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MODELLING ADAPTIVE COMPLEX BEHAVIOUR WITH AN APPLICATION TO THE STOCK MARKET DYNAMICS

(El Farol Problem Revisited)

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Abstract

In this paper we review a simple agent-based model of adaptive complex behaviour that shows how the interaction of different agent's profit-oriented decisions leads to a wide spectra of organizational possibilities. We comment on some potential applications of this model to the social and life sciences, and later focus on the modelling of the stock market dynamics. We show how some of the features of stock price series, and in particular extreme events such as speculative bubbles and crashes, can be obtained when certain conditions are satisfied by most of the investors' preferences.

Key Words

Nonlinear time series; chaos; complexity; bounded rationality; crashes; bubbles; stock market dynamics.

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1 Introduction

The modelling of *complex nonlinear systems* is a comparatively recent area of research which welcomes the concerted efforts and knowledge of many different disciplines (see for instance, West, 1985, Haken and Mikhailov, 1993; Green and Bossomaier, 1993, Verhulst, 1994; Horgan, 1995, Kauffman, 1995, Bar-Yam, 1997, and Casti, 1997, for a few surveys on the potential applications). Its purpose is the building of mathematical models of cooperative phenomena that are abstractions derived from various branches of science.

Complex systems may differ substantially in certain properties, but they all share a common feature : they all function in a coherent and potentially predictable way while being composed of a considerably large number of interacting and adaptive units. For example, the natural systems of the life sciences are the result of a long-evolutionary selection which has perfected their inner organization. By means of the interaction of these units, new properties may emerge that are absent at the individual level.

There is a widespread interest in the social sciences in developing models of social structure derived from processes of rational choice and the aggregation of individual strategic decisions. An example of one such process, which is representative of a broad class of systems, arises as follows. Suppose a medium-size number, n, of individuals[1] that are willing to visit a place periodically if it is not too crowded. More precisely, the ith individual will independently decide to visit the place at time t if he (she) expects no more than $\theta_{i,t}$ visitors at that time. Notice that it is impossible to be sure in advance of the number of people coming to the place.

The previous problem was introduced by Brian (1994), and was baptised as *"El Farol" problem* to remind its original source of inspiration. "El Farol" is a sort of bar-restaurant in Santa Fe where people gather on Thursday nights to hear good Irish music. The problem arises when most of these people would not attend the bar in the midst of a loudy crowd. That is, each prospective visitor has to decide on whether visiting or not the pub on the next Thursday evening, based on his (her) own forecasts of the next attendance figure, and knowing that it will also depend on the forecasts the others made. In order to make these forecasts, every person constructs an optimal predictor that takes into account his (her) own preferences as regards the maximum number of attendants

[1]The reason why the number of agents, n, is considered to be of medium size is because too small values lead to problems which can be conveniently approached using game-theoretic analysis methods, while too large values need to be treated statistically. By a medium number of agents, we mean that n is neither too small nor too big, but still creates complex patterns of emergent behavior.

that he (she) can tolerate. Once all the individual decisions are made, people converge on the bar. As soon as the latest attendance figure is known during the week, the agents adapt their own prediction rules to the new piece of information, and the process continues for another round. As remarked by Brian, this process leads to self-invalidating expectations and thus to the failure of deductive logic. And the reason is that agents are not *perfectly rational.* To explain, agents cannot foresee either the logical implications of their actions or the reactions of the other agents, or both. As a consequence, they are thrown into the realm of inductive inference.

The key features of "El Farol" are ubiquitous in any problem involving intelligent decision-making, many of which arise in the social and behavioral sciences. The purpose of this paper is to discuss some of the properties of this model of macrobehavior derived from the aggregation of a not too large number of intelligent decision-making units and assumptions about their microbehavior. The paper is structured as follows. In section 2, we present the recursive model for the time series of attendances built upon the aggregation of decision variables, and in section 3, we discuss different features of the model as well as potential applications in the social and the life sciences. Section 4 focusses on an application to the modelling of the *stock market dynamics.* In particular, we show how unusual events such as *crashes* and *speculative bubbles*, usually attributed to incidentals and unexplained by models of *rational expectations*, can be generated with this model when some conditions are satisfied by most investors' preferences. In section 5, we present some simulation experiments to illustrate some of the features of the time series produced with the model. Finally, section 6 is devoted to the conclusions.

2 The model

Suppose that our potential visitors to "El Farol" (here also referred to, indiferently, as *agents* or as *economic units*) follow generally different strategies embodied in a particular *transition probability* from the state "out" (of the place) to the state "in" (the place), say from state "0" to state "1". Each agent's transition probability will depend on a (generally different) history of the series of attendances. We will assume that this series, say x_t, is obtained by regular sampling (the potential visitors only plan to visit the place each thursday evening) of the *discrete process* of attendances, say $\{X_t\}_{t \geq 0}$. To begin, let $p_{i,t}$ be the ith individual transition probability, which embraces the information on $\theta_{i,t}$. Assume also that, in general, agents do not have access to what other agents are doing or planning to do, when trying to define their own optimal decision rules at any time instant t. This means that the i-th agent will generally base his (her) decisions at t independently of the decisions of the other agents, and thereby will use a generally different predictor for X_t, say $\widehat{X}_{i,t}$. This predictor will combine past information in either a linear or a nonlinear way. That is, we may write $\widehat{X}_{i,t} = W_i(\mathbf{X}_{i,t-1})$, where $\mathbf{X}_{i,t-1} = (X_{t-1}, ..., X_{t-p_i})'$ represents the i-th visitor's *state vector*, which summarizes the useful or the available past history of the series for this agent, while W_i represents the functional form of his (her) forecasting rule. Notice that in the linear prediction case this rule reduces to $W_i(\mathbf{X}_{i,t-1}) = \mathbf{a}_i'\mathbf{X}_{i,t-1}$, where $\mathbf{a}_i = (a_{i,1}, ..., a_{i,p_i})'$ is a vector of coefficients whose elements and dimension may differ from one agent to the other. The fact that the state vector contains only a portion of the history of the process X_t, and that its length will be generally different for each agent, is because agents may be willing to face different costs in data collection, storage, computation and communication.

Now suppose we can write

$$p_{i,t} = 1 - \mu_{i,t}(\widehat{X}_{i,t}, \theta_{i,t}) \tag{1}$$

where $\mu_{i,t}$ (henceforth referred to as the *abhorrence function* for the ith individual at t) is a function that codes the preferences $\theta_{i,t}$ of the ith individual or his (her) degree of crowd aversion at t into a workable rule. Thus each agent's decision rule will be characterized by a triad $(\widehat{X}_{i,t}, \theta_{i,t}, \mu_{i,t})$, consisting of : (a) the predictor, $\widehat{X}_{i,t}$, which produces a credible hypothesis about the uncertain future at t, (b) the preferences, $\theta_{i,t}$, which is the unobserved psychological parameter determining the agent's attitude at t, and (c) the abhorrence function, $\mu_{i,t}$, which formalizes the degree of adherence of the agent towards his (her) preferences at this same instant t. Clearly, we must have $0 \leq |\mu_{i,t}(x, \theta_{i,t})| \leq 1, \forall x, t$.

The agents' preferences could be more or less noisy. This noise is introduced to account for the vagueness of these preferences. At one extreme, agents' may have no preference for a particular value of $\theta_{i,t}$. This amounts at having purely random preferences, that is

$$\theta_{i,t} = \varepsilon_{i,t}, \tag{2}$$

where for each i, $\{\varepsilon_{i,t}\}_t$ (the state noise), is a nonnegative sequence of random variables which, assuming second order stationarity, will have a variance that depends positively on the degree of vagueness about the preferences. At the other extreme, if agents are pretty sure of their needs, their preferences will follow pure deterministically evolving patterns, say

$$\theta_{i,t} = \delta_{i,t}, \tag{3}$$

where for each i, $\delta_{i,t}$ is a deterministic function of t. It is reasonable to assume that, in general, we will have a mixture of these two extreme models.

Similarly, the abhorrence function $\mu_{i,t}$ may transduce these preferences with varying degrees of fuzziness. This fuzziness is introduced to model the limitations of the agent's logical apparatus, or in other words, to model the agent's degree of reliability on his (her) subjective reasoning. For example, if agents have limited confidence on the adequacy of their predictors, they may use a fuzzy abhorrence law such as

$$\mu_{i,t}(x, \theta_{i,t}) = 1 - \exp(-x/\theta_{i,t}), \ x \geq 0. \tag{4}$$

On the opposite, if agents fully trust their predictors, we will have

$$\begin{aligned} \mu_{i,t}(x, \theta_{i,t}) &= \mathbf{1}(x > \theta_{i,t}), \ \forall x, \\ &= \begin{cases} 1, & \text{if } x > \theta_{i,t} \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{5}$$

In principle, agents would be willing to consider using the tools of deductive logic to design their optimal clear-cut decision programs (objective reasoning). However, the imposibility to foresee the consequences of their own actions as well as the actions of the other agents, and the costs of collecting, processing and communicating information (*bounded rationality*), make them realize the utopy behind this formidable task, and force them into inductive inferences. This uncertainty will affect every element of the triad $(\widehat{X}_{i,t}, \theta_{i,t}, \mu_{i,t})$, and will justify the need for : (a) an *ecology* of *evolutionary predictors* $\widehat{X}_{i,t}$ (continuously adapting as new information arrives), (b) *psychological noise* for $\theta_{i,t}$, and (c) *fuzziness* in $\mu_{i,t}$.

The expected number of people at the pub at time t, given the predictor vector $\widehat{\mathbf{X}}_t = (\widehat{X}_{1,t}, ..., \widehat{X}_{n,t})'$ and the vector of preferences $\boldsymbol{\theta}_t = (\theta_{1,t}, ..., \theta_{n,t})'$ at time t is given by

$$E(X_t|\widehat{\mathbf{X}}_t, \boldsymbol{\theta}_t) = \sum_{i=1}^{n} p_{i,t} \tag{6}$$

$$= n - \sum_{i=1}^{n} \mu_{i,t}(\widehat{X}_{i,t}, \theta_{i,t}) \tag{7}$$

$$= F_{\theta_t}(\widehat{\mathbf{X}}_t). \tag{8}$$

Remark that X_t is a random variable which can be expressed as the sum of n generally dependent Bernouilli random variables with possibly different probability distributions. The mutual dependence between the Bernouilli variables here is due to the fact that potential visitors share the past information on the process of attendances for making their decisions. Of course, if these variables were independent and the transition probabilities identical to p, X_t would be a binomial random variable with parameters n and p. However, in the general case, it is unclear how the series of attendances, x_t, will behave dynamically over time. In the light of the previous formulas, it is possible to write our discrete process as

$$X_t = F_{\theta_t}(\widehat{\mathbf{X}}_t) + \xi_t, \tag{9}$$

where, far from the *barriers* at $x = 0$ and $x = n$, the sequence $\{\xi_t\}_t$ behaves as a *martingale difference* sequence (see for instance, Billingsley, 1986, p. 497) with respect to the predictor space $\Im_{\widehat{\mathbf{X}}_t}$ spanned by the component variables in $\widehat{\mathbf{X}}_t$ and given the vector of preferences $\boldsymbol{\theta}_t$ at t. To be explicit, ξ_t is a sequence of random variables verifying

$$E(\xi_t | \Im_{\widehat{\mathbf{X}}_t}, \boldsymbol{\theta}_t) = 0, \tag{10}$$

$$\xi_t + F_{\theta_t}(\widehat{\mathbf{X}}_t) \text{ is an integer in the inteval } [0, n], \ \forall t. \tag{11}$$

Accordingly, the equilibrium error depends on the agents' forecasts of the changes in X_t. Thus both nonlinearity in the mean and in higher conditional moments are possible in the process X_t having barriers at 0 and at n. Remark also that by increasing n, X_t would be free to wander inside a wider range of values and thus the probability that it reaches any of the barriers could be made arbitrarily small.

3 Some features and potential applications of the model

In the model presented in the previous section, the outcome of X_t depends directly on the agents' predictions, thereby allowing the possibility of *self-fulfilling* or *self-invalidating expectations.* In fact, in "El Farol" problem, the current value of the series of attendances will depend negatively on the agents' expectations. Therefore, any common belief for a majority of the agents is always invalidated (if a majority of agents believes most will go, this majority will not go, and if the majority believes most will not go, the majority will go). This imprints a particular feature to the series of attendances, x_t, when the agents do not adapt their predictors immediatley after each new observation of X_t. In this case, x_t will exhibit an oscillatory pattern of behavior, whose amplitude will depend on the common belief of the majority.

As a concrete example, assume $\boldsymbol{\theta}_t = \boldsymbol{\theta} = (\theta, ..., \theta)'$ and $\mu_{i,t} \equiv \mu$, $\forall i, t$. If there was an *equilibrium* in the form of a fixed point, this must satisfy the equation $x = F_{\boldsymbol{\theta}}(x)$, that is

$$\frac{x}{n} = 1 - \mu(x), \text{ for } 0 \leq x \leq n. \tag{12}$$

If the common abhorrence function is $\mu(x) = \mathbf{1}(x > \theta)$, the only fixed points will occur when $\theta > n$ and when $\theta < 0$, and would be $x = n$ and 0, respectively. On the contrary, for $0 \leq \theta \leq n$, the series x_t will oscillate between n and 0.

To show the leading influence of the belief of the majority of agents, suppose again that their abhorrence functions are given by $\mu_{i,t}(x, \theta_{i,t}) = \mathbf{1}(x > \theta_{i,t})$. In this case, the process is deterministic, and one will have

$$\begin{aligned} X_t &= F_{\theta_t}(\widehat{\mathbf{X}}_t) \\ &= n - \sum_{i=1}^{n} \mathbf{1}(\widehat{X}_{i,t} > \theta_{i,t}) \end{aligned} \tag{13}$$

Now, if we denote by $\overline{\mu}_t(\widehat{\mathbf{X}}_t)$ the mean abhorrence function at t accross agents, one has

$$\frac{n - X_t}{n} = \overline{\mu}_t(\widehat{\mathbf{X}}_t). \tag{14}$$

It follows that X_t responds directly to changes in the mean abhorrence regardless of whatever the individual expectations are. Notice that if the predictors (preferences) were fixed and known in advance, it might be possible to estimate non-parametrically $\overline{\mu}_t(x)$ from the time series of attendances, and from this, to track the evolution of the gravity center for the ecology of preferences (predictors).

The fact that when $\mu_{i,t}(x, \theta_{i,t}) = \mathbf{1}(x > \theta_{i,t})$ the dynamics of the process X_t are deterministic should not prevent X_t from exhibiting temporal patterns of varying complexity, depending on how large is n and on the constellation of the component prediction rules in $\widehat{\mathbf{X}}_t$. An interesting case results when the series x_t appears to stabilize around a low-dimensional structure as t grows to infinity. This structure is what in the nonlinear science literature is called an *attractor* (see for instance Granger and Teräsvirta, 1993, p. 53), that is, a subset Ω of $(\Re^+ \cup \{0\})^m$ with the property that if the m-dimensional *state* or *phase-space vector* $\mathbf{X}_t = (X_t, X_{t-1}, ..., X_{t-m+1})'$ belongs to Ω, then $\mathbf{X}_{t+i} = (X_{t+i}, X_{t+i-1}, ..., X_{t+i-m+1})'$ also belongs to Ω, $\forall i \geq 1$. The attractor may be quite simple, such as a stable fixed point in $\Re$, or a set of q unstable fixed points in $\Re$ if x_t has eventually a *limit cycle* of period q. But it could also be so complex as to deserve being qualified as *strange* (after Ruelle and Takens, 1971). Strange attractors may emerge as a result of geometric interaction between the population units (see Hoppensteadt, 1982, p. 7). This sort of interaction is likely in "El Farol" problem, where a large number of visitors at a given time may discourage future attendances to the pub.

If the attractor is the unique stable fixed-point of F_{θ_t}, then this attactor will give us the common optimal predictor of the series when agents are perfectly rational. However, if x_t appears to settle onto a more complex attractor, we cannot conclude that the final agents' predictors are good just because they have simplified the dynamics of X_t. A simpler structure for the series may be very hard to forecast for an agent with no side information about the other agents' decision rules. Thus the asymptotic structure of x_t has, in general, little to do with the performances of the individual predictors. The fact that organized macrobehavior can spring from a mess of microbehaviors, have induced some social researchers to adhere to the philosophical postulate that organizations could conduct their affairs rationally even though individual actors cannot (see for example, Stinchcombe, 1965).

In general, however, the preferences $\theta_{i,t}$ of the potential visitors are contaminated with what we called psychological noise. As we said in the previous section, this component, say $\varepsilon_{i,t}$, somehow quantifies the vagueness or indeterminacy of the agents with respect to their preferences. If we assume a model for $\theta_{i,t}$ that is additive in this perturbation, we may write in general,

$$\theta_{i,t} = \delta_{i,t} + \sigma_{i,t}\varepsilon_{i,t}, \tag{15}$$

where $\delta_{i,t}$ denotes the deterministic part of $\theta_{i,t}$, and $\{\sigma_{i,t}\}_{i=1,n}$ is a family of processes introduced to account for time-varying or *stochastic volatility* (see Harvey, 1993, p. 281), that is, for the randomly or time-varying degree of uncertainty of the agents about the optimality of their inductive inference rules (which may possibly be induced by alternating periods of political or social calm and turmoil, of lower and higher unpredictability of the weather, etc). Under the effect of $\varepsilon_{i,t}$, and even for

$\mu_{i,t}(x, \theta_{i,t}) = \mathbf{1}(x > \theta_{i,t})$, the process X_t becomes stochastic :

$$X_t = n - \sum_{i=1}^{n} \mathbf{1}(X_{t-1} > \delta_{i,t} + \sigma_{i,t}\varepsilon_{i,t}). \tag{16}$$

To show the effect that the presence of this state noise may have on the dynamics of X_t, suppose that in our general model we have $\mu_{i,t} = \mu$, $\forall i, t$, where μ has first-order partial derivatives. We obtain

$$X_t = F_{\theta_t}(\widehat{\mathbf{X}}_t) + \xi_t = n - \sum_{i=1}^{n} \mu(\widehat{X}_{i,t}, \theta_{i,t}) + \xi_t. \tag{17}$$

Further suppose $\boldsymbol{\theta}_t = (\theta_{1,t}, ..., \theta_{n,t})'$, with $\theta_{i,t} = \theta_{i,t-1} + \zeta_{i,t}$ $\forall i, t$, where $\zeta_{i,t} = \sigma_{i,t}\varepsilon_{i,t}$ and satisfies $|\zeta_{i,t}| \ll \theta_{i,t-1} > 0$. A Taylor series expansion of $\mu(\widehat{X}_{i,t}, \theta_{i,t-1} + \zeta_{i,t})$ around $\theta_{i,t-1}$ leads to the first-order approximation

$$X_t \approx F_{\theta_{t-1}}(\widehat{\mathbf{X}}_t) + \sum_{i=1}^{n} \lambda_{i,t}(\widehat{X}_{i,t})\zeta_{i,t} + \xi_t, \tag{18}$$

with

$$F_{\theta_{t-1}}(\widehat{\mathbf{X}}_t) = n - \sum_{i=1}^{n} \mu(\widehat{X}_{i,t}, \theta_{i,t-1}) \tag{19}$$

and

$$\lambda_{i,t}(\widehat{X}_{i,t}) = \left(\frac{\partial F_{\theta_t}(\widehat{\mathbf{X}}_t)}{\partial \zeta_{i,t}} \right)_{\zeta_{i,t} = 0}. \tag{20}$$

Granger (1980) showed that the aggregation of simple, possibly dependent time series models, can produce processes of the *integrated* type (Box and Jenkins, 1970). This class of processes includes nonstationary and infinite-variance processes. Thus if certain data generating mechanisms generate the random processes $\{\zeta_{i,t}\}_{i=1,n}$, the component

$$Z_t = \sum_{i=1}^{n} \lambda_{i,t}(\widehat{X}_{i,t})\zeta_{i,t} \tag{21}$$

will exhibit such features.

There are many potential applications of the model described above to both social and life science problems involving either predation, competition, cooperation or consumption of substitutes. Some simple examples could be the following :

a) X_t = number of voters at t to one of two major political parties (it is assumed that the population of potential voters has fixed size n and that voting to one of these parties is compulsory).

b) X_t = number of acres of land at t of a given territory (of n acres) conquered by one of two species that compete for the same resources.

c) X_t = number of people at t in a town with n inhabitants that have been infected by a contagious disease.

d) X_t = number of smokers at t in a town with n inhabitants, or number of customers of a given telephone company (as opposed to the number of customers of other telephone companies), or number of consumers of a certain class of milk or any other product manufactured by different companies, or number of people subscribing to a given magazine, journal or newspaper (the target populations here are the number of potential customers and subscribers, respectively, assumed to be invariant over time).

e) X_t = number of unemployed people at t in a country with n people with working abilities and willing to work.

f) X_t = number of cancer cells invading an organ's body at t, or the count of platelets (or other blood cells) per mm^3 in an aplastic anemia patient (or a patient of having any other hematologic disorder).

g) X_t = number of shares of stock of a given company owned by the State or by an individual investor at t (where n is the total number of shares of that stock issued and sold by the company).

h) X_t = number of stockholders at t in a group of n potential investors that either invest in stocks or in bonds, but not in both.

Remark that in all the cases above, the preferences are susceptible of control from outside forces. For example, in *a*) preferences can be influenced by propaganda. In *d*) and *g*) this can be done by marketing strategies, while in *b*) an intruder in need of space or other resources (such as man) could be a determining factor. In *c*) and *f*) preferences can be changed by medical intervention, such as effective drug therapy or preventive vaccination. In *e*) employment and environmental policies may have an impact on the evolution of the process by altering the preferences. Finally, in *h*) interest rates is a powerful instrument for either propelling or damping preferences.

So far we have considered a single-population model of aggregate behavior, but this model could be extended to allow for interactions between different populations. For example, consider the case of two coexisting populations X and Y in an ecosystem that can support at most n_x and n_y individuals, respectively, of each type. And suppose that the number of individuals in each population organize according to the interacting aggregation models

$$X_t = n_x - \sum_{i=1}^{n_x} \mathbf{1}(\widehat{X}_{i,t} > \theta_{i,t}^{(x)}(\mathbf{Y}_{t-1})), \tag{22}$$

$$Y_t = n_y - \sum_{i=1}^{n_y} \mathbf{1}(\widehat{Y}_{i,t} > \theta_{i,t}^{(y)}(\mathbf{X}_{t-1})), \tag{23}$$

where $\mathbf{X}_{t-1} = (X_{t-1}, ..., X_{t-m_x})$ and $\mathbf{Y}_{t-1} = (Y_{t-1}, ..., Y_{t-m_y})'$ are state vectors that summarize the past history of the series x_t and y_t, respectively, and which drive the preferences $\theta_{i,t}^{(x)}$ and $\theta_{i,t}^{(y)}$ of the individuals in each population. The form of the interdependencies between the two populations will define the organizational temporal pattern of the bivariate series $(x_t, y_t)'$. For example, if $\theta_{i,t}^{(x)}$ $(\theta_{i,t}^{(y)})$ depends negatively (positively) on Y_{t-1} (X_{t-1}), and if $\theta_{i,t}^{(x)}$ $(\theta_{i,t}^{(y)})$ depends negatively on X_{t-1} (Y_{t-1}) then our model will exhibit some of the features of a bounded-population *predator-prey model* (see for instance Renshaw, 1993, and Hofbauer and Sigmund, 1996). In this case, X (Y) would represent the prey (predator) population. When these two populations are constrained to coexist in an environment with limited resources, both species may run out of food (and therefore begin to starve) if the number of individuals of each becomes too large (close to either n_x or n_y). Moreover, a growing number of predators will follow an increase in their supply of food (preys), while the number of preys will decline as the number of predators to be feeded increases.

The predator-prey organizational behavior is only one possibility among several others in the previous two-population interaction model. Even when sticking to the case of linear inter-dependencies and $m = 1$, that is to

$$\begin{aligned} \theta_{i,t}^{(x)} &= a_0 + a_1 X_{t-1} + a_2 Y_{t-1}, \\ \theta_{i,t}^{(y)} &= b_0 + b_1 X_{t-1} + b_2 Y_{t-1}, \end{aligned} \tag{24}$$

we may have different alternative dynamics for the series x_t and y_t. For example, an extreme case is the configuration

$$\begin{aligned} a_0, b_0 &> 0, \\ a_1, b_2 &< 0, \\ a_2, b_1 &= 0, \end{aligned} \tag{25}$$

for which we obtain two non-interacting populations. Population Y may also feed on the waste products of population X, with no quarrelling or competition between the two species, when

$$\begin{aligned} a_0 &> 0,\ b_0 < 0, \\ a_1, b_2 &< 0, \\ a_2 &= 0,\ b_1 > 0. \end{aligned} \tag{26}$$

Symbiotic behavior will be obtained if, for example, population Y not only lives on population X but also cultivates it (e.g. human farming pigs or fisheries, harvesting cereals, etc.), that is when

$$\begin{aligned} a_0, b_0 &< 0, \\ a_1, b_2 &= 0, \\ a_2, b_1 &> 0. \end{aligned} \tag{27}$$

Finally, notice that a predator-prey model is arrived at with the configuration

$$\begin{aligned} a_0, b_0 &< 0, \\ a_1, b_2 &< 0, \\ a_2 &< 0,\ b_1 > 0. \end{aligned} \tag{28}$$

4 Modelling the stock market dynamics

Most models of the stock market proposed by economic theory rely on the assumption that human behavior could be interpreted as the solution to an optimisation problem, in a way coherent with

the axioms of deductive logic. This is the sort of behavior that economists call "rational". But which beliefs and preferences are rational under conditions of uncertainty ?
There are indeed many aspects of human economic decision-making that are difficult to rationalized. In the economy, these difficulties can be found in both single-person decision problems such as consumer behaviour, and games of strategy such as bidding in auctions. For example, models of the stock market which rely on the perfect rationality hypothesis are hardly able to mimic such extreme events as crashes or bubbles. The hypothesis of rational expectations (Muth, 1961) entails that the price of a security is equal to the present value of its future dividend stream. This contradicts the well-known existence of periods, the so-called speculative booms (crashes), when the prices of some assets are far above (below) this value. Both speculative bubbles and crashes may appear when the expected rate of market price change influences the current market price. In particular, a bubble (crash) will arise when the actual market price depends positively (negatively) on its own expected rate of change.
The shortcomings of the perfect rationality hypothesis have led many economists and social researchers to support the weaker hypothesis of bounded rationality for the agents (see Feldman and Lindell, 1990, Abrahamsson, 1993, Brian, 1994, and Zey, 1998, for recent critical surveys on rational choice theory in organizations and in cooperation processes). Following Magill and Quinzii (1996), an agent is boundedly rational if time and effort on his part are necessary to : (a) gain access to and process information, (b) create a mental image of possible future consequences of his (her) decisions, (c) make the necessary computations to obtain a solution to his (her) strategic problem, and (d) if the agent cannot devote unlimited time and effort to it. These costs underline the importance for the agents of adapting their models as they learn new information (see Holland, 1995, and Börgers, 1996, for a general discussion on the relevance of learning in strategic decisions and of computed-based models of adaptive agents, respectively).

In the following, we consider the model of the previous section in the context of the stock market dynamics, and show how it can generate both crashes and bubbles providing some conditions are satisfied by the preferences of most investors. This arises as a consequence of letting the evolution of our time series (here representing the number of stockholders) depend explicitely on the expectations of the agents (here on the expected rate of market price change).
Other studies on the analysis of these financial extreme events can be found in Flood and Garber

(1994). However, these studies are motivated from a different perspective, and focus mostly on testing for the presence of such events. Brian et al. (1996) present a computer model of stock market behavior which summarizes recent market activity by a collection of descriptors and is able to reproduce the rich behavior seen in real world speculative markets. The model applied in this section is intended to provide just an pproximation to some of the most striking features of the stock market dynamics. The quality of this approximation will depend on the accuracy with which the agents' preferences are modelled. As we pointed in section 3, this approach allows a wide range of potential applications to the social and economic sciences such as the modelling of the number of customers in a service, and that of groups contending for power, such as labor unions, political parties, guerrilla armies, etc. But also the growth and decline of professions, or the cooperative behavior apparently characterizing the beginning of many industries (see for instance, Hannan and Caroll, 1992, Nowak et al., 1995, Casti, 1997 for details).

In what follows, let our population of agents be one of prospective stockholders, and let X_t represent the number of these stockholders (as opposed to the number of investors on bonds) at time t. In the previous section we saw that our model could be written as $X_t = F_{\theta_t}(\widehat{\mathbf{X}}_t)+ \xi_t$, where $\widehat{\mathbf{X}}_t$ is the agents' predictor vector at t, and ξ_t is a martingale difference constrained by the fact that X_t is a discrete process taking values in $[0, n]$.

An important case corresponds to when agents have unlimited computational power, storage, time as well as free access to the full history of the series, and to the plans of the rest of the agents (perfect rationality). In this case, they will select the n-th dimensional optimal predictor vector of X_t given all this information, that is a vector, say $\widehat{\mathbf{X}}_t^{(pr)}$, whose i-th component is

$$\widehat{X}_{i,t}^{(pr)} = E(X_t | I_{t-1}; \widehat{\mathbf{X}}_{-i,t}^{(pr)}; \theta_t), \tag{29}$$

with I_{t-1} denoting the information set reflected in the values of the random variables of the process up to time $t-1$, and $\widehat{\mathbf{X}}_{-i,t}^{(pr)}$ denotes the $(n-1)$-th stacked dimensional vector defined as

$$\widehat{\mathbf{X}}_{-i,t}^{(pr)} = (\widehat{X}_{1,t}^{(pr)}, \widehat{X}_{2,t}^{(pr)}, ..., \widehat{X}_{i-1,t}^{(pr)}, \widehat{X}_{i+1,t}^{(pr)}, ... \widehat{X}_{n,t}^{(pr)})'. \tag{30}$$

Because agents are supposed to know the consequences of their own actions and those of the other agents, the optimal predictor can be deduced from the givens of the problem. This predictor will satisfy

$$\widehat{X}_{i,t}^{(pr)} = \widehat{X}_t^{(pr)} = F_{\theta_t}(\widehat{X}_t^{(pr)}). \tag{31}$$

That is, it is a perfect foresight because it confirms the deductions that went into it. Quoting Brian (1995), "when the agents' expectations induce actions that aggregatively create a world that validates them as predictions, they are in equilibrium and are called *rational expectations*". This equilibrium must be stable, or in other words, the optimal predictor $\widehat{X}_t^{(pr)}$must be self-enforcing, since it is clearly in the interest of an agent to use this predictor when the other agents use it. But for this to occur, there are only two possibilities : (a) the trivial case where F_{θ_t} is the identity map, **I**, and (b) the case where $\widehat{X}_t^{(pr)}$ is a function of t whose values are the *stable fixed-point* of the family of maps $\{F_{\theta_t}\}_t$. In this latter case, the existence and unicity of the common optimal predictor requires that the maps F_{θ_t} be *contractions*, that is, that there exist a sequence of positive real number, $\{\rho_t\}_t$, with $\rho_t < 1$, $\forall t$, and such that (see for example, Lusternik and Sobolev, 1989, p. 46)

$$\|F_{\theta_t}(X) - F_{\theta_t}(Y)\| \leq \rho_t \|X - Y\|, \forall X, Y \in \Re, \ \forall t. \tag{32}$$

It is an easy exercise to see that, in the case where $\mu_{i,t}(x, \theta_{i,t}) = \mathbf{1}(x > \theta_{i,t})$, $\forall i$, the value of $\widehat{X}_t^{(pr)}$ at t would be the stable solution (assuming its existence and unicity) of the equation

$$x = \#_{i \in [1,n]}\{x \leq \theta_{i,t}\}, \tag{33}$$

where $\#_{i \in [1,n]}\{x \leq \theta_{i,t}\}$ denotes the number of agents whose preferences at t satisfy $x \leq \theta_{i,t}$, and x is an integer belonging to the interval $[0, n]$.

When agents use the rational expectations' predictor, one has

$$X_t = \widehat{X}_t^{(pr)} + \xi_t.$$

An interesting result is obtained if the common optimal predictor $\widehat{X}_t^{(pr)}$ equals the "naive" predictor, X_{t-1}, in which case X_t will behave as an integrated process. More precisely, X_t obeys to a discrete *martingale* model with *barriers* at $x = 0$ and $x = n$ (see for instance Mills, 1993, pp. 90-91) :

$$X_t = X_{t-1} + \xi_t. \tag{34}$$

The naive predictor requires a minimum amount of knowledge, time and intelligent power, and is most efficient when the only relevant information in the past values of the series lie in the most recent one. When ξ_t is a sequence of independent and identically distributed random variables, the martingale becomes a random walk, which since the pioneering work of Bachelier (1900) and the seminal paper of Fama (1965), has been widely accepted as a model for time series of prices at frictionless large stock markets. In this context, it would suggest that prices very quickly reflect changes in conditions of demand and supply, so that actual market prices should approximate the equilibrium price (see Holden et al., 1991, ch. 6, for an introduction to forecasting asset market prices). Consequently, any information in past prices cannot generate consistent profits, as they would already be discounted in the price. Moreover, the possibility of extreme events such as price crashes or temporary bubbles would be excluded, since any sudden news would be instantaneously and fully reflected in the price. That is, if a good were predicted to rise in price, some stockholders may desire to bid up the current price in order to cash in on the predicted capital gains, while if it were predicted to fall in price, they may be willing to sell their stocks of that good to avoid the predicted capital losses.

In general, however, ξ_t is just a martingale difference, which suggests the possibility of "beating the market" by using appropriate nonlinear models for this volatility. Alternative models to the random walk are also obtained when full information on the history of x_t is not generally available, and/or when computational and time resources are limited. As a result, the predictor vector $\widehat{\mathbf{X}}_t$ will have components that are generally different from each other, and different from $\widehat{X}_t^{(pr)}$. The naive predictor will be at most suboptimal in this case in which agents will select their predictors as a compromise between efficiency, the time (opportunity) costs, and the data collection and processing costs.

As an example, suppose that the only departure from the perfect rationality of the agents lies in their lack of information about the other agents' future actions, but that for each t, there is a fixed set of hypotheses materialized in a finite number of predictors available to all of them, say $\{\widehat{X}_t^{(k)}\}_{k=1,K}$, that are chosen independently with equal probability accross agents, that is

$$p_{i,t}(k) = P(\text{the } i\text{-th agent selects the } k\text{-th predictor at } t) = p_t(k), \ \forall i. \tag{35}$$

We also assume the existence of a fixed set of evolving preferences, $\{\theta_{i,t}\}_{i=1,n}$.Under these conditions of limited uncertainty, there will be a single common optimal predictor at t for all the agents. This will be given by

$$\widehat{X}_t^{(o)} = \frac{1}{K^{n-1}} \sum_{\substack{k_j \in [1,K] \\ j=1,n \\ j \neq i}} \left[\prod_{j=1}^{n} p_t(k_j) \right] E(X_t | I_{t-1}; \widehat{X}_{j,t} = \widehat{X}_t^{(k_j)}; \boldsymbol{\theta}_t), \tag{36}$$

where $\boldsymbol{\theta}_t = (\theta_{1,t}, \theta_{2,t}, ..., \theta_{n,t})'$ and the sum is extended to all possible choices (K^{n-1}) of predictors for the $n-1$ remaining agents. However, in general,

$$\widehat{X}_t^{(o)} \neq F_{\theta_t}(\widehat{X}_t^{(o)}), \tag{37}$$

but instead

$$X_t = \widehat{X}_t^{(o)} + \eta_t + \xi_t, \tag{38}$$

where the new noise component η_t in the model is introduced to account for the uncertainty of each agent about the other agents' plans. If in the stock price example we had $\widehat{X}_t^{(o)} = X_{t-1}$, the presence of the term η_t would suggest the possibility of obtaining profits with trading rules that anticipate the market psychology.

As a second example, suppose we are interested in finding optimal predictors under bounded rationality. Let us write

$$X_t = \sum_{j=1}^{n} Z_{j,t}, \text{ with } Z_{j,t} = \mathbf{1}(\widehat{X}_{j,t} \leq \theta_{j,t}), \tag{39}$$

and let

$$X_t^{(-i)} = \sum_{\substack{j=1 \\ j \neq i}}^{n} Z_{j,t}. \tag{40}$$

Brian (1995) argues that when some agents use different predictors, predictions of the future outcome of the series will depend on the other agents' different predictions, and others' predictions of others' predictions, thus obtaining a state of ever-changing expectations (out of equilibrium) that leads to the breakdown of rational deduction. Since optimal predictors for boundedly rational agents are therefore self-referential and give rise to an infinite recursion of expectations, we will consider suboptimal predictors in which an agent's predictor only depends on the past history of the series and the behavior that he (she) expects from the other agents' collective action given this history of the series. By disaggregating the i-th agent's behavior from the collective behavior, the i-th agent's suboptimal predictor at t can be expressed as

$$\widehat{X}_{i,t} = E_i\left\{X_t|I_{t-1}; E_i(X_t^{(-i)}|I_{t-1})\right\} \tag{41}$$
$$= E_i(X_t^{(-i)}|I_{t-1}) + \mathbf{1}\left\{E_i(X_t^{(-i)}|I_{t-1}) + 1 \leq \theta_{i,t}\right\}. \tag{42}$$

To simplify further our discussion, suppose that agents realize the complexity of a forecasting rule based on the remote history of the series in the light of their limited computational and time resources, and that therefore they decide to use simply one-step-ahead forecasting rules. We will have

$$E_i(X_t^{(-i)}|I_{t-1}) = E_i(X_t^{(-i)}|X_{t-1}) \tag{43}$$
$$= X_{t-1} - \mathbf{1}(\widehat{X}_{i,t-1} \leq \theta_{i,t-1}). \tag{44}$$

And thus the i-th agent's predictor will evolve dynamically over time as the solution of time-varying nonlinear first-order autoregression :

$$\widehat{X}_{i,t} = X_{t-1} - \mathbf{1}(\widehat{X}_{i,t-1} \leq \theta_{i,t-1}) + \mathbf{1}\left\{X_{t-1} - \mathbf{1}(\widehat{X}_{i,t-1} \leq \theta_{i,t-1}) + 1 \leq \theta_{i,t}\right\}. \tag{45}$$

In a more general situation, it is clear that the order of the nonlinear autoregression to be satisfied by $\widehat{X}_{i,t}$ will be equal to the highest lag of X_t considered in this predictor.

To summarize, we have shown that these suboptimal predictors form an ecology which evolves deterministically and nonlinearly, and which depends on both past and present preferences, and a history of the series. In the stock market context, this suggests again departures from the *market efficiency hypothesis* and thus the possibility of obtaining real profits by using the appropriate nonlinear trading strategy. If now our i-th agent decides to use the linear "naive" predictor X_{t-1} he (she) will commit a prediction error given by

$$\epsilon_{i,t} = \widehat{X}_{i,t} - X_{t-1} \tag{46}$$
$$= \mathbf{1}\left\{X_{t-1} - \mathbf{1}(\widehat{X}_{i,t} \leq \theta_{i,t-1}) + 1 \leq \theta_{i,t}\right\} - \mathbf{1}(\widehat{X}_{i,t} \leq \theta_{i,t-1}). \tag{47}$$

These errors represent some of the information on the market which agents are unable to anticipate. When the preferences are very noisy, these prediction errors will no longer appear to have a systematic component (since masked by the state noise) and thereby will be difficult to detect in the mean behavior of the process X_t. This amounts to saying that the dynamics of the process become approximatively linear in the mean, and that the naive predictor could be the common suboptimal one for practical purposes.

The fact that prices are not purely random walks and the existence of profit opportunities has been pointed in many studies. For example, Lucas (1978) showed that asset prices in equilibrium models do not in general follow martingale processes. Poterba and Summers (1988) found evidence in support of an AR(1) process added to a random walk, and Maheswaran and Sims (1993) showed that there are classes of behavior for prices that are inconsistent with the martingale model. Also, Taylor (1994) reported on empirical evidence against the random walk hypothesis. As we have pointed, rejection of this hypothesis in price series amounts at saying that some information is not well reflected by the price on the day it first became known.

To investigate the model capabilities for simulating extreme events such as bubbles or crashes, we will restrict again our discussion to the case of purely deterministic dynamics, that is when $\mu_{i,t}(x) = \mathbf{1}(x > \theta_{i,t})$. This means that investor i ($i = 1, ..., n$) will buy stocks at time t only if his (her) expected number of stockholders at time t is smaller than $\theta_{i,t}$, or put differently, if he (she) expects stock prices at t to be low enough for him (her) to buy. Thus $\theta_{i,t}$ can be interpreted as a measure of the minimum level of returns from stocks at t that are acceptable for investor i, and this level will depend on the current level of the interest rates r_t. By changing $\theta_{i,t}$, investors adjust their preferences on stocks at t (with respect to bonds). These adjustments may respond to exogeneous events (changes in corporate policy, social, political or even meteorological events) as well as to changes in r_t.

Depending on the investors' preferences, the stock market will show different sorts of behavior. For example, suppose that we have constant interest rates ($r_t = r$, $\forall t$) and that initially we have $\theta_{i,0}(r) = \theta^*$, $\forall i$. If now investor j changed his (her) preferences so that $\theta_{j,1}(r) < \theta^*$, the number of stockholders at $t = 1$ will start to decrease, and the continuing anticipation of this market trend by every agent (which may regard the current number of the stockholders as an indicator of potential earnings) could become self-fulfilling and lead to a crash. This will occur unless interest rates are reduced to a level which discourages any investment on bonds. In fact, it is easy to see that a crash will follow when the condition

$$\frac{\partial \theta_{i,t}(r)}{\partial t} < 0, \tag{48}$$

is satisfied $\forall i, t$. Similarly, speculative bubbles will be obtained if

$$\frac{\partial \theta_{i,t}(r)}{\partial t} > 0, \ \forall i, t. \tag{49}$$

If instead, we have

$$\frac{\partial \theta_{i,t}(r)}{\partial t} = 0, \ \forall i, t, \tag{50}$$

neither crashes or bubbles will develop, and the series x_t will show no upward or downward trending patterns. In practice, the formation of a crash or a bubble needs only that these conditions be satisfied by most investors' preferences most of the time. To see this, let us come back to the general model of previous sections with $\mu_{i,t} = \mu$ and $\widehat{X}_{i,t} = X_{t-1}$, $\forall i, t$ and with $\theta_{i,t} = \theta_{i,t-1} + \delta_{i,t}$, $\forall i, t$, where $|\delta_{i,t}| \ll \theta_{i,t-1} > 0$. If we further assume that the first-order partial derivatives of μ exist, a Taylor expansion of $\mu(X_{t-1}, \theta_{i,t-1} + \delta_{i,t})$ around $\theta_{i,t-1}$ obtains

$$X_t \approx F_{\theta_{t-1}}(X_{t-1}) + \sum_{i=1}^{n} \lambda_{i,t}(X_{t-1})\delta_{i,t} + \xi_t, \tag{51}$$

with

$$\lambda_{i,t}(X_{t-1}) = \left(\frac{\partial F_{\theta_{t-1}}(X_{t-1})}{\partial \delta_{i,t}} \right)_{\delta_{i,t} = 0} > 0. \tag{52}$$

As remarked in the previous section, if the $\{\delta_{i,t}\}_i$ are stochastic processes of a certain type, their aggregation may produce in $\{X_t\}_{t>0}$ infinite variance patterns of integration or *persistence in the mean* behavior. That is the long strides in the sample paths of the process causing very slowly decaying autocorrelations (see for instance, Granger and Teräsvirta, 1993) that are typically found in stock price series.

Notice however that a sustained tendency for negative values of $\delta_{i,t}$ for most investors will enforce the convergence of $E(X_t|X_{t-1})$ to 0 (crash). Similarly, if the expected profits of most investors have an upward trend, $E(X_t|X_{t-1})$ will converge to n (bubble). Thus in spite of being generally different, most of the investors' preferences will happen to behave similarly at certain periods of time where crashes or bubbles occur.

A common economic control instrument intended to prevent the development of crashes and bubbles are interest rates, also referred to as the *value of money*, r. Decreasing (increasing) interest rates make stocks more (less) attractive to investors. Formally, what we have is

$$\frac{\partial \theta_{i,t}(r)}{\partial r} < 0, \ \forall i, t, \tag{53}$$

since investors respond to an increase in the returns of bonds by requesting higher returns from stocks, and thus by investing less on this sort of assets. Moreover, any realistic preference model should be able to respond to a lowering of r by shifting $\partial\theta_{i,t}(r)/\partial t < 0$ to $\partial\theta_{i,t}(r)/\partial t \geq 0$, thus making it possible to move out of a crash. Conversely, when increasing r, the investors' preferences must be such that $\partial\theta_{i,t}(r)/\partial t > 0$ be shifted to $\partial\theta_{i,t}(r)/\partial t \leq 0$, so that a developing bubble could be exploded. This two-fold condition could be written as :

$$\frac{\partial^2 \theta_{i,t}(r)}{\partial r \partial t} < 0, \ \forall i, t. \tag{54}$$

5 Experiment

After discussing some theoretical aspects of the model, in this section we illustrate the different sorts of organizational behavior that are obtained with it. Consider again the case of deterministic dynamics, for which we have :

$$\begin{aligned} X_t &= F_{\theta_t}(X_{t-1}) \qquad (55) \\ \text{with } F_{\theta_t}(X_{t-1}) &= n - \sum_{i=1}^{n} \mathbf{1}(\alpha_{i,t} X_{t-1} > \theta_{i,t}) \\ \text{and } \theta_{i,t} &= a_{i,t} r_t + b_{i,t} r t + c_{i,t} t, \end{aligned}$$

with different choices for the parameters $\alpha_{i,t}$, $a_{i,t}$, $b_{i,t}$ and $c_{i,t}$, and with $n = 10$. In this model r_t represents the time-varying interest rate. Notice that

$$\begin{aligned} \frac{\partial \theta_{i,t}}{\partial r} &= a_{i,t}, \qquad (56) \\ \frac{\partial \theta_{i,t}}{\partial t} &= c_{i,t} + b_{i,t} r, \\ \frac{\partial^2 \theta_{i,t}}{\partial r \partial t} &= b_{i,t}. \end{aligned}$$

Therefore, according to our discussion in the previous section, we must have $a_{i,t} < 0$, and $b_{i,t} < 0$, $\forall i, t$. Furthermore, if we assume $r_t = r = 0.02$, $b_{i,t} = b = -50$, then $c_{i,t} < 1$ for a crash, $c_{i,t} > 1$ for a bubble, and $c_{i,t} = 1$ for a stable pattern of behavior. In the sequel, except when explicitly stated, we take $a_{i,t} = a = -50$, $r_t = r = 0.02$ and $c = 1$.

Figures 1 to 9 describe several forms of organizational behavior resulting from different modifications of the previous configuration for the parameters in the model. The plots show the series on top, the cross plot of x_{t-1} versus x_t (bottom left), and the initial preferences $\theta_{i,0}$ (bottom right). The specific parameter choices in the simulation experiments are listed below for each of the following figures.

1. Example of simple attractor (limit cycle) with $\alpha_{i,t} = 1$.

2. Effect of partially random preferences with $\alpha_{i,t} = 1$.

Here $\theta_{i,t} = ar + brt + ct + 10\,\varepsilon_{i,t}$ con $\varepsilon_{i,t} \sim U\{[-0.5, 0.5]\}$ independent random variables, with $U\{[a, b]\}$ representing the uniform distribution in the interval $[a, b]$.

3. Effect of randomly time-varying sensitivity to the interest rate with $\alpha_{i,t} = 1$.

Here $a_{i,t} = -50\,(1 + \varepsilon_{i,t})$ con $\varepsilon_{i,t} \sim U\{[-0.5, 0.5]\}$ independent random variables.

4. Example of crash with $\alpha_{i,t} = 1$, and $c = 0.9$.

5. Example of controlled crash with $\alpha_{i,t} = 1$, and $c = 0.9$.

Here r is reduced by a factor of 0.95 at each time t for which one has $\frac{1}{6}\sum_{i=0}^{5} X_{t-i} < 0.1\,n$.

6. Example of bubble with $\alpha_{i,t} = 1$, and $c = 1.1$.

7. Example of controlled bubble with $\alpha_{i,t} = 1$, and $c = 1.1$.

Here r is increased by a factor of 1.005 at each time t for which on has $\frac{1}{6}\sum_{i=0}^{5} X_{t-i} > 0.8\,n$.

8. Example of persistence in the mean behavior.

a) Here $\alpha_{i,t} = \alpha_i \in (0, 1)$ selected uniformely at random, and $c_{i,t} \sim U\{(0.5, 1.5)\}$ independent random variables.

b) Here $\alpha_{i,t} = \alpha_i \in (-0.5, 0)$ initally, and $c_{i,t}$ were selected as in a). The agents were allowed to change their one-step-ahead predictors at t in the following way. If $\alpha_{i,t}\ y_{t-1} < y_t$ ($> y_t$) then $\alpha_{i,t+1} = -\alpha_{i,t}$. On the other hand, $\alpha_{i,t+1} = \begin{cases} \alpha_{i,t}+d_i(y_t-\alpha_{i,t}y_{t-1})\alpha_{i,t} & \text{if } \alpha_{i,t}>0, \\ \alpha_{i,t}+d_i(y_t-\alpha_{i,t}y_{t-1})\alpha_{i,t} & \text{if } \alpha_{i,t}>0, \end{cases}$ where d_i are the coefficients of the adaptation rule, selected at random from a uniform distribution in the range $[0, 0.001]$.

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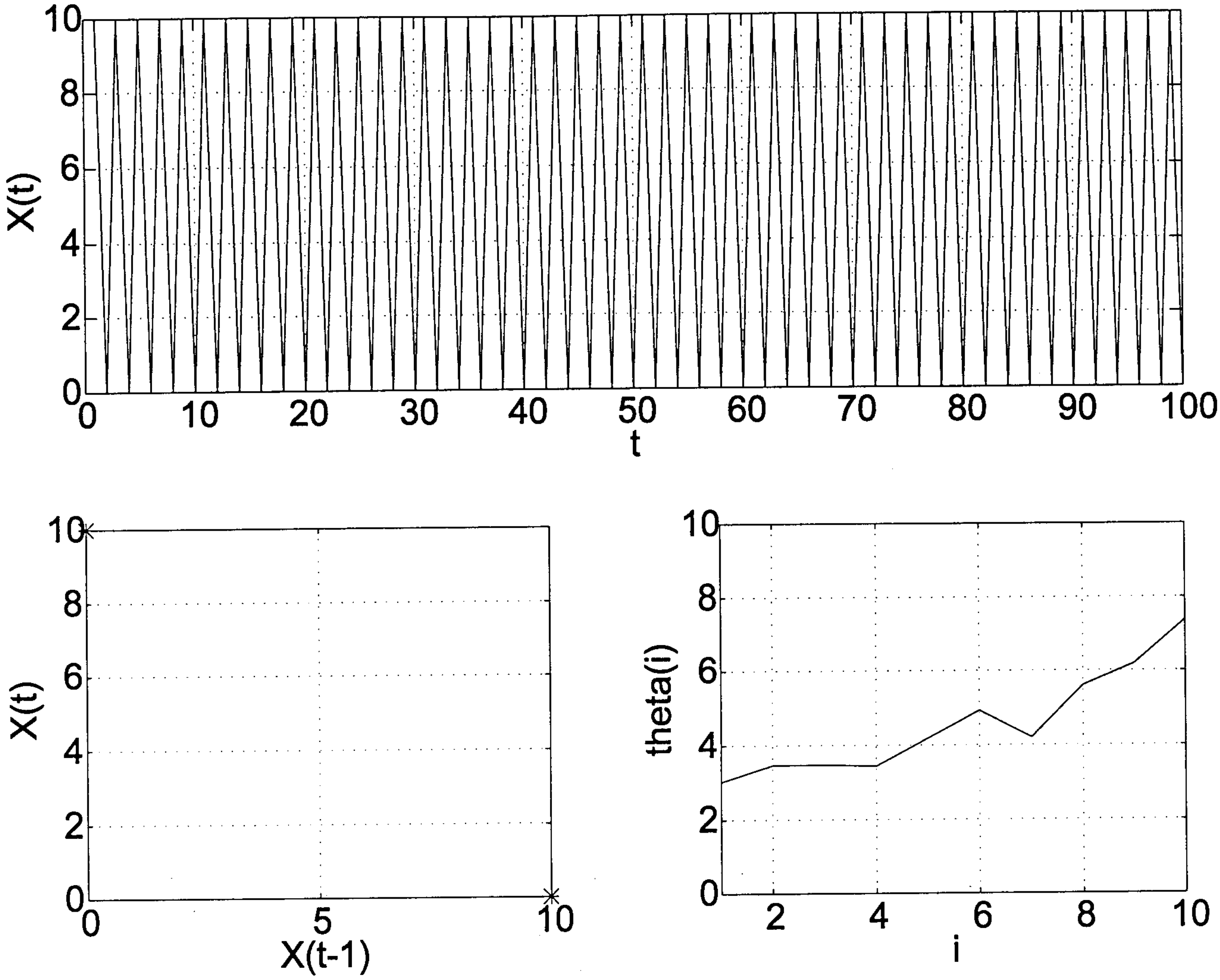

Figure 1: Limit cycle.

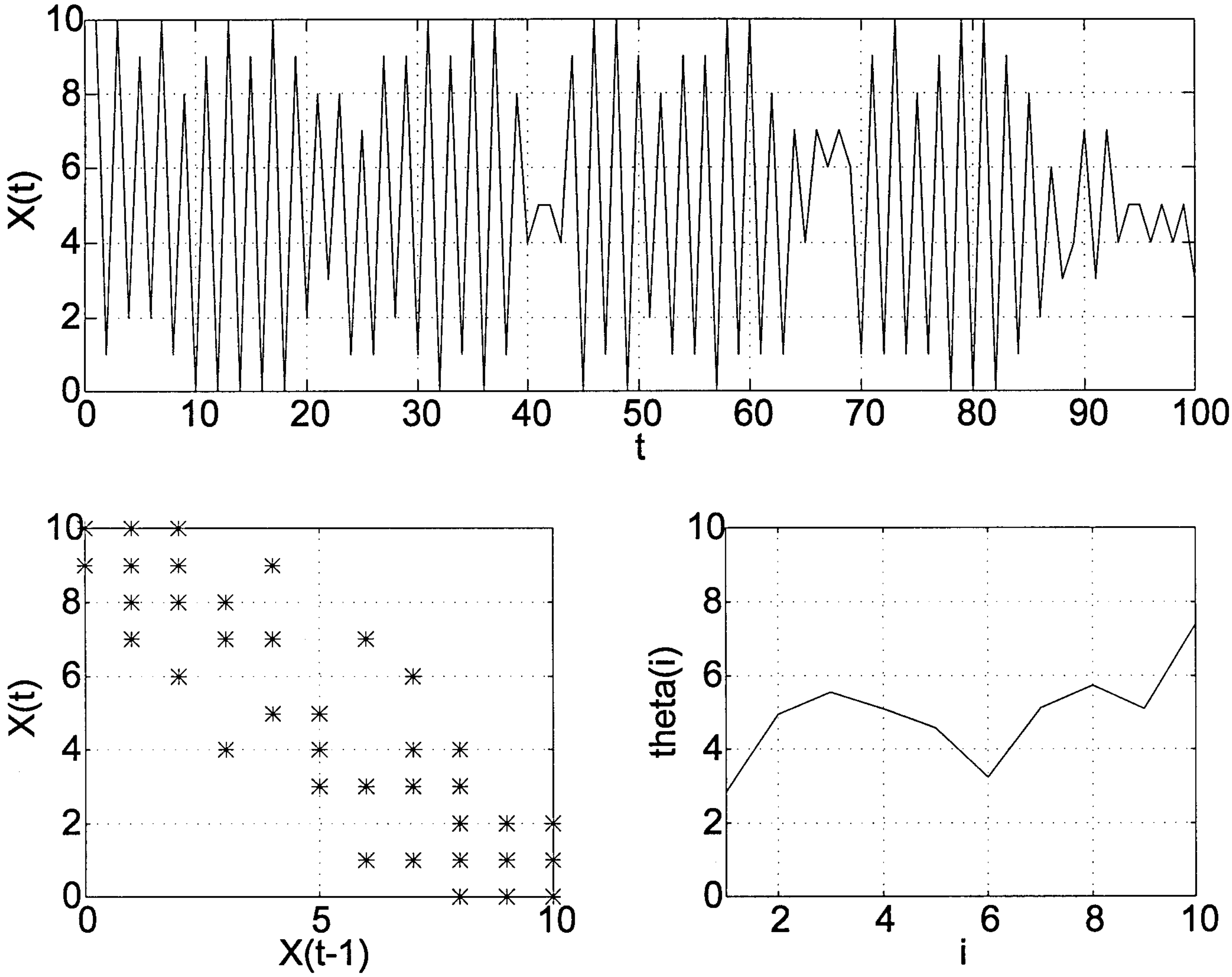

Figure 2: Partially random preferences.

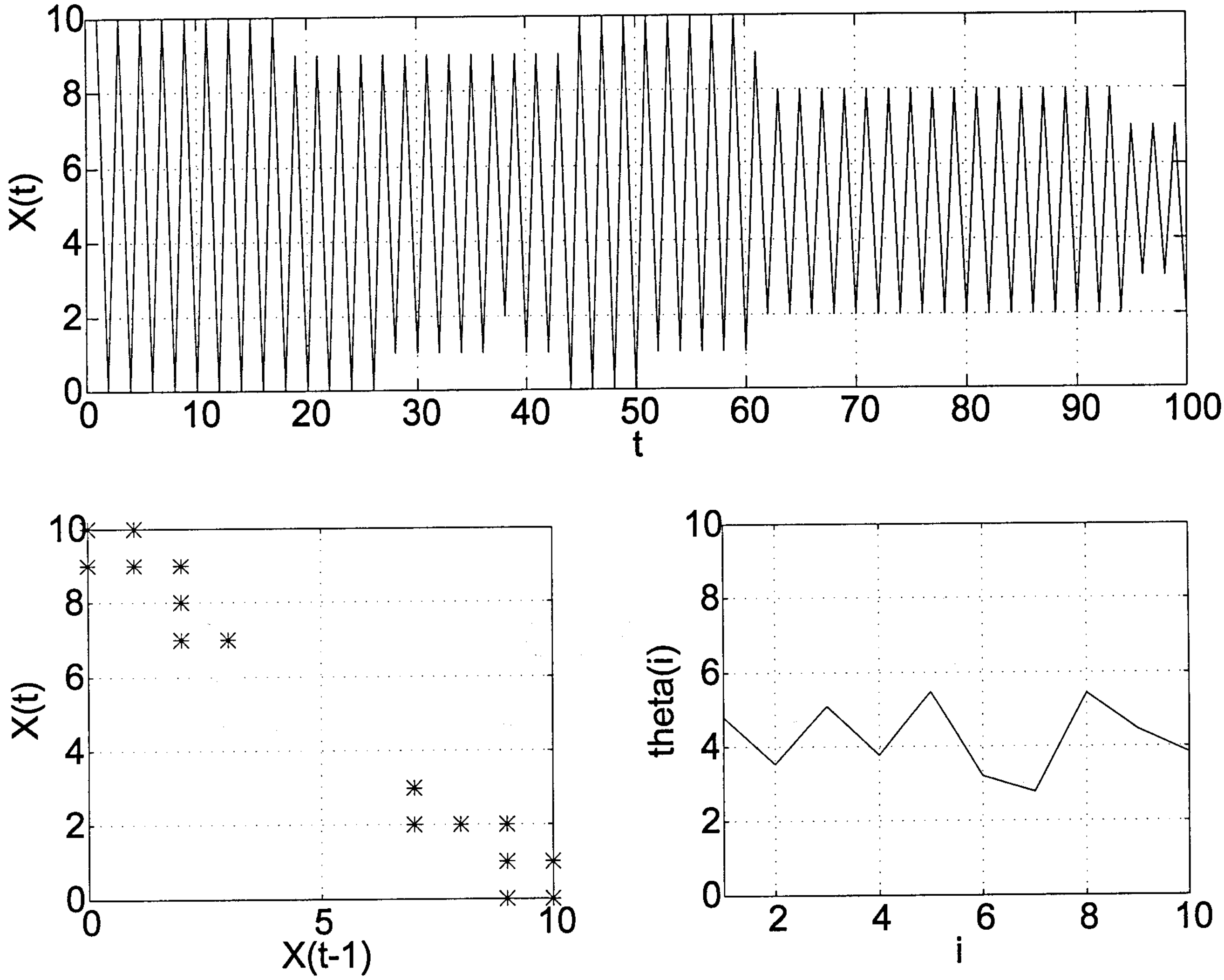

Figure 3: Random sensitivity to interest rates.

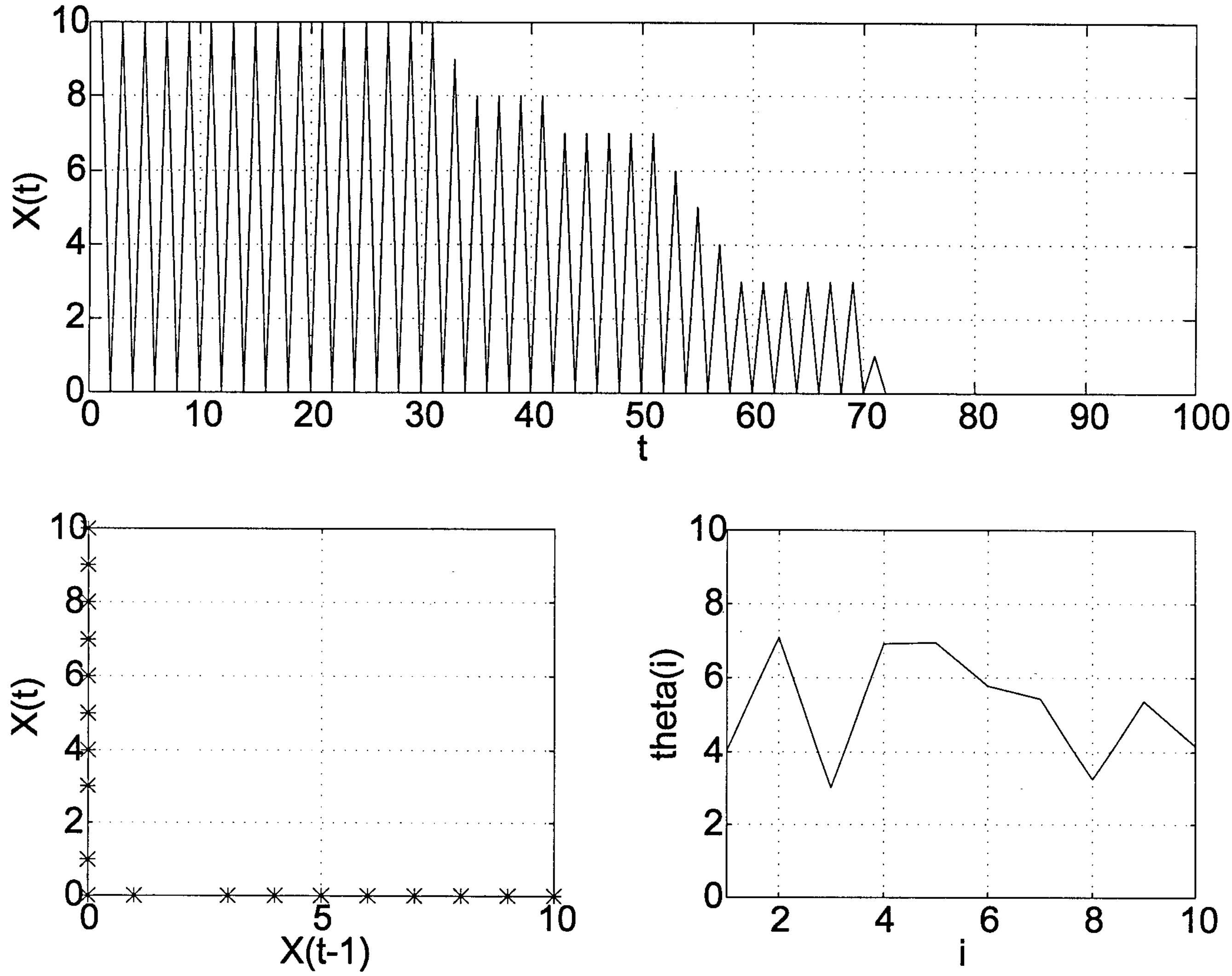

Figure 4: Crash.

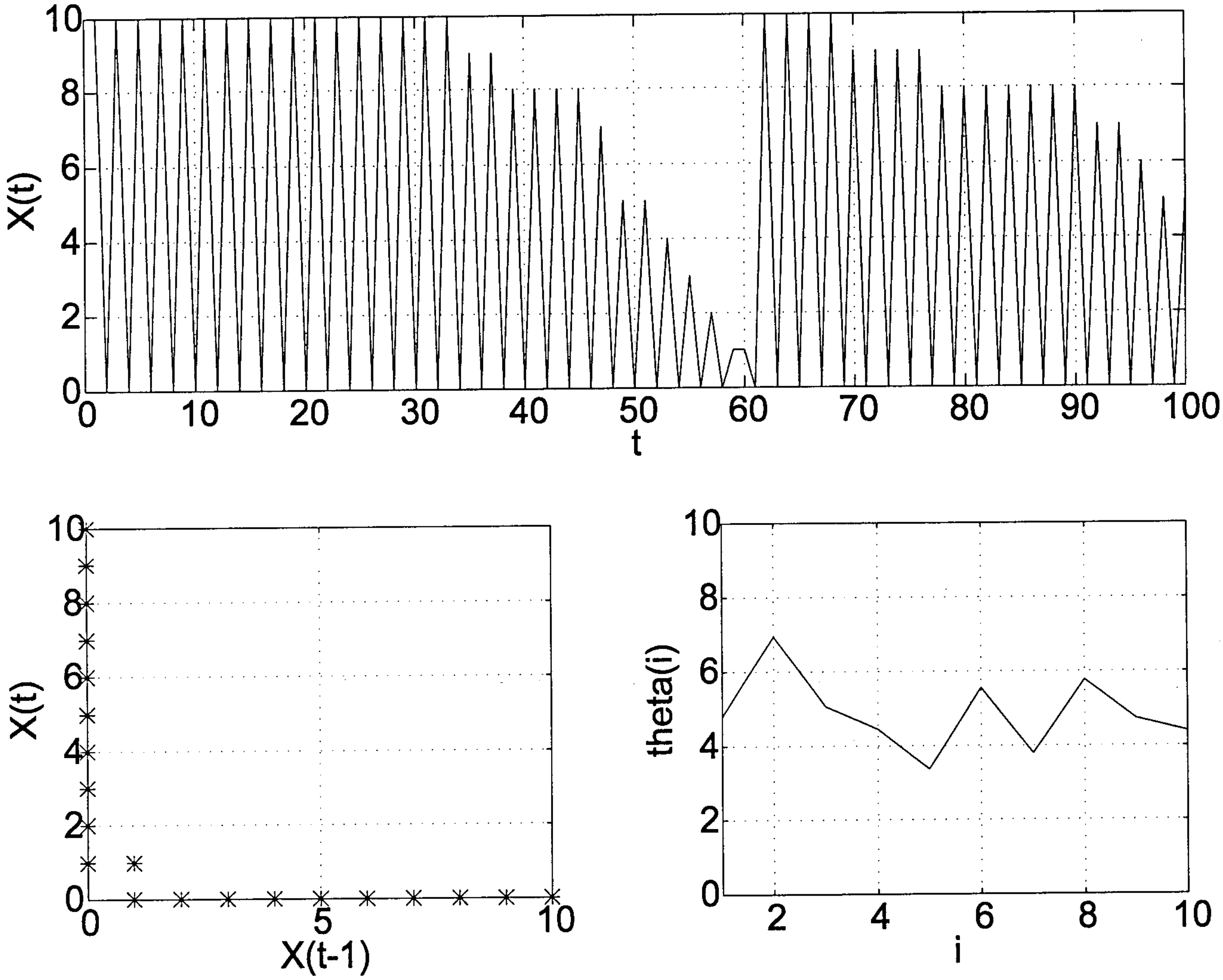

Figure 5: Controlled crash.

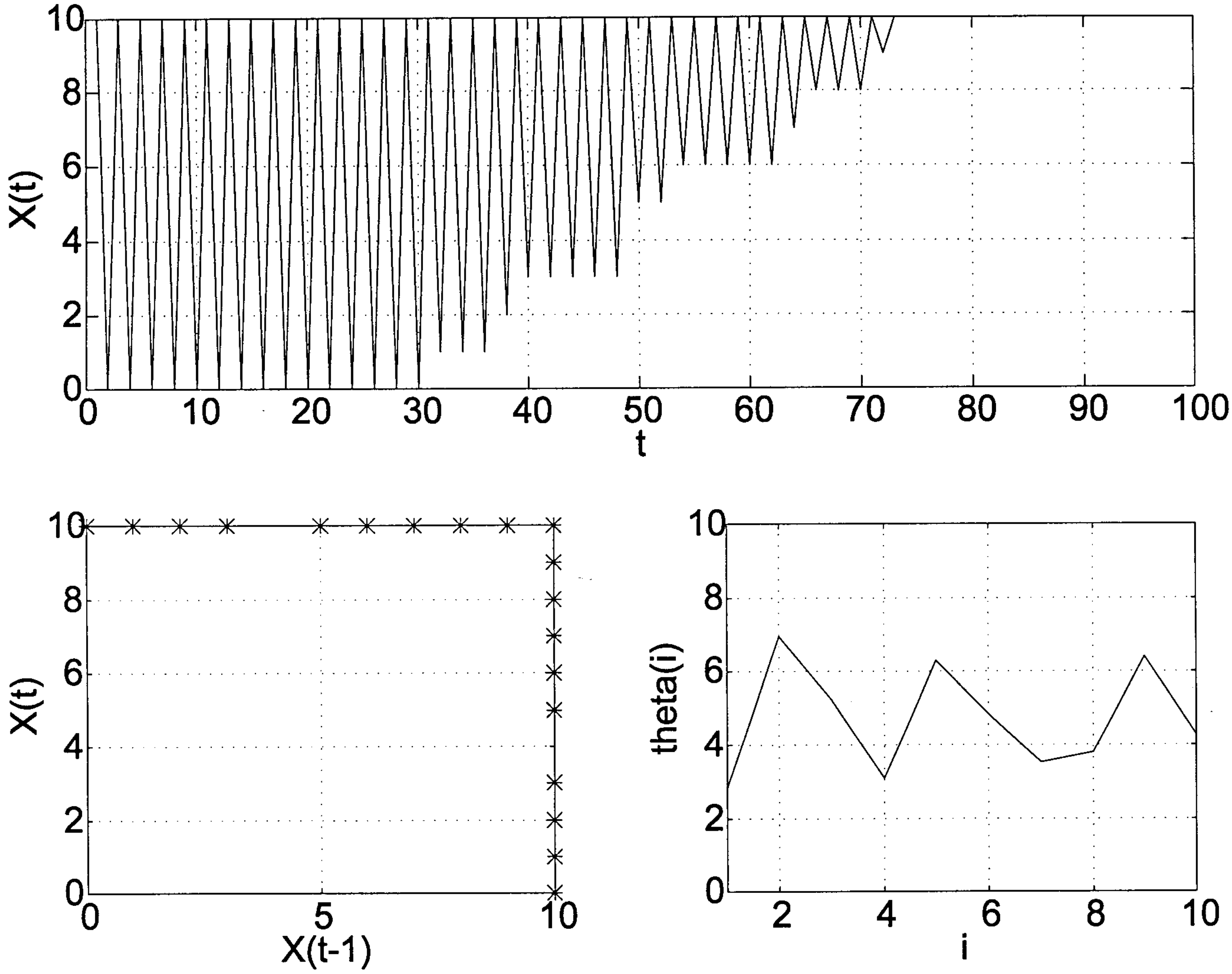

Figure 6: Bubble.

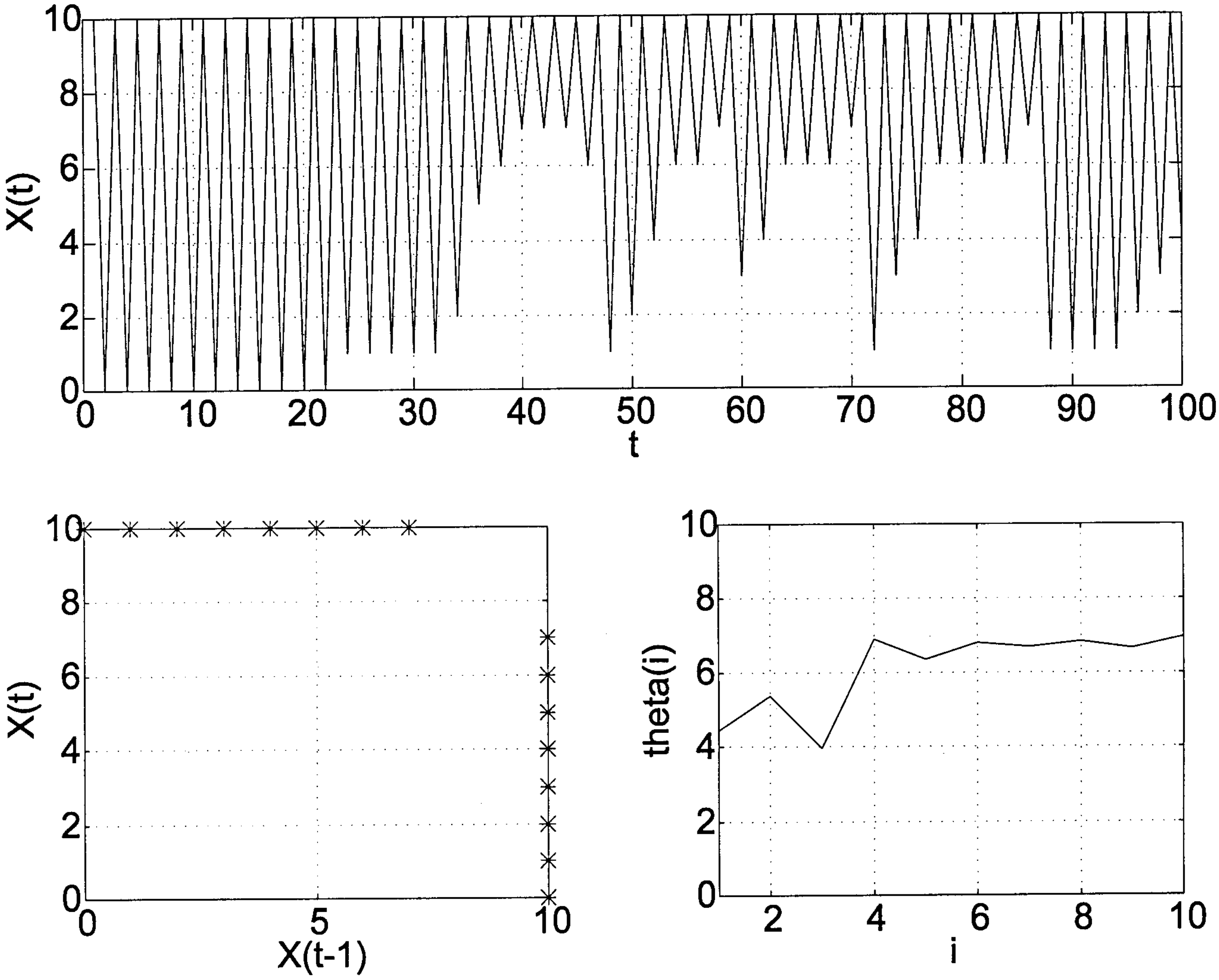

Figure 7: Controlled bubble.

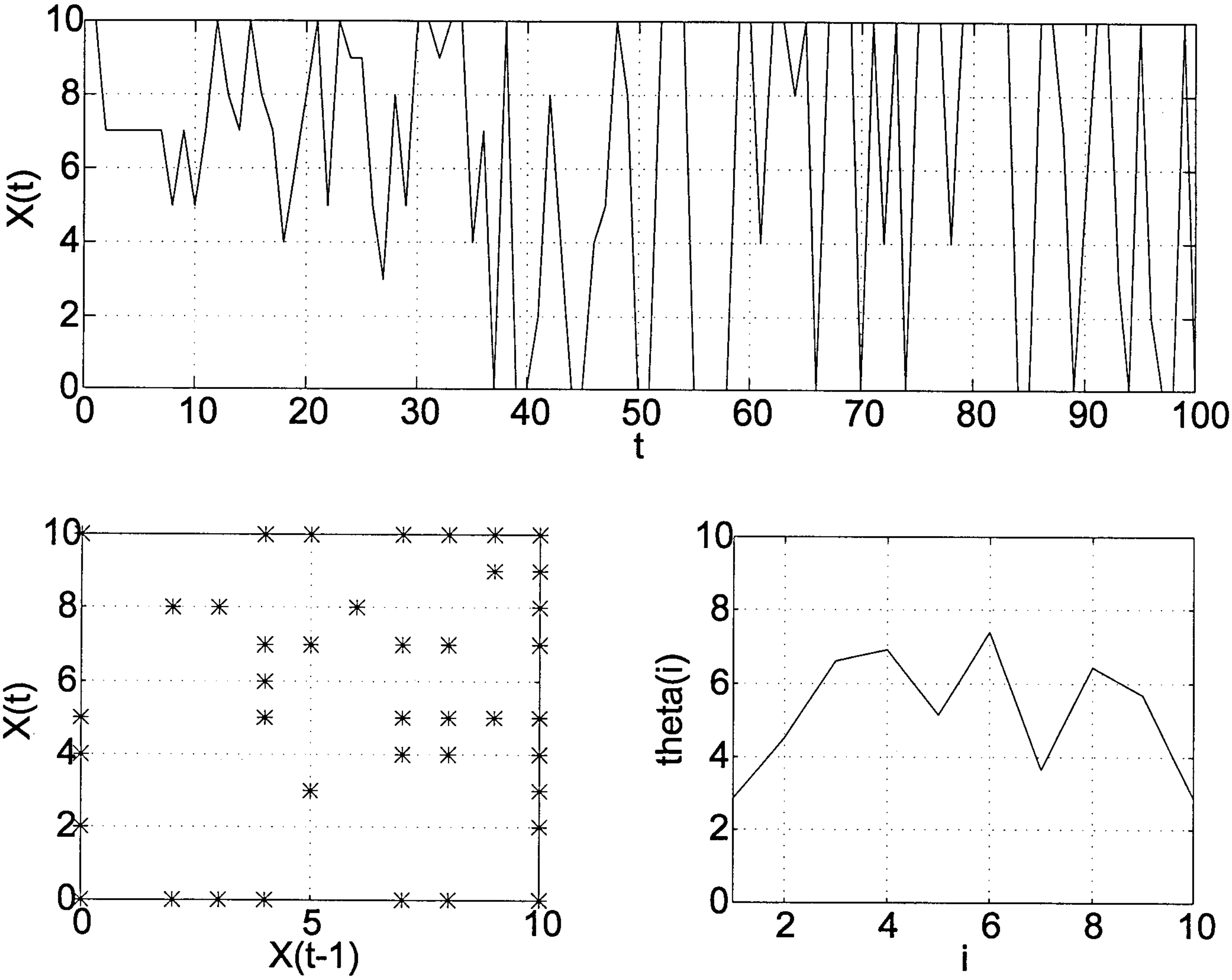

Figure 8: Persistence (a).

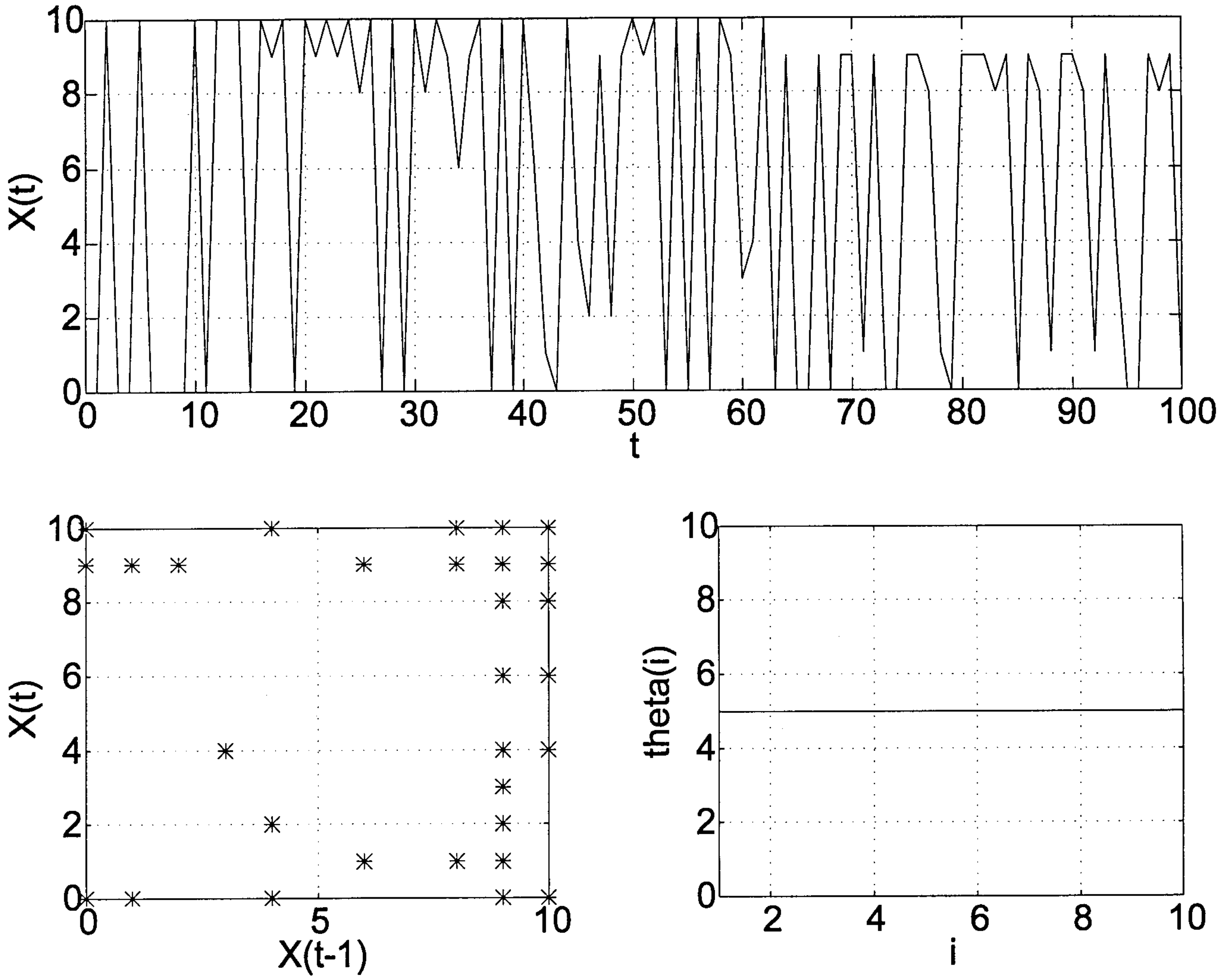

Figure 9: Persistence (b).