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OPTIMAL DURATION OF MAGAZINE PROMOTIONS*

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Abstract

The planning of promotions and other marketing events frequently requires manufacturers to make decisions about the optimal duration of these activities. Yet manufacturers often lack the support tools for decision making. We assume that customer decisions at the aggregated level follow a state-dependent Markov process. On the basis of the expected economic return associated with dynamic response to stimuli, we determine the ideal length of marketing events using dynamic programming optimization and apply the model to a complex promotion event. Results suggest that this methodology could help managers in the publishing industry to plan the optimal duration of promotion events.

Keywords: Optimal duration of promotion events, Markovian process, Dynamic programming.

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1. Introduction

Short-term promotions can suffer from many hidden pitfalls if they are not well planned and controlled (see Strang, 1976). The planning of promotion events should be based on objective procedures such as the maximization of profits or other marketing objectives linked to a particular promotion. The critical decisions of manufacturers are often concerned with the time dimension of promotion events: that is, their frequency (number of events in the planning horizon), timing (dates for each event), and duration (the length of time an event should last). Yet the literature on manufacturers' promotions focuses on timing and frequency rather than duration. This situation exists partly because the duration of promotion events is usually predetermined by the trade, particularly in the case of price discounts (with or without feature and display), the duration of which is fixed by retailers at one week. This focus on frequency and timing is reflected in various approaches to promotion planning, from the perspective of both retailers and manufacturers (Little 1975, Neslin and Shoemaker 1983, Cooper et al., 1999). Within this framework researchers have examined the number of times a brand or category should be promoted, the interval between consecutive promotions, and the regularity or homogeneity of inter-promotion times. For price promotions, in fact, the duration of the promotion event has been analyzed only in the context of coupons, the finding being that short-term promotions accelerate purchases, whereas long-term promotions seem not to affect purchase acceleration (Aggarwal and Vaidyanathan, 2003).

In the case of manufacturers' non-price promotions, however, the time dimension is more customized and the duration of a particular promotion is more of a decision factor for the manufacturer. This is the case of value-packaging, in-pack premiums, manufacturing couponing, and other promotion events decided by manufacturers. In particular, manufacturers are less restricted by trade when planning the length of value-pack promotions than when planning the length of price promotions. Unfortunately, despite the large number of academic studies on promotions (e.g. Blattberg and Neslim, 1990; Chandon, 1995), non-price promotions have received limited attention by marketing researchers. And, when price promotions are either impossible to implement or inadequate, existing research provides little guidance to decision makers (Lichtenstein et al., 1997).
Two conditions are present in a wide variety of marketing activities, particularly in the area of promotions; most of the promotion events show a steady effectiveness decay of economic returns, as represented in Figure 1 (Blattberg and Neslin 1990, p. 361) and have direct costs associated with the duration of the event.

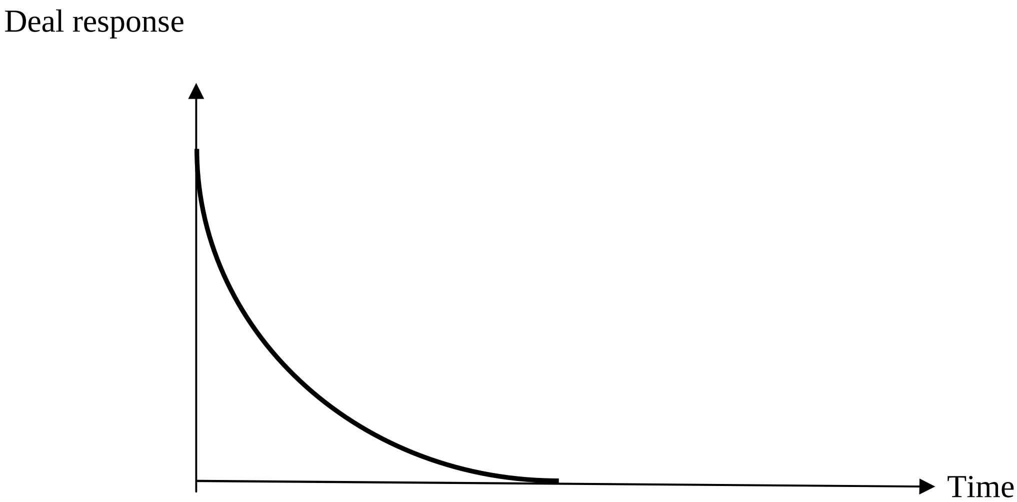

Figure 1: Decay of deal response to the marketing stimuli

Since the 1990s, hazard models have been the tool most frequently used for solving duration problems in marketing (see Helsen and Schmittlein, 1993, for a review of duration time problems). In marketing research, these models are used to study inter-purchase times in various contexts (Gupta, 1991; Vilcassim and Jain, 1991; Wedel et al., 1995) and the timing of the adoption of an innovation (see e.g. Sinha and Chandrashekaran, 1992). But in hazard models, the market responses to a stimulus – for example, customer arrivals – occur randomly and follow a point process (usually, a Poisson process). This model specification implicit in hazard models are not appropriate for tackling problems in which customer arrival times are not observed or inter-purchase times are irrelevant (e.g. in periodical magazines, what matters is the total sales of each issue rather than when they are purchased).

The objective of this paper is to fill this gap by providing a model that, from the perspective of economic returns, should help to plan the optimal duration of promotion events and other marketing actions which could be implemented when two circumstances occur: a) the marketing stimuli have a steady declining effect, and b) there are direct costs to maintaining the stimuli. In order to illustrate the potential of the model, we consider *serial promotions* in the publishing industry, a complex promotional type that will illustrate the model's capability of tackling the duration decision issue in a wide variety of marketing decisions and environments. This empirical setting presents two points of view that make the application of the methodology attractive: 1) It is a type of promotion that has not been studied before but which has shown to be effective in diminishing the decline rate of periodical sales, and 2) it is a complex promotion that, we demonstrate, cannot be modeled with the methodological approaches currently used in marketing.

The paper is structured as follows. First, we present the general methodology in order to provide a broad view of its potential applicability to marketing decisions. Second, we consider a complex type of promotion event and apply the model with two aims: to show that the flexibility of this modeling approach allows for adaptations to complex situations, and to overcome the limitations of the existing methodology in marketing – primarily hazard models assuming Poisson distributions – when facing some of the contingencies of promotion events that affect the decision of duration. Finally, we discuss the results and provide the conclusions and extensions of the research.

2 The methodological background

The study of optimal duration has attracted considerable attention in many scientific disciplines. From the perspective of dynamic optimization, the analysis of the optimal length of time spent in a particular event is called *Optimal Stopping Time problem* – a type of optimization problem for a random process involving only two possible actions: to stop or to continue. The literature in this area is extensive (see e.g. Bertsekas, 1987; Shiryayev, 1978). These techniques have been applied to several disciplines, including probability and statistics (e.g. Freeman, 1983) and economics and finance (e.g. Myneni, 1992), among others. Although these techniques have enormous potential for application in the marketing context, they have yet to be applied to assist managers in those decisions in which the duration of a marketing activity is involved.

Let's consider a marketing dynamic activity, defined by certain set S=$\{s_1,...,s_d\}$ of possible states. Let X be a Markov random process[1], which summarizes the successive states achieved by the process on the states space S. The Markov process is defined over the times {0,1,2,...,T} and denoted by $X=\{X_k\}_{k=0}^{T}$, where T can take a finite or an infinite value. Marketing decision makers can apply certain stimuli to modify the probability law of process X. When the stimulus is applied, the evolution of process X is ruled by a matrix B of transition probability distributions and managers can decide to stop the stimulus at any time based on the experience up to that time. At period k, the control variable can be "applying stimulus" or "not applying stimulus". In case of not applying stimulus at time k, the reward in the state X_k is $f(X_k)$. Using Bellman's maximum principle, when the stimulus is applied, $V(k,X_k)$ is the optimal reward at time k and the optimal strategy at time k satisfies $V(k,X_k)=\max\{f(X_k),E_B[V(k+1,X_k)]\}$. As a consequence, an optimal rule is "not applying stimulus" if the reward $f(X_k)$ is greater than the expected optimal reward $E_B[V(k+1,X_k)]$ and "applying stimulus", otherwise. Therefore, the optimal duration of the stimulus is $k^*=\min\{k:V(k,X_k)=f(X_k)\}$.

Sometimes the optimal stopping rule can be implemented at any time; e.g., discount promotions. However, in many marketing activities duration must be planned in advance and k^* is an indicative duration choice for future marketing activities; e.g., coupons which are a series of attached tickets often to be collected periodically and needed to obtain a discount or gift on merchandise. Pre-promotion planning involves setting some promotional-test, gathering information about market responses, building a decision model and determining the optimal length of future marketing activities. The stability of the optimal duration can be followed up on the basis of future responses.

Intuitively, many marketing activities appear to be suitable settings for exploring the potential of these models. In general, the model is an appropriate optimization tool when the following conditions are presented: a) a steady declining effect of the marketing stimuli and b) direct costs to maintaining the stimuli.

One effective way to capture the steady declining effect of marketing stimuli is to consider that the consumers' response $\{X_k\}$ follows a state-dependent Markovian process. Markovian models have been previously adopted for modeling purchasing behavior (Telser 1963, and Zufryden 1986) and other marketing problems[2]. We say that the consumers' response $\{X_k\}$ follows a Markov process if $\Pr\left\{X_{k+1}\middle|\{X_j\}_{j=0}^{k}\right\}=\Pr\{X_{k+1}|X_k\}$ for each time $k>0$. Let $b=\Pr(X_0)$, with $b_i\geq 0$ and $\sum_{i=1}^{d}b_i=1$, denote the probability distribution of being in each state i at time 0 and let B define the transition probability matrix containing the probabilities $\Pr\{X_{k+1}|X_k\}$. Then, the probability distribution of X_k is given by $\Pr\{X_k\}=b'B^k$. The Markov process is stationary if the probability distribution of X_k remains unaltered with the

[1] Markov processes are described in many books on probability (see e.g. Ross, 1996). Statistical inference for discrete time Markov processes was considered by Billingsley (1961) and Telser (1963).
[2] For a review of this topic, see e.g. Leeflang et al. (2000).

passage of time; i.e. $\Pr\{X_k\} = \Pr\{X_{k+1}\}$ for any time k. This only happens if $b' = b'B$ or equivalently $(I - B')b = 0$; i.e. b is an eigenvector (normalized to sum one) associated with a unit eigenvalue[3].

Assume that the consumers' response $\{X_k\}$ to some marketing stimuli and the reward $f(X_k)$ at time k is a random variable which possible outcomes are $f = (f(s_1),..., f(s_d))'$. Some of these outcomes may be negative (the application of the stimulus is not profitable), and others positive. As the promotion outcome is ruled by a markovian process, the expected profit is given by

$$E[f(X_k)] = E\left[E[f(X_k)|X_0]\right] = b'B^k f\ ,$$

evolving according to $b'B^k$. Note that when b is a stationary distribution, the expected profit $E[f(X_k)]$ is constant for any time k. If it is a positive constant, the optimal duration will be infinite, i.e. the stimulus becomes an additional service or characteristic of the product. If it is a negative value, the stimulus should never be applied. But for most of the real world situations, the market responses to marketing stimuli are not stationary. Typically the process starts with $E[f(X_0)] = b'f > 0$ and then it exhibits an exponential trend given by $E[f(X_k)] = b'B^k f$. In the long term the expected profit $b'B^k f$ converge to a limit performance value, that we will call A: if A>0, the performance of the marketing stimulus is always profitable and its duration should be infinite, but if A<0, there is a time k^* when the stimulus is not longer profitable and should be stopped. Being the latter the most frequent type of market response, we will use this markovian process in the model presented in the next section.

2.1. The model

Let's consider a representative potential consumer who may be in one of two states: prior purchase intention, event denoted by x^1; or no prior purchase intention, denoted by x^2. At the same time, manufacturers should analyze the expected performance of the promotion event. This marketing activity cannot be stopped once it has been launched. However, in the planning process, manufacturers consider the dilemma: "Would it better to have designed the promotion in *k-1* periods? (which, in dynamic programming terms, is called *to stop promoting*)" or "Would it better to have designed the promotion in *k* or more periods? (called *to continue promoting*)". This analysis will help manufacturers to plan the duration of future promotion events to achieve the best expected return. If promotion continues, the potential customer decides, based on prior purchase intentions, whether to buy the product, denoted by state z^1; or not to buy the product, denoted by z^2. Manufacturers aim to enhance the purchase intention X of customers, but they only observe the purchase decision outcome Z.

Assuming that the prior purchase intention depends only on the previous intention and not on the consumer's entire purchasing history, we model the decisions process as a Markov chain, the motion of which is governed by the transition probabilities $\Pr\{X_{k+m}|X_k\}$, constant for all

[3] As B has non negative elements and its rows sum one, B has at least a unit eigenvalue.

period *k*. The transition probabilities describe the probability distribution from one decision to another. Let B denote the transition matrix defined as follows:

$$B = \begin{pmatrix} \Pr\left\{X_k = x^1 \middle| X_{k-1} = x^1\right\} & \Pr\left\{X_k = x^2 \middle| X_{k-1} = x^1\right\} \\ \Pr\left\{X_k = x^1 \middle| X_{k-1} = x^2\right\} & \Pr\left\{X_k = x^2 \middle| X_{k-1} = x^2\right\} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \theta & (1-\theta) \end{pmatrix}.$$

This means that there is a proportion $\theta \in (0,1)$ of customers who intended not to buy at the previous period, but who now intend to buy the product.

When a promotion occurs, the probability distribution of purchase decisions depends conditionally on the prior purchase intention at each period *k*, as

$$\Pr\left\{Z_k = z^1 \middle| X_k = x^1\right\} = 1, \quad \Pr\left\{Z_k = z^2 \middle| X_k = x^1\right\} = 0,$$
$$\Pr\left\{Z_k = z^1 \middle| X_k = x^2\right\} = \gamma, \quad \Pr\left\{Z_k = z^2 \middle| X_k = x^2\right\} = 1-\gamma.$$

Therefore, there is a proportion $\gamma \in (0,1)$ of potential customers who change their minds and decide to buy the promoted product.

Computing the optimal promotional duration requires the study of $\pi_k = \Pr\left\{X_k = x^1 \middle| \vec{Z}_k\right\}$, the probability distribution of prior buying intention ($X_k = x^1$) conditional on the history of buying behavior $\vec{Z}_k = \left(Z_0, ..., Z_k\right)'$. Publishers do not usually observe all data *X*, except for the final decision (potential customers either do or do not buy the product). Applying Bayes' rule, we can derive the analytical expression

$$\pi_{k+1} = \Pr\left\{X_{k+1} = x^1 \middle| \vec{Z}_{k+1}\right\} = \frac{\Pr\left\{X_{k+1} = x^1 \middle| \vec{Z}_k\right\} \Pr\left\{Z_{k+1} \middle| \vec{Z}_k, X_{k+1} = x^1\right\}}{\Pr\left\{Z_{k+1} \middle| \vec{Z}_k\right\}}.$$

From the probability properties of the buying decision process, we can prove that $\pi_{k+1} = \Phi\left(\pi_k, Z_{k+1}\right)$, with

$$\begin{aligned} \Phi\left(\pi_k, Z_{k+1}\right) &= \frac{\pi_k + (1-\pi_k)\theta}{\pi_k + (1-\pi_k)\theta + (1-\pi_k)(1-\theta)\gamma} \cdot I\left(Z_{k+1} = z^1\right) \\ &= \frac{1-(1-\pi_k)(1-\theta)}{1-(1-\pi_k)(1-\theta)(1-\gamma)} \cdot I\left(Z_{k+1} = z^1\right), \end{aligned}$$

where $I\left(Z = a\right)$ is one when $Z = a$, and zero when $Z \neq a$. This expression can be used to compute iteratively, using the observed data $\vec{Z}_k$ and the estimations of parameters θ and γ. Note that π_{k+1} is larger than π_k for $Z_{k+1} = z^1$, and vanishes to zero when $Z_{k+1} = z^2$.

Next we focus on the computation of the optimal duration for promotions. Denote by $P' > C' > 0$, the price (P) and cost (C) of the product with promotion; and $P > C > 0$, the price and cost without promotion. We assume that manufacturers aim to maximize the expected economic value of prior purchase intentions. If the promotion has continued to the end of period k+1, the associated expected return associated with the sales-promotion process is given by the value function $V_{K+1}(\pi_{K+1})$. At the end of period k, each manufacturer computes its probability distribution of prior purchase intention π_k and decides whether, in dynamic programming terms, to stop promotion (with the associated expected profit, $\pi_k(P-C)+(1-\pi_k)(-C)$), or to continue promotion (with the associated expected profit, $E\left[V_{K+1}(\pi_{K+1})\right]$). Whenever the expected profit of terminating promotion is greater than the expected profit under promotion; i.e. if

$$\pi_k(P-C)+(1-\pi_k)(-C) > E_{Z_{k+1}}\left[V_{k+1}(\Phi(\pi_k, Z_{k+1}))\right],$$

the manufacturer should have designed a promotion of k periods length. Then, solving the equality $\alpha_k(P-C)+(1-\alpha_k)(-C) = E_{Z_{k+1}}\left[V_{k+1}(\Phi(\alpha_k, Z_{k+1}))\right]$, there is an $\alpha_k \in (0,1)$, such that whenever $\pi_k < \alpha_k$, promotion should have been stopped (see Figure 2).

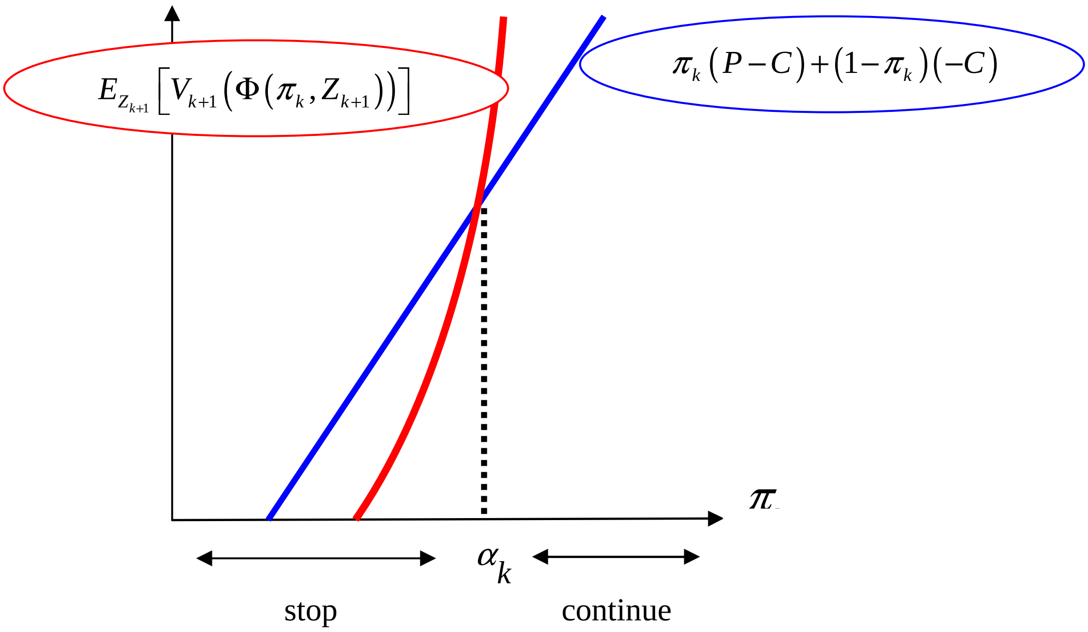

Figure 2: The decision threshold at k-th period

The computation of all the thresholds $\{\alpha_k\}$ is involved (see Appendix for details), so that we consider a simple-to-apply approximation to the solution. Note that $V_k(\pi) \le V_{k+1}(\pi)$ for all k and all $\pi \in (0,1)$ and $V_k(\pi)$ is piecewise linear, concave, and increasing in π. Henceforth, $\alpha_{k-1} \le \alpha_k \le \alpha_{k+1}$ for all k. Publishers should stop production of promoted magazines with negative expected returns, so that $\alpha_k(P-C)+(1-\alpha_k)(-C) \ge 0$, and therefore $\alpha_k \ge C/P$, are valid for all *k*. Because $\alpha = \frac{C}{P} \le \alpha_1 \le \alpha_2 \le ... \le \alpha_k \le ...$, an optimal stopping rule for promotion is given as follows,

$$k^* = \min\left\{k \in \{1,2,3,....\} : \pi_k < \alpha\right\}, \qquad \alpha = \frac{C}{P},$$

where $\alpha \in [0,1]$ since 0<C<P. In other words, the optimal duration of the promotion event is k^* periods, and manufacturers should therefore plan promotions to be no longer than k^* periods based on their expected economic value of purchase intentions.

The decision rule reported here is based on a two-state model. Generalization to multi-state models is clearly desirable to deal with a number of interesting marketing decision situations. Yet in serial promotions this parsimonious modeling is consistent with our problem as illustrated in the next section.

3. The application and results

In the publishing industry, where most products are marketed with a fixed resale price, non-price promotions predominate. This is the case for periodical magazines; a common practice in some European countries is to assemble a value pack containing the magazine plus another product to sell at a price above the price of the magazine but below the sum of the expected prices of the two products. One option for publishers is to serialize the promotion by fractioning the additional product in the value pack across different issues of the magazine: a dictionary fractionated in a collection of CDs, for example. We call this type of marketing activity a *serial promotion*. The marketing objectives of serial promotions are to acquire new customers for the magazine and to increase purchasing frequency among existing customers.

The acquisition of new customers is the basic objective of these promotions, as those who are satisfied with the promoted product have an increased probability of making repeat purchases (see Rothschild and Gaidis, 1981). In fact, new customers may switch from other magazines (secondary demand) during the life of the promotion if they find it to be attractive enough. Alternatively, new customers may not be regular buyers of this type of magazine, but may enter the market (primary demand) when they see an appealing promotion. Hence, it is expected that most of these customers will soon stop participating in the promotion if the value pack does not fulfill their expectations[4].

Regular buyers may skip some issues, either because they decide not to buy any publication on that occasion or because they switch to another publication. Thus an increase in the purchase loyalty of actual customers is another objective pursued through the use of serial promotions, which reinforce customer loyalty by introducing incentives to decrease skipping behavior and by raising barriers against switching. Customers who purchase a value pack with a low-involvement attitude may stop buying if they do not find the added product to be worthy of the premium price. So, during the serial promotion, a large percentage of this type of customer is expected to abandon the purchase of the value pack in the early stages; either they turn to the regular issue or they return to their previous switching behavior.

[4] The long-term effectiveness of the promotion event in the acquisition of new customers is rarely large. In one study, publishers of several newspapers estimated that these types of promotions generate an increase of about 1% of customers (Santana López, 2002).

What publishers actually pursue is the market response portrayed in Figure 1: a positive immediate response to the deal, followed by effectiveness decay. The previous presentation of the objectives of the promotion gives some hints to understanding the shape of the effect decay of the promotion over time. But the basic explanation to this convex shape is that most entries will take place during the first week; thereafter, the decision of any customer to purchase a value pack at a premium price instead of the regular issue depends on whether or not the customer purchases the previous issue. To enter the promotion after the first week or to re-enter the promotion after skipping one or more issues would mean that the customer, who is paying a premium price, would fail to complete the collection. So, although some customers may demonstrate this behavior, their impact on the total sales of every issue would always be exceeded by the number of defections. Defections occur because customers try the promotion and decide that it is not worth the premium price, or because they skip issues for reasons external to the promotion (e.g. boredom, inconvenience). These defections occur primarily during the first weeks of the promotion, and the defection rate decreases over time as the commitment of the remaining customers increases. (Discontent with the fifth issue in a collection of six issues would not stop most of the remaining customers from buying the last issue). Expecting such a deal response, the relevant question for publishers is: Following a profit maximization criterion, into how many issues should the added value of the promotion be fractionated?

The optimal stopping time rule previously described is an appropriate tool for determining the ideal length of serial promotions in periodical magazines as their inter-purchase times are fixed, not allowing consumers to purchase at any time during the length of the promotion event, and difficult to be registered, and there are remarkable state-dependence effects on consumers' purchase intentions and final purchase decisions.

We apply the optimal stopping time rule to two specialized magazines, Magazine A and Magazine B, which are published by a multinational publishing company and distributed monthly at different prices, with and without promotion. For Magazine A, which is the leader in the category of *Science and Nature* magazines, we consider a data sequence that begins November, 1999 and ends September, 2003. For Magazine B, which has the second highest market share in the *Business* category, data begin April, 2001 and end September, 2003. In particular, we are using their sales data as a measure of consumer responses to promotion in each magazine. Due to the confidentiality policies of the publishing company, sales data were provided as a percentage of the market potential, hereafter called *potential market share*. Figures 3 and 4 show the values of the potential market share for Magazines A and B, respectively, in which vertical lines denote the end of promotion (promotions are consecutively implemented within the sample period and the exponential decay of their effect is clearly observed in all of them). The duration time of serial promotions planned by the publishing industry is 6 or 7 weeks, what is considered a good rule of thumb based on its experience.

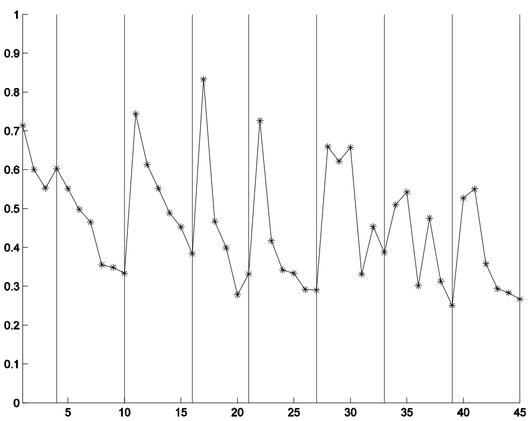

Figure 3: Sales potential quota of the promoted Magazine A

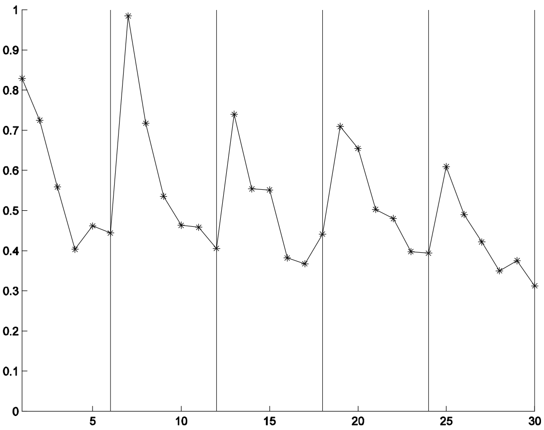

Figure 4: Sales potential quota of the promoted Magazine B

Next, following the profit maximization criterion inhered in our model we determine the optimal duration length of serial promotion for Magazines A and B.

Two steps are involved in the task of developing an adequate estimation approach of the parameters θ and γ. First, we define the decision variables that describe the consumer behavior. Given a threshold at each period *k,* we set the prior purchase intention to buy $X_k = x^1$, if the sales potential quota of the non-promoted magazine is larger than the threshold; and $X_k = x^2$, otherwise. Analogously, we define the variable Z_k using the sales potential quota of the promoted magazine. The second step consists of estimating the parameters θ and γ as follows,

$$\hat{\theta} = \frac{\sum_{k=2}^{K} I\left(X_k = x^1\right) I\left(X_{k-1} = x^2\right)}{\sum_{k=2}^{K} I\left(X_{k-1} = x^2\right)}, \quad \hat{\gamma} = \frac{\sum_{k=2}^{K} I\left(Z_k = x^1\right) I\left(X_k = x^2\right)}{\sum_{k=2}^{K} I\left(X_k = x^2\right)}.$$

For this problem, we set the threshold at 0.5, which is the central value of the sales potential quota in both magazines. For Magazine A, the estimation of parameters θ and γ is $\hat{\theta}^A = 0.5833$ and $\hat{\gamma}^A = 0.4167$, respectively; for Magazine B, we obtain $\hat{\theta}^B = 0.5385$ and $\hat{\gamma}^B = 0.2308$, respectively.

The aim of launching a promotion is to predispose potential customers to buy a product. However, its effect on sales does not last forever. The length of the promotion should be efficiently chosen according to the sales-promotion process. Accounting for consumer responses to promotion, $\hat{\theta}$ and $\hat{\gamma}$, manufacturers should plan the length of promotion that maximizes their expected return over the planning horizon. Next, we compute the probabilities $\{\pi_k\}$ and the parameter α associated with each magazine in order to determine the optimal promotional duration k^* for Magazines A and B.

Figure 5 shows the values of $\{\pi_k\}$ and α for Magazine A, which reveals that the optimal duration for promoting Magazine A is $k^* = 6$, as $\pi_k < \alpha$, for *k*=6, 7, 8... To analyze this result, we consider the actual average length of promotion, which is 6 (see Figure 3, in which vertical lines denote the end of promotion). These data allow us to conclude that, in the case of Magazine A, the duration of serial promotions planned by the publishing company is the optimal length.

In the case of Magazine B, the optimal duration of serial promotions computed by the model is $k^* = 4$ (see Figure 6). Currently, the duration time planned by the publisher is 6 weeks (see Figure 4, denoting the end of promotion with a vertical line), 2 weeks longer than the length suggested by the model.

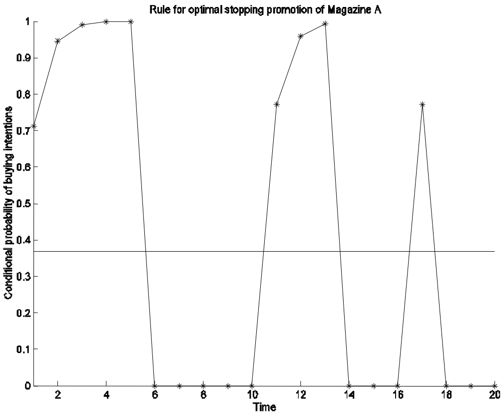

Figure 5: Sequence π_k and line α for promoted Magazine A

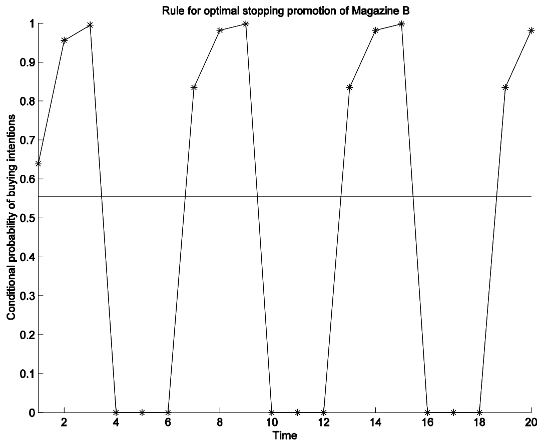

Figure 6: Sequence π_k and line α for promoted Magazine B

The application of our model to the publishers' decisions suggests that the serial promotion of Magazine B should be lengthened by two weeks and that Magazine A's promotion should be maintained at the current six weeks' duration.

As Figures 3 and 4 show, serial promotion can be sequentially implemented with a fixed length *k*. Next we show that this methodology also is an effective tool for dynamic decision making in the long term. Assume that promotions are sequentially implemented. The return of the first promotion is given by $R_{1,k} = f\left(X_k\right)$ and the return rate of the *n*-th promotion is $R_{n,k} = f\left(X_{nk}\right) / f\left(X_{(n-1)k}\right)$, that we assume, stationary and ergodic with $E\left[\left|\ln R_{1,k}\right|\right] < \infty$. Thus, the return after *n* promotions is $f\left(X_{nk}\right) = R_{n,k} R_{(n-1),k} \ldots R_{1,k}$ and taking logarithms, we have:

$$\ln f\left(X_{nk}\right)^{1/n} = \frac{1}{n}\sum_{i=1}^{n} \ln R_{i,k} \xrightarrow{a.s.} M_k = E\left[\ln R_{1,k}\right],$$

by the ergodic theorem. Therefore, the return after *n* promotions growths exponentially as $f\left(X_{nk}\right) \approx e^{nM_k}$ (in the long term planning, a discount factor e^{-rn} should multiply the previous expression). In summary, the long term performance of successive promotions is determined by the return rate of a single promotion M_k, suggesting that the optimal k^* is an optimal rule for all successive promotions.

4. Conclusions and managerial implications

Despite advances in modeling promotion events, much can be done to enlarge the catalogue of techniques for determining the optimal promotion duration. Further development in this direction has practical relevance for marketing managers. Most of the research has overlooked the issue of duration and has focused, rather, on timing (when?) and frequency (how many?), so that models available to assist the short-term planning of promotion events have still not covered the other basic time dimension: duration.

As we have discussed, promotional models available in the marketing literature are of limited use in the context of those promotion events or other marketing initiatives in which customer arrivals, sales, or other phenomena do not occur as a random series of events, but rather in fixed periodical points of time.

We propose a model for fixed timing customer entries. These problems may be found when trying to model the market response of any promotion in which the marketing effect (e.g. sales, arrivals, attendance) is distributed in a time-discrete fashion. Examples can be found in the publishing and entertainment industries (such as periodical publications, TV and broadcasting series, entertainment, and sports events) and in other industries such as transportation (scheduled flights). The analysis also incorporates the state-dependence of purchase decisions by modeling periodical decisions of customers through a Markovian process. Using dynamic programming optimization, we tackle the question of the optimal duration of magazine promotions, taking into account the economic value of purchase intentions.

We show that this methodology is an effective and useful tool for dynamic decision making in the short and long term planning of promotion events. We apply the method to two magazines, and our analysis suggests that this approach can determine the optimal duration of promotion events in both and that promotion length should be modified in one of them.

One potential limitation of our approach is the requirement of sales data with and without promotion to infer purchase intentions and actual purchase decision. However, when non-promoted sales are unavailable, this methodology can still be applied by means of a scenario-case analysis for different conjectured parameters θ and γ, using the observed purchases as input.

We believe that the model and methodology employed in this paper are broadly applicable to other types of promotions and to other industry sectors that require decisions to be made about the optimal duration of marketing activities. In particular, our approach can be applied to a wide range of marketing decision situations that are characterized by a steady decay response to marketing stimuli, including situations involving advertising effectiveness, broadcast audiences, and such exhibition attendance as movies.

Appendix: Stopping Time Rule

Assume a planning horizon of N periods. If the promotion has continued to the end of the Nth period, the expected return associated with the sales-promotion process is

$$V_N(\pi_N) = \pi_N(P' - C') + (1 - \pi_N)(-C').$$

At the end of period N-1, each manufacturer computes its probability distribution of prior purchase intention π_{N-1} and decides whether to stop promotion (with the associated expected profit, $\pi_{N-1}(P - C) + (1 - \pi_{N-1})(-C)$) or to continue promotion (with the associated expected profit, $E_{Z_N}\left[V_N\left(\pi_N\right)\right]$). As a consequence, the optimal decision will be one that maximizes the expected return, i.e.,

$$\begin{aligned}V_{N-1}\left(\pi_{N-1}\right) &= \max\left\{\pi_{N-1}\left(P-C\right)+\left(1-\pi_{N-1}\right)\left(-C\right), E_{Z_N}\left[V_N\left(\pi_N\right)\right]\right\} \\ &= \max\left\{\pi_{N-1}\left(P-C\right)+\left(1-\pi_{N-1}\right)\left(-C\right), E_{Z_N}\left[V_N\left(\Phi\left(\pi_{N-1}, Z_N\right)\right)\right]\right\}.\end{aligned}$$

This rule can be extrapolated to all stage k, substituting N for k+1, so that

$$V_K\left(\pi_K\right) = \max\left\{\pi_k\left(P-C\right)+\left(1-\pi_k\right)\left(-C\right), E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\pi_k, Z_{k+1}\right)\right)\right]\right\},$$

In order to compute $E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\pi_k, Z_{k+1}\right)\right)\right]$, we consider

$$\begin{aligned}E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\pi_k, Z_{k+1}\right)\right)\right] &= Pr\left\{Z_{k+1}=z^1\left|\pi_k\right.\right\}E_{Z_{k+1}}\left[V_{K+1}\left(\frac{1-\left(1-\pi_k\right)\left(1-\theta\right)}{1-\left(1-\pi_k\right)\left(1-\theta\right)\left(1-\gamma\right)}\right)\right] \\ &\quad +\left(1-Pr\left\{Z_{k+1}=z^1\left|\pi_k\right.\right\}\right)E_{Z_{k+1}}\left[V_{K+1}\left(0\right)\right],\end{aligned}$$

and since $Pr\left\{Z_{k+1}=z^1\left|\pi_k\right.\right\}=1-\left(1-\theta\right)\left(1-\gamma\right)\left(1-\pi_k\right)$, then

$$\begin{aligned}E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\pi_k, Z_{k+1}\right)\right)\right] &= \left(1-\left(1-\theta\right)\left(1-\gamma\right)\left(1-\pi_k\right)\right)V_{K+1}\left(\frac{1-\left(1-\pi_k\right)\left(1-\theta\right)}{1-\left(1-\theta\right)\left(1-\gamma\right)\left(1-\pi_k\right)}\right) \\ &\quad +\left(1-\theta\right)\left(1-\gamma\right)\left(1-\pi_k\right)V_{K+1}\left(0\right),\end{aligned}$$

particularly, for the last period, $E_{Z_N}\left[V_N\left(\Phi\left(\pi_{N-1}, Z_N\right)\right)\right]=\left(1-\left(1-\theta\right)\left(1-\pi_{N-1}\right)\right)P'-C'$, so that, $V_{N-1}\left(\pi_{N-1}\right)=\max\left\{\pi_{N-1}\left(P-C\right)+\left(1-\pi_{N-1}\right)\left(-C\right),\left(1-\left(1-\theta\right)\left(1-\pi_{N-1}\right)\right)P'-C'\right\}$. For previous periods, this expectation could be iteratively computed backwards.

Consider the decision problem at time k, and the value function $V_K\left(\pi_K\right)$. Whenever the expected profit of terminating promotion is greater than the expected profit under promotion; i.e. if $\pi_k\left(P-C\right)+\left(1-\pi_k\right)\left(-C\right)>E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\pi_k, Z_{k+1}\right)\right)\right]$, the manufacturer should stop the promotion. Then, solving the equality $\alpha_k\left(P-C\right)+\left(1-\alpha_k\right)\left(-C\right)=E_{Z_{k+1}}\left[V_{k+1}\left(\Phi\left(\alpha_k, Z_{k+1}\right)\right)\right]$, there is an $\alpha_k\in\left(0,1\right)$, such that whenever $\pi_k<\alpha_k$, promotion should be stopped.
For example at time N-1, assuming $P'>\left(C'-C\right)/\theta$, promotion should be stopped if $\pi_{N-1}<\alpha_{N-1}$, where α_{N-1} solves $\alpha_{N-1}\left(P-C\right)+\left(1-\alpha_{N-1}\right)\left(-C\right)=\left(1-\left(1-\theta\right)\left(1-\alpha_{N-1}\right)\right)P'-C'$, i.e., $\alpha_{N-1}=\left(C-C'\right)+\theta P'/P-\left(1-\theta\right)P'$. The computation of all the thresholds $\left\{\alpha_k\right\}$ is involved, so that we consider a simple approximation to the solution, described in Section 3.

References

Aggarwal, P., and R. Vaidyanathan. (2003). "Use it or lose it: Purchase acceleration effects of time-limited promotions", Journal of Consumer Behaviour 2(4), 393-403.

Bertsekas, D. P. (1987). Dynamic programming, deterministic and stochastic models. Englewood Cliffs: Prentice-Hall.

Billingsley, P (1961). Statistical inference for Markov Processes. Chicago: University of Chicago Press.

Blattberg, R. C., and S. A. Neslin. (1990). Sales promotion: concepts, methods and strategies. New Jersey: Prentice-Hall.

Chandon, P. (1995). "Consumer Research on Sales Promotion: A State-of-the-Art Literature Review", Journal of Marketing Management 11(5), 419-441.

Cooper, L. G., P. Baron, W. Levy, M. Swisher, P. Gogos. (1999). "PromoCast: A New Forecasting Method for Promotion Planning", Marketing Science 18(3), 301 –316.

Freeman, P. R. (1983). "The secretary problem and its extensions: A review", International Statistical Review 51, 189-206.

Helsen, K., and D. C. Schmittlein. (1993). "Analyzing duration times in marketing: Evidence for the effectiveness of hazard rate models", Marketing Science 12(4), 395-414.

Leeflang, P. S. H., D. R. Wittink, M. Wedel and P. A. Naert. (2000). Building Models for Marketing Decisions. International Series in Quantitative Marketing. Boston: Kluwer.

Lichtenstein, D. R., S. Burton, and R. G. Netemeyer. (1997). "An examination of deal proneness across sales promotion types: A consumer segmentation perspective", Journal of Retailing 73(2), 283-297.

Little, J. D.C. (1975), "BRANDAID: A Marketing-Mix Model. Part I: Structure; Part II: Implementation", Operations Research 23(4), 628-673.

Myneni, R. (1992). "The pricing of American Option", The Annals of Applied Probability 2(1), 1-23.

Neslin, S. A. and Shoemaker, R. W. (1983). "A model for evaluating the profitability of coupon promotions", Marketing Science, 2, 361-388.

Ross, S. M. (1996). Stochastic processes (2nd ed.) Wiley series in Probability and Mathematical Statistics. New York: John Wiley and Sons.

Rothschild, M. L., and W. C. Gaidis. (1981). "Behavioural Learning Theory: Its Relevance to Marketing and Promotions", Journal of Marketing 45(2), 70-78.

Santana López, Eva. (2002). Las promociones publicitarias en la prensa. Evolución y análisis sistemático de las promociones realizadas entre los años 1990-1998 en la prensa de información general de Cataluña. Universitat Autonoma de Barcelona, Spain.

Shiryayev, A. N. (1978). Optimal Stopping Rules. New York: Springer.

Strang, R. A. (1976). “Sales, promotion: Fast growth, faulty management”. Harvard Business Review 54, 115-124.

Telser, L., (1963). “Least squares estimates of transition probabilities”. In: *Measurement in economics*, Chirst et al. (Stanford, California: Standford University Press), 270-292.

Vilcassim, N. J. and D.C. Jain, (1991). “Modeling purchase timing and brand switching behavior incorporating explanatory variables and unobserved heterogeneity”, Journal of Marketing Research 28, 29-41.

Wedel, M., W.A. Kamakura, W.S. DeSarbo and F. ter Hofstede, (1995). “Implications for asymmetry, non proportionality and heterogeneity in brand switching models from piece-wise exponential mixture hazard models”, Journal of Marketing Research 32, 457-463.

Zufryden, F. S. (1986). “Multibrand transition probabilities as a function of explanatory variables: estimation by a least-squares-based approach”. Journal of Marketing Research 22, 177-183.