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Heterogeneous Effects of Risk-Taking on Bank Efficiency: A Stochastic Frontier Model with Random Coefficients*

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Abstract

We estimate a stochastic frontier model with random inefficiency parameters, which allows us not only to identify the role of bank risk-taking on driving cost and profit inefficiency, but also to recognize heterogeneous effects of risk exposure on banks with different characteristics. We account for an integral group of risk exposure covariates including credit, liquidity, capital and market risk, as well as bank-specific characteristics of size and affiliation. The model is estimated for the Colombian banking sector during the period 2002-2012. Results suggest that risk-taking drives inefficiency and its omission leads to over (under) estimate cost (profit) efficiency. Risk-taking is also found to have different effects on efficiency of banks with different size and affiliation, and those involved in mergers and acquisitions. In particular, greater exposures to credit and market risk are found to be key profit efficiency drivers. Likewise, lower liquidity risk and capital risk lead to higher efficiency in both costs and profits. Large, foreign and merged banks benefit more when assuming credit risk, while small, domestic and non-merged banks institutions take advantage of assuming higher market risk.

Keywords: *Bank Efficiency; Bayesian Inference; Heterogeneity; Random Parameters; Risk-Taking; Stochastic Frontier Models.*

JEL classification: C11; C23; C51; D24; G21; G32

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1. Introduction

After the global financial crisis, understanding bank risk-taking has gained more attention among researchers and practitioners. In particular, due to the regulatory framework proposed in Basel III, which is intended to limit and monitor bank risk-taking by imposing higher capital requirements and more liquid assets holdings in banks' portfolios (BIS, 2010).[1] These higher requirements may reduce bank risk exposure but also force them to be more cost-efficient in order to be profitable. In the case of emerging economies, there is an additional factor associated to the effects of the increasing risk appetite exhibited recently by financial institutions, which is mainly associated to higher capital inflows from advanced economies, where financial fragility coexists with prolonged lower interest rates, especially in the Euro area (Bruno and Song, 2013; Ahmed and Zlate, 2013). As a result, banks from emerging economies take advantage from lower interest rates abroad to fund mergers and acquisitions (M&A) in search of cost efficiency, while expanding their loans and investments motivated by risk-taking incentives. Thus, most of the financial authorities in those economies have been facing the challenges of having large banks (i.e. concentrated markets) with higher market and credit risk exposures. In this context, the analysis of bank efficiency incorporating risk exposure may contribute to the proper design of macroprudential policies to enhance financial stability.

The study of bank risk-taking and its impact on banks performance has been a recurrent issue in the literature. The seminal work of Berger and DeYoung (1997) showed that banks with lower efficiency tend to exhibit higher ratio of bad loans and in turn those banks are more prone to default than banks with higher efficiency and lower share of bad loans. Thus, bank efficiency indicators have been used as a potential measures of bank failures (Podpiera and Weill, 2008). Likewise, the modern banking theory highlights that risk-taking is an inherent element of the banking production which should be properly modeled into the efficiency measurement (Hughes et al., 2001). Recent studies in this field have showed that failing to account for risk-taking leads to biased estimations of bank efficiency as well as mislead estimates of scale economies and cost elasticities (Hughes and Mester, 2013; Koetter, 2008; Malikov et al., 2013). However, studies that incor-

[1] The Basel III framework promotes higher and better-quality capital, risk coverage and leverage ratios to increase resilience in periods of stress. Likewise, states the introduction of a Liquidity Coverage Ratio for short term (30 days) and a Net Stable Funding Ratio (NSFR) for long-term (one year), which will be implemented gradually during the 2015-2018 period according with the evolution of the economic activity in each member country. The recent initiatives by the European Systemic Risk Board (ESRB), the International Organization of Securities Commissions (IOSCO) and the Financial Stability Board (FSB) are also aligned to limit the risk-taking behavior and contagion in financial market through micro and macro-prudential policies.

porate bank risk-taking in efficiency estimations have traditionally included only proxies of credit risk exposure (i.e. non-performing loans or loan losses provisions), omitting other important risks faced by banks (e.g. liquidity, market and capital exposures, among others).

Credit risk proxies are usually included into costs and profit functions as a measure of output quality that directly affects the technology (Mester, 1996; Hughes and Mester, 1998) or as an undesirable output where reductions are desirable (see some applications in Park and Weber, 2006; Zago and Dongili, 2011; Assaf et al., 2013). Under that approach, risk-taking is assumed as an exogenous component of the banking production process. This contrasts with recent empirical literature that illustrates how most of bank inefficiency corresponds to either poor management or riskier strategies reflected in a higher ex-post credit risk (i.e. elevated share of NPLs) (see Lepetit et al., 2008).

When risk-taking is modeled as endogenous, we can find two approaches in the literature: Firstly, structural models of banking production that account for managerial risk preferences and endogenous risk-taking. In these models, bank performance is measured in the risk-return space that incorporates the trade-off between expected profit and risk.[2] The second approach is stochastic frontier analysis (SFA), first introduced by Aigner et al. (1977) and Meeusen and van den Broeck (1977). In particular, SFA models in which firm-specific characteristics are modeled as elements that affect the inefficiency distribution. This framework avoids additional assumptions on banks behavior and their impact on the production technology. Recently, Radić et al. (2012) applied the later approach to assess cost and profit efficiency of G-7 investment banks. These authors included a set of measures of risk exposure and other firm-specific and macro-related factors and found that those variables affect the inefficiency distribution rather than the production technology. Moreover, they identified that omitting risk-taking from the efficiency estimation leads to underestimate profit efficiency, and that liquidity and capital are the most relevant risk exposures explaining efficiency.

Recent literature recognizes that risk exposure may also affect banks with different characteristics in different ways. Foreign banks may deal better with risk exposure given cheaper access to funding sources or more diversification (see Chen and Liao, 2011). Similar effects could be faced by large institutions or those operating in different markets, mainly associated to scale economies (Bos and Kool, 2006; Wheelock and Wilson, 2012). Moreover, recent studies show that large banks face lower cost on both deposits and interbank funds mainly because their creditors infer that those banks are *too-important-to-fail* and will be saved by the

[2] These models were developed by Hughes et al. (1996, 2001) and have been recently applied by Koetter (2008).

government in case of failure to avoid contagion into the financial system (Bertay et al., 2013; Santos, 2014; IMF, 2014). In addition, there is evidence supporting the fact that high-leveraged (or low capitalized) institutions tend to take more risk when they can adjust their capital structures or in the presence of market power (Borio and Zhu, 2012; Dell'Ariccia et al., 2011).Banks with a higher risk appetite may choose to produce less fixed interest bearing loans and engage more in securities or derivatives trading, increasing their market risk exposure. Likewise, lower capitalized banks may increase their risk of insolvency due to credit losses or sudden security price deterioration (Mester, 2008). Risky banks tend to attract more deposits because creditors demand higher interest rates as a way to exert market discipline (Demirgüç-Kunt and Huizinga, 2004). Therefore, it is relevant not only to account for risk exposure measures as possible inefficiency drivers, but also to recognize differences in the way risk exposure may affect different banks. Indeed, several studies reveal that estimates of bank efficiency can be biased if bank heterogeneity is ignored (Mester, 1996; Bos et al., 2005; Galán et al., 2014c; Pestana-Barros and Williams, 2013).

In this context, our contribution to the literature is twofold. Firstly, we propose an alternative approach to model observed and unobserved bank heterogeneity within a Bayesian stochastic frontier framework. Secondly, we provide evidence on the importance to properly account for the effects of risk-taking in bank efficiency estimations. Particulary, we present a novel stochastic frontier model with random inefficiency coefficients, which is able to identify not only the effects of observed covariates in the inefficiency but also the type of banks that are more affected by each of these characteristics. This specification for the inefficiency may capture unobserved heterogeneity sources related to risk exposure among banks. Thus the proposed specification allows us to identify the role of risk-taking on driving inefficiency and different effects of risk-taking on the efficiency of banks involved in M&A and banks with different size and affiliation. We account for an integral group of risk exposure covariates (i.e. risks of credit, liquidity, capital, and market). Unlike previous studies, we use internal loan ratings as measure of ex-ante credit risk which is a more accurate indicator than the traditional NPLs measure that captures ex-post realizations of credit risk. The inference of the model is carried out via Bayesian methods that allows to formally incorporate parameter uncertainty and to derive posterior densities of cost and profit efficiency for each bank. We compare our proposed random coefficients specification to models with fixed coefficients, with risk exposure variables in the frontier and omitting risk-taking measures. We identify that risk-taking has a different impact on cost and profit efficiency of each bank depending on their specific characteristics. To the best of our knowledge, our proposal constitutes the first SFA model that incorporates risk-taking within a Bayesian framework and that accounts for firm-specific effects of covariates in the inefficiency.

The proposed model is estimated for the Colombian banking sector using bank-level-data for the period 2002-2012 and provides the first empirical evidence on the effects of risk-taking on the efficiency of the Colombian banking industry. Results are important given that as other emerging economies, the Colombian banking sector has experienced a growing expansion in recent years. During this period, the value of loans has grown 300% and the ratio investments to assets have doubled their share. Likewise, several M&A processes have been carried out, concentrating financial services in few but large institutions. As a result, risk exposure has presented important increases.[3] This has led regulators to monitor closely credit and market risks and to face the challenges of having systemic financial institutions (?León et al., 2012). Recent studies on efficiency of the Colombian banking sector have evidenced an improvement on technical efficiency and productivity change among banks. However, none of these studies have yet incorporated the impact of risk-taking on efficiency, which plays a major role for explaining banks performance within a risk-production environment.

The rest of the paper contains five additional sections. In Section 2 we present a review of literature on bank risk-taking and efficiency. In Section 3, we present the proposed specification, the Bayesian inference for this model and some model comparison criteria. In Section 4, we describe the data and the empirical model. In Section 5 we present and analyze the main results of the application to the Colombian banking sector. Section 6 concludes the paper.

2. Literature Review

2.1. Risk-taking and bank efficiency

The seminal study of Berger and DeYoung (1997) postulated intuitive hypotheses to explain the relationship among bank risk-taking, equity capital and bank efficiency. On the one hand, they defined a *bad management hypothesis* which states that poor managed banks face higher operative costs and may have difficulties in their evaluation of credit risk. As a result, lower efficiency may causes an increase of NPLs. On the contrary, the *bad luck hypothesis* argues that because of adverse economic conditions (beyond the banks control) bad loans may increase and then banks have to expend more resources to recover them. Thus, an increase of risky loans leads banks to employ more resources for monitoring loans and then to lower their cost efficiency. Similarly, they suggest that skimping on loan

[3] In May of 2013 the Colombian Treasury Bills (TES) prices decreased 20% in two weeks as a result of the uncertainty related to the FED's exit strategy. This downward led to bank losses of COP 2,32 billion that represent 4.87% of their equity capital (BR, 2013).

monitoring and the presence of moral hazard incentives (because of lower capitalization) are also associated to higher ex-post credit risk and risk-taking incentives, respectively. They use Granger causality tests to model the inter-temporal relationship among these measures of efficiency, risk and capital. Overall, they find that there is a negative relation between cost efficiency and credit risk in the U.S. failed banks. Likewise, they show that highly capitalized banks are more efficient than thinly capitalized banks. Thus, one direct consequence of these results is that banks should enhance capital equity and that efficiency may constitutes an important indicator of potential bank failures.

Recent studies provide evidence on these hypothesis for the banking sector in European and emerging economies by using alternative approaches to identify inter-temporal relationships among efficiency, risk and capital. Most of them show that because of banks can choose their level of operating costs, monitoring cots and capitalization to manage their level of risk exposure (e.g. adequate proportion of riskier loans), the consequences of a bad management or higher risk-taking, by skimping or moral hazard incentives, would lead to both inefficiency and higher bad loans (see Williams, 2004; Altunbas et al., 2007; Lepetit et al., 2008; Podpiera and Weill, 2008; Tabak et al., 2011).In general, they find a negative relationship between NPLs and cost efficiency and concur that banking supervisors should focus on enhancing bank cost efficiency and bank capital in order to reduce the probability of bank failures and to support financial stability objectives. However, little is known about how differences in risk exposure affect the efficiency of banks with different characteristics. The efficiency frontier models can be used to address this issue.

2.2. Frontier efficiency methods and bank risk-taking

Under frontier efficiency analysis bank risk-taking can be modeled as a component of the technology or as a factor that influence the inefficiency. The traditional approach used by most studies on bank efficiency is to incorporate credit risk into the production function as a bad output where reductions are desirable. This method, in which loan losses (the proxy of ex-post credit risk) are included as an undesirable output that affects the production process, was initially introduced in Berg et al. (1992). Other studies have followed the same approach using the share of NPLs as a measure of credit risk exposure and alternative cost functions or input distance functions (Mester, 1996; Hughes and Mester, 1998; Park and Weber, 2006; Zago and Dongili, 2011; Barros et al., 2012; Assaf et al., 2013).

Recent developments in banking theory and production economics include risk-taking as a determinant of bank performance by assuming that banks incorporate credit risk into their expected production plans. These structural models were developed by Hughes et al. (1996) and extended by Hughes et al. (2001) under the managerial utility maximizing approach by employing an almost ideal demand system from the consumer theory. In this framework, bank performance account for the risk-return trade-off by computing a risk-return efficiency measure. (see Altunbas et al., 2000; Koetter, 2008; Hughes and Mester, 2013, for studies on scale, cost and profit efficiency adjusted by risk-return for the banking industries of Japan, Germany, and the U.S, respectively). These studies find that efficiency measures adjusted by risk-return are lower than those computed in standard models given that banks choose their optimal risk-return trade-offs, and they remark that risk-taking play a major role in explaining bank efficiency.

In recent stochastic frontier applications, bank risk exposure has been treated as an environmental factor that can be modeled according with its influence on the production process or the inefficiency distribution. Radić et al. (2012) follow the general approach proposed by Coelli et al. (1999) to test the impact of several risks and market conditions on the cost and profit efficiency of investment banks.[4] The authors evaluate a sample of 800 banks of the G-7 countries during the period 2001-2007 and find that measures of risk-taking influence the inefficiency distribution rather than the bank technology (see Glass and McKillop, 2006, for evidence using this approach for large credit unions in the U.S.). They also find that omitting bank risk-taking from the efficiency estimation leads to underestimate profit efficiency. In particular, liquidity and capital risk exposures are found to be the most relevant factors.

The mentioned studies that incorporate risk into efficiency estimation focuses only on credit risk exposure, usually by including NPLs as an undesiderable output. However, banks face other important risks that affect their performance, which have been less explored in the literature of bank efficiency. Liquidity risk is an essential risk that reflects the typical maturity mismatch of banks because of their business of borrow short (take deposits from savers) and lend long (assign loans to borrowers). Similarly, traditional measures of market risk exposure such as securities over total assets and capital risk exposure (equity capital over total assets) may help to capture market and regulatory conditions that affects banks performance. These risks have been incorporated in estimations of cost and profit functions in some studies showing its importance on bank performance (Altunbas et al., 2000; Athanasoglou et al., 2008; Brissimis et al., 2008; Lepetit et al., 2008). Overall, banks face different types of risks that influence their performance, and they should be accounted for in bank efficiency measurements.

[4]Coelli et al. (1999) test a model including heterogeneity or environmental factors into the production function and one including them in the inefficiency distribution against a model with effects in both parts.

2.3. Evidence from the Colombian banking sector

The efficiency of the Colombian banking sector has been widely studied because of two main reasons: firstly, the increase on M&A in the banking industry as a result of the growing affluence of capital flows from advanced and other emerging economies. Secondly, regulatory purposes trying to identify micro and macro prudential measures to reduce bank default episodes and to mitigate contagion among financial institutions.[5]

Some studies have applied the standard stochastic frontier approach using alternatively Cobb-Douglas or translog functions to characterize the technology and find evidence of low cost efficiency during the 90's (Estrada and Osorio, 2004; Clavijo et al., 2006). The most recent studies for the Colombian banking sector find a general improvement of both technical and cost efficiency along with a greater heterogeneity among banks. Sarmiento et al. (2013) use a non-parametric frontier model to evaluate the efficiency of Colombian banks for the period 2000-2009. They found that technical efficiency gradually improved during the decade up to the global financial crisis of 2008-09, when all estimated measures of efficiency decreased and a negative productivity change was found. Further, a high heterogeneity in efficiency scores was observed among banks irrespective of their size and affiliation, and M&A were found to have a significant and positive impact on bank efficiency for merged or acquired banks.

For the same period, Galán et al. (2014c) estimated input-oriented technical efficiency using a dynamic Bayesian SFA model. They find that foreign ownership has positive and persistent effects on efficiency, while the effects of size are positive but rapidly adjusted. They also identified high inefficiency persistence in Colombian banks with important differences between institutions. In particular, merged banks were found to exhibit low costs of adjustment that allow them to recover rapidly the efficiency losses derived from merging processes.

Moreno and Estrada (2013) studied the role of market power in explaining efficiency gains in Colombian banks during the 2004-2012 period. By using alternative SFA and non-parametric models, they found that there is a positive relationship between market power and efficiency, which is explained by the product differentiation that allows banks to gain in efficiency while they do not charge excessive credit prices.

However, none of these studies have yet incorporated the role of bank risk-taking on the banking production to estimate efficiency.

[5]Several bank regulatory measures were adopted by the end of nineties as a result of the Mexican and Asian crisis that also affected the Colombian banking sector (see Clavijo et al., 2006, for a detailed review on the evolution of M&A, regulation and performance of the Colombian banking industry during the regional and local financial crises).

3. Methodology

3.1. A stochastic frontier model with random inefficiency coefficients

Distinguishing inefficiency from heterogeneity is an important issue in the efficiency frontier literature. Omitting heterogeneity variables has been identified to lead to biased estimations of inefficiency. In the banking literature, Bos et al. (2009) identify important effects of observed heterogeneity on efficiency levels and rankings, while Feng and Zhang (2012) find that failure to consider unobserved heterogeneity results in misled efficiency rankings and mismeasured technical efficiency, productivity growth, and returns to scale. Observed and unobserved heterogeneity sources are important to be considered. In the first case, covariates are usually included either in the frontier or in the parameters of the inefficiency distribution (see Kumbhakar and Lovell, 2000, for a complete review). In the second case, unobserved heterogeneity has been mainly modeled in the frontier and then, assumed to distinguish firms because of differences in the technology rather than in the inefficiency. Greene (2005) proposes different methods to deal with this kind of heterogeneity under the frequentist approach.

In the Bayesian context, Tsionas (2002) proposes a model with random coefficients in the frontier, which captures different effects of technology factors for every firm. The modeling of these unobserved sources of heterogeneity in the inefficiency has been less explored. Recently, Galán et al. (2014b) propose the inclusion of a random parameter that can be modeled along other observed covariates and which is found to perform well in capturing latent heterogeneity. However, here we are more interested in studying different effects of observed heterogeneity variables (i.e. risk exposure) in the inefficiency of banks with different characteristics. That is, a similar approach to that of Tsionas (2002) but including random coefficients in the covariates of the inefficiency distribution rather than in the frontier.

The proposed stochastic frontier model is the following:

$$\begin{aligned} y_{it} = \; & \mathbf{x_{it}}\boldsymbol{\beta} - u_{it} + v_{it} \\ & v_{it} \sim N(0, \sigma_v^2) \\ & u_{it} \sim Exp(\lambda_{it}) \\ & \lambda_{it} = \exp\left(\begin{pmatrix} \boldsymbol{\gamma} \\ \gamma_i^* \end{pmatrix}' \begin{pmatrix} \mathbf{z_{it}} & \mathbf{0} \\ \mathbf{0} & \mathbf{z^*_{it}} \end{pmatrix}\right), \end{aligned} \tag{1}$$

where y_{it} represents the output for firm i at time t, $\mathbf{x}_{it}$ is a row vector that contains the input quantities, $\boldsymbol{\beta}$ is a vector of parameters, v_{it} is an idiosyncratic error assumed to follow a normal distribution, and u_{it} is the inefficiency component. The inefficiency is assumed to follow an exponential distribution with a firm specific and time-varying parameter λ_{it}, which depends on a vector including two sets of parameters and a matrix that includes two types of heterogeneity variables. $\boldsymbol{\gamma}$ is a vector of parameters which are common to all firms, including the constant; and, $\boldsymbol{\gamma}_i^*$ is a vector of firm-specific parameters intended to capture differences in the effects of covariates across firms on the inefficiency. Therefore, $\mathbf{z}_{it}$ is a vector of heterogeneity variables whose effects are assumed to be constant across firms, and $\mathbf{z}_{it}^*$ contains a set of heterogeneity variables with firm-specific effects. In the case of assessing cost or profit efficiency, y_{it} would represent costs or profits, and for the cost efficiency case the sign of the inefficiency component is reversed.

The random specification for the inefficiency coefficients is intended to capture differences in the way risk exposure affects cost and profit efficiency of different types of banks. Thus, the model is able to identify, not only the effects of observed covariates in the inefficiency, but also the type of banks that are more affected by each of these characteristics.

3.2. Bayesian inference

The inference of the model is carried out using Bayesian methods. This approach was introduced in stochastic frontier models by van den Broeck et al. (1994) and allows us to formally incorporate parameter uncertainty and to derive posterior densities of cost and profit efficiency for every individual bank.

We assume proper but relatively disperse prior distributions throughout. In particular, the distributions assumed for the parameters in the frontier are: $\boldsymbol{\beta} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}})$ where $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1}$ is a precision diagonal matrix with priors set to 0.001 for all coefficients. The variance of the idiosyncratic error term is inverse gamma, that is equivalent to $\sigma_v^{-2} \sim G(a_{\sigma_v^{-2}}, b_{\sigma_v^{-2}})$ with priors set to 0.01 for the shape and rate parameters, respectively.

Regarding the inefficiency component, its distribution is assumed to be exponential: $u_{it}|\boldsymbol{\gamma}, \boldsymbol{\gamma}^*, \mathbf{z_{it}}, \mathbf{z_{it}^*} \sim Exp(\exp(\mathbf{z_{it}}\boldsymbol{\gamma} + \mathbf{z_{it}^*}\boldsymbol{\gamma}_i^*))$. The prior distribution of the vector of common parameters $\boldsymbol{\gamma}$ is chosen to be centered in a given prior mean efficiency value r^* following the procedure in Griffin and Steel (2007), where $\exp(\boldsymbol{\gamma}) \sim Exp(-\ln r^*)$. For the firm-specific inefficiency heterogeneity coefficients, a hierarchical structure is defined, where $\exp(\boldsymbol{\gamma}_i^*) \sim Exp(\boldsymbol{\gamma}^*)$, and $\boldsymbol{\gamma}^* \sim Exp(-\ln r^*)$. Therefore, the firm-specific parameters are centered a priori in a common parameter, which at its turn, is centered in a given prior mean efficiency value. In this particular application, r^* is set at 0.65, following other Bayesian SFA studies in banking (see Marzec and Osiewalski, 2001; Tabak and Tecles, 2010). Sensitivity analysis is performed to the use of a normal prior distribution for the inefficiency parameters such that $\boldsymbol{\gamma}$ is $N(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\gamma}})$ with priors for the diagonal precision matrix $\boldsymbol{\Sigma}_{\boldsymbol{\gamma}}^{-1}$ equal to 0.1 for all the coefficients.[6] In this case, the hierarchical structure used for the firm-specific parameters is: $\boldsymbol{\gamma}_i^* \sim N(\boldsymbol{\gamma}^*, \boldsymbol{\Sigma}_{\boldsymbol{\gamma}^*})$ where $\boldsymbol{\gamma}^*$ is defined in

[6] This centers efficiencies in a value of 0.37.

the same way that $\boldsymbol{\gamma}$. Results show convergence to roughly the same values after the number of iterations described below.

Markov Chain Monte Carlo (MCMC) methods and in particular the Gibbs Sampling algorithm with data augmentation, as presented by Koop et al. (1995) for stochastic frontier models, can be used here.[7] The MCMC algorithm involves 50.000 iterations where the first 10.000 are discarded and a thinning equal to 4 is used to remove autocorrelations. Therefore, 10.000 iterations are used for the posterior inference.

We assess the fit and predictive performance of the different models using a version of the Deviance Information Criterion (DIC) called DIC_3 and the Log Predictive Score (LPS) (see Griffin and Steel, 2004; Ferreira and Steel, 2007; Galán et al., 2014b; Galán and Pollitt, 2014a, for applications of these criteria to Bayesian SFA models). The former is a stable variant of the within sample measure of fit introduced by Spiegelhalter et al. (2002) commonly used in Bayesian analysis. Defining the deviance of a model with parameters θ as $D(\theta) = -2\log f(\mathbf{y}|\theta)$, where $\mathbf{y}$ are the data, then $DIC = 2\overline{D(\theta)} - D(\bar{\theta})$. However, using an estimator of the density $f(\mathbf{y}|\theta)$ instead of the posterior mean $\bar{\theta}$ is more stable. This alternative specification was first proposed by Richardson (2002) and presented by Celeux et al. (2006) to overcome problems when the original DIC is implemented to random effects and mixture models.[8] The formulation for this criterion is:

$$DIC_3 = -4E_\theta[\log f(\mathbf{y}|\theta)|\mathbf{y}] + 2\log \widehat{f}(\mathbf{y}) \tag{2}$$

Regarding LPS, it is a criterion for evaluating the out-of-sample behaviour of different models. This criterion was first introduced by Good (1952) and is intended to examine model performance by comparing its predictive distribution with out-of-sample observations. For this purpose the sample is split into a training and a prediction set. Our prediction set consists of observations corresponding to the last two observed years of every firm in the sample, and the training set contains all the rest. The formula is the following:

$$LPS = -\frac{1}{k}\sum_{i=1}^{k} \log f(y_{i,t_i}|\text{previous data}), \tag{3}$$

where y_{i,t_i} represents the observations in the predictive set for the k firms in the sample and t_i represents the penultimate time point with observed data for firm i.

[7]The implementation of our models is carried out using the WinBUGS package (see Griffin and Steel, 2007, for a general procedure).

[8]Li et al. (2012) also remark on the lack of robustness of the original DIC in models with data augmentation

4. Data and empirical model

We employ annual data from 31 commercial banks for the period 2002-2012. This is an unbalanced panel data set from the local central bank (*Banco de la República*) and the financial supervisory agency (*Superintendencia Financiera de Colombia*). We follow the financial intermediation approach in which banks employ deposits, labor and physical capital to produce loans, securities investments and other financial services (Berger, 2007).[9] We consider as input prices: the price of deposits (p_1), which is the ratio of interest expenses divided by total deposits; the price of labor (p_2), which is personnel expenses divided by the total number of employees, and the price of physical capital (p_3), which is calculated as the ratio of operating expenses (i.e. non-interest reduced by personnel) to total fixed assets. As outputs we consider: loans (y_1) including consumer, commercial, mortgage, and microcredit; securities (y_2), which includes public and private bonds holdings, and other securities investments; and off-balance-sheet (OBS) activities (y_3) measured as the ratio of non-interest income over total income. Non-interest income includes securitization, brokerage services, and management of financial assets for clients which represent an important source of income for Colombian banks.[10] Total costs are considered as the sum of interest and non-interest costs and total profit as the earned net profit.

We define a set of bank-specific characteristics including: size (z_1), measured as the level of total assets; and foreign ownership (z_2), which is a binary variable taking the value of 1 if more than 50% of bank shares are foreign owned, and 0 otherwise. As aforementioned, these effects have been found to be relevant inefficiency drivers in previous studies.

Additionally, we include several specific measures of credit, liquidity, capital and market risk according to recent literature, the Colombian financial regulation and the Basel III standards. Credit risk (z_1^*) is measured as risky loans over total loans. We use risky loans instead of NPLs because it is a measure of ex-ante credit risk assumed by banks when they assign loans, which is based on internal loan ratings associated to their probability of default.[11] In addition, the regulation establishes that loan losses provisions are required for each loan according with its rating. Thus, higher credit risk exposure is associated to more provisions for potential loan losses. As we mentioned before, the use of NPLs in the estimation

[9]Hughes and Mester (1993) provide evidence that confirm that deposits should be treated as inputs (see Sealey and Lindley, 1977, for a discussion on the intermediation approach).

[10]In a recent study, Tabak and Tecles (2010) find that omitting OBS as an output over (under) estimate cost (profit) efficiency results.

[11]This measure of ex-ante credit risk has been used in the literature to identify bank risk-taking in the credit market (Ioannidou and Penas, 2010).

of bank efficiency may lead to biased estimates of bank technology (Malikov et al, 2013). Liquidity (z_2^*) is measured as liquid assets over total assets.[12] Higher liquid assets prevent banks from losses due to rapid price deterioration and also for the maturity mismatch. Capital risk exposure (z_3^*) is measured as capital equity over total assets. Capital risk is considered as a proxy for regulatory conditions that may affect bank inefficiency. Lower capitalization is usually associated to higher inefficiency.[13] Finally, market risk exposure (z_4^*) is measured as securities investments over total assets. Operating costs associated to securities investments are generally lower that those involved in monitoring and assessing of loans which may induce less efficient banks to engage on more securities investments. However, higher holdings of securities by banks also entail higher market risk exposure.

Table 1 exhibits the summary statistics of the main variables described above, where all monetary values are expressed in thousands of U.S. dollars at constant prices of year 2012.

Table 1: Summary statistics

Variable	**Mean**	**SD**	**Min**	**Max**
Total loans	3,342,012	4,206,436	11,553	28,267,020
Securities	1,265,349	1,339,794	563	6,461,458
OBS	0.0354	0.0299	0.0266	0.0587
Price of deposits	0.0248	0.0121	0.0009	0.0923
Price of labour	36.44	22.30	3.13	142.03
Price of capital	1.92	2.66	0.29	17.30
Total assets	5,503,680	6,425,746	39,699	41,786,469
Credit risk exposure	0.0988	0.0667	0.0019	0.3839
Liquidity risk exposure	0.2296	0.0667	0.0019	0.3839
Capital risk exposure	0.1211	0.0757	0.0448	0.7854
Market risk exposure	0.2381	0.1368	0.0013	0.7478
Total cost	1,132,776	1,402,621	15,673	7,722,227
Total profit	76,927	377,974	- 784,642	2,809,771

Source: Colombian central bank and financial supervisory agency.

We evaluate cost and profit efficiency for the Colombian banking sector. Thus, we use cost and profit functions for the frontier specification in (1), and we represent them with translog multi-product functions. The estimated model is:

[12] Liquid assets include cash holdings, negotiable and available to sell public and private debt instruments and pledged collateral in repurchase agreement operations.

[13] We use both Tier I and Tier II capital requirements as measure of capital equity.

$$
\begin{aligned}
\ln c_{it} = \quad & \beta_0 + \sum_{m=1}^{3} \beta_m \ln y_{m_{it}} + \sum_{r=1}^{2} \delta_r \ln p_{r_{it}} + \tfrac{1}{2} \sum_{m=1}^{3} \sum_{n=1}^{3} \beta_{mn} \ln y_{m_{it}} \ln y_{n_{it}} \\
& + \tfrac{1}{2} \sum_{r=1}^{2} \sum_{s=1}^{2} \delta_{rs} \ln p_{r_{it}} \ln p_{s_{it}} + \sum_{m=1}^{3} \sum_{r=1}^{2} \eta_{mr} \ln y_{m_{it}} \ln p_{r_{it}} + \kappa_1 t \\
& + \tfrac{1}{2} \kappa_2 t^2 + \sum_{m=1}^{3} \phi_m t \ln y_{m_{it}} + \sum_{r=1}^{2} \varphi_r t \ln p_{r_{it}} + \sum_{j=1}^{4} \omega_j z^*_{j_{it}} + v_{it} + uit \\
& v_{it} \sim N(0, \sigma_v^2) \\
& u_{it} \sim \exp(\lambda_{it}) \\
& \lambda_{it} = exp(\gamma_0 + \sum_{h=1}^{2} \gamma_h z_{h_{it}} + \sum_{j=1}^{4} \gamma^*_{j_i} z^*_{j_{it}}),
\end{aligned}
\tag{4}
$$

where c_{it} is the total cost or the total profit, y are outputs, p are input prices and t is a time trend in order to account for technological change. Linear homogeneity of the cost function is achieved by normalizing total costs and input prices by the price of capital (p_3). We include two types of heterogeneity variables: i) those related to size (z_1) and foreign ownership (z_2), which are modeled in the inefficiency distribution and have common effects to all banks; and, ii) those capturing banks risk-exposure ($z_1^*, z_2^*, z_3^*, z_4^*$), which may be included either in the frontier or in the inefficiency. In the latter case, they are able to be modeled either with common or firm-specific effects on banks inefficiency. In order to overcome the problem of calculations of logarithms of negative profits, we correct profit values by a factor corresponding to the absolute value of the lowest profit plus one (see Tecles and Tabak, 2010). Symmetry of the cross-effects is accomplished by imposing $\beta_{mn} = \beta_{nm}, \delta_{rs} = \delta_{sr}$.

5. Results

From the general model in (4) we estimate four models intended to evaluate cost efficiency (C1 to C4) and four models assessing profit efficiency (P1 to P4). Models C1 and P1 do not include any risk-exposure variable, so $\omega_1, \omega_2, \omega_3, \omega_4 = 0$ and $\gamma^*_{1i}, \gamma^*_{2i}, \gamma^*_{3i}, \gamma^*_{4i} = 0$. Models C2 and P2 include the risk-exposure variables only in the frontier and then $\gamma^*_{1i}, \gamma^*_{2i}, \gamma^*_{3i}, \gamma^*_{4i} = 0$. Models C3 and P3 include the risk covariates only in the inefficiency but restrict them to have a common effect on the inefficiency of all banks; thus, $\omega_1, \omega_2, \omega_3, \omega_4 = 0$ and $\gamma^*_{1i}, \gamma^*_{2i}, \gamma^*_{3i}, \gamma^*_{4i} = \gamma^*_1, \gamma^*_2, \gamma^*_3, \gamma^*_4$. Finally, our proposed specification to model random inefficiency coefficients is estimated in models C4 and P4 ($\omega_1, \omega_2, \omega_3, \omega_4 = 0$). This allows the effects of risk exposure to be different among banks.

Table 2: Posterior mean and 95% probability intervals of parameter distributions in cost models

	Model C1		**Model C2**		**Model C3**		**Model C4**	
	No risk		Risk in frontier		Risk in inefficiency		Random coefficients	
	Mean	95% PI	Mean	95% PI	Mean	95% PI	Mean	95% PI
Frontier								
β_0	5.656	[4.457, 6.995]	6.183	[4.998, 7.337]	5.879	[4.918, 6.876]	5.344	[4.101, 6.739]
β_1 y_1	0.053	[0.002, 0.138]	0.132	[0.055, 0.188]	0.029	[0.004, 0.062]	0.087	[0.004, 0.175]
β_2 y_2	0.093	[0.002, 0.218]	0.099	[0.009, 0.251]	0.079	[0.007, 0.217]	0.040	[0.002, 0.108]
β_3 y_3	0.048	[0.002, 0.118]	0.032	[0.001, 0.074]	0.052	[0.004, 0.109]	0.057	[0.002, 0.181]
β_{11} y_1^2	0.071	[0.019, 0.128]	0.124	[0.073, 0.174]	0.078	[0.029, 0.129]	0.087	[-0.003, 0.160]
β_{12} y_1y_2	0.019	[-0.038, 0.069]	-0.075	[-0.133, -0.018]	0.012	[-0.039, 0.059]	0.002	[-0.069, 0.084]
β_{13} y_1y_3	-0.005	[-0.011, 0.001]	-0.001	[-0.007, 0.004]	-0.004	[-0.009, 0.001]	-0.003	[-0.009, 0.003]
β_{22} y_2^2	0.012	[-0.035, 0.059]	0.122	[0.058, 0.184]	0.016	[-0.026, 0.058]	0.003	[-0.075, 0.069]
β_{23} y_2y_3	0.002	[-0.002, 0.006]	-0.001	[-0.005, 0.002]	0.002	[-0.002, 0.005]	0.001	[-0.003, 0.005]
β_{33} y_3^2	0.001	[-0.001, 0.003]	0.001	[-0.001, 0.002]	0.001	[-0.001, 0.003]	0.001	[-0.001, 0.003]
δ_1 p_1	0.154	[0.005, 0.435]	0.244	[0.005, 0.516]	0.148	[0.011, 0.359]	0.096	[0.003, 0.285]
δ_2 p_2	0.180	[0.007, 0.618]	0.113	[0.004, 0.248]	0.173	[0.013, 0.409]	0.152	[0.005, 0.472]
δ_{11} p_1^2	0.219	[0.086, 0.322]	0.136	[-0.014, 0.254]	0.186	[0.058, 0.288]	0.039	[-0.157, 0.217]
δ_{12} p_1p_2	-0.221	[-0.301, -0.132]	-0.165	[-0.247, -0.076]	-0.206	[-0.277, -0.129]	-0.147	[-0.255, -0.034]
δ_{22} p_2^2	0.201	[0.098, 0.301]	0.120	[0.018, 0.221]	0.186	[0.098, 0.272]	0.183	[0.075, 0.281]
η_{11} y_1p_1	0.151	[0.099, 0.201]	0.140	[0.089, 0.189]	0.149	[0.104, 0.193]	0.162	[0.089, 0.233]
η_{12} y_1p_2	-0.029	[-0.077, 0.015]	-0.022	[-0.062, 0.019]	-0.029	[-0.068, 0.011]	-0.035	[-0.093, 0.028]
η_{21} y_2p_1	-0.017	[-0.074, 0.027]	-0.047	[-0.097, 0.001]	-0.028	[-0.086, 0.016]	-0.098	[-0.170, -0.023]
η_{22} y_2p_2	-0.084	[-0.129, -0.037]	-0.052	[-0.095, -0.009]	-0.075	[-0.117, -0.032]	-0.044	[-0.101, 0.013]
η_{31} y_3p_1	0.001	[-0.006, 0.009]	0.001	[-0.005, 0.007]	0.002	[-0.004, 0.008]	0.003	[-0.003, 0.012]
η_{32} y_3p_2	0.004	[-0.004, 0.010]	0.005	[-0.001, 0.011]	0.002	[-0.004, 0.008]	-0.002	[-0.010, 0.006]
κ_1 t	-0.346	[-0.595, -0.098]	-0.362	[-0.575, -0.151]	-0.359	[-0.567, -0.149]	-0.309	[-0.526, -0.098]
κ_2 t^2	0.002	[-0.007, 0.011]	-0.001	[-0.010, 0.008]	0.002	[-0.005, 0.009]	0.005	[-0.003, 0.013]
ϕ_1 ty_1	0.036	[0.014, 0.059]	0.036	[0.016, 0.055]	0.037	[0.018, 0.056]	0.036	[0.018, 0.056]
ϕ_2 ty_2	-0.034	[-0.051, -0.018]	-0.030	[-0.045, -0.015]	-0.035	[-0.049, -0.021]	-0.036	[-0.049, -0.021]
ϕ_3 ty_3	0.001	[-0.001, 0.002]	0.000	[-0.001, 0.001]	0.000	[-0.001, 0.001]	-0.001	[-0.001, 0.001]
φ_1 tp_1	-0.040	[-0.068, -0.012]	-0.042	[-0.068, -0.016]	-0.042	[-0.065, -0.018]	-0.039	[-0.065, -0.013]
φ_2 tp_2	0.017	[-0.007, 0.041]	0.011	[-0.009, 0.032]	0.017	[-0.003, 0.036]	0.009	[-0.011, 0.031]
ω_1 $z_1^*(cred.)$			0.134	[-0.553, 0.835]				
ω_2 $z_2^*(liq.)$			-0.188	[-0.590, 0.215]				
ω_3 $z_3^*(cap.)$			0.734	[-0.107, 1.536]				
ω_4 $z_4^*(mkt.)$			-1.090	[-2.119, 0.001]				
Inefficiency								
γ_0	1.035	[0.014, 0.059]	0.532	[-1.743, 2.083]	0.895	[-0.439, 2.074]	1.009	[-0.537, 2.218]
γ_1 z_1	-0.198	[-0.291, -0.082]	-0.173	[-0.271, -0.056]	-0.232	[-0.347, -0.114]	-0.156	[-0.338, -0.009]
γ_2 z_2	-0.814	[-1.958, -0.094]	-1.170	[-4.274, -0.026]	-0.687	[-1.650, -0.014]	-0.192	[-0.256, -0.096]
γ_1^* $z_1^*(cred.)$					0.201	[-1.807, 1.728]	-0.001	[-2.781, 1.852]
γ_2^* $z_2^*(liq.)$					0.369	[-0.722, 1.411]	0.485	[-1.646, 1.961]
γ_3^* $z_3^*(cap.)$					1.538	[0.446, 2.423]	3.181	[0.307, 6.780]
γ_4^* $z_4^*(\mathrm{mkt.})$					0.043	[-1.875, 1.504]	-0.017	[-2.054, 1.635]
Eff.	0.893		0.909		0.892		0.710	
DIC_3	2982.76		2916.44		2497.59		2007.75	
LPS	-9.62		-29.34		-65.19		-90.67	

Note: Values for γ_1^* to γ_4^* in Model C4 correspond to the hyperparameter

Table 3: Posterior mean and 95% probability intervals of parameter distributions in profit models

	Model P1		**Model P2**		**Model P3**		**Model P4**	
	No risk		Risk in frontier		Risk in inefficiency		Random coefficients	
	Mean	95% PI	Mean	95% PI	Mean	95% PI	Mean	95% PI
Frontier								
β_0	9.789	[-3.249,24.840]	9.873	[-6.084,23.52]	3.698	[-9.591,20.04]	7.205	[-1.754,18.03]
β_1 y_1	3.025	[0.6511,5.195]	3.495	[1.263,5.555]	4.031	[1.639,6.182]	2.914	[1.242,4.282]
β_2 y_2	3.391	[1.609,5.336]	4.122	[2.310,5.757]	4.586	[2.841,6.214]	3.538	[2.436,4.606]
β_3 y_3	-0.199	[-0.391,0.013]	-0.218	[-0.418,0.030]	-0.261	[-0.436,0.064]	-0.212	[-0.339,0.077]
β_{11} y_1^2	-0.457	[-0.713,-0.201]	-0.566	[-0.774,-0.301]	-0.511	[-0.775,-0.251]	-0.399	[-0.584,-0.171]
β_{12} y_1y_2	0.267	[0.0725,0.459]	0.384	[0.189,0.557]	0.307	[0.092,0.515]	0.231	[0.052,0.388]
β_{13} y_1y_3	0.017	[0.0024,0.033]	0.018	[0.001,0.032]	0.015	[0.004,0.027]	0.013	[0.003,0.023]
β_{22} y_2^2	0.010	[-0.108,0.146]	-0.078	[-0.198,0.081]	0.029	[-0.107,0.183]	0.048	[-0.071,0.195]
β_{23} y_2y_3	-0.003	[-0.019,0.013]	-0.005	[-0.015,0.004]	-0.001	[-0.013,0.010]	0.001	[-0.005,0.008]
β_{33} y_3^2	0.001	[-0.003,0.004]	0.002	[-0.002,0.006]	0.001	[-0.002,0.004]	-0.001	[-0.004,0.000]
δ_1 p_1	-1.678	[-4.807,2.347]	-2.966	[-6.609,0.055]	-4.381	[-7.473,-0.846]	-3.307	[-5.343,-1.122]
δ_2 p_2	0.820	[-1.730,3.045]	1.633	[-0.616,4.027]	2.272	[-0.244,4.200]	1.645	[-0.389,2.898]
δ_{11} p_1^2	0.179	[-0.141,0.571]	0.102	[-0.149,0.423]	-0.029	[-0.301,0.264]	-0.043	[-0.253,0.171]
δ_{12} p_1p_2	-0.001	[-0.372,0.318]	0.044	[-0.257,0.281]	0.095	[-0.182,0.331]	0.083	[-0.075,0.235]
δ_{22} p_2^2	-0.077	[-0.443,0.343]	-0.174	[-0.488,0.222]	-0.150	[-0.419,0.177]	-0.204	[-0.381,-0.031]
η_{11} y_1p_1	0.170	[-0.110,0.416]	0.245	[0.012,0.497]	0.319	[0.015,0.547]	0.235	[0.072,0.394]
η_{12} y_1p_2	0.123	[-0.071,0.319]	0.063	[-0.137,0.237]	0.036	[-0.144,0.223]	0.111	[-0.009,0.226]
η_{21} y_2p_1	0.005	[-0.115,0.140]	-0.004	[-0.114,0.096]	-0.044	[-0.154,0.075]	-0.030	[-0.107,0.044]
η_{22} y_2p_2	-0.124	[-0.272,0.008]	-0.095	[-0.224,0.029]	-0.090	[-0.207,0.028]	-0.114	[-0.188,-0.034]
η_{31} y_3p_1	-0.009	[-0.029,0.017]	-0.012	[-0.036,0.009]	-0.016	[-0.033,0.005]	-0.009	[-0.020,0.003]
η_{32} y_3p_2	-0.006	[-0.023,0.013]	0.001	[-0.017,0.024]	0.004	[-0.014,0.020]	-0.003	[-0.014,0.008]
κ_1 t	-0.336	[-1.018,0.411]	-0.363	[-0.941,0.216]	-0.543	[-1.096,0.015]	-0.511	[-0.849,-0.169]
κ_2 t^2	0.007	[-0.013,0.028]	0.003	[-0.014,0.022]	-0.005	[-0.019,0.010]	-0.001	[-0.011,0.009]
ϕ_1 ty_1	0.066	[0.010,0.119]	0.059	[0.004,0.110]	0.083	[0.033,0.127]	0.078	[0.043,0.109]
ϕ_2 ty_2	-0.023	[-0.055,0.014]	-0.016	[-0.045,0.020]	-0.022	[-0.049,0.009]	-0.030	[-0.051,-0.009]
ϕ_3 ty_3	-0.003	[-0.007,0.000]	-0.003	[-0.006,0.000]	-0.002	[-0.006,0.000]	-0.002	[-0.003,-0.001]
φ_1 tp_1	0.049	[-0.021,0.128]	0.034	[-0.023,0.102]	0.044	[-0.010,0.101]	0.024	[-0.008,0.057]
φ_2 tp_2	-0.054	[-0.136,0.030]	-0.047	[-0.113,0.009]	-0.052	[-0.107,0.005]	-0.044	[-0.073,-0.013]
ω_1 $z_1^*(cred.)$			-0.757	[-2.004,0.522]				
ω_2 $z_2^*(liq.)$			-1.149	[-1.943,0.321]				
ω_3 $z_3^*(cap.)$			-0.468	[-1.911,1.134]				
ω_4 $z_4^*(mkt.)$			0.124	[-1.666,1.888]				
Inefficiency								
γ_0	-0.967	[-2.439,0.306]	-1.453	[-3.093,0.224]	-1.224	[-3.244,0.625]	-1.414	[-3.681,0.546]
γ_1 z_1	0.056	[0.022,0.149]	0.083	[0.019,0.180]	0.072	[0.031,0.164]	0.141	[0.004,0.276]
γ_2 z_2	1.012	[0.687,1.359]	1.133	[0.753,1.519]	1.041	[0.680,1.395]	1.018	[0.270,1.728]
γ_1^* $z_1^*(cred.)$					-3.261	[-5.328,-1.242]	-1.015	[-1.702,-0.609]
γ_2^* $z_2^*(liq.)$					0.245	[-0.662,1.160]	0.991	[-0.241,2.072]
γ_3^* $z_3^*(cap.)$					1.618	[0.627,2.467]	0.958	[0.056,1.823]
γ_4^* $z_4^*(mkt.)$					-0.928	[-1.914,0.045]	-0.822	[-1.927,0.036]
Eff.	0.515		0.531		0.507		0.641	
DIC_3	3168.01		3085.10		2466.83		2368.70	
LPS	-180.01		-282.42		-305.94		-401.56	

Note: Values for γ_1^* to γ_4^* in Model P4 correspond to the hyperparameter

Results of the models using both the cost and profit functions derived from (4) are presented in tables 2 and 3, where posterior means and probability intervals are presented for all the parameters. We observe that loans, investments and OBS affect cost positively in all models as well as input prices. In the case of profits, the relationship is also positive for loans and investments but negative, although not significant, for OBS. This result for OBS was also found by Tabak and Tecles (2010) in an application to the Indian banking sector. However, they found loans and investments to be not significant when OBS is included in both cost and profit models. Regarding input prices the coefficients are not relevant in any of the profit models.

We also found decreasing returns to scale in all the models, which may suggest low margin for more M&A processes in the sector. We analyze scale economies by groups of banks with similar characteristics of size, ownership, and involvement in M&A. We find that while large, domestic and merged institutions operate at decreasing returns to scale; small, foreign and non-merged banks exhibit increasing returns to scale.[14] These results coincide with those reported by Galán et al. (2014c) which suggests that M&A processes carried out mainly by domestic and large institutions may led them to be oversized; while small and foreign banks may still present some potential scale gains.

We observe that size and foreign ownership are important inefficiency drivers in all the models. Their effects are negative on cost inefficiency and positive on profit inefficiency. This suggests that large and foreign banks are more cost efficient but less profit efficient than their counterparts. Previous studies have found similar effects. Chen and Liao (2011) found that foreign banks perform better than local banks because they may deal better with risk exposure given cheaper access to funding sources or more diversification. Fries and Taci (2005) found similar results for banks with majority of foreign ownership in emerging economies. Regarding size, previous studies have found that large institutions tend to exhibit greater efficiency associated to higher scale economies (Bos and Kool, 2006; Glass and McKillop, 2006; Wheelock and Wilson, 2012; Hughes and Mester, 2013). In previous applications to Colombian banks, both foreign and large banks have also been found to be more cost efficient than local and small banks (Moreno and Estrada, 2013; Galán et al., 2014c; Sarmiento et al., 2013). However, the fact that larger banks are found to operate on decreasing returns to scale while they exhibit higher cost efficiency may suggest that those banks benefit from lower funding costs. This relatively advantage over smaller banks has been recently reported in the literature as evidence of the *too-important-to-fail* dilemma where larger banks take advantage of their size for funding at lower cots and for

[14]Small and large banks are those below and above the median of the total assets, respectively.

taking on more risk (IMF, 2014; Santos, 2014). Bertay et al. (2013) analyzed a large sample of banks for 90 countries during the period 1992-2011 and found that banks interest costs tend to decline with systemic size. This result was found for all banks except for those with very low capitalization level. Interestingly, as it will be mentioned further, we also find that lower capitalization (i.e. higher capital risk exposure) is associated with lower cost and profit efficiency.

Regarding the effects of risk exposure, posterior results for risk coefficients and model comparison indicators lead to similar conclusions in both the cost and profit models.[15] That is, models including risk exposure improve from a model omitting these variables; and, from these models, the one including these covariates in the inefficiency distribution exhibit better fit and predictive performance. Moreover, no important effects of any risk measure are observed when they are modeled in the frontier, while some risk exposures are found to be very relevant when they are included in the inefficiency distribution. This suggests that risk-taking is an important driver of banks inefficiency. Also, cost and profit efficiency are found to be over and under estimated, respectively, when risk exposure measures are not modeled in the inefficiency distribution.

We identify that greater capital risk exposures lead to lower cost and profit efficiency. There is evidence showing that highly capitalized banks tend to be more efficient than thinly capitalized banks (Kwan and Eisenbeis, 1997). This may be associated to the fact that lower capitalized banks may increase their risk of insolvency due to credit losses or sudden security price deterioration.[16]

Credit and market risks are also found to be key drivers of profit efficiency, providing evidence in favor of risk-taking in both the credit and securities markets. This may be related to the *skimping* hypothesis in Berger and DeYoung (1997), showing that when banks relax credit standards (or expend lower resources in the analysis of loan applications) they try to increase their quantity of loans and perform better (more profitable). This ex-ante credit risk is reflected in a growing proportion of risky loans which also tend to increase during periods of lower interest rates (Dell'Ariccia et al., 2013). A similar strategy is adopted by banks in the securities market. Banks with a higher risk appetite may choose to produce less fixed interest bearing loans and engage more in securities, increasing their market risk exposure (Mester, 2008).

[15] Lower values for DIC_3 and LPS indicate better fit and predictive performance.

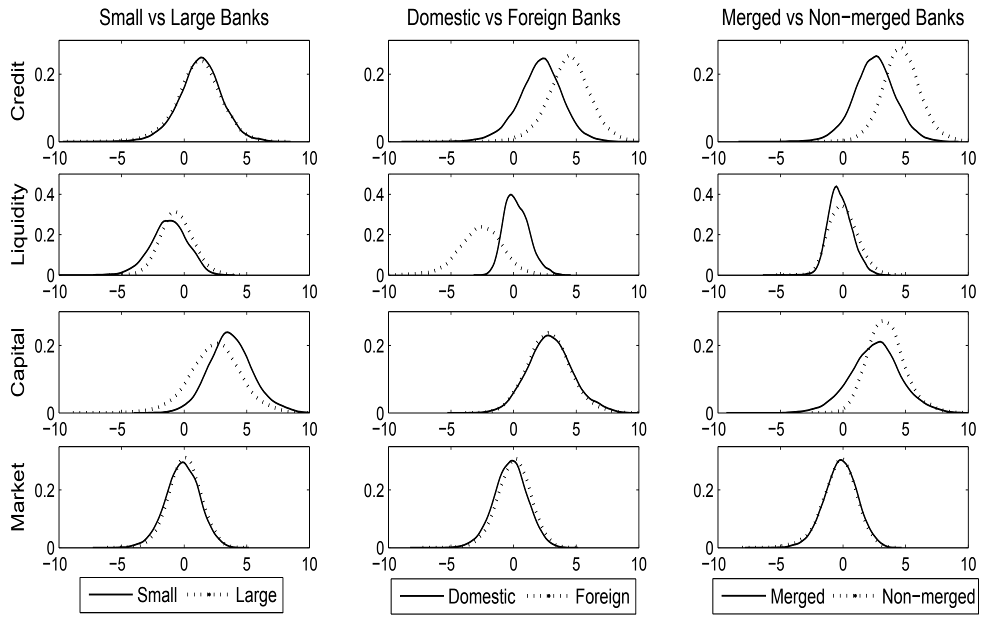
[16] See evidence for the U.S banks in Hughes and Mester (2001) and for German banking industry in Koetter (2008). However, Altunbas et al. (2007) find the opposite relationship for European banks (i.e. less efficient banks hold more capital and more liquid assets than higher efficient banks).

5.1. Analysis of risk random coefficients

Results of DIC_3 and LPS favor our proposed inefficiency specification with random coefficients for risk covariates in both cost and profit models. These results suggest not only that risk exposure measures are important inefficiency drivers but also that risk has different effects on cost and profit inefficiency of banks with different characteristics.

Figures 1 and 4 exhibit the posterior distributions of the four risk exposure bank-specific coefficients averaged by groups of banks in models C4 and P4, respectively. The analysis is performed by groups of banks with different characteristics of size (small vs large banks), ownership (domestic vs foreign banks), and involvement in M&A processes (merged vs non-merged banks). We observe two main results when heterogeneity in the way risk affects inefficiency is accounted for: firstly, some groups of banks are more affected than others at the same risk exposure levels; and secondly, some types of risk become relevant as inefficiency drivers for some groups of banks.

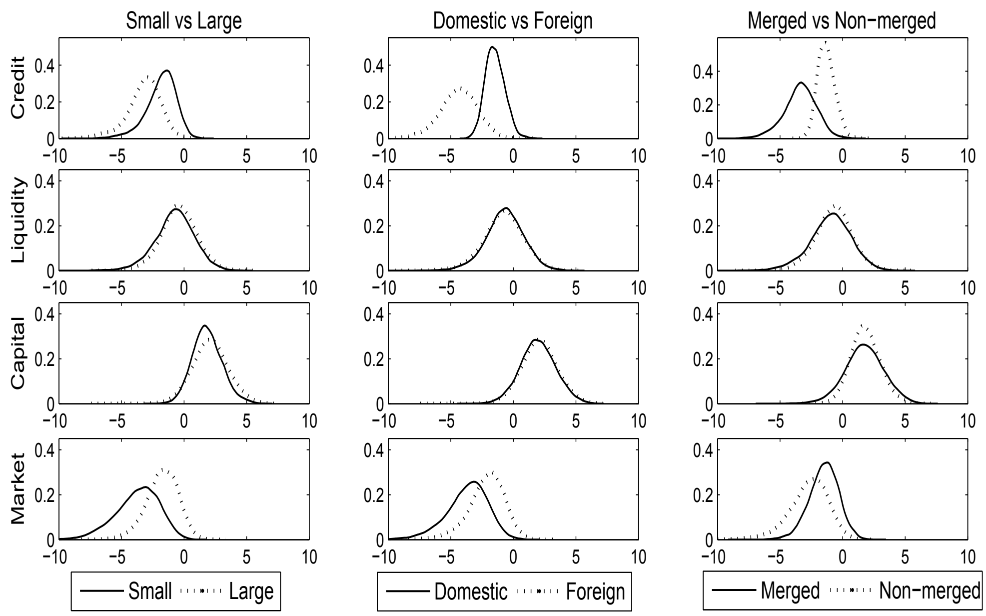
Figure 1: Average posterior distributions of risk random coefficients by groups of banks under cost model C4



In particular, credit risk is identified as a key cost inefficiency driver for foreign and non-merged banks, in the sense that higher exposures to credit risk reduce cost efficiency of these types of banks. Likewise, liquidity exposure is only identified as having relevant negative effects on cost inefficiency of foreign banks. Moreover, the effect on domestic banks is almost nonexistent. This result indicates that holding less liquid assets is more costly for foreign banks which could be associated to their incentives to engage on more risk-taking. Regarding capital risk exposures, the positive effects on cost inefficiency are similar between types of banks. However, increasing capital in the same proportion is more likely to affect more non-merged institutions. Finally, market risk is not relevant for any of the analyzed types of banks following the conclusion obtained from the fixed coefficients model.

Regarding bank-specific effects of risk on profit efficiency, it is observed in Figure 4 that credit risk affects more large, foreign and merged banks. Thus, these types of banks benefit more by assuming the same credit risk exposures than their counterparts. On the other hand, small, domestic and non-merged banks find more benefits when they increase market risk in the same proportions. As to liquidity and capital exposures, no differences are identified between these groups of banks. As in the fixed coefficients model, liquidity continues to be non-relevant explaining profit efficiency; while increasing capital has similar positive effects on profit inefficiency for all banks.

Figure 2: Average posterior distributions of risk random coefficients by groups of banks under profit model P4

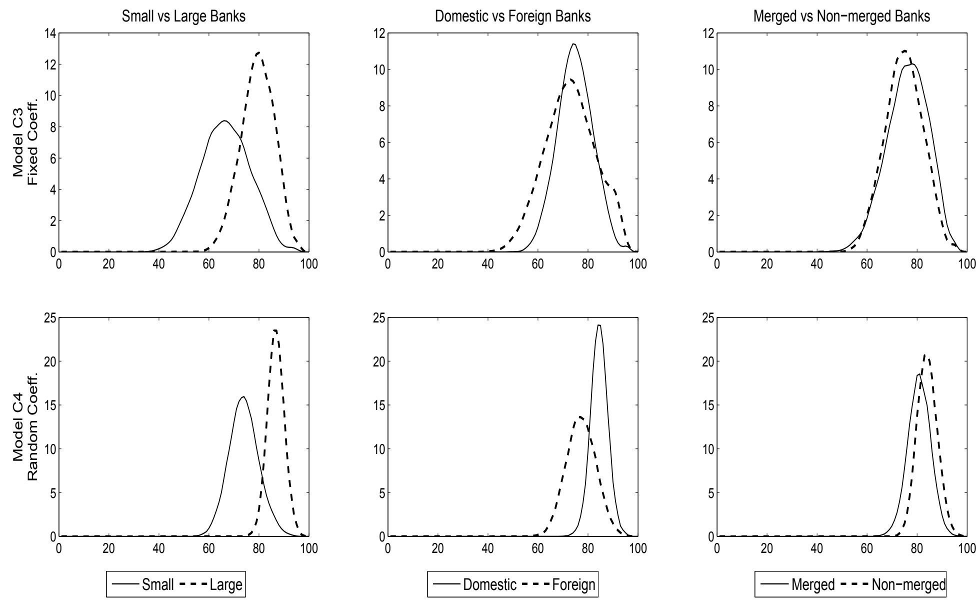


5.2. Analysis of efficiency

The most important changes in the posterior efficiency estimations are observed when the model is allowed to estimate bank-specific risk coefficients in the inefficiency distribution. In these cases, the average posterior cost efficiency decreases and profit efficiency increases with respect to the other models. This suggests that considering heterogeneity in the way risk affects inefficiency has important effects on estimations. It is also observed that in both cost and profit models, the dispersion of the posterior efficiency presents important increases in the random coefficient models. This suggests that these models are recognizing differences between banks in terms of their risk exposure and that these differences have effects on their efficiency estimations.

Figures 3 and 4 exhibit the average posterior distributions of cost and profit efficiency, respectively. Posterior efficiency is analyzed by groups of banks and results from models with fixed and random coefficients of risk covariates are presented.

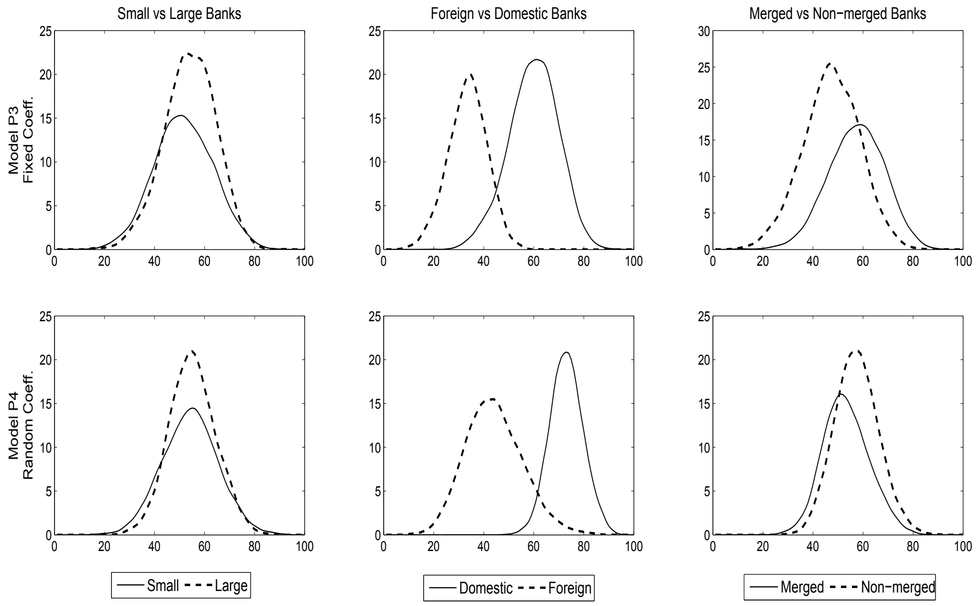
Figure 3: Average posterior distributions of cost efficiency by groups of banks in Models C3 and C4



In general, bank-specific characteristics are found to be relevant factors that differentiate banks performance. Large banks exhibit higher costs efficiency levels than small banks in both fixed and random coefficients models. However, the random coefficient models show a higher difference among large and small institutions. As we mentioned before, a possible explanation for the differences between banks with different size may be associated to the fact that large banks are considered by creditors as *too-important-to-fail* and then, they are willing to offer funds at lower costs. In the case of small banks the result can be seen as opposite in the sense that creditors and depositors may ask for higher returns from those "risky-small-banks" as a way to exert market discipline (see evidence in Wheelock and Wilson, 2012; Bertay et al., 2013; Hughes and Mester, 2013; IMF, 2014). Regarding affiliation, domestic institutions present higher costs efficiency than foreign banks but this difference is only important under the random coefficients model, suggesting that domestic banks benefit from the differences in the way credit and liquidity risk affect these banks. On the other hand, no important differences are observed in cost efficiency between merged and non-merged banks. As presented further, differences are more evident in the evolution of their efficiency.

Regarding profit efficiency, the random coefficients model identifies some differences in the location and dispersion of the posterior efficiency distributions. The main difference in profit efficiency is observed between domestic and foreign banks. Domestic banks almost double profit efficiency of foreign banks in both models.

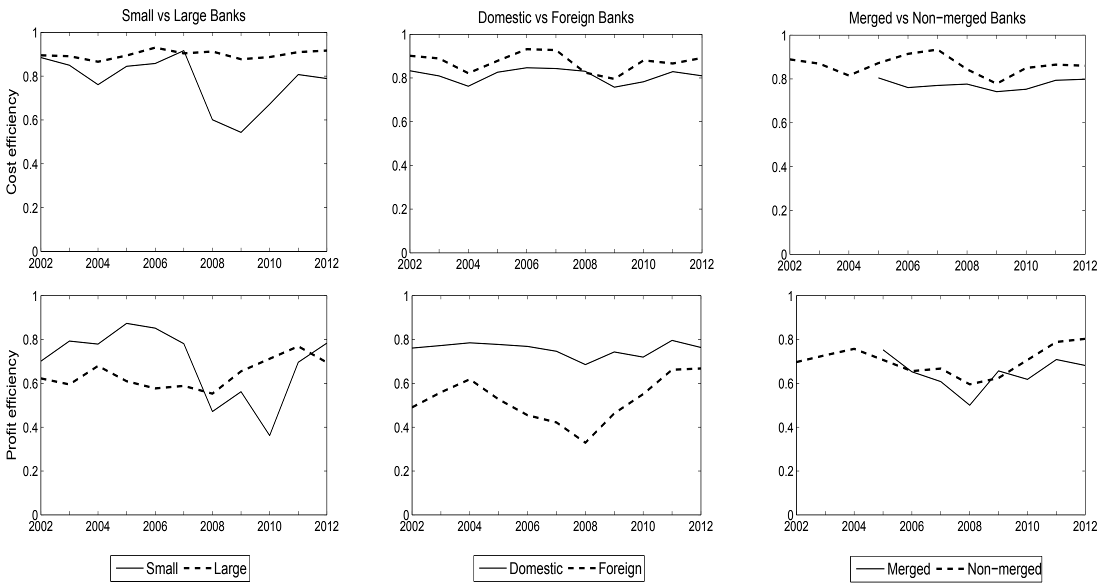
Figure 4: Average posterior distributions of profit efficiency by groups of banks in Models P3 and P4



Finally, focusing on the results of our preferred models with random coefficients, the evolution of cost and profit efficiency over time is presented in Figure 5 by groups of banks. Small banks have been more volatile in both cost and profit efficiency over time, while large banks have been more stable and present higher cost efficiency during all the period. This may suggest that large banks are less sensitive to environmental conditions. Foreign banks present lower profit efficiency with the lowest value presented in 2008 coinciding with the global financial crisis. This suggest that foreign institutions could be affected by their operations and investments in international markets. Nevertheless, in the last years, foreign banks exhibit an increasing trend in profit efficiency and their scores are very close to those of local banks.

Regarding merged banks, we observe that they present decreases in cost and profit efficiency in the first years after these processes have been carried out (2005 - 2006). However, in the last two years they seem to recover part of these losses. This pattern was also found in Galán et al. (2014c) by using an input-oriented technical efficiency approach under a dynamic SFA model. They find that merged banks are able to recover very fast their efficiency levels and present higher efficiency than non-merged institutions due to lower adjustment costs. Cuesta and Orea (2002) had also found a similar pattern in merged Spanish banks after evaluating output-oriented technical efficiency. Here, we find that these effects are even more evident when we assess integrally costs and revenues in a profit efficiency analysis.

Figure 5: Evolution of mean posterior cost and profit efficiency by groups of banks in random coefficient models



6. Concluding remarks

Risk-taking is an inherent condition of the banking business. However, traditional studies on bank efficiency had assumed that risk is incorporated on bank output without explicitly modeling its role in explaining inefficiency. Recent studies show that failing to account for risk-taking leads to biased estimations of bank efficiency as well as mislead estimates of scale economies and cost elasticities. Likewise, the literature has focused mainly on credit risk, omitting other important risks faced by banks.

We present a stochastic frontier model, which is able to model unobserved heterogeneity sources through random coefficients in the inefficiency distribution and incorporates bank-specific characteristics and measures of credit, liquidity, capital and market risk exposures. It also provides the first empirical evidence on the role of bank risk-taking in the inefficiency of the Colombian banking industry. This is particularly important because during the last years the banking industry in this emerging economy has expanded their financial services along with greater credit and market risk exposure. Recent studies on efficiency of the Colombian banking sector have showed an improvement on technical efficiency and productivity change among banks. However none of these studies have incorporated the impact of bank risk-taking behavior on inefficiency, which plays a major role in the banking production.

Our findings remark the importance of considering different types of bank risk exposure as cost and profit inefficiency drivers. In particular, large and foreign banks exhibit higher costs efficiency, which are unrelated to scale economies, suggesting evidence of *too-important-to-fail* considerations that may benefit large banks from lower deposit and funding costs. Regarding risk effects on inefficiency, we find that greater exposures to credit and market risks are found to be key drivers of profit efficiency. These findings suggest that banks may have incentives for risk-taking in both the credit and securities markets. We also find evidence to support that lower capital risk (i.e. higher capitalization) leads to higher efficiency in both costs and profits. Finally, our proposal to include random coefficients in the inefficiency capture differences in the way risk affects cost and profit efficiency of banks involved in M&A processes, and banks with different size and type of ownership. We identify large, foreign and merged banks to benefit more from taking on the same credit risk exposures than their counterparts; while small, domestic and non-merged banks institutions take advantage of higher market risk.

Banks cost and profit efficiency under risk-taking may be a useful indicator for financial stability considerations given that banks with lower efficiency have been found to be more prone to future bank fails (Berger and DeYoung, 1997). In this context, regulators should not only take into account the impact of requirements in capital and liquid assets on cost and profit efficiency of banks, but also that these policies have different, and sometimes opposite, effects on banks with different characteristics.

Moreover, the fact that large and merged banks are found to face lower costs and to have incentives to take on more risk in credit and securities markets constitute a signal for regulators to monitor the behavior of these type of banks and their riskiness. Regulators should also consider alternative measures to limit risk-taking incentives associated to the fact that larger banks exploit the benefits from being considered as *too-important-to-fail*. This is even more important given the recent local expansion of financial conglomerates, which makes more difficult for regulators to monitor their behavior and may enhance regulatory arbitrage through non-banking institutions boosting financial fragility. The lack of market discipline by depositors and the difficulties of regulators to be effective monitors of banks has enhanced the role of market-based-monitoring. Under this approach, riskier banks are forced to pay higher funding costs given that their peers are able to identify those banks in the market. Work is currently in progress on this area and on the relationship between risk-taking and the *too-important-to-fail* dilemma in interbank markets.

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