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Improved performance of LDPC-coded MIMO systems with EP-based soft-decisions

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Abstract—Modern communications systems use efficient encoding schemes, multiple-input multiple-output (MIMO) and high-order QAM constellations for maximizing spectral efficiency. However, as the dimensions of the system grow, the design of efficient and low-complexity MIMO receivers possesses technical challenges. Symbol detection can no longer rely on conventional approaches for posterior probability computation due to complexity. Marginalization of this posterior to obtain per-antenna soft-bit probabilities to be fed to a channel decoder is computationally challenging when realistic signaling is used. In this work, we propose to use Expectation Propagation (EP) algorithm to provide an accurate low-complexity Gaussian approximation to the posterior, easily solving the posterior marginalization problem. EP soft-bit probabilities are used in an LDPC-coded MIMO system, achieving outstanding performance improvement compared to similar approaches in the literature for low-complexity LDPC MIMO decoding.

Index Terms—MIMO communication systems, Low Complexity receiver, Expectation Propagation, LDPC.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are getting to a mature stage with a significant deployment in several wireless communication systems [1]. MIMO systems increase capacity (throughput) and improve reliability (reduced symbol error rate and outage), and these gains scale with the dimension of the MIMO system, roughly with the number of transmit/receive elements. However, some limitations prevent the widespread deployment of high-dimensional MIMO systems. Specifically, spatial restrictions for antenna deployment and the signal processing complexity at both ends. Despite these issues, novel studies [2] suggest benefits from incorporating a large number of antennas and they point out some feasible solutions for its practical implementation.

Soft-output symbol detection for soft-decoding is a sensitive process in MIMO systems, particularly as the system dimensions grow. In an additive white Gaussian noise $r \times m$ MIMO scenario, a memoryless channel and uniformly distributed transmitted symbols, given observation $\mathbf{y} \in \mathbb{C}^r$, we need to marginalize the posterior probability distribution $p(\mathbf{u}|\mathbf{y})$ over all possible transmitted symbols $\mathbf{u} \in \mathcal{A}^m$ to obtain the per-antenna symbol posterior probability $p(u_i|\mathbf{y})$ $i = 1, \dots, m$. The evaluation of the m marginals is NP-hard [3] and constitutes a bottleneck even for not so large MIMO systems.

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In this paper, we focus on soft-output MIMO detectors that have competitive complexity with respect to the minimum-mean-squared error (MMSE) MIMO detector [4], i.e., the overall complexity is dominated by a $m \times m$ matrix inversion. Indeed, the MMSE solution can be cast as a soft-output detector since it computes the mode of an approximate to the posterior probability $p(\mathbf{u}|\mathbf{y})$ where the discrete uniform prior $p(\mathbf{u})$ is replaced by a zero-mean and E_s -variance independent multivariate Gaussian distribution. However, it is well known that MMSE solution provides poor performance [5]. Other alternatives also compute the marginal probability distribution based on MMSE and then use an importance sampling algorithm to correct the distribution [6]. The Gaussian tree approximation (GTA) algorithm ignores the discrete nature of the prior $p(\mathbf{u})$ and computes the Gaussian tree approximation that minimizes the KL divergence with respect to $p(\mathbf{y}|\mathbf{u})$ [5]. The hard-decision performance of GTA can be significantly outperformed by implementing a successive interference cancellation (SIC) procedure [7]. GTA-SIC is able to outperform the best linear detectors for MIMO detection proposed in the literature in the past years, such as MMSE and MMSE-SIC with lattice reduction using the Lenstra-Lenstra-Lovász (LLL) algorithm [8]. However, GTA-SIC is not designed to provide soft-outputs, which ultimately restrains the system performance when combined with modern coding system such as LDPC codes [9].

In this contribution, we propose the expectation propagation (EP) algorithm [10] to compute at low-complexity an accurate Gaussian approximation to the symbol vector posterior $p(\mathbf{u}|\mathbf{y})$, easily solving the posterior marginalization problem. EP generalizes belief propagation (BP) in two ways. First, EP can naturally and efficiently work with continuous distributions by moment matching (BP needs to propagate the full distribution) and it powerfully deals with more complex and versatile approximating functions (e.g. tree or forests). The proposed approximation for the posterior has been shown to be an efficient solution for high-order high-dimensional MIMO hard-decision symbol detection [11], improving MMSE-SIC an GTA-SIC at similar complexity.

In this work, we show that the MIMO system using EP as soft-output detector is able to achieve performance gains of one order of magnitude compared to GTA and MMSE soft-output detectors for rate-1/2 LDPC codes. Using the same LDPC code, any possible gain in performance is only explained by the fact that EP is able to provide much more reliable estimates to the symbol posterior probabilities than

that of GTA and MMSE. Thus, the EP algorithm emerges as a powerful and efficient method to implement the receiver detector in high-order high-dimensional MIMO scenarios.

Notation: Capital and lowercase boldface symbols represent matrices and vectors respectively. $(\cdot)^\top$ is the transpose. The operator $\text{diag}(\cdot)$ when applied to a vector, e.g. $\text{diag}(\mathbf{x})$, returns a diagonal matrix with diagonal given by \mathbf{x} and for a given square matrix \mathbf{X} , e.g. $\text{diag}(\mathbf{X})$, denotes its diagonal vector.

II. SYSTEM MODEL FOR MIMO-LDPC

A binary word $\mathbf{b} = [b_1, b_2, \dots, b_k]^\top$ is encoded by an (n, k) LDPC encoder giving the binary codeword $\mathbf{c} = [c_1, c_2, \dots, c_n]^\top$. Codeword \mathbf{c} is Gray-mapped and modulated into a set of M-QAM m -dimensional symbol vectors $[\mathbf{u}[1], \mathbf{u}[2], \dots, \mathbf{u}[L]]$, where $\mathbf{u}[l] = \mathbf{a}[l] + j\mathbf{b}[l] \in \mathcal{A}^m$. We assume we can choose the system parameters so that $\log_2(M) mL = n$. The symbols are transmitted over a memoryless flat-fading complex MIMO channel defined by the $r \times m$ matrix $\mathbf{H}[l]$, where each coefficient is drawn according to a proper complex zero-mean unit-variance Gaussian distribution and m and r are respectively the number of transmitting and receiving antennas. The channel output is $\mathbf{y}[l] \in \mathbb{C}^r$ is

$$\mathbf{y}[l] = \mathbf{H}[l]\mathbf{u}[l] + \mathbf{w}[l], \quad (1)$$

where $\mathbf{w}[l] \in \mathbb{C}^r$ is an additive white circular-symmetric complex Gaussian noise vector with independent zero-mean components and σ_w^2 -variance. The signal-to-noise ratio is defined as

$$\text{SNR} = 10 \log_{10} \left(m \log_2 M \frac{n E_b}{k \sigma_w^2} \right), \quad (2)$$

where $E_s = \log_2 M \frac{n}{k} E_b$ is the constellation average energy. Inference in graphical model is typically presented using real-valued random variables, instead of complex-valued variables used in signal processing for communications, and we believe the EP algorithm is better understood that way. Consequently, we first reformulate the complex-valued MIMO system into a real-valued one, before presenting the EP algorithm. The system model in (1) can be translated into an equivalent double-sized real-valued representation that is obtained by considering the real $\mathcal{R}(\cdot)$ and imaginary parts $\mathcal{I}(\cdot)$ separately. We define $\tilde{\mathbf{u}}[l] = [\mathbf{a}[l]^\top \ \mathbf{b}[l]^\top]^\top$, $\tilde{\mathbf{y}}[l] = [\mathcal{R}(\mathbf{y}[l])^\top \ \mathcal{I}(\mathbf{y}[l])^\top]^\top$, $\tilde{\mathbf{w}}[l] = [\mathcal{R}(\mathbf{w}[l])^\top \ \mathcal{I}(\mathbf{w}[l])^\top]^\top$ and

$$\tilde{\mathbf{H}}[l] = \begin{bmatrix} \mathcal{R}(\mathbf{H}[l]) & -\mathcal{I}(\mathbf{H}[l]) \\ \mathcal{I}(\mathbf{H}[l]) & \mathcal{R}(\mathbf{H}[l]) \end{bmatrix}. \quad (3)$$

The channel model can now be written as follows:

$$\tilde{\mathbf{y}}[l] = \tilde{\mathbf{H}}[l]\tilde{\mathbf{u}}[l] + \tilde{\mathbf{w}}[l], \quad (4)$$

where $\sigma_w^2 = \sigma_w^2/2$ is the variance of the real and imaginary components of the noise and we define $\tilde{\mathcal{A}}$ as the new alphabet for the real and imaginary components of the symmetric M-QAM signal, i.e. $\tilde{\mathbf{u}}[l] \in \tilde{\mathcal{A}}^{2m}$, with energy $\tilde{E}_s = E_s/2$. In the rest of this paper we adopt the real-valued channel model formulation in (4) and we drop the model indicator (\cdot) to keep the notation uncluttered.

A. Soft-output detection for soft binary decoding

The system model can be observed in Fig. 1. At the receiver, a soft-output detector computes the posterior probability for each antenna, which are then used to obtain soft information for the LDPC coded bits. Given the model above, the posterior probability of the transmitted symbol vector $\mathbf{u}[l]$ can be formulated as follows:

$$p(\mathbf{u}[l]|\mathbf{y}[l]) = \frac{p(\mathbf{y}[l]|\mathbf{u}[l])p(\mathbf{u}[l])}{p(\mathbf{y}[l])} \propto \mathcal{N}(\mathbf{y} : \mathbf{H}[l]\mathbf{u}[l], \sigma_w^2 \mathbf{I}) \prod_{i=1}^{2m} \mathbb{I}_{u_i[l] \in \mathcal{A}}, \quad (5)$$

where $\mathbb{I}_{u_i[l] \in \mathcal{A}}$ is the indicator function that takes value one if $u_i[l] \in \mathcal{A}$ and zero otherwise. Note that $p(\mathbf{u}[l]) \propto \prod_{i=1}^{2m} \mathbb{I}_{u_i[l] \in \mathcal{A}}$ is uniform across all antenna dimensions m , although non-uniform signaling could be handled by the proposed algorithm in this paper.

Without loss of generality we assume a sequential mapping of the coded bits into the symbols, so the bit assigned to j -th position of the Gray encoding at the i -th antenna during the l -th symbol time, i.e., $c_{j+\log_2 M(i-1)+2m \log_2 M(l-1)}$, is renamed to $c_{ji}[l]$, with $j = 1 \dots \log_2 M$, $i = 1 \dots 2m$ and $l = 1 \dots L$. For a given channel observation $\mathbf{y}[l]$ at time l , the posterior probability of the $c_{ji}[l]$ bit can be computed as follows:

$$p(c_{ji}[l] = c|\mathbf{y}[l]) = \sum_{u_i[l] \in \mathcal{B}_j(c)} p(u_i[l]|\mathbf{y}[l]) = \sum_{u_i[l] \in \mathcal{B}_j(c)} \sum_{\mathbf{u}_{-i}[l] \in \mathcal{A}^{2m-1}} p(\mathbf{u}[l]|\mathbf{y}[l]), \quad (6)$$

for $c \in \{0, 1\}$, where $\mathcal{B}_j(c) = \{u \in \mathcal{A} | \text{Gray}_j(u) = c\}$, $\text{Gray}_j(u)$ is the bit in the j -th position of the Gray encoding of symbol u and $\mathbf{u}_{-i}[l]$ are all components in $\mathbf{u}[l]$ except $u_i[l]$. Furthermore the Logarithm Likelihood Ratio (LLR) for the $c_{ji}[l]$ bit is obtained as follows:

$$\text{LLR}(c_{ji}[l]) = \log \frac{p(c_{ji}[l] = 1|\mathbf{y}[l])}{p(c_{ji}[l] = 0|\mathbf{y}[l])}. \quad (7)$$

As discussed in the introduction, computing the symbol posterior probability $p(u_i[l]|\mathbf{y}[l])$ for each antenna $i = 1, \dots, m$ in (6) is prohibitively complex and we have to resort on approximations. In the next section, we present an approximation based on the EP algorithm.

Finally, we use the standard BP algorithm to perform soft-decoding fed with the vector of LLR computed (or estimated) in one shot for each LDPC-coded bit, once L symbols are received in order to obtain the n code length.

III. EXPECTATION PROPAGATION SOFT-OUTPUT DETECTOR

The MMSE approximation to the true posterior distribution $p(\mathbf{u}|\mathbf{y})$ replaces the prior over the transmitted symbols by a zero-mean E_s -variance independent component-wise Gaussian. Intuitively it might make sense to chose the parameters of the Gaussian prior in this way, because it matches the first two moments of the input distribution. It is certainly not the best choice, as we are interested in matching the

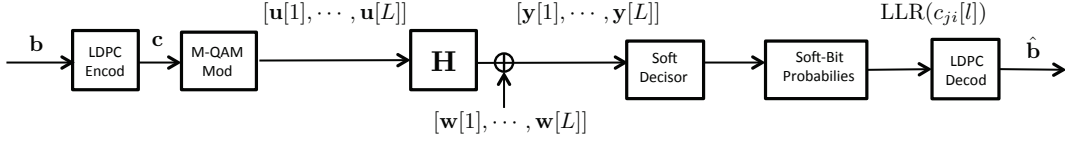


Fig. 1. System model.

posterior distribution. In this paper we propose an algorithm, in which the prior distribution is optimized to ensure that the approximating Gaussian posterior matches the first two moments of the posterior distribution.

Expectation Propagation (EP) [10] is a Bayesian machine-learning technique to construct tractable approximations to a given probability distribution. In our case, we use EP to approximate $p(\mathbf{u}|\mathbf{y})$ by a Gaussian distribution $q_{\text{EP}}(\mathbf{u}) = \mathcal{N}(\mathbf{u} : \boldsymbol{\mu}_{\text{EP}}, \boldsymbol{\Sigma}_{\text{EP}})$ that matches the first two moments of $p(\mathbf{u}|\mathbf{y})$, namely

$$\boldsymbol{\mu}_{\text{EP}} = \mathbb{E}_{p(\mathbf{u}|\mathbf{y})}[\mathbf{u}] \quad (8)$$

$$\boldsymbol{\Sigma}_{\text{EP}} = \text{CoVar}_{p(\mathbf{u}|\mathbf{y})}[\mathbf{u}] \quad (9)$$

This condition is known as *moment matching*. While the direct computation of the $p(\mathbf{u}|\mathbf{y})$ moments requires $|\mathcal{A}|^{2m}$ operations, Minka proposed a sequential algorithm to iteratively approach the solution in (8) and (9) at polynomial time complexity [10], [12]. Once the iterative method has stopped, either by convergence or maximum number of iterations reached, the EP approximation can be used to hard-decide on each of the components of the transmitted symbol [11]:

$$\hat{u}_{i,\text{EP}} = \arg \min_{u_i \in \mathcal{A}} |u_i - \mu_{i,\text{EP}}|^2 \quad (10)$$

or to approximate the LLR for each coded symbol in (7) as follows:

$$\text{LLR}(c_{ji}[l]) = \log \frac{\sum_{u_i \in \mathcal{B}_j(1)} \mathcal{N}(u_i : \mu_{i,\text{EP}}, \Sigma_{i,\text{EP}})}{\sum_{u_i \in \mathcal{B}_j(0)} \mathcal{N}(u_i : \mu_{i,\text{EP}}, \Sigma_{i,\text{EP}})}, \quad (11)$$

where $\mu_{i,\text{EP}}$ is the i -th component of mean vector $\boldsymbol{\mu}_{\text{EP}}$ and $\Sigma_{i,\text{EP}}$ is the i -th element of $\text{diag}(\boldsymbol{\Sigma}_{\text{EP}})$.

A. Parallel EP iterative method

Given the factorization of the posterior in (5), in EP we replace each one of the non-Gaussian factors by an unnormalized Gaussian:

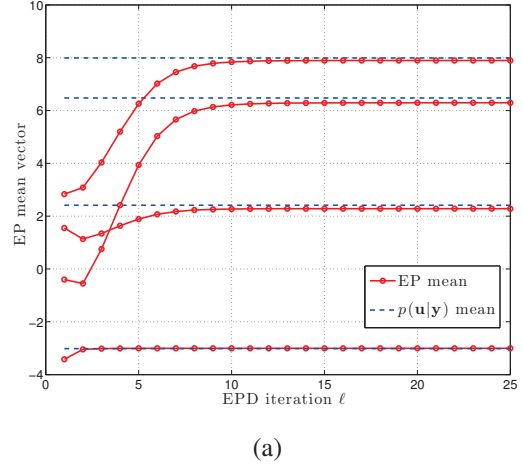
$$q(\mathbf{u}) \propto \mathcal{N}(\mathbf{y} : \mathbf{H}\mathbf{u}, \sigma_w^2 \mathbf{I}) \prod_{i=1}^{2m} e^{\gamma_i u_i - \frac{1}{2} \Lambda_i u_i^2}, \quad (12)$$

where γ_i and $\Lambda_i > 0$ are real constants. For any vector $\boldsymbol{\gamma} \in \mathbb{R}^{2m}$ and $\boldsymbol{\Lambda} \in \mathbb{R}_+^{2m}$, $q(\mathbf{u})$ is a Gaussian with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$:

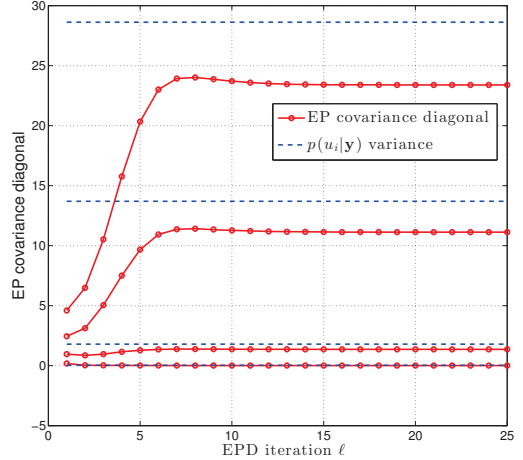
$$\boldsymbol{\Sigma} = (\sigma_w^{-2} \mathbf{H}^\top \mathbf{H} + \text{diag}(\boldsymbol{\Lambda}))^{-1}, \quad (13)$$

$$\boldsymbol{\mu} = \boldsymbol{\Sigma} (\sigma_w^{-2} \mathbf{H}^\top \mathbf{y} + \boldsymbol{\gamma}), \quad (14)$$

EP is able to converge to the solution in (8) and (9) at polynomial complexity by recursively updating the pairs (γ_i, Λ_i) , $i = 1, \dots, 2m$. For each input dimension, we use



(a)



(b)

Fig. 2. In (a), we plot the evolution of each component of the EP mean $\boldsymbol{\mu}^{(\ell)}$ in (14) as EP iterates for a $m = r = 2$ scenario with a 256-QAM constellation and SNR = 15 dB. In dashed lines, we indicate the mean of the true posterior $p(\mathbf{u}|\mathbf{y})$. In (b), for the same scenario, we plot the evolution of each component of the EP covariance $\boldsymbol{\Sigma}^{(\ell)}$ in (13). In dashed lines, we indicate the variance of the marginal symbol posterior $p(u_i|\mathbf{y})$.

a single non-Gaussian factor from the posterior (5) at each iteration. In this paper we follow the EP update rules described in [13], [14]. We initialize $\gamma_i = 0$ and $\Lambda_i = E_s^{-1}$ for all i (this would give the MMSE solution). At each EP iteration all pairs $(\gamma_i^{(\ell+1)}, \Lambda_i^{(\ell+1)})$ for $i = 1, \dots, 2m$ are updated in parallel, where ℓ denotes the EP iteration. Given the i -th marginal of the distribution $q^{(\ell)}(\mathbf{u})$, namely $q_i^{(\ell)}(u_i) = \mathcal{N}(u_i : \mu_i^{(\ell)}, \sigma_i^{2(\ell)})$, the pair $(\gamma_i^{(\ell+1)}, \Lambda_i^{(\ell+1)})$ is computed as follows:

1) Compute the cavity marginal:

$$q^{(\ell)\setminus i}(u_i) = \frac{q^{(\ell)}(u_i)}{\exp(\gamma_i^{(\ell)} u_i - \frac{1}{2} \Lambda_i^{(\ell)} u_i^2)} \sim \mathcal{N}(u_i : t_i^{(\ell)}, h_i^{2(\ell)}),$$

where

$$h_i^{2(\ell)} = \frac{\sigma_i^{2(\ell)}}{(1 - \sigma_i^{2(\ell)} \Lambda_i^{(\ell)})}, t_i^{(\ell)} = h_i^{2(\ell)} \left(\frac{\mu_i^{(\ell)}}{\sigma_i^{2(\ell)}} - \gamma_i^{(\ell)} \right).$$

2) Compute the mean $\mu_{p_i}^{(\ell)}$ and variance $\sigma_{p_i}^{2(\ell)}$ of the distribution

$$\hat{p}^{(\ell)}(u_i) \propto q^{(\ell)\setminus i}(u_i) \mathbb{I}_{u_i \in \mathcal{A}_i}. \quad (15)$$

3) Finally, the pair $(\gamma_i^{(\ell+1)}, \Lambda_i^{(\ell+1)})$ is updated so that the following unnormalized Gaussian distribution

$$q^{(\ell)\setminus i}(u_i) \exp(\gamma_i^{(\ell+1)} u_i - \frac{1}{2} \Lambda_i^{(\ell+1)} u_i^2) \quad (16)$$

has mean and variance equal to $\mu_{p_i}^{(\ell)}$ and $\sigma_{p_i}^{2(\ell)}$. A simple calculation shows that the solution is given by:

$$\Lambda_i^{(\ell+1)} = \frac{1}{\sigma_{p_i}^{2(\ell)}} - \frac{1}{h_i^{2(\ell)}}, \quad (17)$$

$$\gamma_i^{(\ell+1)} = \frac{\mu_{p_i}^{(\ell)}}{\sigma_{p_i}^{2(\ell)}} - \frac{t_i^{(\ell)}}{h_i^{2(\ell)}}. \quad (18)$$

Note that the update rules described above only need the marginal for each component. Given $\gamma^{(\ell)}$ and $\Lambda^{(\ell)}$ and once we have computed $\Sigma^{(\ell)}$ and $\mu^{(\ell)}$ using (13) and (14), then all $(\gamma_i^{(\ell+1)}, \Lambda_i^{(\ell+1)})$ pairs for $i = 1, \dots, 2m$ can be updated in parallel.

B. Complexity and moment matching convergence

The marginalization of the posterior in (6) has a significant complexity for discrete constellations. The Gaussian approximation of the posterior provided by EP makes this computation trivial. The complexity of EP per iteration is dominated by the computation of the covariance matrix in (13) and the mean vector in (14). Once the EP marginals $q^{(\ell)}(u_i)$ for $i = 1, \dots, 2m$ have been computed, the parallel update of all pairs $(\gamma_i^\ell, \Lambda_i^\ell) \leftarrow (\gamma_i^{(\ell+1)}, \Lambda_i^{(\ell+1)})$ for $i = 1, \dots, 2m$ has a small computational complexity, linear in $m|\mathcal{A}|$. Thus, if EP is run I iterations, the final complexity is $\mathcal{O}(I(m^3 + m|\mathcal{A}|))$.

With a toy example, we illustrate the EP ability to converge to the moments of the true posterior $p(\mathbf{u}|\mathbf{y})$ in (5). In Fig. 2(a), we show an example of the evolution of the components (in solid lines) of the EP mean vector $\mu^{(\ell)}$ in (14) as EP iterates for a given channel observation \mathbf{y} in a $m = r = 2$ scenario with a 256-QAM constellation and SNR = 15 dB. In dashed lines, we indicate the mean of the true posterior $p(\mathbf{u}|\mathbf{y})$ (real and imaginary parts). In 10 iterations, EP provides an accurate estimate of the posterior mean of the posterior distribution $p(\mathbf{u}|\mathbf{y})$ in (5). For the same scenario and same observed vector \mathbf{y} , Fig. 2(b) shows the evolution of the diagonal components of the EP covariance matrix $\Sigma^{(\ell)}$ in (13) as EP iterates. In dashed lines, we plot the real and imaginary values of

the variance of the marginal symbol posterior $p(u_i|\mathbf{y})$. EP, besides accurately matching the posterior mean, provides a reliable measure of the uncertainty per symbol, identifying which symbols can be decided with high grade of confidence and for which ones the risk of error in hard decision is large. As equal as BP, EP does not guarantee convergence but in the several scenarios tested in [11], this is not a problem at all for EPD. In addition, further results shown in Section IV also confirm that EP-based decisions only needs a few iterations to reach an excellent performance and that no improvement can be observed between $I = 10$ and $I = 100$.

IV. EXPERIMENTAL RESULTS

In this section, we illustrate the performance of the EP algorithm when used to compute hard-decisions in (10) and its ability to provide accurate estimate to the bit posterior probabilities in (11). These probabilities are then fed to the LDPC decoder. We consider a $m = r = 32$ scenario with a 16-QAM modulation and the EP performance is shown in terms of the symbol error rate (SER) when measured before the LDPC channel decoder. After the LDPC decoder, is given in terms of the LDPC word error rate (WER). We have used a (3,6)-regular LDPC code of length $n = 5120$ bits and rate 1/2. All results have been averaged with 5000 realizations of the channel matrix.

In Fig. 3, we compare the performance of an EP detector (EPD) when used to compute hard-decisions with $I = 100$ iterations (EPD 100), EPD with $I = 10$ iterations (EPD 10) and EPD with only $I = 2$ iterations (EPD 2). We compare the results with MMSE, GTA and GTA-SIC. All these algorithm, including EP, have a complexity $\mathcal{O}(m^3)$. Besides, we do not include comparison with respect MMSE-SIC since its tends to overlap with GTA in most of the SNR range [5]. In Fig. 3, observe that EPD is able to significantly outperform GTA-SIC at the same complexity order. Note also that running EPD for 100 iterations does not result in an appreciable gain in performance with respect to the case $I = 10$ and, in addition, we can modulate number of iterations in EP according to our complexity constraints, between $I = 2$ and $I = 10$, without degrading the performance above the GTA-SIC curve.

In Fig. 4, we show the WER system performance after the LDPC decoding stage using the MMSE and GTA and EP soft-outputs. The EP algorithm is run for 10 iterations. The BP is run until it coversges to a valid codeword or up to 200 iterations. We also include the performance corresponding to GTA-SIC with hard LDPC decoding, since GTA-SIC is not able to provide posterior probability estimates.

As expected, the MIMO receiver based on EP for soft-output detection provides the best performance, with gains of close to one order of magnitude. The MMSE performance is quite poor and so it is the combination of GTA-SIC and hard-decoding of the LDPC code. Despite GTA-SIC is a good algorithm for hard-decision at $\mathcal{O}(m^3)$ complexity, see Fig. 3, the lack of soft-information severely degrades the decoding stage. Compared to the GTA algorithm, at similar complexity order, the EP algorithm is able to compute more accurate

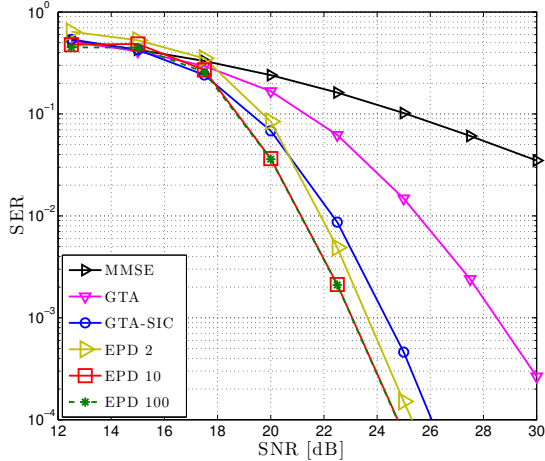


Fig. 3. Hard-decision performance of EPD with $I = 100$ iterations, EPD with $I = 10$ iterations, EPD with $I = 2$ iterations, GTA-SIC, GTA and MMSE for the case $m = r = 32$ and a 16-QAM constellation.

estimates to the QAM symbol posterior probabilities $p(u_i|y)$ $i = 1, \dots, m$ and, consequently, to the posterior probability for each encoded bit. These accurate estimates ultimately explain the significant performance gain after the LDPC decoding stage. These results confirm that the moment matching criterion used by EP to adjust the Gaussian approximation to the discrete posterior $p(\mathbf{u}|y)$ yields a powerful approximation that can be exploited in large MIMO systems.

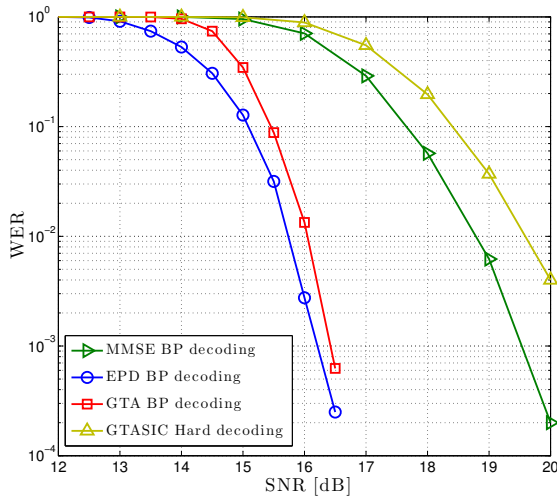


Fig. 4. WER performance of EPD with $I = 10$ iterations, GTA and MMSE with BP channel decoding for the case $m = r = 32$ and a 16-QAM constellation. We also include the case GTA-SIC with hard decoding of the LDPC code.

V. CONCLUSIONS

The design of efficient MIMO digital receivers is a challenging open problem when the complexity grows either for the dimension of the MIMO system or for the order of the

constellation. To this complexity we need to add the problem of providing accurate soft-bit information to the decoder when the marginal of the posterior is difficult to compute if realistic signaling is used.

In this paper, we focus on obtaining soft-decision information that can be further used for symbol detection or for soft-decoding. Expectation Propagation is shown to be a powerful approximate inference technique to construct tractable approximations to a given probability distribution: we put forward a valid approximation for the posterior that solves the receiver complexity problem and that dramatically simplifies the marginal computation for soft-bit information. Among existing methods for soft-output MIMO detection that have cubic complexity in the number of antennas, we have shown EP is able to provide outstanding performance when combined with an LDPC coded system since it provides accurate estimates to the posterior probability for each encoded bit.

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