Working Paper 97-86
Business Economics Series 16
November 1997

Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-9608

WHY DO WE SMILE? ON THE DETERMINANTS OF THE IMPLIED VOLATILITY FUNCTION

Ignacio Peña*, Gonzalo Rubio** and Gregorio Serna*

Abstract

We report simple regressions and rather sophisticated linear and nonlinear Granger causality test in order to understand the pattern of implied volatilities across exercise prices. We employ all calls and puts transacted between 16:00 and 16:45 on the Spanish IBEX-35 index from January 1994 to April 1996. Transaction costs, proxied by the bid-ask spread, seem to be a key determinant of the volatility smile. Moreover, time to expiration, the uncertainty associated with the market and the relative market momentum are also important variables in explaining the smile.

* Universidad Carlos III de Madrid, Departamento de Economía de la Empresa, C/ Madrid, 126, 28903 Getafe (Madrid) Spain. Phone: 34-1-624 96 25. Fax: 34-1-624 96 08. E-mail: ypenya@eco.uc3m.es.

** Universidad del País Vasco, Departamento de Fundamentos del Análisis Económico, Facultad de Ciencias Económicas y Empresariales, Avda. L. Aguirre, 83, 48015 Bilbao, Spain. Phone: 34-4-479 74 70. Fax: 34-4-479 74 74. E-mail: jepruirg@bs.ehu.es.

We would like to thank José Luis Serna and Ainhoa Zárraga for capable computational assistance, and Rafael Salinas of BBV Interactivos for helpful conversations. We have received valuable comments form Lars Norden, Richard Stapleton, as well a seminar participants at the 6th Annual Meeting of the European Financial Management Association, Istambul, and I Foro de Segovia-Finanzas. Partial financial support was provided by Dirección General Interministerial Cienfífica y Técnica (DGICYT) grant PB95-0298 (Ignacio Peña and Gregorio Serna) grant PB94-1373 (Gonzalo Rubio). Ignacio Peña and Gonzalo Rubio would also like to acknowledge the financial support provided by Fundación BBV. The contents of this paper are the sole responsibility of authors.

WHY DO WE SMILE? ON THE DETERMINANTS OF THE IMPLIED VOLATILITY FUNCTION

Ignacio Peña *
Universidad Carlos III de Madrid
Gonzalo Rubio †
Universidad del País Vasco
Gregorio Serna *
Universidad Carlos III de Madrid

October 1997

Abstract

We report simple regressions and rather sophisticated linear and nonlinear Granger causality tests in order to understand the pattern of implied volatilities across exercise prices. We employ all calls and puts transacted between 16:00 and 16:45 on the Spanish IBEX-35 index from January 1994 to April 1996. Transaction costs, proxied by the bid-ask spread, seem to be a key determinant of the volatility smile. Moreover, time to expiration, the uncertainty associated with the market and the relative market momentum are also important variables in explaining the smile.

*Departamento de Economía de la Empresa. Universidad Carlos III de Madrid. Calle Madrid 126, 28903 Getafe, Madrid, Spain. Phone: 34–1–624 96 25. Fax: 34–1–624 96 08. e-mail: ypenya@eco.uc3m.es

†Departamento de Fundamentos del Análisis Económico, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Avda. L. Aguirre 83, 48015 Bilbao, Spain. Phone: 34–4– 479 74 70. Fax: 34–4–479 74 74. e-mail: jepruirg@bs.ehu.es

We would like to thank José Luis Serna and Ainhoa Zárraga for capable computational assistance, and Rafael Salinas of BBV Interactivos for helpful conversations. We have received valuable comments from Lars Nordem, Richard Stapleton, as well as seminar participants at the 6th Annual Meeting of the European Financial Management Association, Istanbul, and I Foro de Segovia–Finanzas. Partial financial support was provided by Dirección General Interministerial Científica y Técnica (DGICYT) grant PB95-0298 (Ignacio Peña and Gregorio Serna) grant PB94-1373 (Gonzalo Rubio). Ignacio Peña and Gonzalo Rubio would also like to acknowledge the financial support provided by Fundación BBV. The contents of this paper are the sole responsibility of the authors.

1 Introduction

There has recently been an incresing number of papers dealing with one prominent feature of the option pricing data. It is well known that after the October 1987 crash the implied volatility computed from options on stock indexes in the US market inferred from the Black–Scholes (1973) formula (BS henceforth) appears to be different across exercise prices. This is the so–called "volatility smile"[1]. In fact, as pointed out by Dumas, Fleming and Whaley (1996) (DFW henceforth), implied volatilities of the S&P 500 options decrease monotonically as the exercise price becomes higher relative to the current level of the underlying asset.

Of course, given the BS assumptions, all option prices on the same underlying security with the same expiration date but with different exercise prices should have the same implied volatility. However, the volatility smile pattern suggests that the BS formula tends to misprice deep in–the–money and deep out–the–money options. There have been various attempts to deal with this apparent failure of the BS valuation model. The stochastic volatility framework of Hull and White (1987) was the first systematic approach in option princing literature to recognize nonconstant volatility[2]. When volatility is stochastic but uncorrelated with the underlying asset price, they show that the price of a European option is the BS price integrated over the probability distribution of the average variance during the life of the option[3]. Unfortunately, however, this framework generally requires a market price of volatility risk. In other words, with stochastic volatility, a second factor is introduced requiring the option to satisfy a bivariate stochastic differential equation. Since the volatility –the second factor– is not spanned by existing securities, arbitrage pricing techniques

[1] See Rubinstein (1994), and Jackwerth and Rubinstein (1996) for a detailed discussion of this empirical regularity.

[2] Ball and Roma (1994) show that the implied variance of the BS price when the true price process is subject to stochastic volatility is quadratic in the out–ness–of–the–money, and that the greatest downward bias occurs for at–the–money options. This suggests that the stochastic volatility option pricing model is consistent with the smile.

[3] Another related (non–stochastic) approach allows the volatility to depend functionally on the underlying security price. Various alternative proposals for the functional volatility process have been suggested. The well known constant elasticity of variance model due to Cox and Ross (1976) is the most prominent one.

are no longer valid. We must therefore introduce the explicit exogenous market price of volatility risk. We face similar problems if we introduce any other non–traded sourse of risks such as systematic jumps or transaction costs.

Recent advances in this literature include Stein and Stein (1991), Heston (1993), and Bates (1996). In particular, Heston (1993) shows that a closed–form solution for a European call can be derived as an integral of the future security price density which itself may be calculated by an inverse Fourier trransform. This method may also be applied when correlation between the increments of the driving Brownian motions of the underlying asset and the volatility is non–zero. Thus, while Hull and White (1987) is an approximation, Fourier inversion methods are potentially more precise. Of course, estimation methods remain quite challenging.

An alternative approach for dealing with nonconstant volatility was suggested by Rubinstein (1994), Jackwerth and Rubinstein (1996) and Jackwerth (1996), and a related series of papers by Derman and Kani (1994), Dupire (1994), Chriss (1995), Derman, Kani and Chriss (1996). Instead of imposing a parametric funtional form for volatility, they construct a binomial or trinomial numerical procedure so that a perfect fit with observed option prices is achieved. This procedure captures (by construction) the most salient characteristics of the data. In particular, the implied tree employed in the numerical estimation must correctly reproduced the volatility smile. The most popular models within this family use recombining binomial trees implied by the smile from given prices of European options. Once the appropriate prices and transition probabilities corresponding to the nodes and links of the tree are calculated, any American path–dependent option may be priced consistently with the market. Also, to eliminate arbitrage opportunities, negative node transition probabilities are not allowed and the branching process must be risk–neutral at each step.

Empririical tests of implied binomial trees have been proposed by DFW (1996) and Jackwerth (1996). DFW point out that none of the previous studies analyze the out–of–sample behavoir of the time–varying volatility function obtained by the in–sample implied binomial trees. The key empirical issue becomes the stability of the volatility function. Surprisingly enough, DFW find that the pricing (and hedging) out–of–sample performance of the implied binomial trees is worse than of an *ad hoc* BS model with variable implied volatilities. They suggest that the BS model may be pertectly correct, but trading costs combined with option series clienteles may produce systematic patterns in implied volatilities[4]. The point is that these patterns may have no relation with the distributional characteristics of the underlying asset.

On the other hand, Jackwerth (1996) tests the pricing performance of implied binomial trees, the BS model, and the constant elasticity of variance model. He chooses the parameters of these models to fit the observed prices of longer term options best and then price shorter options with those parameters. In the post–crash period, Jackwerth favours the pricing of the impied binomial trees.

Finally, the papers by Corrado and Su (1996a, 1996b) contain a related way to cope with the smile effect featured by the option data. It is well understood that volatility smiles are a consequence of empirical violations of the normality assumption in the BS model. In other words, skewness and kurtosis in the option–implied distributions of stock returns are the source of volatility smiles. This is, of course, closely related to stochastic volatility models which can nicely explain the behaviour of option prices in terms of the underlying distribution of returns. In particular, the correlation between the Brownian motions associated with the underlying asset and the volatility affects the skewness of returns, while the volatility of volatility is directly related with kurtosis[5]. Following this reasoning, Corrado and Su suggest an extended version of the BS model to account for biases induced by nonnormal skewness and kurtosis in stock return distributions. Their valuation formula is given by the sum of the BS option price plus adjustment terms for nonnormal skewness and kurtosis. They find that their adjusted formula yields significantly improved pricing performance for deep in–the–money or deep out–of–the–money options.

Despite the fact that we have theoretical models consistent with the smile pattern across exercise prices, it is also true that the empirical smiles are about twice as large

[4]For an alternative discussion of trading costs, see Longstaff (1995).

[5]See the excellent discussion provided by Heston (1993). Hull (1997) also contains a general analysis of these issues.

as predicted by theory[6]. It seems quite clear that something else is going on. In this regard, given the evidence provided by Longstaff (1995) and DFW (1996), a serious candidate to explain the pronounced pattern of volatility estimates across exercise prices might be related to liquidity and trading costs. In fact, Corrado and Su may be explaining the effects of trading costs rather than an actual deficiency of the BS model.

These remarks are at the origin of our research project. What is clearly missing in the extant literature is an analysis of the determinants of the implied volatility function. Surprisingly, none of the papers above has tried to explain directly the determinants of the smile, although this is a relevant issue. Otherwise, we may be missing an important point here; i.e., the reasons behind the "apparent" failure of the BS pricing model. Hence, the main objective of this paper is to study directly the determinants of the volatility smile. We employ an extensive database of intraday transaction prices for options on the Spanish IBEX–35 stock exchange index. This is one of the most popular option contracts traded in Europe. Given that we are particularly concerned with trading costs and liquidity effects, it may be relevant to explore alternative option markets which are probably narrower than the fully investigated S&P 100 index options traded at the Chicago Board Options Exchange (CBOE).

Our empirical results show that transaction costs, proxied by bid–ask spreads, and variables related to the uncertainty about the return of the underlying asset and to the relative market momentum seem to be key aspects regarding the shape of the implied volatility function. Moreover, complex and nonlinear causality effects on the dynamic interrelations between these variables and the volatility smile are also found.

This paper is organized as follows: the next section contains a brief summary of the Spanish option market. The data are described in Section 3. Some previous general results are reported in Section 4. In particular, smile seasonality is briefly discussed throughout this section. Section 5 presents the empirical results regarding

[6]See Ghysels, Harvey and Renault (1996).

the determinants of the smile volatility. Finally, we conclude with a summary and discussion.

2 The Spanish IBEX–35 Index Options

The Spanish IBEX–35 index is a value–weighted index comprising the 35 most liquid Spanish stocks traded in the continuous auction market system. The official derivative market for risky assets, which is known as MEFF, trades a futures contract on the IBEX–35, the equivalent option contract for calls and puts, and individual option contracts for blue–chip stocks. Trading in the derivative market started in 1992. The market has experienced tremendous growth from the very beginning. Relative to the volume traded in the Spanish continuous market, trading in MEFF represented 40% of the regular continuous market in 1992, 156% in 1994, and 170% in 1995. The number of all traded contracts in MEFF relative to the contracts traded in the CBOE reached 20% in1995.

The IBEX–35 option contract is a cash settled European option with trading during the three nearest consecutive months and the other three months of the March–June– September–December cycle. The expiration day is the third Friday of the contract month. Trading occurs from 10:30 to 17:15. During the sample period covered by this research, the contract size is 100 Spanish pesetas times the IBEX–35 index, and prices are quoted in full points, with a minimum price change of one index point or 100 pesetas[7]. The exercise prices are given by 50 index point intervals.

It is important to point out that liquidity is concentrated in the nearest expiration contract. Thus, during 1995 almost 90% of crossing transactions occurred in this type of contracts. Finally, it should be noticed that option and futures contracts are clearly associated. The futures contract has exactly the same contract specifications as the IBEX–35 options. This will allow us to employ the futures price rather than the spot price in our empirical exercise. In fact, this is what is

[7] This has recently been changed to 1,000 pesetas.

usually done by practitioners.

3 The data

Our database is comprised of all call and put options on the IBEX–35 index traded daily on MEFF during the period January 1994 through April 1996. Given the concentration in liquidity, our daily set of observations includes only calls and puts with the nearest expiration day. Moreover, we eliminate all transactions taking place during the last week before expiration. In other words, for each monthly expiration date cycle, we only take into account prices for the first three weeks of the cycle.

As usual in this type of research, our primary concern is the use of simultaneous prices for the options and the underlying security. The data, which are based on all reported transactions during each day throughout the sample period, do not allow us to observe simultaneously enough options with the same time–to–expiration on exactly the same underlying security price but with different exercise prices. In order to avoid large variations in the underlying security price, we restrict our attention to the 45–minute window from 16:00 to 16:45. It turns out that almost 25% of crossing transactions occur during this interval. Figure 1 contains the average hourly percentages of crossing transactions during the whole sample period. Moreover, care was also taken to eliminate the potential problems with artificial trading that are most likely to occur at the end of the day. Thus, all trades after 16:45 were eliminated so that we avoid data which may reflect trades to influence market maker margin requirements. At the same time, using data from the same period each day avoids the possibility of intraday effects in the IBEX–35 index options market. Finally, we eliminate from the sample all call and put prices that violate the well known arbitrage bounds.

These exclusionary criteria yield a final daily sample of 7,947 observations. The implied volatility for each of our 7,947 options is estimated next. Note that we take as the underlying asset the average of the bid and ask price quotation given for each futures contract associated with each option during the 45–minute interval[8]. Recall that we are allowed to use futures prices given that the expiration day of the futures and option contracts systematically coincides during the expiration date cycle. Moreover, note that dividends are already taken into account by the futures price. To proxy for riskless interest rates, we use the daily series of annualized repo T–bill rates with either one week, two weeks or three weeks to maturity. One of these three interest rates will be employed depending upon how close the option is to the expiration day.

As discussed by French (1984), volatility appears to be a phenomenon that is basically related to trading days. However, interest rates are paid by the calendar day. We therefore employ Black's (1976) option pricing formula adjusted by two time measures to reflect both trading days and calendar days until expiration.

We next observe all calls and puts with the same exercise price for each day in the sample and for our 45–minute interval. We average all implied volatilities previously estimated for each level of the exercise price available during each daily window. All underlying futures prices associated with each exercise price level are averaged to obtain the corresponding level of the underlying asset associated with each average implied volatility. We define moneyness as the ratio between the exercise price and the average of the futures price relative to each average implied volatility as previously obtained. We can now estimate our daily volatility smile. It should be pointed out that the number of observations within a day may vary according to the number of crossing transactions associated with different exercise prices available for each day. In any case, this procedure reduces our sample to 3,016 observations from January 1994 to April 1996. This implies that, on average, we have between 5 and 6 options available for alternative exercise prices during each day.

Figures 2, 2.2 and 2.3 present the representative smiles for the whole period and two consecutive subperiods. We employ five fixed intervals for the degree of

[8]There might be that lack of liquidity in the futures market is responsible for the lack of variation in the price of the underlying asset during the 45–minute window. However, this is not the case. In fact, the futures market is, at least, as liquid as the spot market in terms of comparable measures of trading volume.

moneyness, and compute the median over the alternative subperiods of the implied volatility within that fixed interval. These intervals are given by the following degrees of moneyness: 0.8598–0.9682; 0.9682–0.9913; 0.9913–1.0101; 1.0101–1.0321; 1.0321–1.1875. It is interesting to note that the Spanish market seems to be "smiling" independently of the subperiod employed in the estimation.

Finally, Figure 3 reports similar evidence when the smile is obtained for calls and puts independently. As before, a rather well defined smile seems to be a typical phenomenon in the Spanish options market. However, it should be recognized that a somewhat clearer picture emerges for puts than for calls.

This generally well behaved smile contrasts with the evidence found in the US market where the typical shape of the volatility function after the 1987 crash is closer to a "sneer". Formal tests among the two alternatives are performed in the following section.

4 The implied volatility function and smile seasonality

We next investigate the determinants of the smile. The idea is to estimate the volatility function by fitting the implied volatility through six alternative structural forms:

$$\begin{aligned}
Model\ \ 1:\sigma &= b_0+\epsilon \\
Model\ \ 2:\sigma &= b_0+b_1X+\epsilon \\
Model\ \ 3:\sigma &= b_0+b_1X+b_2X^2+\epsilon \\
Model\ \ 4:\sigma &= b_0+b_1U+b_2D^2+\epsilon \\
Model\ \ 5:\sigma &= b_0+b_1U+b_2X^2+\epsilon \\
Model\ \ 6:\sigma &= b_0+b_1U+b_2X^2+b_3D+\epsilon
\end{aligned} \tag{1}$$

where X is the degree of moneyness; this is to say, the exercise price divided by the futures price. Let K be the exercise price and F the futures price associated to a particular level of the underlying futures price, then X is equal to K/F. Thus, model 1 is the volatility function of the BS constant volatility model. Model 2 posits a linear relation between volatility and the degree of moneyness. Model 3 incorporates a quadratic term to capture the typical smile shape. Finally, Models 4 to 6 employ three different ways of recognizing potential asymmetries in the shape of the volatility function. In particular, Model 4 assumes that the left side of the volatility function is linear on the degree of moneyness, but a quadratic term is necessary to capture some degree of curvature in the right side of the function. Thus:

$U = (U_1, \ldots, U_n)$ and $D = (D_1, \ldots, D_n)$ where:

$$U_i = \begin{cases} X_i & \text{if } X_i < 1 \\ 0 & \text{if } X_i \geq 1 \end{cases} \qquad D_i = \begin{cases} 0 & \text{if } X_i < 1 \\ X_i & \text{if } X_i \geq 1 \end{cases}$$

where n is the total number of exercise levels for a given day within our 45–minute window.

For each day in the sample, we run the regressions given by (1). Given the number of observations available during each day, not all models can be run for every day. Table 1 contains the average adjusted R^2 weighted by the number of observations available for each day within each model. The results are reported for the whole sample period, two different subperiods, and for all available quarters. The results suggest that model 3 is the best model in capturing variation in implied volatility attributable to moneyness. This quadratic model explains almost 63% of the variability of implied volatility. It should also be noted that Model 6 explains, in general, as well as the quadratic model.

Given that the behaviour of the implied volatility pattern seems to be different in Spain than in the U.S. market, it was decided to run a formal test to compare statistically the different performance of the "sneer" (model 2) and the smile (model 3)[9]. In order to investigate this issue, the regression below is estimated by stacking

[9]It should be pointed out that the evidence found in the U.S. market is best described by a "straight sneer". Rubinstein's (1994) findings are a good example. In this sense, model 2 becomes the relevant benchmark.

all of the observations and using OLS procedures:

$$\sigma_{jt} = a_0 + a_1 X_{jt} + a_2 X_{jt}^2 + \epsilon_{jt}; \quad j = 1, ..., J; \quad t = 1, ..., T \tag{2}$$

where σ_{jt} is the implied volatility of each option j available during the 45–minute window and for each day t in our sample, and X is the degree of moneyness.

The idea is to test whether the coefficient associated with the quadratic term, a_2, is statistically different from zero. Since our two models are nested, we are able to use a LM statistic that is asymptotically distributed as a chi-squared with one degree of freedom. It turns out that this statistic is equal to 353.29 (p-value = 0.0000). This implies that the estimate of a_2 is statistically different form zero, so that we favour model 3 relative to model 2[10].

Finally, we also look at both call and put options separately. Given that the put–call parity relationship implies that European call and put options of identical moneyness and maturity should have identical implied volatilities, there is no theoretical reasons to expect a significantly different behaviour between calls and puts. In fact, this turns out to be the case[11]. Model 3 presents the highest average adjusted R^2 weighted by the number of observations available for each day within each model for both call and put options. Moreover, when we run regression (2) stacking all available observations, we find that the Lagrange Multiplier statistic equals to 87.46 (p-value = 0.0000) and 148.16 (p-value = 0.0000) for calls and puts respectively.

Given these results, we will focus on the coefficients estimated with Model 3. It is important to emphasise that Model 3 is estimated every day in the sample period with enough observations. In other words, to run the corresponding regression for Model 3 every day we need to have enough levels of exercise prices throughout the 45–minute interval. In particular, for Model 3 and using all call and put options at the same time, we have 446 days with enough observations. Therefore, in this case and on a daily basis, we can cross–sectionally estimate 446 coefficients for

[10]It should be noted that Lagrange Multipler Test $\leq$ Likelihood Ratio Test $\leq$ Wald Test. Thus, we would also reject the null with either one of the alternative statistics.

[11]This evidence is consistent with the empirical findings reported by Bates (1991).

b_0, b_1 and b_2.

Daily return seasonalities have been a very popular research topic in recent years. Moreover, daily microstructure seasonalities have also been investigated by Foster and Viswanathan (1993) for the US market, and Lehmann and Modest (1994) for the Tokyo Stock Exchange market among others. They conclude that volume is lowest on Mondays, reflecting both reduced demand for liquidity traders who may fear increased adverse selection, and the higher trading costs on Monday since this is the day when bid–ask spreads are clearly largest. In the Spanish continuous auction market, Rubio and Tapia (1996) find both higher bid–ask spreads and lower depth on Monday. They conclude that liquidity is unambiguously lower on Monday.

These findings may imply that the smile volatility function does not remain stable throughout all week days. There may be seasonalities in the shape of the volatility smile which may reflect different degrees of liquidity, institutional arrangements or a continuous learning process of market makers throughout the week which may suggest a different implied volatility function at the beginning of the week.

These issues are investigated by running the following regressions:

$$b_{it} = \beta_{MO} MON_t + \beta_{TU} TUE_t + \beta_{WE} WED_t + \beta_{TH} THU_t + \beta_{FR} FRI_t + \epsilon_t \qquad (3)$$

where b_{it} is either b_{0t}, b_{1t} or b_{2t} and MON, TUE, WED, THU and FRI are dummy variables for Monday through Friday. The estimates of b_{MO}, b_{TU},...,b_{FR} are the sample means corresponding to each day of the week for the three coefficients of Model 3. Newey-West consistent standard errors with five lags are employed in all estimations. Moreover, in this case the statistic to jointly test seasonalities across week days follows a χ^2 distribution asymptotically, under the null hypothesis.

The results are reported in Table 2. Average coefficients for b_0, b_1 and b_2 are significantly different from zero throughout the sample period. This confirms the characteristics of the volatility smile intuitively suggested by Figure 2. It seems certainly the case that, on average, the Spanish smile is characterized by a large degree of curvature. At the same time, independently of the day of week, all three coefficients are, on average, significantly different from zero. However, their magnitude seems quite different from one day to another. In particular, the results suggest that Monday presents a lower slope and lesser degree of curvature than other week days. In fact, our χ^2 statistic significantly rejects the equality of coefficients across all days.

To formally test daily seasonality, we run the following regressions:

$$b_{it} = \beta_1 + \beta_2 TUE_t + \beta_3 WED_t + \beta_4 THU_t + \beta_5 FRI_t + \epsilon_t \tag{4}$$

where TUE, WED, THU, and FRI are dummy variables for Tuesday through Friday. The estimate of β_1 is the sample mean for Monday, while the estimates of the remaining coefficients are equal to the difference between the sample mean for each day and the sample mean for Monday. The results are contained in Table 3. The reported figures in the last line of the table are obtained by running the following regression:

$$b_{it} = \beta_1^* + \beta_2^* MON_t + \epsilon_t \tag{5}$$

where β_1^* is sample mean for all days except Monday, and β_2^* is the difference between Monday and the rest of the week.

The results clearly suggest that the slope of the implied volatility function on Monday is statistically lower than in the rest of the week. It turns out that the difference becomes more and more relevant to the end of the week. A similar finding is obtained relative to the degree of curvature. Monday presents a statistically significant lower degree of curvature than the rest of the week. As before, this characteristic becomes more evident as we get closer to the end of week. Thus, Friday has the highest degree of curvature and the highest slope relative to the beginning of the week. It should be pointed out that there is an almost perfectly negative correlation coefficient between b_1 and b_2. If we take the derivative of Model 3 relative to the degree of moneyness, X, and equate to zero to find the minimum level of X, we note that $X_{min} = -b_1/2b_2$. It turns out that X_{min} is very close to one most of the time. That is to say, the minimum implied volatility on a daily basis is generally very close to at-the-money implied volatility. Hence, the estimate of b_1 should be approximately equal to minus two times the estimate of b_2. These characteristics are reflected in the results reported in Table 3.

In summary, we may conclude that the volatility smile is statistically different on Monday relative to the rest of the week. Both the slope and the curvature are different. This implies a significant daily seasonality in the shape of the volatility smile. It may be interesting to have a single series summarising the three series of coefficients given by Model 3. It was decided to synthesise these coefficients computing the first principal component of the 446x3 matrix of our daily estimates. Given that, as discussed above, the correlation of the estimated parameters are very high, the first principal component explains almost 100% of the variability of these series[12].

The analysis of the seasonality of the principal component of the volatility smile is carried out by the same regressions given by equations (3), (4) and (5). The results are reported in Tables 4 and 5. As expected, relative to the rest of the week, our results indicate a significantly different behaviour of the principal component on Monday. It is quite striking to observe how the principal component is negative on Monday and becomes progressively positive towards the end of the week. The Spanish options market smiles very differently on Monday than on other week days. More specifically, the Spanish smile is statistically different at the beginning of the week relative to the end of the week. We may even conclude that, if this behaviour is directly associated with the failure of the BS option pricing framework, the BS model might be working better at the beginning of the week while its performance gets worse as the week nears its end. The next section investigates the reasons behind this surprising seasonality and other determinants of the implied volatility function.

5 On the determinants of the implied volatility function

As argued in the introduction, the key issue of this paper concerns the direct analysis of the reasons explaining the volatility smile. It is important to emphasize that,

[12]The principal component is positively correlated with b_0 and b_2, and negatively correlated with b_1.

given that the IBEX–35 option contract is a European option, the pattern of implied volatilities across different exercise prices provides direct evidence of the shape of the risk–neutral density, relative to the lognormal benchmark. Of course, this is because the second derivative of the European call (put) option price with respect to the exercise price is proportional to the appropriate risk–neutral probability density. This argument implies that, in fact, our objective is to explain the true implicit distribution in actual option prices.

Under this line of reasoning, the results from our previous sections suggest that the implicit distribution in the Spanish market is leptokurtic in (both) the right and the left tail of the distribution. This means that out–of–the–money calls (in–the–money puts) and puts (in–the–money calls) which pay off under realizations in the tails are more valuable than predicted by the BS model with its lognormal distribution assumption. An important point of our research is therefore to investigate the characteristics of the (deep) out–of–the–money calls (in–the–money puts) and puts (in–the–money calls).

5.1 Data and preliminary findings

Analysis of the determinants of the volatility smile is based on three categories of economic variables. The economic determinants should include relevant characteristics of the underlying asset, economic variables that help to predict the future stock market, and some characteristics of the options market itself. In particular, violations of a constant implied volatility function may be due to the effects of trading costs or to the degree of options market liquidity. Proxies for these characteristics should be included in the list of relevant variables.

To capture the possibility of market–related effects on option pricing, we include the annualised standard deviation of the IBEX for each day in the sample estimated with minute by minute observations, and the natural log of the number of shares traded (volume) by the components of the IBEX during the 45–minute interval for which we have option pricing data. The idea is to incorporate both a measure of uncertainty and a measure of the level of activity in the underlying asset.

Two variables are employed in order to incorporate variables that may help in predicting the market. Both measures reflect the relative market momentum of the underlying economic situation of the Spanish economy. The idea is to construct variables that reflect levels of asset prices. This is obviously somewhat arbitrary. However, there is well known evidence that suggests some useful instruments in predicting general market conditions and expected returns of risky assets[13].

Our first variable of this type is the log relative treasury bill rate (RTB) given by the following expression:

$$RTB_t = \log \frac{r_t}{\frac{1}{60} \sum_{\tau=t-1}^{t-61} r_\tau} \tag{6}$$

where r_t is the one week Treasury bill repo rate available at day t. It provides a relative measure of the interest rate levels with respect to its three–month (60 trading days) moving average.

The second variable (MKT) is the log of the ratio of the previous short–run level of the IBEX, given by its three–month moving average, to its current level:

$$MKT = \log \frac{\frac{1}{60} \sum_{\tau=t-1}^{t-61} IBEX_\tau}{IBEX_t} \tag{7}$$

where $IBEX_t$ is the level of the value-weighted Spanish stock exchange index at the end of day t.

The underlying justification for including both types of determinants in our analysis (two relevant characteristics of the underlying asset, and two economic variables that help to predict the future stock market) lies in the possibility of their having path–dependent effects on option pricing. If such effects exist, they may impact the market valuation of out–of–the–money calls (in–the–money puts) and puts (in–the–money calls).

The last group of variables which may be relevant in explaining implied volatility patterns across exercise prices is associated with the characteristics of the option

[13]See for example, Keim and Stambaugh (1986) and Campbell (1996).

market itself. As a measure of transaction costs, we employ the daily average relative bid–ask spread for the options transacted during the 45–minute interval. They reflect the market–making costs and adverse selection risks faced by agents participating in the option market. Finally, as a measure of the level of activity in the option market, we include the natural log of the number of option contracts negotiated during the 45–minute interval employed in this paper. It provides a reasonable estimate of the general liquidity of the option market.

Before presenting a time–series regression analysis relating the main characteristics of the volatility smile to the variables described above, we must analyze the potential non–stationarities in our chosen variables. Augmented Dickey–Fuller (DF) tests for unit roots are reported in Table 6. The tests are also performed for the first principal component of the 446x3 matrix of coefficients b_0,b_1 and b_2 characterizing the smile over time. Independently of the specification employed in the analysis, the results imply that the log relative Treasury bill repo rate (RTB) is nonstationary, while for the rest of our chosen variables, we are able to reject the existence of a unit–root. In the tests below, we therefore use the first daily differences of the log relative repo rate.

Our first test consists of simple regressions, with Newey–West robust standard errors, relating the variables described previously to either the principal component of the smile or the coefficients themselves. This section of the paper analyzes several factors (potentially) related to the volatility smile, but it does not test for *causes* of the smile. The hypothesis merely involves correlation between the volatility smile and some other variables.

In the regressions below, we also include a dummy variable for Monday, and two other control variables for moneyness and time to expiration. In particular, the average degree of moneyness of all options used in the analysis, and the time–to–expiration of the options employed in our database are taken into account. Note that, for a given day, all options available throughout the 45–minute interval have the same time–to– expiration. However, the volatility smile may be changing throughout the life of the options[14].

In order to explain the variability of the principal component and the coefficients which characterizes the smile, the following time–series regressions are run:

$$pc_{0t}(b_{it}) = \beta_0 + \beta_1 MON_t + \beta_2 MKT_t + \beta_3 SIGMA_{t-1} + \beta_4 VMKT_{t-1} + \beta_5 DRTB_t +$$
$$+ \beta_6 BA_t + \beta_7 VOPT_{t-1} + \beta_8 TIME_t + \epsilon_t \quad (8)$$

where:

- pc is the principal component of the 446x3 matrix of coefficients (b_0, b_1 and b_2) characterizing the volatility smile throughout the time period employed in the analysis;
- MON is the dummy variable for Mondays;
- MKT is the log of the relative market momentum given by expression (7);
- $SIGMA$ is the annualized standard deviation of the IBEX for each day in the sample estimated by minute by minute observations;
- $VMKT$ is the log of the number of shares traded by the individual stocks conforming the IBEX calculated during the 45–minute interval;
- $DRTB$ is the first daily difference of the log relative Treasury bill repo rate given by expression (6);
- BA is the daily average relative bid–ask spread for the options transacted during the 45–minute interval considered in the analysis;
- $VOPT$ is the log of the number of options contracts negotiated during the 45–minute interval; and
- $TIME$ is the annualized number of days to expiration of the options transacted during the 45–minute interval.

[14] Given that the degree of moneyness does not have any significant influence in the results, it is not included in the regressions shown in the paper.

The regressions are run with a one–period lag for the volatility variable, and for the volume related variables in the option and stock markets. In principle, it would be desirable to use available information at each day t if we want to make stronger statements about these regressions. However, the results reported are based on the contemporaneous bid–ask spread and the contemporaneous market momentum. It should be pointed out that very similar results are found when we also use a one–period lag for the market momentum variable and the bid–ask spread. Somewhat better statistical fit values are obtained, however, when we run the regression model given by (8)[15].

The results are shown in Table 7. They suggest that the principal component (and therefore the degree of curvature) of the volatility smile is positively and significantly related to transaction costs represented by the bid–ask spread. On average, whenever the bid–ask spread tends to increase, the degree of curvature of the volatility smile increases (and the slope increases). Alternatively, when market makers tend to face higher adverse selection risks, out–of–the–money calls (in–the–money puts) and puts (in–the–money calls) are more highly valued by the market relative to the BS model.

On the other hand, the principal component (the degree of curvature) is negatively and significantly related to the historical volatility of the underlying asset, and to time to expiration. It is interesting to point out that options with short times to expiration tend to have a higher degree of curvature (and higher slope) in the implied volatility pattern across exercise prices. It is also interesting that high volatility periods tend to be associated with lower curvature (lower slope) of the smile.

Finally, the relative momentum of the market seems to be weakly related to the degree of curvature and the principal component of the smile. Whenever the current level of the stock market improves relative to the past, we find that, on average, the degree of curvature of the smile increases (the slope increases).

[15]Stepwise regressions are also employed in deciding the variables and the number of lags to be included in the regressions reported in the paper.

The results regarding the correlations between market momentum, historical volatility and the shape of the volatility smile suggest that relatively calm periods but with, at the same time, increasing current levels of the market stock exchange index tend to be associated with a higher degree of curvature (higher slope) of the volatility smile. This suggests that at these particular moments of time out–of–the–money puts (in–the–money calls) are asymmetrically valued by the market relative to in–the–money puts (out–of– the–money calls). Alternatively, the pattern across exercise prices becomes more flat (and with a less degree of curvature) whenever the volatility of the underlying asset goes up, and the relative market momentum gets worse. At these periods of time, out–of–the–money puts (in–the–money calls) become more symmetrically valued by the market relative to in–the–money puts (out–of–the–money calls).

To finish the discussion of this preliminary evidence, it should be pointed out that the Monday dummy variable does not seem to be significant once other variables are taken into account by the analysis.

In summary, we may conclude that transaction costs influence the relative valuation of out–of–the–money puts (in–the–money calls) and in–the–money puts (out–of–the–money calls). Higher transaction costs are associated with higher market values of extreme (in term of moneyness) options, but in a rather asymmetric way. These costs seem to affect more out–of–the–money puts (in–the–money calls) than in–the–money puts (out–of–the–money calls). However, higher uncertainty seems to be associated with a more symmetric valuation (and lower slope) of extreme options. Hence, out–of–the money puts (in–the– money calls) are relatively more valued than in–the–money puts (out–of–the–money calls) at periods of time for which there is a decrease in uncertainty and the relative market level goes up.

Therefore, it seems that two quite distinct forces are associated with the shape of the volatility smile. On the one hand, transaction costs are related to higher market valuation of extreme (in terms of moneyness) options relative to the BS model; they are particularly associated with the degree of curvature of the smile, but given the asymmetric relative valuation of extreme options, they are also associated with a higher slope. Secondly, relative market momentum and the uncertainty of the market, proxied by the volatility of the underlying asset, also affect the shape of the smile. With relatively high index levels and low volatility, the market might be giving more value to out–of–the–money puts (in–the–money calls) relative to the values of the in–the–money puts (out–of–the–money calls). This asymmetric valuation and the effects of market conditions might be related to skewness effects on option prices. As shown by Heston (1993), the correlation between volatility and the spot asset's price is a key issue for explaining skewness. In particular, negative skewness which is consistent with asymmetric GARCH effects found in the IBEX-35 index by León and Mora (1996) might be the explanation of our preliminary evidence[16].

5.2 Linear Granger causality tests and the volatility smile

The general idea behind causality tests is that they can provide useful information on whether knowledge of past values of the variables employed in the previous section improves short–run forecasts of current and future variability on the shape of the volatility smile. In this section, we employ traditional Granger tests to investigate the presence of linear predicting power between the variables discussed previously and the shape of the volatility smile.

Let us assume that we observe two stationary and ergodic time series, X_t, and Y_t. Let $F(X_t \mid Z_{t-1})$ be the conditional probability distribution of X_t given a bivariate information set Z_{t-1}. This information set is formed with the Lx-length vector of past values of X_t, X_{t-Lx}, and the Ly-length vector of past values of Y_t, Y_{t-Ly}. Given these lags, the series, Y_t, does not strictly Granger cause X_t if:

$$F(X_t \mid Z_{t-1}) = F(X_t \mid (Z_{t-1} - Y_{t-Ly})); \quad t = 1, 2, ... \tag{9}$$

Alternatively, of course, if the equality (9) does not hold, then knowledge of past

[16]Negative skewness or higher downside volatility might be explained by either the well known leverage effects or by wealth effects. The later consists of economic agents becoming more risk averse as prices (wealth) go down. Hence, the arrival of new information cause a greater reaction among agents, so that volume of trading and volatility increase.

values of Y_t is useful in predicting current and future values of X_t, and therefore Y is said to strictly Granger cause X[17].

In order to implement this test, we estimate the following bivariate vector autoregression (VAR) model:

$$pc_{0t} = \alpha + A_{11}(L)pc_{0t} + A_{12}(L)DET_t + U_t$$

$$DET_t = \beta + A_{21}(L)pc_{0t} + A_{22}(L)DET_t + W_t \tag{10}$$

where, α and β are two constant terms, pc_{0t} is the principal component of the volatility smile[18], and DET represents the variables employed in the previous section of the paper. In particular, DET can be one of the following variables: the log relative market momentum (MKT), the annualized historical volatility of the underlying asset (SIGMA), the log of the number of shares traded in the underlying asset ($VMKT$), the first differences of the log relative Treasury bill repo rate ($DRTB$), the log of the number of contracts negotiated in the option market ($VOPT$), and the average relative bid–ask spread of the options transacted in our 45 minutes interval (BA). Moreover, $A_{11}(L)$, $A_{12}(L)$, $A_{21}(L)$, and $A_{22}(L)$ are lag polynomials of the same order in the lag operator L, and the residuals U_t and W_t are assumed to be mutually independent and individually i.i.d. variables with zero mean and constant variance.

To test for linear Granger causality from DET (MKT, $SIGMA$,...,BA) to the principal component or, alternatively, to the set of coefficients characterizing the smile, a standard joint F test of exclusion restrictions is carried out to determine whether lagged values of DET have significant linear predicting power for the principal component (or the smile coefficients). The appropriate number of lags is determined in each case on the basis of four alternative information criteria: the Akaike information criterion, the Schwarz specification test, the final prediction error criterion, and the Hannan–Quinn test. When conflicts are found, the Akaike criterion

[17]When the bivariate information set includes the current level of Y, we have the concept of instantaneous Granger causality.

[18]Similar regressions were run for the three coefficients characterizing the smile, b_0, b_1 and b_2.

is employed[19]. The null hypothesis that DET does not Granger cause the principal component (the smile coefficients) is rejected if the coefficients on the elements in $A_{12}(L)$ are jointly significantly different from zero. When feedback causality exists, then the coefficients on the elements in both $A_{12}(L)$ and $A_{21}(L)$ are jointly different from zero.

Table 8 reports the results of testing linear Granger causality between the principal component of the smile (and the coefficients) and the average relative bid–ask spread (BA)[20]. By analyzing rejections of the null hypothesis of Granger linear noncausality at the 5% level, our tests indicate that there is clear unidirectional causality from transaction costs, proxied by the bid–ask spread, to the principal component of the smile. Moreover, the evidence of unidirectional causality is also found for each of the coefficients characterizing the smile: the intercept, the slope and the degree of curvature. The conclusion is quite clear: the bid–ask spread does Granger cause the shape of the volatility smile. This is also the case when we analyze the case of instantaneous linear Granger causality. Finally, an important point is that the shape of the volatility smile also Granger causes transaction costs. Bi–directional causality is therefore found between transaction costs and the volatility smile.

Table 9 reports the same linear Granger causality tests for the rest of the variables used in our analysis. Granger noncausality from the alternative variables to the principal component cannot be rejected at the 5% significance level.

These results suggest that transaction costs, represented by the average bid–ask spread, is a key determinant of the shape of the implied volatility function. A quick way of checking the consistency of these results is reported in Table 10. This is not a formal test, but it provides an intuitive explanation of the results found in the paper. The table employs the five fixed intervals for the degree of moneyness used throughout the paper. The average of the relative bid–ask spread within each of the fixed intervals is calculated. As expected, given the empirical evidence reported

[19]Similar results across all criteria are generally obtained for all variables used in the analysis. In each case, the number of lags is always the same for the principal component and DET (MKT, $SIGMA$, ... , BA).

[20]The same results are found when using White standard errors and a χ^2 test of exclusion.

in this paper, the extreme options (in terms of moneyness) have the highest bid–ask spreads. In other words, deep out–of–the–money (in–the–money) options have the highest transaction costs, while the at–the–money options present the lowest transaction costs. This pattern of transaction costs seems to be reflected in the pricing of options. They are precisely the options most highly valued on average relative to the BS model. It may easily be the case that these higher transaction costs reflect higher adverse selection costs faced by market–makers when negotiating these options.

As previously mentioned, smile patterns are consistent with leptukortic distributions. However, it seems difficult to accept that transaction costs, proxied by bid–ask spreads, cause leptukortic distributions relative to the lognormal benchmark. In fact, as pointed out by Bates (1996), extremely high values of the volatility of volatility are necessary to obtain implicit leptokurtosis of a magnitude consistent with the empirically observed volatility smile. Our results suggest that the missing variable to explain the actual pattern of implied volatility across exercise prices is proxied by the bid–ask spread. Therefore, on the one hand, we have a theoretical justification for the smile –the volatility of volatility or leptokurtosis–, and on the other hand, transaction costs seem to be the ultimate reason behind the actual magnitude of the smiles.

Finally, it should be recognized that the linear causality found in the tests also runs in the opposite direction. In other words, the fact that these extreme options are more highly valued seems also to Granger cause higher transaction costs.

5.3 Nonlinear Granger causality tests and the volatility smile

There is increasing interest in the study of nonlinearities in the dynamic interrelations between financial time series. The point is that by removing linear predictive power with a linear VAR model of equation (10), there may be a remaining incremental predictive power of one residual series to another. In this case, this incremental predictive power is considered to be nonlinear.

We follow the modified version of Baek and Brock's (1992) nonlinear Granger causality tests as suggested by Hiemstra and Jones (1994). Let us consider two strictly stationary and weakly dependent time series U_t and W_t[21]. Denote the m-length lead vector of U_t by U_{t+m}, the Lu-length vector of past values of U_t, by U_{t-Lu}, and the Lw- length vector of past values of W_t, by W_{t-Lw}.

For a given value of m, Lu, and Lw ≥ 1 and for $\delta > 0$, W does not strictly Granger cause U if:

$$\begin{aligned} & Pr\left(\|U_{t+m} - U_{s+m}\| < \delta \quad \mid \quad \|U_{t-Lu} - U_{s-Lu}\| < \delta, \quad \|W_{t-Lw} - W_{s-Lw}\| < \delta\right) \\ & \qquad = Pr\left(\|U_{t+m} - U_{s+m}\| < \delta \quad \mid \quad \|U_{t-Lu} - U_{s-Lu}\| < \delta\right) \end{aligned} \tag{11}$$

where $Pr(.)$ represents probability and $\|.\|$ is the maximum norm. This definition indicates that the conditional probability that two m–length lead vectors of U_t are within a distance δ of each other, given that the Lu–length lag vectors of U_t <u>and</u> Lw–length lag vectors of W_t are within δ of each other is the same as the conditional probability that two m-length lead vectors of U_t are within a distance δ of each other, given that the Lu–length lag vectors of U_t are within δ of each other.

Hiemstra and Jones express the conditional probabilities in terms of the corresponding ratios of joint probabilities. Let $C1(m + Lu, Lw, \delta)/C2(Lu, Lw, \delta)$ and $C3(m + Lu, \delta)/C4(Lu, \delta)$ be the ratios of joint probabilities corresponding to the LHS and RHS of equation (11). Recall that the conditional probability $Pr(X \mid Y)$ can be expressed as $Pr(X \cap Y)/Pr(Y)$ and that, by the definition of the maximum norm:

$$Pr\left(\|U_{t+m} - U_{s+m}\| < \delta, \quad \|U_{t-Lu} - U_{s-Lu}\| < \delta\right) = Pr\left(\|U_{t+m-Lu} - U_{s+m-Lu}\| < \delta\right)$$

Then, these joint probabilities can be defined as:

$$\begin{aligned} C1(m + Lu, Lw, \delta) &\equiv Pr\left(\|U_{t+m-Lu} - U_{s+m-Lu}\| < \delta, \quad \|W_{t-Lw} - W_{s-Lw}\| < \delta\right) \\ C2(Lu, Lw, \delta) &\equiv Pr\left(\|U_{t-Lu} - U_{s-Lu}\| < \delta, \quad \|W_{t-Lw} - W_{s-Lw}\| < \delta\right) \\ C3(m + Lu, \delta) &\equiv Pr\left(\|U_{t+m-Lu} - U_{s+m-Lu}\| < \delta\right) \\ C4(Lu, \delta) &\equiv Pr\left(\|U_{t-Lu} - U_{s-Lu}\| < \delta\right) \end{aligned} \tag{12}$$

[21] In the application, these series correspond to the residuals of the VAR model of equation (10).

Rewriting the condition for nonlinear noncausality given by (11), we say that, for a given value of m, Lu, and Lw ≥ 1 and for $\delta > 0$, W does not strictly Granger cause U if:

$$\frac{C1(m+Lu, Lw, \delta)}{C2(Lu, Lw, \delta)} = \frac{C3(m+Lu, \delta)}{C4(Lu, \delta)} \tag{13}$$

In order to implement the test based on equation (13), Hiemstra and Jones suggest using the correlation–integral estimators of the joint probabilities in equation (12). Denote the time series of realizations on U_t and W_t by u_t and w_t. Moreover, let $I(X_1, X_2, \delta)$ be the kernel that equals 1 when the two vectors X_1 and X_2 are within the maximum–norm distance δ of each other and zero otherwise. Correlation–integral estimators can then be expressed as:

$$\begin{aligned}
\hat{C}1(m+Lu, Lw, \delta, n) &= \frac{2}{n(n-1)} \sum_{t<s}\sum I(u_{t+m-Lu}, u_{s+m-Lu}, \delta) \cdot I(w_{t-Lw}, w_{s-Lw}, \delta) \\
\hat{C}2(Lu, Lw, \delta, n) &= \frac{2}{n(n-1)} \sum_{t<s}\sum I(u_{t-Lu}, u_{s-Lu}, \delta) \cdot I(w_{t-Lw}, w_{s-Lw}, \delta) \\
\hat{C}3(m+Lu, \delta, n) &= \frac{2}{n(n-1)} \sum_{t<s}\sum I(u_{t+m-Lu}, u_{s+m-Lu}, \delta) \\
\hat{C}4(Lu, \delta, n) &= \frac{2}{n(n-1)} \sum_{t<s}\sum I(u_{t-Lu}, u_{s-Lu}, \delta)
\end{aligned} \tag{14}$$

where $t, s = max(Lu, Lw) + 1, \ldots, T - m + 1; \quad n = T + 1 - m - max(Lu, Lw)$

Given these joint probability estimators, we can test the nonlinear Granger noncausality condition in equation (11). For a given value of m, Lu, and Lw ≥ 1 and for $\delta > 0$, if W_t does not strictly Granger cause U_t then:

$$\sqrt{n}\left(\frac{\hat{C}1(m+Lu, Lw, \delta, n)}{\hat{C}2(Lu, Lw, \delta, n)} - \frac{\hat{C}3(m+Lu, \delta, n)}{\hat{C}4(Lu, \delta, n)}\right) \rightsquigarrow N\left(0, \sigma^2(m, Lu, Lw, \delta)\right) \tag{15}$$

where $\sigma^2(m, Lu, Lw, \delta)$ and a consistent estimator for it are given by Hiemstra and Jones in their appendix. A significant positive value of the statistics given by (15) implies the existence of a nonlinear causality from W to U. In the application, this would suggest a nonlinear causality of any of the variables under DET in equation (10) to the shape of the volatility smile[22].

[22]Hiemstra and Jones (1994) argue that the modified Baek and Brock test has good finite–sample size and power properties against a variety of linear and nonlinear causal and noncausal relations. Note that by allowing the errors to be weakly dependent, the key difference between the original test of Baek and Brock and the test employed in this paper lies in the estimators for $\sigma^2(m, Lu, Lw, \delta)$.

To implement the test of equation (15), we set the lead length for all cases at $m = 1$, and set $Lu = Lw$, using a common lag length of 1 to 4. Recall that these tests are based on the residuals of the VAR system of equation (10), so that the appropriate number of lags in the VAR has already been chosen according to our information criteria. Moreover, each series of residuals is standardized so that the two series have the same standard deviation, i.e., $\sigma = 1$.

The scale parameter, δ, is chosen to be either $\delta = 1.5\sigma$ or $\delta = 0.5\sigma$, where $\sigma = 1$ is the standard deviation of the standardized time series of residuals. Note that, since we standardized the series, all of them share a common scale parameter. It should be pointed out that the joint probabilities given by (12) should be lower whenever the scale parameter is smaller. The (joint) probabilities of two vectors being within a given distance of each other will be smaller whenever the distance becomes smaller. As expected, this is actually the case in the application below. However, this does not imply a systematic effect on either the statistic of equation (13) or the significance level of the test given by (15). In other words, there is no *a priori* monotone relation between the distance imposed and the acceptance of the null hypothesis.

The results are reported in Tables 11, 12 and 13 for the three sets of variables employed in this work to explain the shape of the volatility smile, i.e., the characteristics of the options market itself (the bid–ask spread and the volume negotiated in the derivative market), the economic characteristics of the underlying asset (the volatility and its volume), and our economic variables that help to predict the stock market (market momentum and the relative level of interest rates). Panel A of these tables refers to a given δ of 1.5, while Panel B imposes a δ of 0.5.

Interestingly, the results are not robust to alternative scale parameters. For a δ of 1.5, there seem to be no clear signs of nonlinear causality. The null hypothesis of noncausality cannot be rejected when evaluated with right–tailed critical values of the asymptotic $N(0,1)$ distribution. If anything, there is some slight evidence of unidirectional nonlinear Granger causality between market momentum and the principal component of the smile.

On the other hand, for $\delta = 0.5$, independently of the number of lags assumed in the estimation for $Lu = Lw$, we find seemingly strong evidence of nonlinear Granger causality from the bid–ask spread, volume in both the underlying asset and the option market, market momentum, to the principal component of the smile. It is also the case that for shorter lags, there seems to be evidence of nonlinear causality from the volatility of the underlying asset and even the relative level of the interest rates to the principal component of the smile.

Unfortunately, the interpretation of the results is rather complicated. It may the case that, for this particular application, the nonlinear predictive power improves significantly when we use not only the variable itself, but also other variables such as the bid–ask spread or market momentum, as long as the required distance in the tests becomes smaller.

In any case, it is certainly relevant to find evidence of nonlinear causality, particularly for those cases for which traditional linear Granger tests have not been able to detect evidence of causality. Once again, the substantial differences found in this paper between linear and nonlinear causality tests suggest the relevance of testing for both linear and non–linear predicting power between economic variables.

Our results imply that the dynamic interrelations between the implied volatility function and economic variables such as transaction costs or market momentum and other relevant variables such as the volatility of the underlying asset are, to a certain extent, non–linear. Future research should probably be concentrated on using non–linear theoretical mechanisms when developing models of microstructure dynamic interrelations between information flow and the pricing of extreme (in terms of moneyness) options.

6 Conclusions

To the best of our knowledge, this work analyzes for the first time the underlying determinants of the well known pattern of implied volatilites across exercise prices for otherwise identical options, i.e., the so called volatility smile. We employ a database comprised of all call and put options on the IBEX–35 Spanish index traded daily during the 45–minute interval from 16:00 to 16:45 from January 1994 to April 1996. Contrary to the US market, where the smile is rather a (straight) sneer, we find that the Spanish market tends to smile consistently throughout the sample period.

In order to understand the behavior of the implied volatility function, formal tests are performed under a simple regression framework together with more sophisticated tecniques of both linear and nonlinear Granger causality tests. Our results suggest a strong seasonal behavior in the volatility smile. However, this seasonality tends to disappear when we include several economic variables in the analysis. In particular, transaction costs proxied by the bid–ask spread of the negotiated options, the volatility of the underlying asset, time to expiration and relative market momentum seem to be key variables in explaining the variability of the implied volatility function over time.

Linear causality tests point to bidirectional linear Granger causality between the shape of the smile and transaction costs. No other economic variable seems to linearly cause the smile. Somewhat surprisingly, however, nonlinear nonparametric tests also suggest that levels of activity in both the derivative market and the underlying asset market, the volatility of the stock market index, and the relative market momentum and relative interest rates present evidence of nonlinear causality to the shape of the smile. More research to understand these nonlinearities is clearly justified.

As a more general conclusion, it seems that current market conditions proxied by both volatility of the underlying asset and relative market momentum, play a relevant role in shaping the smile. High levels of the market index and, on average, the corresponding low level of volatility as a consequence of negative skewness effects might be pushing the behavior of the smile towards a rather asymmetric valuation of extreme options. Out–of–the–money puts (in–the–money calls) are more highly valued whenever market conditions get better relative to in–the–money puts (out–of–the–money calls). This suggests that, whenever the market is relatively high (and, on average, volatility is relatively low), economic agents assign a higher probability (relative to a normal distribution) to weaker future market conditions.

It should be pointed out, however, that the key finding of this paper is associated with the importance of the bid–ask spread in explaining the shape of the volatility function. Transaction costs are a key determinant of the smile. They cause a higher degree of curvature and, at the same time, a higher slope in the implied volatility function.

To conclude, both forces –transaction costs and market conditions– play a simultaneous role in explaining the shape of the implied volatility pattern across exercise prices. Taking into account the significant and inverse relationship between time to expiration and degree of curvature, we are tempted to conclude that market conditions and transaction costs are relatively more important whenever there exists a short way to go in the life of the option.

References

[1] Baek, E. and W. Brock (1992). "A general test for non–linear Granger causality", Working Paper, Iowa State University of Wisconsin, Madison.

[2] Bates, D. (1991). "The crash of '87: was it expected? The evidence from options markets". *Journal of Finance* 46, pp. 1009–1044.

[3] Bates, D. (1996). "Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options", *Review of Financial Studies* 9, pp. 69–107.

[4] Black, F. (1976). "The pricing of commodity contracts", *Journal of Financial Economics* 3, pp. 167–179.

[5] Black, F. and M. Scholes (1973). "The pricing of options and corporate liabilities", *Journal of Political Economy* 81, pp. 637–659.

[6] Campbell, J. (1996). "Understanding risk and return", *Journal of Political Economy* 104, pp. 298–345.

[7] Chriss, N. (1995). "How to grow a smiling tree", Harvard University, Department of Mathematics Working Paper.

[8] Corrado, C. and T. Su (1996a). "Skewness and kurtosis in S&P 500 index returns implied by option prices", *Journal of Financial Research* 19, pp. 179–192.

[9] Corrado, C. and T. Su (1996b). "Implied volatility skews and stock return skewness and kurtosis implied by stock option prices", *European Journal of Finance*, forthcoming.

[10] Cox, J. and S. Ross (1976). "The valuation of options for alternative stochastic processes", *Journal of Financial Economics* 3, pp. 145–160.

[11] Derman, E. and I. Kani (1994). "Riding on a smile", *Risk* 7, pp. 32–39.

[12] Derman, E., I. Kani and N. Chriss (1996). "Implied trinomial trees of the volatiltiy smile", *Journal of Derivatives* 3, pp.7–22.

[13] Dumas, B., J. Fleming and R. Whaley (1996). "Implied volatility functions: empirical tests", National Boureau of Economic Research, Inc., Working Paper 5500.

[14] Dupire, B. (1994). "Pricing with a smile", *Risk* 7, pp. 18–20.

[15] Foster, D. And S. Viswanathan (1993). "Variations in trading volume, return volatility, and trading costs: evidence on recent price formation models", *Journal of Finance* 48, pp. 187–211.

[16] French, D. (1984). "The weekend effect on the distribution of stock prices: implications for option pricing", *Journal of Financial Economics* 13, pp. 547–559.

[17] Ghysels, E., A. Harvey and E. Renault (1996). "Stochastic volatility". In Statistical Methods in Finance, Handbook of Statistics 14, edited by G. S. Maddala and C. R. Rao, North Holland.

[18] Heston, S. (1993) "A closed–form solution for options with stochastic volatility with applications to bond and currency options", *Review of Financial Studies* 6, pp. 327–344.

[19] Hiemstra, C. and J. Jones (1994). "Testing for linear and nonlinear Granger causality in the stock price–volume relation", *Journal of Finance* 49, 1639–1664.

[20] Hull, J. (1997). "Options, futures and other derivative securities", Prentice Hall, third edition, New Jersey.

[21] Hull, J. and A. White (1987). "The pricing of options on assets with stochastic volatilities", *Journal of Finance* 42, pp. 281–300.

[22] Jackwerth, J.C. (1996). "Implied binomial trees: generalizations and empirical tests", University of California at Berkeley, Working Paper RPF–262.

[23] Jackwerth, J.C. and M. Rubinstein (1996). "Recovering probability distributions from option prices", *Journal of Finance* 51, pp. 1611–1631.

[24] Keim, D. and R. Stambaugh (1986). "Predicting returns in the stock and bond markets", *Journal of Financial Economics* 17, pp. 357–390.

[25] Lehmann, B. and D. Modest (1994). "Trading and liquidity on the Tokyo Stock Exchange: a bird's eye view", *Journal of Finance* 49. pp. 951–984.

[26] León, A. and J. Mora (1996). "Modelling conditional heteroskedasticity: application to the IBEX–35 stock–return index", Working Paper AD 96–11, Instituto Valenciano de Investigaciones Económicas, Universidad de Valencia.

[27] Longstaff, F. (1995). "Option pricing and the martingale restriction", *Review of Financial Studies* 8, pp. 1091–1124.

[28] Rubinstein, M. (1994). "Implied binomial trees", *Journal of Finance* 49, pp. 771–818.

[29] Rubio, G. and M. Tapia (1996). "Adverse selection, volume and transactions around dividend announcements in a continous auction system", *European Financial Management* 2, pp. 39–67.

[30] Stein, E. and J. Stein (1991). "Stock price distributions with stochastic volatility: an analytical approach", *Review of Financial Studies* 4, pp. 727–752.

FIGURE 1:

AVERAGE HOURLY PERCENTAGE OF CROSSING TRANSACTIONS

January 1994–April 1996

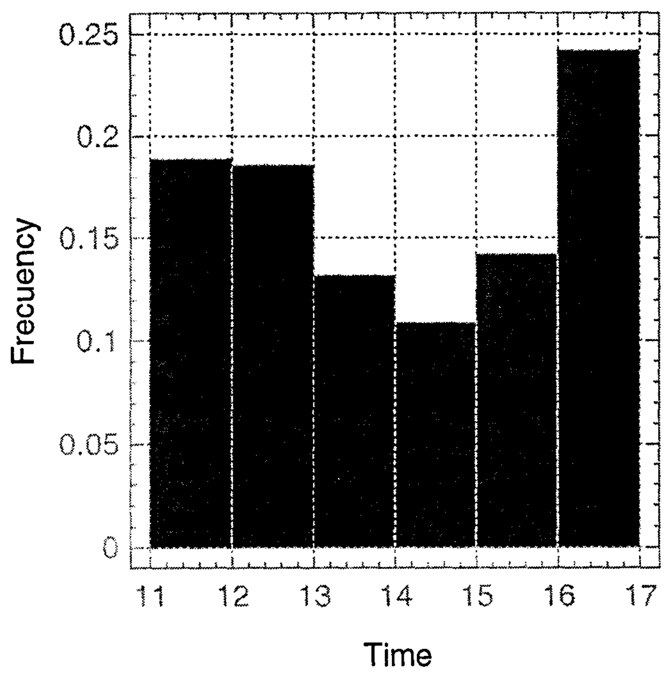
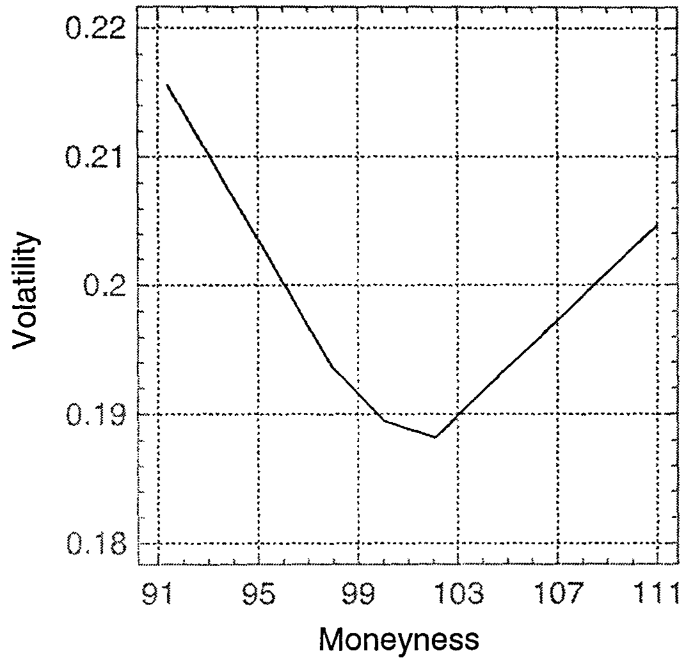


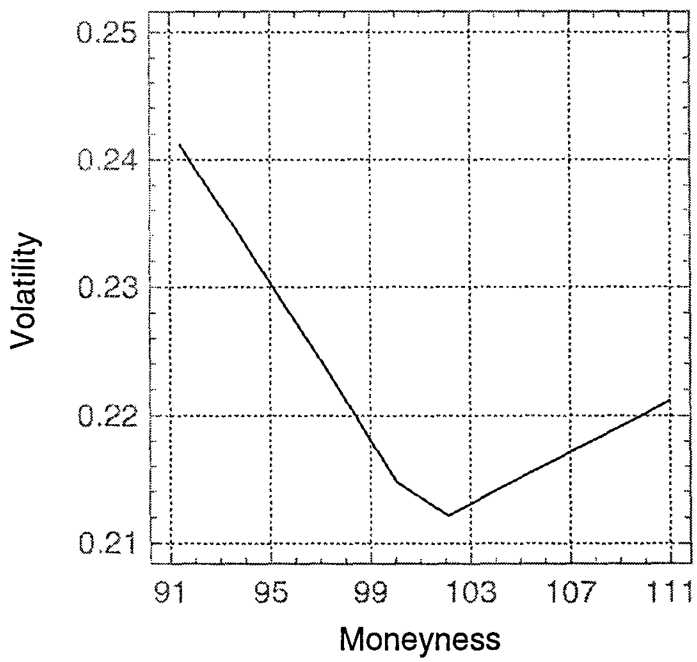
FIGURE 2:

AVERAGE SMILES OVER THE SAMPLE PERIOD

2.1 January 1994–April 1996



2.2 January 1994–February 1995



2.3 March 1995–April 1996

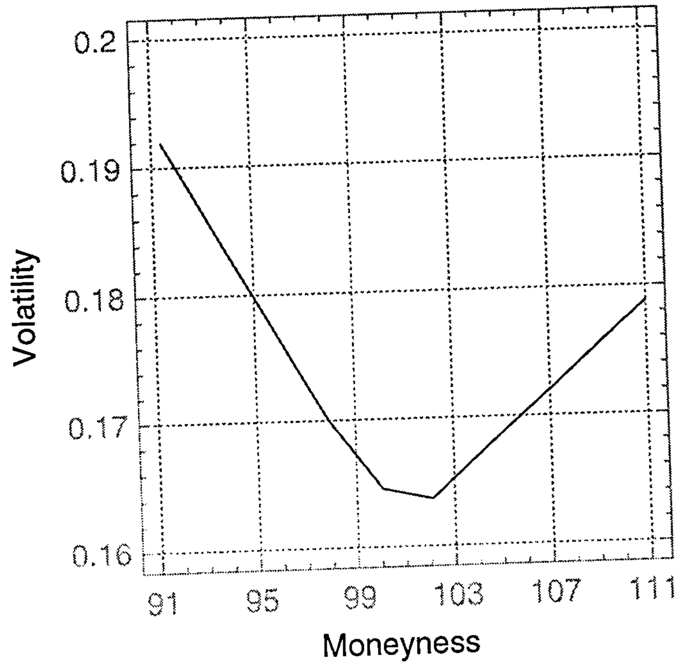
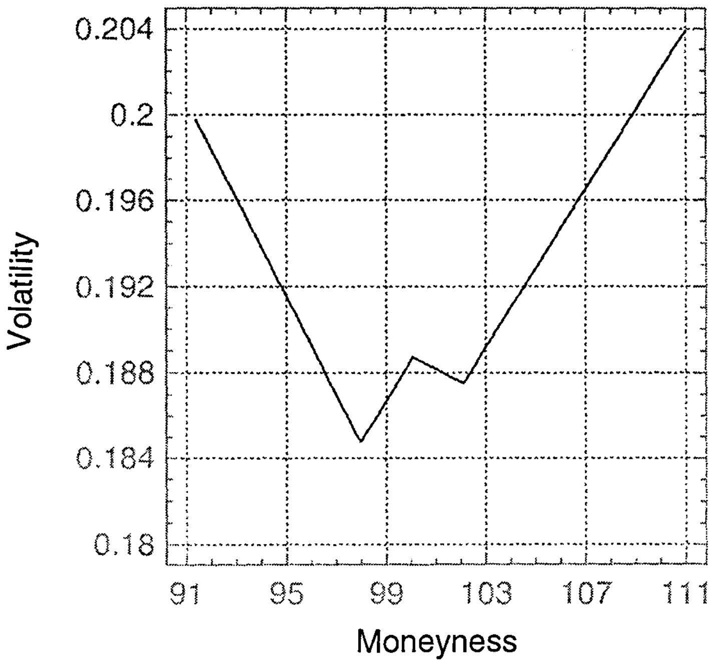


FIGURE 3:
AVERAGE SMILES OVER THE SAMPLE PERIOD FOR CALLS AND PUTS. JANUARY 1994–APRIL 1996

SMILE CALLS



SMILE PUTS

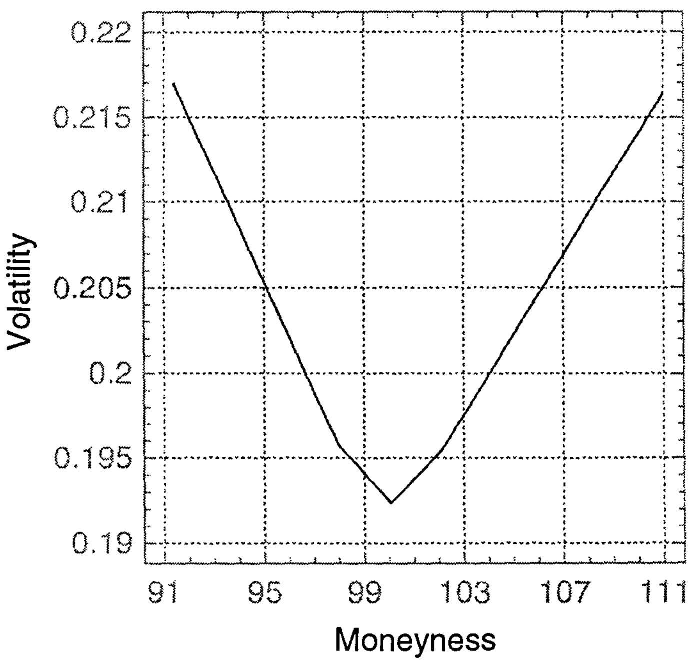


TABLE 1
HOW DO WE SMILE?
ADJUSTED R^2 WEIGHTED BY THE NUMBER OF OBSERVATIONS AVAILABLE FOR EACH DAY WITHIN EACH OF 5 ALTERNATIVE MODELS AND DIFFERENT PERIODS

The five models are the following:

$$\begin{aligned} Model\ \ 2:\sigma &= b_0 + b_1 X + \epsilon \\ Model\ \ 3:\sigma &= b_0 + b_1 X + b_2 X^2 + \epsilon \\ Model\ \ 4:\sigma &= b_0 + b_1 U + b_2 D^2 + \epsilon \\ Model\ \ 5:\sigma &= b_0 + b_1 U + b_2 X^2 + \epsilon \\ Model\ \ 6:\sigma &= b_0 + b_1 U + b_2 X^2 + b_3 D + \epsilon \end{aligned}$$

These regressions are run from January 1994 to April 1996, where X is equal to the exercise price divided by the underlying asset value (K/F), and U and D capture asymmetries in the smile. All calls and puts over the 45 minutes interval from 16:00 to 16:45 are employed in the estimation.

PERIODS	MOD2	MOD3	MOD4	MOD5	MOD6
FULL SAMPLE PERIOD January 1994–April 1996	0.4802	$\boxed{0.6293}$	0.4211	0.4865	0.6257
FIRST SUBPERIOD January 1994–February 1995	0.4640	0.5889	0.4223	0.4728	$\boxed{0.6161}$
SECOND SUBPERIOD March 1995–April 1996	0.4973	$\boxed{0.6727}$	0.4198	0.5010	0.6635
FIRST QUARTER	0.5861	0.7232	0.5495	0.5960	0.7276
SECOND QUARTER	0.4027	0.5686	0.3486	0.4029	0.5154
THIRD QUARTER	0.5989	0.7169	0.5984	0.6468	0.7748
FOURTH QUARTER	0.3106	0.3187	0.2588	0.3075	0.3936
FIFTH QUARTER	0.3804	0.5660	0.2674	0.3345	0.4886
SIXTH QUARTER	0.3618	0.5221	0.4021	0.4696	0.5470
SEVENTH QUARTER	0.4259	0.5845	0.2967	0.3793	0.5711
EIGHTH QUARTER	0.4526	0.7035	0.3340	0.4114	0.7104
NINETH QUARTER	0.6879	0.8324	0.6154	0.7063	0.8168
MONDAYS ONLY	0.5218	$\boxed{0.6152}$	0.4528	0.5096	0.5967
REST OF WEEK DAYS	0.4700	$\boxed{0.6327}$	0.4132	0.4807	0.6321

TABLE 2
SMILE DAILY SEASONALITY

A quadratic model relating implied volatility and moneyness is fitted from January 1994 to April 1996: $\sigma = b_0 + b_1 X + b_2 X^2$, where $X = K/F$, the exercise price divided by the underlying asset value. All calls and puts over the 45 minutes interval from 16:00 to 16:45 are employed in the estimation. The reported coefficients are the mean of the daily estimates of b_i; $i = 0, 1, 2$. The following regression is run:

$$b_{it} = \beta_{MO} MON_t + \beta_{TU} TUE_t + \beta_{WE} WED_t + \beta_{TH} THU_t + \beta_{FR} FRI_t + \epsilon_t$$

where MON, TUE, WED, THU and FRI are dummy variables for Monday, through Friday. Newey–West robust standard errors with five lags are employed (t–statistics in parenthesis). The use of a consistent covariance matrix implies that the test statistic under the null follows asymptotically a χ^2 distribution.

DAYS	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	*NO.OBS.*
MONDAY	3.428	−6.220	2.982	81
	(5.48)	(−5.01)	(4.84)	
TUESDAY	5.167	−9.698	4.722	94
	(3.14)	(−2.96)	(2.90)	
WEDNESDAY	4.951	−9.297	4.537	87
	(5.66)	(−5.34)	(5.23)	
THURSDAY	6.060	−11.48	5.613	92
	(5.93)	(−5.65)	(5.56)	
FRIDAY	7.062	−13.48	6.608	92
	(5.94)	(−5.67)	(5.57)	
ALL DAYS	5.384	−10.14	4.943	446
	(9.65)	(−9.12)	(8.93)	

Test statistics for the term b_0:
$\chi^2(1)\{\beta_{MO} = (\beta_{TU} + \beta_{WE} + \beta_{TH} + \beta_{FR})/4\} = 14.490; \quad p - value = 0.00014$

$\chi^2(4)\{\beta_{MO} = \beta_{TU} = \beta_{WE} = \beta_{TH} = \beta_{FR}\} = 15.981; \quad p - value = 0.00304$

Test statistics for the term b_1:
$\chi^2(1)\{\beta_{MO} = (\beta_{TU} + \beta_{WE} + \beta_{TH} + \beta_{FR})/4\}$=14.680; p-value=0.00013

$\chi^2(4)\{\beta_{MO} = \beta_{TU} = \beta_{WE} = \beta_{TH} = \beta_{FR}\} = 16.002; \quad p - value = 0.00302$

Test statistics for the term b_2:
$\chi^2(1)\{\beta_{MO} = (\beta_{TU} + \beta_{WE} + \beta_{TH} + \beta_{FR})/4\} = 15.016; \quad p - value = 0.00011$

$\chi^2(4)\{\beta_{MO} = \beta_{TU} = \beta_{WE} = \beta_{TH} = \beta_{FR}\} = 16.093; \quad p - value = 0.00290$

TABLE 3
SMILE DAILY SEASONALITY: MONDAY VS. THE REST OF THE WEEK

A quadratic model relating implied volatility and moneyness is fitted from January 1994 to April 1996: $\sigma = b_0 + b_1 X + b_2 X^2$, where $X = K/F$, the exercise price divided by the underlying asset value. All calls and puts over the 45 minutes interval from 16:00 to 16:45 are employed in the estimation. The reported coefficients are the mean of the daily estimates of $b_i = 0, 1, 2$. The results are estimated by running the following regression:

$$b_{it} = \beta_1 + \beta_2 TUE_t + \beta_3 WED_t + \beta_4 THU_t + \beta_5 FRI_t + \epsilon_t$$

where TUE, WED, THU and FRI are dummy variables for Tuesday through Friday. The estimate of β_1 is the sample mean for Monday, while the estimates of the remaining coefficients are equal to the difference between the sample mean for each day and the sample mean for Monday. The reported figures in the last line are obtained by running the following regression:

$$b_{it} = \beta_1^* + \beta_2^* MON_t + \epsilon_t$$

where β_1^* is the sample mean for all days except Monday, and β_2^* is the difference between Monday and the rest of the week[1]. Newey–West robust standard errors with five lags are employed (t–statistics in parenthesis).

DAYS	b_0	b_1	b_2
MONDAY	3.428	−6.220	2.982
	(5.48)	(−5.01)	(4.84)
TUE–MON	1.1740	−3.478	1.739
	(1.01)	(−1.01)	(1.01)
WED–MON	1.523	−3.077	1.555
	(1.55)	(−1.57)	(1.60)
THU–MON	2.632	−5.258	2.631
	(2.29)	(−2.31)	(2.33)
FRI–MON	3.634	−7.259	3.626
	(2.84)	(−2.84)	(2.84)
MON–REST WEEK	−2.391	4.784	−2.396
	(−2.75)	(2.76)	(−2.78)

[1]Monthly seasonality was also investigated. August seems to be a month with a higher slope and a more pronounced curvature. However, the differences are not statistically significant.

TABLE 4
SMILE DAILY SEASONALITY: THE PRINCIPAL COMPONENT APPROACH

A quadratic model relating implied volatility and moneyness is fitted from January 1994 to April 1996: $\sigma = b_0 + b_1 X + b_2 X^2$, where $X = K/F$, the exercise price divided by the underlying asset value. All calls and puts over the 45 minutes interval from 16:00 to 16:45 are employed in the estimation. We have 446 daily observations available. A 446x3 matrix of coefficients, b_0, b_1 and b_2, is formed and its first principal component is estimated.The reported coefficients are the mean of the daily estimates of the principal component, pc_0. The following regression is run:

$$pc_{0t} = \beta_{MO} MON_t + \beta_{TU} TUE_t + \beta_{WE} WED_t + \beta_{TH} THU_t + \beta_{FR} FRI_t + \epsilon_t$$

where MON, TUE, WED, THU and FRI are dummy variables for Monday, through Friday. Newey–West robust standard errors with five lags are employed (t–statistics in parenthesis). The use of a consistent covariance matrix implies that the test statistic under the null follows asymptotically a χ^2 distribution.

DAYS	PRINCIPAL COMPONENT	NO. OBS.
MONDAY	−4.796	81
	(−3.15)	
TUESDAY	−0.536	94
	(−0.13)	
WEDNESDAY	−1.027	87
	(−0.48)	
THURSDAY	1.646	92
	(0.66)	
FRIDAY	4.095	92
	(1.41)	

Test statistics for the principal component pc_0:
$\chi^2(1)\{\beta_{MO} = (\beta_{TU} + \beta_{WE} + \beta_{TH} + \beta_{FR})/4\} = 34.792; \quad p - value = 0.00000$
$\chi^2(4)\{\beta_{MO} = \beta_{TU} = \beta_{WE} = \beta_{TH} = \beta_{FR}\} = 16.016; \quad p - value = 0.002997$

TABLE 5
SMILE DAILY SEASONALITY AND THE PRINCIPAL COMPONENT APPROACH: MONDAY VS. THE REST OF THE WEEK

A quadratic model relating implied volatility and moneyness is fitted from January 1994 to April 1996: $\sigma = b_0 + b_1 X + b_2 X^2$, where $X = K/F$, the exercise price divided by the underlying asset value. All calls and puts over the 45 minutes interval from 16:00 to 16:45 are employed in the estimation. A 446x3 matrix of coefficients, b_0, b_1 and b_2, is formed and its first principal component estimated.The reported coefficients are the mean of the daily estimates of the principal component, pc_{ot}. The following regression is run:

$$pc_{0t} = \beta_1 + \beta_2 TUE_t + \beta_3 WED_t + \beta_4 THU_t + \beta_5 FRI_t + \epsilon_t$$

where TUE, WED, THU and FRI are dummy variables for Tuesday through Friday. The estimate of β_1 is the sample mean for Monday, while the estimates of the remaining coefficients are equal to the difference between the sample mean for each day and the sample mean for Monday. The reported figures in the last line are obtained by running the following regression:

$$pc_{0t} = \beta_1^* + \beta_2^* MON_t + \epsilon_t$$

where β_1^* is the sample mean for all days except Monday, and β_2^* is the difference between Monday and the rest of the week[1]. Newey–West robust standard errors with five lags are employed (t–statistics in parenthesis).

DAYS	PRINCIPAL COMPONENT
MONDAY	−4.976
	(−3.15)
TUE–MON	4.260
	(1.01)
WED–MON	3.769
	(1.58)
THU–MON	6.442
	(2.31)
FRI–MON	8.891
	(2.84)
REST OF WEEK	1.064
	(0.61)
MON–REST OF WEEK	−5.860
	(−2.77)

[1] We also tested whether the average coefficient was different than the average coefficient for the rest of week assuming that the variance during the rest of week was three times the variance for Monday. This is known as the Behrens–Fisher problem in Statistics. The result was again significant. Monthly seasonality was also investigated. August seems to be a month with a higher average principal component. However, the difference with other months is not statistically significant.

TABLE 6
TESTS FOR UNIT ROOTS

The following (augmented) DF models are run with daily data from January 1994 to April 1996:

$$\Delta X_t = a + (\rho_1 - 1)X_{t-1} + \sum_{\tau=1}^{K} \delta_\tau X_{t-\tau} + v_t \tag{1}$$

$$\Delta X_t = a + bt + (\rho_1 - 1)X_{t-1} + \sum_{\tau=1}^{K} \delta_\tau X_{t-\tau} + v_t \tag{2}$$

where X is the variable for which the test is performed, and the number of lags K is previously determined in each case for each variable using the Akaike information criterion. The test statistic is given by: $(\hat{\rho}_1 - 1)/SE(\hat{\rho}_1)$. The empirical cumulative distribution is tabulated by Dickey and Fuller. The variables analized are the following: MKT is the logarithm of the ratio of the previous short-run level of the IBEX (three-month moving average) to its current level, $SIGMA$ is the annualized standard deviation of the IBEX during each day in the sample estimated with minute by minute observations, $VMKT$ is the logarithm of the number of shares traded by the components of the IBEX calculated during the 45–minute interval, RTB is the log relative (with respect to its three–month moving average) treasury bill rate, BA is the daily average relative bid–ask for the options available during the 45–minute interval, $VOPT$ is the logarithm of the number of option contracts negotiated during the 45–minute interval, and PC is the first principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time

VARIABLES	DF MODEL (1)	DF MODEL (2)
MKT	−3.027	−3.462
SIGMA	−8.854	−9.133
VMKT	−5.820	−5.835
RTB	−1.289*	−1.108*
BA	−10.373	−10.511
VOPT	−5.932	−7.506
PC	−10.509	−10.535

* We cannot reject the existence of a unit root. At the 0.05 level and for 500 observations, the critical values given by Dickey and Fuller for models (1) and (2) are -2.87 and -3.42 respectively.

TABLE 7
THE DETERMINANTS OF THE IMPLIED VOLATILITY FUNCTION

A quadratic model relating implied volatility and moneyness is fitted from January 1994 to April 1996: $\sigma = b_0 + b_1 X + b_2 X^2$, where $X = K/F$, the exercise price divided by the underlying asset value. All calls and puts over the 45-minute interval from 16:00 to 16:45 are employed in the estimation. We have 446 daily observations available. A 446x3 matrix of coefficients, b_0, b_1 and b_2, is formed and its first principal component is estimated. Time-series regressions are run to explain the variability of the principal component and the coefficients which characterized the volatility smile. The following regressions are run:

$$pc_{0t}(bit) = \beta_0 + \beta_1 MON_t + \beta_2 MKT_t + \beta_3 SIGMA_{t-1} + \beta_4 VMKT_{t-1} + \beta_5 DRTB_t +$$
$$+ \beta_6 BA_t + \beta_7 VOPT_{t-1} + \beta_8 TIME_t + \epsilon_t$$

where MON is a dummy variable for Monday, MKT is the logarithm of the ratio of the previous short-run level of the IBEX (three-month moving average) to its current level, $SIGMA$ is the annualized standard deviation of the IBEX for each day in the sample estimated by minute by minute observations, $VMKT$ is the logarithm of the number of shares traded by the componentes of the IBEX calculated during the 45-minute interval, $DRTB$ is the first daily differences of the log relative (with respect to its three-month moving average) treasury bill rate, BA is the daily average relative bid-ask for the options available during the 45-minute interval, $VOPT$ is the logarithm of the number of option contracts negotiated during the 45-minute interval, and $TIME$ is the annualized number of days to expiration of the options available in the sample. Newey-West robust standard errors are employed (t-statistics in parenthesis). The reported coefficients are the estimated coefficients divided by 100. The (average) adjusted R^2 is 0.19.

COEFFICIENTS	PRINCIPAL COMPONENT	SMILE INTERCEPT (b_0)	SMILE SLOPE (b_1)	SMILE CURVATURE (b_2)
INTERCEPT	−0.063	0.025	−0.050	0.026
	(−0.34)	(0.33)	(−0.33)	(0.35)
MONDAY(t)	−0.027	−0.011	0.022	−0.011
	(−1.34)	(−1.34)	(1.33)	(−1.34)
MKT(t)	−0.465	−0.188	0.382	−0.188
	(−1.76)	(−1.73)	(1.76)	(−1.75)
SIGMA(t-1)	−0.406	−0.165	0.331	−0.166
	(−2.18)	(−2.18)	(2.18)	(−2.17)
VMKT(t-1)	0.027	0.011	−0.022	0.011
	(1.12)	(1.15)	(−1.12)	(1.09)
DRTB(t)	−1.364	−0.555	1.115	−0.558
	(−1.29)	(−1.28)	(1.29)	(−1.29)
BA(t)	0.302	0.123	−0.246	0.124
	(2.36)	(2.35)	(−2.36)	(2.38)
VOPT(t-1)	0.003	0.001	−0.003	0.001
	(0.43)	(0.49)	(−0.43)	(0.38)
TIME(t)	−0.033	−0.014	0.027	−0.013
	(−6.10)	(−6.10)	(6.10)	(−6.09)

TABLE 8
LINEAR GRANGER CAUSALITY TEST RESULTS BETWEEN THE RELATIVE BID–ASK SPREAD AND THE CHARACTERISTICS OF THE VOLATILITY SMILE: JANUARY 1994-APRIL 1996

This table reports the results of the linear Granger causality test based on the following bivariate VAR model:

$$pc_{0t} = \alpha + A_{11}(L)pc_{0t} + A_{12}(L)BA_t + U_t$$

$$BA_t = \beta + A_{21}(L)pc_{0t} + A_{22}(L)BA_t + W_t$$

where BA is the daily average relative bid–ask for the options available during the 45–minute interval, and pc_0 is the first principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time. Similar analysis are performed with respect to the intercept (b_0), the slope (b_1) and the curvature (b_2) of the smile. The results are based on exclusion tests relative to an $F(q, T-K)$ where q is the number of excluded (lags) variables and $T-K$ is the number of observations minus the number of independent variables. Hence, the p-value denotes the marginal significance level of the computed F-statistic used to test the zero restrictions implied by the null hypothesis of Granger noncausality. All lag lengths are set on the basis of four alternative criteria: the Akaike information criterion, the Schwarz specification test, the Final Prediction Error Criterion, and the Hannan–Quinn test. Similar results across all criteria are generally obtained; when conflicts are found, the Akaike test is employed.

H_0: BA does not cause PC_0			H_0: PC_0 does not cause BA		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	3.442	0.018	3	9.145	0.000
H_0: BA does not cause instantaneously PC_0			H_0: PC_0 does not cause instantaneously BA		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	2.841	0.025	3	5.206	0.001
H_0: BA does not cause the intercept (b_0)			H_0: The intercept (b_0) does not cause BA		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	3.448	0.017	3	9.114	0.000
H_0: BA does not cause the slope (b_1)			H_0: The slope (b_1) does not cause BA		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	3.436	0.018	3	9.143	0.000
H_0: BA does not cause the curvature (b_2)			H_0: The curvature (b_2) does not cause BA		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	3.455	0.017	3	9.180	0.000

TABLE 9
LINEAR GRANGER CAUSALITY TEST RESULTS BETWEEN CHARACTERISTICS OF THE UNDERLYING STOCK INDEX, MARKET CONDITIONS AND OPTION ACTIVITY, AND THE VOLATILITY SMILE: JANUARY 1994-APRIL 1996

This table reports the results of the linear Granger causality test based on the following bivariate VAR model:

$$pc_{0t} = \alpha + A_{11}(L)pc_{0t} + A_{12}(L)DET_t + Ut$$
$$DET_t = \beta + A_{21}(L)pc_{0t} + A_{22}(L)DET_t + W_t$$

where DET represents the following variables: MKT is the logarithm of the ratio of the previous short–run level of the IBEX (three–month moving average) to its current level, $SIGMA$ is the annualized standard deviation of the IBEX for each day in the sample estimated by minute by minute observations, $VMKT$ is the logarithm of the number of shares traded by the components of the IBEX calculated during the 45–minute interval, $DRTB$ is the first daily differences of the log relative (with respect to its three–month moving average) treasury bill rate, $VOPT$ is the logarithm of the number of option contracts negotiated during the 45–minute interval, and pc_0 is the first principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time. The results are based on exclusion tests relative to an $F(q, T-K)$ where q is the number of excluded (lags) variables and $T-K$ is the number of observations minus the number of independent variables. Hence, the p–value denotes the marginal significance level of the computed F–statistic used to test the zero restrictions implied by the null hypothesis of Granger noncausality. All lag lengths are set on the basis of four alternative criteria: the Akaike information criterion, the Schwarz specification test, the Final Prediction Error Criterion, and the Hannan–Quinn test. Similar results across all criteria are generally obtained; when conflicts are found, the Akaike test is employed.

H_0: MKT does not cause PC_0			H_0: PC_0 does not cause MKT		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
2	0.498	0.608	2	0.673	0.511
H_0: $SIGMA$ does not cause PC_0			**H_0: PC_0 does not cause $SIGMA$**		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
4	0.984	0.417	4	0.589	0.671
H_0: $VMKT$ does not cause PC_0			**H_0: PC_0 does not cause $VMKT$**		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
2	0.238	0.788	2	0.294	0.746
H_0: $DRTB$ does not cause PC_0			**H_0: PC_0 does not cause $DRTB$**		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	1.390	0.247	3	2.063	0.105
H_0: $VOPT$ does not cause PC_0			**H_0: PC_0 does not cause $VOPT$**		
Lengh	Stat–F	p–value	Lengh	Stat–F	p–value
3	1.434	0.233	3	2.165	0.092

TABLE 10
AVERAGE TRANSACTION COSTS ACROSS FIVE FIXED INTERVALS FOR THE DEGREE OF MONEYNESS: JANUARY 1994-APRIL 1996

This table reports the average transaction costs (relative bid–ask spreads) across five fixed intervals for the degree of moneyness. We define moneyness as the ratio between the exercise price level and the average of the future price relative to each average implied volatility.

CLASIFICATION	MONEYNESS	RELATIVE BID-ASK SPREAD
Deep OMP (IMC)*	0.8598 – 0.9682	0.3333
OMP (IMC)	0.9682 – 0.9913	0.1946
AMP (AMC)	0.9913 – 1.0101	0.1616
IMP (OMC)	1.0101 – 1.0321	0.2166
Deep IMP (OMC)	1.0321 – 1.1875	0.3668

*OMP is out–of–the–money puts; IMC is in–the–money calls; AMP (AMC) is at–the–money puts (calls); IMP is in–the–money puts; OMC is out–the–money calls.

TABLE 11
NONLINEAR GRANGER CAUSALITY TEST RESULTS BETWEEN THE OPTION MARKET CHARACTERISTICS AND THE VOLATILITY SMILE: JANUARY 1994–APRIL 1996

This table reports the results of the modified Baek and Brock nonlinear Granger causality test applied to the VAR residuals corresponding to the principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time and the relative bid–ask spread (and number of option contracts negotiated) of the options available in the sample. BA is the daily average relative bid–ask for the options available during the 45–minute interval employed in the analysis, and $VOPT$ is the logarithm of the number of option contracts negotiated during the 45–minute interval. $L_u = L_w$ denotes the number of lags on the residuals series used in the test. In all cases reported below, the tests are applied to unconditionally standardized series, the lead length, m, is set to unity, and the length scale, δ, is set to either 1.5 or 0.5. DIF and $STAT$, respectively, denote the difference between the two ratios of joint probabilities of the Baek and Brock nonlinear test in equation (12) and the standardized tests statistic (the modified Baek–Brock test) in equation (14). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0,1)$. The test statistic should be evaluated with right–tailed critical values.

PANEL A: $\delta = 1.5$

	H_0: BA does not cause pc_0		H_0: $VOPT$ does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00036	1.0106	0.00068	1.7315
2	0.00091	1.4665	0.00062	1.3888
3	0.00103	1.4939	0.00038	0.7962
4	0.00122	1.5878	0.00065	1.0426

PANEL B: $\delta = 0.5$

	H_0: BA does not cause pc_0		H_0: $VOPT$ does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00484	2.7882	0.00553	2.8246
2	0.00582	2.9094	0.00582	2.5921
3	0.00439	2.4558	0.00514	2.1477
4	0.00495	2.6637	0.00619	1.9667

TABLE 12
NONLINEAR GRANGER CAUSALITY TEST RESULTS BETWEEN THE UNDERLYING INDEX CHARACTERISTICS AND THE VOLATILITY SMILE: JANUARY 1994–APRIL 1996

This table reports the results of the modified Baek and Brock nonlinear Granger causality test applied to the VAR residuals corresponding to the principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time and the annualized volatilty (and number of shares negotiated). *SIGMA* is the annualized standard deviation of the IBEX for each day in the sample estimated by minute by minute observations, and *VMKT* is the logarithm of the number of shares traded by the components of the IBEX calculated during the 45–minute interval. $L_u = L_w$ denotes the number of lags on the residuals series used in the test. In all cases reported below, the tests are applied to unconditionally standardized series, the lead length, m, is set to unity, and the length scale, δ, is set to either 1.5 or 0.5. *DIF* and *STAT*, respectively, denote the difference between the two ratios of joint probabilities of the Baek and Brock nonlinear test in equation (12) and the standardized tests statistic (the modified Baek–Brock test) in equation (14). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0,1)$. The test statistic should be evaluated with right–tailed critical values.

PANEL A: $\delta = 1.5$

	H_0: *SIGMA* does not cause pc_0		H_0: *VMKT* does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00099	1.4967	0.00082	0.7990
2	0.00191	1.5446	0.00089	0.7390
3	0.00202	1.6389	0.00083	0.7367
4	0.00213	1.6747	0.00103	0.7958

PANEL B: $\delta = 0.5$

	H_0: *SIGMA* does not cause pc_0		H_0: *VMKT* does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00686	3.0978	0.00773	2.9144
2	0.00761	1.8324	0.00710	2.2880
3	0.00733	1.7750	0.00771	2.6617
4	0.00657	1.5366	0.00752	2.2336

TABLE 13
NONLINEAR GRANGER CAUSALITY TEST RESULTS BETWEEN MARKET CONDITIONS AND THE VOLATILITY SMILE: JANUARY 1994–APRIL 1996

This table reports the results of the modified Baek and Brock nonlinear Granger causality test applied to the VAR residuals corresponding to the principal component of the 446x3 matrix of coefficients b_0, b_1 and b_2 characterizing the smile over time and the relative index level (and changes in relative interest rate levels). MKT is the logarithm of the ratio of the previous short–run level of the IBEX (three–month moving average) to its current level, and $DRTB$ is the first daily differences of the log relative (with respect to its three–month moving average) treasury bill rate. $L_u = L_w$ denotes the number of lags on the residuals series used in the test. In all cases reported below, the tests are applied to unconditionally standardized series, the lead length, m, is set to unity, and the length scale, δ, is set to either 1.5 or 0.5. DIF and $STAT$, respectively, denote the difference between the two ratios of joint probabilities of the Baek and Brock nonlinear test in equation (12) and the standardized tests statistic (the modified Baek–Brock test) in equation (14). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0, 1)$. The test statistic should be evaluated with right–tailed critical values.

PANEL A: $\delta = 1.5$

	H_0: MKT does not cause pc_0		H_0: $DRTB$ does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00180	1.6067	0.00054	1.4509
2	0.00297	1.9904	0.00090	1.6447
3	0.00311	1.8869	0.00095	1.3978
4	0.00363	2.0384	0.00114	1.4861

PANEL B: $\delta = 0.5$

	H_0: MKT does not cause pc_0		H_0: $DRTB$ does not cause pc_0	
$L_u = L_w$	DIF	STAT	DIF	STAT
1	0.00920	3.3671	0.00657	2.8353
2	0.00962	2.6405	0.00702	2.0536
3	0.00920	2.2795	0.00572	1.3509
4	0.00964	2.2093	0.00679	1.4376

WORKING PAPERS 1997

Business Economics Series

97-18 (01) Margarita Samartín
"Optimal allocation of interest rate risk"

97-23 (02) Felipe Aparicio and Javier Estrada
"Empirical distributions of stock returns: european securities markets, 1990-95"

97-24 (03) Javier Estrada
"Random walks and the temporal dimension of risk"

97-29 (04) Margarita Samartín
"A model for financial intermediation and public intervention"

97-30 (05) Clara-Eugenia García
"Competing trough marketing adoption: a comparative study of insurance companies in Belgium and Spain"

97-31 (06) Juan-Pedro Gómez and Fernando Zapatero
"The role of institutional investors in international trading: an explanation of the home bias puzzle"

97-32 (07) Isabel Gutiérrez, Manuel Núñez Niekel and Luis R. Gómez-Mejía
"Executive transitions, firm performance, organizational survival and the nature of the principal-agent contract"

97-52 (08) Teresa García and Carlos Ocaña
"The role of banks in relaxing financial constraints: some evidence on the investment behavior of spanish firms"

97-59 (09) Rosa Rodríguez, Fernando Restoy and Ignacio Peña
"A general equilibrium approach to the stock returns and real activity relationship"

97-75 (10) Josep Tribo
"Long-term and short-term labor contracts versus long-term and short-term debt financial contracts"

97-79 (11) Sandro Brusco
"Perfect bayesian implementation: one round of signaling is not enough"

97-80 (12) Sandeep Baliga and Sandro Brusco
"Collusion, renegotiation and implementation"

97-81 (13) Sandro Brusco
"Unique implementation of the full surplus extraction outcome in auctions with correlated types"

97-82 (14) Sandro Brusco and Matthew O. Jackson
"The optimal design of a market"