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Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249607

A NONLINEAR PRODUCT DIFFERENTIATION MODEL À LA COURNOT: A NEW LOOK TO THE NEWSPAPERS INDUSTRY.*

Mercedes Esteban-Bravo[1], José M. Vidal-Sanz[2]

Abstract

In this work, we develop a new model for competition in markets with differentiated products. In addition, we present a consumer model designed to produce a flexible nonlinear inverse demand system that resembles the classical Multinomial Logit model, and discuss several extensions. We characterize firms competition in quantities based on the inverse demand system. The model is applied to the Spanish newspaper industry. This is a highly competitive two-sided market whose revenues are generated from sales and to a larger extent from advertising driven by its circulation. We then characterize the Perfect Equilibrium by conditional moment conditions, and estimate the parameters using the Generalized Method of Moments.

Keywords: newspapers, differentiated products, dynamic equilibrium, Generalized Method of Moments.Advertising Expenditure, time series, persistence, cointegration, structural changes.

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[1] Mercedes Esteban-Bravo, Department of Business Administration, University Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), Spain; tel: +34 91 624 8921; fax: +34 91 624 8921; e-mail: mesteban@emp.uc3m.es

[2] Jose M. Vidal-Sanz, Department of Business Administration, University Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), Spain; tel: +34 91 624 8642; fax: +34 91 624 9607; e-mail: jvidal@emp.uc3m.es

1 Introduction

In a Monopolistic Competition (MC), several companies produce similar but differentiated products, and each company sets its price and sales quantity constrained by the market demand system. This form of competition was initially studied by Chamberlin (1933) and to some extent by Robinson (1933), but this model was never formulated in terms of analytical equations. A second and more successful wave within MC literature was spurred by the seminal papers of Spence (1976) and especially by Dixit and Stiglitz (1977). This approach is workable to compute the demand faced by each firm. It is built under the premise that a representative consumer maximizes a CES utility function over the substitute products subject to a budget constraint. In a Nash equilibrium all companies maximize their profits in prices, and their decisions are compatible in the demand system. Similar results have been obtained by considering heterogenous consumers (e.g. Sattinger 1984, Hart 1985a, 1985b). Zhelobodko et al. (2012) discuss some limitations of this model. The third wave of MC models was based on McFadden's (1981, 1984) multinomial and nested Logit models, where consumers face mutually exclusive choices maximizing a stochastic utility function (see Perloff and Salop 1985, Berry 1994 and Berry et al. 1995). Additional refinements have been considered such as McFadden's (1978) GEV models (e.g. Bresnahan et al., 1997), and mixed Multinomial Logits (see McFadden and Train, 2000). This approach is widespread in the *new empirical industrial organization* (see e.g. Berry et al. 1996, 1999 and 2004) and the marketing literature (Bensanko et al. 1988, Allenby and Rossi 1991); and to a lesser extent in economic theory, due in part to the micro-foundation weakness when dealing with the budget constraint in the consumer decision problem and the simplification of rent-effects (see Berry et al. 1994 and Petrin 2002). A crucial characteristic of both the Spence-Dixit-Stiglitz and the stochastic utility models is that they produce an aggregated demand system. Friedman (1977) and Shubik with Levitan (1980) actually postulated to use an aggregate demand system directly in MC oversimplification of the individual consumer underpinnings.

Independently of the attention given to the behavior of representative/heterogeneous consumers, all these MC models compute the aggregated demand for each firm as a function of their own price and the competitor ones. Then, the equilibrium is solved in prices (as a Bertrand competition). Therefore, costs become the central issue in the discussion about profitability, and the potential entrance of additional competitors. The way in which products are differentiated is linked to these costs, and this problem is often endogenized in the model considering of a two-step game for product design and prices. In these models, the market power often leads to a low level of strategic interaction, where the decision of each firm has little impact on their competitors' payoffs. But price competition may be inadequate for many MC industries where the companies fix the price strategically from time to time, print it on the cover of the product, and adapt their sales to consumers' orders. For example, this is the case with newspapers. Unfortunately, given the difficulty to obtain an inverse aggregated demand system (unless we consider a linear demand system), Cournot equilibrium is generally not considered in the MC context.

In this paper we present a product differentiation model where firms compete in quantities. The model is based on a new consumer decision model, specifically designed to produce a flexible nonlinear inverse demand system appropriate for its use in MC where firms compete in quantities. A key advantage of the presented inverse demand system is its convenience for applied work, as it resembles the classical Multinomial Logit model. We discuss several conceptual extensions of the model, in particular the case where consumers' decision are partitioned over different subcategories of substitutive products.

We apply the proposed to the Spanish newspaper industry. The newspaper industry is highly competitive and exposed to environmental threats, and their revenues are dependent on interrelated streams provided by sales and advertising. The media industry can also be considered as a particular type of two-sided market. These are markets characterized by bilateral network externalities. The demand on one side (advertisers) depends on the consumption of agents on the other market side (readers). Two sided markets posses specific features in terms of pricing principles and externalities. There has been a recent surge of interest in two-sided markets in Industrial Organization (IO) after the seminal papers by Rochet and Tirole (2002, 2003), Caillaud and Jullien (2003), and Armstrong (2006).

Newspaper's advertising spending depends on its expected readership; i.e. the expected total circulation multiplied by the average number of readers per copy. A newspaper's circulation is the number of copies that it distributes on an average day. Circulation is not the same as copies sold, since some copies of newspapers are distributed for free. To increase circulation (or the advertising share of the market), newspapers give away free copies distributed by hand, postal delivery, or placing racks in well-transited locations such as public transport stations, hospitals, shopping centers, universities and so on. Publishers can also cut prices (subsidize prices) to increase circulation. Higher circulation attracts advertisers, and these revenues enable a spiral down of prices and a spiral up of sales, with advertising accounting for 70-80% of their revenues. Newsstand sales account for nearly 15-20% revenues, and subscriptions just for the 1-3%. The efficacy of these strategies to attract advertising is heterogeneous, for it depends on the average number of newspaper readers per copy.

We model the Spanish newspapers market taking into account all the interactions of these factors, as: (1) The traditional newspapers' returns at each period depend on current sales, and the share of advertising determined by previous circulation (sales and given-away issues). Given-away issues introduce a negative externality on current demand; (2) All firms compete in quantity, and we use the proposed MC model to address this goal; (3) To account for the role of information in the competitiveness of the market, our modelling framework considers the effects of rational expectations, anticipating how many other competing brands are likely to be produced and what the associated residual inverse demand is. The model can be estimated by the Generalized Method of Moments (GMM). Our results prove how this approach allows companies in a monopolistic competition to set production, adjusting their prices implicitly.

2 Benchmark model with a representative consumer

Consider an economy consisting of a representative consumer and L firms competing for quantities q_l of a non-homogeneous product.

Consumers. Consumer preferences over consumption bundles $(q_1, ..., q_L)$ are represented by a utility function

$$u(q) = \sum_{l=1}^{L} \frac{1}{\alpha_l} \exp(\mu_l + \alpha_l \, q_l), \tag{1}$$

where $q_l \geq 0$ is the quantity chosen for product l. The parameters $\{\mu_l, \alpha_l\}_{l=1}^{L}$ reflect taste differences over the substitutive products. Note that $u(q_l)$ is monotonously increasing in q_l for $\alpha_l \neq 0$, since

$$\frac{\partial u}{\partial q_l} = \exp(\mu_l + \alpha_l\, q_l) > 0.$$

For each product l, the coefficients must satisfy that $\alpha_l < 0$, so that $\partial^2 u / \partial q_l^2 < 0$. The utility takes negative values for negative α's, but a monotonous transformation could re-scale u into the positive half-line without loss of generality (by adding $-\sum_{l=1}^{L} \alpha_l^{-1}$). To avoid overparametrization, we typically normalize the coefficient $\mu_L = 0$.

The representative consumer's expenditure constraint is given by $\sum_{l=1}^{L} p_l\, q_l \leq m$, where p_l is the price of product l at and m is the representative consumer's total shopping budget. From the optimality First Order Conditions (FOC), the representative consumer demands a bunch of products satisfying:

$$\exp(\mu_l + \alpha_l q_l) - \lambda\, p_l = 0, \qquad l = 1, ..., L, \tag{2}$$

where λ is the Lagrange multiplier of the budget constraint. The ratio of any l, j optimality conditions yields

$$\frac{p_l}{p_j} = \frac{\exp(\mu_l + \alpha_l\, q_l)}{\exp\left(\mu_j + \alpha_j\, q_j\right)}. \tag{3}$$

Taking logarithms, a linear structural demand model in q is satisfied:

$$\ln p_l - \ln p_j = \mu_l - \mu_j + \alpha_l\, q_l - \alpha_j\, q_j, \tag{4}$$

for all l, j. These equations together with the budget constraint define a linear system in quantities that can be used to solve the Marshallian demand system (see Appendix A1 for details). Notice that from (4), the demand price elasticity is negative, and given by

$$\varepsilon_l = \frac{dq_l/q_l}{dp_l/p_l} = \frac{dq_l}{d\ln p_l}\frac{1}{q_l} = \left(q_l \frac{d\ln p_l}{dq_l}\right)^{-1} = \frac{1}{\alpha_l q_l} < 0.$$

The cross elasticities are given by

$$\varepsilon_{l,j} = \frac{dq_l/q_l}{dp_j/p_j} = \frac{dq_l}{d\ln p_j}\frac{1}{q_l} = \left(q_l \frac{d\ln p_j}{dq_l}\right)^{-1} = -\frac{1}{\alpha_j q_l} > 0,$$

meaning that when the price of product j goes up the quantity demanded of product l will increase; in other words, the two goods l, j are substitutes for each other.

Next we focus on determining the inverse demand function. Given the mean of prices $\pi = L^{-1}\sum_{l=1}^{L} p_l$, and using the optimality conditions (3), we have

$$\frac{L\pi}{p_j} = \sum_{l=1}^{L} \frac{p_l}{p_j} = \frac{\sum_{l=1}^{L} \exp\left(\mu_l + \alpha_l q_l\right)}{\exp\left(\mu_j + \alpha_j q_j\right)}.$$

Thus, the inverse of demand function for any product $j = 1, \ldots, L$ is given by

$$p_j = \frac{\exp\left(\mu_j + \alpha_j q_j\right)}{\sum_{l=1}^{L} \exp\left(\mu_l + \alpha_l q_l\right)} L\pi. \tag{5}$$

Notice also that, from the FOC conditions (2), the Lagrange multiplier is given by,

$$\lambda = \frac{\exp\left(\mu_l + \alpha_l q_l\right)}{p_l} = \sum_{l=1}^{L} \exp\left(\mu_l + \alpha_l q_l\right) / L\pi.$$

The extension of the model to the case of heterogeneous consumers is straightforward (see Appendix A2).

Note that we can consider normalized prices in the simplex, setting $L\pi = 1$, then we obtain the inverse demand function for each product

$$p_j = \frac{\exp\left(\mu_j + \alpha_j q_j\right)}{\sum_{l=1}^{L} \exp\left(\mu_l + \alpha_l q_l\right)}, \qquad j = 1, \ldots, L. \tag{6}$$

However, in many instances the mean price π should be included to account for exogenous nominal price effects. Therefore, the inverse demand system has analytical expression that resembles that of a Multinomial Logistic conditional probability distribution, and therefore it is quite flexible and can be easily estimated. A price inelastic product L can be also considered as a limit case, if $\mu_L, \alpha_L, \beta_L \to 0$, yielding

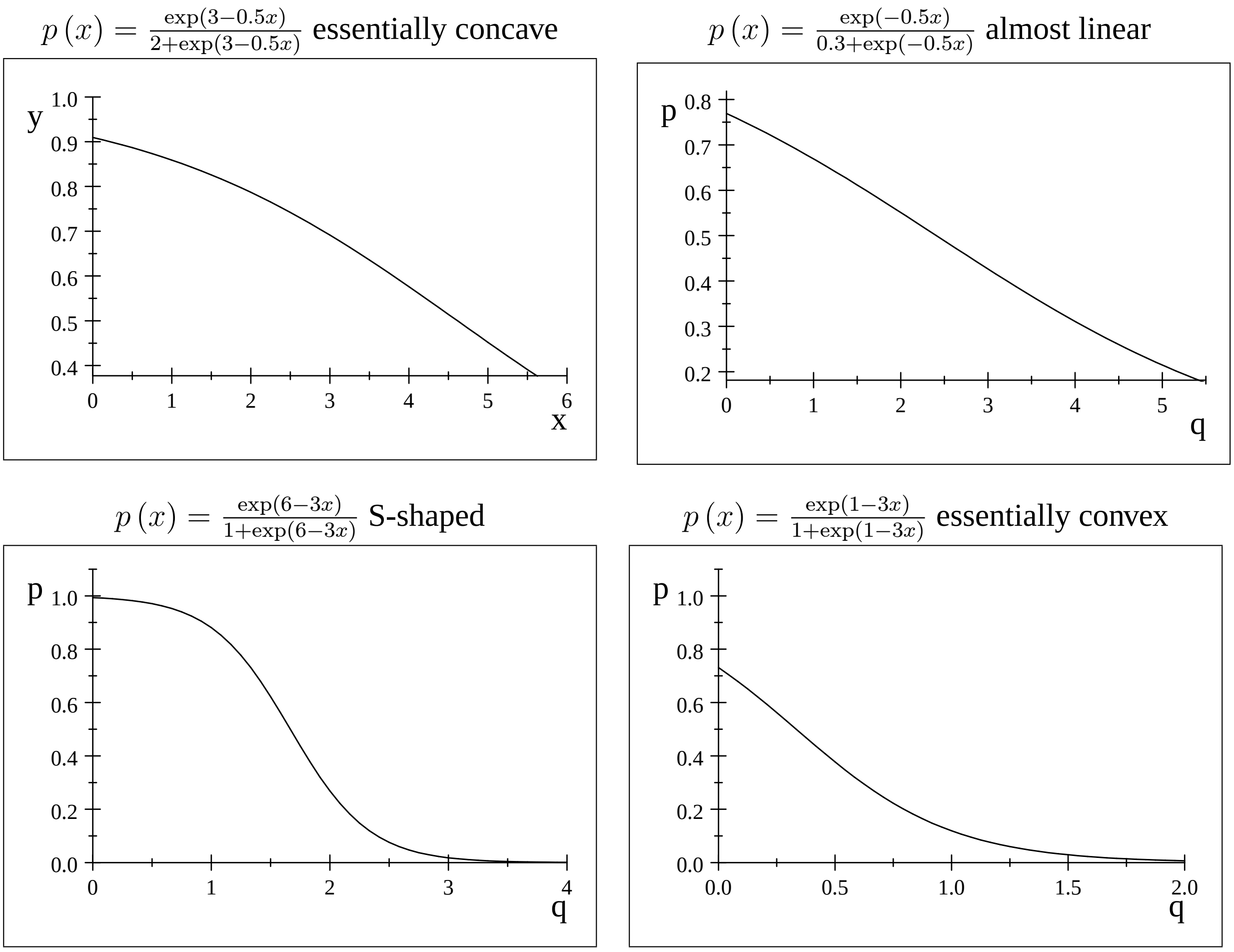
$$\begin{aligned} p_L &= \left(1 + \sum_{l=1}^{L-1} \exp \mu_l + \alpha_l q_l\right)^{-1} \\ p_j &= \frac{\exp\left(\mu_j + \alpha_j q_j\right)}{1 + \sum_{l=1}^{L-1} \exp\left(\mu_l + \alpha_l q_l\right)}, \qquad j = 1, ..., L-1. \end{aligned}$$

Notice that in a *ceteribus paribus* context, the residual inverse demand of firm j can be expressed as

$$p_j = \frac{\exp\left(\mu_j + \alpha_j q_j\right)}{K_j + \exp\left(\mu_j + \alpha_j q_j\right)}$$

for a constant $K_j = \sum_{l \neq j} \exp\left(\mu_l + \alpha_l q_l\right)$. One of the advantages of this model is the flexibility of the Logistic function to fit many different patterns often observed in empirical contexts. Figure 1 shows different shapes of inverse demand functions based on the proposed specification.

Figure 1: Shapes of inverse demand functions for different parameters



Producers and Market equilibrium: Next we study the market equilibrium in a Cournot-type competition based on the inverse demand system (5), where each company is profit-maximizer in quantities q_j of a differentiated product with different cost functions $c_j(\cdot)$ monotonically nondecreasing and convex. Thus, each company faces the following problem:

$$\pi_j(q) = p_j(q)\, q_j - c_j(q_j) = \left(\frac{\exp\left(\mu_j + \alpha_j q_j\right)}{\sum_{l=1}^{L} \exp\left(\mu_l + \alpha_l q_l\right)} L\pi\right) q_j - c_j(q_j). \tag{7}$$

The solution to the FOC for maximization of (7) satisfies the system of equations:

$$\frac{\partial \pi_j(q)}{\partial q_j} = p_j(q) + \alpha_j\, q_j\, p_j(q)\ \left(1 - \frac{p_j(q)}{L\pi}\right) - c_j'(q_j) = 0,$$

where $c_j'(q_j)$ denotes the partial derivative of the cost function. The solution is the unique maximizer of the profit function (7) provided that the second-order condition for profit maximization:

$$\begin{aligned}
\frac{\partial^2 \pi_j(q)}{\partial^2 q_j} &= \frac{\partial p_j}{\partial q_j} + \left[\alpha_j\, p_j\ \left(1 - \frac{p_j}{L\pi}\right)\right] + \alpha_j\, q_j\, \frac{\partial p_j}{\partial q_j}\ \left(1 - \frac{p_j}{L\pi}\right) \\
&\quad + \alpha_j\, q_j\, p_j\ \left(-\frac{1}{L\pi}\right) \frac{\partial p_j}{\partial q_j} - c_j''(q_j) \\
&= \frac{\partial p_j}{\partial q_j}\left(2 + \alpha_j\, q_j\ \left(1 - \frac{p_j}{L\pi}\right) - \frac{\alpha_j}{L\pi}\, q_j\, p_j\right) - c_j''(q_j) \\
&= \frac{\partial p_j}{\partial q_j}\left(2 + \alpha_j\, q_j\ \right) - c_j''(q_j) < 0,
\end{aligned}$$

is satisfied, which holds e.g. if $(2 + \alpha_j\, q_j\ ) \geq 0$ as the derivative of the inverse demand is negative:

$$\begin{aligned}
\frac{\partial p_j}{\partial q_j} &= \frac{\partial}{\partial q_j}\left(\frac{\exp\left(\mu_j + \alpha_j q_j\right)}{\exp\left(\mu_j + \alpha_j q_j\right) + \sum_{l\neq j} \exp\left(\mu_l + \alpha_l q_l\right)} L\pi\right) \\
&= \alpha_j p_j - \alpha_j \frac{1}{L\pi} p_j^2 = \alpha_j\, p_j\ \left(1 - \frac{1}{L\pi} p_j\right) \leq 0.
\end{aligned}$$

We have compared the outcome of the Cournot type equilibrium, with the result provided by a Bertrand competition. Notice that from the first first-order optimality condition, it is satisfied that

$$\frac{p_j(q) - c_j'(q_j)}{L\pi} = -\alpha_j\, q_j\, \frac{p_j(q)}{L\pi}\ \left(1 - \frac{p_j(q)}{L\pi}\right) \geq -0.5\, \alpha_j\, q_j = \frac{1}{-2\varepsilon_j},$$

as $(p_j(q)/L\pi)\;(1-(p_j(q)/L\pi)) \leq 0.5$. In particular, for $c_j'(q_j) = c_j$ is constant, then we obtain a lower bound for the unit margin,

$$(p_j - c_j) \geq \frac{L\pi}{2}\left|\varepsilon_j\right|^{-1}.$$

In contrast, the competition in prices renders the Lender condition

$$(p_j - c_j) = p_j\left|\varepsilon_j\right|^{-1}$$

meaning that competing higher Firms' unit margins are in quantities than in prices whenever $L\pi > 2p_j$ (which is the usual case as the sum or prices by all competitors is usually higher than two times the price charged by one of them). Firms would more likely prefer competing in quantities than in prices.

3 Extensions of the benchmark model

The proposed consumer model generates a flexible inverse demand system that can be adapted to a variety of cases. Here we discuss some possible extensions.

Multi attribute models: In the benchmark model, product differentiation is reflected in the parameters of the utility function. Notice that instead of setting different parameters $\left\{\mu_j, \alpha_j\right\}_{j=1}^{J}$ in utility function we can introduce a vector x of product attributes. For example, if $u(q) = \sum_{l=1}^{L}\alpha_l^{-1}\exp\left(\mu + \alpha q_l + \gamma' x_l\right)$, the inverse demand would be given by,

$$p_j = \frac{\exp\left(\mu + \alpha q_j + \gamma' x_j\right)}{\sum_{l=1}^{L}\exp\left(\mu + \alpha q_l + \gamma' x_l\right)} L\pi, \qquad j = 1, \ldots, L. \tag{8}$$

In particular, if x is quality we can study vertical product differentiation using this model, but we can use location attributes instead. Note also that setting $q_j = 1$ for all j, model (8) can be used to justify a new type of nonlinear Hedonic Regression explaining unit prices for all substitutive products through differences in products' attributes. Competition can be also considered in terms of product attributes. But this paper does not delve into this idea. Notice also that any externality affecting the utility of the consumers can end up included in the inverse demand function, similarly to the variable x.

Multiple categories. In our model, similarly to the Dixit-Stiglitz framework, the relative optimal consumption of two distinct varieties is a function of their relative prices (and vice versa). This property is meaningful provided that changes in the consumption of third products do not affect substitutability preferences for two given products, which is not necessarily true. There are some extensions that can modify this property. For example, consider a utility model over K different bunches of product categories,

$$u_i(q) = \sum_{k=1}^{K} \frac{1}{\beta_k} \exp\left(\mu_k + \sum_{l \in B_k} \frac{\beta_k}{\alpha_{l_k}} \exp(\mu_l + \alpha_l q_l)\right).$$

where $\{B_k\}_{k=1}^{k}$ is a partition of all goods in K categories of nested products, and $\beta_k, \alpha_k < 0$.

To compute the inverse demand system, we parametrize the budget constraint in a convenient form. We denote the normalized prices on each bundle by $\{w_l\}_{l \in B_k}$, with $\sum_{l \in B_k} w_l = 1$. Then, the price of a product q_l can be expressed as the product $p_k w_l$ where p_k is the relative value of bundle k which is also normalized. Therefore, we can parametrize the budget constraint as

$$\sum_{k=1}^{K} p_k \left(\sum_{l \in B_k} w_l q_l\right) = m,$$

where we consider an overall simplex normalization $\sum_{k=1}^{K} p_k \left(\sum_{l \in B_k} w_l\right) = 1$ over the prices $\{p_k\}$. Now we can compute the inverse demand functions for a bundle price, and for products inside a bundle similarly to the general case.

Computing the first order conditions, if $l, j \in B_k$, then for the optimal consumption plan,

$$\frac{w_l}{w_j} = \frac{\exp(\mu_l + \alpha_l q_l)}{\exp(\mu_j + \alpha_j q_j)}.$$

leading to the inverse of demand for products in category k,

$$w_j = \frac{\exp(\mu_j + \alpha_j q_j)}{\sum_{l \in B_k} \exp(\mu_l + \alpha_l q_l)}. \tag{9}$$

For elements $j \in B_k$ and $l \in B_{k'}$ in different nests, we obtain that

$$\frac{p_{k'} w_l}{p_k w_j} = \frac{\exp\left(\mu_k + \sum_{l \in B_{k'}} \frac{\beta_{k'}}{\alpha_l} \exp\left(\mu_l + \alpha_l q_l\right)\right) \exp\left(\mu_l + \alpha_l q_l\right)}{\exp\left(\mu_k + \sum_{l \in B_k} \frac{\beta_k}{\alpha_l} \exp\left(\mu_l + \alpha_l q_l\right)\right) \exp\left(\mu_j + \alpha_j q_j\right)}.$$

We define the utility of the optimal nest $U_k = \sum_{l \in B_k} \frac{1}{\alpha_{l_k}} \exp\left(\mu_l + \alpha_l q_l\right)$, then

$$\frac{p_{k'} w_l}{p_k w_j} = \frac{\exp\left(\mu_{k'} + \beta_{k'} U_{k'}\right) \exp\left(\mu_l + \alpha_l q_l\right)}{\exp\left(\mu_k + \beta_k U_k\right) \exp\left(\mu_j + \alpha_j q_j\right)}.$$

adding similar conditions and using that $\sum_{k=1}^{K} p_k \left(\sum_{l \in B_k} w_l\right) = 1$, we obtain that

$$\frac{1}{p_k w_j} = \frac{\sum_{k'=1}^{K} p_{k'} \left(\sum_{l \in B_{k'}} w_l\right)}{p_k w_j} = \frac{\sum_{k'=1}^{K} \exp\left(\mu_{k'} + \beta_{k'} U_{k'}\right) \left(\sum_{l \in B_{k'}} \exp\left(\mu_l + \alpha_l q_l\right)\right)}{\exp\left(\mu_k + \beta_k U_k\right) \exp\left(\mu_j + \alpha_j q_j\right)}.$$

Therefore, inverting the expression

$$p_k w_j = \frac{\exp\left(\mu_k + \beta_k U_k\right) \exp\left(\mu_j + \alpha_j q_j\right)}{\sum_{k'=1}^{K} \exp\left(\mu_{k'} + \beta_{k'} U_{k'}\right) \left(\sum_{l \in B_{k'}} \exp\left(\mu_l + \alpha_l q_l\right)\right)},$$

substituting $w_j = \exp\left(\mu_j + \alpha_j q_j\right) / \sum_{l \in B_k} \exp\left(\mu_l + \alpha_l q_l\right)$, and canceling terms we obtain that

$$\begin{aligned} p_k &= \frac{\exp\left(\mu_k + \beta_k U_k\right) \left(\sum_{l \in B_k} \exp\left(\mu_l + \alpha_l q_l\right)\right)}{\sum_{k'=1}^{K} \exp\left(\mu_{k'} + \beta_{k'} U_{k'}\right) \left(\sum_{l \in B_{k'}} \exp\left(\mu_l + \alpha_l q_l\right)\right)} \\ &= \frac{\sum_{l \in B_k} \exp\left(\mu_k + \beta_k U_k + \mu_l + \alpha_l q_l\right)}{\sum_{k'=1}^{K} \sum_{l \in B_{k'}} \exp\left(\mu_{k'} + \beta_{k'} U_{k'} + \mu_l + \alpha_l q_l\right)}. \end{aligned} \tag{10}$$

The actual price of a commodity $j \in B_k$ based on the inverse demand is given by the product $p_k w_j$. The extended version of the model based on (9) and (10) is particularly relevant when the overall consumer expenditure on different product categories is modeled together with the choice of differentiated product on each category. Utility functions with more than two sequential levels of nests can be handled alike.

4 Empirical application to the Newspaper Industry

In this section we present an empirical application to the Spanish newspaper industry, where revenue streams are drawn from newsstand and subscription paid copy sales and from advertising. Advertising revenues depends on previous circulation. Spanish Newspapers usually give away some free copies to increase circulation, aimed at differentiated individuals to avoid sales cannibalization. We propose a model where sold units and given away copies are used as strategic tools.

There is a piece of classical economic literature investigating different features of newspaper competition, (see e.g., Reddaway (1963), Ferguson (1963, 1983), Telser (1966), and Rosse (1967, 1970)). It is based on monopolistic competition models where firms compete in price, in both sales and attracting advertising. But price competition has limitations in this framework. Newspaper prices show small variability, and since revenues are drawn from sales, and advertising driven by expected circulation, publisher decisions on the two key strategic variables, sales and given-away issues, maximize forward expected returns. In this paper, we consider the competition in two-sided markets with differentiated products and sales competition "a la Cournot". To this end, we propose a new model for competition in markets with differentiated products, providing a flexible nonlinear inverse demand system.

The dynamics of this market are a relevant issue. Newspapers are edited daily; today's newspapers will be nothing more than wrapping paper tomorrow, so that every day the demands for sold copies are essentially static and independent. On the contrary, the demand for advertising space depends on expected circulation and therefore has a dynamic component. The objective of this paper is to gain insight into the nature of the competitive strategies in a market with differentiated products, in which firms compete for quantities over time, and have rational expectations about their rivals' actions and optimize their strategic response at each time. Perfect equilibrium in dynamic games can be used to study market trends and cycles (Pakes and Ericson 1998, Pakes and MacGuire 2001). The key idea is to combine strategic foresight and interaction effects in the model; this is particularly relevant in two-sided markets.

4.1 Data

We have used data from Spanish daily national newspapers. Circulation data were gathered by the auditing organization *Oficina de Justificación de la Difusión (OJD)* for montly sales and given away newspapers, owned by *Introl S.L*, and advertising data have been provided by *InfoAdex*, which is the main advertising auditing company in Spain. The sample period begins January, 1995 and ends December, 2004. We have considered monthly sales, advertising revenues and given away units of the main Spanish national newspapers during this period (*ABC*, *El Mundo* and the leader *El País*). Table 1 shows the average sales and the average given-away units of newspapers within the sample period. The leader El País is left-wing oriented, whereas the second player El Mundo is right-wing oriented, advocating modern style liberalism. ABC is the oldest, with a right wing perspective linked to traditional conservatism (monarchist and Catholic).

Table 1: Averages sales and giving away units of newspapers

	Monthly Sales	**Given away units**	**% given away over sales**
ABC	244,333	9,168	3.75%
El Mundo	253,956	8,920	3.51%
El País	401,451	12,132	3.02%

Advertising expenditure on each newspapers is quite seasonal. Although there is some effect on sales, it is not so strong. We have extracted the seasonal component in all the considered time series. To that in the end we have used two linked seasonal adjustment programs: TRAMO (Time series Regression with ARIMA noise, Missing observations, and Outliers) and SEATS (Signal Extraction in ARIMA Time Series). The programs TRAMO/SEATS, were developed by Agustin Maravall and Victor Gomez at the Bank of Spain. TRAMO provides automatic ARIMA modeling, while SEATS computes the components for seasonal adjustment based on TRAMO ARIMA-model. SEATS uses signal extraction based on the ARIMA filters which fit to the series by TRAMO to extract the seasonal components.

4.2 The newspapers model

Newspaper publishers compete with closely related, but differentiated products over the periods of time $t = 0, 1, 2 \ldots$. Let index $l = 1$ denote the newspaper ABC, index 2 the newspaper El Mundo, and 3 the leader El País. Their dynamic decision problem where the strategic space of the firms consists of the sold quantities q_{jt} and the number of giveaways s_{jt} at each period of time t. It is a two-sided market, the advertising market is dynamic (expected advertising depends on previous circulation), and the other side of the market is essentially static (sold copies are marketed daily). The utility function of the representative customer at time t is given by

$$u\left(q_t | E_t\right) = \sum_{l=1}^{3} \frac{1}{\alpha_l} \exp\left(\mu_l + \alpha_l q_{lt}\right), \tag{11}$$

with $\mu_3 = 0$. We assume that the newspapers' quality and printing characteristics, market area, and scale of printing plant are given decisions. We will consider that firms $l = 1, 2, 3$ behave strategically, competing in quantities and subsidies (given away products), where current subsidies enhance future demands. Under monopolistic competition, the residual demand of firm j when its rivals set $\{q_{lt}^e\}_{l \neq j}$ is given by

$$p_{jt}\left(q_{jt}\right) = \frac{\exp\left(\mu_j + \alpha_j q_{jt}\right)}{\exp\left(\mu_j + \alpha_j q_{jt}\right) + K_{jt}^e} L \pi_t, \tag{12}$$

$$K_{jt}^e = \sum_{l \neq j} \exp\left(\mu_l + \alpha_l q_{lt}^e\right) \tag{13}$$

Since we are in a partial equilibrium setup we do not introduce money demand as a numeraire, nor consider the price simplex, but consider that the nominal level or mean newspapers prices π_t in the economy is exogenously given. In addition newspapers receive a large part of their profits from advertising. We consider that the revenues obtained by firm $j = 1, 2, 3$ are given by,

$$a_{jt} = w_{j,t-1} \cdot A_t,$$

where A_t is the total advertising expense received by the industry which is considered exogenous, and $w_{j,t-1}$ is the share of the advertising market which depends on the total circulation in the previous period $\left(q_{jt-1} + s_{jt-1}\right)$, relatively to that of other competitors. In particular, we now consider a classical Multinomial Logistic regression model, where

$$w_{j,t-1} = w_j\left(q_{jt-1} + s_{jt-1}\right) = \frac{\exp\left\{\upsilon_j + \phi_j\left(q_{jt-1} + s_{jt-1}\right)\right\}}{\sum_{l=1}^{L}\exp\left\{\upsilon_l + \phi_l\left(q_{lt-1} + s_{lt-1}\right)\right\}}.$$

with $\phi_j \geq 0$, and to ensure identification $\upsilon_3 = 0$. In each period newspapers decide $\{q_{jt}, s_{jt}\}$ before knowing the exogenous variables and the decisions of their competitors. Regarding the overall advertising budget A_t, we generally consider that it follows an exogenous process driven by its past and other exogenous variables (e.g., the inter-annual growth or rate in the GDP).

4.2.1 Market equilibria

Assume that mean price and advertising $\{A_t, \pi_t\}_{t\geq 0}$ follow a predictable exogenous stochastic process, and denote by $c_{jt}(\cdot)$ the cost function of firm j. Under the rationality assumption, each firm aims to maximize its expected profit, given the conjectures. In the Nash equilibrium, each firm j aims to maximize its discounted profit

$$\Pi_j\left(\{q_{jt}, s_{jt}\}_{t\geq 0}\right) = E_0\left[\sum_{t\geq 0}\delta^t\,\Pi_{jt}\right],$$

$$\Pi_{jt} = \left(p_{jt}\left(q_{jt}\right)\ q_{jt} - \left(c_{jt}\left(q_{jt}\right) + k_{jt}\left(s_{jt}\right)\right) + a_{jt}\left(q_{jt-1} + s_{jt-1}\right)\right).$$

where $\delta = (1+i)^{-1}$ is the discount parameter ($i > 0$ is the interest rate), $E_t[\cdot]$ is the conditional expectation available at time t, $c_{jt}(q_{jt})$ and and $k_{jt}(s_{jt})$ are respectively the cost functions for sold and given away issues (they will be assumed linear but different, as the distribution channels are very different for both types of products), $a_{jt}(q_{jt-1} + s_{jt-1}) = w_j(q_{jt-1} + s_{jt-1})\cdot A_t$, and the optimal decision is compatible with their rivals' decisions, i.e. $q^e_{lt} = q_{lt}$ and $s^e_{lt} = s_{lt}$ in (12).

The dynamic process for $\{A_t, \pi_t\}$ is forecasted by firms using rational expectations. The arrival of new information sometimes turns the strategic response suboptimal (i.e. the Nash equilibrium involves "incredible" threats). A commonly-used refinement is given by the notion of Subgame Perfect Equilibrium, i.e. an equilibrium in which the strategies are a Nash equilibrium within each subgame (defined by the information set I_{t-1}). A subgame perfect equilibrium is the long-run outcome of dynamic learning equilibrium paths. In a subgame perfect equilibrium, at each time t each firm $j = 1, 2, 3$ maximizes its conditional expected profit given the available information, i.e. we require that at any time $s \geq 0$, each firm j maximize its discounted profit

$$\Pi_{js}\left(\{q_{jt}, s_{jt}\}_{t\geq s}\right) = E_s\left[\sum_{t\geq s} \delta^{t-s}\, \Pi_{jt}\right],$$

and the optimal decision is compatible with their rivals decisions. Notice that the conditional expectations only affect to p_{jt} and a_{jt} replacing π_t, A_t by their conditional expectations; we will consider this implicitly in the following equations.

The equilibrium can be characterized by the first order conditions. Using that $\partial\{p_{jt}\cdot q_{jt}\}/\partial q_{jt} = p_{jt} + q_{jt}\partial p_{jt}/\partial q_{jt}$, at each time s the decisions of firm j satisfy for any $t \geq s$,

$$\begin{aligned} E_s\left[\frac{\partial \Pi_{js}}{q_{jt}}\right] &= \delta^{t-s} E_s\left[p_{jt} + q_{jt}\frac{\partial p_{jt}}{\partial q_{jt}} - \frac{\partial c_{jt}}{\partial q_{jt}} + \delta\frac{\partial a_{jt+1}}{\partial q_{jt+1}}\right] = 0, \\ E_s\left[\frac{\partial \Pi_{js}}{s_{jt}}\right] &= \delta^{t-s} E_s\left[-\frac{\partial k_{jt}}{\partial s_{jt}} + \delta\frac{\partial a_{jt+1}}{\partial s_{jt+1}}\right] = 0. \end{aligned}$$

where $E_s[\cdot]$ is the conditional expectation to available information on states previous to s, that we consider symmetric for all players. Therefore, the subgame perfect equilibrium satisfies at any time $t \geq 0$ the conditional moment conditions:

$$\begin{gathered} E_{t-1}\left[p_{jt} + q_{jt}\,\alpha_j p_{jt}\left(1 - p_{jt}/L\pi_t\right) - c'_{jt}\left(q_{jt}\right) + \delta\phi_j w_{jt}\left(1 - w_{jt}\right) A_{t+1}\right] = 0, \\ E_{t-1}\left[-k'_{jt}\left(s_{jt}\right) + \delta\phi_j w_{jt}\left(1 - w_{jt}\right) A_{t+1}\right] = 0, \end{gathered} \tag{14}$$

Subtracting both equations in (14) we can rewrite the first equation as

$$E_s\left[p_{jt} + q_{jt}\,\alpha_j\, p_{jt}\left(1 - p_{jt}/L\pi_t\right) - \left(c'_{jt}\left(q_{jt}\right) - k'_{jt}\left(s_{jt}\right)\right)\right] = 0.$$

In particular, if we consider a linear cost functions $(c_{jt}q_{jt} + k_{jt}s_{jt})$, we can conclude that

$$E_s\left[p_{jt} + q_{jt}\,\alpha_j\, p_{jt}\left(1 - p_{jt}/L\pi_t\right) - \left(c_{jt} - k_{jt}\right)\right] = 0. \tag{15}$$

and $k_{jt} = E_{t-1}\left[\delta\phi_j w_{jt}\left(1 - w_{jt}\right) A_{t+1}\right]$. The most relevant condition for out inference analysis is (15), where we will consider $\kappa = (c_{jt} - k_{jt})$ as a constant cost parameter to estimate, and we will assume that it is common for all newspapers.

4.3 Estimation

We have estimated the parameters of the model discussed in the previous section using the Generalized Method of Moments (GMM) estimation, for details see Hansen (1982), where as usual the estimation is carried out in two steps: first we estimate the parameters (using the instruments variance as weighting matrix) and in the second step we update the weights to achieve asymptotic efficiency (using a Newey-West type weight matrix with 12 lags). The procedure is then iterated ten times. Notice that if the solution X_t of an economic model with parameters θ^0 satisfies the a set of conditional moments $E_{t-1}\left[g\left(\theta^0, X_t\right)\right] = 0$, then the Law of iterated expectations implies the orthogonality conditions

$$E\left[g\left(\theta^0, X_t\right) \otimes W_t\right] = 0, \tag{16}$$

for any vector of instruments W_t in the conditioning information set, where $\otimes$ denotes the Kronecker product. The GMM estimator typically estimates θ^0 minimizing a quadratic form based on the empirical analogous to the system (16). For the discount parameter $\delta = (1+i)^{-1}$ we have considered a monthly rate is $i = 0.1/12$, so that the annual rate is approximately 10%.

We have focused on sold newspapers. Notice that the conditional moment conditions (15) are satisfied in the equilibrium for all period of time $t = 0, 1, \ldots, T$ and $j = 1, 2, 3$. In addition, the actual prices P_{jt} should fit the inverse demand model p_{jt}, and we have considered the condition $E_{t-1}\left[(P_{jt} - p_{jt})\right] = 0$, so that

$$E\left[(P_{jt} - p_{jt}) \otimes W_t\right] = 0, \quad j = 1, 2, 3. \tag{17}$$

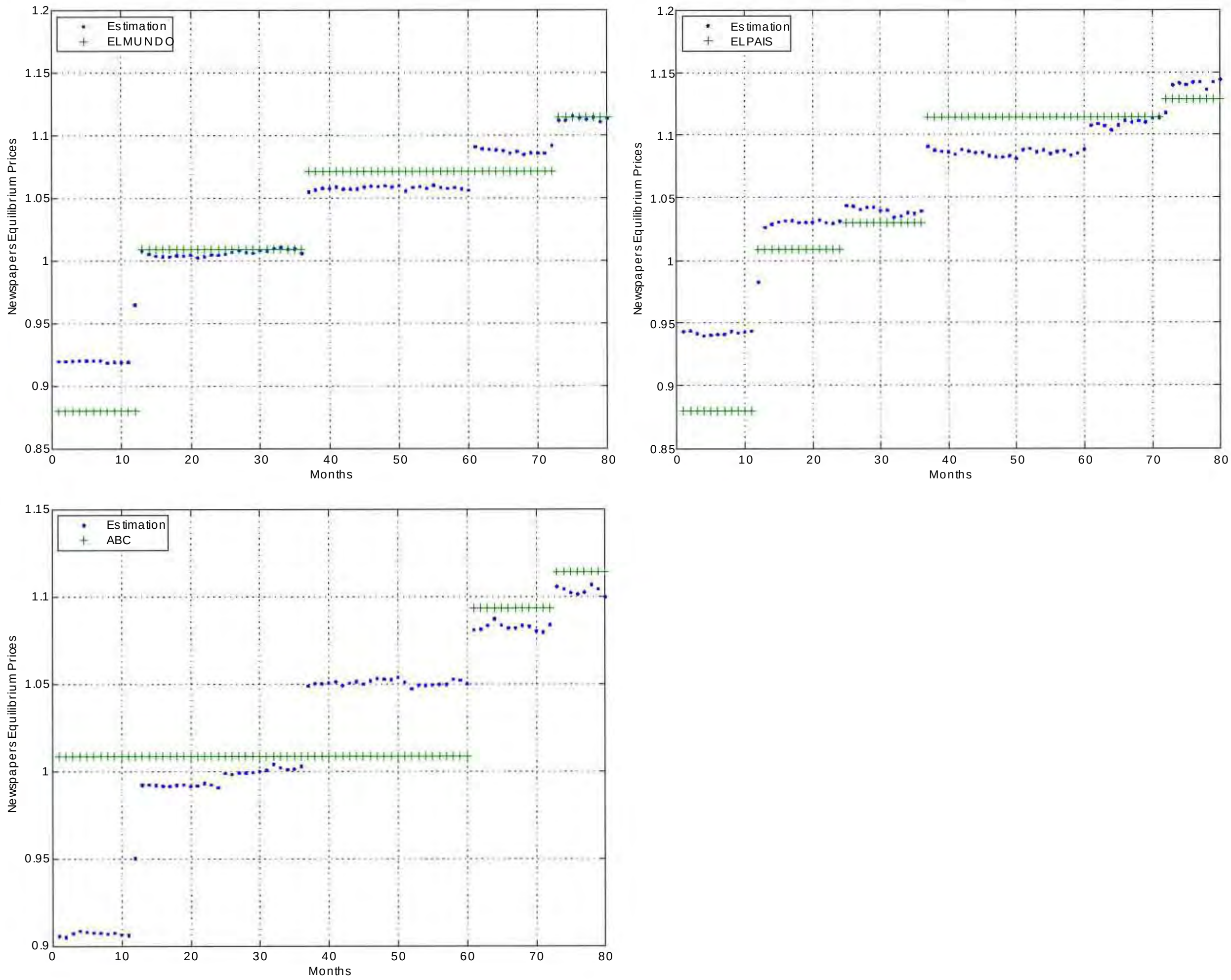
using and linear symmetric cost $\kappa = (c_{jt} - k_{jt})$. The instruments are $W_t = \left(1, q_{t-1}'\right)'$, i.e. one lag for all sales. Table 2 shows the GMM estimation results with moment conditions (15) and (17).

Table 2: Parameter estimations

Parameter	Estimation	Std. Err	Stat. t	p value
μ_1	-0.08463	0.014	-5.88	0.00
μ_2	-0.0636	0.004	-14.78	0.00
α_1	-1.4×10^7	1.8×10^8	-7.7	0.00
α_2	-1.7×10^7	1.0×10^8	-17.17	0.00
α_3	-2.0×10^7	1.2×10^8	-16.08	0.00
κ	1.01	0.0042	235	0.00

The t-statistics and the p-values are testing the null hypotheses that all parameters are equal to zero, implying all parameters are significant. Note that the number of moment conditions is higher than the number of parameters. The value of Hansens' overidentification statistics is $J = 12.93$, and $\Pr\left\{\chi^2_{18} > J\right\} = 0.7954$ and we accept the over-identifying moment conditions. Figure 2 shows the inverse demand price forecast for each of the three newspapers, compared with actual data. Looking at the results in Figure 2, we find that on average the observed prices are close to the optimal ones based on the model. The fit is remarkable, given the little variability in prices that we have observed in the sample. The largest difference between observed and fitted prices corresponds to the newspaper *ABC*. In particular, we observe that *ABC* applied a suboptimal price between 37th sample month (January 2002) and the 60th one (December 2004). In January 2002, the Euro currency was introduced, and the *ABC* response was perhaps motivated by monetary illusion considerations. Eventually, *ABC* upgraded prices to a level slightly above the optimal value.

Figure 2: Actual price data and model forecast



We have estimated the advertising response functions. We have denoted A_{jt} as actual advertising revenues of publisher $j = 1, 2, 3$ and $A_t = \sum_{j=1}^{3} A_{jt}$. We have required the advertising model $a_{jt} = w_{jt-1}A_t$ to satisfy $E_{t-1}[(A_{jt} - a_{jt})] = 0$, and from here we can also obtain the orthogonality conditions

$$E[(A_{jt} - w_{jt-1}A_t) \otimes W_t] = 0, \quad j = 1, 2, 3.$$

We have estimated the parameters of the demand for advertising by GMM using this system, using $W_t = (1, A_{t-1}, A_{t-2}, A_{t-3})'$ as instruments. Table 3 shows the GMM estimators. The leader, *El País*, has the lowest advertising elasticity with respect to circulation (with $\phi_3 = 1 \times 10^{-6}$), and *El Mundo* is the most sensitive to circulation. The value of Hansens' overidentification statistics is $J = 18.77$, and $\Pr\{\chi^2_{25} > J\} = 0.80$ and we accept the over-identifying moment conditions. The results validate that the advertising specification is appropriate for this industry.

Table 3: Parameter estimations for the advertising response model

Parameter	Estimation	Std. Err	Stat. t	p value
υ_1	-0.7788	0.3183	-2.45	0.01
υ_2	-3.72	0.6189	-6.02	0.00
ϕ_1	4×10^{-6}	8×10^{-7}	4.95	0.00
ϕ_2	14×10^{-6}	2.5×10^{-7}	5.72	0.00
ϕ_3	1×10^{-6}	4×10^{-7}	2.33	0.01

Figure 3 shows the fit between forecasts based on the estimated weights $\{w_{jt}\}$ and the actual A_{jt}. Recall that the data are deseasonalized, and fluctuations are associated to circulation. Notice that *ABC* advertising revenues were lower during the central period, and they have the higher percentage of given away copies over sold ones (3.75% on average).

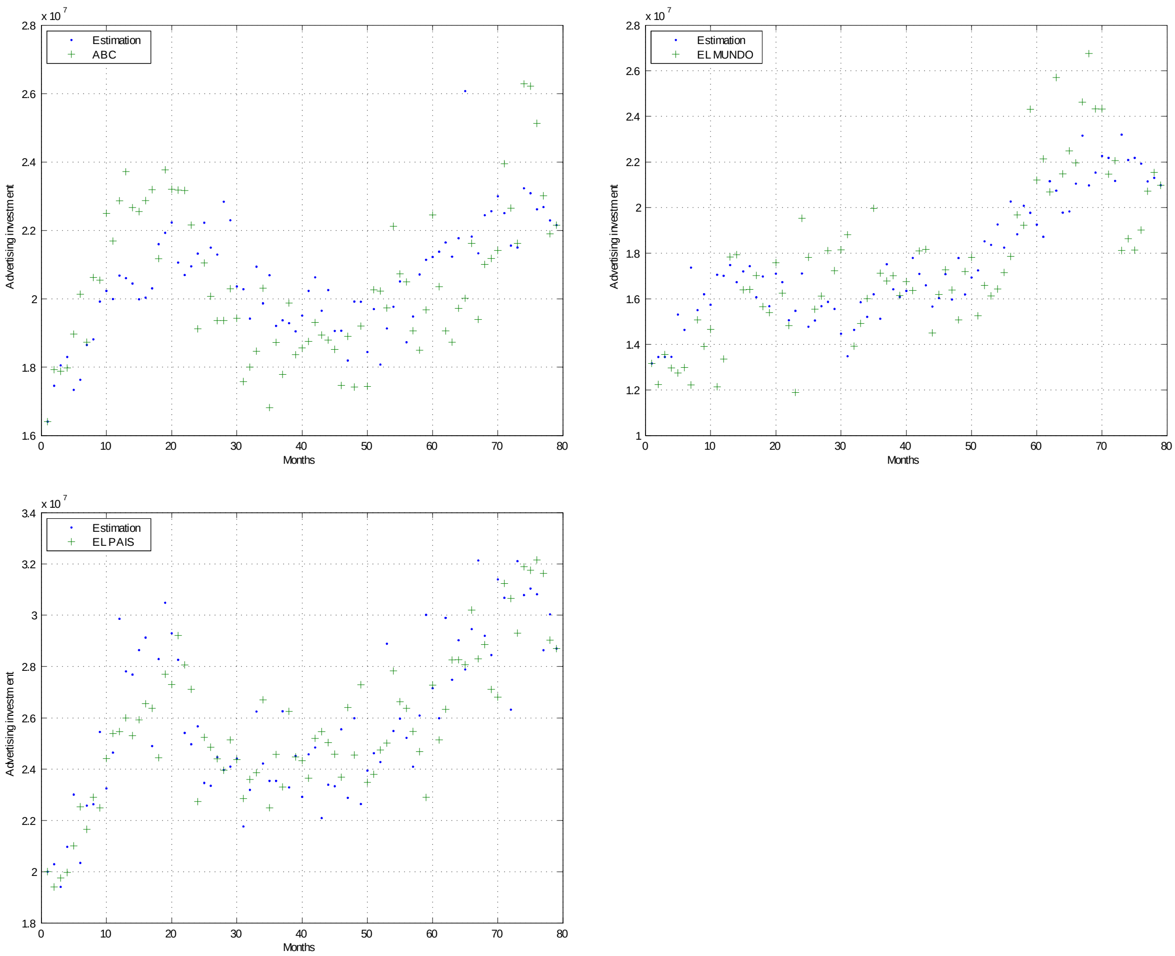
Based on these results we can conclude that the newspapers industry can be modeled with the proposed model. But we also warn about some problems. If we put together all the moment conditions, then we face computational problems minimizing GMM, with multiplicity of local minima. By focusing on each area in a separated way, we do not suffer from that problem. Perhaps with a larger sample, that problem can be overcome, as GMM is sensitive to the inclusion of a large number of moment conditions.

Based on the previous demand estimates, we can now explore the cost parameters. Notice that from the second equation in (14), we can estimate the overall unit cost k of given away issues using GMM, based on the moment condition

$$E_{t-1}\left(\left[-k+\delta\phi_j w_{jt}\left(1-w_{jt}\right)A_{t+1}\right]\otimes W_t\right)=0,$$

for $j = 1, 2, 3$. with $W_t = (1, A_{t-1}, A_{t-2}, A_{t-3})'$ and we can estimate k, where the parameters in w_{jt} are replaced by the estimators. In particular we obtain $k = 0.8843$ and we can recover the estimation of the unit cost of sold copies $c = \kappa + k = 1.8947$. The estimated cost of sold copies is slightly higher than the range of prices observed in the sample, suggesting that during this period advertising is partially subsidizing this side of the market.

Figure 3: Forecasted and actual advertising cash-flows for the three newspapers



Newspapers are differentiated products. An interesting question is whether publishers tend to charge different prices. A price differentiation index can be computed at the equilibrium, based on the notion of "entropy", using the measure

$$H = -\sum_{j=1}^{L} \left(\frac{p_j}{L\pi}\right) \log\left(\frac{p_j}{L\pi}\right).$$

Notice that $H \leq \ln(L)$, with equality if and only if all $p_j = \pi$. Using the inverse demand system, it can be considered as a function of q, but we can simply compute it for the equilibrium prices. The actual mean prices for the period (normalized to the simplex) for publishers $j = 1, 2, 3$ are 0.331, 0.335 and 0.330, respectively, and we obtain an entropy index

$$H = -(0.331 \ln(0.331) + 0.335 \ln(0.335) + 0.330 \ln(0.330)) = 1.0982,$$

quite close to $\ln(3) = 1.0986$. This result confirms that in equilibrium the actual price differentiation is not too strong. But the differences in parameters of the inverse demand system p_j makes this compatible with significant differences in sales, and the attraction of different levels of advertising revenues.

4.4 Limitations and possible extensions

The empirical application is presented as an example of a potentially insightful application, but there are several extensions that could be tackled if the model is implemented in a different time period, or another country. We will discuss some of them.

4.4.1 The Internet

There is a third side of the newspaper industry that we have not considered, as it has developed over the last decade and in our sample it did not yet play such a crucial role. Nowdays, most general newspapers provide online access for free to attract digital advertising revenues (mainly through displays such as banners, and digital classified). Newspapers' digital advertising revenue is relatively small (typically less than 5% of total advertising revenue, albeit growing), as media agencies consider that this type of advertising has less impact than the off-line advertising; perhaps it improves in the future (driven by access to tables and specially mobile devices).

Digital traffic can be measured, and that is the reference to allocate higher shares to different newspapers. However, it cannot be easily controlled by newspapers except for quality and services in online contents (but that is generally a long-run strategic decision, and once it has been settled there are not additional controls except for small tactic adjustments). For some years it has been a relatively exogenous flow of revenues. Note that digital traffic of newspaper j could be included in the externality component E_j. The third side would be more interesting from an economic perspective if papers were willing to charge access. But most general newspapers oppose pricing digital access for fear that the drop in digital adversting revenue flows may be higher than the access revenues streams. Given the worth of online adversting for newspapers, it was a matter of time to reconsider the strategy.

The situation is currently changing, driven by exogenous problems faced by this industry. During the last decade newspapers' global circulation has declined slowly in most developed countries. In 2006, "The Economist" asked on its cover who had "killed the newspaper." Nowadays there are roughly 1,350 surviving U.S. English-language daily newspapers, down from about 1,400 five years ago, and only 70 of them have circulations above 100,000 (see, Rosenstiel et al., 2012). But things are now getting better. To stem losses, many papers have been raising their subscription price and newsstand copies. In the USA, advertising revenues from circulation are still declining but at a slower rate, and circulation revenues are stabilizing. In addition, most papers are reconsidering the digital business model. Even if they do not charge for digital access, they are increasingly charging readers for online content with "Paywalls" methods (see: "News adventures," *The Economist*, Dec. 8th, 2012). Newspapers acknowledge now that some pay systems can work. A price decision can be used to control the digital third-side of the market, and could be modeled using a classical price competition with differentiated products. The whole industry of the future can be modeled as four-sided extension to the presented model, with two digital sides (access and online advertising) and two off–line sides (circulation and printed advertising), including externalities between the classical circulation market and digital traffic (print and online papers are sometimes found to be substitutive rather than complementary, see Gentzkov, 2007, but some articles report contradictory results, see Deleersnyder et al., 2002). This is an interesting issue for future research.

4.4.2 Local competitors

There is another limitation in the empirical analysis. The analysis has focused on the main nationwide newspapers, i.e., we consider a market with a moderate number of firms. Nevertheless, there are many local newspapers that can attract readerships and have not been included. To study the impact of having a large number of small competitors, we will consider the context where the number of firms $L \to \infty$, so that we can assume a continuous of products q_γ, and a utility function

$$U = \int \exp\left(\mu_\gamma + \alpha_\gamma q_{\gamma t}\right) dF\left(\gamma\right),$$

and budget constraint $\int p_{\gamma t} q_{\gamma t} dF\left(\gamma\right) \leq m_t$. Then we can consider the inverse demand for each firm is given by,

$$p_{jt} = \frac{\exp\left(\mu_j + \alpha_j q_{jt}\right)}{\int \exp\left(\mu_\gamma + \alpha_\gamma q_{\gamma t}\right) dF\left(\gamma\right)},$$

with $\int p_{\gamma t} q_{\gamma t} dF\left(\gamma\right) = 1$. In this context, firm actions do not affect the denominator (any firm has zero mass) and the first order condition is simplified since

$$\frac{\partial}{\partial q_{jt}} p_{jt} = \frac{\partial}{\partial s_{jt}} p_{jt} = \frac{\alpha_j \exp\left(\mu_j + \alpha_j q_{jt}\right)}{\int \exp\left(\mu_\gamma + \alpha_\gamma q_{\gamma t}\right) dF\left(\gamma\right)} = \alpha_j p_{jt} < 0,$$

i.e. when quantities increase the price is deceased. To discuss the behavior of the model, assume that there are not subsides and neither advertising revenues, then under perfect information the maximization of profits $\pi_{jt} = p_{jt} \cdot \left(q_{jt}\right) - c_{jt}\left(q_{jt}\right)$ leads to

$$\frac{\partial}{\partial q_{jt}} \left(p_{jt} q_{jt} - c_{js}\left(q_{js}\right)\right) = \alpha_j p_{jt} q_{jt} + p_{jt} - c_{jt} = 0,$$

implying that

$$p_{jt} = \left(1 + \alpha_j q_{jt}\right)^{-1} c_{lt}.$$

using $\alpha_j q_{jt} < 0$, so that $p_{jt} > c_{lt}$. If advertising revenues are included,

$$a_{jt} = \frac{\exp\left\{\upsilon_j + \phi_j\left(q_{jt-1} + s_{jt-1}\right)\right\}}{\int \exp\left\{\upsilon_\gamma + \phi_\gamma\left(q_{\gamma t-1} + s_{\gamma t-1}\right)\right\} dF\left(\gamma\right)} A_t = w_{jt} A_t,$$

The myopic maximization of profits under perfect information requires that,

$$\frac{\partial}{\partial q_{jt}}\left(p_{jt}q_{jt}-a_{jt}-c_{ls}\left(q_{js}\right)\right)=\alpha_j p_{jt}q_{jt}+p_{jt}+\phi_j w_{jt}A_t-c_{jt}=0,$$

and $p_{jt} = (1+\alpha_j q_{jt})^{-1}\left(c_{lt}-\phi_j w_{jt}A_t\right)$, suggesting that p_{jt} can be partially subsidize by advertising. In these cases, it could be worth subsidizing the product completely, even with added gifts. This is a commonly adopted strategy by some players in the newspaper market.

4.4.3 Free press

An extreme give-away strategy is followed by the *free newspapers* which generate all their revenue from advertising, who exclusively give-away copies that usually have small unit costs (the issues have less contents). In the last decade, free daily newspapers have been introduced in most developed countries, and in some of then the market has been turned into a battlefield for advertising. Internationally, the free press sector leaders are the Swedish company *Metro International* (www.metro.lu), and the Norwegian company *20 Minutos A.G.* (www.20minutes.com) controlled by the Norwegian media group *Shibsted*. In Spain these groups are also the free press leaders. The first local free newspapers were launched in the eighties, but the Spanish free press took off in 2000. The most read free newspaper is now the leader and pioneer *20 Minutos* with over 2.4 million readers (higher than the traditional press leader *El País*). It started in February, 2000 with several dailies and in June 2001, a new owner (20 Minutos Holding) changed its name. *Que*, *ADN* and *Metro* were launched in 2005. The competition was fierce and in 2009 the Spanish diary *Metro* closed. Our sample does not include data beyond 2004, and up to that time the impact on traditional press was not so high (it was comparable to that of a small local Newspaper), therefore we have not included it in the model.

In any case, the model can be modified to accommodate this phenomenon. If there $r = 1, ..., F$ free newspapers, their advertising revenues at time t are $a_{rt} = w_{r,t-1} \cdot A_t$ where the shares of advertising are distributed between both types of newspapers as:

$$
\begin{aligned}
w_{j,t-1} &= w_j\left(q_{jt-1}+s_{jt-1}\right)=\frac{\exp\left\{\upsilon_j+\phi_j\left(q_{jt-1}+s_{jt-1}\right)\right\}}{\sum_{l=1}^{L}\exp\left\{\upsilon_l+\phi_l\left(q_{lt-1}+s_{lt-1}\right)\right\}+\sum_{f=1}^{F}\exp\left\{\upsilon_f+\phi_f s_{f,t-1}\right\}}, \\
w_{r,t-1} &= w_r\left(s_{jt-1}\right)=\frac{\left\{\upsilon_f+\phi_f s_{f,t-1}\right\}}{\sum_{l=1}^{L}\exp\left\{\upsilon_l+\phi_l\left(q_{lt-1}+s_{lt-1}\right)\right\}+\sum_{f=1}^{F}\exp\left\{\upsilon_f+\phi_f s_{f,t-1}\right\}}
\end{aligned}
$$

The profits of paid Newspapers $j=1,...L$ at time t would be given by

$$
\pi_j=\left(p_{jt}\cdot q_{jt}+a_{jt}-\left(c_{jt}\left(q_{jt}\right)+k_{jt}\left(s_{jt}\right)\right)\right),
$$

where c_{jt} and k_{jt} are the cost functions (edition and distribution) of sold and given away units, and the returns of free Newspapers $f=1,...,F$ are given by $\pi_{ft}=\left(a_{f,t}-k_{ft}\left(s_{f,t}\right)\right)$, which production and distribution costs are typically smaller than those of paid for press (as they only use news agency which have few journalists, and distribute in selected places). In equilibrium, the free Newspapers $f=1,...,F$ maximize,

$$
\begin{aligned}
\Pi_f\left(\left\{s_{ft}\right\}_{t\geq 0}\right) &= E_0\left[\sum_{t\geq 0}\delta^t\cdot\pi_{ft}\right], \\
\pi_{ft} &= \left(a_{f,t}-k_{ft}\left(s_{f,t}\right)\right),
\end{aligned}
$$

In a perfect equilibrium, free newspapers decision s_{ft} satisfies the first order condition

$$
E_t\left[\delta\frac{\partial\pi_{ft}}{s_{ft}}\right]=\delta E_t\left[\phi_f w_{ft}\left(1-w_{ft}\right)A_{t+1}-k'_{ft}\left(s_{ft}\right)\right]=0.
$$

Moment equations can be based on this expression, and included in the GMM objective function.

The big threat for the free press is that their returns depends dramatically on A_t, and they are relatively sensitive to falls in overall advertising budget A_t due to economic crisis, compared to classical paid Newspapers. Notice that if A_t declines due, e.g., to an economic crisis, all regular newspapers can raise copy prices for the readers' market and compensate for the loss. However free newspapers do not have the possibility to compensate the effect of contractions in A_t, which possess a significant threat to their survival.

5 Concluding Remarks

Monopolistic competition is a powerful conceptual framework to study firms' interactions, generally more accurately than assuming perfect competition or a classical homogeneous-product oligopoly. In most sectors there are a number of key firm players which tend to differentiate their product to obtain certain levels of market power. But modeling MC competition is not straightforward. Essentially, MC literature has considered price as the key decision variable, given the difficulty of producing flexible nonlinear inverse demand systems required to model competition in quantities. The main contribution of this paper is methodological. We have presented an inverse demand system capable of approximating many different data shapes, and have discussed how this model can be applied in MC when firms compete in quantity. We also consider extension of the model to work with alternative product categories. The model is essentially static, but it can be implemented in dynamic setups.

We have presented an empirical application to show the usability of the model. The empirical application is stochastic and dynamic, and it shows how the benchmark model can be implemented in relatively complex settings. We believe that the model and methodology employed in this paper are broadly applicable to other types of industries in which a few firms compete in the same market with a closely related but not homogeneous product.

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Appendix

A1 Marshallian demand system

The demand system can be obtained using the linear system (4) and the budget constraint. For example, when $L = 3$, setting (4) for l, j equal to $1, 2$ and $1, 3$ the demand system can be solved from the linear system

$$\begin{pmatrix} \alpha_1 & -\alpha_2 & 0 \\ \alpha_1 & 0 & -\alpha_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \ln p_1 - \ln p_2 - \mu_1 + \mu_2 \\ \ln p_1 - \ln p_3 - \mu_1 + \mu_3 \\ m \end{pmatrix},$$

where we just need to compute the inverse

$$\begin{pmatrix} \alpha_1 & -\alpha_2 & 0 \\ \alpha_1 & 0 & -\alpha_3 \\ p_1 & p_2 & p_3 \end{pmatrix}^{-1} = \frac{1}{\alpha_1\alpha_2 p_3 + \alpha_1\alpha_3 p_2 + \alpha_2\alpha_3 p_1} \begin{pmatrix} \alpha_3 p_2 & \alpha_2 p_3 & \alpha_2\alpha_3 \\ -(\alpha_3 p_1 + \alpha_1 p_3) & \alpha_1 p_3 & \alpha_1\alpha_3 \\ \alpha_1 p_2 & -(\alpha_2 p_1 + \alpha_1 p_2) & \alpha_1\alpha_2 \end{pmatrix}.$$

With $L = 4$ the result is analogous, but now the relevant inverse is

$$\begin{pmatrix} \alpha_1 & -\alpha_2 & 0 & 0 \\ \alpha_1 & 0 & -\alpha_3 & 0 \\ \alpha_1 & 0 & 0 & -\alpha_4 \\ p_1 & p_2 & p_3 & p_4 \end{pmatrix}^{-1} = \frac{1}{\alpha_1\alpha_2\alpha_3 p_4 + \alpha_1\alpha_2\alpha_4 p_3 + \alpha_1\alpha_3\alpha_4 p_2 + \alpha_2\alpha_3\alpha_4 p_1} \times$$

$$\begin{pmatrix} \alpha_3\alpha_4 p_2 & \alpha_2\alpha_4 p_{3t} & \alpha_2\alpha_3 p_4 & \alpha_2\alpha_3\alpha_4 \\ -\begin{pmatrix} \alpha_1\alpha_3 p_4 + \alpha_1\alpha_4 p_3 \\ +\alpha_3\alpha_4 p_1 \end{pmatrix} & \alpha_1\alpha_4 p_3 & \alpha_1\alpha_3 p_4 & \alpha_1\alpha_3\alpha_4 \\ \alpha_1\alpha_4 p_2 & -\begin{pmatrix} \alpha_1\alpha_2 p_4 + \alpha_1\alpha_4 p_2 \\ +\alpha_2\alpha_4 p_1 \end{pmatrix} & \alpha_1\alpha_2 p_4 & \alpha_1\alpha_2\alpha_4 \\ \alpha_1\alpha_3 p_{2t} & \alpha_1\alpha_2 p_{3t} & -\begin{pmatrix} \alpha_1\alpha_2 p_3 + \alpha_1\alpha_3 p_2 \\ +\alpha_2\alpha_3 p_1 \end{pmatrix} & \alpha_1\alpha_2\alpha_3 \end{pmatrix}$$

Let B_L denote the analogous matrix for L goods. The element i, j of the inverse is

$$B^{-1}_{i,j} = \frac{1}{\sum_{l=1}^{L} p_l \left(\prod_{j \neq l} \alpha_j\right)} \times \begin{cases} p_{(j+1)} \left(\prod_{l \notin \{(j+1),1\}} \alpha_j\right) & i = 1,\ j < L \\ -\left(\sum_{l \neq (j+1)} p_l \left(\prod_{l \notin \{(j+1),l\}} \alpha_j\right)\right) & i > 1, j < L \\ p_{(j+1)} \left(\prod_{l \notin \{(j+1),i\}} \alpha_j\right) & i > 1, j < L \\ \prod_{l \neq i} \alpha_l & j = L \end{cases}$$

A2 Inverse Demand Robustness to Consumer's Heterogeneity

Consider N heterogeneous consumers ($i = 1, .., N$), with preferences (1) defined by different parameters (α^i, μ^i) ($i = 1, .., N$). Without loss of generality, we assume that the parameters (α^i, μ^i) are independently drawn from a probability density function $g(\alpha, \mu)$ on a closed interval, and each consumer has an inverse demand system. Let $q^{(\mu,\alpha)}(p)$ denote the demand of an individual with parameters (α, μ) given a price p, and $p^{(\mu,\alpha)} = p\left(q^{(\mu,\alpha)}\right)$ the inverse demand system.

The market inverse demand can be derived using that the j-th price is defined by the geometric mean of individuals' inverse demands. In particular for the j-th product, we obtain that

$$
\begin{aligned}
\overline{p}_j &= \left(\prod_{i=1}^{N} p_j^{\left(\mu^i,\alpha^i\right)}\right)^{\frac{1}{N}} = \prod_{i=1}^{N}\left(\frac{\exp\left(\mu_j^i+\alpha_j^i q_j^i\right)}{\sum_{l=1}^{L}\exp\left(\mu_l^i+\alpha_l^i q_l^i\right)}\right)^{\frac{1}{N}} \\
&= \frac{\exp\left(N^{-1}\sum_{i=1}^{N}\left(\mu_j^i+\alpha_j^i q_j^i\right)\right)}{\sum_{l=1}^{L}\exp\left(N^{-1}\sum_{i=1}^{N}\left(\mu_l^i+\alpha_l^i q_l^i\right)\right)} \\
&\rightarrow \frac{\exp\left(\int\left(\mu+\alpha q_j^{\alpha}\right) g_j\left(\mu,\alpha\right) d\mu d\alpha\right)}{\sum_{l=1}^{L}\exp\left(\int\left(\mu+\alpha q_l^{\alpha}\right) g_l\left(\mu,\alpha\right) d\mu d\alpha\right)} \qquad \text{(A-1)}
\end{aligned}
$$

where $g_j\left(\mu,\alpha\right)$ denotes the marginal density of $\left(\mu_j^i,\alpha_j^i\right)$, and we have assumed that the Strong Law of Large Numbers can be applied so that

$$
N^{-1}\sum_{i=1}^{N}\left(\mu_j^i+\alpha_j^i q_j^i\right) \rightarrow_{a.s.} E\left[\left(\mu+\alpha q_j^{(\mu,\alpha)}\right)\right] = \int\left(\mu+\alpha q_j^{(\mu,\alpha)}\right)\ g_j\left(\mu,\alpha\right)\ d\left(\mu,\alpha\right),
$$

for a large N. Applying the Second Mean Value Theorem for integrals[1], we can express (A-1) as

$$
\begin{aligned}
&= \frac{\exp\left(\overline{\mu}_j+\overline{\alpha}_j\int q_j^{\alpha}\ g_j\left(\alpha\right)\ d\alpha\right)}{\sum_{l=1}^{L}\exp\left(\overline{\mu}_l+\overline{\alpha}_l\int q_l^{\alpha}\ g_l\left(\alpha\right)\ d\alpha\right)} \\
&\approx \frac{\exp\left\{\overline{\mu}_j+\overline{\alpha}_j\left(N^{-1}\sum_{i=1}^{N} q_j^i\right)\right\}}{\sum_{l=1}^{L}\exp\left\{\overline{\mu}_l+\overline{\alpha}_l\left(N^{-1}\sum_{i=1}^{N} q_l^i\right)\right\}}
\end{aligned}
$$

for some $\left(\overline{\alpha},\overline{\beta}\right)$. The term N^{-1} can be included in the parameters $\overline{\alpha}$ and we obtain an expression identical to (6), providing a relationship between geometric mean of individual inverse demands and the aggregated demanded quantities.

[1]The Second Mean Value Theorem for Integrals, state that if $f\left(x\right)$ and $g\left(x\right)$ are continuous on $[a,b]$ and $g\left(x\right)\geq 0$ for any $x\in[a,b]$, then there exists $c\in(a,b)$ such that

$$
\int_a^b f\left(x\right) g\left(x\right) dx = f\left(c\right)\int_a^b g\left(x\right) dx.
$$

The number $f\left(c\right)$ is called the $g\left(x\right)$-weighted average of $f\left(x\right)$ on the interval $[a,b]$.