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The Closed Primaries versus the Top-two Primary*

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Abstract

The top-two primary recently approved in states like Washington, California, and Alaska eliminates the closed party primaries and creates instead a single ballot in which the first and second place winners pass to the general election. We compare the electoral consequences of the top-two primary with those of the closed primaries. We present a model where each primary procedure induce a sequential game with three stages: candidate-entry stage, primary election, and general election. We analyze the equilibria of these games and show that the top-two primary contributes to political moderation and may increase the number of swing states.

Keywords: Voting system; Closed primaries; Open primaries; Top-two primary; Political moderation; Sequential voting.
JEL Classification Numbers: C72; D72.

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1 Introduction

In this paper, we provide a positive analysis of the consequences of applying different voting systems to select the nominees for general elections. We analyze and compare the electoral outcomes derived from the closed primaries with the electoral outcome derived from the top-two primary, with the first representing partisan primaries and the second representing non-partisan primaries.

The primary elections describe the process by which the electorate chooses its nominees (or leaders) for general elections. The origins of the primaries can be traced back to the Progressive Movement in the U.S., which intended to introduce more intraparty competition in the selection of candidates.[1] On the one hand, empirical evidence has shown that primaries have fostered competition, especially in those states that lacked of two-party competition (Key, 1958; Grau, 1981; Jewell and Olson, 1978). On the other hand, more than a century of primaries in U.S. politics has shown some of their faults. In this line, Ansolabehere et al (2010) and Hirano et al. (2010) highlight the decline of competition in U.S. primary elections. Among other reasons, their evidence shows that the rise in the value of incumbency has contributed to less competition in the primary elections.

Despite these long running negative effects, primary elections are of key interest as there is a growing number of political parties in Western democratic countries as well as in Latin American countries, with interests in incorporating such procedures to their governing constitutions. Kenig (2009) shows that the selection of party leaders has gone through a considerable shift during the last three decades and some of the political parties in Denmark, France, Finland, Greece, Italy, Israel, Japan, Norway and the U.K. have incorporated primary elections to select their leaderships.[2] Carey and Polga-Hecimovich (2006) show a similar trend for Latin American countries.

Primary elections can be classified as lying somewhere on a scale from open primaries to closed primaries. In an open primary, registered voters can vote in any party's primary regardless of their party affiliation (these are also called blanket primaries). In the closed primaries, only those voters that are officially registered members of that party are eligible to vote in the primary. In a semi-closed primary, unaffiliated voters can participate as well.[3]

Recently, several states in the U.S. have approved and incorporated an alternative open primary to their governing constitutions: the *top-two primary election*.[4] This is the primary approved by voters in 2004 for Washington State, in 2010 for California, and in 2011 for Alaska. Depending on the state, the

[1] The Progressive Movement represented by Robert La Follette, governor of Wisconsin from 1901 until 1906, established direct primary elections in which voters, instead of party officials, had the right to select their candidates. Prior to this, candidates had been selected by private caucuses and conventions rather than by a direct vote by electors (Hofstadter, 1955; Lovejoy, 1941; Merriam, 1909; Merriam and Overacker 1928; Ranney, 1975).

[2] Hazan (1997) analyzes the case of Israel; Wauters (2010) analyzes the Belgian case.

[3] Gerber and Morton (1998) describe the difference between closed and open primaries. Cain and Gerber Ed., (2002), also analyze these primaries.

[4] These primaries are also known as nonpartisan blanket primaries.

top-two primary applies to the State Senate, House of Representatives, State Legislature, and Governor among others. Louisiana has been using a similar system since 1975 and other states, such as Arizona, New York or Wisconsin, keep an open debate on the convenience of modifying their primaries by incorporating a similar top-two system.[5]

The top-two primary election eliminates the closed party primaries from the electoral process and creates a system where all voters (partisan or not) equally participate at every stage. In the top-two primary, all the candidates, whatever their affiliation (if any), are placed on the same ballot, and only the first and second place winners pass to the general election. Candidates have the option to add their party affiliation to their name on the ballot, or they may choose not to be identified by party. Among other cases, two members of the same party can move forward to the general election.

The approval of this alternative primary system has been surrounded by strong controversy. Lawsuits have been filed against the law approving the top-two primary in the states of Washington, California, and Alaska.[6] Supporters of the top-two primary elections have seen in this system the possibility to free their democracies from partisan gridlock. They argue that the system will give more choices to the electorate, and that it will result in more moderate politicians given that moderate candidates have more options to win votes from members of the other party. The proponent´s argument is as follows: "This new system will elect state officials who are less extreme on the right or left. In districts with heavy Democratic voter registration, for example, the two candidates who move on to contest the general election may both be Democrats. Republicans would be able to vote for the more moderate Democratic candidate in the run off, rather than having only a choice, for example, between a very liberal Democrat and a very conservative Republican. Thus, the more moderate Democratic candidate may win".[7] On the other hand, opponents argue that this system reduces election choices because the top two vote-getters may be of the same party resulting in only one party being represented on the ballot. All things considered, the top-two primaries have opened the debate on the roll of political parties and its influence in the election of nominees.

The purpose of this paper is to check, in a clear theoretical model, the main statement defended by top-two supporters. In particular, we want to provide an answer to the following puzzle: does the top-two primary select more moderate politicians than the closed primaries? In solving this question we describe a game-theory setting in which each party runs its separate primary election to select its nominee. We refer to this setting as the traditional primary system. We then compare the results of this traditional primary system with those derived

[5] The difference of the Louisiana primaries with respect to the top-two is that if a candidate wins a simple majority in the first round there is no second round. Other states such as Alabama, Arkansas, Georgia, Mississippi, South Carolina and Texas have closed party primaries in which a runoff between the top two is required when the candidates do not reach certain threshold (Bullock and Johnson, 1992; Engstrom and Engstrom, 2008).

[6] Concerning the lawsuit seeking to declare the law unconstitutional in Washington State, the U.S. Supreme Court declared finally that the top-two system is constitutional.

[7] Proponents of Proposition 14 on California´s June 2010 Ballot.

from the alternative scenario in which there is a top-two primary system. Two relevant features of our analysis are the endogenous entry of candidates and the strategic voting decisions of the electorate.[8]

When measuring party identification in the U.S., political scientists use the Likert Scale. According to this scale, Strong Democrats and Strong Republicans are located at the two extremes of the scale. In between the two extremes are all the other identifications with the following order: Weak Democrat, Lean Democrat, Independent, Lean Republican, Weak Republican. Thus, we refer to *moderate politicians* as those located closer to the middle positions in the proposed scale.

Outline of the model and the results

We consider four potential candidates whose ideologies are located on the real line. These candidates are labeled following this order: extreme democrat, moderate democrat, moderate republican, extreme republican, i.e., each party has two potential candidates, one extreme and another moderate.[9] Voters preferences over the four candidates are single-peaked with respect to the ideological location of the candidates. We consider six different profiles of preferences, labeled as strong, weak, and lean either democrat or republican. For example, a lean democrat has the moderate democrat as its top option and the moderate republican, the extreme democrat, the extreme republican as its subsequent options.[10] The single-peak preferences over the ideologies guarantee that there is a well-defined median voter which we assume, for simplicity, to be unique.

We describe the *traditional election system* as the one in which each party holds a separate closed primary. Those voters whose most preferred candidate is a democrat, vote in the democratic primary whereas those voters whose most preferred candidate is a republican, vote in the republican primary. Events unfold as follows: first, the candidates decide whether to run or not; second, each party simultaneously runs its primary; third, voters cast their ballot for one out of the two party-nominees.

We describe the *top-two election system* as the one in which there is a single primary. Every voter casts their ballot in the unique primary for one of the self-declared candidates. The first and second place winners move to the general election. Thus, events unfold as follows: first, the candidates decide whether to

[8] The endogenous entry is the key assumption in the citizen-candidate approach (see Osborne and Slivinsky, 1996; Besley and Coate, 1997). According to these authors, there is a first stage in which the citizens choose whether or not to run as candidates, and a second stage where voters elect one of these candidates. In contrast, we introduce an intermediate stage with the primary election.

[9] We analyze the two cases in which the moderate candidate identifies with either a weak partisan or a lean partisan.

[10] We omit the description of those voters identified as independent. In fact, according to opinion polls, a substantial fraction of these voters do not respond and if they do, they split their vote between the two candidates (http://www.electionstudies.org/nesguide/2ndtable/t9a_1_1.htm). They do not participate in the closed primaries (as they lack any party-affiliation) and we omit their relevance in the top-two primaries.

run or not; second, the single primary takes place; third, voters cast their ballot for one out of the two nominees.

Both the traditional election system and the top-two system can be described as a sequential game form with three stages. We solve each of these games according to the subgame perfect Nash equilibrium concept. Besides, we require that, at each stage, the players do not play weakly dominated strategies given the equilibrium continuation strategies. This is a minimal requirement that refines the equilibrium concept and reduces the number of equilibria (see Bag et al., 2009).

For each of the two models we describe the set of candidates running in the primaries and the candidate winning in equilibrium in terms of the possible locations of the median voter (Theorems 1 and 2). When comparing the results, we conclude that the top-two election system may contribute to political moderation. In general, any equilibrium winner under the traditional election system is also an equilibrium winner under the top-two election system. There is only one exception to this rule: if the overall median voter is weak partisan but the median voter within his/her party is strong (and some additional mild assumptions hold), then the equilibrium winner according to the traditional election system is an extreme candidate, while the equilibrium winner according to the top-two election system is a moderate one. On the other hand, not all equilibrium winners under the top-two election system are also equilibrium winners under the traditional election system. In particular, if the overall median voter is strong partisan, then the equilibrium winner under the traditional election system is an extreme candidate, while under the top-two election system, there are also equilibria where a moderate candidate wins.

The top-two election system also provides certain chances of winning to candidates whose ideology differs from that of the median voter. Thus, if the median voter is lean partisan (and some additional conditions hold), then the moderate candidate whose ideology differs from that of the median voter can win in the top two system (but not in the traditional election system). Therefore, political parties that dominate in safe states could be negatively affected by the top-two primary system.

We also study the case in which there is a cost of running for election (Theorems 3 and 4). The results here also support the idea that the top-two election system contributes to political moderation. In particular, it is still the case that when the overall median voter is weak partisan and the median voter within his/her party is strong, then the winner under the traditional system is extremist, while the winner under the top-two system is moderate. The most controversial point when there is a cost of running is that, if the overall median voter is lean partisan and the median voter within his/her party is strong, the traditional election system has no equilibrium. Nevertheless, we can interpret the absence of equilibrium as indicating that all candidates (in particular, extremist ones) have a positive probability of winning. Since the equilibrium winner under the top-two election system in this case is a moderate candidate, this result also supports the idea that the top-two election system elects less extreme candidates.

Related literature

Several authors analyze, from a theoretical perspective, the benefits or costs associated with adopting primary elections. Adams and Merrill (2008) develop a two-stage election model in which candidates with uncertain campaigning skills strategically locate their platforms.[11] The analyzed closed primaries are proved to benefit both parties, the strong and the weak. The former benefits from the strategic motivation to locate closer to the center of the general election, and the later benefits from the selection of high quality candidates. Serra (2011) and Hortala-Vallve and Muelle (2012) focus on the party elites' decision on whether or not to hold a primary. The former author shows that primaries increase the valence of the nominee at the expenses of an extra cost of moving policy position. The later authors highlight that primaries can act as a mechanism that prevents political parties from splitting into more homogenous groups. Hirano et al. (2010) show that the primary election systems do not appear to generate polarization of the political parties, in contrast to widespread arguments defending the opposite. Snyder and Ting (2011) show from a combined empirical and theoretical perspective that, on the one hand, primaries raise the expected quality of a party's candidate and, on the other hand, primaries hurt the ex-ante preferred party in a competitive electorate by increasing the chances of revealing the opposing party's candidate as superior.

Closely related to our motivation, we know of two other contributions that compare different candidate selection procedures in terms of the induced electoral outcome. Gerber and Morton (1998) show, according to evidence based on U.S. primary elections, that representatives from closed primaries take policy positions that are furthest from their district's estimated median voters, whereas semi-closed primaries select even more moderate representatives than open primaries. Jackson et al. (2007) develop a two-stage model with a first nomination stage and a second general election stage. They analyze three different scenarios according to the procedure to elect nominees for general elections: (i) nomination by party leaders, (ii) nomination by the members of the parties, and (iii) nomination by campaign spending. In their second scenario, nominees satisfy certain "group stability" criteria by which there cannot be another nominee preferred by a majority of the party given the other party nomination.[12] Similarly to Gerber and Morton, they argue that more open selection induces more centrist candidates.[13] In contrast to our analysis, they do not propose a concrete primary election procedure and their equilibrium concept does not account for an endogenous entry of candidates.

The rest of the paper is organized as follows. Section 2 presents the model describing a common setting for the analysis of both election systems. Section

[11] In contrast to our model, we account for candidates with fixed positions with endogenous entry decision.

[12] This is a desirable normative property that we think deserves more attention in terms of the currently used primary procedures.

[13] In particular, they show that more extreme outcomes can emerge from spending competition than from nominations by votes or by party leaders, and that non-median outcomes can result from any of these processes.

3 analyzes the equilibria according to the traditional election system. Section 4 analyzes the equilibria according to the top-two election system. Section 5 studies the case in which there is a cost of running. Section 6 compares the results of the previous sections. Section 7 concludes. All proofs are in the Appendix.

2 The model

Let $\mathcal{C} = \{D^+, D^-, R^+, R^-\}$ be a group of candidates running to become a representative in the legislature. We use the letters D and R to refer to the **democratic** and **republican** candidates. The superscripts $+$ and $-$ mean **extremist** and **moderate**. For example, D^+ refers to the extreme democratic candidate and R^- does to the moderate republican one. General elements of $\mathcal{C}$ are denoted by x, y, etc. Each $x \in \mathcal{C}$ is identified with a fixed position in the interval $[0, 1]$ as in Figure 1, so that $\mathcal{C}$ is an ordered set with $D^+ < D^- < R^- < R^+$.

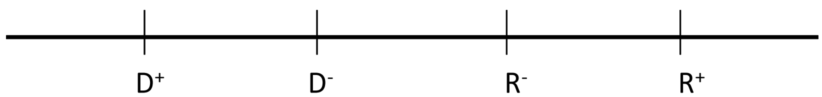

Figure 1 Position of the candidates once they have decided to run.

Let $\mathcal{V} = \{1, \ldots, v\}$ be the set of voters that must choose one of the candidates in $\mathcal{C}$. General elements of $\mathcal{V}$ are denoted by i, j, etc. Each voter $i \in \mathcal{V}$ has a (strict) single-peaked preference relation over the set of candidates, $\succ_i$: there is one candidate, called peak and denoted by $p(\succ_i)$, such that, for all x, $y \in \mathcal{C}$, if $y < x < p(\succ_i)$ or $p(\succ_i) < x < y$, then $x \succ_i y$. We call **democratic partisans** the voters whose peaks are a democratic candidate and **republican partisans** those voters whose peaks are a republican candidate. We suppose that democratic partisans always prefer the extreme democratic candidate over the extreme republican candidate, and republican partisans always prefer the extreme republican candidate over the extreme democratic candidate.[14] Then, the admissible preferences for each voter i over $\mathcal{C}$ are those represented in Table 1 (higher candidates in the table are preferred to lower candidates).

$\succ_{D^+}$	$\succ^1_{D^-}$	$\succ^2_{D^-}$	$\succ^2_{R^-}$	$\succ^1_{R^-}$	$\succ_{R^+}$
D^+	D^-	D^-	R^-	R^-	R^+
D^-	D^+	R^-	D^-	R^+	R^-
R^-	R^-	D^+	R^+	D^-	D^-
R^+	R^+	R^+	D^+	D^+	D^+

Table 1 Admissible preferences for the voters.

[14] Thus, the single-peaked preference relations $\succ^*_{D^-}$ and $\succ^*_{R^-}$ such that $D^- \succ^*_{D^-} R^- \succ^*_{D^-} R^+ \succ^*_{D^-} D^+$ and $R^- \succ^*_{R^-} D^- \succ^*_{R^-} D^+ \succ^*_{R^-} R^+$ are not admissible. This is a simplifying assumption that can be interpreted as a consistency requirement over the preferences.

Let $\mathbb{P} = \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$ be the set of admissible preference relations and let $\succ = (\succ_i)_{i \in \mathcal{V}} \in \mathbb{P}^v$ be a preference profile for voters in $\mathcal{V}$. Let $\mathcal{V}_D$ and $\mathcal{V}_R$ be the sets of democratic and republican partisans respectively; i.e., $\mathcal{V}_D = \{i \in \mathcal{V} : \succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}\}$ and $\mathcal{V}_R = \{i \in \mathcal{V} : \succ_i \in \{\succ_{R^+}, \succ^1_{R^-}, \succ^2_{R^-}\}\}$. Within each group, each type of voter is labeled as **strong** (when preferences are $\succ_{D^+}$ or $\succ_{R^+}$), **weak** (when preferences are $\succ^1_{D^-}$ or $\succ^1_{R^-}$), and **lean** (when preferences are $\succ^2_{D^-}$ or $\succ^2_{R^-}$).

Let $\succ_{D^+} < \succ^1_{D^-} < \succ^2_{D^-} < \succ^2_{R^-} < \succ^1_{R^-} < \succ_{R^+}$ be the order for the elements of $\mathbb{P}$. Given this order, and for each $\succ \in \mathbb{P}^v$, let $\succ^m$ be the median of the elements of $\mathbb{P}$ at $\succ$; *i.e.*, $\succ^m \in \mathbb{P}$ is such that $\#\{i \in \mathcal{V} : \succ_i \leq \succ^m\} \geq \frac{v}{2}$ and $\#\{i \in \mathcal{V} : \succ_i \geq \succ^m\} \geq \frac{v}{2}$. Suppose, for the sake of simplicity, that $\succ^m$ is unique. We call $\succ^m$ the **median voter's preferences**. Notice that, for all $x, y \in \mathcal{C}$ such that $x \succ^m y$, either (1) $x \succ_i y$ for all $i \in \mathcal{V}$ such that $\succ_i \leq \succ^m$, or (2) $x \succ_i y$ for all $i \in \mathcal{V}$ such that $\succ_i \geq \succ^m$. Hence, when comparing any two candidates x and y, if the median voter prefers x to y, then a majority of voters also prefer x to y.[15]

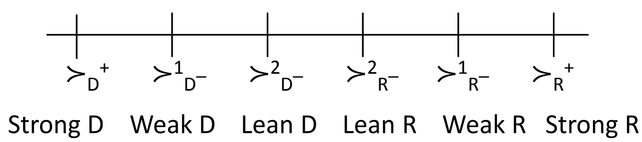

Figure 2 Order of the voters' preferences.

For each $\succ \in \mathbb{P}^v$, let $\succ^m_D$ be the median of the elements of the set $\{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}$; *i.e.*, $\succ^m_D \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}$ is such that $\#\{i \in \mathcal{V}_D : \succ_i \leq \succ^m_D\} \geq \frac{v_D}{2}$ and $\#\{i \in \mathcal{V}_D : \succ_i \geq \succ^m_D\} \geq \frac{v_D}{2}$. We call $\succ^m_D$ the **median democratic partisan's preferences**. The **median republican partisan's preferences**, $\succ^m_R \in \{\succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$, are defined in a similar way. Suppose, for simplicity, that $\succ^m_D$ and $\succ^m_R$ are unique. Abusing notation, we write $\succ^m_D = \succ_{D^-}$ and $\succ^m_R = \succ_{R^-}$ to denote $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$ and $\succ^m_R \in \{\succ^1_{R^-}, \succ^2_{R^-}\}$, respectively. Note that there exists a relationship between the median voter and the median partisans: (i) if $\succ^m = \succ_{D^+}$ then $\succ^m_D = \succ_{D^+}$ (if $\succ^m = \succ_{R^+}$ then $\succ^m_R = \succ_{R^+}$), and (ii) if $\succ^m = \succ^1_{D^-}$ then $\succ^m_D \in \{\succ_{D^+}, \succ^1_{D^-}\}$ (if $\succ^m = \succ^1_{R^-}$ then $\succ^m_R \in \{\succ_{R^+}, \succ^1_{R^-}\}$).

Each candidate $x \in \mathcal{C}$ also has a (strict) single-peaked preference relation over $\mathcal{C}$, $\succ_x \in \mathbb{P}$, such that $p(\succ_x) = x$ (i.e., the peak of each candidate is his/her

[15] For this result to be true, it is crucial that the median of the elements of $\mathbb{P}$ is defined with respect to the order $\succ_{D^+} < \succ^1_{D^-} < \succ^2_{D^-} < \succ^2_{R^-} < \succ^1_{R^-} < \succ_{R^+}$. Suppose, for instance, that we define the median with respect to the order $\succ_{D^+} < \succ^1_{D^-} < \succ^2_{D^-} < \succ^1_{R^-} < \succ^2_{R^-} < \succ_{R^+}$. Suppose that $\succ^m = \succ^1_{R^-}$ and compare candidates R^+ and D^-. Although $R^+ \succ^m D^-$, we cannot ensure that a majority of voters also prefer R^+ to D^-. To see this note that neither $R^+ \succ_i D^-$ for all i such that $\succ_i \leq \succ^1_{D^-}$ (since $D^- \succ^2_{D^-} R^+$), nor $R^+ \succ_i D^-$ for all i such that $\succ_i \geq \succ^1_{D^-}$ (since $D^- \succ^2_{R^-} R^+$). A similar problem occurs if $\succ^m = \succ^2_{R^-}$. Note also that, if the preference relations $\succ^*_{D^-}$ and $\succ^*_{R^-}$ defined in Footnote 14 were admissible, there would not be any order for the elements of $\mathbb{P}$ for which the median voter predicts the winner of a majoritarian election.

self). Thus, the preference relations of candidates D^+ and R^+ are the preferences $\succ_{D^+}$ and $\succ_{R^+}$ defined in Table 1, respectively. Similarly, $\succ^1_{D^-}$ and $\succ^2_{D^-}$ are admissible preference relations for candidate D^-, while $\succ^1_{R^-}$ and $\succ^2_{R^-}$ are admissible preference relations for candidate R^-.

In the election systems described below, candidates decide whether to run or not. We denote $\emptyset$ the situation where no candidate is running and assume that (i) for each $i \in \mathcal{V}$, $p(\succ_i) \succ_i \emptyset$, and (ii) for each $x \in \mathcal{C}$, $x \succ_x \emptyset$.

Traditional election system

The traditional election system consists of three stages. In the first stage, the four candidates simultaneously decide whether to run or not. In the second stage, the republican and the democratic parties hold their conventions. In the republican (democratic) party convention, only republican (democratic) partisans can vote. In the third stage, all voters elect one winner between the republican and democratic nominees.

Next, we formally define the sequential game induced by the traditional election system. At the first stage, each candidate $x \in \mathcal{C}$ has to choose between running (Y) or not (N). Let $S^1_x = \{Y, N\}$ denote the strategy space of candidate x. We call $s^1_x \in S^1_x$ a strategy of candidate x and $s^1 \in S^1 = \times_{x \in \mathcal{C}} S^1_x$ a strategy profile played by the four candidates.[16]

Let $2^{\mathcal{C}}$ be the set of all subsets of $\mathcal{C}$. Let $\mathcal{C}^r \in 2^{\mathcal{C}}$ be the set of candidates who are running. Let $\mathcal{C}^r_D = \{D^+, D^-\} \cap \mathcal{C}^r$ and $\mathcal{C}^r_R \subseteq \{R^+, R^-\} \cap \mathcal{C}^r$ be the sets of democratic and republican candidates who are running.

Each voter $i \in \mathcal{V}$ has to cast their vote at the second and third stages and, therefore, their strategy has two components, $s_i = (s^2_i, s^3_i)$. At the second stage, i knows $\mathcal{C}^r$. For each democratic partisan voter, $i \in \mathcal{V}_D$, $s^2_i : 2^{\mathcal{C}} \longrightarrow \{D^+, D^-, \emptyset\}$ is a mapping such that, for each $\mathcal{C}^r \in 2^{\mathcal{C}}$, $s^2_i(\mathcal{C}^r) \in \mathcal{C}^r_D$ is the candidate for whom i will vote in the primaries of the democratic party at the second stage if the candidates who decided to run at the first stage are $\mathcal{C}^r$. Let S^2_D denote the set of all these mappings. For each republican partisan voter $i \in \mathcal{V}_R$, we define in a similar way the mapping $s^2_i : 2^{\mathcal{C}} \longrightarrow \{R^+, R^-, \emptyset\}$ and the set S^2_R. Let $S^2 = \times_{i \in \mathcal{V}} S^2_i$ (where $S^2_i = S^2_D$ if $i \in \mathcal{V}_D$ and $S^2_i = S^2_R$ if $i \in \mathcal{V}_R$), and let $s^2 = (s^2_i)_{i \in \mathcal{V}} \in S^2$.

Let $x^n_D \in \mathcal{C}^r_R$ and $x^n_R \in \mathcal{C}^r_R$ be the democratic and republican nominees, *i.e.*, the candidates who get the most votes in the democratic and republican primaries, respectively. Suppose that, if there is a tie in the primaries, any of the two candidates is equally likely to be the nominee.[17] At the third stage, each voter $i \in \mathcal{V}$ knows both x^n_D and x^n_R. Let $s^3_i : \{D^+, D^-, \emptyset\} \times \{R^+, R^-, \emptyset\} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ be a mapping such that, for each pair of nominees x^n_D and x^n_R, $s^3_i(x^n_D, x^n_R) \in \{x^n_D, x^n_R\}$ is the candidate for whom i will vote in the general

[16] Throughout the paper, only pure strategies are considered.

[17] For instance, if R^+ and R^- are involved in a tie in the republican primaries and D^- wins the democratic primaries, then the confrontations R^+ versus D^- and R^- versus D^- are equally likely in the third stage.

election.[18] Let S_i^3 denote the set of all these mappings, $S^3 = \times_{i \in \mathcal{V}} S_i^3$, and $s^3 = (s_i^3)_{i \in \mathcal{V}} \in S^3$. For each $s^1 \in S^1$, $s^2 \in S^2$ and $s^3 \in S^3$, let $x(s^1, s^2, s^3) \in \{D^+, D^-, R^+, R^-, \emptyset\}$ be the candidate who wins the general election; *i.e.*, the candidate who collects the most votes at the third stage. If there is a tie in the general election any of the two candidates is equally likely win.

Top-two election system

The top-two election system also consists of three stages. In the first stage, the four candidates simultaneously decide whether to run or not. The second stage is the so-called open primary. In this stage all voters cast their votes for one of the candidates who decided to run. The third stage is the general election between the two candidates who got the most votes in the second stage.

Analogously to the traditional election system, we model the top-two election system as a sequential game. The first stage is identical to that of the traditional election system. For each $x \in \mathcal{C}$, $T_x^1 = \{Y, N\}$ denotes the strategy space of candidate x, $t_x^1 \in T_x^1$ is the strategy of candidate x, and $t^1 \in T^1 = \times_{x \in \mathcal{C}} T_x^1$ is a candidates' strategy profile.

At the second stage, each voter $i \in \mathcal{V}$ has to vote for one of the candidates who are running. A strategy at this stage for i is a mapping $t_i^2 : 2^{\mathcal{C}} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ where, for each $\mathcal{C}^r \in 2^{\mathcal{C}}$, $t_i^2(\mathcal{C}^r) \in \mathcal{C}^r$ is the candidate for whom i will vote in the open primary if the running candidates are $\mathcal{C}^r$. The strategy space for i at the second stage, T_i^2, is the set of all these mappings. Let $t^2 \in T^2 = \times_{i \in \mathcal{V}} T_i^2$ be a profile of strategies for the voters at the second stage.

Let $x_1^n, x_2^n \in \{D^+, D^-, R^+, R^-, \emptyset\}$ be the two candidates who get the most votes at the second stage. We call these candidates the nominees. We assume that, if there is a tie, any potential pair of candidates is equally likely to move to the third stage.[19] At the third stage, each voter $i \in \mathcal{V}$ knows who the nominees are. A strategy at the third stage for i is a mapping $t_i^3 : \{D^+, D^-, R^+, R^-, \emptyset\} \times \{D^+, D^-, R^+, R^-, \emptyset\} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ such that, for each pair $x_1^n, x_2^n \in \{D^+, D^-, R^+, R^-, \emptyset\}$, $t_i^3(x_1^n, x_2^n) \in \{x_1^n, x_2^n\}$ is the candidate for whom i votes in the general election.[20] Let T_i^3 be the set of all these mappings, $T^3 = \times_{i \in \mathcal{V}} T_i^3$, and $t^3 = (t_i^3)_{i \in \mathcal{V}} \in T^3$. For each $t^1 \in T^1$, $t^2 \in T^2$, and $t^3 \in T^3$, let $x(t^1, t^2, t^3) \in \{D^+, D^-, R^+, R^-, \emptyset\}$ be the candidate who gets the most votes at the third stage. If there is a tie, any of the two candidates is equally likely win.

Equilibrium concept

Since the voting games that we are considering have a dynamic structure, we will consider subgame perfect Nash equilibria. In addition, as is common in the literature on voting, we need to eliminate choices that are weakly dominated.

[18] In particular, $s_i^3(x_D^n, x_R^n) = \emptyset$ if and only if $x_D^n = \emptyset$ and $x_R^n = \emptyset$.

[19] For instance, if R^+ is the candidate who gets more votes and D^+ and D^- are tied for second place, then the confrontations R^+ versus D^+ and R^+ versus D^- are equally likely in the third stage. Similarly, if R^+, D^+, and D^- are tied for first place then the confrontations R^+ versus D^+, R^+ versus D^-, and D^+ versus D^- are equally likely in the third stage.

[20] In particular, $t_i^3(x_1^n, x_2^n) = \emptyset$ if and only if $x_1^n = x_2^n = \emptyset$.

Otherwise there is a large number of trivial equilibria in which each voter's choice is immaterial. For this reason, following Bag et al. (2009), we require that, at each stage of the game, the strategies of each player are not weakly dominated given the equilibrium continuation strategies in future stages. Note that this equilibrium notion is stronger than the undominated subgame perfect equilibrium (a weakly undominated strategy may be weakly dominated if we consider that in the continuation game the players play equilibrium strategies).[21]

Consider the traditional election system. For any $s^1 \in S^1$ and $x \in \mathcal{C}$, let $s^1_{-x} \equiv (s^1_y)_{y \in \mathcal{C} \setminus \{x\}}$ be the list of strategies of the profile s^1 for all candidates except x. Denote the set of such s^1_{-x} by S^1_{-x}. Similarly, for any $s^k \in S^k$ ($k \in \{2, 3\}$) and $i \in \mathcal{V}$, let s^k_{-i} be the list $(s^k_j)_{j \in \mathcal{V} \setminus \{i\}}$ and let S^k_{-i} denote the set of such s^k_{-i}. Any equilibrium profile of strategies $s^* = (s^{*1}, s^{*2}, s^{*3}) \in S^1 \times S^2 \times S^3$ must have the following properties. In any subgame at the third stage, s^{*3} must be a weakly undominated Nash equilibrium in the subgame. In any subgame starting at the second stage, the voters' strategies s^{*2} must be an undominated Nash equilibrium in the subgame given that the voters play according to s^{*3} in the continuation game. At the first stage, the candidates' strategies s^{*1} must be an undominated Nash equilibrium given that the voters play according to s^{*2} and s^{*3} in the continuation game.

Definition: *A profile of strategies $s^* = (s^{*1}, s^{*2}, s^{*3}) \in S^1 \times S^2 \times S^3$ is an equilibrium of the traditional election system if:*

(a) Subgame perfection: in any subgame, s^* is a Nash equilibrium.

(b) Non weak domination in the continuation strategy in future stages:

(b.1) for each $x \in \mathcal{C}$, there is no $s^1_x \in S^1_x$ such that:

$x((s^1_x, s^1_{-x}), s^{*2}, s^{*3}) \succeq_x x((s^{*1}_x, s^1_{-x}), s^{*2}, s^{*3})$ *for all* $s^1_{-x} \in S^1_{-x}$, *and*
$x((s^1_x, s^1_{-x}), s^{*2}, s^{*3}) \succ_x x((s^{*1}_x, s^1_{-x}), s^{*2}, s^{*3})$ *for some* $s^1_{-x} \in S^1_{-x}$.

(b.2) for each $s^1 \in S^1$ and $i \in \mathcal{V}$, there is no $s^2_i \in S^2_i$ such that:

$x(s^1, (s^2_i, s^2_{-i}), s^{*3}) \succeq_i x(s^1, (s^{*2}_i, s^2_{-i}), s^{*3})$ *for all* $s^2_{-i} \in S^2_{-i}$, *and*
$x(s^1, (s^2_i, s^2_{-i}), s^{*3}) \succ_i x(s^1, (s^{*2}_i, s^2_{-i}), s^{*3})$ *for some* $s^2_{-i} \in S^2_{-i}$.

(b.3) for each $s^1 \in S^1$, $s^2 \in S^2$, and $i \in \mathcal{V}$, there is no $s^3_i \in S^3_i$ such that:

$x(s^1, s^2, (s^3_i, s^3_{-i})) \succeq_i x(s^1, s^2, (s^{*3}_i, s^3_{-i}))$ *for all* $s^3_{-i} \in S^3_{-i}$, *and*
$x(s^1, s^2, (s^3_i, s^3_{-i})) \succ_i x(s^1, s^2, (s^{*3}_i, s^3_{-i}))$ *for some* $s^3_{-i} \in S^3_{-i}$.

The definition of an equilibrium of the top-two election system, $t^* = (t^{*1}, t^{*2}, t^{*3}) \in T^1 \times T^2 \times T^3$, is analogous and we omit it in the interest of space.

3 Equilibria of the traditional election system

In this section, we make a detailed analysis of the equilibria of the sequential game induced by the traditional system. We are particularly interested in figur-

[21] Not surprisingly, if we simply impose undominated subgame perfection, any candidate may win the election. That is, for each candidate, there is an undominated subgame perfect Nash equilibrium such that this candidate results as the winner of the election process.

ing out who will win the general election in equilibrium. Our analysis, however, gives us additional information about the profiles of equilibrium strategies.

Third stage of the traditional election system

We start analyzing the last stage of the sequential game. At this stage, the democratic (x_D^n) and republican (x_R^n) nominees compete in the general election. There are nine different types of subgames beginning at the third stage depending on who the nominees are: $(x_D^n, x_R^n) \in \{(\emptyset,\emptyset),(D^+,\emptyset),\ (D^-,\emptyset),(\emptyset,R^+),(\emptyset,R^-),$ $(D^+,\ R^+),(D^+,R^-),(D^-,R^+),\ (D^-,\ R^-)\}$. Any profile of equilibrium strategies is such that, in each of these subgames, the median voter's favorite candidate between x_D^n and x_R^n wins the election. For instance, in any subgame starting at the third stage where the two nominees are D^+ and R^-, if $\succ^m = \succ^2_{D^-}$, then any profile of equilibrium strategies is such that R^- wins the general election, since the median voter prefers R^- to D^+, and so do a majority of voters.

Proposition 1 *Any profile of equilibrium strategies of the traditional election system is such that the candidates winning the general election in the subgames beginning at the third stage are as described in Table 2.*

Nominees		Median voter					
x_D^n	x_R^n	$\succ_{D^+}$	$\succ^1_{D^-}$	$\succ^2_{D^-}$	$\succ^2_{R^-}$	$\succ^1_{R^-}$	$\succ_{R^+}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	$\emptyset$	D^+	D^+	D^+	D^+	D^+	D^+
D^-	$\emptyset$	D^-	D^-	D^-	D^-	D^-	D^-
$\emptyset$	R^+	R^+	R^+	R^+	R^+	R^+	R^+
$\emptyset$	R^-	R^-	R^-	R^-	R^-	R^-	R^-
D^+	R^+	D^+	D^+	D^+	R^+	R^+	R^+
D^+	R^-	D^+	D^+	R^-	R^-	R^-	R^-
D^-	R^+	D^-	D^-	D^-	D^-	R^+	R^+
D^-	R^-	D^-	D^-	D^-	R^-	R^-	R^-

Table 2 Results of Proposition 1: winner in equilibrium in the subgames beginning at the third stage of the traditional election system.

Second stage of the traditional election system

At the second stage, the parties simultaneously hold their conventions to pick their candidates for the general election. In the republican (democratic) party convention, each republican (democratic) partisan has to vote for one of the republican (democratic) candidates who decided to run.

There are sixteen different types of subgames beginning at the second stage depending on who the running candidates are. Our next proposition shows who wins the general election in equilibrium in each of these subgames depending on the median voter's preferences, the median democratic partisan's preferences, and the median republican partisan's preferences. If there is at most one candidate from each party, then there is no decision to be made at the second stage and the favorite between them for $\succ^m$ wins the general election. If only the two democratic (republican) candidates are running, then the favorite between them for $\succ^m_D$ ($\succ^m_R$) wins the democratic (republican) primary and the general election. If the two democratic (republican) candidates and only one republican (democratic) candidate are running, then the favorite for $\succ^m_D$ ($\succ^m_R$) between the candidates who would win the two potential confrontations in the third stage wins the general election. If the four candidates are running, then the favorite candidate for the partisan median voter of the party of the median voter wins the general election (for example, if $\succ^m = \succ^1_{D^-}$ and $\succ^m_D = \succ_{D^+}$, then the favorite candidate for $\succ_{D^+}$ wins the general election). Furthermore, if $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$, there are also equilibria in which R^- wins (the reason is that voting for D^+ is not weakly dominated at the second stage for voters of type $\succ_{D^+}$ given the continuation equilibrium strategies and, therefore, it can be the case that D^+ and R^- are the candidates moving to the third stage in equilibrium). Similarly, if $\succ^m = \succ^2_{R^-}$ and $\succ^m_R = \succ_{R^+}$, there are also equilibria in which D^- wins.

Proposition 2 *Any profile of equilibrium strategies of the traditional election system is such that the candidates winning the general election in the subgames beginning at the second stage are as described in Table 3.*[22]

Remark 1 *In the subgames beginning at the second stage of the traditional election system where all candidates are running, if $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$, there are equilibria in which the candidate winning the general election is R^- and equilibria in which the candidate winning the general election is D^-. However, there is no equilibrium where both candidates have a positive probability of winning the general election. A similar result holds for the case in which $\succ^m = \succ^2_{R^-}$ and $\succ^m_R = \succ_{R^+}$.*

[22] For example, if the median voter is $\succ^2_{D^-}$ any equilibrium is such that, in every subgame beginning at the second stage where the running candidates are D^+ and R^-, the candidate winning the general election is R^-. If the median voter is $\succ^1_{D^-}$ any equilibrium is such that, in every subgame beginning at the second stage where the running candidates are D^+, D^-, and R^-, the candidate winning the general election is D^+ if the median democratic partisan is $\succ_{D^+}$, and D^- if the median democratic partisan is $\succ^1_{D^-}$ or $\succ^2_{D^-}$. If the median voter is $\succ^2_{R^-}$ and the median republican partisan is $\succ_{R^+}$ any equilibrium is such that, in every subgame beginning at the second stage where all candidates are running, the candidate winning the general election is either D^- or R^- (*i.e.*, there exist both, equilibria resulting in D^- and equilibria resulting in R^-).

Candidates		Median partisan	Median voter					
$\mathcal{C}_D^r$	$\mathcal{C}_R^r$		$\succ_{D^+}$	$\succ^1_{D^-}$	$\succ^2_{D^-}$	$\succ^2_{R^-}$	$\succ^1_{R^-}$	$\succ_{R^+}$
$\emptyset$	$\emptyset$		$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	$\emptyset$		D^+	D^+	D^+	D^+	D^+	D^+
D^-	$\emptyset$		D^-	D^-	D^-	D^-	D^-	D^-
$\emptyset$	R^+		R^+	R^+	R^+	R^+	R^+	R^+
$\emptyset$	R^-		R^-	R^-	R^-	R^-	R^-	R^-
D^+	R^+		D^+	D^+	D^+	R^+	R^+	R^+
D^+	R^-		D^+	D^+	R^-	R^-	R^-	R^-
D^-	R^+		D^-	D^-	D^-	D^-	R^+	R^+
D^-	R^-		D^-	D^-	D^-	R^-	R^-	R^-
$\emptyset$	R^+R^-	$\succ^m_R=\succ_{R^+}$	R^+	R^+	R^+	R^+	R^+	R^+
		$\succ^m_R=\succ_{R^-}$	R^-	R^-	R^-	R^-	R^-	.
D^+D^-	$\emptyset$	$\succ^m_D=\succ_{D^+}$	D^+	D^+	D^+	D^+	D^+	D^+
		$\succ^m_D=\succ_{D^-}$	.	D^-	D^-	D^-	D^-	D^-
D^+	R^+R^-	$\succ^m_R=\succ_{R^+}$	D^+	D^+	R^-	R^+	R^+	R^+
		$\succ^m_R=\succ_{R^-}$	D^+	D^+	R^-	R^-	R^-	.
D^-	R^+R^-	$\succ^m_R=\succ_{R^+}$	D^-	D^-	D^-	R^-	R^+	R^+
		$\succ^m_R=\succ_{R^-}$	D^-	D^-	D^-	R^-	R^-	.
D^+D^-	R^+	$\succ^m_D=\succ_{D^+}$	D^+	D^+	D^+	D^-	R^+	R^+
		$\succ^m_D=\succ_{D^-}$	.	D^-	D^-	D^-	R^+	R^+
D^+D^-	R^-	$\succ^m_D=\succ_{D^+}$	D^+	D^+	D^-	R^-	R^-	R^-
		$\succ^m_D=\succ_{D^-}$	.	D^-	D^-	R^-	R^-	R^-
D^+D^-	R^+R^-	$\succ^m_D=\succ_{D^+}$	D^+	D^+	R^- or D^-			
		$\succ^m_D=\succ_{D^-}$	.	D^-	D^-			
		$\succ^m_R=\succ_{R^+}$				D^- or R^-	R^+	R^+
		$\succ^m_R=\succ_{R^-}$				R^-	R^-	.

Table 3 Results of Proposition 2: winner in equilibrium in the subgames beginning at the second stage of the traditional election system.

First stage of the traditional election system

At the first stage, the four candidates simultaneously decide whether to run (Y) or not (N). From the analysis of the third and second stages, we know who wins the general election depending on who is running. Theorem 1 uses this information to calculate which candidates run and which of them win the general election in equilibrium.

Regardless of who the median voter is, there is always an equilibrium in which all candidates are running. In this equilibrium, the median voter's favorite candidate wins the general election, except in the case in which $\succ^m=\succ^1_{D^-}$ and $\succ^m_D=\succ_{D^+}$ ($\succ^m=\succ^1_{R^-}$ and $\succ^m_R=\succ_{R^+}$), where D^+ (R^+) wins. If $\succ^m=\succ^2_{D^-}$ and $\succ^m_D=\succ_{D^+}$ ($\succ^m=\succ^2_{R^-}$ and $\succ^m_R=\succ_{R^+}$), there is another equilibrium in which D^+ (R^+) is not running, although the median voter's favorite still wins the general election. The reason is that, in this case, running is not a weakly dominant strategy for D^+ (R^+) at the first stage if the equilibrium continuation strategies are such that R^- (D^-) wins the general election if all candidates are running. We make a detailed analysis of these results in Section 6.

Theorem 1 *If the voting system is the traditional election system then equilibrium always exists. The candidates running and the candidate winning the general election in any equilibrium are as described in Table 4.*

Median voter	Candidates running in equilibrium	Winner in equilibrium
$\succ^m = \succ_{D^+}$	D^+, D^-, R^-, R^+	D^+
$\succ^m = \succ^1_{D^-}$	D^+, D^-, R^-, R^+	If $\succ^m_D = \succ_{D^+}$: D^+ If $\succ^m_D = \succ_{D^-}$: D^-
$\succ^m = \succ^2_{D^-}$	D^+, D^-, R^-, R^+ D^-, R^-, R^+ (if $\succ^m_D = \succ_{D^+}$)	D^- D^-
$\succ^m = \succ^1_{R^-}$	D^+, D^-, R^-, R^+	If $\succ^m_R = \succ_{R^+}$: R^+ If $\succ^m_R = \succ_{R^-}$: R^-
$\succ^m = \succ^2_{R^-}$	D^+, D^-, R^-, R^+ D^+, D^-, R^- (if $\succ^m_R = \succ_{R^+}$)	R^- R^-
$\succ^m = \succ_{R^+}$	D^+, D^-, R^-, R^+	R^+

Table 4 Results of Theorem 1: winner and candidates running in equilibrium in the traditional election system.

4 Equilibria of the top-two election system

Next, we analyze the equilibria of the top-two election system. For that, we solve the sequential game induced by this voting system starting from the last stage.

Third stage of the top-two election system

The two candidates who got the most votes in the open primary at the second stage, x_1^n and x_2^n, compete in the general election. There are eleven different types of subgames beginning at the third stage depending on who the nominees are (there are two more types of subgames than in the traditional election system since now the nominees may also be D^+ and D^- or R^+ and R^-). As in the traditional election system, in each of these subgames the median voter's favorite candidate between x_1^n and x_2^n wins the election.

Proposition 3 *Any profile of equilibrium strategies of the top-two election system is such that the candidates winning the general election in the subgames beginning at the third stage are as described in Table 5.*

	Median voter					
Nominees	$\succ_{D^+}$	$\succ^1_{D^-}$	$\succ^2_{D^-}$	$\succ^2_{R^-}$	$\succ^1_{R^-}$	$\succ_{R^+}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	D^+	D^+	D^+	D^+	D^+	D^+
D^-	D^-	D^-	D^-	D^-	D^-	D^-
R^+	R^+	R^+	R^+	R^+	R^+	R^+
R^-	R^-	R^-	R^-	R^-	R^-	R^-
D^+R^+	D^+	D^+	D^+	R^+	R^+	R^+
D^+R^-	D^+	D^+	R^-	R^-	R^-	R^-
D^-R^+	D^-	D^-	D^-	D^-	R^+	R^+
D^-R^-	D^-	D^-	D^-	R^-	R^-	R^-
D^+D^-	D^+	D^-	D^-	D^-	D^-	D^-
R^+R^-	R^-	R^-	R^-	R^-	R^-	R^+

Table 5 Results of Proposition 3: winner in equilibrium in the subgames beginning at the third stage of the top-two election system.

Second stage of the top-two election system

At the second stage the open primary is held. All voters cast their votes for one of the candidates who decided to run. A republican (democratic) partisan needs not to vote for a republican (democratic) candidate. The two candidates who get the most votes will advance to the third stage.

There are sixteen different types of subgames beginning at the second stage depending on who the running candidates are. In Proposition 4 we analyze who wins the general election in equilibrium in each of these subgames. If there are at most two candidates running, then the voters do not have to take any decision at the second stage and the favorite between them for the median voter wins the general election. If only three candidates are running, again the favorite between them for the median voter wins the general election. If the four candidates are running, then there are multiple equilibria. In particular, if $\succ^m = \succ_{D^+}$, then there are equilibria where D^+ wins and equilibria where D^- wins (voting for D^+ and voting for D^- in the open primary are the only two strategies that are not weakly dominated for voters of type $\succ_{D^+}$ given the equilibrium continuation strategies, and then more than $\frac{v}{4}$ of the voters vote for D^+ and/or D^- in the open primary). If $\succ^m = \succ^1_{D^-}$, we have to distinguish two cases, depending on whether more than half of the voters are of type $\succ^1_{D^-}$ or not. In the former case, there are equilibria where D^+ wins and equilibria where D^- wins (voting for D^+ and voting for D^- in the open primary are the only two strategies that are not weakly dominated for voters of type $\succ^1_{D^-}$ given the equilibrium continuation strategies, and then more than $\frac{v}{4}$ of the voters vote for D^+ and/or D^- in the open primary). In the latter case, in addition to these two types of equilibria, there are also equilibria where R^- wins the general election (voting for R^+ and voting for R^- in the open primary are not weakly dominated for any voter who is not of type $\succ^1_{D^-}$ given the equilibrium continuation strategies, and then R^+ and R^- may be the two most voted candidates in the open primary). If $\succ^m = \succ^2_{D^-}$, we have to distinguish two cases depending on whether more than half of the voters are of type $\succ^2_{D^-}$ or not. In the former case, there are equilibria where D^- wins and equilibria where R^- wins (voting for D^- and voting for R^- in the open primary are the only two strategies that are not weakly dominated for voters of type $\succ^2_{D^-}$ given the equilibrium continuation strategies, and then more than $\frac{v}{4}$ of the voters vote for D^- and/or R^- in the open primary). In the latter case, in addition to these two types of equilibria, there are also equilibria where D^+ wins the general election (there are equilibria where D^+ and R^+ are the two most voted candidates in the open primary). The cases $\succ^m = \succ_{D^+}$, $\succ^m = \succ^1_{D^-}$, and $\succ^m = \succ^2_{D^-}$ are analogous.

Proposition 4 *Any profile of equilibrium strategies of the top-two election system is such that the candidates winning the general election in the subgames beginning at the second stage are as described in Table 6.*

	Median voter					
Candidates	$\succ_{D^+}$	$\succ^1_{D^-}$	$\succ^2_{D^-}$	$\succ^2_{R^-}$	$\succ^1_{R^-}$	$\succ_{R^+}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	D^+	D^+	D^+	D^+	D^+	D^+
D^-	D^-	D^-	D^-	D^-	D^-	D^-
R^+	R^+	R^+	R^+	R^+	R^+	R^+
R^-	R^-	R^-	R^-	R^-	R^-	R^-
D^+R^+	D^+	D^+	D^+	R^+	R^+	R^+
D^+R^-	D^+	D^+	R^-	R^-	R^-	R^-
D^-R^+	D^-	D^-	D^-	D^-	R^+	R^+
D^-R^-	D^-	D^-	D^-	R^-	R^-	R^-
D^+D^-	D^+	D^-	D^-	D^-	D^-	D^-
R^+R^-	R^-	R^-	R^-	R^-	R^-	R^+
$D^+D^-R^-$	D^+	D^-	D^-	R^-	R^-	R^-
$D^+D^-R^+$	D^+	D^-	D^-	D^-	R^+	R^+
$D^+R^-R^+$	D^+	D^+	R^-	R^-	R^-	R^+
$D^-R^-R^+$	D^-	D^-	D^-	R^-	R^-	R^+
$D^+D^-R^-R^+$	D^+ or D^-	D^- or D^+ or $R^{-(1)}$	D^- or R^- or $D^{+(2)}$	R^- or D^- or $R^{+(3)}$	R^- or R^+ or $D^{-(4)}$	R^+ or R^-

(1) Only if $\#\{i \in V : \succ_i = \succ^1_{D^-}\} < v/2$ (2) Only if $\#\{i \in V : \succ_i = \succ^2_{D^-}\} < v/2$
(3) Only if $\#\{i \in V : \succ_i = \succ^2_{R^-}\} < v/2$ (4) Only if $\#\{i \in V : \succ_i = \succ^1_{R^-}\} < v/2$

Table 6 Results of Proposition 4: winner in equilibrium in the subgames beginning at the second stage of the top-two election system.

Remark 2 *In the subgames beginning at the second stage of the top-two election system where all candidates are running, there are different equilibria in which the candidate winning the general election is not the same. However, there is no equilibrium where two or more candidates have a positive probability of winning the general election. For example, when* $\succ^m = \succ^1_{D^-}$ *and less than* $\frac{v}{2}$ *voters are of type* $\succ^1_{D^-}$*, there are equilibria where* D^+ *wins the general election, equilibria where* D^- *wins the general election, and equilibria where* R^- *wins the general election. Nevertheless, there is no equilibrium where more than one of these candidates have a positive probability of winning the general election.*

First stage of the top-two election system

From our previous analysis we know which candidates win the general election depending on who is running and who is the median voter. Theorem 2 uses this information to calculate which candidates will run and which of them may win the general election in equilibrium.

Regardless of who the median voter is, there is always an equilibrium in which all candidates are running and the median voter's favorite wins the general election. If $\succ^m \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, there are other equilibria in which not all candidates are running but the median voter's favorite still wins the general election. Additionally, if $\succ^m = \succ_{D^+}$, there are equilibria where all candidates are running and D^- wins the general election (the intuition of this result is that, in this case, voting for D^- in the open primary is not weakly dominated for voters of type $\succ_{D^+}$ given the equilibrium continuation strategies, and then there are equilibria where D^- passes to the third stage but D^+ does not). If $\succ^m = \succ^1_{D^-}$, the preferences of candidate D^- are of type $\succ^1_{D^-}$, and less than half of the voters are of type $\succ^1_{D^-}$, there is another type of equilibrium where all candidates but D^- are running and D^+ wins the general election (the intuition of this result is that, if the equilibrium strategies at the second stage are such that R^- wins the general election if all candidates are running, then D^- prefers not to run if the other three candidates are running). Finally, if $\succ^m = \succ^2_{D^-}$, the preferences of candidate D^- are of type $\succ^2_{D^-}$, and less than half of the voters are of type $\succ^2_{D^-}$, there is another type of equilibrium where all candidates except D^- are running and R^- wins the general election (the intuition of this result is that, in this case, if the equilibrium strategies at the second stage are such that D^+ wins the general election if all candidates are running, then D^- prefers not to run if the other three candidates are running). The cases $\succ^m = \succ_{D^+}$, $\succ^m = \succ^1_{D^-}$, and $\succ^m = \succ^2_{D^-}$ are analogous.

Theorem 2 *If the voting system is the top-two election system then equilibrium always exists. The candidates running and the candidate winning the general election in any equilibrium are as described in Tables 7 and 8.*

Median voter	Who would win if all candidates were running	Candidates running in equilibrium	Winner in equilibrium
$\succ^m = \succ_{D^+}$		D^+, D^-, R^-, R^+	D^+ or D^-
$\succ^m = \succ^1_{D^-}$	D^+	D^+, D^-, R^-	D^-
		D^+, D^-, R^+	D^-
		D^+, D^-	D^-
	D^-	D^+, D^-, R^-, R^+	D^-
	$R^{-(1)}$	If D^- is type $\succ^1_{D^-}$: D^-, R^-, R^+	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, R^-, R^+	D^+
		If D^- is type $\succ^2_{D^-}$: D^-, R^-, R^+	D^-
$\succ^m = \succ^2_{D^-}$	$D^{+(2)}$	If D^- is type $\succ^1_{D^-}$: D^+, D^-, R^-	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, D^-, R^+	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, D^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-, R^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-, R^+	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, R^-, R^+	R^-
	D^-	D^+, D^-, R^-, R^+	D^-
	R^-	D^-, R^-, R^+	D^-

(1) Only if $\#\{i \in V : \succ_i = \succ^1_{D^-}\} < v/2$ (2) Only if $\#\{i \in V : \succ_i = \succ^2_{D^-}\} < v/2$

Table 7 Results of Theorem 2: winner and candidates running in equilibrium in the top-two election system when the median voter is democratic.

Median voter	Who would win if all candidates were running	Candidates running in equilibrium	Winner in equilibrium
$\succ^m=\succ_{R^+}$		D^+, D^-, R^-, R^+	R^+ or R^-
$\succ^m=\succ^1_{R^-}$	R^+	D^-, R^-, R^+	R^-
		D^+, R^-, R^+	R^-
		R^-, R^+	R^-
	R^-	D^+, D^-, R^-, R^+	R^-
	$D^{-(1)}$	If R^- is type $\succ^1_{R^-}$: D^+, D^-, R^-	R^-
		If R^- is type $\succ^1_{R^-}$: D^+, D^-, R^+	R^+
		If R^- is type $\succ^2_{R^-}$: D^+, D^-, R^-	R^-
$\succ^m=\succ^2_{R^-}$	$R^{+(2)}$	If R^- is type $\succ^1_{R^-}$: D^-, R^-, R^+	R^-
		If R^- is type $\succ^1_{R^-}$: D^+, R^-, R^+	R^-
		If R^- is type $\succ^1_{R^-}$: R^-, R^+	R^-
		If R^- is type $\succ^2_{R^-}$: D^-, R^-, R^+	R^-
		If R^- is type $\succ^2_{R^-}$: D^+, R^-, R^+	R^-
		If R^- is type $\succ^2_{R^-}$: R^-, R^+	R^-
		If R^- is type $\succ^2_{R^-}$: D^+, D^-, R^+	D^-
	R^-	D^+, D^-, R^-, R^+	R^-
	D^-	D^+, D^-, R^-	R^-

(1) Only if $\#\{i \in V : \succ_i=\succ^1_{R^-}\} < v/2$ (2) Only if $\#\{i \in V : \succ_i=\succ^2_{R^-}\} < v/2$

Table 8 Results of Theorem 2: winner and candidates running in equilibrium in the top-two election system when the median voter is republican.

5 The case in which there is a cost of running

In this section we study the case in which there is a cost of running for election. Such a cost is formulated in terms of the following assumption.

Assumption A. Each candidate $x \in \mathcal{C}$ prefers to run if by doing so he/she alters the result of the election and the winner is more preferred for him/her. If the election result is the same whether x is running or not, then x prefers not to run.

For example, under Assumption A, if (i) the election system is the traditional one, (ii) $\succ^m = \succ_{D^+}$, and (iii) candidates D^+, R^-, and R^+ are running, then candidate D^- prefers not to run, because the winner in equilibrium will be D^+, whether D^- runs or not (see Table 3). Similarly, if (i) the election system is the top-two election system, (ii) $\succ^m = \succ_{D^+}$, (iii) candidates D^+, D^-, and R^+ are running, and (iv) the equilibrium strategies in the second and third stages are such that D^- wins the general election if all candidates are running, then candidate R^- prefers to run (even though he/she does not win the election), because if he/she does not run D^+ will win the election in equilibrium, which for them is less preferred than D^- (see Table 6).

The analysis in Sections 3 and 4 for the third and second stage of the traditional and top-two election systems is still valid in this case. Next, we analyze the first stage of the sequential games induced by both election systems when there is a cost of running.

We denote by $s^1 = (s^1_{D^+}, s^1_{D^-}, s^1_{R^-}, s^1_{R^+}) \in S^1$ a strategy profile played by the four candidates (for example, $s^1 = (Y, N, N, Y)$ denotes the situation where D^+ and R^+ are running while D^- and R^- are not). Abusing notation, for any $x \in \mathcal{C}$ and $s^1, \hat{s}^1 \in S^1$, we write $s^1 \succ_x \hat{s}^1$ if one of the two following cases occurs: (i) x prefers any possible equilibrium result in equilibrium after candidates played s^1 in the first stage to any possible equilibrium result after they played $\hat{s}^1$, or (ii) $s^1_x = N$, $\hat{s}^1_x = Y$, and the only possible equilibrium result after candidates played s^1 in the first stage coincides with the only possible equilibrium result after they played $\hat{s}^1$.

Theorem 3 shows who runs and who wins in equilibrium in the traditional election system when there is a cost of running. These results are very similar to those obtained in the case where there was not a cost of running (Theorem 1). There are, however, two main differences. The first one is that now all equilibria are such that only one candidate is running. The second difference is that when $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$ there is no equilibrium. Recall that, if there is no cost of running, there are equilibria in this situation where D^- wins the general election. If there is a cost of running, however, a situation where only D^- is running is not an equilibrium because D^+ would prefer to run and win the general election, and a situation where only D^+ and D^- are running is not an equilibrium either, because D^- would prefer not to run, since D^+ wins anyway. Given the symmetry of our model, if $\succ^m = \succ^2_{R^-}$ and $\succ^m_R = \succ_{R^+}$, there is no equilibrium either.

Theorem 3 *Suppose that Assumption A holds and the voting system is the traditional election system. Then, if (i)* $\succ^m = \succ^2_{D^-}$ *and* $\succ^m_D = \succ_{D^+}$*, or (ii)* $\succ^m = \succ^2_{R^-}$ *and* $\succ^m_R = \succ_{R^+}$*, there is no profile of equilibrium strategies. Otherwise, equilibrium exists. The candidates winning the general election in equilibrium are as described in Table 9 and any equilibrium is such that the winning candidate is the only one running.*

Median voter	Winner in equilibrium
$\succ^m = \succ_{D^+}$	D^+
$\succ^m = \succ^1_{D^-}$	If $\succ^m_D = \succ_{D^+}$: D^+ If $\succ^m_D = \succ_{D^-}$: D^-
$\succ^m = \succ^2_{D^-}$	If $\succ^m_D = \succ_{D^+}$: $\nexists$ Equilibrium If $\succ^m_D = \succ_{D^-}$: D^-
$\succ^m = \succ^1_{R^-}$	If $\succ^m_R = \succ_{R^+}$: $\nexists$ Equilibrium If $\succ^m_R = \succ_{R^-}$: R^-
$\succ^m = \succ^2_{R^-}$	If $\succ^m_R = \succ_{D^+}$: R^+ If $\succ^m_R = \succ_{R^-}$: R^-
$\succ^m = \succ_{R^+}$	R^+

Table 9 Results of Theorem 3: winner in equilibrium in the traditional election system when there is a cost of running.

Theorem 4 shows who wins the general election in equilibrium when the election system is the top-two election system and there is a cost of running. In this case, equilibrium always exists. Moreover, any equilibrium is such that the median voters' favorite candidate is the only one running, therefore winning the general election. Recall that, besides the equilibria where the median voter's favorite candidate wins, if there is no cost of running and $\succ^m = \succ_{D^+}$, there is an equilibrium where all candidates are running and D^- wins the general election. This equilibrium disappears if there is a cost of running (the reason is that, if the strategies are such that when all candidates are running D^- passes to the third stage and wins the general election, then D^+ prefers not to run, since D^- will win anyway; but then R^+ and R^- would prefer not to run either; a situation where only D^- is running is not an equilibrium because D^+ would prefer to run and win the general election). Similarly, the equilibrium where D^+ wins the general election when there is no cost of running, $\succ^m = \succ^1_{D^-}$, the preferences of candidate D^- are of type $\succ^1_{D^-}$, and less than half of the voters are of type $\succ^1_{D^-}$, disappears when there is a cost of running (if all candidates except D^- were running, then R^+ and R^- would prefer not to run either, since D^+ will win anyway; a situation where only D^+ is running is not an equilibrium because D^- would prefer to run and win the general election). Finally, the equilibrium where R^- wins the general election when there is no cost of running, $\succ^m = \succ^2_{D^-}$, the preferences of candidate D^- are of type $\succ^2_{D^-}$, and less than half of the voters are of type $\succ^2_{D^-}$, also disappears when there is a cost of running (if all candidates except D^- were running, then D^+ and R^+ would prefer not to run, since R^- would win anyway; a situation where only R^- is running is not an equilibrium because D^- would prefer to run and win the general election).

Theorem 4 *Suppose that Assumption A holds and the voting system is the top-two election system. Then, equilibrium always exists. The candidates winning the general election in equilibrium are as described in Table 10 and any equilibrium is such that the winning candidate is the only one running.*

Median voter	Winner in equilibrium
$\succ^m = \succ_{D^+}$	D^+
$\succ^m = \succ^1_{D^-}$	D^-
$\succ^m = \succ^2_{D^-}$	D^-
$\succ^m = \succ^2_{R^-}$	R^-
$\succ^m = \succ^1_{R^-}$	R^-
$\succ^m = \succ_{R^+}$	R^+

Table 10 Results of Theorem 4: winner in equilibrium in the top-two election system when there is a cost of running.

6 Comparing the traditional and the top-two election systems

The case in which there is no cost of running

Theorems 1 and 2 describe all the equilibrium configurations induced by the traditional and the top-two election systems when there is no cost of running. First, we make a comparison between the two election systems in terms of the number of candidates running in the primaries.

The traditional election system is such that there are always equilibria where the four candidates are running. There is, however, one case in which only three candidates are running: when the median voter is lean but his/her party's median voter is strong. This equilibrium is such that all candidates except the extreme candidate of the median voter's party run. If that candidate also decides to run, he/she would win his/her party's primary, but he/she would be defeated by the moderate candidate of the opposite party in the general election. Then, the extreme candidate of the median voter's party prefers not to run (see the proof of Proposition 2, Case 4, and the proof of Theorem 1, Case 3).

The top-two election system is such that there are always equilibria where the four candidates are running. Besides, there are equilibria in which only three or two candidates run. In particular, the equilibria where only two candidates are running are such that both candidates belong to the same party. For example, if the median is weak democratic, there is an equilibrium in which only the two democratic candidates are running and the moderate democratic candidate wins the general election. This equilibrium is such that, if the two republican candidates were also running, then the extreme democratic candidate would win the general election (if all candidates are running, voting for the extreme democratic candidate is not weakly dominated at the second stage for the strong and weak democratic voters; see Lemma 2 in the Appendix).

In general, in terms of the number of candidates running for election, the top-two election system generates more equilibria than the traditional election system. The new equilibria of the top-two system are such that the number of candidates who have incentives to enter the race is smaller. Both systems, however, guarantee that two candidates face each other at the general election. Figure 3 summarizes these results.

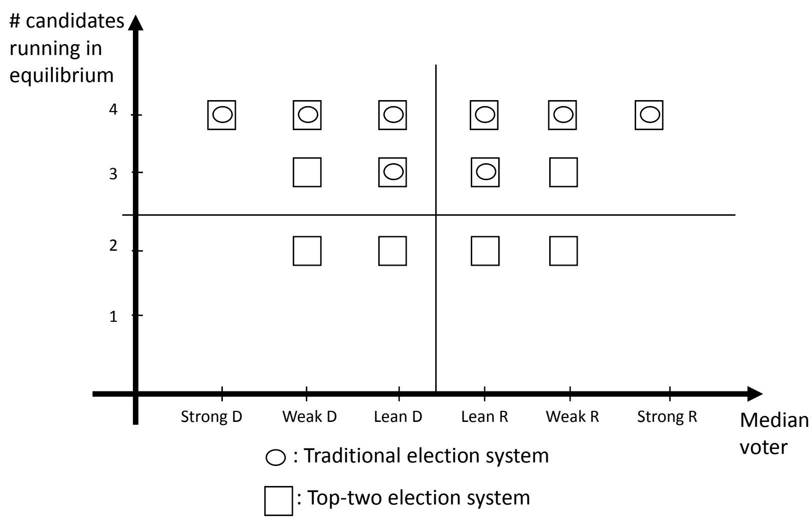

Figure 3 Comparison of the number of candidates who may run in equilibrium when there is no cost of running.

Next, we compare the two election systems in terms of the candidates who win in equilibrium.

The traditional election system is such that, in almost all cases, the median voter's favorite candidate wins the general election in equilibrium. There is only one exception to that rule: the case in which the median voter is weak but his/her party's median voter is strong. In this case, the candidate winning the election in equilibrium is not the moderate candidate preferred by the median voter, but the extremist candidate of the same party. The reason is that the nominee of the party of the median voter will eventually win the general election (no matter who the nominee of the other party is) and, therefore, in the primary of that party all voters have incentives to vote for their most preferred candidate. Since the party's median voter is strong, the strong candidate will win that primary and the general election (see Proposition 2, Case 4, and Theorem 1, Case 2). However, if the median voter is lean, the extreme candidate of his/her party cannot win. The reason is that, if the extreme candidate becomes nominee, he/she will face the moderate candidate of the opposite party in the general election (and, if the median voter is lean partisan, he/she prefers the moderate candidate of the opposite party rather than the extreme candidate of his/her own party).

In the top-two election system, there are always equilibria in which the median voter's favorite candidate wins the general election. Besides, there are other equilibria where the winner is different.

Firstly, if the median voter is strong, there are equilibria where the moderate candidate of the median voter's party wins the general election. The reason is that, in contrast to what occurs in the traditional election system, now voting for the moderate candidate in the open primary is not weakly dominated for the median voter given the continuation equilibrium strategies (there are situations where the median voter prefers to vote for his/her moderate candidate in the open primary, thus ensuring that candidates's victory, rather than voting for his/her extreme candidate, because in the latter case a tie occurs in the open primary and both the moderate candidate of the median voter's party and the moderate candidate of the opposite party, may win the general election with probability 0.5; see Lemma 1 in the Appendix).

Secondly, as happens in the traditional election system, if the median voter is weak, there are equilibria where the extreme candidate of the median voter's party wins the general election. For this equilibrium to happen, it is necessary that (i) the moderate candidate of the median voter's party is also weak (*i.e.*, if that candidate does not win, he/she prefers the extreme candidate of his/her party rather than the moderate candidate of the opposite party), and (ii) less than half of the voters have the median voter's preferences. In this case, if all candidates were running, voting for either candidate of the median voter's opposite party in the open primary is not weakly dominated for a majority of voters given the continuation equilibrium strategies (in particular, it is not dominated for the strong partisans of the median voter's party; see Lemma 2). Therefore, the two candidates of the median voter's opposite party can pass to the general election, in which case the moderate one would win. Then, the moderate candidate of the median voter's party prefers not to run, since in this case the extreme candidate of his/her party wins the general election.[23]

[23] This equilibrium is supported by crossover voting since, if the four candidates were in the race, some voters whose party-affiliation coincide with that of the median voter would vote for candidates of the opposite party. The underlying crossover voting strategies are not weakly dominated, and the complete lack of coordination devices in the Nash equilibrium concept generates this type of out of equilibrium strategies.

Finally, if the median voter is lean, there are equilibria of the top-two system where the moderate candidate of the median voter's opposite party wins the general election. For this type of equilibrium to happen it is necessary that (i) the moderate candidate of the median voter's party is also lean and (ii) less than half of the voters have the median voter's preferences. In this case, if all candidates were running, voting for the extreme candidate of either party in the open primary is not weakly dominated for a majority of voters given the continuation equilibrium strategies (see Lemma 3). Therefore, the two extreme candidates can pass to the general election, in which case the extreme candidate of the median voter's party would win. Then, the moderate candidate of the median voter's party prefers not to run, since in this case the moderate candidate of the opposite party wins the general election.

Note that, in general, if a candidate can win in equilibrium in the traditional election system, he/she can also win in equilibrium in the top-two election system. There is only one exception to this rule: when (i) the median voter is weak but his/her party's median voter is strong and, either (ii) the moderate candidate of the median voter's party is not weak, or (iii) more than half of the voters have the median voter's preferences. In this case, the only candidate winning in equilibrium in the traditional election system is the extremist candidate of the median voter's party, while the only candidate winning in equilibrium in the top-two system is the moderate candidate of the median voter´s party (*i.e.*, in this case, the top-two election system elects less extreme candidates).

On the other hand, not all candidates who can win in equilibrium in the top-two election system can also win in equilibrium in the traditional election system. In particular, if the median voter is strong, the moderate candidate of his/her party can win in equilibrium in the top-two election system, but not in the traditional election system (*i.e.*, in this case, the top-two election system may elect less extreme candidates). Finally, the top-two election system provides certain chances of winning to those moderate candidates whose ideologies differ from that of the median voter: if (i) the median voter is lean, (ii) the moderate candidate of the median voter's party is also lean, and (iii) less than half of the voters have the median voter's preferences, then the moderate candidate whose ideology differs from that of the median voter can win in equilibrium in the top two system, but not in the traditional election system.

Figure 4 summarizes the results obtained in Theorems 1 and 2 concerning the candidates who can win the general election in equilibrium as a function of the median voter.

The case in which there is a cost of running

As Theorems 3 and 4 state, when there is a cost of running, any equilibrium of the traditional and the top-two election systems is such that only one candidate is running.

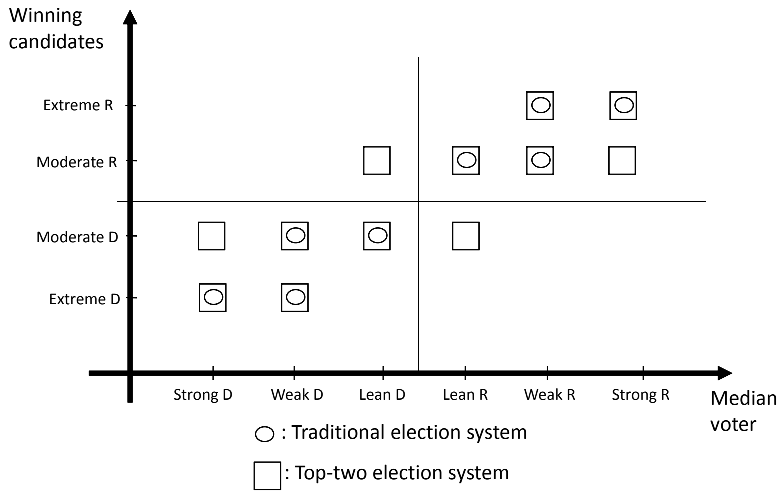

Figure 4 Comparison of the candidates who may win in equilibrium when there is no cost of running.

The analysis of which candidates win the general election in equilibrium in the traditional election system is very similar to the case in which there is no cost of running. The only difference is that now, if the median voter is lean but his/her party's median voter is strong, then there is no equilibrium (in all other cases, equilibrium in the traditional election system exists and the winning candidates are the same as when there is no cost of running). The reason for this lack of equilibrium is as follows: (i) when there is a cost of running, there is no equilibrium where more than one candidate is running; (ii) a situation where only one of the candidates of the median voter's opposite party is running is not an equilibrium, since the moderate candidate of the median voter's party would prefer to run and win the general election; (iii) a situation where only the extremist candidate of the median voter's party is running is not an equilibrium either, since the moderate candidate of the opposite party would prefer to run and win the general election; (iv) a situation where only the moderate candidate of the median voter's party is running is not an equilibrium, since the extremist candidate of the same party would prefer to run and win the party primary and the general election.

In the top-two election system, when there is a cost of running, the candidate winning the general election in equilibrium is always the median voter's favorite candidate (in this case, equilibrium always exists).

Note that, in this setting, the two election systems elect the same candidates in equilibrium except in two cases: (1) when the median voter is weak but his/her party's median voter is strong, and (2) when the median voter is lean but his/her party's median voter is strong. In the first case, the extremist candidate of the median voter's party wins the general election under the traditional election system, while the moderate candidate of the median voter's party wins the general election under the top-two election system (*i.e.*, in this case, the top-two election system elects less extreme candidates). In the second case, the traditional election system has no equilibrium, while the moderate candidate of the median voter's party wins the general election in equilibrium under the top-two election system (if we interpret the lack of equilibrium as the possibility that extremist candidates may end up winning the general election, then we could say that the top-two election system also elects less extreme candidates in this case). Figure 5 summarizes these results.

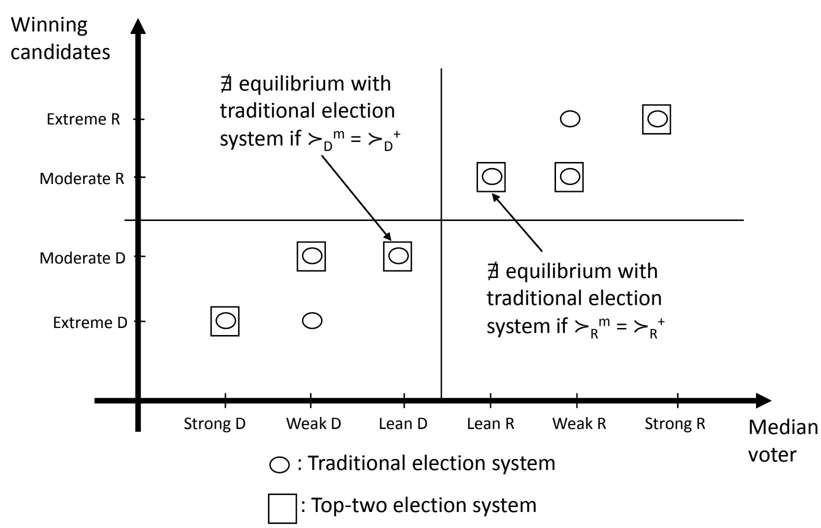

Figure 5 Comparison of the candidates who may win in equilibrium when there is a cost of running.

7 Conclusion and final remarks

Our analysis disentangles and compares some of the consequences of two different primary election procedures used to select nominees (or leadership) in representative democracy.

We present a new stylized model in which political partisanship is divided into two groups, democrats and republicans. In this setting, four potential candidates labeled as extreme and moderate partisans, and six different types of voters labeled as strong, weak and lean partisans, participate in the electoral process to select a representative. We compare two election systems: one in which parties select nominees according to closed primaries (traditional election system) and another in which nominees are selected according to the top-two primary (top-two election system). We model these settings as sequential games and we solve them according to the subgame perfect Nash equilibrium concept in which every equilibrium strategy is an undominated strategy given the equilibrium continuation strategies of the game.

As a result of the comparison of the two systems, we conclude that the top-two election system may contribute to political moderation. In particular, when the overall median voter is weak but his/her party's median partisan is strong (and some additional mild assumptions hold), then the only candidate winning the general election in equilibrium under the traditional election system is an extremist candidate, while the only candidate winning the general election under the top-two election system is a moderate one. Furthermore, if the median voter is strong, then the only winner in equilibrium under the traditional election system is an extremist candidate but, under the top-two election system, there are some equilibria where a moderate candidate wins. These results support the idea proposed by Gerber and Morton (1998) and Jackson et al. (2007) that the open selection of candidates produces more centrist candidates. In the particular case of the top-two primary, we show that this system may contribute to political moderation.

Another observation regarding the top-two election system is noticeable: the party-affiliation of the median voter may not coincide with the party-affiliation of the winning candidate when the median voter is lean and the moderate candidate of his/her party is also lean. This possibility is totally discarded in the traditional election system. Therefore, changing the closed primaries system to the top-two primary system may increase the number of swing states (the moderate candidate in the opposition under the closed primaries system may have some chances of winning under the top-two primary system). According to this last argument, political parties that dominate in safe states could be negatively affected by the top-two primary system.

Under the assumption that there is a cost of running (so that a candidate only wants to run for election if, by doing so, he/she ensures that the winning candidate is better for him/her), some of the equilibrium results change. However, there are still reasons to support the idea that the top-two election system contributes to political moderation. In particular, there are situations where the only candidate winning in equilibrium under the traditional election system is extremist while the only candidate winning in equilibrium under the top-two system is moderate (this happens when the median voter is weak but his/her party's median partisan is strong). Furthermore, if the median voter is lean but his/her party's median voter is strong, the traditional election system has no equilibrium when there is a cost of running, while the moderate candidate of the median voter's party wins under the top-two election system. If we interpret the absence of equilibrium as implying that every candidate has a positive probability of winning, then we could say that the top-two election system also elects less extreme candidates in this case.

The top-two election system shares some similarities with the runoff voting system used in several countries to elect a president (France, Poland, Argentina, Brazil, and Colombia, among others). In a runoff system, each candidate either has the support of a political party or is independent, and there cannot be a party supporting two different candidates. Moreover, if a candidate receives an absolute majority, there is no need for a second round. In contrast, the top-two election system does not restrict the candidates to be members of different political parties, and there is always a second voting round (the general election). Obviously, these differences between the two systems generate different strategic considerations. The analysis of the runoff-system focuses on information aggregation and its comparison with a simple plurality rule, as well as its properties in terms of Condorcet efficiency (see Cox, 1997; Martinelli, 2002; Bouton, 2013).[24]

All in all, our setting provides formal arguments based on rigorous strategic analysis that can be used in favor of or against the electoral use of the top-two primary. Our predictions are open to empirical scrutiny but, given that the top-two primary has recently been incorporated as a candidate selection procedure in some states of the U.S., there are not yet sufficient observations to determine its impact.

[24] In contrast to these authors, we account for endogenous entry of candidates and we have a specific institutional structure in which candidates and voters have party identification.

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Appendix

PROOF OF PROPOSITION 1: It follows from the fact that, in any subgame beginning at the third stage, voting for his/her most preferred candidate, x_D^n or x_R^n, is a weakly dominant strategy for each voter. ■

PROOF OF PROPOSITION 2: We distinguish four cases.

Case 1. The subgame is such that $\mathcal{C}_D^r \neq \{D^+, D^-\}$ and $\mathcal{C}_R^r \neq \{R^+, R^-\}$.

This case is trivial, since there is no decision to be made at the second stage.

Case 2. The subgame is such that (i) $\mathcal{C}_D^r = \emptyset$ and $\mathcal{C}_R^r = \{R^+, R^-\}$, or (ii) $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = \emptyset$.

In any subgame beginning at the second stage with $\mathcal{C}_D^r = \emptyset$ and $\mathcal{C}_R^r = \{R^+, R^-\}$, the candidate who wins the republican primaries eventually wins the general election at the third stage. Therefore, voting for his/her most preferred candidate, R^+ or R^-, is a weakly dominant strategy for each republican partisan $i \in \mathcal{V}_R$. Hence, the candidate winning the election in equilibrium will be R^+ if $\succ_R^m = \succ_{R^+}$ and R^- if $\succ_R^m = \succ_{R^-}$. The case in which $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = \emptyset$ is symmetric.

Case 3. The subgame is such that (i) $\mathcal{C}_D^r = D^+$ and $\mathcal{C}_R^r = \{R^+, R^-\}$, or (ii) $\mathcal{C}_D^r = D^-$ and $\mathcal{C}_R^r = \{R^+, R^-\}$, or (iii) $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = R^+$, or (iv) $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = R^-$.

Suppose that $\mathcal{C}_D^r = D^+$ and $\mathcal{C}_R^r = \{R^+, R^-\}$. In this case the candidate who wins the republican primaries will end up running against D^+. Let $x_{D^+R^+}$ be the candidate who wins the general election in equilibrium at the third stage if $(x_D^n, x_R^n) = (D^+, R^+)$ (from Table 2 we know who this candidate is). Let candidate $x_{D^+R^-}$ be defined in an analogous manner. Note that, for each $i \in \mathcal{V}_R$, if $x_{D^+R^+} \succ_i x_{D^+R^-}$ ($x_{D^+R^-} \succ_i x_{D^+R^+}$, respectively), then voting for R^+ (R^-, respectively) in the republican primaries is a weakly dominant strategy at the second stage given the continuation equilibrium strategies at the third stage. Therefore, the favorite candidate between $x_{D^+R^+}$ and $x_{D^+R^-}$ for the median republican partisan will win the election in equilibrium.[25] The cases (ii) $\mathcal{C}_D^r = D^-$ and $\mathcal{C}_R^r = \{R^+, R^-\}$, (iii) $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = R^+$, and (iv) $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = R^-$, are analogous.

Case 4. The subgame is such that $\mathcal{C}_D^r = \{D^+, D^-\}$ and $\mathcal{C}_R^r = \{R^+, R^-\}$.

Suppose first that $\succ^m \in \{\succ_{D^+}, \succ_{D^-}^1\}$. From Table 2 we have that the democratic nominee eventually wins the general election, no matter who the republican nominee is. Therefore, voting for his/her most preferred candidate, D^+ or D^-, is a weakly dominant strategy for each democratic partisan at the second stage given the continuation equilibrium strategies at the third stage. Hence, the favorite candidate between D^+ and D^- for the median democratic partisan

[25] For instance, if $\succ^m = \succ_{R^-}^2$ then $x_{D^+R^+} = R^+$ and $x_{D^+R^-} = R^-$. Therefore, (i) if $\succ_R^m = \succ_{R^-}$, R^- will win the election in equilibrium, and (ii) if $\succ_R^m = \succ_{R^+}$, R^+ will win the election in equilibrium.

will win the election in equilibrium. The case in which $\succ^m \in \{\succ_{R^+}, \succ^1_{R^-}\}$ is analogous.

Suppose now that $\succ^m = \succ^2_{D^-}$. From Table 2 we have that (i) if $x^n_D = D^-$, then D^- will win the general election no matter who x^n_R is, (ii) if $x^n_D = D^+$ and $x^n_R = R^+$, then D^+ will win the election, and (iii) if $x^n_D = D^+$ and $x^n_R = R^-$, then R^- will win the election. On the one hand, since for all $i \in \mathcal{V}_R$, $R^- \succ_i D^+$, then voting for R^- is a weakly dominant strategy for each republican partisan at the second stage given the continuation equilibrium strategies at the third stage. On the other hand, since for all democratic partisan $i \in \mathcal{V}_D$ such that $\succ_i \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, $D^- \succ_i D^+$ and $D^- \succ_i R^-$, then voting for D^- is a weakly dominant strategy for those voters at the second stage given the continuation equilibrium strategies at the third stage. Then, if $\succ^m_D = \succ_{D^-}$, D^- will win the election in equilibrium, and if $\succ^m_D = \succ_{D^+}$, both D^- and R^- can be sustained as an equilibrium.[26] The case in which $\succ^m = \succ^2_{R^-}$ is analogous. ■

PROOF OF REMARK 1: Suppose that $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$. From the proof of Proposition 2, we know that R^- wins the republican primary. Therefore, the only possibility that R^- and D^- had a positive probability of winning the general election would be that D^+ and D^- were involved in a tie in the democratic primary (then the confrontations R^- versus D^+ and R^- versus D^- would be equally likely in the third stage and, since $\succ^m = \succ^2_{D^-}$, R^- would win the general election with probability 0.5 and D^- would win the general election with probability 0.5). In this case, since $\succ^m_D = \succ_{D^+}$, at least one voter whose preferences are of type $\succ_{D^+}$ should be voting for D^+ in the democratic primary. This would not be an equilibrium since this voter would be better off if he/she voted for D^- in the democratic primary (in that case, D^- would win the general election with probability equal to one). ■

PROOF OF THEOREM 1: We distinguish three cases.

Case 1. $\succ^m \in \{\succ_{D^+}, \succ_{R^+}\}$.

Suppose that $\succ^m = \succ_{D^+}$. From Proposition 2, we know who wins the general election depending on who is running. Table 11 summarizes this information. Each of the four tables there corresponds to a different situation regarding candidates R^+ and R^-. For example, the top-left table corresponds to the case where both R^+ and R^- are running. Each cell of this table shows who wins in equilibrium depending on whether, additionally, D^+ and D^- are running or

[26] If $\succ^m_D = \succ_{D^+}$, there are Nash equilibria in the game that begins at the second stage where all democratic partisans with preferences $\succ_{D^+}$ vote for D^- and all republican partisans vote for R^- (and then D^- will win the election), since a single voter cannot benefit from unilateral deviating. Moreover, in these equilibria, the strategies of all voters are undominated given the equilibrium continuation strategies in the third stage. Similarly, there are equilibria where all democratic partisans with preferences $\succ_{D^+}$ vote for D^+ and all republican partisans vote for R^- (and then R^- will win the election). Note that, once we have eliminated the strategy of voting for R^+ in the republican primaries, then the strategy of voting for D^+ is weakly dominated (given the continuation equilibrium strategies at the third stage) for each democratic partisan. In this paper, however, we only consider one round of deletion of weakly dominated strategies.

not. In particular, if R^+ and R^- are running and D^+ and D^- are not, the winner in equilibrium depends on who the median republican partisan is (R^+ if $\succ_R^m = \succ_{R^+}$ and R^- if $\succ_R^m = \succ_{R^-}$).

From Table 11 it can be observed that Y is a weakly dominant strategy for each candidate at the first stage given the continuation equilibrium strategies. The case in which $\succ^m = \succ_{R^+}$ is analogous.

Case 2. $\succ^m \in \{\succ_{D^-}^1, \succ_{R^-}^1\}$.

Suppose that $\succ^m = \succ_{D^-}^1$. From Proposition 2, we know who wins the general election depending on who is running. Table 12 summarizes this information. It can be observed that Y is a weakly dominant strategy for each candidate at the first stage given the continuation equilibrium strategies. Then, any profile of equilibrium strategies is such that all candidates are running, D^+ wins the general election if $\succ_D^m = \succ_{D^+}$ and D^- wins the general election if $\succ_D^m = \succ_{D^-}$. The case in which $\succ^m = \succ_{R^-}^1$ is analogous.

Case 3. $\succ^m \in \{\succ_{D^-}^2, \succ_{R^-}^2\}$.

Suppose that $\succ^m = \succ_{D^-}^2$. From Proposition 2, we know who wins the general election depending on who is running. Table 13 shows this information (note that if all candidates are running and $\succ_D^m = \succ_{D^+}$, then both R^- and D^- can win the general election in equilibrium).

		R^-: Y		R^-: N	
R^+	D^+	D^-: Y	D^-: N	D^-: Y	D^-: N
Y	Y	D^+	D^+	D^+	D^+
Y	N	D^-	$R^{+(1)}$ / $R^{-(2)}$	D^-	R^+
N	Y	D^+	D^+	D^+	D^+
N	N	D^-	R^-	D^-	$\emptyset$

$^{(1)}$If $\succ_R^m = \succ_{R^+}$ $\quad$ $^{(2)}$If $\succ_R^m = \succ_{R^-}$

Table 11 Winner in equilibrium in the traditional election system depending on who is running when $\succ^m = \succ_{D^+}$.

R^+	R^-	D^+	D^-: Y	D^-: N
Y	Y	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
Y	Y	N	D^-	$\frac{R^{+(3)}}{R^{-(4)}}$
Y	N	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
Y	N	N	D^-	R^+
N	Y	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
N	Y	N	D^-	R^-
N	N	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
N	N	N	D^-	$\emptyset$

$^{(1)}$If $\succ_D^m = \succ_{D^+}$ $^{(2)}$If $\succ_D^m = \succ_{D^-}$ $^{(3)}$If $\succ_R^m = \succ_{R^+}$ $^{(4)}$If $\succ_R^m = \succ_{R^-}$

Table 12 Winner in equilibrium in the traditional election system depending on who is running when $\succ^m = \succ^1_{D^-}$.

R^+	R^-	D^+	D^-: Y	D^-: N
Y	Y	Y	$\frac{R^- \text{ or } D^{-(1)}}{D^{-(2)}}$	R^-
Y	Y	N	D^-	$\frac{R^{+(3)}}{R^{-(4)}}$
Y	N	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
Y	N	N	D^-	R^+
N	Y	Y	D^-	R^-
N	Y	N	D^-	R^-
N	N	Y	$\frac{D^{+(1)}}{D^{-(2)}}$	D^+
N	N	N	D^-	$\emptyset$

$^{(1)}$If $\succ_D^m = \succ_{D^+}$ $^{(2)}$If $\succ_D^m = \succ_{D^-}$ $^{(3)}$If $\succ_R^m = \succ_{R^+}$ $^{(4)}$If $\succ_R^m = \succ_{R^-}$

Table 13 Winner in equilibrium in the traditional election system depending on who is running when $\succ^m = \succ^2_{D^-}$.

It can be observed that Y is a weakly dominant strategy for candidates D^-, R^-, and R^+ at the first stage given any possible continuation equilibrium strategies. Moreover, if $\succ_D^m = \succ_{D^-}$ then Y is also a weakly dominant strategy for candidate D^+ at the first stage given any possible continuation equilibrium strategies. In this case, any profile of equilibrium strategies in the traditional election system is such that all candidates are running and D^- wins the general election. If $\succ_D^m = \succ_{D^+}$, however, Y is not a weakly dominant strategy for D^+ at the first stage given any possible continuation equilibrium strategies.[27] In this case, there are two possible types of equilibrium strategies: one in which D^+ is not running while D^-, R^-, and R^+ are running and D^- wins the general election, and another one in which all candidates are running and D^- wins the general election.[28] Then, if $\succ^m = \succ_{D^-}^2$, any profile of equilibrium strategies in the traditional election system is such that D^-, R^-, and R^+ are running and D^- wins the general election. The case in which $\succ^m = \succ_{R^-}^2$ is analogous. ■

PROOF OF PROPOSITION 3: It follows from the fact that voting for his/her most preferred candidate, x_1^n or x_2^n, is a weakly dominant strategy for each voter in the general election. ■

PROOF OF PROPOSITION 4: To prove Proposition 4 we need three previous lemmas.

Lemma 1 *If $\succ^m = \succ_{D^+}$, any subgame beginning at the second stage of the top-two election system where all candidates are running is such that, for any voter i with $\succ_i = \succ_{D^+}$:*

(1) voting for R^+ and voting for R^- in the open primary are weakly dominated strategies (given the equilibrium continuation strategies) for i, and

(2) voting for D^+ and voting for D^- in the open primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for i.

Proof. Let i be such that $\succ_i = \succ_{D^+}$. First, note that voting for D^+ in the open primary weakly dominates to voting for R^+ and to voting for R^- (given the equilibrium continuation strategies) for i. We omit the proof of this point in the interest of brevity. It follows from the fact that, since $\succ^m = \succ_{D^+}$, if D^+ is one of the candidates passing to the next round, then D^+ will win the general election (see Table 5).

Suppose, without loss of generality, that $v = 100$. Now we prove that voting for D^- in the open primary is not weakly dominated by any other strategy (given the equilibrium continuation strategies) for i. First, we show that voting

[27] In particular, if (i) D^-, R^-, and R^+ decide to run and (ii) the continuation equilibrium strategies are such that when all candidates are running R^- wins the general election, then candidate D^+ is strictly better off not running than running (since $D^- \succ_{D^+} R^-$).

[28] Note that there is no equilibrium where all candidates are running and R^- wins the general election since, in that case, candidate D^+ would prefer to deviate and not run (because in this case, given any possible continuation equilibrium strategies, D^- would win the general election).

for D^+ in the open primary does not weakly dominates to voting for D^- for i. Consider a profile of strategies in the open primary for the other 99 voters, t^2_{-i}, such that 11 of them are voting for D^+, 29 are voting for D^-, 30 are voting for R^-, and 29 are voting for R^+. For simplicity, we denote as $(D^+, D^-, R^-, R^+) = (11, 29, 30, 29)$ this situation. If i votes for D^-, then D^- and R^- pass to the third stage and, since $\succ^m = \succ_{D^+}$, D^- wins the general election. If i votes for D^+, then D^- and R^+ are involved in a tie for second place and the confrontations R^- versus D^- and R^- versus R^+ are equally likely in the third stage. Since $\succ^m = \succ_{D^+}$, D^- will win the general election with probability 0.5 and R^- will win the general election with probability 0.5. Since $\succ_i = \succ_{D^+}$, this situation is worse for i than the one where D^- wins the general election with probability 1.

Voting for D^- is not weakly dominated for i by voting for R^- either. To see this, consider a profile t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$. If i votes for D^- in the open primary then, using a similar argument to the previous one, we obtain that D^- will win the general election. If instead of doing that i votes for R^-, then R^- will win the general election, what is worse for i.

To see that voting for D^- is not weakly dominated for i by voting for R^+, consider a profile t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$. If i votes for D^- in the open primary then D^- will win the general election. If instead of doing that i votes for R^+, then R^- will win the general election, what is worse for i.

Finally, we prove that voting for D^+ in the open primary is not weakly dominated by any other strategy (given the equilibrium continuation strategies) for i. That voting for D^+ is not weakly dominated by voting for R^+ or voting for R^- for i follows immediately from point (1) of this lemma. To see that voting for D^+ is not weakly dominated for i by voting for D^-, consider a profile t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (20, 20, 50, 9)$. If i votes for D^+ in the open primary then D^+ will win the general election. If instead of doing that i votes for D^-, then D^- will win the general election, what is worse for i. ■

Lemma 2 *If $\succ^m = \succ^1_{D^-}$, any subgame beginning at the second stage of the top-two election system where all candidates are running is such that:*

(1) voting for R^+ and voting for R^- in the open primary are weakly dominated strategies (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^1_{D^-}$,

(2) voting for R^+ and voting for R– in the open primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$, and

(3) voting for D^+ and voting for D^- in the open primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}\}$.

Proof. First note that, for any voter i such that $\succ_i = \succ^1_{D^-}$, voting for D^- in the open primary weakly dominates to voting for R^+ and to voting for R^- (given the equilibrium continuation strategies). We omit the proof of this point. It follows from the fact that, since $\succ^m = \succ^1_{D^-}$, if D^- is one of the candidates passing to the next round, then D^- (the most preferred candidate for any voter with preferences type $\succ^1_{D^-}$) will win the general election (see Table 5).

Suppose, without loss of generality that $v = 100$. Now we prove point (2). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (40, 20, 20, 19)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 12, 28)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 10, 19)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (70, 10, 9, 10)$ then i will be better off voting for R^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 28, 12)$ then i will be better off voting for R^- than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 19, 10)$ then i will be better off voting for R^- than voting for R^+.

Let i be such that $\succ_i = \succ^2_{D^-}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 10, 50, 19)$ then i will be better off voting for R^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 9, 31, 29)$ then i will be better off voting for R^+ than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 30, 29, 30)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (30, 9, 30, 30)$ then i will be better off voting for R^- than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$, then i will be better off voting for R^- than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 30, 30, 29)$, then i will be better off voting for R^- than voting for R^+.

Let i be such that $\succ_i = \succ^2_{R^-}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20,\ 10,\ 50,\ 19)$ then i will be better off voting for R^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 9, 31, 29)$ then i will be better off voting for R^+ than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 20, 50, 19)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (30, 9, 30, 30)$ then i will be better off voting for R^- than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$ then i will be better off voting for R^- than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 20, 19, 50)$ then i will be better off voting for R^- than voting for R^+.

The proof that voting for R^+ and voting for R^- are not weakly dominated for any voter i such that $\succ_i \in \{\succ^1_{R^-}, \succ_{R^+}\}$ is identical to the proof for the case that $\succ_i = \succ^2_{R^-}$.

Finally, we prove point (3). Let i be such that $\succ_i = \succ^1_{D^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then

i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 50, 20)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for D^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 50, 9)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+.

Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 9, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 50, 20)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for D^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. ■

Lemma 3 *If $\succ^m = \succ^2_{D^-}$, any subgame beginning at the second stage of the top-two election system where all candidates are running is such that:*

(1) voting for D^+ and voting for R^+ in the open primary are weakly dominated strategies (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^2_{D^-}$,

(2) voting for D^+ and voting for R^+ in the open primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$,

(3) voting for D^- in the open primary is not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}$, and

(4) voting for R^- in the open primary is not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$.

Proof. First, note that voting for D^- in the open primary weakly dominates to voting for D^+ and to voting for R^+ (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^2_{D^-}$.We omit the proof of this point. It follows from the fact that, since $\succ^m = \succ^2_{D^-}$, if D^- is one of the candidates passing to the next round, then D^- will win the general election (see Table 5).

Suppose without loss of generality that $v = 100$. Now we prove point (2). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 9, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+

than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 0, 0, 99)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (40, 20, 20, 19)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 12, 28)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 10, 19)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i = \succ^1_{D^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (19, 10, 20, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 0, 0, 99)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i = \succ^2_{R^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (8, 20, 50, 21)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 1, 98, 0)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 10, 20, 19)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 21, 8, 50)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 21, 8)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (8, 21, 50, 20)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i \in \{\succ^1_{R^-}, \succ_{R^+}\}$. The proof that voting for D^+ and voting for R^+ are not weakly dominated for i is identical to the proof in the case that $\succ_i = \succ^2_{R^-}$.

Next, we prove point (3). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^- is not weakly dominated for any i such that $\succ_i = \succ_{D^+}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. To see that voting for D^- is not weakly dominated for i such that $\succ_i = \succ^1_{D^-}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) =$

$(9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. To see that voting for D^- is not weakly dominated for i such that $\succ_i = \succ^2_{D^-}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 50, 9)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+.

Finally, we prove point (4). Let i be such that $\succ_i = \succ^2_{D^-}$. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for R^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (31, 8, 30, 30)$ then i will be better off voting for R^- than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^- than voting for R^+. The proof that voting for R^- is not weakly dominated for any voter i such that $\succ_i \in \{\succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$ is identical to the proof for the case that $\succ_i = \succ^2_{D^-}$. $\blacksquare$

Now, we can prove Proposition 4**.** We distinguish four cases.

Case 1. The subgame is such that, at most, there are two candidates running.

This case is trivial since the voters do not have to take any decision at the second stage.

Case 2. The subgame is such that only three candidates are running.

Suppose first that $\mathcal{C}^r = D^+D^-R^-$. Then, there are three potential pairs of candidates that may pass to the next round: D^+D^-, D^+R^-, and D^-R^-. From Table 5 it can be observed that, in this case, if $\succ^m = \succ_{D^+}$ only two candidates may win the general election: D^+ (if D^+ passes to the third stage) and D^- (if D^+ does not pass to the third stage). Then, voting for D^+ in the open primary is a weakly dominant strategy at the second stage for any voter who prefers D^+ to D^-, given the continuation equilibrium strategies at the third stage. Since $\succ^m = \succ_{D^+}$, a majority of voters prefers D^+ to D^- and D^+ will win the election in equilibrium. Using a similar argument it can be shown that, (i) if $\succ^m = \succ^1_{D^-}$, voting for D^- in the open primary is a weakly dominant strategy for any voter who prefers D^- to D^+, and D^- will win the election in equilibrium; (ii) if $\succ^m = \succ^2_{D^-}$, voting for D^- in the open primary is a weakly dominant strategy for any voter who prefers D^- to R^-, and D^- will win the election in equilibrium; (iii) if $\succ^m = \succ^2_{R^-}$, voting for R^- in the open primary is a weakly dominant strategy for any voter who prefers R^- to D^-, and therefore R^- will win the election in equilibrium; (iv) if $\succ^m = \succ^1_{R^-}$, voting for R^- in the open primary is a weakly dominant strategy for any voter who prefers R^- to D^-, and therefore R^- will win the election in equilibrium; (v) if $\succ^m = \succ_{R^+}$, voting for R^- in the open primary is a weakly dominant strategy for any voter who prefers R^- to D^-, and therefore R^- will win the election in equilibrium.

Suppose now that $\mathcal{C}^r = D^+D^-R^+$. In this case, only the pairs D^+D^-, D^+R^+, and D^-R^+ may pass to the next round. From Table 5 and using a similar argument to previous case it can be shown that: (i) if $\succ^m = \succ_{D^+}$, a majority of voters will vote for D^+ in the primary and D^+ will win the election in equilibrium; (ii) if $\succ^m \in \{\succ^1_{D^-}, \succ^2_{D^-}, \succ^2_{R^-}\}$, a majority of voters will vote for D^- in the open primary and D^- will win the election in equilibrium; (iii) if $\succ^m \in \{\succ^1_{R^-}, \succ_{R^+}\}$, a majority of voters will vote for R^+ in the open primary and R^+ will win the election in equilibrium.

If $\mathcal{C}^r = D^+R^-R^+$, only the pairs D^+R^-, D^+R^+, and R^-R^+ may pass to the next round. From Table 5 and using a similar argument to previous cases it can be shown that: (i) if $\succ^m \in \{\succ_{D^+}, \succ^1_{D^-}\}$, a majority of voters will vote for D^+ in the open primary and D^+ will win the election in equilibrium; (ii) if $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}\}$, a majority of voters will vote for R^- in the open primary and R^- will win the election in equilibrium; (iii) if $\succ^m = \succ_{R^+}$, a majority of voters will vote for R^+ in the open primary and R^+ will win the election in equilibrium.

Finally, if $\mathcal{C}^r = D^-R^-R^+$, only the pairs D^-R^-, D^-R^+, and R^-R^+ may pass to the next round. From Table 5 and using a similar argument to previous cases it can be shown that: (i) if $\succ^m \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}$, a majority of voters will vote for D^- in the open primary and D^- will win the election in equilibrium; (ii) if $\succ^m \in \{\succ^2_{R^-}, \succ^1_{R^-}\}$, a majority of voters will vote for R^- in the open primary and R^- will win the election in equilibrium; (iii) if $\succ^m = \succ_{R^+}$, a majority of voters will vote for R^+ in the open primary and R^+ will win the election in equilibrium.

<u>Case 3</u>. The subgame is such that the four candidates are running.

<u>Subcase 3.1</u>. $\succ^m \in \{\succ_{D^+}, \succ_{R^+}\}$.

Suppose first that $\succ^m = \succ_{D^+}$. Then, more than half of the voters have preferences of type $\succ_{D^+}$. From Lemma 1, voting for D^+ and D^- are the only two strategies that are not weakly dominated in the open primary for these voters (given the equilibrium continuation strategies). Then more than $\frac{v}{4}$ of the voters vote for D^+ and/or D^- in the open primary. Hence, D^+ and/or D^- go to third stage and, therefore, D^+ or D^- may win the general election in equilibrium.[29] The case in which $\succ^m = \succ_{R^+}$ is symmetric to the case in which $\succ^m = \succ_{D^+}$ (there is also a symmetric version of Lemma 1 for the case in which $\succ^m = \succ_{R^+}$).

<u>Subcase 3.2</u>. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. From points (1) and (3) of Lemma 2 we have that, if more than half of the voters were of type $\succ^1_{D^-}$, then more than $\frac{v}{4}$ of the voters would vote for D^+ and/or D^- in the open primary. In this case, from Table 5, D^+ and D^- would be the only two candidates who might win in equilibrium. The fact that $\succ^m = \succ^1_{D^-}$, however, does not imply that more than $\frac{v}{2}$ voters are of type $\succ^1_{D^-}$. Then, given points (2) and (3) of Lemma 2, we cannot rule out the possibility that any pair of candidates can pass to the next round in equilibrium. In particular, it is possible that the candidates who pass to the next round are R^- and R^+ in which case the winning candidate would be R^-. The only equilibrium result that we can rule out is that R^+ wins the general election (from Table 5, since $\succ^m = \succ^1_{D^-}$, the only chance for R^+ to win would be

[29] For example, a situation where all voters type $\succ_{D^+}$ vote for D^+ is an equilibrium in the second stage resulting in D^+. Similarly, a situation such that (i) all voters type $\succ_{D^+}$ vote for D^- and (ii) the candidate who gets most votes from the rest is not D^+, is an equilibrium in the second stage resulting in D^-.

that all voters were voting for R^+ in the open primary; this situation, however, would never be an equilibrium since any democratic partisan would prefer to vote for D^+ or D^-). Therefore, D^+, D^-, or R^- may win the general election in equilibrium.[30] The case in which $\succ^m = \succ^1_{R^-}$ is symmetric to the case in which $\succ^m = \succ^1_{D^-}$ (there is also a symmetric version of Lemma 2 for the case in which $\succ^m = \succ^1_{R^-}$).

Subcase 3.3. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. From points (1), (3), and (4) of Lemma 3, if more than half of the voters were of type $\succ^2_{D^-}$, then more than $\frac{v}{4}$ of the voters would vote for R^- and/or D^- in the open primary. In this case, from Table 5, R^- and D^- would be the only two candidates who might win in equilibrium. The fact that $\succ^m = \succ^2_{D^-}$, however, does not imply that more than $\frac{v}{2}$ voters are of type $\succ^2_{D^-}$. Given point (2) of Lemma 3, it is possible that the candidates who pass to the next round are D^+ and R^+ in which case the winning candidate would be D^+. As in the case that $\succ^m = \succ^1_{D^-}$, the only equilibrium result that we can rule out is that R^+ wins the general election. Therefore, D^-, R^-, or D^+ may win the general election in equilibrium.[31] The case in which $\succ^m = \succ^2_{R^-}$ is symmetric to the case in which $\succ^m = \succ^2_{D^-}$ (there is also a symmetric version of Lemma 3 for the case in which $\succ^m = \succ^2_{R^-}$). ■

PROOF OF REMARK 2: Suppose first that $\succ^m = \succ_{D^+}$. As we have shown in Proposition 4, in the subgames beginning at the second stage of the top-two election system where all candidates are running, there are equilibria in which D^+ wins the general election and equilibria in which D^- wins the general election. However, there is no equilibrium in which there is a tie in the open primary and both, D^+ and D^-, have a positive probability of winning the general election. The only possibility for that to happen would be that, in the open primary, D^- was the most voted candidate and D^+ was one of the second most voted candidates tied with at least another candidate.[32] In that case, since

[30] For example, a situation where all voters type $\succ_{D^+}$ and $\succ^1_{D^-}$ vote for D^- while the rest vote for R^- would be an equilibrium in the second stage resulting in D^- (since $\succ^m = \succ^1_{D^-}$, more than a half of the voters are of type $\succ_{D^+}$ or $\succ^1_{D^-}$). Similarly, a situation where all voters type $\succ_{D^+}$ and $\succ^1_{D^-}$ vote for D^+, while the rest vote for R^- would be an equilibrium in the second stage resulting in D^+. Finally, if less than a half of the voters are of type $\succ^1_{D^-}$, a situation where half of the voters type $\succ^1_{D^-}$ vote for D^-, the other half of the voters type $\succ^1_{D^-}$ vote for D^+, half of the rest of voters vote for R^-, while the other half vote for R^+, would be an equilibrium in the second stage resulting in R^-.

[31] For example, a situation where all voters type $\succ_{D^+}$, $\succ^1_{D^-}$, and $\succ^2_{D^-}$ vote for D^- while the rest vote for D^+ would be an equilibrium in the second stage resulting in D^- (since $\succ^m = \succ^2_{D^-}$, more than a half of the voters are of type $\succ_{D^+}$, $\succ^1_{D^-}$, or $\succ^2_{D^-}$). Similarly, a situation where all voters type $\succ^2_{D^-}$, $\succ^2_{R^-}$, $\succ^1_{R^-}$, and $\succ_{R^+}$ vote for R^-, while the rest vote for D^+ would be an equilibrium in the second stage resulting in R^- (since $\succ^m = \succ^2_{D^-}$, more than a half of the voters are of type $\succ^2_{D^-}$, $\succ^2_{R^-}$, $\succ^1_{R^-}$, or $\succ_{R^+}$) Finally, if less than a half of the voters are of type $\succ^2_{D^-}$, a situation where half of the voters type $\succ^2_{D^-}$ vote for R^-, the other half of the voters type $\succ^2_{D^-}$ vote for D^-, half of the rest of voters vote for R^+, while the other half vote for D^+, would be an equilibrium in the second stage resulting in D^+.

[32] Note that in this case a triple or a quadruple tie in the first position is not possible.

$\succ^m = \succ_{D^+}$, at least one voter with preferences type $\succ_{D^+}$ should be voting for a candidate different from D^+ in the open primary. Note that this would not be an equilibrium since this voter would be better off if he/she voted for D^+ in the open primary (in that case, D^+ would win the general election with probability equal to one).

Suppose now that $\succ^m = \succ^1_{D^-}$. As we have shown in Proposition 4, in the subgames beginning at the second stage of the top-two election system where all candidates are running, there are equilibria in which D^- wins the general election, equilibria in which D^+ wins the general election, and equilibria in which R^- wins the general election. Nevertheless, there is no equilibrium in which there is a tie in the open primary and more than one candidate have a positive probability of winning the general election. To see this note that:

(1) There is no equilibrium in which there is a tie in the open primary and both, D^+ and D^-, have positive probability of winning the general election (but not R^-). The only possibility for that to happen would be that, in the open primary, D^- was involved in a tie in the first or second positions. Since $\succ^m = \succ^1_{D^-}$, at least one voter whose preferences are not of type $\succ_{D^+}$ should be voting for a candidate different from D^- in the open primary. This would not be an equilibrium since this voter would be better off if he/she voted for D^- in the open primary (in that case, D^- would win the general election with probability equal to one).

(2) There is no equilibrium in which there is a tie in the open primary and both, D^- and R^-, have positive probability of winning the general election (but not D^+). The only possibility for that to happen would be that, in the open primary, D^- was involved in a tie in the first or second positions. Since $\succ^m = \succ^1_{D^-}$, at least one voter whose preferences are of type $\succ_{D^+}$ or $\succ^1_{D^-}$ should be voting for a candidate different from D^- in the open primary. Note that this voter prefers D^- to R^-. Then, this would not be an equilibrium since this voter would be better off if he/she voted for D^- in the open primary (in that case, D^- would win the general election with probability equal to one).

(3) There is no equilibrium in which there is a tie in the open primary and both, D^+ and R^-, have positive probability of winning the general election (but not D^-). The only possibility for that to happen would be that, in the open primary, D^+ was involved in a tie in the first or second positions and D^- was in the last position. Since $\succ^m = \succ^1_{D^-}$, at least one voter whose preferences are of type $\succ_{D^+}$ or $\succ^1_{D^-}$ should be voting for a candidate different from D^+ in the open primary. Note that this voter prefers D^+ to R^-. Then, this would not be an equilibrium since this voter would be better off if he/she voted for D^+ in the open primary (in that case, D^+ would win the general election with probability equal to one).

(4) There is no equilibrium in which there is a tie in the open primary and D^-, D^+, and R^- have positive probability of winning the general election. There would be three possibilities for that to happen: (4.1) R^+ was in the first position and D^+, D^-, and R^- were involved in a tie in the second position, (4.2) R^- was in the first position and D^+, D^-, and R^+ were involved in a tie in the second position, and (4.3) D^+, D^-, R^-, and R^+ were involved in a tie in

the first position. Suppose that case (4.1) occurs. For that situation being an equilibrium, all voters type $\succ_{D^+}$ should be voting for D^+ (otherwise they could improve by deviating unilaterally and voting for D^+) and all voters type $\succ^1_{D^-}$ and $\succ^2_{D^-}$ should be voting for D^- (otherwise they could improve by deviating unilaterally and voting for D^-). Since $\succ^m = \succ^1_{D^-}$, $\#\{i \in \mathcal{V} : \succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}\} > \frac{v}{2}$. Therefore, the number of voters who were voting for D^+ and/or D^-should be greater than $\frac{v}{4}$. Hence, since D^+, D^-, and R^- were involved in a tie, the number of voters who were voting for each of these candidates should be greater than $\frac{v}{4}$, which contradicts that R^+ was in the first position. The proofs that cases (4.2) and (4.3) cannot occur are analogous.

Suppose now that $\succ^m = \succ^2_{D^-}$. As we have shown in Proposition 4, in the subgames beginning at the second stage of the top-two election system where all candidates are running, there are equilibria in which D^- wins the general election, equilibria in which R^- wins the general election, and equilibria in which D^+ wins the general election. However, there is no equilibrium in which there is a tie in the open primary and more than one candidate have a positive probability of winning the general election. To see this note that:

(1) There is no equilibrium in which there is a tie in the open primary and both, D^- and R^-, have positive probability of winning the general election (but not R^-). The only possibility for that to happen would be that, in the open primary, D^- was involved in a tie in the first or second positions. Since $\succ^m = \succ^2_{D^-}$, at least one voter whose preferences are of type $\succ_{D^+}$, $\succ^1_{D^-}$, or $\succ^2_{D^-}$ should be voting for a candidate different from D^- in the open primary. This would not be an equilibrium since this voter would be better off if he/she voted for D^- in the open primary (in that case, D^- would win the general election with probability equal to one).

(2) There is no equilibrium in which there is a tie in the open primary and both, D^- and D^+, have positive probability of winning the general election (but not R^-). The only possibility for that to happen would be that, in the open primary, D^- was involved in a tie in the first or second positions. Since $\succ^m = \succ^2_{D^-}$, at least one voter whose preferences are not of type $\succ_{D^+}$ should be voting for a candidate different from D^- in the open primary. Note that this voter prefers D^- to D^+. Then, this would not be an equilibrium since this voter would be better off if he/she voted for D^- in the open primary (in that case, D^- would win the general election with probability equal to one).

(3) There is no equilibrium in which there is a tie in the open primary and both, R^- and D^+, have positive probability of winning the general election (but not D^-). The only possibility for that to happen would be that, in the open primary, R^- was involved in a tie in the first or second positions and D^- was in the last position. Since $\succ^m = \succ^2_{D^-}$, at least one voter whose preferences are of type $\succ^2_{D^-}$, $\succ^2_{R^-}$, $\succ^1_{R^-}$, or $\succ_{D^+}$ should be voting for a candidate different from R^- in the open primary. Note that this voter prefers R^- to D^+. Then, this would not be an equilibrium since this voter would be better off if he/she voted for R^- in the open primary (in that case, R^- would win the general election with probability equal to one).

(4) There is no equilibrium in which there is a tie in the open primary and

D^-, R^-, and D^+ have positive probability of winning the general election. There would be three possibilities for that to happen: (4.1) R^+ was in the first position and D^+, D^-, and R^- were involved in a tie in the second position, (4.2) D^+ was in the first position and D^-, R^-, and R^+ were involved in a tie in the second position, and (4.3) D^+, D^-, R^-, and R^+ were involved in a tie in the first position. Suppose that case (4.1) occurs. For that situation being an equilibrium, all voters type $\succ^1_{D^-}$ and $\succ^2_{D^-}$ should be voting for D^- (otherwise they could improve by deviating unilaterally and voting for D^-) and all voters type $\succ^2_{R^-}$, $\succ^1_{R^-}$, and $\succ_{R^+}$ should be voting for R^- (otherwise they could improve by deviating unilaterally and voting for R^-). Since $\succ^m = \succ^2_{D^-}$, $\#\{i \in \mathcal{V} : \succ_i \in \{\succ^1_{D^-}, \succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}\} > \frac{v}{2}$. Therefore, the number of voters who were voting for D^- and/or R^-should be greater than $\frac{v}{4}$. Hence, since D^+, D^-, and R^- were involved in a tie, the number of voters who were voting for each of these candidates should be greater than $\frac{v}{4}$, which contradicts that R^+ was in the first position. The proofs that cases (4.2) and (4.3) cannot occur are analogous.

The cases where $\succ^m = \succ_{R^+}$, $\succ^m = \succ^1_{R^-}$, and $\succ^m = \succ^2_{R^-}$ are mirror images of cases where $\succ^m = \succ_{D^+}$, $\succ^m = \succ^1_{D^-}$, and $\succ^m = \succ^2_{D^-}$, and the corresponding arguments apply. ■

PROOF OF THEOREM 2: We distinguish three cases.

Case 1. $\succ^m \in \{\succ_{D^+}, \succ_{R^+}\}$.

Suppose first that $\succ^m = \succ_{D^+}$. From Proposition 4, we know who wins the general election depending on who is running in this case. Table 14 summarizes this information. Note that Y is a weakly dominant strategy for each candidate at the first stage given the continuation equilibrium strategies. In this case, any profile of equilibrium strategies in the top-two election system is such that all candidates are running and D^+ or D^- win the general election. The case in which $\succ^m = \succ_{R^+}$ is analogous.

R^+	D^+	R^-: Y, D^-: Y	R^-: Y, D^-: N	R^-: N, D^-: Y	R^-: N, D^-: N
Y	Y	D^+ or D^-	D^+	D^+	D^+
Y	N	D^-	R^-	D^-	R^+
N	Y	D^+	D^+	D^+	D^+
N	N	D^-	R^-	D^-	$\emptyset$

Table 14 Winner in equilibrium in the top-two election system depending on who is running when $\succ^m = \succ_{D^+}$.

Case 2. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose first that $\succ^m = \succ^1_{D^-}$. From Proposition 4, we know who wins the general election depending on who is running in this case. Table 15 summarizes this information.

R^+	R^-	D^+	D^-: Y	D^-: N
Y	Y	Y	D^+ or D^- or $R^{-(*)}$	D^+
Y	Y	N	D^-	R^-
Y	N	Y	D^-	D^+
Y	N	N	D^-	R^+
N	Y	Y	D^-	D^+
N	Y	N	D^-	R^-
N	N	Y	D^-	D^+
N	N	N	D^-	$\emptyset$

$^{(*)}$ Only if $\#\{i \in V : \succ_i = \succ^1_{D^-}\} < v/2$

Table 15 Winner in equilibrium in the top-two election system depending on who is running when $\succ^m = \succ^1_{D^-}$.

We distinguish three subcases:

Subcase 2.1. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^+ wins the general election (i.e., D^+ is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for candidates D^+ and D^- at the first stage given the continuation equilibrium strategies. Given this, in Table 15 can be seen that there are three types of equilibria: one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running (all candidates running is not an equilibrium because in that case both, R^+ and R^-, have incentives to unilaterally deviate). The three types of equilibrium yields the same result: D^- wins the general election.

Subcase 2.2. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^- wins the general election (i.e., D^- is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for all candidates at the first stage given the continuation equilibrium strategies, and then D^- wins the general election.

Subcase 2.3. The equilibrium strategies in the second and third stages are such that, if all candidates are running, R^- wins the general election (i.e., R^- is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for candidates R^+ and R^- at the first stage given the continuation equilibrium strategies. Moreover, Y is a weakly dominant strategy for candidate D^- if his/her preferences are of type $\succ^2_{D^-}$, but not if his/her preferences are of type $\succ^1_{D^-}$ (in the latter case, if the other three candidates are running, D^- prefers not to run, since he/she prefers D^+ to R^-). Then, if the preferences of candidate D^- are of type $\succ^2_{D^-}$, candidates R^+, R^-, and D^- will run and, given this, D^+ prefers not to run. This equilibrium results in D^- winning the general election. If the preferences of candidate D^- are of type $\succ^1_{D^-}$, then there are two types of equilibria: one in which all candidates except D^+ are running (which result in D^- winning the general election), and one in which all candidates except D^- are running (which result in D^+ winning the general election).

The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

<u>Case 3</u>. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

From Proposition 4, we know who wins the general election depending on who is running in this case. Table 16 summarizes this information.

			R^-			
			Y		N	
			D^-		D^-	
			Y	N	Y	N
R^+	Y	D^+ Y	$D^{+(*)}$ or D^- or R^-	R^-	D^-	D^+
		D^+ N	D^-	R^-	D^-	R^+
	N	D^+ Y	D^-	R^-	D^-	D^+
		D^+ N	D^-	R^-	D^-	$\emptyset$

$^{(*)}$ Only if $\#\{i \in V : \succ_i = \succ^2_{D^-}\} < v/2$

Table 16 Winner in equilibrium in the top-two election system depending on who is running when $\succ^m = \succ^2_{D^-}$.

We distinguish three subcases:

<u>Subcase 3.1</u>. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^+ wins the general election (i.e., D^+ is in the upper left cell in Table 16).

In this case, Y is a weakly dominant strategy for candidate D^+ at the first stage given the continuation equilibrium strategies. If the preferences of candidate D^- are of type $\succ^1_{D^-}$, then Y is also a weakly dominant strategy for him/her given the continuation equilibrium strategies, and there are three types of equilibria: one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running (all candidates running is not an equilibrium because in that case both, R^+ and R^-, have incentives to unilaterally deviate). The three types of equilibrium yields the same result: D^- wins the general election. If the preferences of candidate D^- are of type $\succ^2_{D^-}$, then Y is not a weakly dominant strategy for him/her given the continuation equilibrium strategies (if the other three candidates are running, D^- prefers not to run, since he/she prefers R^- to D^+). In this case, there are four types of equilibria: one in which all candidates except D^- are running, one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running. In the first type of equilibrium R^- wins the general election, while in the other three types of equilibria D^- wins the general election.

Subcase 3.2. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^- wins the general election (i.e., D^- is in the upper left cell in Table 16).

In this case, Y is a weakly dominant strategy for all candidates at the first stage given the continuation equilibrium strategies, and then D^- wins the general election.

Subcase 3.3. The equilibrium strategies in the second and third stages are such that, if all candidates are running, R^- wins the general election (i.e., R^- is in the upper left cell in Table 16).

In this case, Y is a weakly dominant strategy for candidates D^-, R^-, and R^+ at the first stage given the continuation equilibrium strategies. Then, candidates D^-, R^-, and R^+ will run and, given this, D^+ prefers not to run. This equilibrium results in D^- winning the general election.

The case in which $\succ^m = \succ^2_{R^-}$ is analogous. ■

PROOF OF THEOREM 3: We distinguish three cases:

Case 1. $\succ^m \in \{\succ_{D^+}, \succ_{R^+}\}$.

Suppose that $\succ^m = \succ_{D^+}$. Table 11 shows who wins the general election depending on who is running in this case. Observe that Y is a weakly dominant strategy for D^+ given the continuation equilibrium strategies. Unlike what happens when there is no cost of running, however, now Y is not a weakly dominant strategy for D^-, R^-, and R^+ given the continuation equilibrium strategies (for example, if the other three candidates are running, D^- prefers not to run, since the fact that he/she runs does not change the result of the general election). Note that $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$, $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(Y,N,Y,N) \succ_{D^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$, and $(Y,N,N,N) \succ_{D^+} (N,N,N,N)$. Therefore, any profile of equilibrium strategies is such that $s^1 = (Y,N,N,N)$ and D^+ wins the general election. The case in which $\succ^m = \succ_{R^+}$ is analogous.

Case 2. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. In this case, Table 12 shows who wins the general election depending on who is running when the election system is the traditional election system. Note that $(Y,Y,Y,N) \succ_{R^+} (Y,Y,Y,Y)$, $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(Y,N,N,N) \succ_{D^+} (N,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if $\succ_D^m = \succ_{D^+}$, then $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^+} (N,Y,N,N)$. Similarly, if $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, then $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$. Therefore: (i) if $\succ_D^m = \succ_{D^+}$, any profile of equilibrium strategies is such that $s^1 = (Y,N,N,N)$ and D^+ wins the general election, and (ii) if $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[33] The case in which $\succ^m = \succ_{R^-}^1$ is analogous.

Case 3. $\succ^m \in \{\succ_{D^-}^2, \succ_{R^-}^2\}$.

Suppose that $\succ^m = \succ_{D^-}^2$. In this case, Table 13 shows who wins the general election depending on who is running when the election system is the traditional election system. Note that $(N,Y,Y,Y) \succ_{D^+} (Y,Y,Y,Y)$, $(Y,N,Y,N) \succ_{R^+} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(N,Y,Y,N) \succ_{D^+} (Y,Y,Y,N)$, $(N,N,Y,N) \succ_{D^+} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(Y,N,Y,N) \succ_{R^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if $\succ_D^m = \succ_{D^+}$, then $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^+} (N,Y,N,N)$, and hence there is no profile of equilibrium strategies. If $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, however, then $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, and therefore any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election. The case in which $\succ^m = \succ_{R^-}^2$ is analogous.[34] ■

PROOF OF THEOREM 4: We distinguish three cases:

Case 1. $\succ^m \in \{\succ_{D^+}, \succ_{R^+}\}$.

Suppose that $\succ^m = \succ_{D^+}$. In this case, Table 14 shows who wins the general election depending on who is running when the election system is the top-two system. Note that $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$, $(Y,Y,N,N) \succ_{D^+} (N,Y,N,N)$, and $(Y,N,N,N) \succ_{D^+} (N,N,N,N)$. Moreover, if the equilibrium strategies in the second and third stages are such that

[33] Note that, in this case, Y is not a weakly dominant strategy for R^- and R^+ given the continuation equilibrium strategies. Moreover, if $\succ_D^m = \succ_{D^+}$, then Y is not a weakly dominant strategy for D^-, and if $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, then Y is not a weakly dominant strategy for D^+ (given the continuation equilibrium strategies).

[34] Note that, in this case, (i) Y is not a weakly dominant strategy for R^- and R^+, (ii) if $\succ_D^m = \succ_{D^+}$, then Y is not a weakly dominant strategy for D^-, and (iii) if $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, then Y is not a weakly dominant strategy for D^+ (given the continuation equilibrium strategies).

D^+ (respectively D^-) wins the general election if all candidates are running, then $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$ (respectively $(N,Y,Y,Y) \succ_{D^+} (Y,Y,Y,Y)$). Therefore, any profile of equilibrium strategies is such that $s^1 = (Y,N,N,N)$ and D^+ wins the general election.[35] The case in which $\succ^m = \succ_{R^+}$ is analogous.

Case 2. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. In this case, Table 15 shows who wins the general election depending on who is running when the election system is the top-two system. Note that $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if the equilibrium strategies in the second and third stages are such that D^+ (respectively D^- or R^-) wins the general election if all candidates are running, then $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$ (respectively $(N,Y,Y,Y) \succ_{D^+} (Y,Y,Y,Y)$). Therefore, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[36] The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

Case 3. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. Table 16 shows who wins the general election depending on who is running when the election system is the top-two system. Note that $(N,N,Y,Y) \succ_{D^+} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(N,N,Y,N) \succ_{D^+} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if the equilibrium strategies in the second and third stages are such that D^+ or D^- (respectively R^-) wins the general election if all candidates are running, then $(Y,Y,N,Y) \succ_{R^-} (Y,Y,Y,Y)$ (respectively $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$). Therefore, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[37] The case where $\succ^m = \succ^2_{R^-}$ is analogous. ■

[35] Observe that, in this case, Y is not a weakly dominant strategy for D^-, R^- and R^+ given the continuation equilibrium strategies. Moreover, if the equilibrium strategies in the second and third stages are such that D^- wins the general election if all candidates are running, then Y is not a weakly dominant strategy for D^+ either.

[36] Observe that, in this case, Y is not a weakly dominant strategy for D^+, R^- and R^+ given the continuation equilibrium strategies. Moreover, if the equilibrium strategies in the second and third stages are such that D^+ wins the general election if all candidates are running (or R^- wins the general election and the preferences of candidate D^- are type $\succ^2_{D^-}$), then Y is not a weakly dominant strategy for D^+ either.

[37] Observe that, in this case, Y is not a weakly dominant strategy for D^+, R^- and R^+ given the continuation equilibrium strategies. Moreover, if the equilibrium strategies in the second and third stages are such that R^- wins the general election if all candidates are running (or D^+ wins the general election and the preferences of candidate D^- are type $\succ^2_{D^-}$), then Y is not a weakly dominant strategy for D^+ either.

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