven on-line by signals[24] that have been previously fine tuned within a test environment.

Test environment: the whole trading platform should be available for the strategists to calibrate the algorithms within the scenarios that they contemplate – e.g. they could be interested on seeing which combination of parameters or weights across algorithms could be best for the platform as a whole. Back-testing, stress testing and fine tuning/calibration shall respect the risk model (stop losses, take profits, volatility signals, etc), latency, slippage and the rest of the features of the trading platform in order to benefit from a realistic approach[25]. For these reasons the algorithms are said to be tailored to the platform.

Production: it refers to the on-line detection of the patterns defined in the test environment. This is, it is an instance of the distribution of experiments ran by the strategists[26] and usually supervised by the portfolio managers. Efficiency is key in production not only in terms of the robustness of the code (any bug could cause a relevant loss if not properly risk-managed) but also in terms of its optimization[27].

[24] Both, entry and exit signals whether using event-driven data (tick by tick) or aggregated in time intervals or volume intervals. The strategists need to be aware of Diamond and Verrecchia (1987) and Easley and O'Hara (1992) who suggest that in the presence of short-sale constraints, inter-trade duration can indicate that the duration between subsequent data arrivals carries information.

[25] A usual variable that is not entirely taken into account is the market impact that should be carried forward into the data feed to see how the impact of one of the platform's strategies execution itself could affect the rest of the strategies subsequently.

[26] A typical error is not to be consistent with what was back-tested. It usually occurs due to an underestimation of some of the variables of the production model that would affect the exploitation of the pattern, e.g. the latency and prioritization by timestamp order in the matching. It would lead to random deviations from the expected performance.

[27] For instance, latency-sensitive patterns would typically leverage among other things on memory management, i.e the use of cache instead of hard disk since its access typically takes only a few processor clock cycles, whereas access to main memory may take tens or even hundreds of cycles.

Inventory management

In this area, there is not a standard policy as to what is best for the trader but a set of good practices instead. For example, the trader could pool together the positions that generate long-lived deviation risk to benefit from portfolio diversification while keeping individual tracking for those mid and high frequency trading, i.e. each asset should be indexed to identify to which particular strategy it corresponds. Moreover, the balance sheet generated by the long-lived deviation risk can be used to reduce the borrow rates on the included stocks (literally, driving them to zero) and to optimize the collateral management – instead of cash the trader can use the long run positions across assets to post collateral on leveraged trades (typically, swaps and futures).

Also, positions should be correctly evaluated controlling the price discovery (last traded price[28]) by the market impact at the frequency of the strategy to which the asset belongs (again, indexation becomes relevant in inventory management), e.g. account for the market impact from a day to a couple of weeks on inelastic flow (typically large sizes, hence market impact is not negligible) and for the rest of the day at most on elastic flow (typically small sizes but high urgency, hence market impact is not negligible either).

Risk management

Extreme Value Theory is one of the main tools for risk management. It is important for the market maker to be aware of the risk that a non-full-replication hedge would imply especially in the case of inelastic flow as extreme non-Gaussian outcome renders a non-negligible risk[29] – remember that elastic flow is more Gaussian and as such the Central Limit Theorem and the Law of Large Numbers are more likely to apply. As said, this flow tends to be one-sided hence tracking error exposure would be built up and usually unwind in blocks at the client's convenience – i.e. patterns can be tracked but not managed dynamically. As a result, it seems apparent that choosing a hedge that among other parameters minimizes the expected extreme loss given a level of confidence can be an appropriate policy for the trader – and this will be the approach that I will follow along the rest of the thesis.

Every type of risk should be properly understood and managed, e.g. not all the types of risk have a symmetric distribution that adds volatility to the performance of the strategies. Follows a set of examples and the main concerns that could be highlighted across them:

- **Market risk:** it is the most popular of the managed risks. Trade-out rules are defined within this block and scanned during production. Many of the indicators are herited from portfolio management such are performance ratios (Sharpe, Omega, etc) or extreme value indices (VaR, cVaR and maximum drawdown[30]).

[28] Or the mid for instance when the last traded price is not realistic any more for having deviated from the latest best ask or below the latest best bid.

[29] *Black Swan*, *Fat Tails*, etc are all terms for the same concept: extreme values typically happen more often than a Gaussian distribution would suggest.

[30] Maximum drawdown risk is typically minimized using options on the VIX (whose delta and vega have to be

- **Model risk:** calibration and slippage[31] are typical examples of the model risk, present by definition on any back-tested strategy. In order to reduce it, I suggest along the thesis the market maker to give up to part of the highest stress-tested performance of data-driven models by adding more theoretical constraints (regularization). This should enhance the out-of-sample robustness of the strategies. Furthermore, I propose in Chapter 5, a calibration method that would take into account the trading style of the portfolio manager through autonomous agents.

- **Credit risk:** within the sell side not only adding the client's risk profile would be prudent but it is also mandatory given the proposals on Basel III about counterparty valuation adjustment (CVA). This type of risk usually arises on inelastic flow where clients typically trade through swaps (as opposed to exchanges). A correct management of the credit risk should reduce the extreme events[32].

- **Liquidity risk:** not only the volatility of the market impact generates liquidity risks but also the running costs mentioned above such are the borrow rates[33] and the funding.

- **Operational risk:** it is a highly non-symmetrical risk in market making. This is, when an error is produced in a quote being its level less aggressive than it should, probably nothing would happen beyond incurring into an opportunity cost. However, when it is more aggressive than it should, there would likely be a trade that would hit the performance of the strategy. The trader shall establish several rules to make sure that in the case of an undesired situation there is a maximum loss to be hit. Some types of operational risks involved with market access can be also be tested when brokers allow for paper trading.

Many financial institutions run stress test scenarios that involve applying simultaneous moves in multiple risk factors to a portfolio. The simultaneous changes in these multiple risk factors, such as interest rates, exchange rates and stock prices, reflect a risk scenario or event that a risk manager believes is plausible in the near future. For example, under normal market conditions, equity and bond prices often move in opposite directions to each other. Financial institutions can offset (to some extent) losses in equity markets with gains in bond markets, or *vice-versa*. In times of stress, however, the negative correlation may change and positions in both equity and bond markets may be adversely affected.

Single-factor stress testing is particularly appropriate at trading desk level to indicate a dealer the effect of a large move in a single risk factor on her position. However, stress events rarely impact on one factor alone: when a stress event occurs, it is common that multiple risk factors change simultaneously.

managed dynamically). By doing so, the trader can benefit from large movements of volatility that typically add randomness to the performance of the strategies put into place.

[31] The deviation between the expected execution and the real one.

[32] Credit risk became especially relevant during the 2008's turmoil when several institutions (from private to supranational) collapsed affecting subsequently their creditors.

[33] In October 2008 those who shorted Volkswagen found out that Porsche held positions of more than 74% of Volkswagen shares. The fall triggered a sudden rally of these shares due to the effect of the former when trying to cover their short positions with urgency. Such rally made it briefly the most valuable company in the world – an increase in price driven by a liquidity issue that should have been properly monitored.

Trading

The trading building block involves both the trade optimization and its execution.

- **Trade optimization**: it is the process where the information of the previous building blocks is put together towards their comprehensive optimization especially when several strategies compete for the same money to be invested. Financial models of asset allocation such is Black-Litterman (Black and Litterman (1992)) can be used to restrict a trade on an asset (typically sectors in the case of a market maker) whose exposure is larger than the one desired. Other approaches such is the Kelly Criterion (Kelly (1956)) can be used to decide how much budget can be allocated across the frequencies on a certain asset. Machine learning techniques, such is Reinforcement Learning can be used to define shortcuts in time-consuming calculations, or to non-linearly optimize overall process such are genetic algorithms and particle swarms as we shall see below.
- **Algorithmic trading**: as already mentioned in order to minimize market impact, to minimize costs and to target the best prices across pools of liquidity it is important to systematically execute through algorithms. Several decisions have to be made at this stage (not only the algorithm itself but also its urgency and number of waves to run its strategies) before routing the order to the right market.

Direct market access

Usually the FIX messages received from the exchange (and sent to it) are managed through a layer of complex event processing (CEP) to favour a fast reaction to the latest information of the order book.

Fragmentation has triggered the need to access the markets through smart order routers (SOR)[34] and this also allows the trader benefit from dark pools of liquidity where interests from their members (institutions and brokers) can be matched anonymously and benefit from arbitrage opportunities (one of the main sources of revenues in index/ETF arbitrage usually leverages on this access). Moreover, DMA typically implies collocation, the allocation of the servers as close as possible to the exchange's servers, to achieve the lowest latency when accessing a local market. Collocation is hence essential in latency-sensitive strategies, typically UHFT. In that respect, Gifford (2011) further states that around 15% of buy side and sell side agents' most common strategies depend entirely on latency. On the other extreme, latency makes little or no difference (i.e. non-sensitive) to 40% of the buy side agents and 20% of the sell side agents.

One of the least known constraints that a market maker faces in HFT is the queuing of the FIX messages that are sent to the exchange. Not only do they need to be prioritized but also the trader has to decide which ones won't even be sent – typically a maximum number of messages per

[34] More subtly, HFT has been favoured across exchanges due to fragmentation, as in the case of Canada's stock exchange that had to improve its HFT features in order not to lose its flow towards the US pools of liquidity in Canadian stocks – these are called depositary receipts (DRs).

"exchange user" is fixed[35]. As already mentioned, the market maker has thus to decide whether to use a message to send a limit order, a market order or a double quote (that substitutes in a single message both the best bid and best offer previously sent to the market).

[35]Hundreds per second for all the quoted instruments – which is more constraining in the sell side than in the buy side given the nature of market making.

Appendix B

The trader's Q-learning matrix, Υ

The trader's Q-learning matrix, Υ, used in the experiment from Chapter 5 obeys to the following rationale:

- From allPositive: "keep mode if it remains profitable, mix randomly if it starts to decay". As a result:

allPositive $\rightarrow$ allPositive
$\Lambda = 1$
$\pi = 0.51$

m	**1**	**0**	**-1**
1	100	0	-
0	100	0	0
-1	-	0	100

allPositive $\rightarrow$ mix
$\Lambda = 0$
$\pi = 0.49$

m	**1**	**0**	**-1**
1	100	100	-
0	0	100	0
-1	-	100	100

Table B.1: Possible scenarios from allPositive.

Note that in this matrix the future transition probabilities for each state, π, show the level of confidence of the trader in the strategy from a theoretical point of view.

- From mix: "keep the mode if it improves profits and freeze it if remains being random". Hence:

mix $\rightarrow$ allPositive
$\Lambda = 1$
$\pi = 0.27$

m	**1**	**0**	**-1**
1	100	0	-
0	100	0	0
-1	-	0	100

mix $\rightarrow$ mix
$\Lambda = 0$
$\pi = 0.5$

m	**1**	**0**	**-1**
1	0	100	-
0	0	100	0
-1	-	100	0

mix $\rightarrow$ allNegative
$\Lambda = -1$
$\pi = 0.23$

m	**1**	**0**	**-1**
1	0	100	-
0	0	0	100
-1	-	100	0

Table B.2: Possible scenarios from mix.

- From allNegative: "if the strategy starts rendering losses, stop trading it". What can be modelled as follows:

allNegative → mix
$\Lambda = 0$
$\pi = 0.6$

m	**1**	**0**	**-1**
1	0	100	-
0	0	100	0
-1	-	100	0

allNegative → allNegative
$\Lambda = -1$
$\pi = 0.4$

m	**1**	**0**	**-1**
1	-	-	-
0	0	100	0
-1	-	100	0

Table B.3: Possible scenarios from allNegative.

Note that the first row in the last table, i.e. $m = 1$, cannot occur as allNegative can only be generated by mix or allNegative. From mix only $m = 0, -1$ are feasible. And from allNegative only $m = 0$.

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