

Working Paper 99-36
Economics Series 16
October 1998

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-98-75

INTERGENERATIONAL TRANSFER INSTITUTIONS: PUBLIC EDUCATION AND PUBLIC PENSIONS *

Michele Boldrin[†] and Ana Montes Alonso[‡]

Abstract

In a world in which credit markets to finance investments in human capital are rare, the competitive equilibrium allocation generally cannot achieve either static or dynamic efficiency. When generations overlap, this inefficiency can be overcome by properly designed institutions. We study the working of two such institutions: Public Education and Public Pensions. We argue that, when established jointly, they implement an intergenerational dynamic game of taxes and transfers through which public education for the young and public pensions for the elderly support each other. Through the public financing of education, the young borrow from the middle age to invest in human capital. When employed, they pay back their debt by means of a social security tax on labor income. The proceedings of the latter are used to finance pension payments to the now elderly lenders. We also show that such intergenerational agreement can be supported as a subgame perfect equilibrium of, relatively straightforward, majority voting games. While the intertemporal allocation so obtained does not necessarily reach full dynamic efficiency it does so under certain restrictions and it always improves upon the *laissez-faire* allocation. We test the main predictions of our model by using micro and macro data from Spain. The results are surprisingly good.

Keywords: Intergenerational contract, efficiency, human capital, political equilibria.

[†] Boldrin (boldrin@eco.uc3m.es), Departamento de Economía, Universidad Carlos III de Madrid and CEPR.

[‡] Montes (amontes@eco.uc3m.es), Departamento de Economía, Universidad Carlos III de Madrid.

* Financial support from the Fundación BBV, DGICYT (PB94-0378) and CICYT (SEC96-0738) is gratefully acknowledged. Our thanks to Gary Becker, Marco Celentani, Ignacio Conde, Sergi Jimenez and Franco Peracchi for useful discussions and insights. Participants at the CEPR/IEG PBM Workshop on "Rethinking the Welfare Society", La Coruña, 27-28 September 1998 also provided us with very useful comments. The mess that remains is all our doing.

1. Introduction

The following, rather stylized facts, characterize most advanced countries.

Fact 1. Human capital and physical capital are complementary factors of production. An increase in the availability of either one tends, *ceteris paribus*, to increase the productivity of the other.

Fact 2. Credit markets for financing investment in human capital are very rare and, for the primary and secondary school levels, altogether absent.

Fact 3. Human capital accumulation is largely financed by public taxation, at least at the primary and secondary level and, in many countries, even at more advanced levels. Actual provision of schooling does take place in a variety of different forms, some of which involves private producers, but costs are generally covered by taxes.

Fact 4. Old-age public pension systems are, almost always, of the pay-as-you-go type and are usually financed by means of a social security tax which is levied only on labor income.

Fact 5. The amount of public resources going into education is established annually by means of the budget legislation. These amounts are relatively stable on a year by year base but they vary, sometime substantially, over longer time horizons. Public education is financed, mostly, by general taxation.

Fact 6. Many public pension systems are of the “defined benefit type” in which a certain pension amount is being promised to elderly citizens satisfying certain contributive requirements. A number of other pension systems are of the “defined contribution type”, in which the actual payments are not explicitly set beforehand, but depend upon circumstances that will be fully determined only at retirement time. In either case, the actual pension payments and social security contributions are determined by ordinary legislation and, very often, by the yearly government budget.

Fact 7. Constitutions and ordinary legislation do not establish any explicit link between the public education system and the public pension system. Barring the aggregate resource constraint, there is no clear connection between the amount spent on education in one year and the level of pension payments during that or any subsequent year. Likewise, we are not aware of any welfare system linking the amount of public education received with the level of social security contributions levied during one’s working life.

The ongoing political and economic debate on the financial crisis of public pension systems concentrates on three themes:

- (i) that demographic evolution has made pay-as-you-go pension systems not longer viable;
- (ii) that current pensions provide the elderly with a rate of return on their past social security contributions which is economically unsustainable and socially unjustifiable;
- (iii) that a transition to a fully funded pension system, in which workers' contributions are invested in the capital market and pensions paid out of the capitalized value, would lead to a socially superior outcome.

We do not question either the logical or the empirical validity of these statements. Indeed, we believe there are serious arguments supporting each of them (compare, *e.g.* World Bank (1994)). Nevertheless, we suggest here a different point of view for evaluating pay-as-you-go pension systems. Our point of view stresses the economic and political links between public pensions and publicly financed education systems. This has both positive and normative implications. In section 5 we use our model to evaluate the “intergenerational fairness” of the current Spanish Public Education and Public Pension (PEPP) system. The same model may also be adopted as a guideline for redesigning the institutions of the welfare state in a Pareto improving direction. Our own appraisal of such normative implications is contained in the concluding section. We move next to discuss the motivations for the model we use and its relation with the stylized facts listed above.

Markets in which credit can be obtained to finance investment in individual's human capital are not frequent. Indeed, there are well understood reasons for which such markets are difficult to set up and sustained over time (see, *e.g.* Becker (1975)). It is also well understood that, in the absence of such borrowing-lending opportunities, the competitive equilibrium will not realize an efficient allocation of resources, either static or dynamic. Further, in a context in which continuous accumulation of human capital is necessary for persistent economic growth, the lack of credit markets may also be the cause of economic stagnation. In our model we try to capture these facts by setting up an overlapping generations model in which individuals live for three periods, in which both human and physical capital are useful in the production of aggregate output, and in which the young agents have no physical resources to invest in acquiring their own education. We also assume parents and grandparents (middle age and old individuals, respectively) are selfish and do not have an incentive for investing directly in their own descendants.

The reader may suspect that the last, somewhat unrealistic, assumption delivers the bulk of our conclusions. This is not so. Parental altruism, to the extent that it does not fully internalize the welfare of all future generations thereby transforming the overlapping

generation economy into one of infinitely lived dynasties, would only attenuate but not eliminate the inefficiencies we mentioned. In particular, parental altruism by itself cannot, in general, provide the “right” amount of investment in human capital. This is because parents, even when they care for the consumption or the human capital level of their progenies, cannot internalize the impact on the productivity of physical capital of an increase in the *aggregate* stock of human capital.

When markets are complete, the equilibrium of our model displays persistent growth and both static and dynamic efficiency. When markets are not complete, economic growth is reduced or eliminated altogether and the equilibrium allocation is inefficient. Such a negative outcome could be overcome if the members of subsequent generations were capable of implementing a repeated sequence of intergenerational transfers.

Long run intergenerational public contracts involving the members of different cohorts are seldom, if ever, explicitly embodied in ordinary legislation or even constitutional laws.[†] In spite of this, it is not obvious that such contracts could not emerge by themselves, as implicit in the equilibria of properly defined political games among citizens of successive cohorts. Voting rules and public institutions may be designed to define such games and to favor the selection of certain, socially desirable, equilibrium strategies. When this happens, one would expect to see actual legislation supporting such equilibria “de facto” even if not “de jure”. We show here that this conjecture, suggested by Becker and Murphy (1988) in an informal context, holds true in a formal model of sequential political choice and explains remarkably well the current Spanish PEPP system.

As a matter of history, during the last century and a half we have witnessed first the introduction and then the expansion of public education and public pension systems in most advanced countries. Formally speaking the two systems were introduced independently one from the other. On the other hand, their adoption and expansion were strictly sequential. More precisely, in all cases we are aware of, public education was introduced before, sometime one or even two generations before, public insurance for the elderly. The same ordering was followed in most subsequent enlargements of the two systems. Public discussion over these issues, as well as common sense, often suggests a link between the “generosity” of one generation with its progenies and the implicit “debt” of gratitude that the latter owes the former. The metaphorical image of a society as a “family” in which

[†] Many constitutions mention explicitly either one or both kinds of intergenerational transfer. Still we have not found a single one in which public education and public pension provision are linked to each other.

parents care for their children in the expectation that the latter will reciprocate later on, is too often used, and maybe abused, to need reminding here. Becker and Murphy (1988) use these and other arguments to support the idea that PEPP systems may be held together by an implicit “intergenerational contract” through which agents of different generations achieve the efficient outcome by attributing to the state the allocational role which was played by the family in the past and that, for the aforementioned reasons, cannot yet be exercised by the markets. We share this intuition. Still, those same arguments leave wide open the question of *how* such an advantageous intergenerational contract can be implemented when a benevolent central planner is not around to dictate it.

In this paper, the model sketched above is used to address this question. Assuming that credit markets to finance the young are absent we ask if, under a mechanism for collective decision making other than the benevolent central planner, a PEPP system would be implemented and how close it could come to achieve full efficiency. We let our agents play a dynamic, majority voting game and look at the subgame perfect equilibria of this game. In fact we study two specifications of this game, where the differences are dictated by Fact 7 above, and characterize their equilibrium outcomes. In our view the two games capture, in a stylized fashion, the most important institutional characteristics of modern welfare states.

Modeling the PEPP system along the lines we have adopted is, without doubts, quite restrictive. From the point of view of political theory, one cannot rule out constitutions in which taxes other than those we consider could be viable means to implement intergenerational transfer schemes. Also, there is no reason that simple majority voting be adopted as a decision rule, voting does not need to take place once per generation, and so on. We make no claim that the particular form of intergenerational contract we are studying is the only possible equilibrium of some general voting game. We claim, instead, that the equilibria described below in Section 3 and 4 are supported by a set of social institutions and generational beliefs that are not unlike those we observe in reality. The, relatively detailed and careful, analysis of the Spanish case which we carry out in Section 5 provides a surprisingly strong support to the latter statement.

The remaining of the paper contains the following material. In section 2 we introduce the economic model, characterize the competitive equilibrium when markets are complete and when credit markets for financing human capital are missing. Section 3 illustrates first a lump-sum tax and transfer scheme that would restore efficiency when markets are

incomplete. We introduce next the voting games our agents play and characterize their properties. We show under which circumstances the complete market allocation may and may not be supported as a SPE of our dynamic political games. Section 4 illustrates these conditions by means of two examples. Section 5 reports empirical findings based on Spanish data, they appear to support the predictions of our model. Section 6 concludes.

2. The Basic Model

2.1 Complete Markets

We study an economy composed of overlapping generations of identical agents living for three periods. Each generation is composed by a continuum, of size one, of identical individuals.

In each period $t = 0, 1, 2, \dots$, physical, k_t , and human, h_t , capital are owned, respectively, by the old and the middle age individuals. Aggregate output of the homogenous commodity is $y_t = F(h_t, k_t)$, where $F(h, k)$ is a constant return to scale and neoclassical production function.

At the beginning of each period a new generation of young agents is born. They own no productive capital. Instead, they are endowed with a stock h_t^y of basic knowledge which can be used to acquire human capital. Hence, if they spend time and money at school their human capital becomes $h_{t+1} = h(d_t, h_t^y)$ when middle age. Here d_t is the amount of homogenous good invested in the educational process. It is meant to comprise both direct and opportunity costs. The function $h(d, h^y)$ is a constant return to scale neoclassical production function which also satisfies a number of additional technical properties to be specified momentarily. During the second period of their life, individuals work and carry out consumption-saving decisions. When old, they consume the total return on their savings before dying.

We assume agents draw utility from (c_t^m, c_{t+1}^o) denoting, respectively, consumption when middle age and old. Neither leisure nor the welfare of their descendants affect utility.

Let the homogenous commodity be the numeraire. In each period $t = 0, 1, 2, \dots$, output y_t is allocated to three purposes: aggregate consumption ($c_t = c_t^m + c_t^o$), accumulation of next period's physical capital (k_{t+1}) and investment in education (d_t^d). Human and physical capital are hired by firms at competitive prices equal, respectively, to $w_t = F_1(h_t, k_t)$ and $1 + r_t = F_2(h_t, k_t)$. Aggregate saving is allocated, through competitive credit markets,

to finance investments in physical and human capital ($s_t = k_{t+1} + d_t^s$), accruing a total return equal to $(1 + r_{t+1})s_t = R_{t+1}s_t$.

The life-cycle optimization problem for the agent born in period $t - 1$ is

$$U_{t-1} = \max_{d_{t-1}^d, s_t} u(c_t^m) + \delta u(c_{t+1}^o) \quad (2.1)$$

$$\begin{aligned} \text{subject to : } 0 &\leq d_{t-1}^d \leq \frac{w_t h_t}{R_t} \\ c_t^m + s_t + R_t d_{t-1}^d &\leq w_t h_t \\ c_{t+1}^o &\leq R_{t+1} s_t \\ h_t &= h(d_{t-1}^d, h_{t-1}^y) \end{aligned}$$

Consumption-saving behavior is summarized by the two first order conditions:

$$u' [w_t h(d_{t-1}^d, h_{t-1}^y) - s_t - R_t d_{t-1}^d] = \delta R_{t+1} u' [s_t R_{t+1}] \quad (2.2a)$$

$$u' [w_t h(d_{t-1}^d, h_{t-1}^y) - s_t - R_t d_{t-1}^d] [w_t h_1(d_{t-1}^d, h_{t-1}^y) - R_t] = 0 \quad (2.2b)$$

Competitive equilibrium is defined by the following set of equalities (subscripts, as usual, indicate partial derivatives):

$$F(h_t, k_t) = c_t^m + c_t^o + s_t \quad (2.3a)$$

$$F_1(h_t, k_t) = w_t \quad (2.3b)$$

$$F_2(h_t, k_t) = R_t = w_t h_1(d_{t-1}^d, h_{t-1}^y) \quad (2.3c)$$

$$s_t = d_t^s + k_{t+1} \quad (2.3d)$$

$$d_t^d = d_t^s = d_t \quad (2.3e)$$

Solving (2.2) and (2.3) yields a dynamical system $\Phi : (k_t, h_t) \mapsto (k_{t+1}, h_{t+1})$ which, given the initial conditions (k_0, h_0) and d_{-1} , induces the equilibrium path $\{(k_t, h_t)\}_{t=0}^\infty$.

The algebra leading from (2.2) and (2.3) to Φ can be simplified through a number of technical assumptions. Note first that, given the hypothesis that the aggregate production function is smooth and neoclassical, the rental-wage ratio R/w is, in equilibrium, a well defined and monotone decreasing function of the factor intensity ratio $x = k/h$, i.e.

$$\frac{R}{w} = \frac{f'(x_t)}{f(x_t) - x_t f'(x_t)} = \frac{R(x_t)}{w(x_t)} = \omega(x_t)$$

where $f(x) = F(1, k/h)$. The more technical hypotheses are listed next:

Assumption 1 The function $h : \mathfrak{R}_+^2 \mapsto \mathfrak{R}_+$ is smooth. The function $g : \mathfrak{R}_+^2 \mapsto \mathfrak{R}_+$ satisfying $h_1[g(x, h^y), h^y] - \omega(x) = 0$ exists and it is well defined and continuous.

Assumption 2 The utility function $u : \mathfrak{R}_+ \mapsto \mathfrak{R}_+$ is strictly monotone increasing, strictly concave and smooth. Given numbers I and R both larger than zero, the function $V(I - z, Rz) = u(I - z) + \delta u(Rz)$ is such that $\arg \max_{0 \leq z \leq I} V(I - z, Rz) = S(R, I)$ has the form $S(R, I) = s(R) \cdot I$, with $s(\cdot)$ monotone increasing.

Assumption 3 For all period $t = 0, 1, 2, \dots$ the initial endowment, h_t^y , of the young generation satisfies $h_t^y = \mu h_t$, $\mu > 0$.

Under these hypotheses, tedious but straightforward algebra shows that, given d_{t-1} , the two-dimensional implicit function problem

$$\begin{aligned} h_{t+1} - h[g(x_{t+1}, h_t), h_t] &= 0 \\ s[R(x_{t+1})][w(x_t)h_t - R(x_t)d_{t-1}] - k_{t+1} - g(x_{t+1}, h_t) &= 0 \end{aligned}$$

has a well defined solution

$$h_{t+1} = \Phi^1(h_t, k_t) \tag{2.4a}$$

$$k_{t+1} = \Phi^2(h_t, k_t) \tag{2.4b}$$

Standard methods can be used to show that, given (h_t, k_t) and d_{t-1} , the equilibrium choice of (h_{t+1}, k_{t+1}) is unique and induces a Pareto efficient allocation of resources in period t . This amounts to “static efficiency”: in each period aggregate savings are allocated to equalize rates of return between the investments in physical and human capital. Things are less immediate when it comes to dynamic efficiency. In this case one asks if, given (h_0, k_0) and d_{-1} , there exists some feasible path $\{(k_t, h_t)\}_{t=0}^\infty$, other than the competitive equilibrium, which delivers more consumption during some period t without requiring less consumption during any other period. Assumptions 1-3 are sufficient to apply the characterization of dynamically efficient paths established by Cass [1972]. The original argument needs to be modified to account for the unboundedness of the feasible consumption levels. To do this, re-normalize all variables by a factor which grows at the equilibrium balanced growth rate and then apply Cass criterion to the normalized, and therefore bounded, economy. †

† Technical details are available from the authors upon request.

Depending upon the assumptions one is willing to make about the extent to which human and physical capitals can be increased from one period to the next, the dynamical system (2.4) may or may not have a fixed point

$$\begin{aligned} h^* &= \Phi^1(h^*, k^*) \\ k^* &= \Phi^2(h^*, k^*), \end{aligned}$$

other than the one at the origin. If one assumes, as we do, that both $h(d, h^y)$ and $F(h, k)$ are homogenous of degree one, then unbounded equilibrium paths are feasible and there exist preferences for which unbounded accumulation of both k_t and h_t is an equilibrium.

To illustrate this, consider the two examples we use in this paper.

Example 1. Let $u(c) = \log c$, $F(h, k) = k^\alpha h^{1-\alpha}$ and $h(d, h^y) = \lambda(h^y)d^\beta$, $\lambda : \mathfrak{R}_+ \mapsto \mathfrak{R}_+$ is continuous and monotone increasing. Manipulating the first order conditions we get

$$\begin{aligned} s_t &= \frac{\delta}{1+\delta} [w_t h_t - (1+r_t)d_{t-1}^d] \\ d_{t-1}^d &= \frac{\beta(1-\alpha)}{\alpha} k_t \end{aligned}$$

which, setting $\frac{\beta(1-\alpha)}{\alpha} = \gamma$ and using the market clearing condition for saving and investment, gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}.$$

Aggregate saving is therefore equal to

$$s_t = \left[\frac{\delta(1-\alpha)(1-\beta)}{1+\delta} \right] \left[k_t^\alpha h_t^{1-\alpha} \right]$$

The latter implies

$$k_{t+1} = \eta [k_t^\alpha h_t^{1-\alpha}] \tag{2.5a}$$

$$h_{t+1} = \lambda(h_t^y) (\gamma\eta)^\beta [k_t^\alpha h_t^{1-\alpha}]^\beta \tag{2.5b}$$

where $0 < \eta = \frac{\delta}{1+\delta} \frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Now let $h_t^y = h_t$. Picking specific functional forms for $\lambda(\cdot)$ yields different patterns of long-run behavior. One, none or more than one interior steady states may exist and they may be either asymptotically stable or unstable. Similarly, balanced growth may or may not be an equilibrium. A useful specification is $\lambda(h) = h^{1-\beta}$. The dynamical system (2.5) now reads:

$$k_{t+1} = \eta(k_t^\alpha h_t^{1-\alpha}) \quad (2.6a)$$

$$h_{t+1} = (\gamma\eta)^\beta (k_t^{\alpha\beta} h_t^{1-\alpha\beta}) \quad (2.6b)$$

The only rest point of (2.6) is the origin. The ray in the (h_t, k_t) plane

$$x^* = \frac{k_t}{h_t} = \left[\frac{\eta}{(\gamma\eta)^\beta} \right]^{\frac{1}{1-\alpha(1-\beta)}} \quad (2.7)$$

defines a balanced growth path. Straightforward algebra shows that for all initial conditions $(h_0, k_0) \in \mathfrak{R}_+^2$ iteration of (2.6) leads (h_t, k_t) to the ray x^* .

Along the balanced growth path the two stocks of capital expand (or contract) at the factor

$$1 + g = \eta \left[\frac{(\gamma\eta)^\beta}{\eta} \right]^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which is larger than one when $\eta > (1/\gamma)^{(1-\alpha)}$.

A sufficient condition for the equilibrium path to be dynamically efficient is that the gross rate of return on capital be larger or equal than one plus the growth rate of output. With linearly homogenous production functions the rate of return on capital is determined by the factor intensity ratio. Hence we need

$$x^* \leq \left(\frac{\alpha}{1+g} \right)^{1/(1-\alpha)}$$

i.e. that

$$\alpha \geq \eta \iff \frac{\alpha + \beta(1-\alpha)}{(1-\alpha)(1-\beta)} > \frac{\delta}{1+\delta}$$

Example 2. The use of a linear utility function will, in some circumstances, help our intuition. Therefore, we work out briefly the case $u(c) = c$, and keep the same production functions as before. The first order conditions of households and firms give

$$d_{t-1}^d = \frac{\beta(1-\alpha)}{\alpha} k_t$$

and s_t is $[w_t h_t - (1+r_t)d_{t-1}^d]$, or the interval $[0, w_t h_t - (1+r_t)d_{t-1}^d]$, or 0, if $-1 + \delta(1+r_t)$ is positive, zero, or negative respectively. As before, market clearing gives

$$d_{t-1} = \frac{\gamma s_{t-1}}{1+\gamma}$$

and

$$s_t = (1 - \alpha)(1 - \beta) \left[k_t^\alpha h_t^{1-\alpha} \right]$$

The latter yields

$$k_{t+1} = \bar{\eta} \left[k_t^\alpha h_t^{1-\alpha} \right] \quad (2.8a)$$

$$h_{t+1} = (\gamma \bar{\eta})^\beta \left[k_t^{\alpha\beta} h_t^{1-\alpha\beta} \right] \quad (2.8b)$$

where $0 < \bar{\eta} = \frac{(1-\alpha)(1-\beta)}{1+\gamma} < 1$. Once again, the only rest point of (2.8) is the origin. The ray in the (h_t, k_t) plane

$$\bar{x}^* = \frac{k_t}{h_t} = \left[\frac{\bar{\eta}}{(\gamma \bar{\eta})^\beta} \right]^{\frac{1}{1-\alpha(1-\beta)}} \quad (2.9)$$

defines a balanced growth path. Straightforward algebra shows that for all initial conditions $(h_0, k_0) \in \mathfrak{R}_+^2$ solutions to (2.8) converge to the ray \bar{x}^* .

Along the balanced growth path the two stocks of capital expand (or contract) at the rate

$$1 + \bar{g} = \bar{\eta} \left[\frac{(\gamma \bar{\eta})^\beta}{\bar{\eta}} \right]^{\frac{1-\alpha}{1-\alpha(1-\beta)}}$$

which is larger than one if $\bar{\eta} > (1/\gamma)^{(1-\alpha)}$.

The parameter values at which dynamic efficiency obtains are

$$\alpha \geq \bar{\eta} \iff \frac{\alpha + \beta(1 - \alpha)}{(1 - \alpha)(1 - \beta)} > 1$$

2.2 Equilibrium when Credit Markets are Missing

As we pointed out in the introduction, markets to finance education are very rare. In our model, lack of borrowing opportunities for the young generation implies that $d_t = 0$ for all t and, therefore, $h_{t+1} = h(0, h_t^y)$. This restriction would make the complete market equilibrium allocation not feasible and, by drastically reducing the investment in human capital, lead the economy to an equilibrium with a much lower aggregate growth rate. The final outcome would depend, in general, upon the specific properties of the production function $h(d, h^y)$. For example, in the Cobb-Douglas case the economy collapses to the steady state with zero of both capital stocks in just one period. Should we assume $h(d, h^y)$ to be a CES production function such that $h(0, h^y) > 0$, then $h_t > 0$ could still be maintained and positive production and accumulation could still take place even in the absence of credit markets. In any case, the associated allocation would not be efficient and the long run growth rate of aggregate consumption would be lower.

3. Introducing the State

Our analysis shows that, in the absence of well functioning credit markets profitable intergenerational trades are available which could make all citizens better off but which cannot be carried out without an outside intervention. In this section we investigate if such trades could be achieved through a simple system of intergenerational taxes and transfers. Second, and perhaps more important, we ask if, absent a benevolent planner, such system can be sustained as the equilibrium of a well defined voting game.

3.1 Public Financed Education and Pay-As-You-Go Pensions

Consider the following scheme. In each period t two lump-sum taxes are levied to provide resources for two simultaneous lump-sum transfers.

The first tax is levied only on the middle age generation and its proceedings are used to pay out a pension to the old-age, retired individuals. We assume a period-by-period balanced budget, hence

$$T_t^p = P_t \quad (3.1)$$

The second tax is instead levied on all income recipients, middle age and elderly alike, and its revenues are used to finance investment in the education of the young generation. Balanced budget, again, implies

$$T_t^e = E_t \quad (3.2)$$

The period-by-period budget constraint for the representative member of the generation born in period $t - 1$ is therefore

$$0 \leq d_{t-1} \leq E_{t-1} \quad (3.3a)$$

$$c_t^m + s_t \leq w_t h_t - T_t^p - T_t^{em} \quad (3.3b)$$

$$c_{t+1}^o \leq R_{t+1} s_t + P_{t+1} - T_{t+1}^{eo} \quad (3.3c)$$

$$h_t = h(d_{t-1}, h_{t-1}^y) \quad (3.3d)$$

where $T_t^e = T_t^{em} + T_t^{eo}$ and the latter denote, respectively, the lump-sum education tax levied on the middle age and the lump-sum education tax levied on the old individuals at time t .

Let starred symbols, *e.g.* d_t^* , w_t^* , etc., denote the equilibrium quantities and prices that would arise in the complete market model of Section 2.1. From now on, we will

refer to this allocation as “the complete market allocation”. Comparison of equations (3.3) with the budget restrictions in problem (2.1) shows that, if the lump-sum amounts $(T_t^p, P_t, T_t^{em}, T_t^{eo}, E_t)$ are chosen to satisfy

$$E_t = d_t^{d*}, \quad T_t^{em} + T_t^{eo} = d_t^{s*}, \quad P_t - T_t^{eo} = d_{t-1}^* R_t^*, \quad (3.4)$$

then the competitive equilibrium achieves the complete market allocation. In other words, a benevolent planner can restore efficiency and improve long-run growth by establishing publicly financed education and pay-as-you-go pensions and by linking the two flows of payments via the market interest rate factor.

The scheme just described contains a redundant “double transfer” and, therefore, a degree of indeterminacy. In the complete market allocation the middle age generation is the only one investing in public education. For this reason, in the tax-and-transfer scheme it must ultimately be paying for the whole amount E_t . Hence, one can either set $T_t^{eo} \equiv 0$ and $E_t = T_t^{em}$ or let $T_t^{eo} > 0$ be an arbitrary number and set $P_t = T_t^{eo} + d_{t-1}^* R_t^*$ as in (3.4). The two settings are equivalent but the second allows the interpretation of E_t as financed by a general income tax.

A “Public Education and Public Pension” scheme (PEPP) satisfying restrictions (3.1), (3.2) and (3.4) would also be actuarially fair in the following sense. The pension payments (contributions) that a typical citizen receives (pays) during the third (second) period of his life correspond to the capitalized value of the educational taxes (transfers) he contributed (received) during the second (first) period of his life. These quantities are capitalized at the market rate of interest. †

$$E_t R_{t+1}^* = T_{t+1}^p - T_{t+1}^{eo} \quad (3.5a)$$

$$T_t^e R_{t+1}^* = P_{t+1} - T_{t+1}^{eo} \quad (3.5b)$$

Our next step is to inquire if the efficient allocation could be implemented by a simple, dynamic game of collective decision making based upon period-by-period majority voting.

† In the applied literature on contribution-based Social Security systems the issue of “actuarial fairness” between contributions paid and pensions received is an actively debated topic. Our model suggests that, maybe, one should look for actuarial fairness somewhere else, that is between contributions paid and amount of public funding for education received on the one hand, and between taxes devoted to public education and pension payments on the other.

3.2 Voting upon Public Education and Public Pensions

Consider the same framework as before but assume that the amounts $P_t - T_t^{eo}$ and E_t are not set by a benevolent planner. Instead assume that, in each period t , the members of the middle age and old generations vote on whether and at which level those amounts should be levied. Imagine that all conceivable combinations of (positive) taxes $T_t^p, T_t^{em}, T_t^{eo}$ and transfers P_t, E_t satisfying the budgetary restrictions (3.1) and (3.2) are listed in the ballot and that individual voters, being of negligible measure, vote sincerely in favor of the proposal they like the best. Votes are then counted and the proposal being supported by a (simple) majority of the population is implemented until further voting takes place. To avoid the intricacies of voting theory, which are not our concern here, and to break the tie between the two generations assume the middle age population is $\epsilon > 0$ larger than the old, for ϵ whatever small. Then the median coincides with the middle age voter.

This cursory description of the majority voting mechanism is too vague. Indeed, it is easy to see that many different “rules of the game” are consistent with the verbal description we have just given. One can think of these different sets of rules as different “constitutions”, allowing the median voter the right to make some decisions and forbidding other.

We study two such intergenerational voting games which, we believe, come close to capturing some important features of real world political decision making on public education and pensions. In particular, our two games are consistent with the following facts: (i) the amounts invested in education are established year-by-year or over longer spans of time, but, in no circumstance, are they fixed once and for all at the outset of the system; (ii) pension payments made in any given period are sometimes already established by previous legislation and sometimes established by contemporary legislation.

In both games players cast two independent votes in each period, one upon education and one upon pension payments. In the first game, call it the *defined benefit game*, player t also establishes an amount which he should receive, in the form of a net pension, in period $t + 1$. Not receiving such payment is tantamount to the pension system defaulting upon its commitment. Default is possible because the period $t + 1$ player must approve or disapprove the pension payment. In the second game, instead, player t is not allowed to legislate any promised amount. In each period voters pick the two amounts to be paid, respectively, to the old and young generation. We call this the *defined contribution game*;

to the extent that pensions in period t are paid out of social security contributions levied in the same period, the name is justified.

3.2.1 The defined benefit game

If, at time t , a PEPP does not exist voters must decide to either remain without one or to introduce it. To remain without one is equivalent to a (simple) majority of voters to favor the $(0, 0)$ pair. To introduce a PEPP when one does not yet exist, voters must decide (i) which lump-sum education tax T_t^e should be levied to provide the resources E_t , (ii) the net transfer (if any) to be paid to the currently old generation, $P_t - T_t^{eo}$, and, (iii) a “proposal” to be made for the net transfer $P_{t+1} - T_{t+1}^{eo}$ the currently middle age individuals will receive next period.

If a PEPP is already in place the citizens will be asked if they like to disband it or not. Keeping it entails paying the promised amount to the old people, deciding an amount to transfer to the young and setting a promise for the (net) payment the current player is entitled to claim next period.

Individuals are homogeneous within a given generation and the restrictions introduced so far imply only that

$$0 \leq P_t - T_t^{eo} \leq w_t h_t; \quad 0 \leq E_t \leq w_t h_t; \quad E_t + P_t - T_t^{eo} \leq w_t h_t \quad (3.6)$$

Given sincere voting, the decisive vote is cast by a member of the middle age generation. In period t given (k_t, h_t) and E_{t-1} , the middle age voter is faced by a triple choice:

- (a) pay or not pay to the elderly the amount $P_t - T_t^{eo}$ they expect;
- (b) choose how much, if anything, to invest in the education of the young;
- (c) establish the amount $P_{t+1} - T_{t+1}^{eo}$ to expect next period.

Paying the promised pension is equivalent to an investment for the middle age. Its profitability depends upon the equilibrium strategy adopted by the next player: will he pay $P_{t+1} - T_{t+1}^{eo}$ to her if she pays $P_t - T_t^{eo}$ to the elderly? If the answer is “yes”, she must also compare the amount $P_{t+1} - T_{t+1}^{eo}$ with the payoffs obtainable by investing the amount $P_t - T_t^{eo}$ somewhere else. Only when the former dominates the latter the pension payment will be approved.

The payment in (b) is also an investment but only a portion of its return depends upon the equilibrium strategy adopted by the following player. Investing in the education

of the young pays off in *two* ways for the middle age. The first is through the relation, if any, that may be established between the amount E_t and the amount she can reasonably expect to receive as a net pension when old. This portion of the total return on E_t does depend upon the action of other players and, among other things, may also depend upon the choice made at time t of paying or not the amount $P_t - T_t^{eo}$ to the current elderly. On the other hand, investing in human capital has a second, direct payoff for the middle age generation insofar as it increases the equilibrium rate of return on physical capital R_{t+1} . As long as the middle age voter expects $s_t > 0$ she can try to maximize her total return $k_{t+1}R_{t+1}$ by wisely choosing E_t .

Finally, the payoff of (c) depends on the equilibrium strategies of the future players, on the action of the current player (has she paid the pension in (a)? How much has she invested in (b)?) and upon the possibility for player $t+1$ to find a $P_{t+2} - T_{t+2}^{eo}$ which looks profitable to him and approvable by the next generation, and so on for the infinite future.

3.2.2 The defined contribution game

In period t , given the two stocks of capital k_t and h_t , voters are supposed to pick *both* amounts E_t and $P_t - T_t^{eo}$ under the restrictions (3.6). The middle age voter is the decisive one in this case as well. Given sincere voting, she elects two quantities

- (a') how much should be paid to the old generation via the pension system, $P_t - T_t^{eo}$;
- (b') how much, if anything, to invest in the education of the young, E_t .

Paying a pension still can be seen as an investment for the middle age. Its profitability depends entirely upon the equilibrium strategy being played by the next generation, more precisely it depends upon the rule adopted to determine $P_{t+1} - T_{t+1}^{eo}$. Does this generate a payoff higher than the one achievable by investing the amount $P_t - T_t^{eo}$ somewhere else? Only when the former dominates the latter the pension payment will be positive.

As in the defined benefit game, the return from (b') depends *both* upon the effect of E_t on the total return on capital $k_{t+1}R_{t+1}$ and upon the equilibrium strategy adopted by the next player.

3.3 Voting

We now analyze equilibrium voting strategies in the two PEPP games.

3.3.1 Voting in the defined benefit game

Following along the lines of Boldrin and Rustichini (1995), we begin with a simplified version of the game. There are countably many players who move sequentially. At time t the designated player can choose to give, out of her pocket, an amount $P_t - T_t^{eo} \geq 0$ to the previous player and an amount $E_t \geq 0$ to the player following her. We allow for two independent decisions here: player t may decide to give $P_t - T_t^{eo}$ to player $t - 1$ and deny the gift to player $t + 1$ or, viceversa, decide to give only E_t to player $t + 1$ and deny $P_t - T_t^{eo}$ to the player before her. The logic through which the precise values of $P_t - T_t^{eo}$ and E_t are selected will be determined later and those amounts should be taken as given until later in this subsection when the complete defined benefit game will be introduced. The action of each player is perfectly observable by all those following her. The actions available to each player are

$$a_t = (a_t^e, a_t^p) \in \left\{ \{\mathcal{Y}, \mathcal{N}\} \times \{\mathcal{Y}, \mathcal{N}\} \right\}$$

where \mathcal{Y} stands for *yes* and \mathcal{N} stands for *no*. For a given sequence $\{E_t, (P_t - T_t^{eo})\}_{t=0}^{\infty}$ the interpretation of the four different actions is the following. $(\mathcal{Y}, \mathcal{Y})$ corresponds to accepting both public education and public pensions; $(\mathcal{Y}, \mathcal{N})$ corresponds to accepting public education but not public pensions; symmetrically $(\mathcal{N}, \mathcal{Y})$ shuts down public education and maintains public pensions while, finally, $(\mathcal{N}, \mathcal{N})$ turns down both transfer systems.

A history \mathcal{H}_{t-1} of the game at time t , when it is player's t turn to move, is:

$$\mathcal{H}_{t-1} = (a_1, a_2, \dots, a_{t-1})$$

so that a strategy for player t is a map

$$\sigma_t = (\sigma_t^e, \sigma_t^p) : (a_1, a_2, \dots, a_{t-1}) \mapsto \left\{ \{\mathcal{Y}, \mathcal{N}\} \times \{\mathcal{Y}, \mathcal{N}\} \right\}.$$

Identify \mathcal{Y} with 1 and \mathcal{N} with 0. The payoff for player t is determined by the value function $V_t : (\{0, 1\} \times \{0, 1\})^2 \mapsto \mathfrak{R}$ defined by

$$V_t(a_t, a_{t+1}) \equiv \max u(c_t^m) + \delta u(c_{t+1}^o)$$

$$\text{subject to : } c_t^m + s_t \leq w_t h_t - [P_t - T_t^{eo}]a_t^p - E_t a_t^e$$

$$\text{and : } c_{t+1}^o \leq R_{t+1} s_t + [P_{t+1} - T_{t+1}^{eo}]a_{t+1}^p$$

Barring the uninteresting case in which the proposed sequences cannot improve upon generational autarchy, any of the following three situations may occur

$$V_t[(1, 0); (1, 0)] \geq V_t[(0, 0); (0, 0)] \quad \text{for all } t. \quad (3.7a)$$

$$V_t[(0, 1); (0, 1)] \geq V_t[(0, 0); (0, 0)] \quad \text{for all } t. \quad (3.7b)$$

$$V_t[(1, 1); (1, 1)] \geq V_t[(0, 0); (0, 0)] \quad \text{for all } t. \quad (3.7c)$$

Inequality (3.7a) is by far the weakest and can be interpreted as follows. The amount E_t is such that, over the life cycle, the middle age is better off investing in the education of the young, even when the latter chooses not to pay her a pension. This relies on the fact that an increase in h_{t+1} and a decrease in k_{t+1} (which is what E_t accomplishes) may raise $R_{t+1}k_{t+1}$ by enough to compensate for the loss in consumption at time t . As (3.7a) implies $V_t[(1, 0); (0, 0)] \geq V_t[(0, 0); (0, 0)]$, each generation of middle age voters will have an incentive to introduce a public education system even when all past and future voters are electing to do nothing. These circumstances have been considered in some details in Boldrin (1994).

Inequality (3.7b), on the other hand, means that the proposed pension payments are such that, by paying $P_t - T_t^{eo}$ when middle age and receiving $P_{t+1} - T_{t+1}^{eo}$ when old, each generation achieves a higher life time utility than under generational autarchy. Such utility gain has two sources. One is the well known reduction of capital over-accumulation (Samuelson (1958), Diamond (1977)) while the second is, in analogy with (3.7a), a large enough increase in $R_{t+1}k_{t+1}$ brought about by the reduction in k_{t+1} . These are the cases considered in Boldrin and Rustichini (1995).

Restriction (3.7c) combines these two effects together. Notice though that, for a given sequence $\{E_t, (P_t - T_t^{eo})\}_{t=0}^{\infty}$ this does not imply that the l.h.s of (3.7c) should dominate the l.h.s. of either (3.7a) or (3.7b). This is important for our analysis. We have shown in Section 2 that the sequence $\{E_t^*, (P_t - T_t^{eo})^*\}_{t=0}^{\infty}$ implementing the complete market allocation requires, in general, that *both* E_t^* and $(P_t - T_t^{eo})^*$ be strictly positive. The complete market allocation can therefore be sustained as a subgame perfect equilibrium (SPE) of our voting game only if the equilibrium outcome is $\{a_t^e = 1, a_t^p = 1\}_{t=0}^{\infty}$ when $\{E_t^*, (P_t - T_t^{eo})^*\}_{t=0}^{\infty}$ is being voted upon. It is easy to see that, for this to be the case, restriction (3.7c) is necessary but not sufficient. Indeed we have

Proposition 1 *Let the candidate sequence $\{E_t, (P_t - T_t^{eo})\}_{t=0}^{\infty}$ be given and assume (3.7) are true. Furthermore, let*

$$V_t[(1, 1); (1, 1)] \geq V_t[(1, 0); (1, 0)] \quad \text{for all } t.$$

and

$$V_t[(1, 1); (1, 1)] \geq V_t[(0, 1); (0, 1)] \quad \text{for all } t.$$

Then, for all σ_0 , the strategy:

$$\sigma_t^e = 1, \sigma_t^p = \min_t \{a_1^p, a_2^p, \dots, a_{t-1}^p\}$$

for all $t \geq 1$ is a SPE of the defined benefit game.

Proof: By definition of SPE, we have to prove that for any subgame given by a history \mathcal{H}_{t-1} , the strategy profile $(\sigma_t, \sigma_{t+1}, \dots)$ restricted to the history \mathcal{H}_{t-1} is a Nash equilibrium. Notice first that (3.7a – c), the non negativity restrictions on $\{E_t, (P_t - T_t^{eo})\}_{t=0}^{\infty}$ and the specification of σ_t^p imply that $\sigma_t^e = 1$ is a SPE. As for σ_t^p , we have two possible cases.

Case 1: $\min\{a_1^p, \dots, a_{t-1}^p\} = 0$; then $\min\{a_1^p, \dots, a_{t-1}^p, a_t^p\} = 0$ for any a_t ; so $\sigma_{t+1}^p(\mathcal{H}_{t-1}, a_t^p) = 0$, and so (since $V_t[(\cdot, 0), (\cdot, 0)] > V_t[(\cdot, 1), (\cdot, 0)]$), the best choice for player t is 0, which is equal to $\min\{a_1^p, \dots, a_{t-1}^p\}$.

Case 2: $\min\{a_1^p, \dots, a_{t-1}^p\} = 1$; then $\min\{a_1^p, \dots, a_{t-1}^p, a_t^p\} = a_t^p$ for all a_t ; then $\sigma_{t+1}^p(\mathcal{H}_{t-1}, a_t^p) = a_t^p$. Now from the assumption that $V_t[(\cdot, 1), (\cdot, 1)] \geq V_t[(\cdot, 0), (\cdot, 0)]$ the best choice for t is 1, and the claim follows. **Q.E.D.**

Our two additional assumptions are very strong. Nevertheless, they are necessary to sustain the complete market allocation as a SPE of the political game we are studying, as the examples of section 4 will show. The argument in the proof also suggests a strong asymmetry in the behavior of the two components of a PEPP system under a defined benefit rule. While the public financing of education is a rather “stable” institution, the same is not true of pay-as-you-go pension plans. Indeed the model suggests that, when one is in place, a one-period default is enough to destroy the credibility of the system for the very long future. This is consistent with our discussion of the different nature of the investment in education and the investment in pensions: the return on the second depends entirely on the equilibrium behavior of the other players while the return on the first is, at least in part, independent from other players’ action.

In the simple version of the defined benefit game that we have analyzed, the amounts involved are exogenously given. To obtain a complete model of the PEPP we should

endogenize the choice of the lump-sum amounts used. Write $\omega_t = w_t h_t$. In the full defined benefit game, a history \mathcal{H}_{t-1} at time t is a sequence:

$$\mathcal{H}_{t-1} = \{(a_1, T_1), (a_2, T_2) \dots, (a_{t-1}, T_{t-1})\}$$

where for each $s \in (1, \dots, t-1)$, $T_s = (E_s, P_{s+1} - T_{s+1}^{eo}) \in [0, \omega_s) \times [0, \omega_{s+1})$ and $a_s \in \{\mathcal{Y}, \mathcal{N}\}$.

The interpretation of the action space is the following. Player t chooses a level $E_t \geq 0$ of public education financing and proposes a net pension payment $P_{t+1} - T_{t+1}^{eo}$ she should receive next period. Finally, the binary action a_t amounts to approving or not the payment $P_t - T_t^{eo}$ she is being requested by the currently old generation.

Take now any infinite history $\mathcal{H} \equiv \{(a_1, T_1), \dots, (a_t, T_t), \dots, \}$. We say:

Definition 1 *The defined benefit game is the extensive form game where:*

- (1) *players are indexed by $t = (1, 2, \dots)$;*
- (2) *the action set of each player is $\{\mathcal{Y}, \mathcal{N}\} \times [0, \omega_t] \times [0, \omega_{t+1}]$; the strategy of player t is a map $\sigma_t : \mathcal{H}_{t-1} \mapsto \{\mathcal{Y}, \mathcal{N}\} \times [0, \omega_t] \times [0, \omega_{t+1}]$;*
- (3) *for every history \mathcal{H} of actions, the payoff to player t is equal to the lifetime utility of the representative agent born at time $t - 1$ in the competitive equilibrium of the economy with $\{E_t, (P_{t+1} - T_{t+1}^{eo})\}_{t=0}^\infty$ in the budget constraints (3.3).*

Definition 2 *For a given initial condition (k_0, h_0, d_{-1}) , a **political equilibrium** is a sequence $\{a_t, T_t, w_t, R_t, c_t^m, c_{t+1}^o, h_t, k_t\}_{t=0}^\infty$ such that:*

- (1) *the sequence $\{w_t, R_t, c_t^m, c_{t+1}^o, h_t, k_t\}_{t=0}^\infty$ is a competitive equilibrium given $\{a_t, T_t\}_{t=0}^\infty$;*
- (2) *there exists a sequence of strategies $\{\sigma_t\}_{t=0}^\infty$ for the defined benefit game, which is a subgame perfect equilibrium, and such that $\{T_t\}_{t=0}^\infty$ is the sequence of effective lump-sum taxes and transfers associated with the equilibrium history.*

One can use the latter definition to re-interpret Proposition 1. It says that *any* sequence of taxes and transfers $\{E_t, (P_t - T_t^{eo})\}_{t=0}^\infty$ satisfying

$$V_t[(E_t, P_t - T_t^{eo}); (E_{t+1}, P_{t+1} - T_{t+1}^{eo})] \geq V_t[(0, 0); (0, 0)] \quad \text{for all } t. \quad (3.8a)$$

$$V_t[(E_t, P_t - T_t^{eo}); (E_{t+1}, P_{t+1} - T_{t+1}^{eo})] \geq V_t[(\tilde{E}_t, 0); (\tilde{E}_{t+1}, 0)] \quad (3.8b)$$

$$V_t[(E_t, P_t - T_t^{eo}); (E_{t+1}, P_{t+1} - T_{t+1}^{eo})] \geq V_t[(0, \tilde{P}_t - \tilde{T}_t^{eo}); (0, \tilde{P}_{t+1} - \tilde{T}_{t+1}^{eo})] \quad (3.8c)$$

for all alternative sequences $0 \leq \tilde{E}_t + (\tilde{P}_t - \tilde{T}_t^{eo}) \leq \omega_t$ for all t , may be an equilibrium of the defined benefit game.

As it is often the case in repeated or dynamic games, the set of SPE is very large. It should be clear from the previous discussion that the complete market allocation may be one of them, under certain parametric restrictions. The nature of these restrictions will be illustrated in section 4, by means of our two examples. We will discuss momentarily, in subsection 3.4, the predictions of our model for the case in which the complete market allocation obtains as a SPE. The empirical analysis of Section 5 is meant to shed some light upon the extent to which such predictions are corroborated by actual data.

3.3.2 Voting in the defined contribution game

As in the previous game there are countably many players who move sequentially. Their actions are perfectly observable by all following players and predictable by all previous players. At time t the designated player chooses to give, out of her income $w_t h_t = \omega_t$, an amount $P_t - T_t^{eo} \geq 0$ to the previous player and an amount $E_t \geq 0$ to the player following her. Also in this case the two choices are, a-priori, independent.

The set of actions available to each player is

$$\mathcal{A}_t = \{a_t = (a_t^e, a_t^p) \in [0, \omega_t]^2; \quad a_t^e + a_t^p \leq \omega_t\}$$

A history \mathcal{H}_{t-1} of the game at time t , when it is player's t turn to move, is:

$$\mathcal{H}_{t-1} = (a_1, a_2, \dots, a_{t-1})$$

and a strategy for player t is a map

$$\sigma_t = (\sigma_t^e, \sigma_t^p) : \mathcal{H}_{t-1} \mapsto \mathcal{A}_t.$$

The payoff for player t is determined by the value function $U_t : (\mathcal{A}_t \times \mathcal{A}_{t+1}) \mapsto \mathfrak{R}_+$ defined by

$$\begin{aligned} U_t(a_t, a_{t+1}) &\equiv \max u(c_t^m) + \delta u(c_{t+1}^o) \\ &\text{subject to : } c_t^m + s_t \leq \omega_t - a_t^p - a_t^e \\ &\text{and : } c_{t+1}^o \leq R_{t+1} s_t + a_{t+1}^p \end{aligned}$$

Take now any infinite history $\mathcal{H} \equiv \{a_1, \dots, a_t, \dots\}$. Formally we say:

Definition 3 *The defined contribution game is the extensive form game where:*

- (1) *players are indexed by $t = (1, 2, \dots)$;*
- (2) *the action set of each player is \mathcal{A}_t ; the strategy of player t is a map $\sigma_t : \mathcal{H}_{t-1} \mapsto \mathcal{A}_t$; the payoff of each player is the function $U_t : (\mathcal{A}_t \times \mathcal{A}_{t+1}) \mapsto \mathfrak{R}_+$*
- (3) *for every history \mathcal{H} of actions, the payoff to player t is equal to the lifetime utility of the representative agent born at time $t - 1$ in the competitive equilibrium of the economy with $\{E_t = a_t^e, (P_t - T_t^{eo}) = a_t^p\}_{t=0}^\infty$ in the budget constraints (3.3).*

A political equilibrium can be easily defined by appropriately modifying Definition 2 of the previous subsection. We now proceed to characterize one, important, class of equilibria for the defined contribution game.

We begin by noticing that, in the defined contribution game, a class of relatively simple Markovian strategies is compatible with sub-game perfectness. To put it formally

Proposition 2 *Let k_0 and h_0 be given. Assume the sequence of non-negative real valued functions $\{a_t(k_t, h_t) = (a_t^e(k_t, h_t), a_t^p(k_t, h_t))\}_{t=0}^\infty$ satisfy*

$$U_\tau \left[a_\tau(k_\tau, h_\tau); a_{\tau+1}(k_{\tau+1}, h_{\tau+1}) \right] \geq U_\tau \left[(\tilde{a}_\tau^e, \tilde{a}_\tau^p); (a_{\tau+1}^e(\tilde{k}_{\tau+1}, \tilde{h}_{\tau+1}), 0) \right] \quad (3.10)$$

for all $\tau \geq 0$, where

- *The pair $(\tilde{a}_\tau^e, \tilde{a}_\tau^p)$ is a one-period deviation from $(a_\tau^e(k_\tau, h_\tau), a_\tau^p(k_\tau, h_\tau))$;*
 - *$\{(k_t, h_t)\}_{t=0}^\infty$ are the competitive equilibrium stocks of capital for an economy as in (3.3), with initial conditions (k_0, h_0) and sequences of taxes and transfers equal to $E_t = a_t^e(k_t, h_t)$ and $P_t - T_t^{eo} = a_t^p(k_t, h_t)$.*
 - *The pair $(\tilde{k}_{\tau+1}, \tilde{h}_{\tau+1})$ obtains from (k_τ, h_τ) under the one-period deviation $(\tilde{a}_\tau^e, \tilde{a}_\tau^p)$.*
- Then the strategies $\{\sigma_t^* = (\sigma_t^{e*}, \sigma_t^{p*})\}_{t=0}^\infty$ defined as*

$$\begin{aligned} \sigma_t^{e*} &= a_t^e(k_t, h_t), \\ \sigma_t^{p*} &= a_t^p(k_t, h_t); \quad \text{if } a_{t-1} = \sigma_{t-1}^*, \\ \sigma_t^{p*} &= 0; \quad \text{otherwise.} \end{aligned}$$

are a SPE of the defined contribution game.

Proof:

The proof is straightforward once we realize that inequality (3.10) also implies that

$$U_\tau \left[(a_\tau^e(\tilde{k}_\tau, \tilde{h}_\tau), 0); a_{\tau+1}(k_{\tau+1}, h_{\tau+1}) \right] \geq U_\tau \left[(\tilde{a}_\tau^e, \tilde{a}_\tau^p); (a_{\tau+1}^e(\tilde{k}_{\tau+1}, \tilde{h}_{\tau+1}), 0) \right]$$

where the pair $(\tilde{k}_{t+1}, \tilde{h}_{t+1})$ obtains from (k_t, h_t) under the deviation $(\tilde{a}_t^e, \tilde{a}_t^p)$. The latter means that punishing a deviator is better than entering into a further deviation. **Q.E.D.**

The equilibrium strategies considered in proposition 2 are quite different from those derived, for the defined benefit game, in proposition 1. In fact they are Markovian, *i.e.* current actions only depend upon the current state of the world and the actions of the immediately previous player. The current state of the economy matters in determining the current action, as the amounts a_t^e and a_t^p are functions of (k_t, h_t) and change with them. Notice, in particular, that a one-period deviation moves k and h away from the values they would have assumed if the deviation had not occurred. Hence, equilibrium quantities a_t^e and a_t^p must depend upon realized stocks of capital. Markovianity has an important, practical, implication. In proposition 1 the equilibrium strategies depend upon the whole past history; hence a one-time deviation implies a dismissal of the pension system forever. This is not true in the case of proposition 2: a deviation in period t only requires a punishment in period $t + 1$. Payment of the equilibrium pension may resume in period $t + 2$. This prediction appears to be less farfetched than the former. The equilibrium generated by σ_t^* allows the generations following a deviating one to re-capture the benefits of the PEPP system. and predicts that the political game would not lead players to leave unexploited some profitable intergenerational arrangements.

An important application of proposition 2 is the following. Let k_0, h_0 and h_{-1} be given and denote with starred symbols the quantities and prices that would arise in a competitive equilibrium with complete markets starting at (k_0, h_0, h_{-1}) . Denote with E_t^* and $P_t^* = P_t - T_t^{eo}$ the sequences of lump-sum taxes and transfers satisfying (3.1), (3.2) and (3.4). Recall, again, that both E_t^* and P_t^* are functions of the current stocks k_t and h_t . Assume that in each period t and for any feasible deviation $(\tilde{a}_t^e, \tilde{a}_t^p)$

$$U_t \left[(E_t^*, P_t^*); (E_{t+1}^*, P_{t+1}^*) \right] \geq U_t \left[(\tilde{a}_t^e, \tilde{a}_t^p); (E_{t+1}^*(\tilde{k}_{t+1}, \tilde{h}_{t+1}), 0) \right] \quad (3.11)$$

where the pair $(\tilde{k}_{t+1}, \tilde{h}_{t+1})$ obtains from (k_t^*, h_t^*) under the deviation $(\tilde{a}_t^e, \tilde{a}_t^p)$. Then the strategy

$$\begin{aligned} \sigma_t^{e*} &= E_t^*(k_t, h_t), \\ \sigma_t^{p*} &= R_t^* E_{t-1}^*, \quad \text{if } a_{t-1} = \sigma_{t-1}^*; \\ \sigma_t^{p*} &= 0, \quad \text{otherwise.} \end{aligned}$$

satisfies proposition 2 and, therefore, is a SPE of the defined contribution game. When (3.11) holds this strategy supports the complete market allocation as an outcome of the political equilibrium.

3.4 The efficient allocation as an equilibrium of the political game

The analysis carried out in the previous subsection suggests that the complete market allocation may be achievable even when a benevolent planner is not around to dictate it. It also shows, though, that when each generation is allowed to choose taxes and transfers many other equilibria are achievable, which are not necessarily efficient, and that each generation is faced by incentives that only under certain parametric restrictions may lead it to choose the complete market quantities E_t^* and P_t^* .

When the complete market allocation satisfies conditions (3.8) it will emerge as an equilibrium of defined benefit game. Similarly, when it satisfies (3.11) it will be chosen in the defined contribution game. Both conditions are relatively strong and, as the examples of section 4 show, they are satisfied only for certain subsets of the parameter space. Nevertheless, there exists one, crucial, implication of our results which is clearly observable. That is

$$P_{t+1}^* = R_{t+1}^* T_t^{e*} \quad (3.12a)$$

and, therefore

$$T_{t+1}^{p*} = R_{t+1}^* E_t^* \quad (3.12b)$$

Proposition 3 *If the political equilibrium induced by a PEPP system supports the complete market allocation, the following should be observed. The implicit rate of return i_t that, along the life cycle, equalizes the discounted value of educational services received to the discounted value of social security contributions paid is equal to the implicit rate of return π_t that, along the life cycle, equalizes the discounted value of contributions to public education paid to the discounted value of pension payments received. Furthermore, if $R_t^* = 1 + r_t^*$ denotes the rate of return on physical capital we have*

$$1 + r_t^* = 1 + \pi_t = 1 + i_t.$$

In section 5 we test these predictions by using Spanish data.

4. Examples of the political equilibria

While conditions (3.8) and (3.11) are derived from two different games inspection shows that they are algebraically equivalent for a given set of preferences, technology and initial conditions. To avoid repetition, we will therefore consider their parametric implications only for the, relatively more straightforward, defined contribution game and just add a few remarks on the defined benefit game.

4.1 Sustainability of the efficient allocation in the defined contribution game

Example 1 (Continue). Assume the economy has followed the complete market allocation until period t . Let k_t^* , h_t^* and E_{t-1}^* be given. Consider the alternative payoffs open to agent t . If she follows the candidate equilibrium strategy her payoff is

$$U_t \left[(E_t^*, P_t^*); (E_{t+1}^*, P_{t+1}^*) \right] = \log[\omega_t^* - k_{t+1}^* - E_t^* - P_t^*] + \delta \log \left[R_{t+1}^* (E_t^* + k_{t+1}^*) \right],$$

whereas by leaving the pension system she can save $P_t^* = R_t^* E_{t-1}^*$ and still invest in the education of the young. If she chooses $\tilde{a}_t^p = 0$, the equilibrium strategy implies that $\sigma_{t+1}^{p*} = a_{t+1}^{p*} = 0$. Denoting with starred symbols the values associated to the candidate (complete market) equilibrium and with a tilde those induced by the one-period deviation. The payoff from deviating is

$$U_t \left[(\tilde{a}_t^e, 0); (a_{t+1}^{e*}, 0) \right] = \log[\omega_t^* - \tilde{k}_{t+1} - \tilde{a}_t^e] + \delta \log[\tilde{R}_{t+1} \tilde{k}_{t+1}],$$

with

$$\tilde{a}_t^e = \arg \max_{0 \leq a_t^e \leq \omega_t^*} \log[\omega_t^* - \tilde{k}_{t+1}(\tilde{a}_t^e) - \tilde{a}_t^e] + \delta \log[\tilde{R}_{t+1}(\tilde{a}_t^e) \cdot \tilde{k}_{t+1}(\tilde{a}_t^e)] \quad (4.1)$$

where

$$\tilde{k}_{t+1} = \tilde{s}_t = \frac{\delta}{1 + \delta} (\omega_t^* - \tilde{a}_t^e) \quad \text{and} \quad \tilde{R}_{t+1} = \alpha \left[\frac{(\tilde{a}_t^e)^\beta (h_t^*)^{1-\beta}}{\delta (\omega_t^* - \tilde{a}_t^e)} \right]^{1-\alpha}.$$

Plugging these values in problem (4.1) we get

$$\tilde{a}_t^e = \frac{\gamma \delta \alpha}{1 + \delta(1 + \gamma) \alpha} \omega_t^*, \quad (4.2a)$$

and

$$\tilde{k}_{t+1} = \frac{\delta}{1 + \delta} \frac{1 + \delta \alpha}{1 + \delta(1 + \gamma) \alpha} (1 - \alpha) \left[(k_t^*)^\alpha (h_t^*)^{1-\alpha} \right] \quad (4.2b)$$

$$\tilde{h}_{t+1} = \left(\frac{\gamma \delta \alpha}{1 + \delta(1 + \gamma) \alpha} (1 - \alpha) \right)^\beta \left[(k_t^*)^{\alpha \beta} (h_t^*)^{1-\alpha \beta} \right] \quad (4.2c)$$

The complete market allocation is sustainable as an equilibrium when

$$U_t \left[(E_t^*, P_t^*); (E_{t+1}^*, P_{t+1}^*) \right] \geq U_t \left[(\tilde{a}_t^e, 0); (a_{t+1}^{e*}, 0) \right] \quad (4.3)$$

Comparing the two allocations we have that

$$\tilde{k}_{t+1} > k_{t+1}^* + E_t^* \quad \tilde{c}_t^m > c_t^{m*} \quad \tilde{R}_{t+1} < R_{t+1}^*$$

Hence, the complete market allocation is sustainable only if the higher rate of return it achieves during the second period compensates for both the consumption lost and the smaller total investment undertaken during the first period. Substituting in (4.3), sustainability reduces to

$$(1 + \delta) \log \left[\frac{\omega_t^* - P_t^*}{\omega_t^* - \tilde{a}_t^e} \right] + \delta \log \left[\frac{R_{t+1}^*}{\tilde{R}_{t+1}} \right] \geq 0$$

Algebraic manipulation shows that the latter is equivalent to

$$(1 + \delta) \log(A) + \delta(1 - \alpha)\beta \log \left[\frac{(1 + \alpha\delta)}{\alpha(1 + \delta)} \right] + \delta(1 - \alpha)(1 - \beta) \log \left[\frac{1 + \gamma}{A} \right] \geq 0 \quad (4.4)$$

where

$$A = \frac{(1 - \beta)[1 + \delta(1 + \gamma)\alpha]}{1 + \alpha\delta} < 1.$$

Inequality (4.4) is therefore necessary and sufficient for the efficient allocation to satisfy proposition 2.

Example 2 (Continue). Again, assume the economy has followed the complete market allocation until period t and let k_t^* , h_t^* and E_{t-1}^* be given. If player t follows the candidate equilibrium strategy her payoff is

$$U_t \left[(E_t^*, P_t^*); (E_{t+1}^*, P_{t+1}^*) \right] = R_{t+1}^* [E_t^* + k_{t+1}^*].$$

Her optimal deviation depends upon both initial conditions and parameter values. Equilibrium saving of generation t satisfies

$$\max_{0 \leq s_t \leq \omega_t^*} \omega_t^* - s_t - a_t^e + s_t R_{t+1} \quad (4.5)$$

the solution to which depends upon the equilibrium value R_{t+1} . To determine the latter we need to know the chosen value of a_t^e , which, along a deviation, solves

$$\max_{\tilde{a}_t^e} U_t \left[(\tilde{a}_t^e, 0); (a_{t+1}^{e*}, 0) \right]$$

The latter, together with the first order conditions of problem (4.5), gives

$$\begin{aligned}
\tilde{a}_t^e &= \gamma/(1 + \gamma)\omega_t^*, & \text{if } \tilde{R}_{t+1} > 1; \\
\tilde{s}_t &= 1/(1 + \gamma)\omega_t^*, & \text{if } \tilde{R}_{t+1} > 1; \\
\tilde{a}_t^e + \tilde{s}_t &\in [0, \omega_t^*], & \text{if } \tilde{R}_{t+1} = 1; \\
\tilde{a}_t^e = \tilde{s}_t &= 0, & \text{otherwise.}
\end{aligned}$$

Since $\tilde{R}_{t+1} \geq 1$ if and only if $\tilde{k}_{t+1} \leq (\alpha)^{1/(1-\alpha)}\tilde{h}_{t+1}$ the optimal choice of investment in education depends upon the initial conditions (k_t^*, h_t^*) . We have two cases:

(i) If the initial stocks are such that

$$k_t^* < B \cdot h_t^*, \quad (4.6)$$

where

$$B = \left[\gamma^\beta (\alpha)^{\frac{1}{1-\alpha}} \right]^{\frac{1}{\alpha(1-\beta)}} \left(\frac{1 + \gamma}{1 - \alpha} \right)^{\frac{1}{\alpha}},$$

then the optimal deviation implies

$$\tilde{a}_t^e = \left[\frac{\gamma}{(1 + \gamma)} \right] (1 - \alpha)(k_t^*)^\alpha (h_t^*)^{1-\alpha}$$

and the complete market allocation is sustainable as an equilibrium of the PEPP game if and only if

$$R_{t+1}^* (E_t^* + k_{t+1}^*) \geq \tilde{R}_{t+1} \tilde{k}_{t+1}$$

Since $E_t^* = \gamma k_{t+1}^*$, deviating is not good if

$$y_{t+1}^* (1 + \gamma) \geq \tilde{y}_{t+1}$$

The latter is satisfied when

$$(1 - \beta)^{\alpha + \beta(1-\alpha)} \geq \frac{\alpha}{\alpha + \beta(1 - \alpha)} \quad (4.7)$$

(ii) If the initial stocks of capital do not satisfy inequality (4.6), the optimal deviation for player t is to consume everything immediately. In this case player t leaves the PEPP system if

$$R_{t+1}^* [E_t^* + k_{t+1}^*] \leq \omega_t^*$$

Which gives

$$R_{t+1}^* \leq \frac{1}{(1-\beta)},$$

that is

$$R_{t+1}^* = \alpha(x_{t+1}^*)^{\alpha-1} \leq \frac{1}{(1-\beta)}$$

The rate of return on capital next period is a function of the two stocks of capital in this period. Hence the complete market allocation is sustainable as long as the initial conditions satisfy

$$k_t^* \leq B(1-\beta)^{\frac{\alpha+\beta(1-\alpha)}{\alpha(1-\alpha)(1-\beta)}} \cdot h_t^* \quad (4.8)$$

One can see, though, that our parametric restrictions imply that when (4.8) holds, the optimal deviation consists in not repaying the expected pension and investing in the young generation an amount $\tilde{a}_t^e = \gamma/(1+\gamma)\omega_t^*$.

This shows that, in general, the sustainability of the complete market allocation depends both upon parameter values and initial conditions. More precisely the complete market allocation is a SPE of our game for initial conditions $x_t^* \in (0, B]$ and parameter values (α, β) that satisfy restriction (4.7). When either of these restrictions are not satisfied we have two possible situations: (a) if $x_t^* \in (B, \infty)$ the middle age generation player leaves the system and the economy collapses in just one period to the steady state with zero amount of both stocks; (b) $x_t^* \in (0, B]$ and restriction (4.7) does not hold generation t deviates from the complete market allocation but still invests an amount $\tilde{a}_t^e > 0$ to finance the public education system.

We consider next sustainability of the complete market allocation along a balanced growth path. Condition (4.6) reduces to an inequality involving the growth rate, *i.e.*

$$(1 + \bar{g}) \geq \frac{\bar{\eta}}{B^{1-\alpha}} \quad (4.9)$$

The latter, together with (4.7) and (2.10), defines the subset of the parameter space in which the efficient balanced growth allocation can be supported via a PEPP.

4.2 Sustainability of the efficient allocation in the defined benefit game

To check if the complete market allocation is sustainable as an equilibrium of the defined benefit game we need to find conditions under which the sequence $\{E_t^*, (P_t - T_t^{eo})^*\}_{t=0}^{\infty}$ satisfy restrictions (3.8). In example 1, the use of logarithmic utility functions together with $h(0, h) = 0$ imply that (3.8a) and (3.8c) are always and trivially satisfied. On

the other hand, inequality (3.8b) reduces to restriction (4.4) for the defined contribution game and the parametric restrictions derived there apply also in this case.

In example 2 since, again, $h(0, h) = 0$, inequality (3.8c) always holds. The remaining two inequalities can be easily handled in the following way. On one hand if, along the balance growth path associated to the capital intensity ratio \bar{x}^* , restriction (4.6) is not satisfied then inequality (3.8a) never holds. On the other hand, if \bar{x}^* (or, in the general case, x_t^*) satisfies (4.6) but the parameter values (α, β) do not comply with condition (4.7) then (3.8b) is violated.

Consequently, the complete market allocation is sustainable only for those parameter values at which restrictions (2.9), (4.6) and (4.7) are simultaneously satisfied.

5. The Spanish Case

Our model uses the idea that, in the real world, Pareto improving intergenerational arrangements are not brought about by a benevolent planner. Instead, when achievable, they must be implemented by means of well defined political mechanisms in which individual generations act as selfish, rational players. We have seen that, depending upon the constitutional rules defining the political game and depending upon the constellation of parameter values characterizing the utility and the production functions, the selfish pursuit of generational utility maximization may or may not bring about an efficient outcome.

On the one hand, we have shown that there are large sets of parameter values for which, under either one of the two PEPP games considered, the political equilibrium may not achieve the complete market efficient allocation. On the other hand, we have also proved that the latter is sustainable by means of a relatively simple and, one would be tempted to say, “natural” rule of generational conduct. We have also shown that, if this is the behavioral rule adopted and the efficient allocation is being supported, we should observe an equality between the market rate of interest and the two rates of return implicit in the flows of pension payments, taxes, social security contributions and educational services the representative agent makes or receives over his life-time.

In this section, we use Spanish data to compute the values of i_t and π_t faced by the average Spanish citizen, under the rules in place and the taxes and transfers implemented in the years 1990-91. To carry out this computation some stationarity assumptions must be added to our model. More precisely, we need to assume that the rules of both the Spanish public education and public pension systems will not be changed for the very long future. Also, we need to assume that the aggregate burden of taxation and its intergenerational

distribution have not and will not vary over the life-time of the individuals that were alive in 1990-91. The latter is, indeed, a strong assumption as it practically requires demographic stationarity. This is unlikely to be true, as most demographic studies predict substantial changes in the Spanish demographic structure over the forthcoming 50 years (see *e.g.* Fernández Cordon [1996] and references therein).

5.1 Data sources and methodology[†]

Consider the general case of an individual living for T periods and let p_t denote the probability of survival between age t and $t + 1$. Denote with i_t the interest rate at which young people “borrow” through public education, and with π_t the rate of return the elderly received from their “investment” in public education. The rates i_t and π_t are defined implicitly by

$$\sum_{t=1}^T \left(\prod_{j=1}^t p_j \cdot \prod_{j=t}^T (1 + i_j) \right) \left[E_t^* - T_t^{p*} \right] = 0 \quad (5.1a)$$

$$\sum_{t=1}^T \left(\prod_{j=1}^t p_j \cdot \prod_{j=t}^T (1 + \pi_j) \right) \left[T_t^{e*} - P_t^* \right] = 0 \quad (5.1b)$$

Our model predicts that $r_t = i_t = \pi_t$ should hold, where r_t is some appropriately measured market interest rate. We are interested in verifying the extent to which Spanish data support this prediction.

In order to pursue this objective we need to resort to several kinds of micro and macro data. The choice of the reference year is dictated by the availability of information about individual behavior along the whole life cycle. Measurements of E_t^* , P_t^* , T_t^{e*} and T_t^{p*} at the individual level and for a number of years long enough to reasonably approximate the lifetime of one generation is not available in Spain. At present, there is only one reliable source of microeconomic observations of the allocation of personal time between school, work and retirement at various stages of the life cycle. This is available only for the years 1980-81 and 1990-91 via the Spanish household budget survey (*Encuesta de Presupuestos Familiares*, or EPF). We have used the 1990-91 EPF because the Spanish public pension system underwent a major reform between 1985 and 1987. Short of a few minor changes this reform defined the system which is still in place today.

[†] For a more complete analysis of the data sets we have used and of many findings we are not reporting here, see Montes (1998).

The information in the EPF survey allows the estimation of (1) the amount of public school services received, (2) the amount of direct and indirect taxes paid, (3) the amount of pension contributions paid and, (4) of public contributive pensions received, for each individual in the sample. The information in the EPF also affords the computation, for each age $t = 1, \dots, T$, of the share of the population which is studying, working or unemployed and retired. This life-time distribution of activities is reported in Figure 1. This information is then used to re-construct the life-cycle budget constraints (5.1) for the representative Spanish agent.

To do this we need to compute the life-time distribution of the four kinds of flows which enter equations (5.1). The details of these calculations can be found in the Appendix. We report here only the main steps.

First is the life-time distribution of the educational tax burden. From the EPF we compute weights $\alpha_1, \dots, \alpha_T$, where α_t represents the (relative) burden of education-related taxes charged upon the representative individual at age t . Next we impute to the various ages the total amount of public expenditure in education for the budget year 1990. We set $T_t^{e*} = \alpha_t \cdot T_{90}^e$ for $t = 1, \dots, T$, where T_{90}^e is the total amount of public expenditure in education for the budget year 1990 that is being financed by either direct or indirect taxation. The weights α are computed as

$$\alpha_t = \frac{T_t}{\sum_{t=1}^T T_t L_t}$$

where T_t is the estimated burden of taxation for an individual of age t and L_t is the number of individuals of age t in the EPF.

Secondly, we construct the life-time distribution of total pension payments received by computing weights β_1, \dots, β_T , with β_t denoting the relative pension payment received by the representative individual at age t . We compute the flow of pension receipts at age t using $P_t^* = \beta_t P_{90}$ for $t = 1, \dots, T$, where P_{90} is the total amount of public expenditure on contributive pensions in 1990. The weights β_t are

$$\beta_t = \frac{P_t}{\sum_{t=1}^T P_t L_t}$$

where P_t is the estimated pension received by the representative individual at age t .

In the same way, we construct the life-time distribution of the educational transfers received by the representative individual. We compute weights $\gamma_1, \dots, \gamma_T$ with γ_t denoting the educational transfer received by the representative individual at age t . The flow of

educational transfers is then $E_t^* = \gamma_t \cdot E_{90}$ for $t = 1, \dots, T$, where E_{90} is the total amount of public expenditure on education in 1990. Again, we have

$$\gamma_t = \frac{E_t}{\sum_{t=1}^T E_t L_t}$$

where E_t is the estimated school transfer to the representative individual at age t .

Finally, we construct the life-time distribution of total pension contributions by computing once again weights $\delta_1, \dots, \delta_T$. The flow of pension-related social security contributions along the life-cycle is $T_t^{p*} = \delta_t \cdot T_{90}^p$ for $t = 1, \dots, T$, where T_{90}^p is the total amount of social security taxes levied to finance contributive pension payments in 1990. The weights δ_t are

$$\delta_t = \frac{T_t^p}{\sum_{t=1}^T T_t^p L_t}$$

where T_t^p is the estimated pension contribution, paid by the representative individual at age t . Figure 2 reports the time profile of the four sequences of weights $\alpha_t, \beta_t, \gamma_t, \delta_t$.

To summarize, we use aggregate budget data for: (1) public expenditure on education E_{90} ; (2) amount of taxes devoted to public education T_{90}^e ; (3) total amount of social security contributions devoted to contributive pensions T_{90}^p ; and, (4) total public contributive pension payments P_{90} . We allocate these amounts over the life-cycle of the representative agent by means of life-time activity weights obtained from the EPF. The life-time distribution of these four flows is reported in Figure 3. Equations (5.1) become

$$\sum_{t=1}^{99} \left(\prod_{j=1}^t p_j \right) (1+i)^{99-t} \left[\gamma_t \cdot E_{90} - \delta_t \cdot T_{90}^p \right] = 0 \quad (5.2a)$$

$$\sum_{t=1}^{99} \left(\prod_{j=1}^t p_j \right) (1+\pi)^{99-t} \left[\alpha_t \cdot T_{90}^e - \beta_t \cdot P_{90} \right] = 0 \quad (5.2b)$$

where we have set $T = 99$ [†] and where, given our stationarity assumption, i and π are treated as constants. Expressions (5.2) are then solved numerically to compute the two implicit rates of return associated to the Spanish PEPP in 1990-91.

[†] The 1990-91 EPF contains no observation for individuals older than 99 years.

5.2 Findings

Our point estimate of the implicit rate of return on educational investment, received by the retired people via pension payments, is

$$\pi = 0.040673.$$

Our point estimate of the implicit rate of interest at which young people borrow, by attending either public schools or public funded schools, is more ambiguous. It depends upon the convention one adopts to handle the budget surpluses and deficits of the various Spanish social security administrations. In the budget year 1990, the social security administration for workers of the private sector (INSS) realized a surplus of pension contributions over pension outlays [‡], while the social security administration for public employees (RCP) realized a deficit. The latter was covered by a transfer of funds from the general government budget. Recall that our model assumes year by year balanced budget, *i.e.* $P_t = T_t^p$ for all t .

One possibility is to get rid of both the INSS surplus and of the RCP deficit by assuming that the total amount of social security contributions devoted to contributive pensions (T_{90}^p) was equal to the public contributive pension payments made in that year (P_{90}). In this case our point estimate is

$$i_1 = 0.0363599.$$

A second possibility is to use the actual social security contributions paid to the two administrations in 1990 (T_{90}^p). In this case we have

$$i_2 = 0.0383995.$$

Finally, a third alternative is to add to the total contributions paid in 1990 (T_{90}^p) the amount transferred from the general government budget to cover the RCP deficit. Adopting this wider definition of social security contributions, the implicit rate of interest is computed to be

$$i_3 = 0.0422453.$$

Averaging these three point estimates gives

[‡] The INSS is divided further in six different funds, some of which exhibited a deficit and other a surplus during the same year. Our micro-data do not allow to consider this finer partition.

$$i = 0.039$$

which is remarkably close to our point estimate for π .

Over the period 1990-98, the average real rate of return on Spanish 10-year Treasury bonds was equal to $r = 0.0514$, which is substantially higher than any of the values we just computed for either π or i . It should be noted, though, that, by historical standards, the real interest rate on Spanish debt was extremely high in the early part of this decade and it has followed a clear downward trend since the middle nineties. So for example, the real rate of interest on medium and long term Spanish debt peaked at 6.4% in 1995 and it is now equal to 3.1% and moving further down if anywhere. Assuming that in the long run the real interest rate will stay constant at this lower value, we can compute its average over an horizon roughly comparable to that of the median voter in 1990. The Spanish median voter is a person aged about 45 years, with a conditional life expectancy of about 35 years as of 1990. We have $r = 0.0407$ over the period 1990-2008, $r = 0.0390$ over the period 1990-2012 and $r = 0.0364$ over the period 1990-2025.

6. Conclusions

We have studied a three period overlapping generation model with production and accumulation of physical and human capital. When the young generation cannot borrow to finance investment in human capital, the competitive equilibrium outcome does not satisfy either static or dynamic efficiency and the aggregate growth rate of output and consumption is lower than under the complete market allocation. We have shown that a simple intergenerational transfer agreement could eliminate this problem and induce a fully efficient allocation.

The intergenerational transfer agreement we study is inspired by the argument advanced in Becker and Murphy [1988]. Accordingly we interpret public funding for education as a loan from the middle age to the young generation. The latter uses this loan to finance its accumulation of human capital. Symmetrically, the pay-as-you-go public pension system can be seen as a way for the former borrowers to repay the capitalized value of their educational debt to the previous generation. In this interpretation the two institutions of the welfare state, public education and public pensions, support each other and achieve a more efficient allocation of resources over time.

We have argued that, while a benevolent planner could easily implement such a system of lump-sum taxes and transfers, it is not obvious that a benevolent planner is behind the design of modern welfare state institutions. Hence it is worth investigating if the same equilibrium allocation would arise when the various generations behave in a non-cooperative fashion and taxes and transfers are decided on a period by period basis by means of a majority voting mechanism. In this paper we study two possible constitutional systems defining the rules of the dynamic voting game played by the various generations. The stylized properties of the two games we consider are such that one resembles a defined benefit system and the second is similar to a defined contribution system.

We characterize classes of subgame perfect equilibria (SPE) for the two games. In both cases the complete market allocation may result in an equilibrium outcome when certain restrictions are satisfied. The efficient allocation is, by no means, the only allocation that may be supported as a SPE of the political games we study. Nevertheless we are able to characterize two sets of strategies that would implement the efficient allocation. We show that both classes of strategies provides testable implications about the rates of return implicit in the intergenerational flows of taxes and transfers supporting public education and public pension.

In the last section we test this predictions by computing the “borrowing” and “lending” rates implicit in the Spanish public education and public pension system. We use microeconomic and aggregate data for the years 1990-91. The model predicts that the borrowing and the lending rates should equal each other and be equal, in turn, to the market interest rate. Our point estimates give 0.039 for the borrowing rate, 0.040 for the lending rate and 0.039 for a comparable rate of return on long term public debt.

So far, our reasoning has concentrated on the positive predictions of the model. Nevertheless, there are also important, normative implications of our analysis that may be worth mentioning here. The discussion in section 2 and in the first part of section 3, suggests that an efficient allocation of resources may be obtained by explicitly linking the design of public school financing to that of public pensions provision.

In particular, our model suggests that, abstracting from redistributive considerations, utilization of either public or publicly financed schools and universities should be considered as voluntary accumulation of individual debt. Such debt, capitalized at the market rate of interest should be paid back, along the citizen’s life-time, by means of social security contributions levied upon his or her labor income. Repayment of the educational debt can

be achieved either by means of a voluntary mortgage plan or by means of a compulsory tax. Either choice has some obvious incentive and redistributive implications which are, nevertheless, not dissimilar from those faced by the current arrangement for financing public education. On the side of retirement pensions, the model requires earmarking some tax (paid by individuals) as a source of resources for the public financing of education and to capitalize at the the market rate of interest, the amounts paid by each single citizen. The capital so accumulated should then be paid out, in form of annuities, to the same citizen once retirement age is reached.

We are not aware of any country in which the welfare legislation expressly establishes such a linkage between public education and pensions. Still, the empirical analysis of section 5 shows that, at least for the case of Spain, the average amounts generated by the current systems are not so far from respecting the fundamental relationship our model suggests should characterize an efficient welfare system. In a period, such as the present, in which various proposals for reforming the welfare state are on the table, the scheme we have so briefly described here may deserve a more careful consideration.

Bibliography

- Becker, Gary S. (1975), *Human Capital*, Chicago, IL : The University of Chicago Press.
- Becker, Gary S. and K. Murphy (1988), "The Family and the State", *Journal of Law and Economics* **XXI**, 1-18 .
- Boldrin, Michele (1994), "Public Education and Capital Accumulation", C.M.S.E.M.S. Discussion Paper 1017, Northwestern University; forthcoming in *Research in Economics*.
- Boldrin, Michele and Aldo Rustichini (1995), "Equilibria with Social Security", mimeo, Universidad Carlos III de Madrid and CORE, Louvain; forthcoming in *Review of Economic Dynamics*.
- Cass, David (1972), "Distinguishing Inefficient Competitive Growth Paths: A Note on Capital Overaccumulation and Rapidly Diminishing Future Value of Consumption in a Fairly General Model of Capitalistic Production", *Journal of Economic Theory* **4** 224-240.
- Diamond, Peter (1977), "A Framework for Social Security Analysis", *Journal of Public Economics* **8**, 275-298.
- Fernández Cordón, Juan A. (1996), *Demografía, Actividad y Dependencia en España*, Bilbao, Fundación Banco Bilbao Vizcaya.
- Intervención General de la Administración del Estado (IGAE) (1991a), *Actuación Económica y Financiera de las Administraciones Públicas 1990*, Madrid, Ministerio de Economía y Hacienda.
- Intervención General de la Administración del Estado (IGAE) (1991b), *Cuentas de las Administraciones Públicas 1990*, Madrid, Ministerio de Economía y Hacienda.
- Instituto Nacional de Estadística (INE) (1992), *Encuesta de Presupuestos Familiares 1990-91. Metodología*, Madrid, Instituto Nacional de Estadística.
- Instituto Nacional de Estadística (INE) (1992), *Encuesta sobre Financiación y Gasto de la Enseñanza Privada en España*, Madrid, Instituto Nacional de Estadística.
- Ministerio de Educación y Ciencia (1995), *Estadística del Gasto Público en Educación*, Madrid, Ministerio de Educación y Ciencia.
- Montes, Ana (1998), "Educación para los jóvenes y Pensiones para los mayores: ¿ Existe alguna relación? Evidencia para España", mimeo, Universidad Carlos III de Madrid, October.

Samuelson, Paul (1958), "An Exact Consumption Loan Model with or without the Social Contrivance of Money", *Journal of Political Economy* **66**, 467-482.

World Bank (1994), *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth*, World Bank, Washington, D.C. .

Appendix: Data sources and their treatment

A.1 Data sources

Our sources of data are the following.

We obtain the aggregate expenditure on public education from the *Estadística del Gasto Público en Educación* (EGPE 1995, in Ministerio de Educación y Ciencia (1995)) and the *Encuesta sobre Financiación y Gasto de la Enseñanza Privada* (EFGEP 1990-91, in INE (1992)). The first data base contains public expenditure for each schooling level, the second reports the amount of public funding going to private schools (*centros concertados*). Aggregate tax revenues are obtained from the *Cuentas de las Administraciones Públicas* (IGAE (1991b)). From this we extract the share of total tax revenues allocated to financing public expenditure on education, excluding the fraction that is covered with public debt.

Aggregate flows of public pension payments are also obtained from the *Cuentas de las Administraciones Públicas* (IGAE (1991b)) and *Actuación Económica y Financiera de las Administraciones Públicas* (IGAE (1991a)).

The conditional survival probabilities at each age are equal to those obtained by the latest mortality tables published by the National Statistical Institute (INE) with reference to the year 1990

The aggregate data do not allow the study of individual life-cycle behavior. To do this we use a Spanish household budget survey (*Encuesta de Presupuestos Familiares*, or EPF (INE (1991)) carried out by INE in 1990-91. This survey contains data on individual income, expenditure, personal characteristics and demographic composition for 21,155 households and 72,123 Spanish citizens. This survey is representative of the entire Spanish population and is calibrated on the Spanish Census data.

A.2 Treatment of the data

A.2.1 Life-time distributions

We now detail how, using the data in the EPF, we calculated the life-time distribution of the four flows associated to the two public systems.

The information in the EPF allows the estimation of the contributions and payments associated to the two public systems for each individual in the sample. These contributions and payments depend upon the labor market condition of the individual. Thus, we

have considered five states in which each individual can be. For each state we compute contributions and payments the individual receives or makes. These five states are:

- (\mathcal{E}) Student. If the individual is enrolled in a school or university receiving public funds. The individual is then receiving a transfer (E_t^i), of an amount equal to the average cost of a pupil of his/her age attending a school of the kind he/she specifies, during the fiscal year 1990-91. The same individual contributes toward financing of public education through a portion of his/her direct and indirect taxes, (T_t^i).
- (\mathcal{W}) Worker. If the individual works, he pays social security contributions, (T_t^{pi}) and taxes to support public education, (T_t^i).
- (\mathcal{R}) Retired. We consider as retired only those individuals receiving a contributive pension (P_t^i). Retired individuals are also financing the public educational system with a portion of their taxes (T_t^i).
- (\mathcal{U}) Unemployed. If an individual receives unemployment benefits he/she is financing the public pension system through the social security contributions paid, (T_t^{pi}). Once again, the unemployed are also financing the public education system with a portion of their taxes (T_t^i).
- (\mathcal{I}) Inactive. Here we include all the individuals that are not in any of the previous four states. These individuals only pay taxes (T_t^i), if this is recorded in the EPF.

These five states are mutually exclusive. For the very rare cases in which the same individual in the EPF reports to be in two or more of them, we create two or more “artificial” individuals and increase correspondingly the sample size. We define the universe of states to be $\mathcal{S} = \{\mathcal{E}, \mathcal{W}, \mathcal{P}, \mathcal{U}, \mathcal{I}\}$. The total population at each age $t = 1, \dots, T$ is $\sum_{s \in \mathcal{S}} L_t(s)$, with $L_t(s)$ equal to the number of individuals of age t that are in state s . Denote the share of the population of age t in state s as $\mu_t(s) = L_t(s) / \sum_{s \in \mathcal{S}} L_t(s)$, with $\sum_{s \in \mathcal{S}} \mu_t(s) = 1$. For each t and $s \in \mathcal{S}$, $\mu_t(s)$ is the probability that an individual be in state s at age t .

A.2.1 Public education system

In Spain, public financing of education is allocated in part to public schools and in part to a special kind of private schools, *centros concertados*, by means of school vouchers to students. At the compulsory school level (up to age 14 in 1990, 16 in the current legislation) schooling is completely free. After that, students attending public institutions pay only a small fraction of the total cost, the rest being born by general tax revenues. Students attending private institutions bear the full cost.

Cost of public schooling

For each educational level (primary, secondary, higher and other) we have computed the real, per-pupil public expenditure on education for various types of schools (public and *concertados*) and for the public universities.

The EPF reports if an individual is enrolled at school, the type of school (public or private) and the level he/she is attending. This information is enough to compute the total number of students in each level, type of school and age group.

The criterion we followed to compute the cost of schooling for each “kind” of student (age t , level j , type k of school) is the following. From the EGPE and the EFGEP we obtain the actual total amount of public expenditures for each kind (kj) of school. We divide these amounts by the total number of pupils attending each. This gives us the effective per-student cost for each kind kj of school, E^{jk} . From the EPF we compute how many students of age t are attending a school of kind kj . Using this, we estimate public school expenditure on the representative individual at each age t as

$$E_t = \mu_t(\mathcal{E}) \sum_{k \in TC} \sum_{j \in NE} \mu_t(\mathcal{E}^{jk}) E_t^{jk} = \mu_t(\mathcal{E}) \bar{E}_t$$

where $\mu_t(\mathcal{E})$ denotes the fraction of the population of age t which is attending school, NE is the universe of educational levels, TC is the universe of types of schools. Finally $\mu_t(\mathcal{E}^{jk})$ is the portion of students of age t enrolled in the educational level j in a school of type k .

The age distribution of public education “borrowing” is

$$\gamma_t = \frac{E_t}{\sum_{t=1}^T E_t L_t}$$

Hence, γ_t is the share of (life-time total) education-related transfers the representative individual receives at age t .

Financing of the public education system

On the financing side we need to compute the amount of education-related taxes paid by the representative individual at age t . The taxes we consider are the following: personal income tax (*Impuesto sobre la Renta de las Personas Físicas*, or IRPF), Value Added Tax (VAT), special and other local taxes.

The EPF provides detailed information about the income flow of each individual, and the wealth and consumption baskets of each household. This allows a detailed reconstruction of the various taxes paid by an individual, which we then aggregate in a total burden of taxation (T_t^i), for individual i of age t . We calculate the average tax paid by a person of age t as

$$T_t = \sum_{s \in \mathcal{S}} \mu_t(s) \frac{\sum_{i \in s} T_t^i}{L_t(s)} = \sum_{s \in \mathcal{S}} \mu_t(s) \bar{T}_t^s$$

where \bar{T}_t^s is the average tax paid by an individual in state s , at age t .

Given the values T_t for $t = 1, \dots, T$ the computation of the life-time distribution of the total investment in public education is straightforward,

$$\alpha_t = \frac{T_t}{\sum_{t=1}^T T_t L_t}$$

Hence α_t represents the relative burden of taxation charged upon the representative individual at age t , for $t = 1, \dots, T$. Call this the age distribution of the total tax burden.

To impute the flow of real expenditures in education to the various years of one's life we need to scale the coefficients α_t by the actual public expenditure in education. We retrieve this from IGAE (1991b), call it T_{90}^e . Then we compute as $T_t^{e*} = \alpha_t \cdot T_{90}^e$ for $t = 1, \dots, T$, the investment in public education for the representative agent.

A.2.2 Public pensions

Public contributory pensions are provided by the following programs. The "General Social Security Regime" (*Régimen General de la Seguridad Social*, or RGSS) is the main one and cover most private sector employees plus a (small but growing) number of public employees. The five plans included in the "Special Social Security Regimes" (*Regímenes Especiales de la Seguridad Social*, or RESS) are, respectively, for the self-employed (*Régimen Especial de Trabajadores Autónomos* or RETA), the agricultural workers and small farmers (*Régimen Especial Agrario* or REA), the domestic employees (*Régimen Especial de Empleados de Hogar* or REEH), the sailors (*Régimen Especial de Trabajadores de Mar* or RETM) and the coal miners (*Régimen Especial de la Minería del Carbón* or REMC). Finally, there exists a seventh, special pension system for the public employees (*Régimen de Clases Pasivas*, or RCP).

Financing the public contributive pension system

All seven pension regimes are of the pay-as-you-go-type and, presumably, self-financing[†]. To estimate the life-time distribution of social security payments we identified all individuals in the EPF paying social security contributions, and split them among the seven plans. For each individual we have enough information, either from the EPF or from current legislation (*e.g.* for public employees) to compute the "fictitious income" (*bases de cotización* and *haberes reguladores*) upon which pension contributions are being charged. To each of the fictitious incomes we apply the social security contribution rate, as specified by the 1990-91 legislation, for the pension regime in which the individual was enrolled. Aggregating these amounts over all the individuals of age t , we obtain, for each $t = 1, \dots, T$, the amount of social security contributions paid by individuals in state \mathcal{W} ($T_t^{\mathcal{W}}$) and state \mathcal{U} ($T_t^{\mathcal{U}}$). The social security contribution paid by the representative agent at age t is then

$$T_t^p = \mu_t(\mathcal{W}) \cdot T_t^{\mathcal{W}} + \mu_t(\mathcal{U}) \cdot T_t^{\mathcal{U}}$$

[†] The RGSS shows a surplus. The five special regimes show small deficits.

Also in this case we compute weights by setting

$$\delta_t = \frac{T_t^p}{\sum_{t=1}^T T_t^p L_t}$$

Finally, from IGAE (1991a) and IGAE (1991b) we obtain the total amount of social security contributions paid to the seven plans during the year 1990, T_{90}^p . In our simulation we use

$$T_t^{p*} = \delta_t \cdot T_{90}^p$$

Benefits of the public pension system

The Spanish social security system provides five types of contributive pensions: old-age, disability, widowers, orphans, and other relatives. We have not considered payments of non-contributive pensions as part of our scheme, as they are not financed by means of social security contributions.

In the EPF we are told if an individual is a pension recipient, what kind of pension he or she is receiving and in which amount. The average contributive pension received at each age t is therefore easily computed as

$$P_t = \mu_t(\mathcal{P}) \cdot \sum_{k \in TP} \mu_t(\mathcal{P}^k) \cdot \frac{\sum_{i \in k} P_t^i}{L_t(\mathcal{P}^k)} = \mu_t(\mathcal{P}) \bar{P}_t$$

where $\mu_t(\mathcal{P})$ is the fraction of the population of age t receiving a contributive pension, TP is the universe of kinds of contributive public pensions, $\mu_t(\mathcal{P}^k)$ is the portion of pensioners at age t receiving a pension of type k , P_t^i is the actual pension received by individual i of age t and $L_t(\mathcal{P}^k)$ is the number of individuals of age t receiving a pension of type k .

As in the previous cases, the life-time weights are computed as

$$\beta_t = \frac{P_t}{\sum_{t=1}^T P_t L_t}$$

Finally, from IGAE (1991a) and (1991b) we obtain the total contributive pension payments effectively made, by the seven regimes, during the year 1990, P_{90} . The amounts used in our calculations are, therefore, $P_t^* = \beta_t \cdot P_{90}$.

Figure 1: Life-time distribution among activities.

E= student, W= worker, U= unemployed, R= retired. 1-E-W-U-R= inactive, not reported.

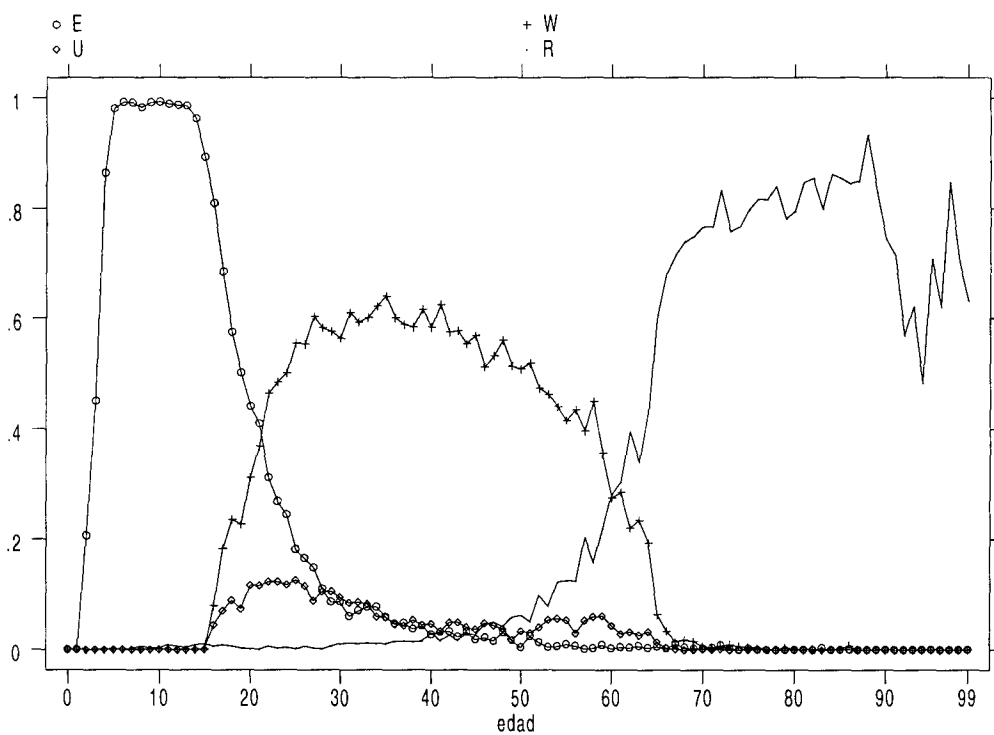


Figure 2: Life-time distribution of tax-transfer shares.
Units are percentage of total national amounts, per citizen.

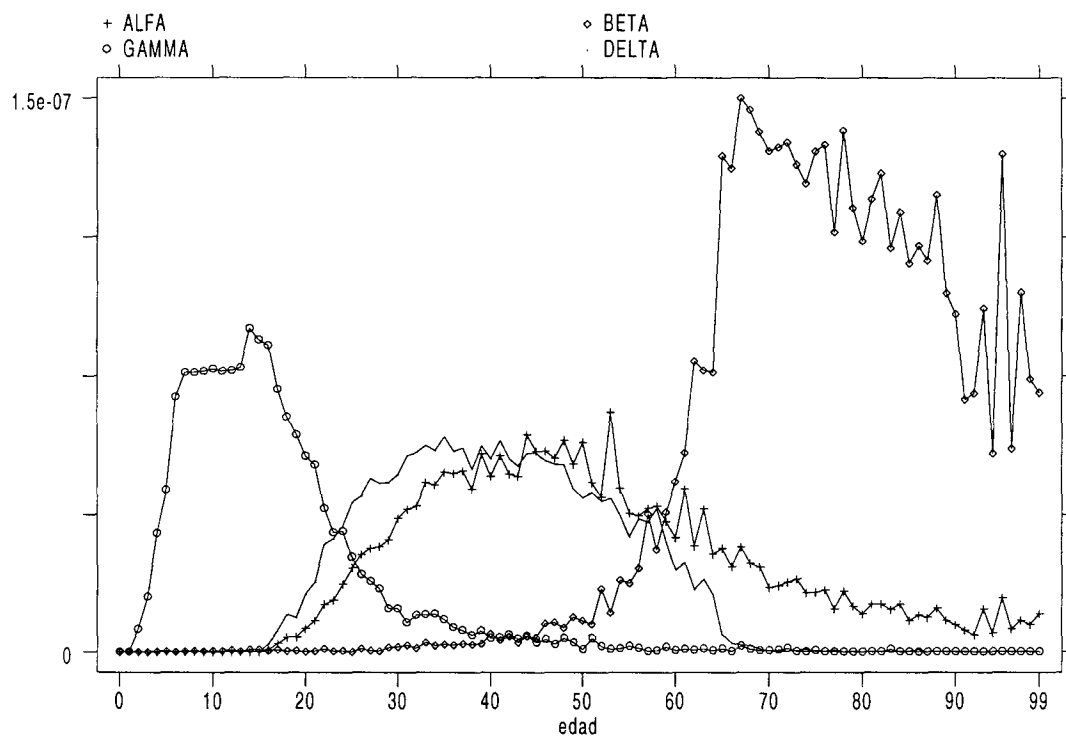


Figure 3: Life-time distribution of tax and transfer flows.
Units are pesetas per capita.

