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Assessment of Fitts' Law for Quantifying Combined Rotational and Translational Movements

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Objective: To develop a model for human performance in combined translational and rotational movements based on Fitts' law. **Background:** Fitts' law has been successfully applied to translational movements in the past, providing generalization beyond a specific task as well as performance predictions. For movements involving both translations and rotations, no equivalent theory exists, making comparisons of input devices for these movements more ambiguous. **Method:** The study consisted of three experiments. In the first two, participants performed either pure translational or pure rotational movements of 1 degree of freedom. The third experiment involved the same movements combined. **Results:** On average, the performance times for combined movements were equal to the sum of the times for equivalent separate rotational and translational movements. A simple Fitts' law equivalent for combined movements with a similar slope as the separate com-ponents was proposed. In addition, a significant degree of coordination of the combined movements was found. This had a strong bias toward a parallel execution in 12 out of 13 participants. **Conclusion:** Combined movements with rotations and translations of 1 degree of freedom can be approximated using a simple Fitts' law equivalent. The rotational and translational components appear to be coordinated by the central nervous system to generate a parallel execution. **Application:** The results may help drive human interface designs and provide insights into the coordination of combined movements. Future extensions may be possible for the movements of higher degrees of freedom used in robot teleoperation and virtual reality applications.

INTRODUCTION

Since its original publication, Fitts' law (Fitts, 1954) has been an important tool in modeling the speed-accuracy trade-off in simple human movements. As seen in Equation 1, the model predicts that the mean time (MT) to complete a movement varies linearly with the index of difficulty (ID). This index is a function of the distance moved and the accuracy requirement, or tolerance, on the movement. These are denoted as the amplitude of movement, A, and the width of the target area, W, respectively. Typically, the application of the law is in simple left-to-right movements of the hand, here denoted as translational movements with 1 degree of freedom (DOF). The term *translation* is used to indicate the lack of a rotational requirement on the movement itself, although rotations of one or more joints will be necessary to perform the task. The coefficients a and b are determined experimentally with the use of a linear regression analysis. The slope coefficient b then becomes a measure of the rate of change of completion time with change in the difficulty of the task and the reciprocal, $1/b$, is known as the index of performance (IP). In other words, human performance for a task with a given distance and accuracy requirement can be predicted on the basis of observations of other such combinations.

Equation 1 illustrates Fitts' law in its original formulation (Fitts, 1954).

$$\mathrm{MT} = a + b[\mathrm{ID}],$$
$$\mathrm{ID} = \log_2\left(\frac{2A}{W}\right). \tag{1}$$

Equation 2 illustrates Fitts' law according to the Shannon formulation (MacKenzie, 1989).

$$\begin{aligned} \mathrm{MT} &= a + b[\mathrm{ID}], \\ \mathrm{ID} &= \log_2\left(\frac{A}{W} + 1\right). \end{aligned} \tag{2}$$

The form of Fitts' law used in this article was first proposed by MacKenzie (1989) and is shown in Equation 2. This version, which is also known as the Shannon formulation, has a better correspondence with the underlying information theory basis of the law and has been shown to provide a better fit to experimental data than the one originally presented by Fitts (1954). It is also the basis for performance testing in ISO 9241-9 (International Organisation for Standardisation [ISO], 2002), which covers ergonomic requirements for nonkeyboard computer input devices. This standard also includes a recommendation for performing the adjustment for accuracy. This implies the calculation of an effective target width for each participant and each condition based on the standard deviation of endpoints found during testing. An effective amplitude equivalent can be calculated from the actual distance moved by each participant for each condition. Substituted for *W* and *A* in Equation 2, an effective ID can then be defined for each case. This should more closely represent the actual difficulty of the task for each participant and can be used to define a new measure, namely, throughput (TP). Although TP is sometimes used to denote IP, the definition of TP used here follows that in Soukoreff and MacKenzie (2004), as seen in Equation 3. This form is also known as the mean-of-means TP, a notation that will be used in the rest of this article. Although similar to the IP described earlier, it is considered to give a better representation of the actual performance of the participants, as it combines the speed and accuracy of the movement performance into one dependent measure.

Equation 3 shows the calculation of the mean-of-means TP.

$$\mathrm{TP} = \frac{1}{y}\sum_{i=1}^{y}\left(\frac{1}{x}\sum_{j=1}^{x}\frac{\mathrm{ID}e_{ij}}{\mathrm{MT}_{ij}}\right) \tag{3}$$

Fitts' law has been used extensively in the field of human-computer interaction (HCI) to quantify performance and drive graphical user interface design. The law is also commonly used in comparisons of input devices, where it provides the capability to generalize about results beyond a specific task. One of the early applications of Fitts' law in HCI was in the favorable comparative evaluation of the mouse with other input devices (originally by Card, English, & Burr, 1978). In fact, Fitts' law remains one of very few hard quantitative tools available to designers of human-machine interfaces, even though it is today considered more an empirical regularity than a model of the underlying mechanics of human movement.

Multidimensional versions of the law have been developed, including those for pointing in 2-D (MacKenzie & Buxton, 1992) and 3-D (Grossman & Balakrishnan, 2004). Rotational movements based on Fitts' law tasks have also been explored. Early studies followed up on Fitts' (1954) original study to determine the best representation of the difficulty of a rotary task (Knight & Dagnall, 1967). It was found that the IP was similar to that found in translational movements (Crossman & Goodeve, 1983) and that Fitts' law could represent rotational tasks reasonably well. Indeed, the law has been extended to represent elbow flexion-extension (Kondraske, 1994) and more recently has been proposed for more complex models of human upper limb performance (Yang, Jin, Zhang, Huang, & Wang, 2001).

The intention behind Fitts' (1954) original experiment was to establish the information capacity of the human motor system. This was inspired by the information-theoretic approaches popular at the time and, more specifically, the effect of noise in limiting the information capacity of a communications channel (for a detailed description, see MacKenzie, 1992). Trying to explain the empirical regularity inherent in the law using theories of human movement has since been an active research field, with a definite conclusion yet to be made. This includes the iterative-corrections model of Crossman and Goodeve (1983), the stochastic optimized-submovement model of Meyer, Abrams, Kornblum, Wright, and Smith (1988), and more recent neurodynamic approaches (Beamish, Bhatti, MacKenzie & Wu, 2006). For the practitioner, however, most of these theories are not easy to apply in the two main uses of Fitts' law, comparing input devices and making movement time predictions. This ease of application to experimental research ensures that Fitts' law will likely remain a useful tool in the design of human-machine interfaces for the foreseeable future. This is also a motivation for the work presented here.

As outlined earlier, the previous applications of Fitts' law have focused on 1-, 2-, or 3-DOF translational tasks or 1-DOF rotational tasks. However, movements that require both translations and rotations are common in many human-machine interface applications. One example is the manipulation of virtual objects in computer-aided modeling and in virtual reality applications. Another is teleoperation of robot manipulators, in which an operator typically controls the translation and rotation of the robot end effector. These applications differ from typical computer workstation tasks in that they require the user to control up to 3-DOF rotational movements and 3-DOF translational movements simultaneously, indeed like many movements of the human hand.

Many different user interfaces exist for this purpose. These include free-flying input devices that directly relate the rotational and translational pose of the input device to the object controlled. Other devices, such as joysticks, are rate based, whereby the deflection or force applied to the device is used to control the velocity of movement. However, determining the most suitable input device for a given 6-DOF application can be difficult because of the many variables involved (Bowman, Kruijff, Laviola, & Poupyrev, 2004). One approach is to test each device in very specific scenarios, for example, peg-in-the-hole tasks. This approach may be sufficient if the tasks are limited and well known, but for most applications, the actual usage of an input device can involve any number of combinations of the translations and rotations available. Each could also have a very specific amplitude and accuracy requirement that needs to be coordinated. This need makes it difficult to come up with a representative set of tasks for a comparison of these input devices. A model that could relate the relevant properties of a 6-DOF task with the completion time could help practitioners generalize to other similar tasks. This model would make comparisons of these input devices less ambiguous and may help resolve lingering issues about their application.

One example is the Remote Manipulator System (RMS) flown on the Space Shuttle, which is controlled with the use of one hand controller for rotations and one for translations. The benefit of separating the DOF between hands was promoted by Hartley, Cwynar, Garcia, and Schein (1986) to help reduce piloting errors arising in high-workload situations. One example given was that a single 6-DOF controller induced unwanted coupling between the controlled axes, such as unwanted roll or yaw rotations accompanying an intentional side-to-side translation command. However, O'Hara (1987) found little difference in errors for similar tasks, and some have argued that one 6-DOF controller is preferable (McKinnon & King, 1988). One question here is the use of one versus two hands. Another is that of coordinating the rotations and translations required for the task. The studies primarily assessed completion time and errors for specific tasks, such as spacecraft docking, a typical approach for studies on high-DOF movements. Other measures have been proposed to quantify the degree of coordination (Zhai & Milgram, 1998). However, a model that can relate task performance with the distance and tolerance parameters of the task would also implicitly describe the coordination of rotations of translations.

The aim of this study was to investigate whether it is possible to extend the speed-accuracy relationship represented by Fitts' law to more complex movements involving both rotations and translations. Movements involving 1-DOF rotation and 1-DOF translation were chosen. Separate 1-DOF rotations and translations have previously been modeled with Fitts' law relationships, for example, in Crossman and Goodeve (1983), but not in movements involving combinations of both. Although simplistic, the choice of only 2 DOF was judged a necessary first step toward modeling more complex 6-DOF movements and allows for direct comparison with the same translational and rotational movements performed separately.

But how can a model for combined movements be deduced from the Fitts' law relationships of the separate movements? One potential approach is shown in Equation 4, in which the MT for a combined movement is assumed to be equal to the sum of the times of the separate movements. This assumption means that the completion time for a task in which both rotation and translation are performed as a combined movement should be approximately equal to the total completion time when the movements are performed separately. However, a further simplification of the model would be beneficial to reduce the number of experimentally determined values. As mentioned earlier, previous studies have indicated that rotational movements have a similar IP as translational movements (Crossman & Goodeve, 1983). If true, the combined model could potentially be reduced to that seen in Equation 5. The rotational distance equivalent here is α, and ω represents the rotational tolerance. This is a simple linear model with two experimentally determined coefficients like Fitts' original law and with an ID that is fully defined by the four parameters describing the combined task. From this model, a combined mean-of-means TP can also be calculated, which describes the speed and accuracy of the combined movement in one dependent variable. From an application point of view, this capability would make it an attractive model for movements including both rotations and translations.

Equation 4 shows the mean time to complete a combined movement in terms of its components.

$$\mathrm{MT}_{\mathrm{combined}} = \mathrm{MT}_{\mathrm{rotation}} + \mathrm{MT}_{\mathrm{translation}} \tag{4}$$

Equation 5 illustrates the proposed model for combined rotational and translational movements.

$$\begin{aligned} \mathrm{MT}_{\mathrm{combined}} &\approx a_{\mathrm{combined}} + b[\mathrm{ID}_{\mathrm{rotation}} + \mathrm{ID}_{\mathrm{translation}}], \\ \mathrm{ID}_{\mathrm{rotation}} &= \log_2\left(\frac{\alpha}{\omega} + 1\right), \\ \mathrm{ID}_{\mathrm{translation}} &= \log_2\left(\frac{A}{W} + 1\right). \end{aligned} \tag{5}$$

Three experiments were performed to evaluate the model for combined movements in Equation 5. Experiments 1 and 2 were straightforward applications of Fitts' law for separate 1-DOF translation and 1-DOF rotation, respectively. These experiments then formed the basis for comparison with the combined movements of 1-DOF translation and 1-DOF rotation performed in Experiment 3. The three experiments each involved a separate set of participants. These groups were made up of 12, 13, and 13 students and staff of the University of Maryland, respectively.

EXPERIMENT 1: TRANSLATIONAL MOVEMENTS

This experiment involved 1-DOF translational movements only, the standard application of Fitts' law. There were 16 levels of ID, via combinations of four translational distances and four translational tolerances. See Table 1 for details.

Method

Participants. The participants were 12 students and staff of the University of Maryland, 6 male and 6 female. None of the participants was included in the other two experiments in this study. All participants were right-handed and were between 18 and 35 years old with a mean of 22.3 years. There were 9 participants with corrected vision, and 8 had previous experience with 3-D input devices. All gave their informed consent to participate according to the regulations of the University of Maryland's institutional review board. Participants not employed by the Space Systems Laboratory were paid $10 for their participation.

Apparatus. The experiment was conducted on an Apple iMac workstation in an office environment. The participants worked on a 20-in. external liquid crystal display monitor (Dell 2007WFP) with a 60-Hz refresh rate. The input device was a magnetic Ascension Flock of Birds sensor system sampling at a rate of 70 Hz. The system has a specified translational and rotational accuracy of 1.8 mm (0.8 mm resolution) and 0.5° (0.1° resolution), respectively. The sensor of this system was held in the participant's dominant hand and measured one translational DOF, the left-to-right position relative to the display in front of the participant. These measurements were used to update the

TABLE 1: All Combinations of Distances (*A* in cm and α in degrees) and Tolerances (*W* in cm and ω in degrees) Used for Experiments 1, 2, and 3

	Experiment 1			Experiment 2			Experiment 3					
Combination	*A*	*W*	ID_t	α	ω	ID_r	*A*	*W*	ID_t	α	ω	ID_r
1	4.8	0.6	3.1	40	3	3.8	12.7	0.8	4.1	50	4	3.8
2	4.8	1.3	2.2	40	6	2.9	12.7	1.6	3.2	50	4	3.8
3	4.8	1.9	1.8	40	9	2.4	4.8	0.8	2.8	50	4	3.8
4	4.8	2.5	1.5	40	12	2.1	4.8	1.6	2.0	50	4	3.8
5	9.5	0.6	4.0	80	3	4.8	12.7	0.8	4.1	50	12	2.4
6	9.5	1.3	3.1	80	6	3.8	12.7	1.6	3.2	50	12	2.4
7	9.5	1.9	2.6	80	9	3.3	4.8	0.8	2.8	50	12	2.4
8	9.5	2.5	2.2	80	12	2.9	4.8	1.6	2.0	50	12	2.4
9	14.3	0.6	4.6	120	3	5.4	12.7	0.8	4.1	130	4	5.1
10	14.3	1.3	3.6	120	6	4.4	12.7	1.6	3.2	130	4	5.1
11	14.3	1.9	3.1	120	9	3.8	4.8	0.8	2.8	130	4	5.1
12	14.3	2.5	2.7	120	12	3.5	4.8	1.6	2.0	130	4	5.1
13	19.1	0.6	5.0	160	3	5.8	12.7	0.8	4.1	130	12	3.6
14	19.1	1.3	4.0	160	6	4.8	12.7	1.6	3.2	130	12	3.6
15	19.1	1.9	3.5	160	9	4.2	4.8	0.8	2.8	130	12	3.6
16	19.1	2.5	3.1	160	12	3.8	4.8	1.6	2.0	130	12	3.6
Minimum	4.8	0.6	1.5	40	3	2.1	4.8	0.8	2.0	50	4	2.4
Maximum	19.1	2.5	5.0	160	12	5.8	12.7	1.6	4.1	130	12	5.1

Note. Corresponding indices of difficulty (IDs) are given for the rotational (subscript *r*) and translational (subscript *t*) components for the respective experiments, based on the Shannon formulation shown in Equation 2.

left-to-right position of a cursor on the display. A calibration was performed to ensure that the apparent displacement of the cursor on the display corresponded with that of the handheld sensor. The participants also manipulated a button with the alternate hand. This button was reading user inputs at a frequency of 100 Hz. See Figure 1 for the physical setup of the experiment. Computer-generated voice feedback was used to inform the participant about the successful or unsuccessful completion of each trial.

Stimuli. The stimuli presented on the display can be seen in Figure 2a. A cursor in the form of a small disc on the display indicated the left-to-right position of the handheld sensor held by the participant, as described. Two vertical lines on the display were used to indicate the area from which each trial should start, and two vertical lines were used to indicate the area within which each trial should finish. The distance between the center of the two areas (movement distance) and the size of the areas (movement tolerance) were varied randomly.

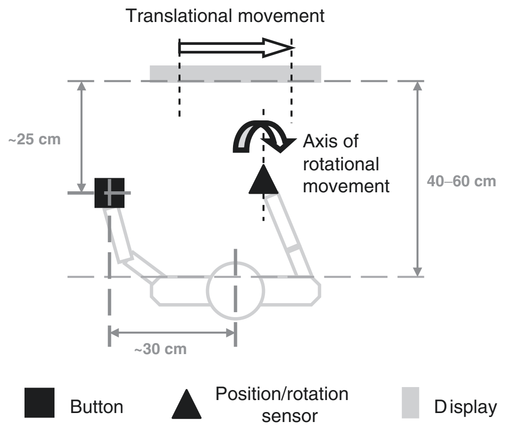

Figure 1. Overhead view of general experiment setup in right-handed configuration.

Procedure. The participants were instructed to start each trial with the disc within the start area on the display. When ready, the participant would then press and hold a button with the alternate hand, indicating the start of the trial.

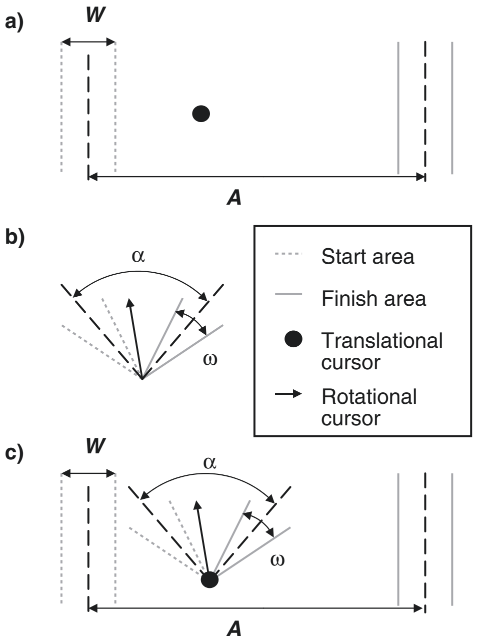

Figure 2. Representations of visual stimuli provided to the participants for Experiments 1, 2, and 3 (a, b, and c, respectively, in figure). Task variables have been added for clarity but were not part of the display shown to the participants in the experiments.

If the button was pressed while the disc was within the start area, a timer was started. If the disc was outside the start area when the button was pressed, the participant was notified and the trial restarted. The successful start of the timer was indicated by a change in color of all objects on the display from blue to green. The participants were told that the goal of each trial was to then move the disc to within the finish area and let go of the button. If the button was released outside the finish area, the trial was restarted, the participant notified, and an error recorded. If the button was released inside the finish area, the timer was stopped and the participant notified of the successful completion of the trial. The movements were from left to right. The participants were instructed to keep the hand holding the sensor (the dominant hand) off the table during movements to prevent obstruction. The participants were instructed to emphasize accuracy versus speed (consistent with instructions given in Fitts, 1954) to achieve an error level of approximately 2%. Feedback on the actual error percentage was given during the initial training session to help the participants adjust their performance.

Each participant's session lasted approximately 1 hr, starting with a thorough brief of the task procedures and a short questionnaire about basic personal information. After completing the task, each participant filled out a questionnaire about the experiment and was also asked to provide suggestions for improving the experiment. Each experiment included 16 different combinations of distances and tolerances, each repeated 12 times for nontraining trials. All participants were first given 48 trials for training to get acquainted with the task and the experimental setup. The participants then performed 96 trials, followed by a 5-min break and 96 more trials. The participants were also allowed to take short breaks in between each trial as needed.

Results and Discussion

The mean completion time across participants for different indices of difficulty can be seen in Figure 3. The relative standard deviation ranged from 20.7% for an ID of 3.09 bits to 30.6% for an ID of 1.52. A linear fit produced a slope of 0.31 and an intercept of 0.22. The square of the correlation coefficient for the fit, r^2, was 0.984. Because of participant error, 2.3% of trials were rerun. The mean-of-means TP was 3.17 bits/s.

The correlation to the linear fit postulated by Fitts' law was similar to that typically reported in the literature for 1-DOF translational tasks (Fitts, 1954; MacKenzie, 1992), in spite of the small but noticeable time delay inherent in the mapping from the sensor to the cursor on the display. Equivalent Flock of Birds systems have been found to exhibit tracker latencies from movement to system response of 23 ms (Mine, 1993).

EXPERIMENT 2: ROTATIONAL MOVEMENTS

This experiment involved rotational movements only. Through combinations of four rotational distances and four rotational tolerances, 16 levels of ID were used. See Table 1 for details.

Method

Participants. The participants were 13 students and staff of the University of Maryland,

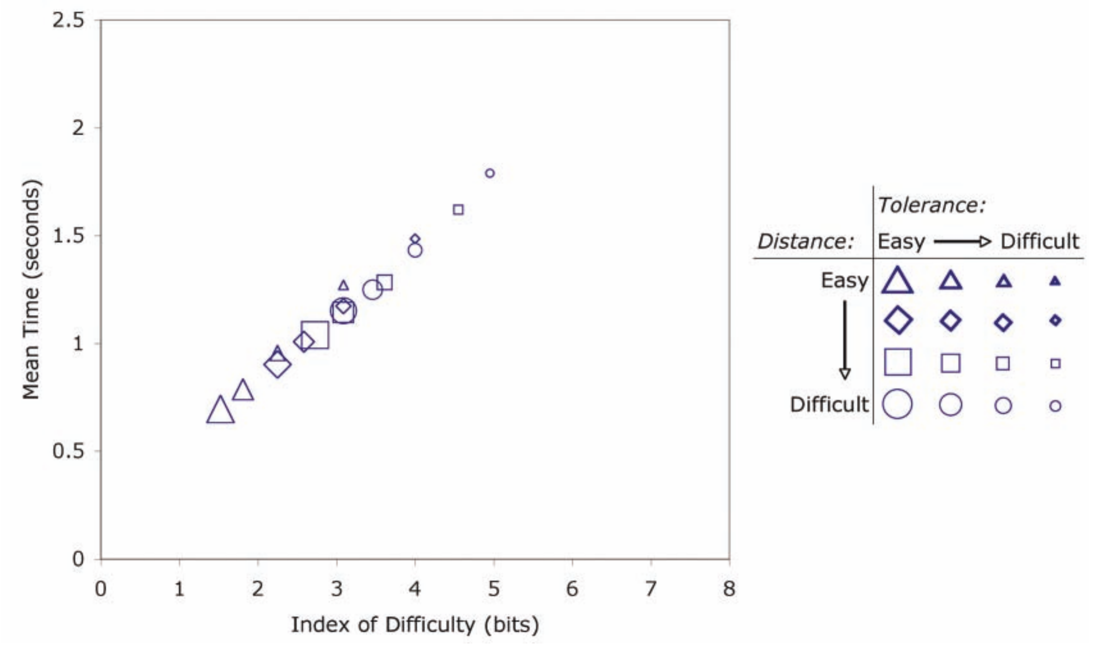

Figure 3. Mean completion time against index of difficulty for translational task in Experiment 1. The levels of tolerance (here, *W*) and distance (here, *A*) are indicated for each task.

5 male and 8 female. None of the participants was included in the other two experiments in this study. Among the participants, 12 were right-handed and 1 was left-handed. All were between 19 and 48 years old, with a mean of 23.5 years. There were 12 participants with corrected vision, and 6 had previous experience with 3-D input devices.

Apparatus. The participants worked with the same physical apparatus as used in Experiment 1. For this experiment, the handheld sensor measured one rotational DOF only, the roll axis of the handheld sensor from the participant's point of view; see Figure 1. Thus, the task required rotary movements mainly about the longitudinal axis of the forearm, similar to turning a doorknob or using a standard screwdriver. No physical restriction was placed on the movement, allowing the participants to use both fingers and wrist to produce the rotation. The measurements from the sensor were used to update the angle of a cursor on the display. The apparent angle of the cursor on the display corresponded with that of the handheld sensor.

Stimuli. The stimuli presented on the display can be seen in Figure 2b. A cursor in the form of a line on the display indicated the angle of the handheld sensor held by the participant, as described earlier. Two radial lines on the display were used to indicate the area from which each trial should start, and two radial lines were used to indicate the area within which each trial should finish. The angle between the center of the two areas (movement distance) and the angle between the two lines representing an area (movement tolerance) were varied randomly.

Procedure. The participants followed the same procedure as in Experiment 1, except for the type of movement performed. The rotational movements were clockwise for right-handed participants. Left-handed participants used a mirror image setup, performing the same forearm-finger supination. The participants were instructed to keep the hand holding the sensor (the dominant hand) off the table during movements to prevent obstruction.

Results and Discussion

The mean completion time across participants for different IDs can be seen in Figure 4. The relative standard deviation ranged from

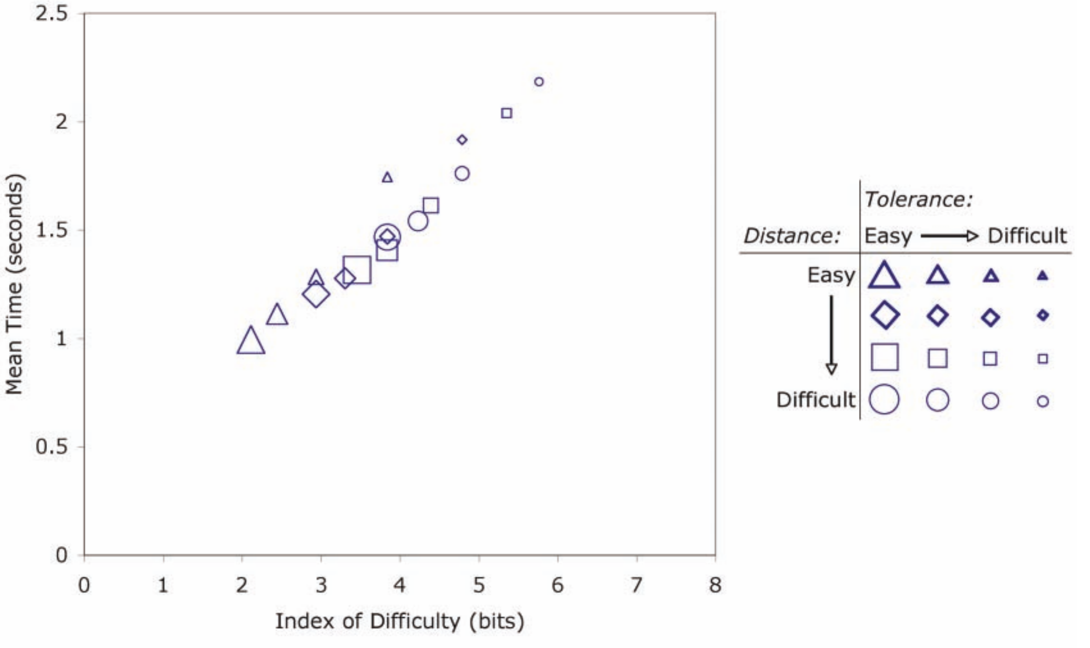

Figure 4. Mean completion time against index of difficulty for rotational task in Experiment 2. The levels of tolerance (here, ω) and distance (here, α) are indicated for each task.

21.9% for an ID of 3.46 bits to 32.2% for an ID of 4.79. A linear fit produced a slope of 0.32 and an intercept of 0.27. The square of the correlation coefficient for the fit, r^2, was 0.930. Because of participant error, 1.7% of trials were rerun. The mean-of-means TP was 3.02 bits/s.

A reasonable correlation to a linear fit was achieved, although less so than for translational movements (for a 95% confidence interval). This can also be seen from Figure 4, in which the results from changes in distance do not line up as well with those from changes in tolerance. However, the slope of the linear fit was very similar to that found for translational movements (differing by 4.4%). Similarly, the mean-of-means TP is 4.9% lower than in Experiment 1. This result, together with evidence from the literature (Crossman & Goodeve, 1983), indicates that these types of translational and rotational movements indeed have approximately the same IP, supporting the model proposed. There is a 24.3% increase in the intercept of the fit for the rotational movements; however, this measure is typically attributed to noninformational aspects of the task (Zhai, 2004). Thus the intercept is not typically affected by the task distance or tolerance but, rather, may indicate the cognitive effort required for initiating a movement or may indicate regression errors.

EXPERIMENT 3: COMBINED MOVEMENTS

This experiment involved movements with a translational and rotational component. A total of 16 levels of the combined ID were used, with combinations of two rotational distances, two rotational tolerances, two translational distances, and two translational tolerances. The model presented in Equation 5 was used to establish the IDs. The sum of the rotational and translational ID ranged from 4.37 to 9.15 bits. See Table 1 for details.

Method

Participants. The participants were 13 students and staff of the University of Maryland, 11 male and 2 female. None of the participants was included in the other two experiments in this study. Of the participants, 12 were right-handed and 1 was left-handed. All were between 18 and 32 years old, with a mean of 22.2 years. There were 12 participants with corrected vision, and 9 had previous experience with 3-D input devices.

Apparatus. The participants worked with the same physical apparatus as used in Experiments 1 and 2. For this experiment, the handheld sensor measured 2 DOF only, the translational DOF used in Experiment 1 and the rotational DOF used in Experiment 2. Thus the sensor measured the left-to-right position relative to the display in front of the participant as well as the roll axis of the handheld sensor. These measurements were used to update the left-to-right position of a translational cursor as well as the angle of a rotational cursor on the display. As in Experiments 1 and 2, the apparent position and rotation of the respective cursors corresponded with those of the handheld sensor.

Stimuli. The stimuli presented on the display can be seen in Figure 2c. A cursor in the form of a disc indicated the left-to-right position, and a cursor in the form of a line indicated the angle of the handheld sensor held by the participant, as described earlier. The rotational axis of the rotational cursor was kept centered at the position of the translational cursor at all times. Thus, the rotational tasks, including the radial lines representing the rotational start and finish areas, moved with the translational cursor. This design was intended to reduce the eye movements needed to coordinate the two tasks and to provide a visualization of the combined movement that was as similar as possible to the actual movements of the handheld sensor. The movement distances and tolerances were varied randomly.

Procedure. The participants followed the same procedure as in Experiments 1 and 2 except for the type of movement performed. The translational movements were from left to right and the rotational movements clockwise for right-handed participants. Left-handed participants used a mirror image setup, performing the same arm abduction and forearm-finger supination. The participants were instructed to keep the hand holding the sensor (the dominant hand) off the table during movements to prevent obstruction.

Results and Discussion

The mean completion time across participants for different IDs can be seen in Figure 5a. The relative standard deviation ranged from 21.8% for an ID of 5.54 bits to 35.1% for an ID of 9.15. A linear fit produced a slope of 0.32 and an intercept of 0.46. The square of the correlation coefficient for the fit, r^2, was 0.817. Because of participant error, 2.8% of trials were rerun. The mean-of-means TP was 3.28 bits/s.

A lower correlation was achieved compared with Experiment 1 but not compared with Experiment 2 (for a 95% confidence interval). Thus, the IP for combined movements has a stronger dependency on the composition of the distances and tolerances of the task than does that for simple translational movements. Although there is still a clear linear trend, this increased dependence means the data provide only partial support for the model proposed. However, the combined slope matched the slopes found in the two first experiments very well, differing by 3.9% from the slope of the translational movements in Experiment 1 and 0.5 % from the slope of the rotational movements in Experiment 2. The mean-of-means TP is 3.2% higher with respect to Experiment 1 and 8.6% higher with respect to Experiment 2.

In Figure 5b, the results for the separate movements in Experiments 1 and 2 were included for comparison. The 256 points representing these results were created by adding the MTs and respective IDs for each combination of separate rotational and translational movements. Thus, the separate movements in the figure represent the application of the combined model (Equation 5) with data from Experiments 1 and 2. A linear fit to this data ($r^2 = 0.953$) produced a slope of 0.32 and intercept of 0.5. This was very close to the result obtained for the combined movements, differing by only 1.5% in slope and 7.8% in intercept. These results indicate that the IP for combined movements on average may be equivalent to that for separate rotational and translational movements.

A four-way analysis of variance (ANOVA) of the results in Experiment 3 was performed to

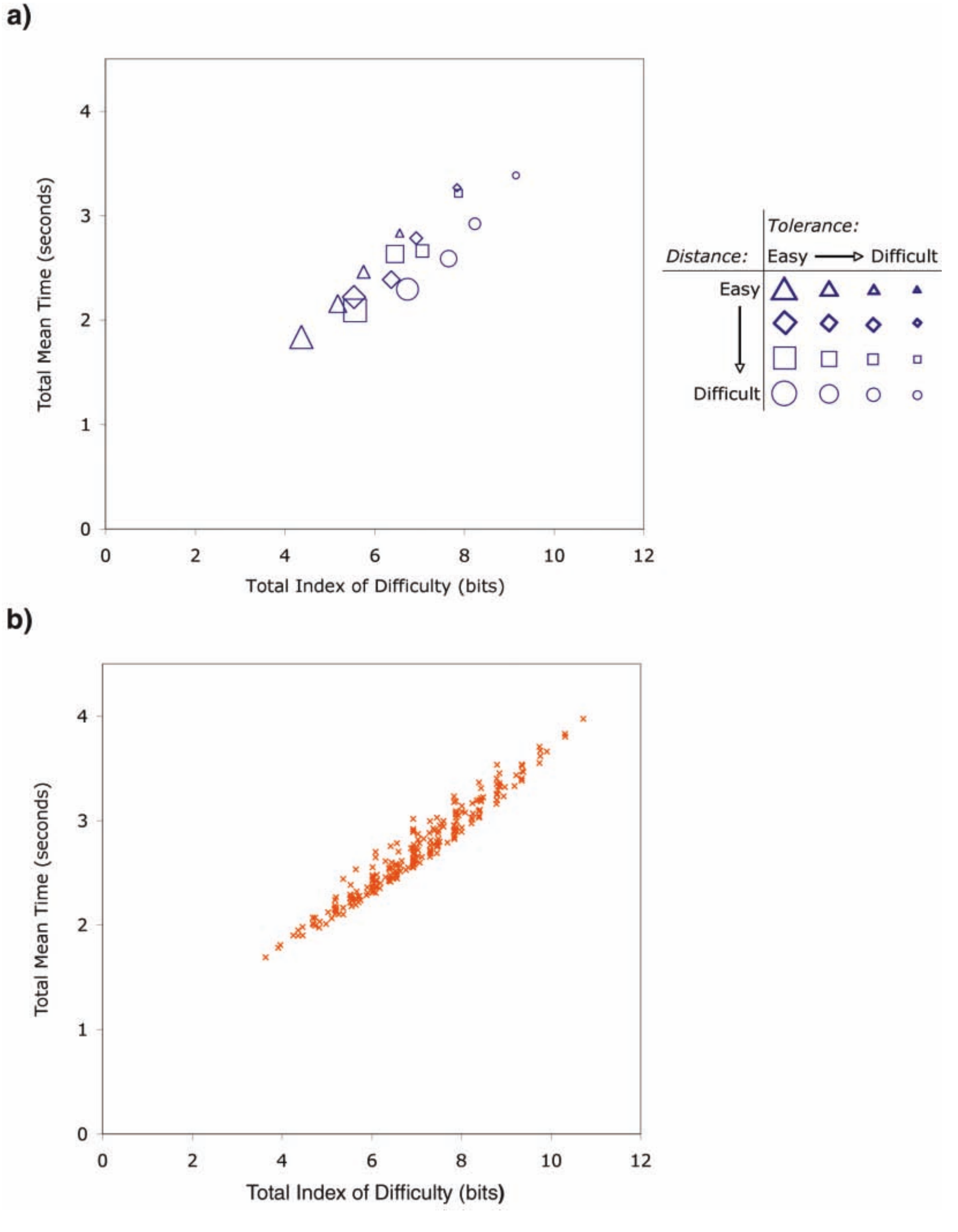

Figure 5. Mean completion time against total index of difficulty for combined task in Experiment 3 (a in figure) compared with summed separate rotational and translational results from Experiments 1 and 2 (b in figure). The levels of tolerance (here, W and ω) and distance (here, A and α) are indicated for each combined task.

assess the contribution of the four task parameters on the results obtained. A mixed model was used, with the two distances (low, high) and two tolerances (low, high) as fixed effects and participants as a random effect. Thus, it was assumed that the participants were sampled at random from a large population and could be modeled as a random variable with zero mean and an unknown variance.

All the main effects were significant on the .05 significance level, yielding F values of $F(1, 2468) = 51.6$, $p < .0001$, $\eta^2 = .010$, for rotational distances; $F(1, 2468) = 170.2$, $p < .0001$, $\eta^2 = .033$, for translational distances; $F(1, 2468) = 804.0$, $p < .0001$, $\eta^2 = .154$, for rotational tolerances; and $F(1, 2468) = 288.9$, $p < .0001$, $\eta^2 = .055$, for translational tolerances. Thus, both distances and both tolerances significantly affected the time taken to perform the combined task, supporting their inclusion in the model. In addition, there were two significant interactions at the .05 significance level, although with relatively low effect size indices. The interaction between the two distance parameters yielded $F(1, 2468) = 17.1$, $p < .0001$, $\eta^2 = .003$, and the interaction between the two tolerance parameters of the task yielded $F(1, 2468) = 8.5$, $p < .005$, $\eta^2 = .002$. In other words, the execution of the rotational movement was affected by the distance and tolerance requirement on the translational movement, and vice versa. This indicates that the participants performed some level of coordinated planning and execution of the two movement components. Indeed, it was found that 12 out of 13 participants performed the combined movements in parallel, and 1 performed them strictly serially. Examples of combined trajectories for a single task (1 out of 12 occurrences) across all 13 participants can be seen in Figure 6. The same pattern was observed across all trials.

To explore this coordination further, a numerical comparison of the relative timing of the rotational and translational movements across all 13 participants was performed. A new measure was introduced: the difference in time at which the participant first crossed the halfway point between the translational and rotational start and finish areas, Δt_{mid}. The halfway point was used to avoid the ambiguity in coding the often multiple crossings of the finish area. The two sampled points flanking the actual boundary were used, and a best estimate of the actual time was found with the use of linear interpolation. The mean of the absolute value of Δt_{mid} across all trials for the 12 participants who performed the movements in parallel was 114 ms (participant means ranging from 67 ms to 247 ms), with a standard deviation of 121 ms. The mean of the absolute value of Δt_{mid} for the participant who performed the movements serially was 1,620 ms, with a standard deviation of 816 ms.

Thus an order-of-magnitude difference in the relative timing of translational and rotational movements was observed between the participants performing in parallel and the one in serial. Few conclusions can be drawn about the latter without more data, however. This participant was 1 of 9 who had previous experience with input devices for 3-D applications, so inexperience is not a likely reason for the difference in execution. For the 12 participants performing the movements in parallel, the rotations and translations were executed with a surprisingly high degree of coordination across participants, despite the physically disparate nature of the two movements. In addition, the two movements had for many trials very different IDs and expected completion time but seemed to be executed so as to start and finish at the same time. Thus, for these participants, the execution can be said to be strictly in phase. The literature on pattern generation in the central nervous system has rich observations of phase interactions in a diverse set of movements. A classic example is the switch from antiphase to in-phase coordination at a given frequency in simultaneous bilateral finger oscillations (Kelso, 1984). Although these oscillatory movements are different from the discrete tasks performed in this study, the idea of coordinating diverse movements using a central rhythmic unit may be applicable.

GENERAL DISCUSSION

One of the main concerns when designing the three experiments was the combination of distances and tolerances used. It was desirable to be able to compare the results from Experiment 3, using the combined model presented in Equation 5, directly with the results from the first two experiments. Another constraint was the size of the virtual object for the combined movements in Figure 2c, which limited the translational movements to approximately 20 cm. The resulting ID levels were

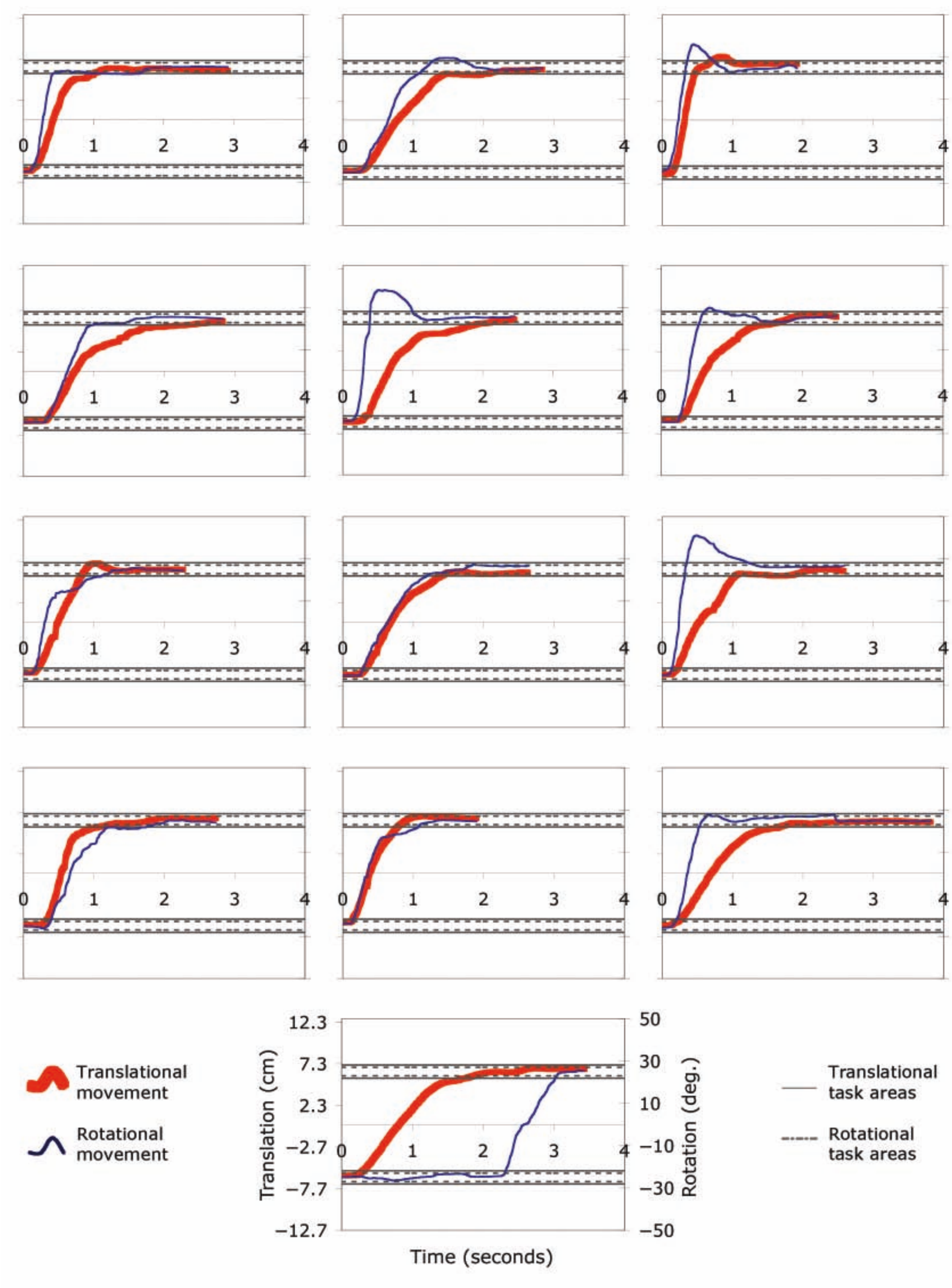

Figure 6. Sample trajectories for 13 participants on a combined task (A = 12.7 cm, W = 1.6 cm, α = 50°, and ω = 4°). Axis scales, labels and units are the same for all plots.

therefore relatively low, with several values less than 3, as seen in Table 1. Gan and Hoffmann (1988) found that a linear model relating MT to the square root of the distance provided a better fit than Fitts' original law (Equation 1) for ID values less than 3. However, the same distance and tolerance combination gives an equivalent ID of only 2.322 with the Shannon formulation used in this study (Equation 2). Thus, 75% of the ID values for the translations in Experiments 1 and 3 were above the threshold, whereas the corresponding percentages for the rotations in Experiments 2 and 3 were 93.75% and 100%, respectively. This finding probably indicates a sufficient manipulation of IDs for the purpose of this study.

Another design decision made was to reflect only the DOF used for each experiment on the display. In addition, the translational DOF and rotational DOF that did not have a requirement for distance and tolerance were not physically constrained. Thus, the participants were, for example, allowed to perform rotations while moving the handheld sensor in Experiment 1, but only left-to-right (and vice versa) translations of the cursor were shown on the display. Constraining the movements physically would be feasible for the simple 2-DOF movements performed in this study but was avoided to enable direct comparison with potential future extensions to movements with several translational and rotational DOF. Constraining these more complex movements to exactly the DOF of interest may prove difficult. In addition, such constraints typically do not exist in the high-DOF input devices for which the work presented here is intended. Another option considered was to provide a graphical representation of all translational and rotational movements on the display to exactly represent the pose of the handheld sensor. A graphical representation used frequently for 6-DOF docking tasks is that of one tetrahedron that is to be aligned with an equal-size target tetrahedron (see, e.g., Zhai & Milgram, 1998). However, it is not clear how to represent clearly the tolerance required on the specific rotational or translational DOF used in this type of 3-D representation. Other issues include occlusions and the need to provide some form of depth perception. Although not directly relevant to this study, given the constrained number of DOF, these issues will need to be dealt with for future extensions of the model to high-DOF movements.

The slope coefficients found for Experiments 1, 2 and 3 were 0.31, 0.32, and 0.32, respectively, with the Shannon formulation (Equation 2). This corresponds to an IP of 3.2, 3.1, and 3.1 bits/s, respectively. The mean-of-means TP performance measures were similar, namely, 3.17, 3.02, and 3.28 bits/s. In comparison, Fitts' original study (Fitts, 1954) with reciprocal hand movements reported an IP of 10.6 bits/s. Another famous example is that of Card et al. (1978), which reported an IP of 10.4 bits/s for a mouse in a text selection task and 4.9 bits/s for a joystick. In general, the IPs reported vary drastically between studies, although most are in the range of 3 bits/s to 12 bits/s (MacKenzie, 1992). Indeed, a more recent survey included nine ISO-conforming studies that reported a mean of mean TP of 0.99 to 2.9 bits/s for touchpad devices and 3.7 to 4.9 bits/s for mice (Soukoreff & MacKenzie, 2004).

In summary, it was found that the time taken to complete a movement consisting of 1-DOF rotational and translational components was equivalent to the sum of the time taken to complete each component separately. Thus, two Fitts' law relationships, one for the rotational part and one for the translational part, can be used to represent the combined movements. However, it was also confirmed that rotational and translational movements have a similar IP and mean-of-means TP, enabling the proposed simplified model for combined movements in Equation 5. This model was found to provide a reasonably accurate estimate of the MT for a combined movement and to allow for the estimation of a single combined IP (and mean-of-means TP) for combined movements. The model proposed thus provides additional value in that it can be used to compare input devices across a range of rotational and translational requirements with a single dependent variable. The model is not a new version of Fitts' law but, rather, a proposal for how to combine the Fitts' law estimates made for the rotational and translational movement components. Further improvements in the accuracy of modeling these movements can probably be made by increasing the number of task parameters or by introducing more empirically determined constants that take into account the observed interactions between the translational and rotational movements. However, doing so may also reduce the scope of tasks for which the model can be used and introduce additional requirements for experimental data. It is hoped that the model in its current form can be useful to human factors practitioners who deal with combined rotational and translational movements while being as simple to apply as Fitts' original law.

For researchers, the finding of a high degree of coordination between the translational and rotational components could be of interest. One potential approach could be to investigate the effect of the instructions given to the participants. For example, future experiments might explicitly specify that the components should be executed in series or in parallel. Another approach could be to compare the results obtained here with an equivalent bimanual experiment. Would the same degree of coordination be observed if the rotational and translational components were split between the two hands? Could this lead to insights into whether to separate the rotational and translational components in high-DOF input devices? The model was tested with a free-flying input device; however, Fitts' law has been shown to be robust across a diverse range of input devices in the past. Does this finding also extend to combined rotational and translational movements?

Among the potential applications could be the expansive field of gesture recognition for mobile devices, driven by the inclusion of accelerometers and other movement sensors into mobile phones and portable music players. Here, physical movements of the device itself are interpreted and converted to interface actions, for example, with the use of 2-DOF rotations to navigate a graphical user interface (see Crossan & Murray-Smith, 2004). Combining rotations and translations could potentially increase the vocabulary of gestures significantly. However, establishing suitable requirements for speed and accuracy of these movements would be made simpler by the use of a model such as the one presented in this article. The model could also be useful in evaluating human-machine interfaces for assistive devices intended for users with disabilities and special needs. These devices include wheelchair-mounted and mobile robotic manipulators for aiding users with varying degrees of disability in their daily activities. One of the main challenges in this field is making interfaces capable of controlling a high-DOF robotic system for users who may have a very limited number of DOF available for use. Allowing the user to control only a subset of the DOF of the system at any time is one approach to the problem. The model proposed could here be used to evaluate the effect of combining rotations and translations for different input devices and user groups. In fact, the original Fitts' law is currently being applied to quantify performance of new brain-computer interfaces (e.g., Felton, Radwin, Wilson, & Williams, 2009) that are intended for users with severe disabilities. These interfaces are currently very limited in TP and are thus usually applied only to low-DOF tasks (Tonet et al., 2008).

The authors also believe that further extensions to the model can be developed for more complex movements. Empirical work is important for determining the most effective use of high-DOF user interfaces as they continue to evolve (Bowman et al., 2004). With a model for high-DOF movements, the comparison of these interfaces will be less ambiguous and should allow for generalization beyond a specific task. It is hoped that extensions of the work performed here may one day provide a theoretical basis for modeling complex high-DOF tasks, such as virtual reality navigation and robot teleoperation.

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