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Make it Challenging: Motivation through Goal Setting.

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Abstract

We study a principal agent model where agents derive a sense of pride when accomplishing production goals. As in classical models, the principal offers a pay-per-performance wage to the agent, determining the agent's extrinsic incentives. However, in our setting, the principal does also want to set goals that affect the agents' intrinsic motivation to work. Agents differ in their personal standard which determines what becomes challenging and rewarding to them, and hence the intensity of their intrinsic motivation to achieve goals. We show that, at the optimal contract, the agents' production, as well as the goals set by the principal, increase with the agents' personal standards. Thus, although goal setting is payoff irrelevant, since it does not directly affect agents' wage, it increases agents' achievement and hence the principal's profits. Moreover, we show that a mediocre standard agent could end up being the most satisfied one. (JEL. D82, D86, M50, Z13)

Keywords: Intrinsic motivation; Goal-setting; Reference dependent preferences

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The object of living is work, experience, and happiness. There is joy in work. All that money can do is buy us someone else's work in exchange for our own. There is no happiness except in the realization that we have accomplished something. Henry Ford, founder of the Ford Motor Company.[2]

1 Introduction

In 1968, the American Pulpwood Association became concerned about how to increase its loggers' productivity as mechanization alone was not increasing the productivity of its logging crews. Two Industrial Organization psychologists –Edwin A. Locke and Gary P. Latham– assured the firm's managers that they had found a way to increase productivity at no financial expense to anyone. The policy seemed too easy; it merely involved setting specific production goals for the loggers. The novelty was that these goals were wage irrelevant, in contrast with classical wage relevant goals such as bonuses. The psychologists argued that introducing a goal that was difficult but attainable, would increase the challenge of the job while making it clear to the workers what was expected from them. Although the managers were quite skeptical at the beginning, the results were surprising: the performance of logging crews increased 18% and the firm's profits rose as well.[3]

This example was followed by many studies in the psychology literature on what is known as "goal setting" (e.g., Yukl & Latham (1978), Shane et al. (2003), Anderson et al. (2010)).[4] The theory states that performance goals are an important determinant

[2] This and other Henry Ford quotes are available at http://www.iwise.com/R5gdr.

[3] We can find this study in Latham & Locke (1979), which also includes similar empirical evidence for the case study with typists.

[4] In management literature, goal setting is known as "management by objectives" (MBO). Several studies find empirical evidence that MBO programs improve workers' performance (e.g., Ivancevich (1974), Bush (1998) and Mosley et al. (2001)).

of employees' motivation to work and hence affect their productivity.[5] (See Locke (1997) and Locke and Latham (2002) for a literature review.)

Our purpose in this paper is to take these kind of motivation theories only addressed in psychology and management and make them precise in standard economic theory. In particular, we propose a model where workers do have a sense of self-accomplishment and may care about pay-off irrelevant goals. This sense of accomplishment is different for workers with different personal standards, which is private information to them. Thus, a worker with a high personal standard can only be motivated to accomplish a sufficiently challenging (difficult) goal.[6]

Before describing the key elements of the model, we start by summarizing the main findings in the goal setting literature. The most important and robust finding is that the more difficult the goal is, the greater the achievement will be. This result applies as long as the individual is committed to the goal (i.e., he cares about it) and has the ability to attain it.[7] The reason why goals affect workers' achievement is that goals affect the challenge of the job and hence the satisfaction workers' obtain from

[5] The goals studied in this literature as well as the one that we use in this paper, are *non-binding goals* since they do not affect the workers' wage. Therefore, these goals do not directly affect the principal's profits (i.e., they are payoff irrelevant). In contrast, *binding goals* (bonuses for example) affect the agent's wage so they are payoff relevant.

[6] We can think of alternative explanations of the goal setting evidence. For instance, a goal may be an implicit benchmark for being retained or for future promotions. However, it is important to clarify a couple of things. First, regarding evidence in the workplace like our previous loggers example, the goal setting policy significantly increased performance even when the supervisor was not present. In this case the supervisor could only observe the crew's performance as a whole, but not the individual performance of each worker. Second, there are numerous laboratory experiments showing that individuals who have been assigned a specific goal solve more arithmetic problems or assemble more tinker toys than do people without goals (See Locke (1997)). Therefore, the evidence indicates that there is an important component of employees motivation through goal setting policies that cannot be explained with classical economic models only.

[7] Our model's set-up allows that higher goals lead to lower achievement. However, under certain conditions this may not happen in equilibrium.

the work itself. As Judge (2000) says:

> *The most effective way an organization can promote job satisfaction of its employees is to enhance the mental challenge in their jobs, and the most consequential way most individuals can improve their own satisfaction is to seek out mentally challenging work.*[8]

Therefore, goals are an important determinant of workers' satisfaction because they help develop a sense of achievement. According to the goal setting literature, goals serve as a reference point of self satisfaction, with harder goals leading to better accomplishments.

Since goals are reference points, it is also plausible that a higher goal lowers the workers' satisfaction. In fact, supporting this reasoning, Mento et al. (1992) have found that those who produce the most, those with difficult goals, are the least satisfied.[9] The question then is why do people accept these goals? According to Locke and Latham (2002), the driving force behind this result is that those people with high goals demand more from themselves, thus they are dissatisfied with less. Therefore, their personal standards are set at a higher level.[10] Similarly, Locke, Latham & Erez (1988) find in an experiment that individuals accept goals if these goals are higher than their personal standard and reject them otherwise. According to this evidence goals affect the challenge of the job differently depending on the individual's standards.

[8] Timothy A. Judge, *Promote Job Satisfaction through Mental Challenge* in The Blackwell Handbook of Principles of Organizational Behavior (2000, Chapter 6, page 107).

[9] This result applies for both, "self set" and "assigned" goals. However, it is important to remark that through the paper we consider assigned goals instead of self set goals. Therefore, people with difficult goals are people who have accepted jobs with high goals instead of people who have set a high goal for themselves in their jobs.

[10] Another example is that even if we consider researchers with the same ability. We usually observe that some of them need to publish their papers in very high ranked journals in order to get a sense of self-achievement while others are happy publishing in low ranked journals.

Finally, an important empirical fact is that demanding goals are more effective with those workers whose personal standards are high. In other words, people who demand more of themselves are the most committed to high goals.[11]

The previous findings are difficult to support with traditional economic models, such as the classical principal agent model, in which only the goals that are directly linked to the agents' wage (e.g., bonuses) affect their incentives to work. Our purpose here is to fill this gap by introducing goal setting into an economic model of managerial incentives. Therefore, we look at the following questions: Can a manager increase the workers' productivity by using goals that are linked to the job's challenge? How should the manager define the workers' goals? What are the determinants of job satisfaction?

To answer these questions we propose a principal agent model where the agent's motivation to work is twofold. First, as in standard models, the agent works in response to *extrinsic* incentives, which in our model are a pay-per-performance wage. Second, the agent has an *intrinsic* motivation to work because he derives an internal sense of achievement from accomplishing goals.[12] Coming back to our introductory example, we can easily imagine harvesting timber to be a monotonous and boring task. However, as we have seen, by setting demanding but attainable production goals, the managers were able to increase the challenge of the job and provide the loggers with a sense of accomplishment that increased their intrinsic motivation to work and hence their performance. In this paper, we capture this effect with a *goal payoff* function, which measures the intrinsic satisfaction that an agent receives from

[11] There are other results in the goal setting literature that we do not describe because they are beyond the scope of this paper, such as the definition of specific or explicit goals, the influence of the individual's self-confidence on the level of the goals accepted, or the importance of feedback showing progress for the effectiveness of the goal.

[12] As Frey (2001) argues there are at least two kinds of worker motivation: extrinsic and intrinsic. The extrinsic motivation is based on incentives coming from outside the worker such as his wage. However, there are other intrinsic motives coming from inside the worker, and that apparently give no reward except the work itself.

his production with respect to the goal set by the principal. Thus, an agent gets a positive goal payoff if he produces above and beyond the set target but a negative goal payoff otherwise. Workers, however, differ in their perception of how challenging goals may be. For instance, we may observe that for loggers who demand more from themselves, only those goals that require a greater amount of timber to be harvested will be found challenging. On the other hand, those loggers who demand little from themselves, lower goals may be just as challenging. We model this *goal commitment* effect with a reference dependent function in which the reference point is the agent's own standard. In particular, we consider the standard as the point up to which an agent considers the goal to be challenging and thus obtains a positive goal commitment.[13]

In our model, agents differ only in their personal standards. Hence, agents with different standards can be motivated differently by the same goal because some of them may consider it to be challenging while others do not. Therefore, the principal will design different contracts (with different goals) for different agent types. We show that at the optimal contract, goals are met by agents and thus they derive a positive intrinsic utility. We also show that the agents' production as well as the goals set by the principal increase with the agents' standard. Thus, in our model, goals that are non-binding for the agent, i.e., they are payoff irrelevant for the principal, increase the principal's profits with respect to the classical principal agent model with no goals. As in classical principal agent models, the principal distorts the low type's contract in such a way that his production decreases with the standard of higher types. With respect to the utility that agents get in equilibrium, we show two important results. First, in our two types model we show that the utility of the high type is an inverted U-shaped function of the agent's standard. Thus, the most satisfied agent is a high type with a mid-ranged standard. Second, in a three types case we show that a

[13] In our model, personal standards do not matter unless there are goals. As we shall see, if the principal does not assign goals, the agents have no intrinsic motivation to work. This is a simplifying assumption.

mid-ranged agent type could be the one most satisfied. In fact, although the highest type achieves the highest production he can receive a zero utility. The intuition is as follows, if the highest type's standard is sufficiently high he does not consider the goals assigned to the other agents to be challenging, thus his informational rents are zero.

While in recent years the problem of goal setting has become an extremely popular topic in psychology and management, the idea of goals that are not linked to the workers' wage may have an economic effect which thus far has received very little attention. Some exceptions deserve to be mentioned. Some papers study the effects of a self-set goal to attenuate the self-control problems of dynamically inconsistent agents. For instance, Hsiaw (2009) studies an optimal stopping problem (or a project termination decision) with hyperbolic discounters in which there is an option value of waiting due to uncertainty. In her model, goals, which act as a reference point up to which agents get an additional positive utility, induce more patient behavior by providing an additional incentive to wait for a higher realization of the project's value. Therefore, the main result is that endogenous goal setting attenuates the impulsiveness of an agent with present-biased time preferences. In our model we use assigned goals in a principal agent model, which makes our research questions and findings completely different.

Köszegi and Rabin (2006) study a model of reference dependent preferences, where the reference point is a person's rational expectations about outcomes. According to this theory, agents are influenced by a "gain-loss sense" that affects the maximum price they are willing to pay. For instance, if a consumer expects to buy a pair of shoes, she experiences a sense of loss if she does not buy them, and this sense of loss increases the maximum price she is willing to pay for the shoes. Daido & Itoh (2007) introduce these preferences in an agency model. They show that under risk aversion, the agent's higher expectation allows the principal to implement greater effort with lower-powered incentives. Moreover, they obtain the two types self-fulfilling prophecy:

the Galatea and the Pygmalion effect. In the former an agent's self-expectation about his performance determines his actual performance, while in the latter the principal's expectation about the agent's performance has an impact on the agent's performance. Although, as in our model, they study a principal agent model with agents' reference dependent preferences, the focus of Daido and Itoh (2007) greatly differs from ours. Firstly, the results of a principal agent model with agents' preferences á la Köszegi and Rabin (2006) can only vary from the standard model if there is common uncertainty about the production function (moral hazard) and not in an adverse selection setting like ours. And more importantly, in our model the agent's reference point (i.e., the goal) is a decision variable of the principal rather than the agent's rational expectations. This allows us to incorporate goal setting as a part of the principal's motivation policy.

Finally, this paper is related to the models that account for the individuals' intrinsic motivation to work. For instance, Bénabou & Tirole (2003) study a principal agent model in which the principal has better information than the agent about the agent's type. The authors show that, although performance incentives lead to an increment of the agent's effort in the short run, they are negative reinforcements in the long run. The idea is that if the principal pays a bonus to induce low ability agents to work (i.e., the principal increases the agent's "extrinsic" motivation), then the agent perceives the bonus as a bad signal about his own ability (which reduces his "intrinsic" motivation). Some papers have also studied the optimal incentive contract when agents have intrinsic motivation. For instance, Fischer & Huddart (2008) study a model where the agents' cost of effort is determined by a social norm; this social norm makes agents work harder in response to an increment in the average effort of their peers. These norms influence the power of financial incentives within an organization. In contrast with this literature, in our model the principal has a more active role since he can directly influence the agent's intrinsic motivation by setting the reference point of his intrinsic utility.

The paper proceeds as follows. Section 2 describes the basic model. In Section 3 we analyze the principal agent relationship by characterizing the optimal contract and studying the two types and the three types cases. Finally, Section 4 concludes.

2 The Model

We study a principal agent model with one risk-neutral employer, the principal, and one worker, the agent. The principal's utility is given by the output produced by the agent, y, minus the wage she has to pay, w.

Output is given by the production function $y = \theta e$, where e is the agent's effort and θ is the agent's ability (i.e., his level of human capital).[14] The agent's disutility of effort, $c(e)$, is a convex function. For simplicity, we assume $c(e) = \frac{e^2}{2}$. We assume θ is observable so that, by observing output, the principal can infer the agent's effort. Thus, we abstract away from moral hazard concerns. The principal offers contracts that are pairs $\{w, g\}$, where w is the wage and g is a production goal. We consider a pay-per-performance wage, $w(y)$, whereas the production goal is a *non-binding goal* since it does not directly affect the agent's wage. We assume that the principal has all the bargaining power so that the contract is a "take-it-or-leave-it" offer.

In this model, there are two ways to motivate the agent to work: an *extrinsic motivation*, which is the difference between the wage and the disutility of effort, and an *intrinsic motivation*, which is the agent's sense of pride in having accomplished goal g with the production y. Therefore, in our setting, challenging goals play the role of inducing the individuals' pride. Moreover, we consider that goals affect the challenge of the job differently depending on what the agents demand from themselves. We capture this effect with the personal standards parameter, s, which is private

[14] We use a standard technology where θ and e are complements. Thus, the greater the agent's ability, the greater the agent's effort productivity. Similar results can be obtained using an additive function where θ and e are independent.

information for the agent, so there is an adverse selection problem. We denote by $V(y, g, s)$ the agent's intrinsic utility function and specify the agent's utility function as

$$U = w(y) + V(y, g, s) - \frac{e^2}{2}.$$

We assume that the intrinsic utility function is of the form $V(y, g, s) = \psi(g, s)\, v(y, g)$ if $g > 0$ and $V(y, g, s) = 0$ if $g = 0$.[15] Where $\psi(g, s)$ is the agent's goal commitment, i.e., the intensity of the intrinsic utility, and $v(y, g)$ is the agent's goal payoff. The *goal payoff function* $v(y, g)$ is the satisfaction that the agent derives from accomplishing output y, when his production goal is g. In order to get closed-form solutions we assume that $v(y, g) = g \ln\left(\frac{y}{g}\right)$. This function satisfies the following properties consistent with empirical facts in the psychology literature:[16]

(i) *Goal dependence*: $v(y, g) \geq 0$ if and only if $y \geq g$;

(ii) *Monotonicity*: $v_1(y, g) > 0$;

(iii) *Complementarity*: $v_{12}(y, g) > 0$; and,

(iv) *Concavity*: $v_{11}(y, g) < 0$.

Property (i) says that the agent obtains a positive goal payoff as long as he meets the goal. Property (ii) says that, for any goal, the agent's goal payoff increases with output. Property (iii) states that goal and output are complements. Therefore, the more difficult attaining the goal is, the greater the marginal payoff from attaining it will be.[17] Finally, property (iv) says that the agent's goal payoff is concave in

[15] From the argument below it is clear that function $v(y, g)$ is not defined for $g = 0$. However, $\lim_{g \to 0} v(y, g) = 0$. Therefore, function $V(y, g, s)$ is continuous for all $g \geq 0$.

[16] See Locke (1997) and Locke and Latham (2002).

[17] Atkinson (1958) finds that if the goal's increment is impossible to attain (or the individual believes that it is impossible), the performance can indeed decrease. Although this "inverse-U" relationship between output and goals is very intuitive, under our conditions that goals may be difficult but attainable a complementarity relationship may best fit with the evidence (see Locke (1997)).

production. Therefore, the marginal goal payoff decreases as the gap between the agent's output and the goal increases.[18]

As we mentioned in the introduction, a necessary condition for goals to influence an agent's performance is that agents are committed to those goals. Although the individuals' goal commitment is a complex theme in the related literature, here we choose an easy and intuitive modelling strategy. The goal commitment is determined by the interaction between goals and personal standards; high personal standards require challenging goals in order for the agent to take pride in accomplishment.[19] Formally, the *goal commitment function*, $\psi(g, s)$, is a reference dependent function, where s is the reference point. For simplicity we consider the following step function:

$$\psi(g, s) = \begin{cases} s & \text{if } g > s, \\ \underline{s} & \text{if } g \leq s. \end{cases}$$

From here on we say that an agent with standard s considers goal g to be *challenging* when $g > s$. In the next proposition, we show an important property of the goal commitment function.

Proposition 1 *In equilibrium* $\frac{de^*}{dg} \geq 0$ *if and only if* $s \geq \underline{s}$.

[18] Imagine for instance that a researcher has the goal of publishing three research papers in top journals. Therefore, he gets a positive intrinsic satisfaction if he attains it whereas he suffers if he fails to do so (property (i)). Moreover, his satisfaction increases with the number of papers published (property (ii)). Obviously, the sense of achievement from attaining this research goal would be lower with an easier goal such as publishing one paper in a lower ranked journal (property (iii)). Finally, if the researcher has already published five papers, the increment in his intrinsic utility if he produces another one is lower than if he only has two or three papers (property (iv)).

[19] There are other determinants of individual's goal commitment that we do not consider here. For instance, there is empirical evidence that core self-evaluations such as self-steem or self-regard, affect the individuals' goal commitment (See, Judge et al. (1998)). Another important determinant of goal commitment is the individuals' participation in the goal setting process (See, Anderson et al. (2010)).

If the goal commitment is greater with a challenging goal than with a non-challenging one, the agent's effort does not decrease with the assigned goal. The intuition is simple, since higher goals increase goal commitment, agents are more motivated to get goal payoff, so they work harder in response to goals. As we have already mentioned, the most consistent empirical fact in the goal setting literature is that agents exert greater effort in response to more challenging and attainable goals. Therefore, from here on, we shall assume that the function $\psi(g, s)$ satisfies:

(v) *Challenging goals are motivational:* $s \geq \underline{s} = 0$.[20]

Note that because of assumption (v), an agent with standard s considers goal g to be challenging $(g > s_i)$ if and only if he is committed to it, i.e., $\psi(g, s) > 0$. This is an intuitive property stating that difficult tasks are motivational,[21] and it is consistent with the findings of Mento et al. (1992) and Locke and Latham (2002) discussed in the introduction, in which the agents' standards are the reference points of their (intrinsic) satisfaction. Moreover note that our goal commitment function satisfies another empirical finding which was discussed in the introduction,

(vi) *Demanding agents are more committed to challenging goals:* $\psi_2(g, s) \geq 0$ iff $g > s$.

As we will see in the next section, this property is important in order to sort agents' types.

Therefore the agent's utility function is given by

$$U = \begin{cases} w + sg \ln\left(\frac{y}{g}\right) - \frac{e^2}{2} & \text{if } g > s, \\ w - \frac{e^2}{2} & \text{if } g \leq s. \end{cases} \tag{1}$$

[20] Our main results still apply if we consider a more general function where $s \geq \underline{s} > 0$.

[21] A similar interpretation would be that a strong commitment to goals is attained when the agent is convinced that they are important, and demanding agents only consider challenging goals to be important (Locke (1997) page 119).

Note that U is discontinuous. If $g \leq s$, the agent obtains zero intrinsic utility; whereas, if $g > s$, he obtains positive intrinsic utility when $y > g$. Thus, in order to get a positive intrinsic motivation ($V(y, g, s) > 0$), an agent not only needs sufficiently high production ($y > g$) to receive a positive goal payoff, but also a sufficiently high goal ($g > s$) to get a positive goal commitment.

The principal does not observe the agent's standard, thus, we have an adverse selection problem. For simplicity we begin by assuming that the personal standard can take two values $s \in \{s_L, s_H\}$, where $s = s_H$ with probability p. In Section 3.4, we extend the analysis to three agent types.

3 The Principal-Agent Relationship

We begin the analysis by characterizing the optimal contract offered by the principal to an agent with goal dependent preferences. Applying the revelation principle, the principal designs one contract for each agent type, $\{w, g\} = \{(w_L, w_H), (g_L, g_H)\}$. Let us define $U(s_i, s_j) = w_j + V(y_j, g_j, s_i) - \frac{e_j^2}{2}$ as the utility of an agent with standard s_i choosing the contract offered to an agent with standard s_j. The principal chooses a wage structure w and sets production goals g that induce efforts $e = (e_L, e_H)$ to maximize expected profit subject to the agent's participation (IR) and incentive compatibility (IC) constraints. Thus, the *principal's problem* is

$$\max_{\{w,g\}} \; p(y_H - w_H) + (1-p)(y_L - w_L)$$

subject to, for all $i, j \in \{L, H\}$

$$w_i + V(y_i, g_i, s_i) - \frac{e_i^2}{2} \geq 0, \qquad (IR)$$

$$w_i + V(y_i, g_i, s_i) - \frac{e_i^2}{2} \geq w_j + V(y_j, g_j, s_i) - \frac{e_j^2}{2}. \qquad (IC)$$

Our first result states that the agent gets a non-negative intrinsic utility in equilibrium.

Lemma 1 *Given a contract* $\{w, g\}$, *in equilibrium* $V(y_i, g_i, s_i) \geq 0$.

The intuition is simple: if agents get a positive intrinsic utility from their job, it is easier to make them participate. If agents receive a negative intrinsic utility, the principal has to pay them higher wages to assure their participation. This can be avoided if the principal offers non-challenging goals ($g_i \leq s_i$) to the agents in such a way that they are not committed to goals ($\psi(g_i, s_i) = 0$). Thus, their intrinsic utility is zero ($V(y_i, g_i, s_i) = 0$).[22]

In order to solve the model, we need to identify a *monotonicity* or *single crossing condition* for the utility function that allows us to sort agent types. Note that this is not obvious in our environment because of the discontinuity of the utility function. We first show that the agent with the high standard will be the one who obtains the highest surplus in equilibrium.

Lemma 2 *Given a contract* $\{w, g\}$, *in equilibrium* $U(s_H, s_H) \geq U(s_L, s_L)$.

By Lemma 2, we can apply standard results in principal agent models which state that the individual rationality of the low type, IR_L, and the incentive compatibility constraints of the high type, IC_H, are binding in equilibrium. Because of this, the next proposition follows.

[22] Therefore, this result is a consequence of our assumption that $\underline{s} = 0$. Thus, agents get zero goal commitment, $\psi(g, s)$, when goals are not challenging for them ($g_i \leq s_i$). If $\underline{s} > 0$, it is possible that in equilibrium $V(y_i, g_i, s_i) < 0$ for some agent i. Therefore, we should study more cases, but our qualitative results would remain unchanged.

Proposition 2 *Given a contract $\{w, g\}$, in equilibrium, IR_L and IC_H bind, i.e.,*

$U(s_L, s_L) = 0,$ *and*

$$U(s_H, s_H) = U(s_H, s_L) = \begin{cases} g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L) & \textit{if } g_L > s_H, \\ 0 & \textit{if } g_L \leq s_H. \end{cases}$$

The low type agent gets zero surplus in equilibrium, and the high type obtains informational rents when the low type's goal is challenging for him (i.e., $g_L > s_H$). Otherwise, the high type agent receives no intrinsic utility from taking the low type contract. Thus, the principal does not need to pay him informational rents.

The next lemma provides a useful result regarding the agents' intrinsic utility in equilibrium.

Lemma 3 *Given a contract $\{w, g\}$, in equilibrium, for all $i \in \{L, H\}$,*

(i) $V(y_i, g_i, s_i) > 0$ *if and only if* $y_i > s_i$,

(ii) $V(y_i, g_i, s_i) = 0$ *if and only if* $y_i \leq s_i$.

By Lemma 3, we know that the agent gets a challenging job in equilibrium, and hence a positive intrinsic utility, if and only if the agent's production is greater than his standard. This is because when $y > s$, the principal can design a goal which is both challenging ($g > s$) and can be successfully accomplished by the agent ($y > g$). Note that this is the best situation for the principal because IR constraints are relaxed and the principal can offer lower wages. However, if $y < s$, there is no way to design a goal that is both challenging and can be successfully accomplished by the agent. In this case, the principal prefers to offer non-challenging goals in order to avoid negative intrinsic utilities.

Since $y = \theta e$, by Lemma 3, it is immediate that when the agent's ability, θ, is high, the principal can always offer a challenging goal to both agent types. This is the content of the next corollary.

Corollary 1 *Given a contract $\{w, g\}$ and the agent's standard s_i, if θ is sufficiently high, in equilibrium, $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$.*

To simplify the analysis, from here on we assume that the condition in Corollary 1 holds, so that agents are intrinsically motivated in equilibrium.[23]

Before setting the equilibrium contracts, we begin by studying the two cases that may arise in equilibrium (see Proposition 2): an *informational rents case*, in which the high type agent gets a positive utility in equilibrium, and a *rent extraction case*, in which both agents obtain a zero utility in equilibrium.

3.1 The Informational Rents Case

As a starting point, we assume that there is an equilibrium in which the low type's goal is challenging for the high type agent ($g_L > s_H$), so that he gets positive informational rents. Then, applying Proposition 2, we have

$$U(s_H, s_L) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L) > U(s_L, s_L) = 0.$$

Therefore, the equilibrium of the model is given by the solution to the principal's problem, where the binding constraints can be rewritten as

$$w_L = \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \tag{IR$_L$}$$

$$w_H = \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right) + g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L). \tag{IC$_H$}$$

Denoting by $\boldsymbol{e}$ Euler's number, the solution to the principal's problem is

$$\begin{aligned} e_H &= \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2(\boldsymbol{e} + s_H), \\ e_L &= \theta\left(1 + \frac{s_L - p s_H}{(1-p)\,\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2\left(\boldsymbol{e} + \frac{s_L - p s_H}{(1-p)}\right). \end{aligned}$$

[23] In the appendix we study the cases that do not satisfy the condition of Corollary 5. We skip these cases here because results are very similar and the intuitions are the same.

Figure 1 shows some comparative statics. We fix the standard of the low type and we plot the results as a function of the high type standard. This allow us to see the effect of the high type standard on the low type contract and hence the informational rents.

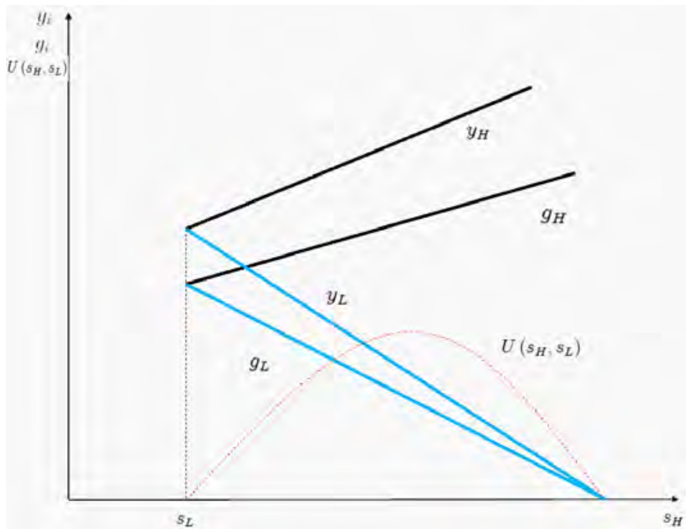

Figure 1. The solution with positive informational rents

Since Corollary 1 is satisfied, i.e., agents are committed to goals in equilibrium, it is immediate that the principal sets goals that maximize the agent's goal payoff given his production, $g_i = \arg\max_{g}.\ v(y_i, g)$, thus $g_i = \frac{y_i}{\boldsymbol{e}}$. Therefore, the principal sets goals that agents can accomplish, $y_i > g_i$. The idea is that the principal uses goals to maximize the agent's intrinsic utility in order to pay lower wages. As we can see in Figure 1, the high type's effort, e_H, as well as his goal, g_H, increase with his standard, s_H. The rationale behind this result is clear: as the agents' standards increase, the principal offers them jobs with demanding goals. By doing so, the principal motivates agents to work hard so that they can reach a high production level. For the low type, both his effort and his goal decrease with the high type standard, s_H. The principal distorts the contract offered to the low type in order to extract greater surplus from the high type. As s_H increases, the high type is more important than the low type for the principal, so he further distorts the low type contract. For the same reason, the lower the proportion of high types, p, the lower the distortion of the low type contract will be. In fact, if $p = 0$, there is no distortion at all.

We can see in Figure 1 that as the high type's standard increases, the production of the high (low) type increases (decreases) at a higher rate than his assigned goal. Thus, in equilibrium, the intrinsic utility of the high (low) type agent is an increasing (decreasing) function of the high type's standard.[24] Therefore, the principal distorts the low type's contract so that his goal payoff, $v(y_L, g_L)$, decreases with s_H.

Regarding the high type's informational rents, we have the following trade-off: On the one hand, as the high type's standard increases, the agent's goal commitment increases as well. This has a direct *positive effect* on the informational rents. On the other hand, we have a *negative effect*, since the greater the high type's standard is, the more will be the principal's distortion of the low type's contract, so that the utility extracted by the high type when choosing the low type contract is lower. Formally, the informational rents function is $v(y_L, g_L)(s_H - s_L)$, where the second part is increasing in s_H and $v(y_L, g_L)$ decreases with s_H as we have just shown. Due to the concavity of the goal payoff function, the negative effect dominates the positive effect when s_H is sufficiently high. This is the intuition of the inverted U shape of the informational rents function illustrated in Figure 1.[25]

To complete the characterization of the contract, we depict the equilibrium wages in Figure 2.

[24] These results hold true if assumptions (i) and (ii) on the function $v(y, g)$ hold.

[25] We can easily check that with a linear goal payoff function the informational rents function is concave and increasing in s_H.

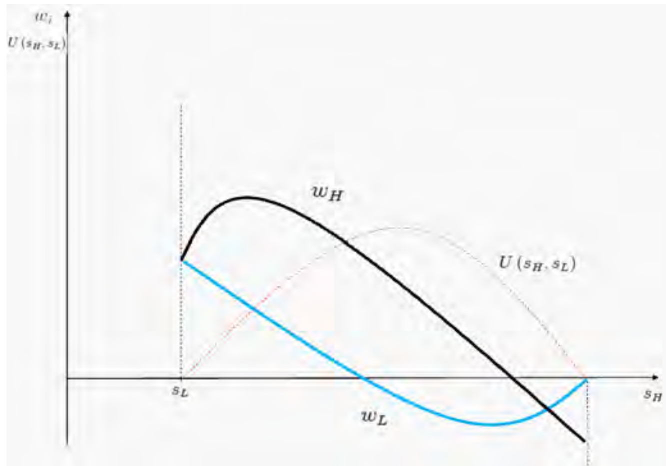

Figure 2. The wages with positive informational rents

Let us recall that from IR_L that

$$w_L = \frac{e_L^2}{2} - V(y_L, g_L, s_L).$$

Thus, the low type agent's wage equals the disutility of effort minus his intrinsic utility. As we have seen, the low type agent's effort, as well as his goal and his intrinsic utility, decrease with the high type's standard, s_H. Due to the concavity of the intrinsic utility function and the convexity of the disutility of effort, the reduction of the intrinsic utility effect dominates the reduction of effort effect if s_H is sufficiently high, so that w_L has a U-shaped form.

Similarly, from IC_H,

$$w_H = \frac{e_H^2}{2} + U(s_H, s_L) - V(y_H, g_H, s_H).$$

Thus, the wage of the high type agent equals the disutility of effort plus the informational rents minus the intrinsic utility. As we know, the high type agent's effort, as well as his goal and intrinsic utility, increase with s_H. If s_H is sufficiently high the intrinsic utility effect dominates the increment in the disutility of effort and the informational rents effect, so that w_H presents an inverted U-shaped form.

Note that if s_H is sufficiently high wages are negative. It is immediate that an agent with no intrinsic motivation (i.e., $V(\cdot) = 0$) and zero productivity (i.e., $\theta = 0$)

receives a zero wage in this model. Therefore, a negative wage means that an intrinsically motivated agent could get a lower wage than an agent with no intrinsic motivation.[26]

3.2 The Rent Extraction Case

Here we study the case in which the low type goal is not challenging for the high type ($g_L \leq s_H$) whereas the high type is given a challenging goal ($g_H > s_H$). Therefore, the informational rents are zero. Note that this case is equivalent to the perfect information case. Moreover, remember that because of Corollary 1 the agents get a challenging goal ($g_1 > s_i$) in equilibrium. Hence we can rewrite the IR_L and IC_H constraints as

$$w_L = \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \tag{IR$_L$}$$

$$w_H = \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \tag{IC$_H$}$$

Therefore, the solution of the principal's problem is, for all $i \in \{L, H\}$

$$\begin{aligned} e_i &= \theta\left(1 + \frac{s_i}{\boldsymbol{e}}\right), \\ g_i &= \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_i), \end{aligned}$$

For both agent types, the effort, e_i, as well as his goal, g_i, increase with his standard, s_i. In this case, the low type's contract does not depend on the high type's standard. In other words, the principal does not distort the low type's contract as in the previous case.

[26] Note that in our model an agent with no intrinsic motivation always gets a zero utility in equilibrium. However, an intrinsically motivated agent may get a positive utility in the form of informational rents.

3.3 The Optimal Contract

In this section, we characterize the optimal contract offered by the principal. Proposition 2 states that one of the two cases studied above may arise in equilibrium: the informational rent case and the rent extraction case. While in the former the low type's goal is challenging for the high type agent, and hence he gets positive informational rents, in the latter the low type's goal is non-challenging for the high type and thus the principal can extract the entire surplus of both agents' types. Let us consider the informational rents case depicted in Figure 1. In this case, the high type agent is committed to the low type's goal, thus $g_L > s_H$ so that $\psi\left(g_L, s_H\right) > 0$. As s_H increases, g_L decreases, therefore there is $s_H = \overline{s}_I$ such that both variables coincides. Thus $\psi\left(g_L, s_H\right) = 0$, which is the rent extraction case.

Note that, in equilibrium, the goal offered to the high type agent, and hence his effort, has the same functional form independently of whether he gets positive informational rents or not, while the contract of the low type is different in the two situations.[27] The next figure illustrates the low type's production and his assigned goal as well as the informational rents as a function of the high type's standard, s_H.

[27] This is the standard "non distortion at the top, distortion at the bottom" result in adverse selection models.

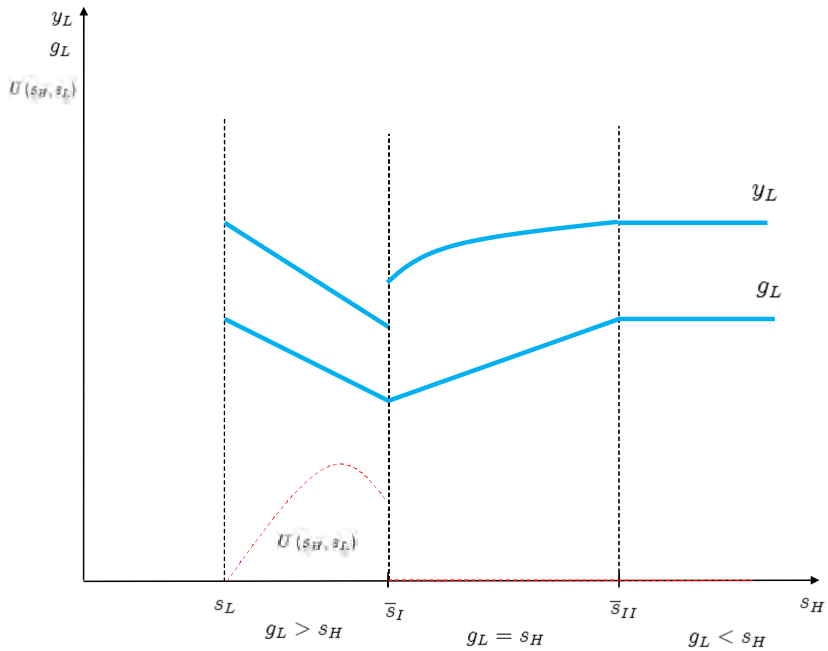

Figure 3. Low type equilibrium.

Thus, if $s_H \in (s_L, \overline{s}_I)$, we are in the informational rents case; whereas, if $s_H \geq \overline{s}_I$, we are in the rent extraction case. If $s_H \in [\overline{s}_I, \overline{s}_{II}]$, we have a corner solution in which $g_L = s_H$, while if $s_H > \overline{s}_{II}$ then $g_L > s_H$.[28] The next proposition fully characterizes the equilibrium.

Proposition 3 *Given $p \in (0,1)$ and $s_L \geq 0$, the optimal production goals are*

$$g_H^* = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H)\,,\; g_L^* = \begin{cases} \left(\frac{\theta}{\boldsymbol{e}}\right)^2 \left(\boldsymbol{e} + \frac{s_L - p s_H}{(1-p)}\right) & \text{if } s_H \in (s_L, \overline{s}_I)\,, \\ s_H & \text{if } s_H \in [\overline{s}_I, \overline{s}_{II}]\,, \\ \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_L) & \text{if } s_H > \overline{s}_{II}. \end{cases}$$

[28] All the technical details are relegated to the Appendix, in which we additionally provide the solution of the cases that violate the condition of Corollary 1, for all of which the high type agent gets zero informational rents.

While the optimal efforts provided by agents are

$$e_H^* = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right),\; e_L^* = \begin{cases} \theta\left(1 + \frac{s_L - p s_H}{(1-p)\boldsymbol{e}}\right) & \text{if } s_H \in (s_L, \overline{s}_I), \\ \frac{\theta + \sqrt{4 s_L s_H + \theta^2}}{2} & \text{if } s_H \in [\overline{s}_I, \overline{s}_{II}], \\ \theta\left(1 + \frac{s_L}{\boldsymbol{e}}\right) & \text{if } s_H > \overline{s}_{II}, \end{cases}$$

where $\overline{s}_I = \frac{\theta^2(s_L + \boldsymbol{e}(1-p))}{p\theta^2 + \boldsymbol{e}^2(1-p)}$ *and* $\overline{s}_{II} = \left(\frac{\theta}{\boldsymbol{e}}\right)^2(\boldsymbol{e} + s_L)$.

The optimal contract gives the maximum informational rents to the high type agent when he has an intermediate standard. This result arises for two reasons. Firstly, because of the inverted U-shaped informational rent function discussed previously, thus if s_H is sufficiently high with respect to s_L, the principal distorts the low type contract so much that the informational rents decrease with s_H. Secondly, because if $s_H \geq \overline{s}_I$, the low type goal designed by the principal is not challenging for the high type and so his intrinsic utility when taking the low type contract (i.e., the informational rents) is zero. Therefore, an agent gets a zero surplus if he is a low type, or he is so demanding that the low type goal is not challenging enough to derive pride in accomplishing it.

It is straightforward to show that in our principal agent model with no goals, which leads to $V(\cdot) = 0$, the effort exerted by the agent is $e = \theta$. In our model we have shown that while goal setting is payoff irrelevant since it does not directly affects the agents' wage, it does increase the agent's output and hence the principal's profits. Moreover we have shown that the higher the agent's standard, the greater the principal's profits will be.

3.4 The Three Types Model

Here we show that the model can be easily extended to a three types case, i.e., $s \in \{s_L, s_M, s_H\}$ with $s_H > s_M > s_L > 0$. First of all, we can check that Lemma

1, Lemma 3 and hence Corollary 1 apply as well to the three types case.[29] For simplicity we consider that the condition of Corollary 1 satisfies such that in equilibrium $V(y_i, g_i, s_i) > 0$. In the next proposition, we find which constraints bind.

Proposition 4 *Given a contract* $\{w, g\} = \{(w_L, w_M, w_H), (g_L, g_M, g_H)\}$*, in equilibrium,* IR_L *and* $IC_{M,L}$ *and* $IC_{H,M}$ *bind, i.e.,*

$$\begin{aligned}
U(s_L, s_L) &= 0, \\
U(s_M, s_M) &= U(s_M, s_L) = \begin{cases} g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_M - s_L) & \text{if } g_L \in (s_M, s_H), \\ 0 & \text{if } g_L \leq s_M. \end{cases} \\
U(s_H, s_H) &= U(s_H, s_M) = \begin{cases} g_M \ln\left(\frac{\theta e_M}{g_M}\right)(s_H - s_M) & \text{if } g_M > s_H, \\ 0 & \text{if } g_M \leq s_H. \end{cases}
\end{aligned}$$

Therefore, our previous results with two agent types are robust to the case of three types. Note that, when $g_L \in (s_M, s_H)$ and $g_M \leq s_H$, the medium type will obtain positive informational rents while the high type will not. Hence, with three consumer types, a mid-ranged agent (not only a mid-ranged standard of the high type as before) could be the most satisfied.

Note that in the classical principal agent model the highest type, the most productive one, has the highest informational rents. However, in our model, the agent who produces the most—the one with the highest standard—may have zero informational rents when he does not consider lower goals to be challenging. In other words, being very demanding can be detrimental.

There is evidence of this effect. In an experiment with undergraduate students, Mento et al. (1992) found that the highest degree of satisfaction is reached by students with a grade goal of C (i.e., students with a mediocre standard) while the lowest one

[29] These results are a consequence of our goal dependent utility function specification rather than the number of agent types.

was attained by students with a grade goal of A (i.e., students with a very high standard). Our results are in line with this empirical evidence.

4 Conclusion

Psychologists and experts in management have long documented the importance of goal setting in worker motivation. In particular, they have found that when workers are committed to challenging but attainable goals, their performance increases even if those goals are not directly linked to wages. In this paper, we have introduced goal setting in a principal agent model of managerial incentives. Agents care about goal setting because achieving those goals creates a sense of pride in accomplishment that modifies their intrinsic motivation to work. We have shown that, in an optimal contract, more challenging objectives increase agents' performance and that the goals set by the principal increase with the agent's standard. Therefore, goals that are payoff irrelevant, since they do not directly affect agents' extrinsic incentives, increase the principal's profits. We have also shown that a mid-ranged standard gives the highest satisfaction to an agent and that a mid-ranged agent type could be the most satisfied among all the agent types. Therefore, being very demanding can be detrimental.

There are some promising lines for future research. First of all, our goal commitment function is a very simple one; an agent is committed to a goal when it exceeds his personal standard sufficiently for him to consider the goal to be challenging. Psychologists have found that there are other determinants of goal commitment that should be studied in an economic model, such as the agents' self-efficacy (i.e., ability confidence) and the agents' participation in the goal setting processes (See Anderson et al. (2010) and Bush (1998)).

A very interesting line of future research is to endogenize the personal standard parameter. There are several ways to do this. First, in a model with different abilities we can imagine that the agent's standard is in part determined by his ability. Second, we can think that the personal standard is determined by the agent's rational expectation about outcomes. This would provide a very good link between the present model with goal dependent preferences and the reference dependent utility from expectations literature (such as models with preferences à la Köszegi and Rabin (2006)). In fact goal setting provides an additional explanation of the formation of reference states. For instance, with an experimental study Matthey (2010) finds evidence that apart from an individual's own past, present and expected future outcomes and the outcomes of relevant others, reference states also depend on environmental factors that do not influence outcomes, i.e., they are payoff irrelevant like the goals studied in this paper.

Another topic would be to introduce competition in the model. If we consider that firms compete for workers, we should reconsider our result that very demanding (and hence productive) agents may be the least satisfied. With competition we should have two opposite effects. On the one hand, we have the effect studied in this paper that very demanding workers may get lower satisfaction than lower types. But, on the other hand, firms compete for more demanding agents offering them higher wages, which has a positive effect on the satisfaction of very demanding agents.

Finally, there is evidence that goal setting policies have more impact on agents' performance as time goes by. In particular, Ivancevich (1974) finds that in a manufacturing company a goal setting program significantly improves workers' performance within six months after implementation. Therefore, it would be interesting to extend our model to allow for dynamic considerations. One possibility is to allow personal standards to be positively related with past goals.

5 References

Anderson, S. W., Deker, H. C. and Sedatole, K. L. (2010), "An empirical Examination of Goals and Performance-to-Goal Following the Introduction of an Incentive Bonus Plan with Participative Goal Setting", Management Science, 56, 90-109.

Bènabou, R. and Tirole, J. (2002), "Self-Confidence and Personal Motivation", The Quarterly Journal of Economics, 117, 871-915.

Bènabou, R. and Tirole, J. (2003), "Intrinsic and Extrinsic Motivation", Review of Economics Studies, 70, 489-520.

Bènabou, R. and Tirole, J. (2006), "Incentives and Prosocial Behavior", American Economic Review, 96, 1652-1678.

Bush, T. (1998), "Attitudes towards Management by Objectives: An Empirical Investigation of Self-Efficacy and Goal Commitment", The Scandinavian Journal of Management, 14, 289-299.

Daido, K. and Itoh, H. (2007), "The Pygmalion and Galatea Effects: An Agency Model with Reference-Dependent Preferences and Applications to Self-Fulfilling Prophecy", Working Paper, Hitotsubashi University.

Dweck, C. S. (2006), "Mindset: The new Psychology of success", Ed. Random House.

Dweck, C. S. and Leggett, E. L. (1988), "A Social-Cognitive Approach to Motivation and Personality", Psychological Review, 95, 256-273.

Ellingsen, T. and Johannesson, M. (2008), "Pride and Prejudice: The Human Side of Incentive Theory", American Economic Review, 98, 990-1008.

Falk, A. and Knell, M. (2004), "Choosing the Joneses: Endogenous Goals and Reference Standards", The Scandinavian Journal of Economics, 106, 417-435.

Fischer, P. and Huddart, S. (2008), "Optimal Contracting with Endogenous Social Norms", American Economic Review, 98, 1459-1475.

Frey, B. S. (1997), "On the relationship between intrinsic and extrinsic work motivation", International Journal of Industrial Organization, 15, 427-439.

Frey, B. S. and Jegen, R. (2001), "Motivation Crowding Theory", Journal of Economic Surveys, 15, 589-611.

Hsiaw, A. (2009), "Goal-Setting, Social Comparison, and Self Control", Working Paper, Princeton University.

Ivancevich, J. M. (1974), "Changes in Performance in a Management by Objectives Program", Administrative Science Quarterly, 19, 563-574.

Judge, T. A., (2000), "Promote Job Satisfaction through Mental Challenge", The Blackwell Handbook of Principles of Organizational Behavior, 75-89.

Judge, T. A., Locke, E. A., Durham, C. C. and Klueger A. N. (1998), "Dispositional Effects on Job and Life Satisfaction: The role of Core Evaluations", Journal of Applied Psychology, 83, 17-34.

Köszegi, B. and Rabin, M. (2006), "A model of Reference-Dependent Preferences", The Quarterly Journal of Economics, 121, 1133-1165.

Latham, G. P. (2000), "Motivate Employee Performance through Goal-Setting", The Blackwell Handbook of Principles of Organizational Behavior, 107-119.

Latham, G. P. and Locke, E. A. (1979), "Goal Setting: A Motivational Technique That Works", Organizational Dynamics, Autumn, 68-80.

Locke, E. A. (1996), "Motivation Through Conscious Goal Setting", Applied & Preventive Psychology, 5, 117-124.

Locke, E. A., Latham, G. P. and Erez, M. (1988), "The Determinants of Goal Commitment", The Academy of Management Review, 13, 23-39.

Locke, E. A. and Latham, G. P. (2002), "Building a Practically Useful Theory of Goal Setting and Task Motivation", American Psychologist, 57, 705-717.

Matthey, A. (2010), "The Influence of Priming on Reference States", Games, 34-52.

Mento, J. A., Klein, J. H. and Locke, E. A. (1992), "Relationship of Goal Level to Valence and Instrumentality", Journal of Applied Psychology, 77, 395-405.

Mosley, H., Schutz, H. and Breyer, N. (2001), "Management by Objectives in European Public Employment Services", Discussion Paper FSI01-203, Social Science Research Centre (WZB).

Murdock, K. (2002), "Intrinsic Motivation and Optimal Incentive Contracts", RAND Journal of Economics, 33, 650-671.

Shane, S., Locke, E. A. and Collins, C. J. (2003), "Entrepreneurial motivation", Human Resource Management Review, 13, 257-279.

Suvorov, A. and van de Ven, J. (2008), "Goal Setting as a Self-Regulation Mechanism", Working Paper, CEFIR.

Yukl, G. A. and Latham, G. P. (1978), "Interrelationships among Employee Participation, Individuals Differences, Goal Difficulty, Goal Acceptance, Goal Instrumentality, and Performance", Personnel Psychology, 31, 305-323.

6 APPENDIX

Proofs of Propositions and Lemmas

Proof of Proposition 1

Firstly we show the *if part*. Thus, given $v(y,g)$, if $s_i \geq \underline{s}$ then $\frac{de^*}{dg} \geq 0$. Note that $s_i \geq \underline{s}$ implies that $\psi_1(g, s_i) \geq 0$. This, jointly with the complementarity condition, i.e., $v_{12}(y,g) \geq 0$, imply that $\frac{dU}{dedg} \geq 0$ which means that $\frac{de}{dg} \geq 0$. Now we show the only if part, if $\frac{de^*}{dg} \geq 0$ then $s_i \geq \underline{s}$. Note that $\frac{de^*}{dg} \geq 0$ implies that effort and goals are complements, i.e., $\frac{dU}{dedg} \geq 0$, which, given $v(y,g)$, implies that $\psi_1(g, s_i) \geq 0$, i.e., $s_i \geq \underline{s}$. ***Q.E.D.***

Proof of Lemma 1

The proof is by way of contradiction. Let $\{w, g\}$ be a contract such that $y < g$ (i.e., $v(y,g) < 0$), so that $V(y,g,s) = \psi(g,s)\, v(y,g) < 0$ as $\psi(g,s) > 0$. The utility of the agent in such a contract is

$$U = w + V(y,g,s) - c < w - c.$$

Because of this, one can design a contract $\left\{w^d, g^d\right\}$ with $g^d < s$ (i.e., $\psi\left(g^d, s\right) = 0$) and $w^d < w$, which is feasible since $U^d = w^d - c \geq U$, and gives larger profits to the principal, as $w^d < w$ and $y^d = y$. ***Q.E.D.***

Proof of Lemma 2

Given a fix pair of agents' standards $\{s_L, s_H\}$, all the possible cases that can arise are:

$(i)\max\{g_L, g_H\} \leq s_L$, $(ii)\, g_L \leq s_L < s_H \leq g_H$, $(iii)\, s_L < g_L \leq s_H < g_H$, $(iv)\min\{g_L, g_H\} > s_H$, $(v)\, g_H < s_L < g_L$, $(vi)\, g_L \leq s_L \leq g_H \leq s_H$, $(vii)\, s_L \leq \{g_L, g_H\} \leq s_H$, $(viii)\, s_L < g_H < s_H < g_L$.

First we show that (vi) - $(viii)$ will not emerge in an optimal contract. In (vi) - $(viii)$, $V(y_H, g_H, s_L) > 0$ as $g_H > s_L$ and we have

$$U(s_L, s_H) = w_H + s_L v(y_H, g_H) - \frac{e_H^2}{2} > w_H - \frac{e_H^2}{2} = U(s_H, s_H).$$

Therefore, as is standard in principal-agent models, in equilibrium, the optimal $\{w, g\}$ satisfies IR binding for the "low" type (here H) and IC binding for the "high" type (here L), i.e., $U(s_H, s_H) = 0$ and $U(s_L, s_L) = U(s_L, s_H) > 0$. However, the following contract is feasible and yields higher profits to the principal

$$\{w^d, g^d\} = \{(w_L^d, w_H), (g_L, g_H^d)\},$$

where $g_H^d < s_L$ (i.e., $V(s_L, s_H) = 0$), $w_L^d = \frac{e_L^2}{2} < w_L = \frac{e_L^2}{2} + s_L v(y_H, g_H)$ and $(y_L^d, y_H^d) = (y_L, y_H)$. Consequently, (vi) - $(viii)$ can be ruled out.

Regarding the remaining cases, note that only in case (iv) we may have positive informational rents because $g_L > s_H$. In the other cases, (i), (ii), (iii) and (v), it is immediate that, in equilibrium, the principal can extract the entire agents' surplus so that $U(s_H, s_H) = U(s_L, s_L) = 0$. Therefore, the monotonicity condition, $U(s_H, s_H) \geq U(s_L, s_L)$, is satisfied in all cases $(i) - (v)$. ***Q.E.D.***

Proof of Proposition 2

By IR and IC the contract $\{w, g\}$ must satisfy $U(s_L, s_L) = 0$ and $U(s_H, s_H) = U(s_H, s_L)$. Therefore, $U(s_H, s_L) = \psi(g_L, s_H) v(y_L, g_L)$, where $\psi(g_L, s_H) > 0$ iff $g_L > s_H$. ***Q.E.D.***

Proof of Lemma 3

By Lemma 1, any optimal contract $\{w, g\}$ satisfies $V(y_i, g_i, s_i) \geq 0$, thus $V(y_i, g_i, s_i)$ is either positive or zero.

(i) First note that the if part, $V(y_i, g_i, s_i) > 0 \Longrightarrow y_i > s_i$, follows straightforwardly. We show the only if part by contradiction. Suppose that $y_i > s_i \Longrightarrow V(y_i, g_i, s_i) = 0$.

For the low type we have $y_L > s_L \Longrightarrow V(y_L, g_L, s_L) = 0$, thus $g_L \leq s_L$ since $\psi(g_L, s_L) = 0$. The following deviation is feasible and yields higher profits,

$$\left\{w^d, g^d\right\} = \left\{\left(w_L^d, w_H\right), \left(g_L^d, g_H\right)\right\},$$

where $g_L^d \in (s_L, s_H)$ (i.e., $V(y_L, g_L, s_L) > 0$ and $V(y_L, g_L, s_H) = 0$), $w_L^d = w_L - V(y_L, g_L, s_L)$ and $\left(y_L^d, y_H^d\right) = (y_L, y_H)$.

For the high type if $y_H > s_H \Longrightarrow V(y_H, g_H, s_H) = 0$, thus $g_H \leq s_H$ since $\psi(g_H, s_H) = 0$. The following deviation is feasible and yields higher profits,

$$\left\{w^d, g^d\right\} = \left\{\left(w_L, w_H^d\right), \left(g_L, g_H^d\right)\right\},$$

where $g_H^d \geq s_H$ (i.e., $V(y_H, g_H, s_H) > 0$), $w_H^d = w_H - V(y_H, g_H, s_H)$ and $\left(y_L^d, y_H^d\right) = (y_L, y_H)$. Moreover $U^d(s_L, s_L) = U(s_L, s_L) = 0$ by Proposition 2.

(ii) First note that the if part, $V(y_i, g_i, s_i) = 0 \Longrightarrow y_i \leq s_i$, follows straightforwardly. We show the only if part by contradiction. Suppose that $y_i \leq s_i \Longrightarrow V(y_i, g_i, s_i) > 0$, thus $s_i < g_i$ since $\psi(g_i, s_i) > 0$. Therefore $y_i < g_i$ which leads to $V(y_i, g_i, s_i) < 0$. ***Q.E.D.***

Proof of Corollary 1

Immediate from Lemma 3. ***Q.E.D.***

Proof of Proposition 3

Under the condition of Corollary 1 we have that $V(y_i, g_i, s_i) > 0$ so that $g_i > s_i$ for all i. Therefore we have four possible cases: (i) $s_L < g_L < s_M < g_M < s_H < g_H$, (ii) $s_L < g_L < s_M < s_H < \min\{g_M, g_H\}$, (iii) $s_L < s_M < \min\{g_L, g_H\} < s_H < g_H$ and (iv) $s_L < s_M < g_M < s_H < \min\{g_H, g_L\}$. However, case (iv) will not emerge in an optimal contract because it does not satisfy incentive compatibility since $U(s_M, s_L) = w_L + s_M v(y_L, g_L) - \frac{e_L^2}{2} > w_L + s_L v(y_L, g_L) - \frac{e_L^2}{2} = U(s_L, s_L)$.

Note that in case (i) agents do not get any intrinsic utility from imitate the others. Therefore agents do not get informational rents and in equilibrium, $U(s_L, s_L) =$

$U(s_M, s_M) = U(s_H, s_H) = 0$. In case (ii) type H is committed to the goal of type M, therefore applying standard results in principal agent models we have that in equilibrium $U(s_H, s_H) = g_M \ln\left(\frac{\theta e_M}{g_M}\right)(s_H - s_M) > U(s_M, s_M) = U(s_L, s_L) = 0$. Finally in case (iii) we have that type M is committed to the goal of type L. Therefore, in equilibrium, $U(s_M, s_M) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_M - s_L) > U(s_H, s_H) = U(s_L, s_L) = 0$. ***Q.E.D.***

Proof of Proposition 4

I follow the same argument used in the proofs of Lemma 2 and Proposition 2. ***Q.E.D.***

The Principal Agent Solution

The Optimal Contract when $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$.

We first solve the principal agent model under the condition of Corollary 1, i.e., $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$. Therefore, cases (iii) and (iv) of Lemma 2 are the only possible cases. Note that now, depending on the location of g_L we may have the following cases in equilibrium.

Assume first $g_L < s_H$. In this case the participation constraint is binding for both agent types. Therefore, the principal's problem simplifies to:

$$\max_{\{e_H, g_H, e_L, g_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \\ w_H &= \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \end{aligned}$$

Denoting by $\boldsymbol{e}$ to the Euler's number, the solution of this problem is:

$$e_H = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H),$$

$$e_L = \theta\left(1+\frac{s_L}{\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2(\boldsymbol{e}+s_L).$$

Note that this case is not feasible when s_H and s_L are sufficiently close, i.e., if $\left(\frac{\theta}{\boldsymbol{e}}\right)^2(\boldsymbol{e}+s_L) \geq s_H$. Under this situation the principal may want to set $g_L = s_H$, so that the high type agent still gets zero information surplus, this is the next situation we analyze. By substituting in the principal's problem g_L by s_H and solving the new principal's problem we get that the contract offered to the high type is the same as the previous case, while the contract offered to the low type is

$$e_L = \frac{1}{2}\left(\theta+\sqrt{\theta^2+4s_Ls_H}\right) \text{ and } g_L = s_H.$$

Assume finally $g_L > s_H$. In this case the high type gets positive informational rents in equilibrium, thus,

$$U(s_H,s_L) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H-s_L) > U(s_L,s_L) = 0.$$

Therefore, the principal's problem becomes:

$$\max_{\{e_H,g_H,e_L,g_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned}
w_L &= \frac{e_L^2}{2} - s_Lg_L\ln\left(\frac{\theta e_L}{g_L}\right),\\
w_H &= \frac{e_H^2}{2} - s_Hg_H\ln\left(\frac{\theta e_H}{g_H}\right) + g_L\ln\left(\frac{\theta e_L}{g_L}\right)(s_H-s_L).
\end{aligned}$$

The solution of this problem is the following:

$$e_H = \theta\left(1+\frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2(\boldsymbol{e}+s_H),$$

$$e_L = \theta\left(1+\frac{s_L-ps_H}{(1-p)\,\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2\left(\boldsymbol{e}+\frac{s_L-ps_H}{(1-p)}\right).$$

The Optimal Contract in the remaining cases.

Previously we have solved cases (iii) and (iv) of Lemma 2, here we proceed by solving cases (i), (ii) and (v).

- Case (i): $\max\{g_L, g_H\} \leq s_L$.

In this case the intrinsic utility of both agent types is zero, thus

$$V(y_i, g_i, s_i) = 0 \text{ for all } i \in \{L, H\}$$

Therefore, applying Proposition 2, we have that the principal's problem is

$$\max_{\{e_H, \overline{y}_H, e_L, \overline{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2}, \\ w_H &= \frac{e_H^2}{2}. \end{aligned}$$

The solution of this problem is

$$e_H = e_L = \theta,$$

$$w_H = w_L = \frac{\theta^2}{2}.$$

- Case (ii): $g_L \leq s_L < s_H \leq g_H$.

In this case we have that $V(y_H, g_H, s_H) = s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right) > 0$ while $V(y_L, g_L, s_L) = 0$. By Proposition 2 we know that in this case the high type gets zero informational rents. Therefore, the principal's problem is

$$\max_{\{e_H, \overline{y}_H, e_L, \overline{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2}, \\ w_H &= \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \end{aligned}$$

The solution entails

$$e_L = \theta, \ e_H = \theta \left(1 + \frac{s_H}{\boldsymbol{e}}\right),$$

with

$$g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + \boldsymbol{s}_H).$$

- Case (v): $g_H < s_L < g_L$.

In this case we have that $V(y_H, g_H, s_H) = 0$ while $V(y_L, g_L, s_L) = s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right) > 0$. By Proposition 2 we know that in this case the high type gets zero informational rents. Therefore, the principal's problem is

$$\max_{\{e_H, \overline{y}_H, e_L, \overline{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \\ w_H &= \frac{e_H^2}{2}. \end{aligned}$$

Whose solution is

$$e_H = \theta, \ e_L = \theta \left(1 + \frac{s_L}{\boldsymbol{e}}\right),$$

with

$$g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + \boldsymbol{s}_H).$$

In the following graph we plot the equilibrium profits as a function of θ, to order all the possible cases.

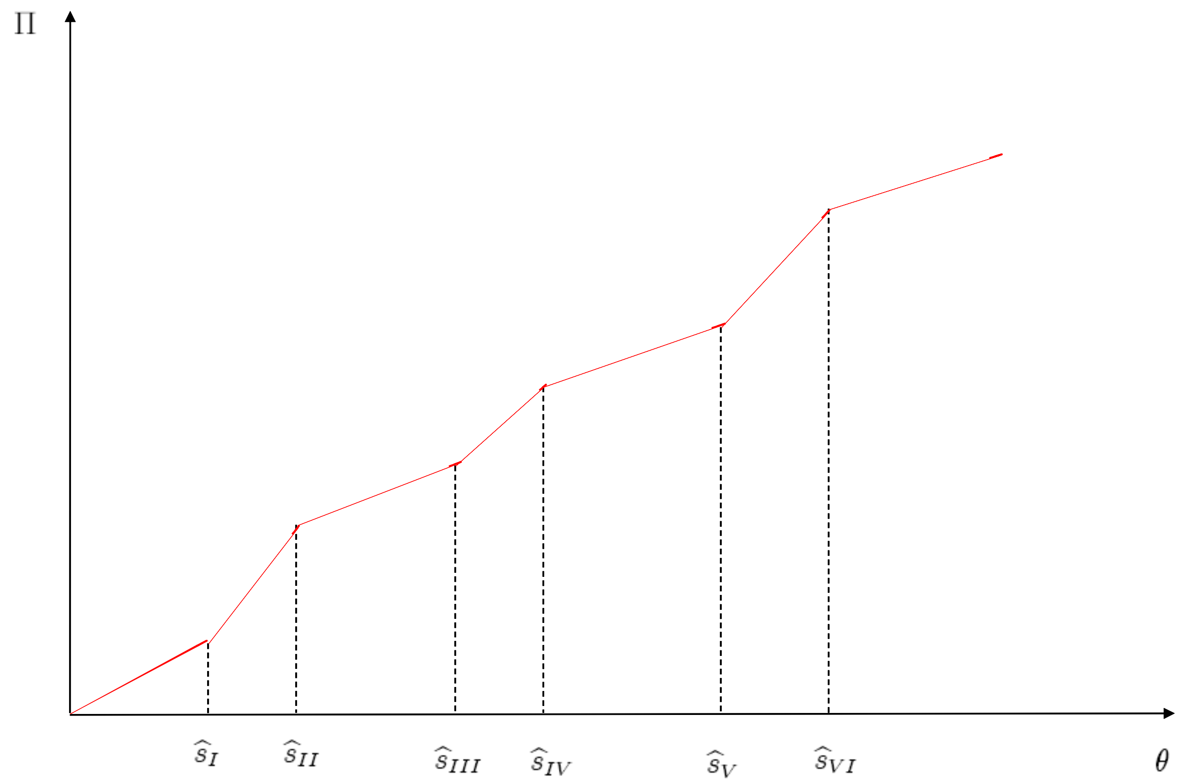

Figure 4. Profits as function of θ.

Since goals are an increasing function of θ, if we rank the cases with respect to θ, keeping the other parameters constant, we have that the first case, i.e., the one that emerges when θ is very low, is case (i). The interior solution of this case emerges when $\theta \in (0, \widehat{s}_I)$, while if $\theta \in (\widehat{s}_I, \widehat{s}_{II})$ we have a corner solution in which.$g_L = s_L$. After this case we have either case (ii) or (v) depending on the other parameter values, it is immediate to check that both cannot hold simultaneously. The interior solution of these cases emerges when $\theta \in (\widehat{s}_{II}, \widehat{s}_{III})$, while if $\theta \in (\widehat{s}_{III}, \widehat{s}_{IV})$ we have a corner solution, i.e., either $g_H = s_H$ or $g_L = s_L$. Finally, when θ is sufficiently high we have the cases studied in the previous section, i.e., cases (iii) and (iv). The interior solution of case (iii) emerges when $\theta \in (\widehat{s}_{IV}, \widehat{s}_V)$, if $\theta \in (\widehat{s}_V, \widehat{s}_{VI})$ we have that $g_L = s_H$, and if $\theta > \widehat{s}_{VI}$ we are in case (iv) which is the only case in which the high type agent gets positive informational rents.