Ambient vibration testing, system identification and model updating of a multiple-span elevated bridge

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ABSTRACT: This paper presents ambient-vibration based investigations conducted on the Newmarket Viaduct, a 12 span, 690m long, curved, segmental, elevated, post-tensioned concrete viaduct located in Auckland, New Zealand, to assess the dynamical behavior of the bridge during construction. The assessment procedure included full-scale ambient vibration testing, modal identification from ambient vibration responses using two different output-only identification methods, finite element (FE) modelling and sensitivity-based model updating-based identification of the uncertain structural parameters of the model. Ambient tests were conducted for the accurate estimation of the dynamic characteristics using Enhanced Frequency Domain Decomposition and Stochastic Subspace Identification. An initial 3D FE model was then developed from the information provided in the design documentation of the bridge. The output-only modal identification results from ambient vibration measurements of the bridge were subsequently used to update the FE bridge model. Different parameters of the model were modified using an automated procedure to improve correlation between the measured and calculated modal parameters. Careful attention was placed on the selection of the parameters to be modified by the updating procedure in order to ensure that the necessary changes are realistic and physically meaningful. A very good match between theoretical and experimental modal parameters was reached. The calibrated FE model reflecting the as-built structural conditions of the bridge will serve as a baseline model for the assessment of structural health using continuous monitoring data.

KEY WORDS: Bridge; Ambient vibration test; Operational modal analysis; FE model; Model updating.

1 INTRODUCTION

The study described in this paper was performed in order to assess the actual dynamic behavior of the Newmarket Viaduct located in Auckland, New Zealand during construction with the ultimate purpose to predict the performance of the bridge when subjected to different live loads, for instance traffic, seismic and wind.

Bridges are special civil structures that due to their long service life, large dimensions, structural complexity and importance need regular condition assessment or continuous structural health monitoring to guarantee their serviceability, safety and reliability. To this end, one way is to measure the actual dynamic characteristics (e.g., natural frequencies and mode shapes) by conducting full scale dynamic tests to assist in understanding of the dynamic behavior under traffic, seismic, wind and other live loads. Full scale dynamic testing of bridges can provide valuable information on the service behavior and performance of structures, as this information can then be used to check the construction quality, conduct condition assessment, and validate or update numerical models of the bridge so that these models can better reflect the as-built, in-situ structural stiffness, boundary conditions, structural connectivity, inertia and energy dissipation properties [1].

For testing and monitoring of large-scale bridges, ambient vibration tests (AVTs) are deemed to be simpler, faster and cheaper than forced vibration tests, and often the only feasible method for the determination of dynamic characteristics. During AVTs it is possible to obtain results using environmental and operational effects such as wind, traffic and other excitations of the bridge. During an AVT, there is often no interference with the normal traffic flow and effects of the non-stationary ambient loads can be minimized by collecting data for sufficiently long periods of time. In the past, AVTs have been successfully applied for assessing the dynamic behavior of different types of full-scale bridges [2-8]. Nevertheless, there is still dearth of experimental and analytical studies for long, multi-span concrete bridges.

AVTs are often performed in difficult conditions, requiring high accuracy both in the control of the test setup and in the analysis of the measured data. As the input excitations are not measured in the AVTs, a remarkable recent development of a new family of modal identification methods under operational conditions, called operational modal analysis (OMA), occurred. The estimated natural frequencies and mode shapes of large bridges are usually used for establishing correlations with numerical predictions or, in some cases, developing and updating of finite element (FE) models [9-14]. However, although there are many methods for OMA [15, 16], even some popular methods face challenges resulting from insufficient ambient excitation.

The dynamic bridge assessment procedure in this paper includes full-scale, in-situ dynamic tests, OMA from ambient vibration response using two identification methods, FE modeling and dynamic-based identification of the uncertain structural parameters of the model via sensitivity-based model updating.

An AVT has been conducted on Newmarket Viaduct with the aim of determining its dynamic response and performing modal system identification. OMA has been carried out both in the frequency domain and in the time domain to extract the dominant frequencies and mode shapes. The application of two well-known identification techniques - Enhanced Frequency Domain Decomposition (EFDD) [17] and Stochastic Subspace Identification (SSI) [18, 19] - yielded very similar results for all the identified modes, providing consistent information for the following FE model updating.

The final goal of this research is to develop a 3D FE model able to match the results of OMA. With this purpose in mind, some uncertain parameters of the model (such as Young's modulus and density of concrete and geometric parameters of sections) were selected as parameters for updating and iteratively modified to minimize the differences in the natural frequencies between the FE model and OMA. The results showed a very good match between the experimental data and the updated FE model for the first eight frequencies and their corresponding mode shapes.

2 DESCRIPTION OF THE BRIDGE AND TESTING PROGRAMME

Newmarket Viaduct (Figure 1), recently constructed in Auckland, New Zealand, is one of the major and most important bridges within the New Zealand road network. It is a horizontally and vertically curved, post-tensioned concrete bridge, comprising two parallel, twin bridges. The total length of the bridge is 690m, with twelve different spans ranging in length from 38.67m to 62.65m and average length of approximately 60m. The superstructure of the bridge is a continuous single-cell box girder of a total width of 30m. The deck of the bridge contains a total of 468 precast box-girder segments and was constructed using the balanced cantilever and prestressed box-beam method. The traffic on the Northbound deck is carried on three lanes, and on four lanes on the Southbound deck. The Southbound Bridge (on the right hand side in Figure 1) is the subject of this paper. At the time of testing described in this paper the two bridges were not structurally linked in any way.

For the purpose of AVTs, the bridge was instrumented with two models of tri-axial wireless USB MEMS accelerometers (www.gcdataconcepts.com) with a user selectable range ±2g - ±6g. Model X6-1A (Figure 2) uses a single AA or D battery and model X6-2 contains an internal hardwired rechargeable Lithium-Polymer battery. The accelerometers do not have data transmission capability; they store the data to a micro SD card and after the test it needs to be downloaded to a computer.

Figure 1. Aerial view of Newmarket Viaduct.

The measurements points were on both sides of the girder (Figure 3) and at approximate distances of span/8. A total of 96 locations inside the bridge girder were chosen to place accelerometers. A maximum of 56 accelerometers for each test setup could be used simultaneously. Of these, eight accelerometers were used as reference accelerometers and their locations in Span 6 and 7 were not changed throughout the tests. The remaining 48 accelerometers were used as roving accelerometers and were moved to cover all the desired locations in five test setups.

The AVT were carried out in November 2011. Data was collected for 1 hour in each setup at a sampling rate of 160 Hz. The raw measurement data from the middle of Span 5 in vertical and transverse direction are presented in Figure 4 as an example of typical levels of vibration observed during the AVT.

Figure 2. MEMS accelerometer used in AVTs.

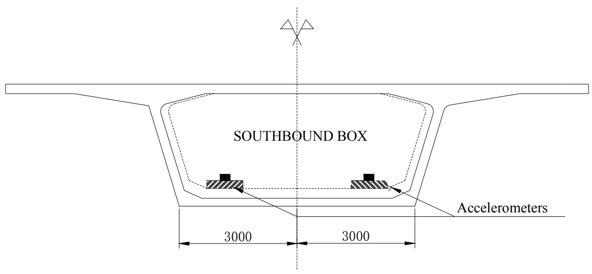


Figure 3. Location of accelerometers inside bridge girder

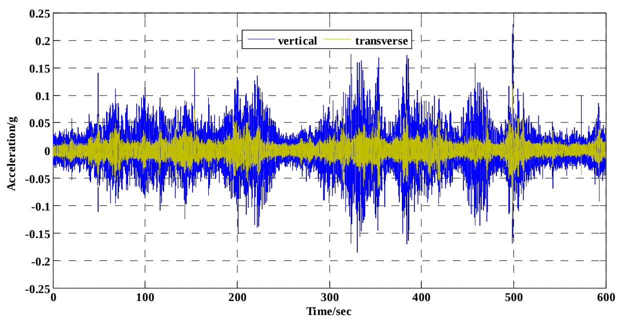


Figure 4. Example of acceleration time series collected in AVTs.

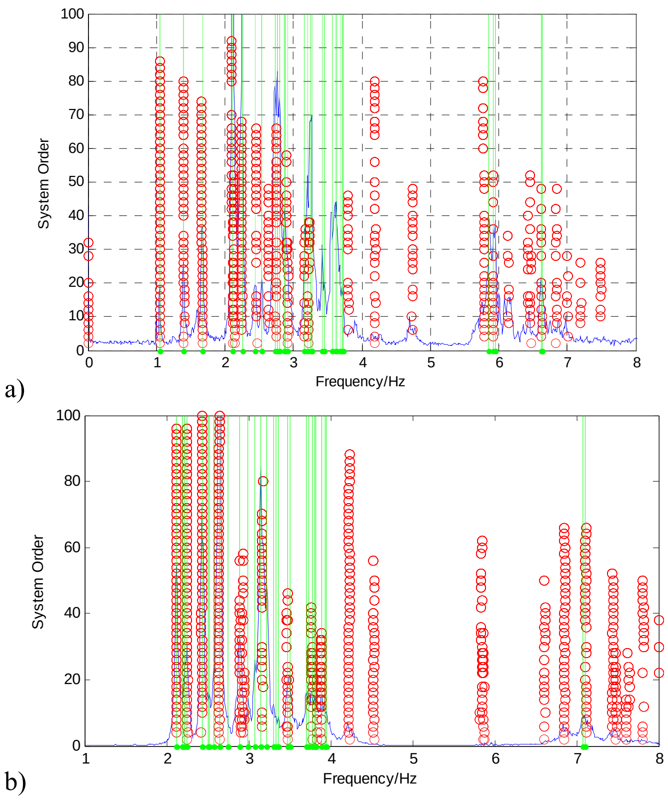


Figure 5. Stabilization diagram for a) transverse, and b) vertical vibration data.

3 DATA PROCESSING AND OMA

On the basis of the results reported in [20], an extended analysis was carried out in order to evaluate the repeatability of the identified frequencies using different segments of data. A total of 10 different data segments sourced from different setups, each 10min long were used. The extraction of the modal parameters from ambient vibration data was carried out using an in-house system identification toolbox written in MATLAB [21]. Several system identification techniques are available in the toolbox including EFDD and SSI. Figure 5 shows two typical stabilization charts: red circles show stable frequencies, damping ratios and mode shapes from SSI, and green lines show the singular values identified by EFDD. Inspection of these diagrams showed more than 20 natural frequencies in the range 0-10Hz.

Table 1 shows the average values and the coefficients of variation (standard deviation/mean) of the first four transverse and vertical frequencies evaluated considering all the 10 data segments and the two identification methods. Figure 6 shows the values of the first transverse and vertical frequencies obtained from the 10 data segments as well as the mean values from the two identification methods. Examining Figure 6 and Table 1, it can be concluded that the frequency values identified from the different data segments are very close; moreover the two identification methods also give very similar results. The mode shapes identified by EFDD related the first eight frequencies are also depicted in Figure 7.

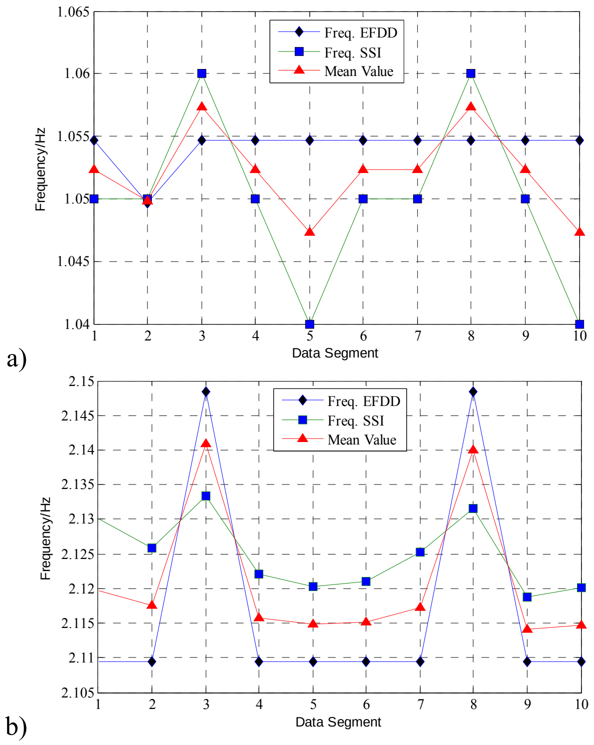


Figure 6. Statistical analysis of a) 1st transverse, and b) 1st vertical frequency estimations.

Table 1. First eight natural frequencies identified from AVT.

Mode	EFDD /Hz	COV /%	SSI /Hz	COV /%	Mean of 2 methods /Hz
1T*	1.05	0.2	1.05	0.7	1.05
2T	1.39	2.0	1.40	1.0	1.39
3T	1.66	2.0	1.66	1.9	1.66
4T	2.11	0.1	2.11	0.5	2.11
5V**	2.12	1.7	2.12	0.5	2.12
6V	2.25	2.0	2.26	1.3	2.26
7V	2.44	2.0	2.43	0. 9	2.43
8V	2.64	2.0	2.64	0.6	2.64

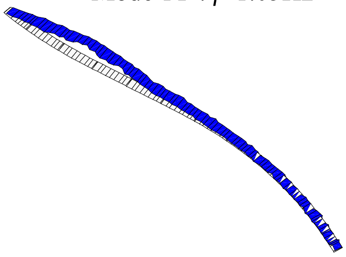
* T=transverse, ** V= vertical

4 FE MODELING

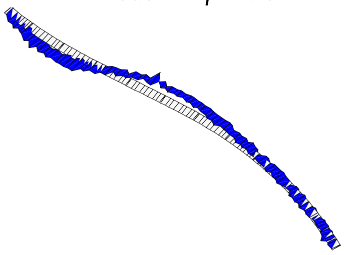
The experimental investigation was accompanied by the development of a 3D FE model based on the design documentation of the bridge and using SAP2000 FE software. SAP2000 is industry standard FE analysis software, which will also be used for structural analysis of the bridge in the later stages of this project. The SAP2000 model was subsequently exported to the FEMtools software for the purpose of model updating. FEMtools is a multi-functional FE analysis tool that has capabilities for validation and updating of FE models.

The 3D FE model developed in SAP2000 is shown in Figure 8a. The superstructure, piers and pier caps were modelled using beam elements, and expansion joints and bearings using spring elements. Fixed boundary conditions were specified at the base of the piers. Then the initial SAP2000 model was imported into FEMtools adjusted (Figure 8b). To simulate the behavior of the three types of bearings

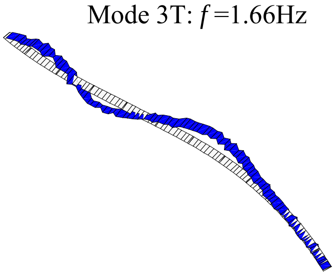
Mode 1T*: *f*=1.05Hz



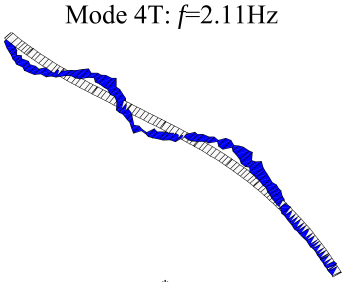
Mode 2T: *f* =1.39Hz



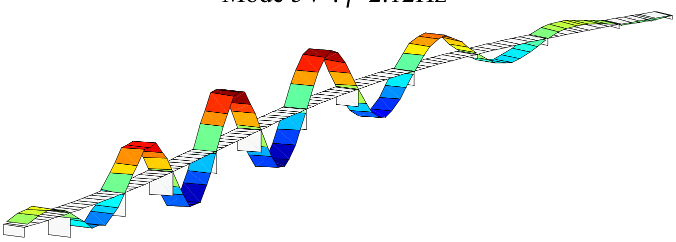
Mode 3T: *f* =1.66Hz



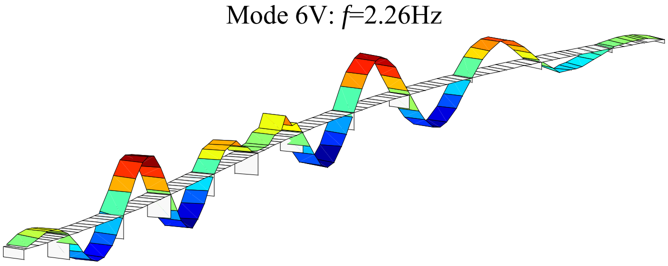
Mode 4T: *f*=2.11Hz



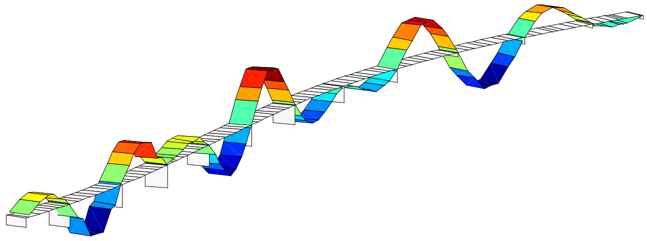
Mode 5V*: *f*=2.12Hz



Mode 6V: *f*=2.26Hz



Mode 7V: *f*=2.43Hz



Mode 8V: *f*=2.64Hz

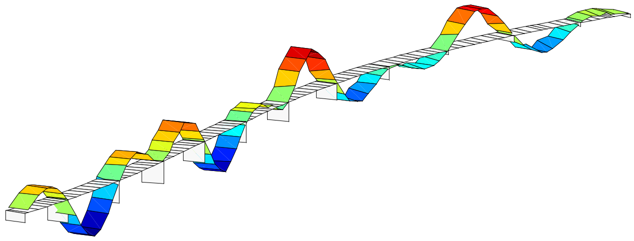
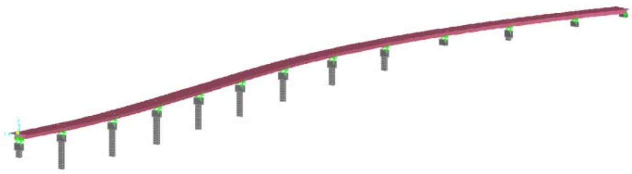
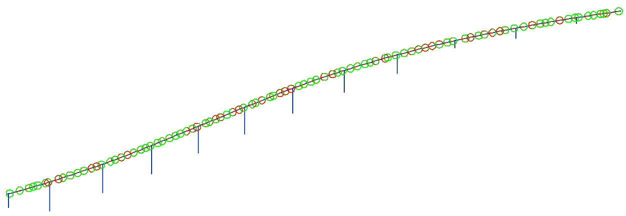


Figure 7. Vibration modes identified from ambient vibration measurement (* T=transverse, ** V= vertical)



a)



b)

Figure 8. 3D FE model of Southbound Bridge: a) SAP2000, and b) FEMtools.

Table 2. AVT and FEtools model results comparison.

Mode	AVT /Hz	FEMtools /Hz	Difference /%
1T*	1.05	0.93	-11.4
2T	1.39	1.12	-19.4
3T	1.66	1.36	-18.1
4T	2.11	1.68	-20.4
5V*	2.12	2.01	-5.2
6V	2.26	2.14	-5.3
7V	2.43	2.35	-3.3
8V	2.64	2.60	-1.5

* T=transverse, ** V= vertical

(pinned, fixed and sliding) in FEMtools, each support was modeled as additional short beam element connecting the deck to the abutments or piers. The roller support can be simulated by choosing a cross-section of the additional beam element with a large section area and small moment of inertia, whereas the rigid support can be simulated in the numerical model by choosing a beam element with a large value of both section area and moment of inertia.

The initial FE frequencies from FEMtools are shown in Table 2 and compared to AVT experimental results. There are rather small differences for the vertical modes of the order of 5%, but more noticeable for the transverse modes of up to approximately 20%.

5 MODEL UPDATING PROCEDURE

Parameters influencing the dynamic response of the structure can be updated to improve the model. For this study, the parameters selected for the updating procedure are concrete Young's modulus and density, cross-sectional area, torsional moment of inertia, transverse bending moment of inertia and vertical bending moment of inertia in different girder segments, and also in bearings beams, pier caps and bent sections. As there are six types of segments that have been used in the girder varying in geometrical properties, there are 24 updating parameters for the girder and 12 for the bearings, 8 for the pier caps and bent sections, adding up to a total 46 parameters considered.

An important part of any updating procedure, which may greatly assist in achieving good results, is a good choice of the parameter starting values. The starting value for Young's modulus (36GPa) was calculated as the average value obtained from 20 specimens collected from the construction site of the bridge (Figure 9) and tested in laboratory; the density was also measured in laboratory using the same specimens (2550kg/m^3). The geometric parameters of cross-sections were obtained from the design documents.

The method used for the normal modes evaluation in the 3D FE model was the Lanczos subspace method [22]. Model parameter updating was carried out by minimizing the differences between the theoretical and experimental natural frequencies because they could be identified with good confidence.

Updating of the 46 parameters previously indicated was carried out using the sensitivity method [23]. The experimental responses are expressed as functions of the structural parameters and a sensitivity coefficient matrix in terms of the first order Taylor series [24] as:

$$\mathbf{R}_e = \mathbf{R}_a + \mathbf{S}(\mathbf{P}_u - \mathbf{P}_0) \tag{1}$$

The above can also be written as:

Figure 9. Concrete samples for laboratory tests.

$$\Delta\mathbf{R} = \mathbf{S}\Delta\mathbf{P} \tag{2}$$

where $\mathbf{R}_e$ is the vector of the reference system responses (experimental data, in the case the eight natural frequencies reported earlier); $\mathbf{R}_a$ is the vector of predicted system responses for a given state $\mathbf{P}_0$ of the parameter values; $\mathbf{P}_u$ is the vector of the updated parameter values (uncertain parameters in the EF model which can include geometric, material properties and boundary and connectivity conditions related to stiffness and inertia); and $\mathbf{S}$ is the sensitivity matrix which can be calculated as:

$$\mathbf{S}_{ij} = \frac{\partial \mathbf{R}_{a,i}}{\partial \mathbf{P}_j} \tag{3}$$

Here $\mathbf{R}_{a,i}$ ($i=1, 2,\ldots,n$) and $\mathbf{P}_j$ ($i=1, 2,\ldots\ \ldots,m$) are the entries of the analytical structural response and the updating structural parameter vectors. Equation (2) can be determined, over-determined or under-determined depending on the number of responses n being equal, larger or smaller than the number of updating parameters m, respectively. It can be solved using a Bayesian technique, pseudo-inverse (least squares) method or weighted least squares method, depending on whether the weighting coefficients are used or not. The applied least squares solution by a pseudo-inverse technique will minimize iteratively the residue $\mathbf{r}$ defined as:

$$\mathbf{r} = \mathbf{S}\Delta\mathbf{P} - \Delta\mathbf{R} \tag{4}$$

with $\Delta\mathbf{P}_{n+1}$ calculated as follows:

$$\Delta\mathbf{P} = \mathbf{S}^T\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\Delta\mathbf{R} \tag{5}$$

The iterative procedure was applied to achieve convergence in a limited number of iterations. The parameters used for updating are bounded according mainly to engineering judgments. The upper and lower limits of cross-section area, torsional moment of inertia, transverse bending moment of inertia and vertical bending moment of inertia were all set as $\pm$10%. The limits for concrete density were set as $\pm$5%. The upper and lower limits of concrete Young's modulus were set as +15% and -5%, respectively.

6 COMPARISON BETWEEN EXPERIMENTAL DATA AND UPDATED MODEL

The comparison of the experimental data with the updated FE model is shown in Table 3 in which the first eight estimated experimental frequencies are listed against the first eight frequencies of the updated model. The percentage errors and the modal assurance criterion values (MAC) are also shown. It can be noticed that the FE model natural frequencies are very close to the experimental ones and the correlation (MAC) between mode shapes shows very good agreement for the bending mode shapes. Moreover, in Figure 10 the comparison between the first eight experimental (red lines with circles) and numerical (blue continuous lines) mode shapes is shown: the good correlation is evident for all the bending modes.

Table 4 shows the design parameters of the bridge before updating and the results of updating process for all the 46 parameters are shown in Table 5. In both tables, A is the cross-section area, I_x is torsional moment of inertia, I_y is the transverse bending moment of inertia, I_z is the vertical bending moment of inertia, E is concrete Young's modulus and ρ is concrete density. Parameter Sets 1-6 are related to the six types of the segments used in the girder, and Sets 7 and 8 are related to the bent sections and pier caps, respectively. Sets 9-11 are related to the three types of short beams modeling the bearings. Analyzing Table 5, it is noted that all the parameters have been substantially changed in updating, but within the maximum set range of the starting values.

Table 3. Comparison of the experimental data with the updated model.

Mode	AVT /Hz	Model /Hz	Difference /%	MAC
1T	1.05	1.08	2.9	0.98
2T	1.39	1.36	-2.2	0.90
3T	1.66	1.67	0.6	0.91
4T	2.11	2.18	3.3	0.87
5V	2.12	2.13	0.5	0.93
6V	2.26	2.25	-0.4	0.97
7V	2.43	2.45	0.8	0.88
8V	2.64	2.66	0.7	0.91

* T=transverse, ** V= vertical

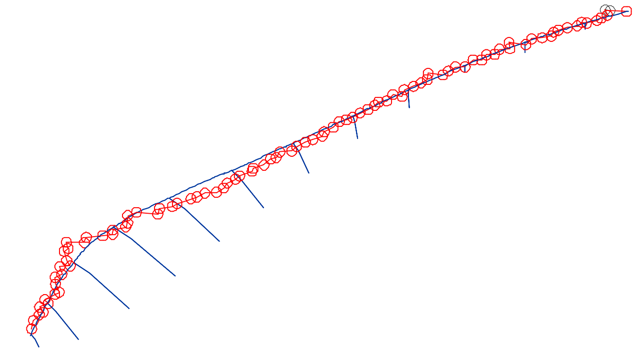
Table 4. Parameters of the bridge before updating.

Set	A /m^2	I_x /m^4	I_y /m^4	I_z /m^4	E /GPa	ρ /kg/m^3
1	22.70	48.81	147.1	20.29		
2	7.27	20.61	92.58	10.37		
3	10.17	24.04	100.85	15.45		
4	6.99	19.38	91.82	9.39		
5	8.34	24.02	98.98	12.32		
6	9.37	26.30	104.81	13.75	36.0	2550
7	5.25	2.88	0.98	5.36		
8	7.13	4.28	1.34	13.39		
9	6.63	2.94	0.86	15.51		
10	0.76	0.04	0.02	3.00		
11	0.76	0.04	0.02	3.00		

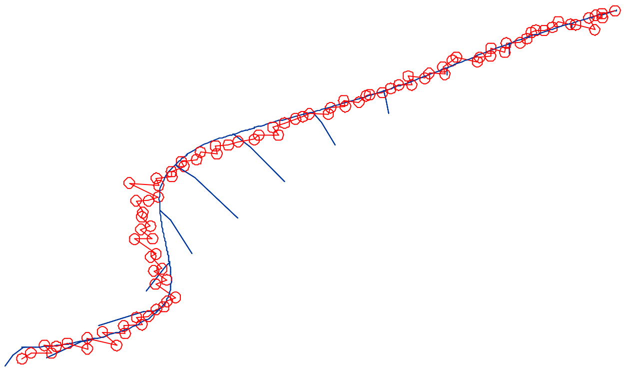
Table 5. Parameters of the bridge after updating.

Set	A /m^2	I_x /m^4	I_y /m^4	I_z /m^4	E /GPa	ρ /kg/m^3
1	23.14	53.33	143.40	18.26		
2	6.54	22.67	86.67	9.33		
3	10.41	25.54	91.49	15.28		
4	6.29	21.30	82.64	9.15		
5	7.50	26.42	92.58	11.09		
6	8.43	29.62	94.32	12.37	39.0	2450
7	4.98	3.01	1.08	5.00		
8	6.41	4.31	1.47	13.39		
9	6.34	2.94	0.90	15.76		
10	0.75	0.04	0.03	2.97		
11	0.54	0.04	0.03	2.90		

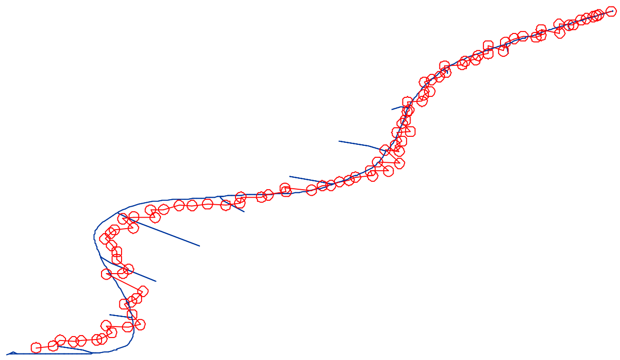
Mode 1T*: f_e^{***}=1.05Hz, f_a^{****}=1.08Hz



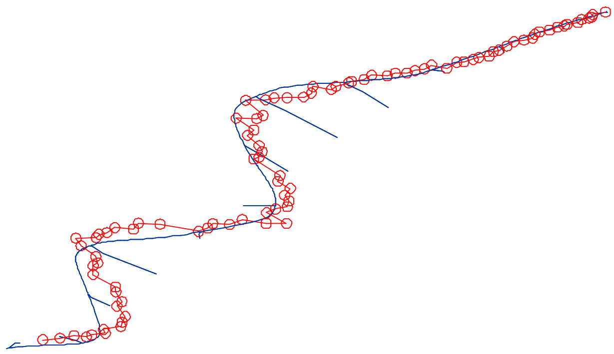
Mode 2T: f_e=1.39Hz, f_a=1.36Hz



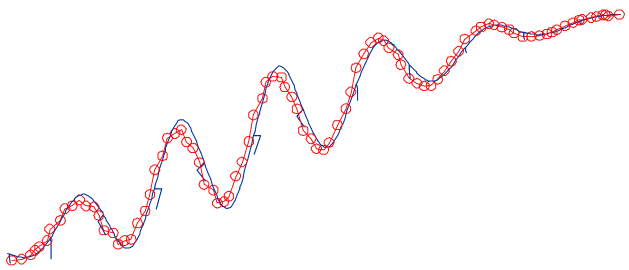
Mode 3T: f_e=1.66Hz, f_a=1.67Hz



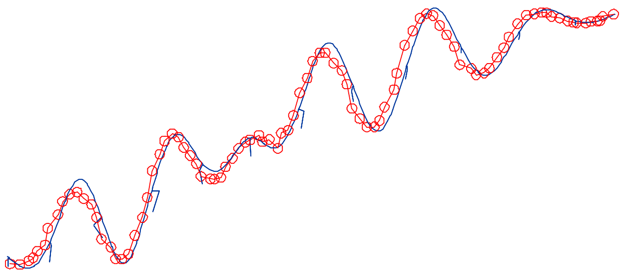
Mode 4T: f_e=2.11Hz, f_a=2.18Hz



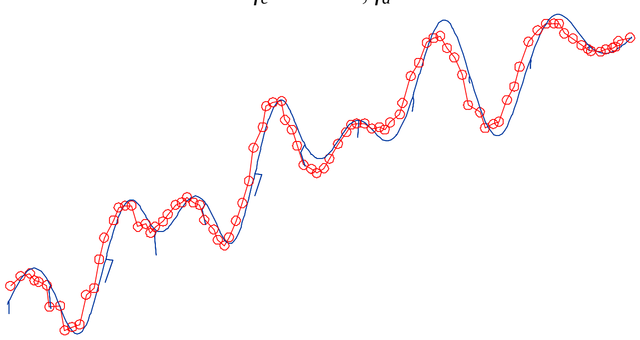
Mode 5V**: f_e=2.12Hz, f_a=2.13Hz



Mode 6V: f_e=2.26Hz, f_a=2.25Hz



Mode 7V: f_e=2.43Hz, f_a=2.45Hz



Mode 8V: f_e=2.64Hz, f_a=2.66Hz

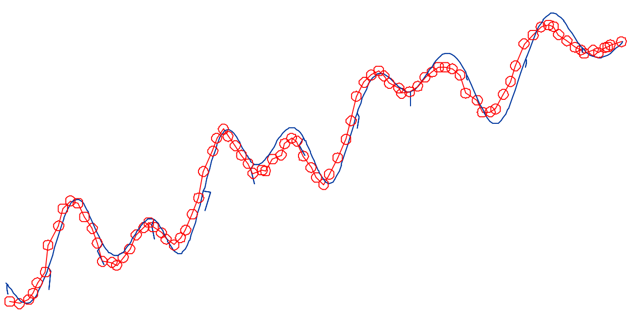


Figure 10. Comparison between the 1st eight experimental and numerical mode shapes (* T=transverse, ** V= vertical, *** f_e = experimental frequency, **** f_a = analytical frequency).

7 CONCLUSIONS

The ambient vibration based investigations carried out to assess the dynamic behavior of the Newmarket Viaduct have been presented in this paper. Five setups of AVTs were conducted for the accurate estimation of the dynamic characteristics evaluating the repeatability of the results. Moreover, a 3D FE model of the bridge was formulated and updated to match the experimental measurements.

The following conclusions can be drawn from this study:

1. Several vibration modes were identified from the ambient vibration tests using the output-only measurements in the frequency range 0-10Hz. It is thus demonstrated that the ambient vibration response measurements are sufficient to identify the most significant modes of such a large concrete bridge with confidence.
2. A very good agreement was found between the modal estimates obtained from the two OMA methods, EFDD and SSI. Also, the identification using different data segments showed that modal frequencies are highly consistent.
3. The good match between the measured and analytical modal parameters was reached using a sensitivity-based updating procedure.
4. Due to the good correlation between experimental results and the updated theoretical FE model, the updated FE model can be expected to provide reliable predictions to assess the structural condition and performance and will serve as a baseline model for assessment of the bridge structural health using continuous monitoring data in future studies.

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