Robot Formations Control Using Fast Marching

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Abstract

This paper presents the application of the Voronoi Fast Marching (VFM) method to the Control of Robot Formations. The VFM method uses the propagation of a wave (Fast Marching) operating on the world model to determine a motion plan over a *viscosity* map (similar to the refraction index in optics) extracted from the updated map model. The computational efficiency of the method allows the planner to operate at high rate sensor frequencies. This method allows us to maintain good response time and smooth and safe planned trajectories. The navigation function can be classified as a type of potential field, but it has no local minima, it is complete (it finds the solution path if it exists) and it has a complexity of order n ($O(n)$), where n is the number of cells in the environment map. The results presented in this paper show how the proposed method behaves with mobile robot formations and generates trajectories of good quality without problems of local minima when the formation encounters non-convex obstacles.

Keywords: robot formation motion planning, formation control, Fast Marching

1. Introduction

Formation control is currently a topic of vast research in literature. Different approaches can be classified according to different criteria. Beard et al [1] consider the different design approaches and classify them into three groups: *leader-following*, where one vehicle is designated as the leader and the others as followers. The leader posture (position and orientation) is determined by a trajectory to be tracked or by external control objectives

(e.g., joysticked by a human) and the followers must track the leader following some prescribed geometry, possibly dynamically changing over time [2]; *behavioral*, where the motion of each vehicle results from a weighted average of several behaviors, ultimately contributing to a desired group behavior [3]; *virtual structure*, where the entire formation is treated as a single structure, whose desired motion is translated into the desired motion of each vehicle [4]. Another possible criterion takes into account the rigidity of the formation geometry: some authors specify the full geometry, e.g., the distances and bearings between the vehicles of the formation and control each vehicle to ensure that these are accurately achieved [5], requiring a coordination architecture to switch between geometries when required by the environment characteristics (e.g., narrow passages, open spaces) [6]; others see the formation as a dynamic geometry structure, that naturally becomes distorted in the presence of obstacles and/or environment geometry changes [7].

We are particularly interested in leader-following deformable formations, where the leader can be virtual and tracks a given trajectory, pulling the followers behind it according to nominal geometry specifications (e.g., desired inter-vehicle distances) that can change within a given range so as to accommodate environment conditions. Leonard and Fiorelli [8] introduced the concept of artificial potential fields between formation vehicles, some of them virtual leaders. The nominal inter-vehicle distance corresponds to minima of the potential functions, representing equilibrium points that balance inter-vehicle repulsion, vehicle-obstacle repulsion, and follower-leader attraction. MacArthur and Crane proposed a similar approach but virtual spring-damper systems are used to "connect" the formation vehicles [9]. The drawback of such approaches is the well-known local minima problem of potential fields, that may lead to breaking the formation in the presence of non-convex shaped obstacles.

In a previous paper [10], we have introduced one possible solution to this problem, where the followers keep track of the n most recent positions of the leader, and not only its current position, to be dragged away of the obstacle trap. However, this method has not been proven to work for all possible situations. We have also introduced a method to avoid potential fields local minima for a single vehicle, using the Voronoi Fast Marching method (VFM) and the Fast Marching Squared method (FM^2) [11],[12],[13].

In this paper we use the Fast Marching (FM) algorithm to control a leader-follower deformable formation, where the trajectory of the leader in an environment cluttered by obstacles is computed using VFM algorithm.

Each follower attempts to reach, at each iteration step, a nominal sub-goal position related to the desired leader trajectory, but takes into account the positions of the other followers and the environment objects, both seen as obstacles. This influences the metrics used by these algorithms, effectively deforming the followers trajectories. This way we ensure that non-convex obstacles do not break the formation, and that the inter-vehicle distances are smoothly deformed while the formation moves from open areas to regions with obstacles, narrow corridors, and narrow passages.

The paper is organized as follows: in Section 2 the Fast Marching method and some of its variants are summarized. Its application to robot formations is described in Section 3. The results of applying the method to two different formation geometries in a diversity of simulation scenarios are presented in Section 4. The paper ends with conclusions and prospects for future work (Section 5).

2. Fast Marching Method and Voronoi Fast Marching Method

2.1. Introduction to Fast Marching and Level Sets

The FM algorithm was introduced by J. Sethian in 1996 and is a numerical algorithm that approximates the viscosity solution of the Eikonal equation

$$|\nabla(D(\mathbf{x}))| = P(\mathbf{x}) \tag{1}$$

i. e., the equation of the light in a non-homogeneous media. The level set $\{\mathbf{x}/D(\mathbf{x}) = t\}$ of the solution represents the wave front advancing with a media velocity $P(\mathbf{x})$, which is the inverse of the media refraction index $R(\mathbf{x})$. Therefore, the Eikonal equation can be written as $|\nabla(D(\mathbf{x}))| = 1/R(\mathbf{x})$. The resulting function D is a distance function, and if the media velocity P is constant, it can be seen as the Euclidean distance function to a set of starting points, usually the goal points. If the media is non-homogeneous and the velocity P is not constant, the function D represents the distance function measured with the metrics $P(\mathbf{x})$ or the arrival time of the wave front to point $\mathbf{x}$.

The FM method is used to solve the Eikonal equation and is very similar to the Dijkstra algorithm that finds the shortest paths on graphs, though it is applied to continuous media. Using a gradient descent of the distance function D, one is able to extract a good approximation of the shortest path (geodesic) in various settings (Euclidean distance with constant P and a weighted Riemanian manifold with varying P).
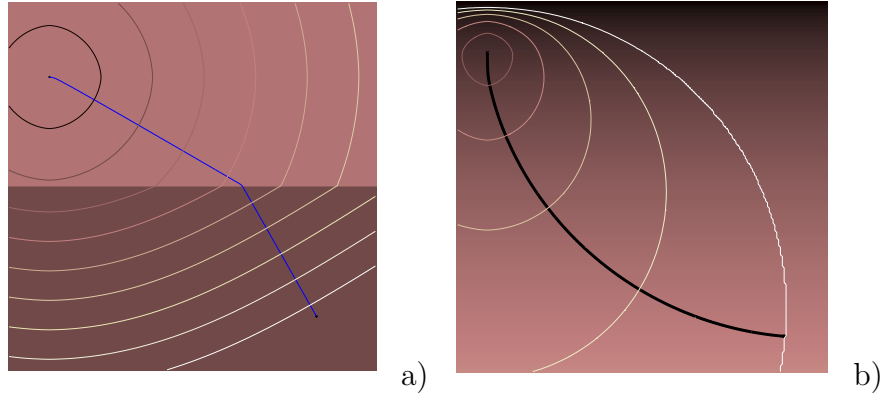
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Figure 1: a) Propagation of a wave and the corresponding minimum time path when there are two media of different slowness (refraction) index; b) the same with a vertical gradient of refraction index.

2.2. Intuitive Introduction to the Fast Marching Motion Planner

To get a Motion Planner for mobile robots with desirable properties, such as smoothness and safety, we can think of attractive potentials. In Nature, there are phenomena with a similar behaviour, e.g., the electromagnetic waves. If there is an antenna in the goal point, that emits an electromagnetic wave and then the robot can drive itself to the destination following the waves to the source. In general, the concept of electromagnetic wave is especially interesting, since the potential and its associated vector field have the good properties desired for the trajectory, such as smoothness and the absence of local minima.

This attractive potential still has some problems. The most important, that typically arises in mobile robotics, is that optimal motion plans may bring robots too close to obstacles or people, which is not safe. This problem has been treated by Latombe [14], and the resulting navigation function is called NF^2. The Voronoi method also tries to follow a maximum clearance map [15]. To obtain a safe path, it is necessary to add a component that repels the robot from obstacles, as proposed by Khatib[16]. In addition, this repulsive potential and its associated vector field should have good properties such as those of electrical fields. If we consider that the robot has an electrical charge of the same sign as obstacles, then the robot would be pushed away from obstacles. The properties of this electric field are very good because it is smooth and there are no singular points in the interest space (C_{free}).

The main problem of attractive and repulsive potentials is how to mix
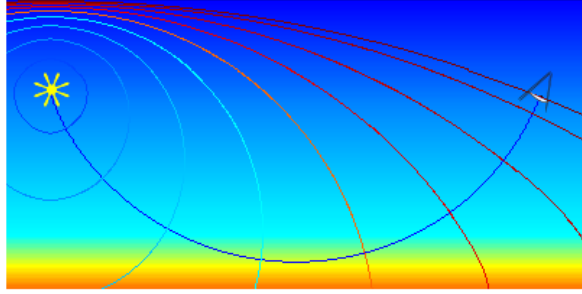
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Figure 2: Hot road surface mirage: Light rays bending due to change of the refraction index in air with higher temperature (and lower refraction index) near the road surface.

the two fields together. This union between an attractive and a repulsive field has been the biggest problem of potential fields in path planning since the works by Khatib [16]. The problem is that the sum, difference, or similar operations provoke the appearance of local minima.

In the proposed algorithm, this problem has been solved in the same way as Nature does: the electromagnetic waves, as the light, have a propagation velocity that depends on the media due to the refraction index. For example, a flint glass has a refraction index of 1.6, while in the air it is approximately of 1. This refraction index of the media is the quotient between the light velocity in the vacuum and its velocity in the media, i.e., the slowness index of the front wave propagation in a media. This idea of mixing the two potentials as Nature does is the main contribution of the VFM and FM^2 methods [11],[12],[13].

According to Fermat's principle, the path taken by light to go from one point to another is the path that minimizes the time. In the case of a homogeneous media, in which the light velocity is constant, the light follows a straight line. The set of points achieved in a fixed time is a circle. Consider two media with different refractive indexes, e.g., air and water. If a ray of light passes from the first media to the second one, the phenomenon known as refraction occurs. The beam appears to bend when the media changes. The light seems to prefer to stay more in the media with greater light velocity, as shown in figure 1a). If there is a continuous change in the refraction index of the media, the path obtained is a curve that goes away from areas with lower light velocity (higher refractive index), as shown in figure 1b) and figure 2. This is the case of the well-known hot road surface mirage, in which it appears
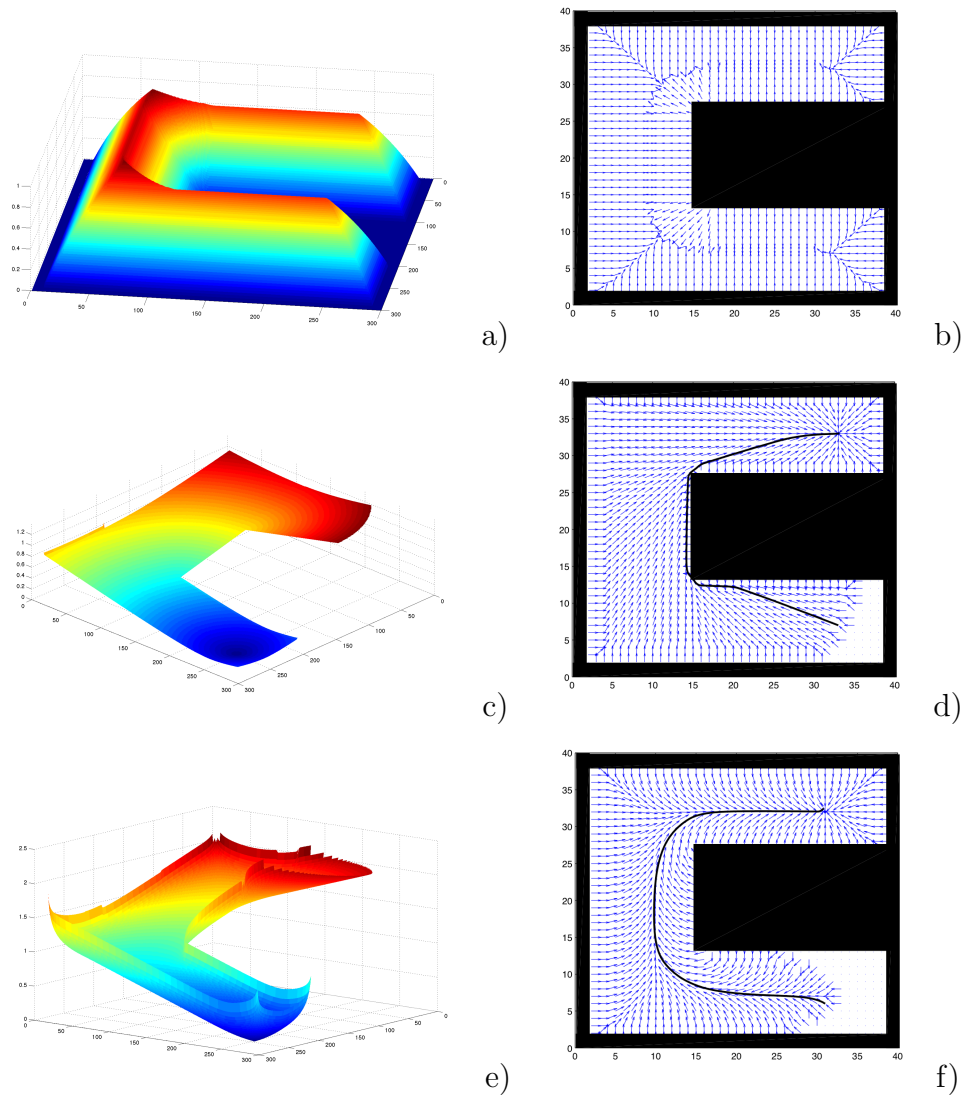
Figure 3: a) Repulsive potential; b) Its associated vector field. This potential can be considered as the metrics, the refraction index, or a viscosity index; c) Attractive potential; d) Its associated vector field and typical trajectory obtained with the attractive potential alone; e) Union of the two potentials: the second one having the first one as refractive index; f) Associated vector field and typical trajectory obtained with this method.

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to be "fake water" on the road and it is produced by the light rays bending due to a change of the refraction index in air with higher temperature (and lower refraction index) near the road surface.

In the proposed methods, the repulsive potential is used as the refraction index of the medium where the wave emitted from the goal point propagates, as shown in figures 3a) and 3b). Figures 3c) and 3d) show the behavior of the attractive potential used alone. When the attractive potential uses the repulsive potential as refraction index, a unique field is obtained and its associated vector field is attracted to the goal point and repulsed from the obstacles, as shown in figures 3e) and 3f). This method inherits the properties of the electromagnetic field, i.e., it is $\mathcal{C}^{\infty}$ if the refraction index is $\mathcal{C}^{\infty}$. Intuitively, the FM method gives the propagation of a front wave in non-homogeneous media.

This repulsive potential can be obtained using the Extended Voronoi Transform (EVT) of the binary image of the map. The EVT computes the Euclidean Distance of the binary image. In a binary image, a pixel is referred to as background if its value is zero. For a given distance metrics, the EVT of an image produces a distance map of the same size. For each pixel in the image, the EVT assigns a number which is the distance to the nearest zero pixel of the image. For each pixel inside the objects in the binary image, the corresponding pixel in the distance map has a value equal to the minimum distance to the background. This is the solution adopted in the VFM method.

Clearly, the EVT is closely related to the Voronoi diagram. The Voronoi diagram concept is involved in many EVT approaches, either explicitly or implicitly.

Another possibility is to build the repulsive potential using FM. This is the solution adopted in the FM^2 method. In this case, a wave is propagated starting from the points representing the obstacles and walls. This wave propagation is achieved through the FM method. The result is a potential map, which can be interpreted as a velocity map (or slowness map) because it gives a clear idea of the robot permissible velocity at each environment cell. This potential map is represented in grey scale, where the walls and obstacles are black and the cells become lighter as long as the distance to these obstacles increases.

2.3. Implementation of a Smooth Slowness Potential

This implementation starts with the calculation of the logarithm of the inverse of the EVT of the 2D environment updated map (*a priori* + sensor

data) (or the inverse of the EVT in case of 3D maps). Each white point of the initial image (which represents free cells in the map) is associated to a level of grey that is the logarithm of the inverse of the 2D distance to the nearest obstacles. As a result of this process, a potential is obtained, proportional to the distance to the nearest obstacles, for each cell.

This function introduces a potential similar to a repulsive electric one (in 2D), that can be expressed as $\phi = c_1 \log(1/r) + c_2$. If $n > 2$, the potential is $\phi = \frac{c_3}{r^{n-2}} + c_4$, where c_1, c_2, c_3, c_4 are constants.

2.4. Fundamentals of the Method

Maxwell's laws govern the propagation of the electromagnetic waves. We can use the most typical simplification of the problem in isotropic and possibly non homogeneous media for a monochromatic wave, considering the so called *almost flat waves*. In this case, the optical wave propagates at wavelengths much smaller than image objects, and therefore the ray optics approximates wave optics. These equations along with the mentioned approach allow us to develop the theories based on rays, such as the geometrical optics, the theory of sound waves, etc. The Eikonal equation is derived from these equations. In the Sethian [17] notation

$$|\nabla(D(\mathbf{x}))| = 1/R(\mathbf{x}) \tag{2}$$

where $D(\mathbf{x})$ represents the distance to the initial set, $R(\mathbf{x})$ is the refraction index of the media, and $\mathbf{x} = (x, y)$ in 2D or $\mathbf{x} = (x, y, z)$ in 3D. In geometrical optics, Fermat's *least time principle* for light propagation in a media with space varying refractive index $R(\mathbf{x})$ is equivalent to the Eikonal equation. The Eikonal solution $D(\mathbf{x})$ is a scalar function whose isolevel contours are normal to the light rays, and the refractive index is the quotient of the light velocity in the vacuum and its velocity in the media whose index calculates. This equation is also known as Fundamental Equation of the geometrical optics.

The most important aspect in this equation, from the path planning point of view, is that the solution is an exponential function of the refractive index

$$D(\mathbf{x}, t) = A(\mathbf{x})e^{j(\frac{\omega}{c}R(\mathbf{x}) - \omega t)} \tag{3}$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency, which is constant because the considered wave is monochromatic, T is the period, $R(\mathbf{x}) = \frac{c}{v(\mathbf{x})}$ is the refraction index, c is the vacuum light velocity, λ is the length wave, $v(\mathbf{x})$ is the

anisotropic media light velocity, and $v = \frac{\lambda}{T}$. In this expression, ω and c are constants and the refraction index is a function of the position, i.e., the considered media is anisotropic.

Since this solution is exponential, if the refraction index or first potential $R(\mathbf{x})$ is $\mathcal{C}^\infty$ then the second potential $D(\mathbf{x})$ is also $\mathcal{C}^\infty$, and therefore the trajectories calculated by the gradient method over this potential would be of the same class.

3. Robot Formations and Fast Marching

Robot formations motion planning deals with the problem of how to find the paths and postures between robots so that the whole formation can adapt to obstacles and other environment characteristics.

In this work, we have considered two types of formations: 1) leader-following, where two robots (followers) follow the leader as if they were body-guards, forming a triangle among the three of them; 2) leader-protecting, a formation of six robots in hexagonal configuration protecting the center of the formation (leader). The leader will not be represented in the leader-protecting figures for the sake of clarity and simplicity.

As stated before, the FM method is based on a potential function without local minima that provides smooth trajectories. The main problem to deal with is how to apply this method to the robot formation motion, keeping its good properties. To solve this problem a repulsive force is needed between the formation robots so that they keep a security distance and do not end up crashing into each other. Besides, an internal attractive force is also required for maintaining the formation while adapting to the environment.

The advantage of using the VFM method, as proposed here, is that each robot, at each time, has one single potential which is attractive to the objective and repulsive from walls, obstacles, and the other robots of the formation, which tries to keep the internal distances following the method described next.

Our first approach to implement the repulsive force was to add small Gaussians to the second potential $\mathbf{D}_i^t$ (distance potential) of each robot in the position of the others in order for every robot to have the rest as obstacles and keep a certain security distance from them. This approach is fast and works for a wide range of cases, though the behaviour is not natural in some special situations such as the ones presented in figure 8 and figure 9, for the

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corridor at the bottom of the images. Another drawback of this solution is the appearance of local minima due to the addition of these potential functions.

Taking this into account, the best solution for a robot to keep the distance from others is to set them as obstacles in the environment map, expressed in a matrix $\mathbf{Wo}_i^t$. The computation cost of this method is higher, since similar matrices $\mathbf{Wo}_i^t$, $\mathbf{W}_i^t$ and $\mathbf{D}_i^t$ (see next section) have to be computed at each step. However, this approach preserves the good properties of FM and avoids the appearance of local minima.

The solution adopted for the implementation of the repulsive force is detailed next and summarized in figure 4.

3.1. Description of the Algorithm of Robot Formations Motion using Fast Marching

3.1.1. Base Method

As shown in the previous section, the VFM method is based on the construction of two potentials: the repulsive or first potential $\mathbf{W}$, that can be interpreted as the metrics, the viscosity, or the refraction index of the media, and the second potential $\mathbf{D}$ with funnel shape, that represents the distance to the goal point measured with the metrics $\mathbf{W}$.

The VFM algorithm consists of an initialization and a loop. In the initialization:

- The initial map of the environment is read.

- The path of the leader is calculated using VFM applied to this map.

Each cycle of the loop represents a step of the movement of the robots. Within the loop:

- For each cycle t, each follower robot i has its own map $\mathbf{Wo}_i^t$ of the environment, which is the initial map, with the other robots of the formation placed as obstacles:

$$\mathbf{Wo}_i^t = f(\mathbf{M}_{GL}^t, \mathbf{M}_{Li}^t)$$

i.e., the map $\mathbf{Wo}_i^t$ is a function of the global map $\mathbf{M}_{GL}^t$ and the local map $\mathbf{M}_{Li}^t$, and the local map $\mathbf{M}_{Li}^t$ is the union (occupancy map) of the map sensed by robot sensors and the map with the other robots positions:

$$\mathbf{M}_{Li}^t = \mathbf{M}_{Si}^t + \mathbf{M}_{r_i, i \neq j}$$

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This matrix $\mathbf{Wo}_i^t$ is different for each follower robot i:

$$\mathbf{Wo}_i^t \neq \mathbf{Wo}_j^t \ \text{ for } \ i \neq j$$

This matrix $\mathbf{Wo}_i^t$ is binary: its entries are 0 (black) in the wall and obstacle locations (including the other vehicles) and 1 (white) in the free space, but also it could be the probability of occupancy. The fact of having the other vehicles as obstacles acts as a repulsive force between these vehicles.

- The metrics matrix for robot i $\mathbf{W}_i^t$ is calculated applying the EVT operator to the matrix $\mathbf{Wo}_i^t$ for each vehicle and then taking the logarithm of the result in order to have a repulsive field similar to the electric one. This matrix is obtained using the obstacles and walls initially set for the FM method. This way, the matrix $\mathbf{W}_i^t$ is composed of real numbers, with value 0 in the walls and obstacles and real grey levels in the rest. The grey levels are darker near walls and obstacles.

$$\mathbf{W}_i^t = log(g_{FM}(\mathbf{Wo}_i^t))$$

The matrix $\mathbf{W}_i^t$ represents the metrics $P(\mathbf{x})$, the viscosity, or the inverse of the refraction in the Eikonal equation.

- The partial goal points are calculated using the leader path and the desired formation. The distance from the partial goals to the leader path is proportional to the grey level of the partial goals position. This way, the formation tends to be near the initial positions and partial goals act as attractive forces between the vehicles.

- The distance matrices $\mathbf{D}_i^t$ are calculated applying the FM method to the metrics matrix $\mathbf{W}_i^t$ for each robot, using as goal the partial goal of the previous step.
$$\mathbf{D}_i^t = h_{FM}(\mathbf{W}_i^t)$$

- The path of minimum distance measured with the metrics $\mathbf{W}_i^t$ from each vehicle to its partial goal is found using its $\mathbf{D}_i^t$ potential and the gradient method.

- All the robots move a fixed number of points on the corresponding path.

- The leader advances its path position a fixed number of points on the initialization path.

The aforementioned attractive and repulsive forces between the vehicles act as glue in order to maintain the formation, but with enough freedom to take the other obstacles, bends, and narrow passages into account. Some interesting aspects of the process are shown in figure 5.

3.1.2. Adding Springs

When there are highly symmetric situations, i.e., the robots have to traverse a small door and they are initially placed orthogonally to the wall, the two follower robots have problems to pass together through the door. In order to solve this problem, it is necessary to introduce a *precedence order*. For example, if there is no room for the two robots to pass, the second one has to pass first, and then the third (the first is the leader).

The solution proposed to this problem is to calculate first the complete trajectory for the second follower and then the complete trajectory for the last follower. This provides an effect similar to using springs with different stiffness between the robots. This effect can be seen in figures 7 and 9.

Other possible solution to solve these highly symmetrical situations is to find the VFM joined trajectory for two followers in four dimensional configuration space resulting from the union of the two followers.

3.1.3. Maximum Energy Configuration

Another interesting problem refers to what the formation must do if the nearest passage is narrower than the width of the robot formation, where the width of the formation is the orthogonal diameter to the movement direction. The solution is to know previously the size of the configuration of minimum distance between the robots (or the maximum energy configuration) and search for another possible passage larger than this size.

The solution proposed is to dilate the walls and obstacles with the minimum radius of the robot formation. Then, the trajectory of the leader is found using this dilated map. This way, it is ensured that the formation passes through the passage used by the leader trajectory. Using this trajectory, the rest of the algorithm is similar to the main method:

- The partial goal points are calculated using the leader path and the desired formation. The distance from the partial goals to the leader path is proportional to the grey level of the partial goals position.
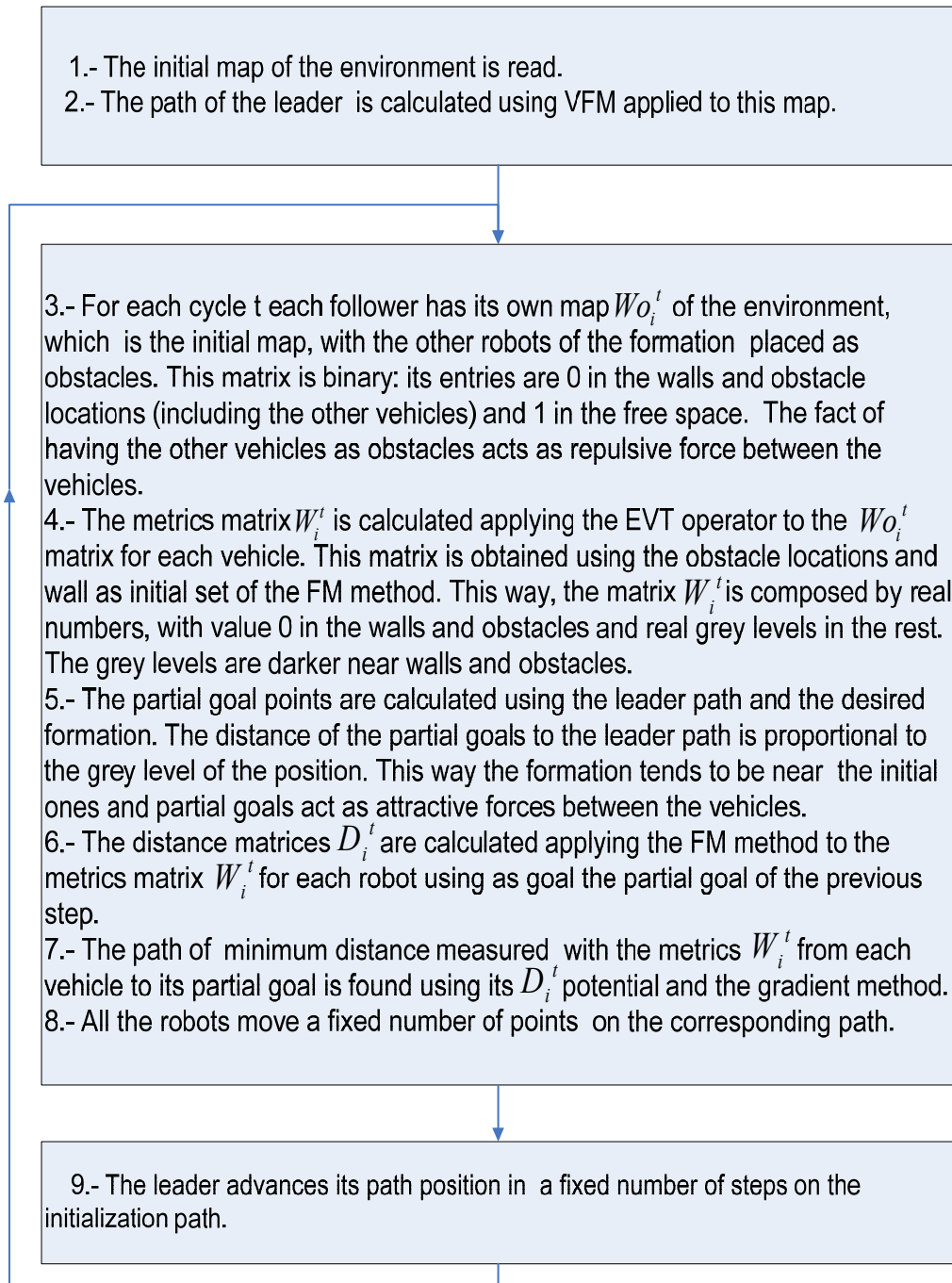
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1.- The initial map of the environment is read.
2.- The path of the leader is calculated using VFM applied to this map.

3.- For each cycle t each follower has its own map Wo_i^t of the environment, which is the initial map, with the other robots of the formation placed as obstacles. This matrix is binary: its entries are 0 in the walls and obstacle locations (including the other vehicles) and 1 in the free space. The fact of having the other vehicles as obstacles acts as repulsive force between the vehicles.

4.- The metrics matrix W_i^t is calculated applying the EVT operator to the Wo_i^t matrix for each vehicle. This matrix is obtained using the obstacle locations and wall as initial set of the FM method. This way, the matrix W_i^t is composed by real numbers, with value 0 in the walls and obstacles and real grey levels in the rest. The grey levels are darker near walls and obstacles.

5.- The partial goal points are calculated using the leader path and the desired formation. The distance of the partial goals to the leader path is proportional to the grey level of the position. This way the formation tends to be near the initial ones and partial goals act as attractive forces between the vehicles.

6.- The distance matrices D_i^t are calculated applying the FM method to the metrics matrix W_i^t for each robot using as goal the partial goal of the previous step.

7.- The path of minimum distance measured with the metrics W_i^t from each vehicle to its partial goal is found using its D_i^t potential and the gradient method.

8.- All the robots move a fixed number of points on the corresponding path.

9.- The leader advances its path position in a fixed number of steps on the initialization path.

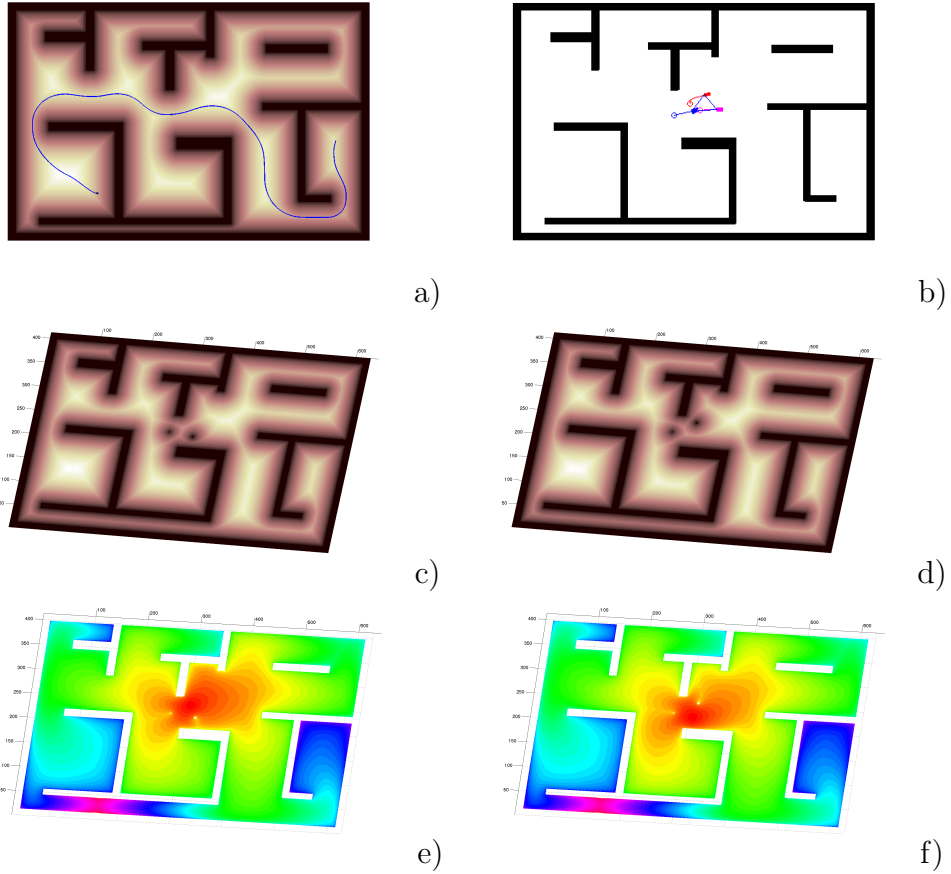Figure 4: Flowchart of the proposed algorithm.

Figure 5: a) Trajectory of the leader obtained with VFM; b) Robot Formation (rectangles) with the corresponding partial objectives (circles) and the partial trajectories of the followers; c) and d) Metrics Matrix $\mathbf{W}_i^t$ for each of the follower robots in a particular step using the method proposed in this paper: the other robots are treated as obstacles in the map, then the metrics matrix $\mathbf{W}_i^t$ is calculated over this map, and finally the distance matrix $\mathbf{D}_i^t$ is calculated using $\mathbf{W}_i^t$; e) and f) The Distance Matrix $\mathbf{D}_i^t$ measured with the metrics $\mathbf{W}_i^t$ for each of the follower robots.

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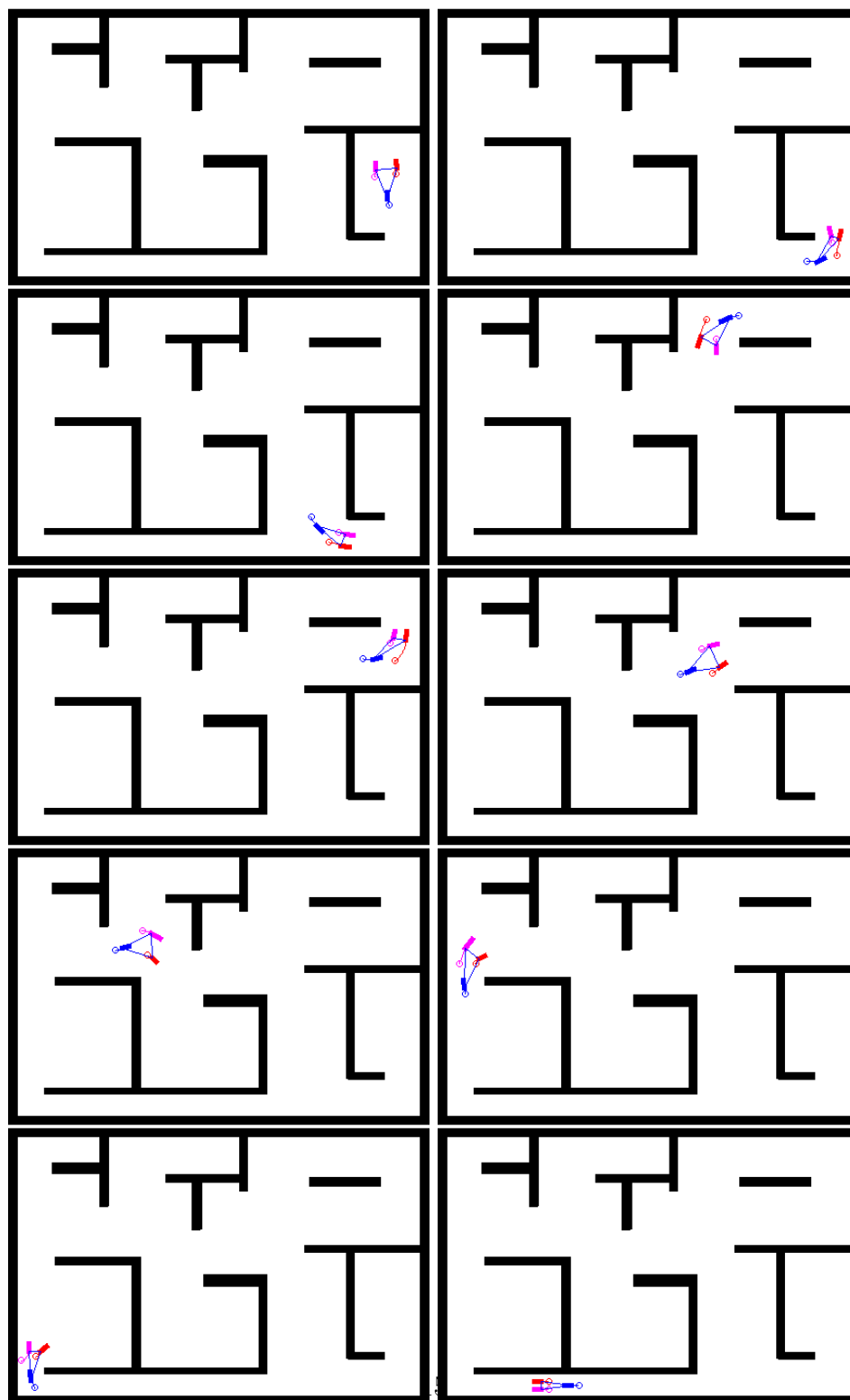
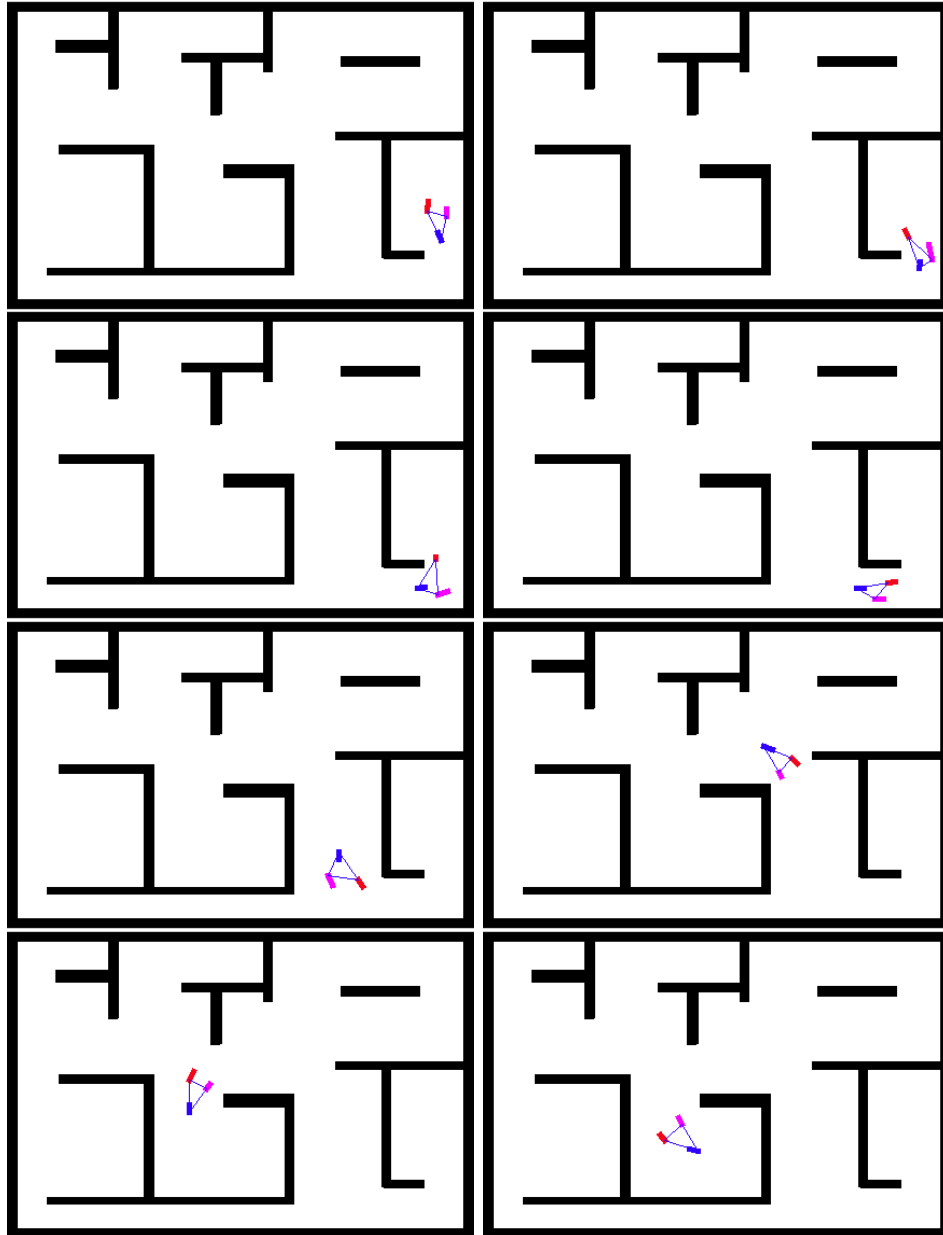Figure 6: Consecutive steps of the robots formation traveling around the maze using the main method.

Figure 7: Consecutive steps of the robots formation traveling around the maze using the method with springs.
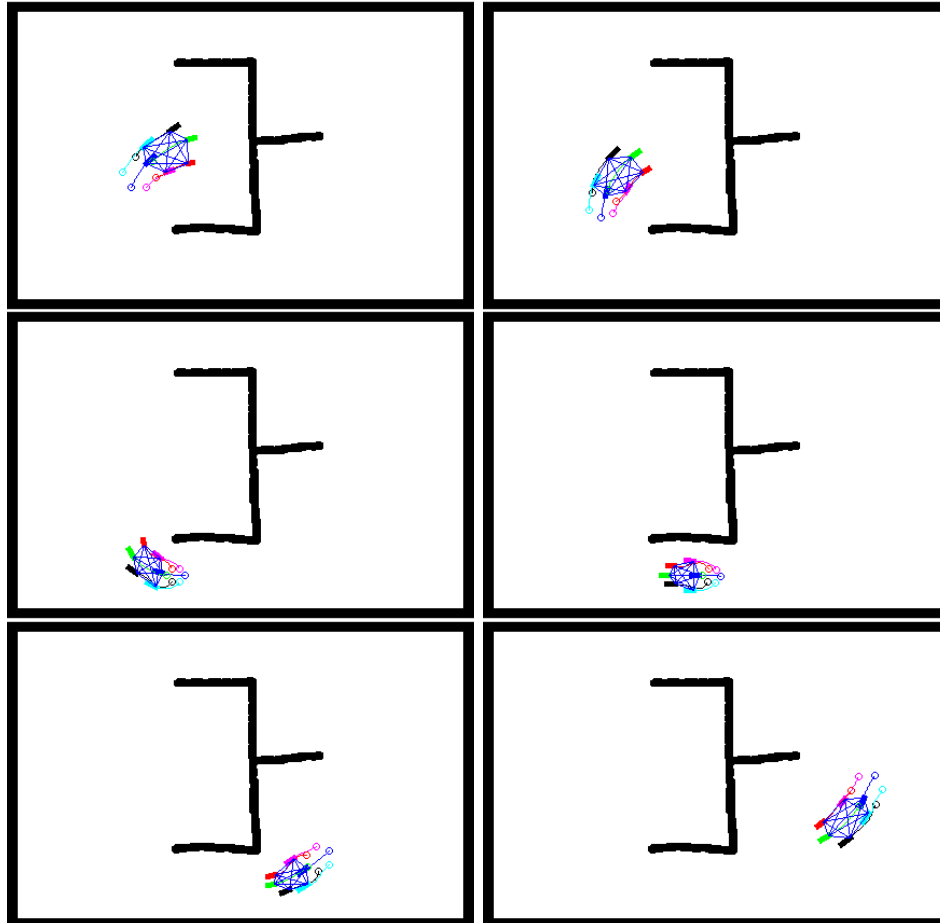
Figure 8: Consecutive steps of the robots formation traveling around the maze using the main method with formation of six robots protecting the center of the formation.

- The distance matrices D are calculated using the metrics matrix W for each robot, using as goal the partial goal of the previous step.

- The minimum distance path to the partial goal is found using the potential D.

- All the robots move a fixed number of steps on the corresponding path.

- The leader advances its path position a fixed number of steps.

3.2. Reduction of the operational cost

Although the FM method is very fast (the computation time is 0.2 sec for figures 6 to , where the map has a size of 628x420 cells), the proposed algorithm for robot formations has to calculate the FM wave propagatio for each follower robot at each cycle of the algorithm. For this reason some techniques should be used to make the algorithm faster.

Since the FM method can be considered as the continuous version of Dijkstra's algorithm, our goal is to turn it into an almost one-dimensional algorithm. To achieve this, the VFM method is applied in a tube around the trajectory calculated for the leader. Thus, the propagation of the FM wave across the map is calculated only the first time to find the trajectory for the leader; the other times (once per cycle and follower robot) it is calculated in a tube around this trajectory, which drastically reduces the computation time. The steps are:

1) Enlargement of the trajectory calculated for the leader to get a tube. To achieve this, this trajectory is dilated to get a tube, and the intersection between this tube and the map obtained from the environment (walls and obstacles) is used as the starting matrix $\mathbf{Wo}_i^t$ (see figure 10).

2) VFM-1st step. Using the map obtained in the previous step, a wave is propagated starting from the points representing the obstacles and walls. This is done with the Extended Voronoi Transform (also called Distance Transform in Computer Vision). The result is a potential map, which can be interpreted as a velocity map (or slowness map), as shown in figure 10c.

3) VFM-2nd step. Based on the previous slowness map, the FM method creates a new potential $\mathbf{D}_i^t$ that represents the propagation of the electromagnetic wave from the goal to the robot position.

A reduction of the computation time even more important is related to the matrix $\mathbf{D}_i^t$, as follows. As the vehicles are very close to the partial objectives,
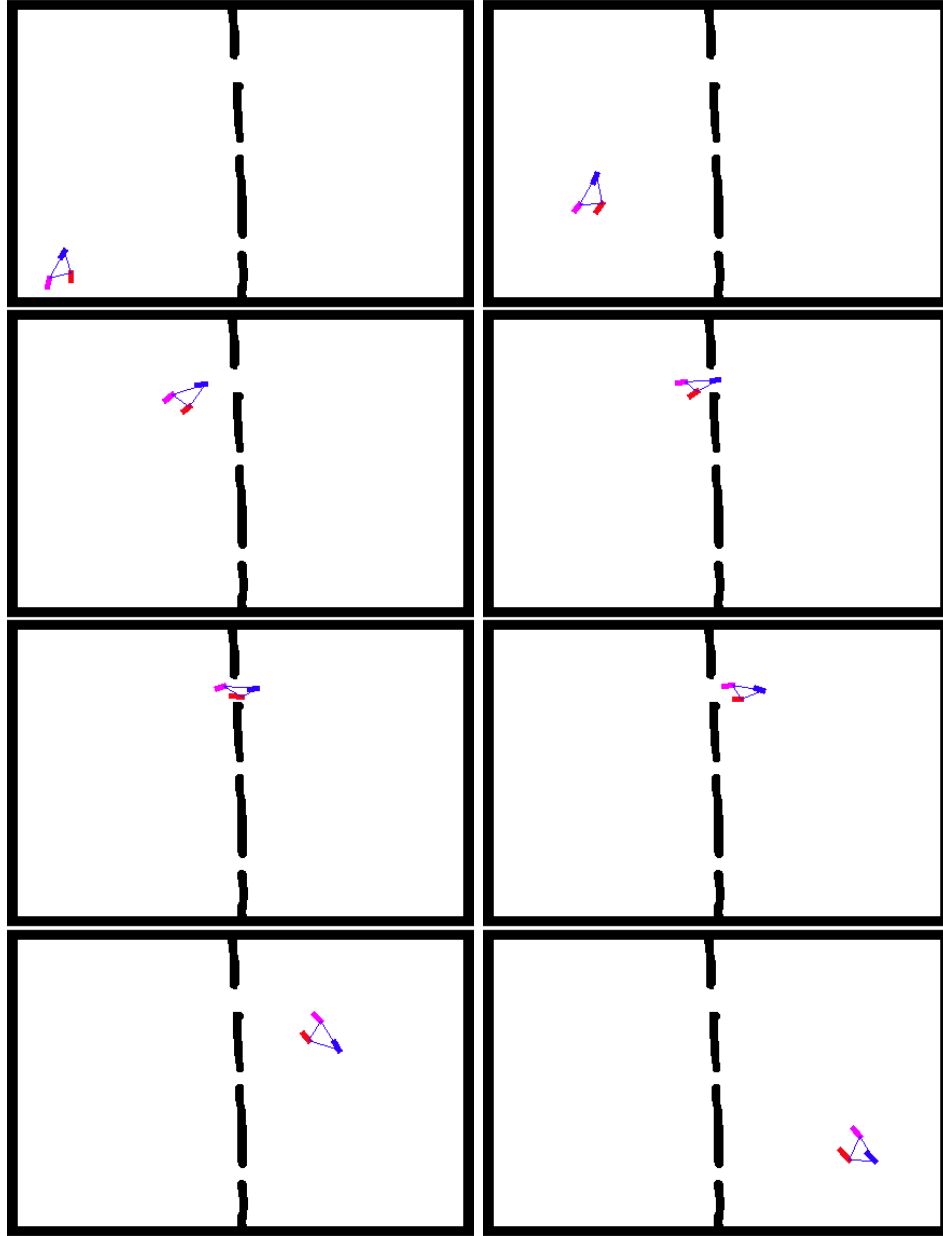
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Figure 9: Consecutive steps of the robots formation using the maximum energy configuration with springs. The formation does not use passages narrower than its maximum energy configuration.
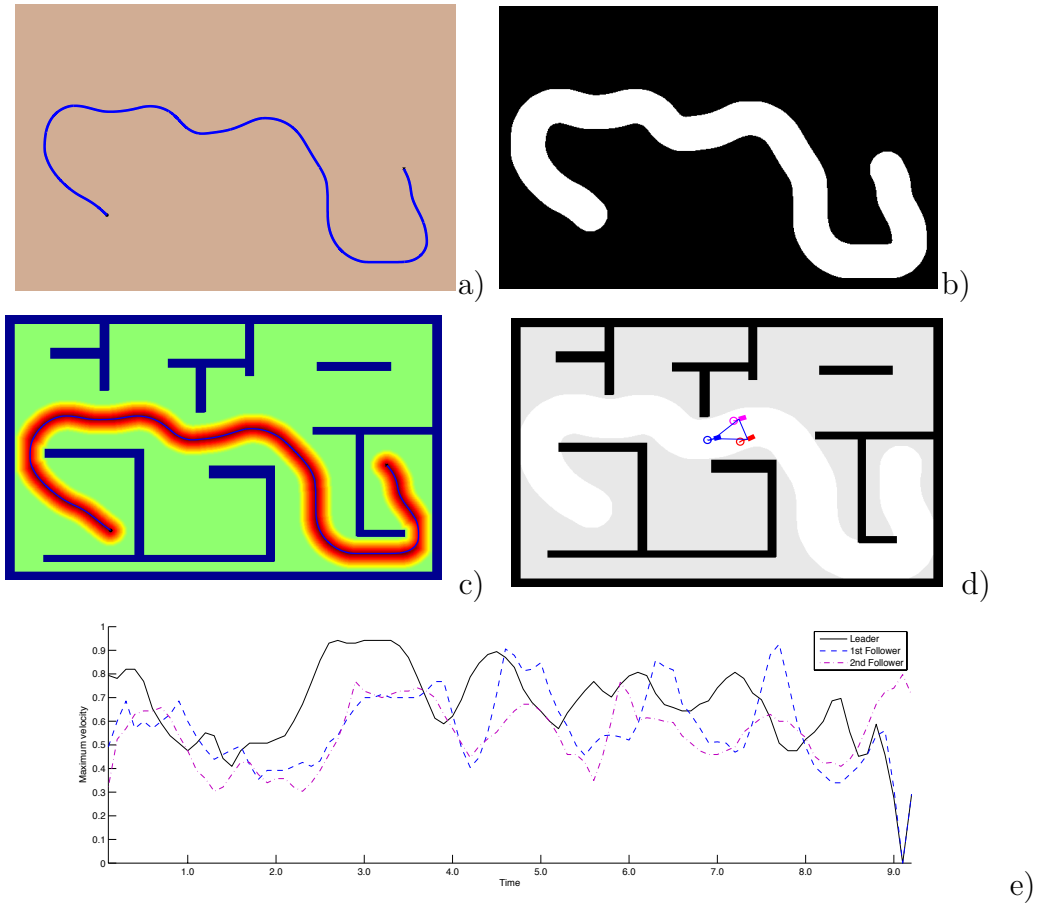
Figure 10: VFM calculated in a tube. a) Trajectory calculated for the leader; b) Intersection between the dilatation of the leader trajectory and the walls and obstacles; c) Extended Voronoi Transform of the tube; d) Robot Formation (rectangles) with the corresponding partial objectives (circles) and the partial trajectories of the followers calculated in the tube. e) Maximum possible velocities of the three robots.

the expansion of the wave over the whole map is not necessary, but just its expansion into the tube, from the partial goal point to the corresponding vehicle. With this change the computation time of the matrix $\mathbf{D}_i^t$, which is the most time consuming part of the algorithm, goes from 0.2 sec to 0.016 sec which implies an algorithm cycle of about 0.5 sec without parallelization (using a MacBook Pro platform at 3.06 GHz). The parallelized version of the algorithm has an algorithm cycle of about 0.3 sec and permits the use of many followers without increasing the computation time.

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4. Simulation Results

The algorithms of the previous sections have been tested using Matlab simulations. A program has been created in which the initial and goal points are given and the VFM method is used for the leader and the rest of the formation. The method uses a nominal desired formation that are the partial goals with a distance orthogonal to the leader trajectory proportional to the grey level of the actual position. The followers trajectories to the partial goals are calculated using the VFM method. The results are shown from figure 6 to figure 8.

The method has been designed for holonomic robots, but it is possible to apply the techniques that we used for non-holonomic robots in [18].

The method has been used giving a sequence of points, joining these points with lines to obtain the leader trajectory. Using this trajectory, the followers trajectories are calculated in the same way as in the main proposed method. In this case, the direction in the given points changes abruptly. It is important to study the behavior of the method in these points, and we can observe that it behaves very well and even smoothes the followers trajectories in comparison with the leader one, as shown in the figures.

The inter-vehicle desired distance has a maximum value in open areas and proportional to the refraction index in the rest. This way, in small corridors the followers can be near each other.

Figure 6 shows an example of a team composed by two vehicles following a moving leader using the main proposed method. The lines connecting each of the three vehicles represent the formation links between them. The circles represent the partial objectives that change at each step of the algorithm. The lines from vehicles to the circles are the partial paths calculated using VFM.

Figure 7 shows a similar case using the proposed method with springs of different stiffness. With this improvement the symmetry of the followers paths is broken and it is easier to pass trough small passages.

Figure 8 shows consecutive steps of the robots formation traveling around the maze using the main method, with a formation of six robots protecting the center of the formation. The EVT can be considered as a viscosity or the inverse of the velocity of the media. This way the method gives a maximum velocity in each point of the trajectory as shown in Figure 8e).

Figure 9 shows consecutive steps of the robots formation using the maximum energy configuration with springs. The formation does not use passages

narrower than its maximum energy configuration.

In the simulations, the algorithms always determine a safe path where the robots avoid obstacles, keep the formation geometry in open space, and deform it when necessary to handle the presence of obstacles. Simulation results show that the method does not have local minima, which naturally results from the method used (light propagation has no local minima) and is a distinctive property with respect to potential fields and other similar methods in the class of motion planning algorithms. Furthermore, the method naturally ensures trajectory tracking by all the formation robots, since it naturally induces the required velocities at each point, if one wants to reach the goal in minimum time, taking into account the constraints imposed by the presence of obstacles and the formation geometry.

5. Conclusions and Future Work

This paper presents a new methodology for the motion of a formation of holonomic robots. The formation is maintained calculating the trajectory of the formation leader. In each loop of the algorithm, using this trajectory, the partial robot objectives are calculated. These partial objectives maintain the formation and have a variable distance between them proportional to the light velocity in that point (inverse of the refraction index). Each robot has an environment map with the other robots as objects. This map is used to construct the refraction index map or metrics $P(\mathbf{x})$ using the FM method. Using this metrics, the distance function $D(\mathbf{x})$ representing the distance function measured with the metrics $P(\mathbf{x})$ is built with the FM method. The partial trajectory of each robot is calculated with the gradient method.

The general trajectories have a behavior like the light trajectory in a space with larger refraction index near the obstacles and walls and with an attraction force that tries to maintain the formation.

The proposed method is highly efficient from a computational point of view, since the Fast Marching complexity is $O(n)$ and the Extended Voronoi Transform complexity is $O(n)$, where n is the number of cells in the environment map.

The main contribution of the method is that it robustly achieves smooth and safe motion plans in real time that can be used at low control levels without an additional smoothing interpolation process. This allows the method to fuse collision avoidance and global planning in only one module, which can simplify the control architecture of the mobile robot, and without local

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minima, as in the case of the potential fields original method and some other algorithms in the same class.

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