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ON THE CORE OF AN ECONOMY WITH DIFFERENTIAL INFORMATION**

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Abstract

We show that the (interim) fine core of an atomless exchange economy with differential information is a subset of the ex-post core of the economy. Moreover, the interim fine core may be empty, and therefore it may be a proper subset of the ex-post core. The inclusion relation does not hold for economies with a finite number of traders.

Keywords: Atomless exchange economies, differential information, fine core, ex-poxt core.

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1 Introduction

In a seminal paper, Wilson (1978) introduced two notions of core of an economy with differential information, the coarse core and the fine core. Since Wilson's work several alternative concepts of core have been proposed in the literature. Among them are the private core introduced in Yannelis (1991), and the weak fine core introduced in Allen (1991) and Koutsougeras and Yannelis (1993). More recently, Vohra (1997) introduced the notion of incentive compatible core, and Volij (1998) the notion of internally consistent core.

The ex-ante fine and weak fine cores were studied by Allen (1991), Koutsougeras and Yannelis (1993), and Einy, Moreno and Shitovitz (1998). In the present work we study the relationship between the (interim) fine core and the ex-post core of an economy with differential information. In Wilson's fine core the traders have the opportunity to communicate (i.e., to exchange information); specifically, the information given to every member of a blocking coalition is the joint information of the members of the coalition. In addition, we assume that a blocking allocation must be measurable with respect to the "joint partition" of the members of the coalition (our main result, however, remains true without this additional assumption).

We consider an atomless exchange economy (see Aumann (1964)) with differential information in which the space of states of nature is finite. We show that under standard conditions, in such economies the (interim) fine core is a subset of the ex-post core (see Theorem 1). (Since we assume that blocking allocations must be measurable with respect to the joint information of the blocking coalition, our interim fine core might be a larger set than Wilson's fine core; therefore this result holds also for Wilson's fine core.) Moreover, under these conditions the ex-post core of the economy is non-empty, while the fine core may be empty (see Example 3.1). Hence the fine core may be a proper subset of the ex-post core. We show that the inclusion result does not hold for economies with a finite number of traders.

2 The Model

We consider a pure exchange economy $\mathcal{E}$ with differential information. The commodity space is $\Re^l_+$. The space of traders is a measure space (T, Σ, μ), where T is a set (the set of *traders*), Σ is a σ-field of subsets of T (the set of *coalitions*), and μ is a measure on Σ. The space of states of nature is a measurable space $(\Omega, \mathcal{F})$, where Ω is a finite set and $\mathcal{F}$ is a field of subsets of Ω. The traders do not necessarily know which state of nature $\omega \in \Omega$ that actually occurred, but they may have some information about the state of nature. We assume that the information of a trader $t \in T$ is described by a measurable partition Π_t of Ω. We denote by $\mathcal{F}_t$ the field generated by Π_t. If ω_o is the true state of nature, trader t observes the member of Π_t which contains ω_0. Every trader $t \in T$ has a probability measure q_t on $\mathcal{F}$ which represents his *prior beliefs.* For simplicity it is assumed that if $A \in \mathcal{F}$ is a non-empty set, then $q_t(A) > 0$ for all $t \in T$. The preferences of a trader $t \in T$ are represented by a *random utility function,* $u_t : \Omega \times \Re^l_+ \to \Re_+$ such that for every $x \in \Re^l_+$, the function $u_t(\cdot, x)$ is $\mathcal{F}$-measurable. It is also assumed that for every $(t, x) \in T \times \Re^l_+$, the mapping $(t, x) \to u_t(\omega, x)$ is $\Sigma \times \mathcal{B}$ measurable, where ω is a fixed member of Ω, and $\mathcal{B}$ is the σ-field of Borel subsets of $\Re^l_+$.

An *assignment* is a function $\mathbf{x} : \Omega \times T \to \Re^l_+$ such that for every $\omega \in \Omega$ the function $\mathbf{x}(\omega, \cdot)$ is μ-integrable on T, and for every $t \in T$ the function $\mathbf{x}(\cdot, t)$ is $\mathcal{F}$-measurable. There is a fixed *initial assignment* $\mathbf{e}$; $\mathbf{e}(\omega, t)$ represents the *initial endowment* of trader $t \in T$ in the state of nature $\omega \in \Omega$. An *allocation* is an assignment $\mathbf{x}$ such that $\int_T \mathbf{x}(\omega, t) d\mu \leq \int_T \mathbf{e}(\omega, t) d\mu$ for every $\omega \in \Omega$.

Since Ω is finite there is a finite number of different information fields $\mathcal{F}_t$, $t \in T$. We denote by $\mathcal{F}_1, \ldots \mathcal{F}_n$ the n distinct information fields of the traders. We assume that $\mathcal{F} = \bigvee_{i=1}^n \mathcal{F}_i$, which means that $\mathcal{F}$ contains no superfluous events about which no trader has information, and therefore cannot affect anyone's consumption decisions.

We use the following notations. For two vectors $x = (x_1, \ldots, x_l)$ and $y = (y_1, \ldots, y_l)$ in $\Re^l$ we write $x \geq y$ when $x_k \geq y_k$ for all $1 \leq k \leq l$, $x > y$ when $x \geq y$ and $x \neq y$, and $x \gg y$ when $x_k > y_k$ for all $1 \leq k \leq l$.

In the rest of the paper an economy $\mathcal{E}$ is an economy with differential information as it has been just described. Also for an economy $\mathcal{E}$ and a state of nature $\omega \in \Omega$, we denote by $\mathcal{E}(\omega)$ the full information economy in which the commodity space is $\Re^l_+$, the space of traders is (T, Σ, μ), and for every trader $t \in T$ his initial endowment is $\mathbf{e}(\omega, t)$ and his utility function is $u_t(\omega, \cdot)$.

3 The Fine and Ex-Post Cores

In this section we define the ex-post core and the interim fine core of an economy with differential information, and we show that if the space of traders of an economy is non-atomic (that is, the population measure μ on (T, Σ) is non-atomic) then the interim fine core is a subset of the ex-post core.

A function $u : \Re^l_+ \to \Re$ is *(strictly) increasing* if for all $x, y \in \Re^l_+$, $(x > y)$ $x \gg y$ implies $u(x) > u(y)$.

Throughout the section we refer to the following conditions.

$(A.1)$ For every $\omega \in \Omega$, $\int_T \mathbf{e}(\omega, t) d\mu \gg 0$.

$(A.2)$ For all $t \in T$ and $\omega \in \Omega$, the function $u_t(\omega, \cdot)$ is continuous, and strictly increasing on $\Re^l_+$.

$(A.3)$ For all $t \in T$ and $\omega \in \Omega$, the function $u_t(\omega, \cdot)$ is continuous, increasing on $\Re^l_+$, and vanishes on the boundary of $\Re^l_+$.

An allocation $\mathbf{x}$ is an *ex-post core allocation* for an economy $\mathcal{E}$ if there does not exist a coalition $S \in \Sigma$, an assignment $\mathbf{y}$, and a state of nature $\omega_0 \in \Omega$ such that

(3.1) $\mu(S) > 0$,

(3.2) $\int_S \mathbf{y}(\omega_0, t) d\mu \leq \int_S \mathbf{e}(\omega_0, t) d\mu$, and

(3.3) $u_t(\omega_0, \mathbf{y}(\omega_0, t)) > u_t(\omega_0, \mathbf{x}(\omega_0, t))$ for almost all $t \in S$.

If there exists S, $\mathbf{y}$, and ω_0 such that $(3.1) - (3.3)$ are satisfied then we say that $\mathbf{y}$ is an *ex-post improvement of S upon $\mathbf{x}$ at ω_0*. The *ex-post core* of an economy $\mathcal{E}$ is the set of all ex-post core allocations of $\mathcal{E}$.

The ex-post core of an economy with differential information and its relationship to the set of rational expectations equilibria were studied by Einy, Moreno and Shitovitz (1998), who show that the ex-post core is non-empty under conditions $(A.1)$ and either

$(A.2)$ or $(A.3)$.

Let $\mathcal{E}$ be an economy with differential information, and let $1 \leq i \leq n$ (recall that n is the number of distinct information fields of the traders of $\mathcal{E}$). Define

$$T_i = \{t \in T \mid \mathcal{F}_t = \mathcal{F}_i\}.$$

We assume that for all $1 \leq i \leq n$ the set T_i is measurable (i.e., $T_i \in \Sigma$) and $\mu(T_i) > 0$. For every $S \in \Sigma$ let

$$I(S) = \{i \mid 1 \leq i \leq n, \text{ and } \mu(S \cap T_i) > 0\}.$$

If $\mathcal{G}$ is a subfield of $\mathcal{F}$, $f : \Omega \to \Re_+$ is an $\mathcal{F}$-measurable function, and $t \in T$ we denote by $E_t(f \mid \mathcal{G})$ the conditional expectation of f with respect to q_t.

An allocation $\mathbf{x}$ is an *interim fine core allocation* for an economy $\mathcal{E}$ if there does not exist a coalition S, an assignment $\mathbf{y}$ and a non-empty event $A \in \bigvee_{i \in I(S)} \mathcal{F}_i$ such that

(3.4) $\mu(S) > 0$,

(3.5) $\int_S \mathbf{y}(\omega, t) d\mu \leq \int_S \mathbf{e}(\omega, t) d\mu$, for every $\omega \in A$,

(3.6) for all $t \in S$, $\mathbf{y}(\cdot, t)$ is $\bigvee_{i \in I(S)} \mathcal{F}_i$-measurable, and

(3.7) for all $\omega \in A$,

$$E_t(u_t(\cdot, \mathbf{y}(\cdot, t)) \mid \bigvee_{i \in I(S)} \mathcal{F}_i)(\omega) > E_t(u_t(\cdot, \mathbf{x}(\cdot, t)) \mid \bigvee_{i \in I(S)} \mathcal{F}_i)(\omega),$$

almost everywhere on S.

If there exists S, $\mathbf{y}$, and $A \in \bigvee_{i \in I(S)} \mathcal{F}_i$ such that $(3.4) - (3.7)$ are satisfied then we say that $\mathbf{y}$ is an *interim fine improvement of S upon $\mathbf{x}$ on A*. The *interim fine core* of an economy $\mathcal{E}$ is the set of all interim fine core allocations of $\mathcal{E}$.

Our definition of interim fine core is that of Wilson's (1978) fine core, with the difference that in Wilson's definition a blocking assignment may not be measurable with respect to the joint information of the blocking coalition. Assuming measurability of a blocking assignment makes it more difficult for a coalition to block an allocation, and therefore the resulting core may be larger than Wilson's fine core. Thus, the inclusion established in Theorem 1 below also holds for Wilson's fine core.

The measurability of the blocking assignment with respect to the joint information of the blocking coalition is also assumed in Allen (1991), Koutsougeras and Yannelis (1993) and Vohra (1997). We now state and prove our result.

Theorem 1. *Assume that $\mathcal{E}$ is an atomless economy (i.e., such that the population measure μ on (T, Σ) is non-atomic) that satisfies conditions $(A.1)$ and either $(A.2)$ or $(A.3)$. Then the interim fine core of $\mathcal{E}$ is a subset of the ex-post core of $\mathcal{E}$.*

Proof: Let $\mathbf{x}$ be an interim fine core allocation of an atomless economy $\mathcal{E}$. Assume, contrary to our claim, that $\mathbf{x}$ is not an ex-post core allocation of $\mathcal{E}$. Then there exists a coalition $S \in \Sigma$ with $\mu(S) > 0$, a state of nature $\omega_0 \in \Omega$, and an assignment $\mathbf{y}$ such that $\int_S \mathbf{y}(\omega_0, t)d\mu \leq \int_S \mathbf{e}(\omega_0, t)d\mu$, and $u_t(\omega_0, \mathbf{y}(\omega_0, t)) > u_t(\omega_0, \mathbf{x}(\omega_0, t))$ for almost all $t \in S$. Define the function $\hat{\mathbf{x}} : T \to \Re^l_+$ by $\hat{\mathbf{x}}(t) = \mathbf{x}(\omega_0, t)$ for all $t \in T$. Then $\hat{\mathbf{x}}$ is an allocation of the full information economy $\mathcal{E}(\omega_0)$. Moreover, $\hat{\mathbf{x}}$ is not in the core of the economy $\mathcal{E}(\omega_0)$. Therefore by the Theorem of Vind (1972) (see also Proposition 7.3.2 in Mas-Colell (1985)), there exists a coalition $Q \in \Sigma$ with $\mu(Q) > \mu(T) - \min\{\mu(T_1), \ldots \mu(T_n)\}$, and an integrable function $\hat{\mathbf{y}} : T \to \Re^l_+$ such that $\int_S \hat{\mathbf{y}}(t)d\mu \leq \int_S \mathbf{e}(\omega_0, t)d\mu$, and $u_t(\omega_0, \hat{\mathbf{y}}(t)) > u_t(\omega_0, \hat{\mathbf{x}}(t))$ for almost all $t \in Q$. Let $A(\omega_0)$ be the atom of the field $\mathcal{F}$ containing ω_0. Define a function $\mathbf{z} : \Omega \times T \to \Re^l_+$ by

$$\mathbf{z}(\omega, t) = \begin{cases} \hat{\mathbf{y}}(t) & \text{if } \omega \in A(\omega_0) \\ \mathbf{e}(\omega, t) & \text{otherwise.} \end{cases}$$

Then $\mathbf{z}(\cdot, t)$ is $\mathcal{F}$-measurable for all $t \in T$. Therefore $\mathbf{z}$ is an assignment in $\mathcal{E}$. We show that $\mathbf{z}$ is an interim fine improvement of Q upon $\mathbf{x}$ on $A(\omega_0)$.

Since $\mathbf{e}(\omega, \cdot) = \mathbf{e}(\omega_0, \cdot)$ for all $\omega \in A(\omega_0)$ (because $\mathbf{e}(\cdot, t)$ is $\mathcal{F}$-measurable for all $t \in T$), we have,

$$\int_S \mathbf{z}(\omega, t)d\mu \leq \int_S \mathbf{e}(\omega, t)d\mu,$$

for all $\omega \in \Omega$.

Now as $\mu(Q) > \mu(T) - \min\{\mu(T_1), \ldots \mu(T_n)\}$, we have $I(Q) = \{1, \ldots n\}$, and $\bigvee_{i \in I(Q)} \mathcal{F}_i = \mathcal{F}$. Therefore $A(\omega_0) \in \bigvee_{i \in I(Q)} \mathcal{F}_i$. Since for all $t \in T$ the function

$u_t(\cdot, \mathbf{x}(\cdot, t))$ is $\mathcal{F}$-measurable, for all $t \in T$ we have

$$E_t(u_t(\cdot, \mathbf{x}(\cdot, t)) \mid \bigvee_{i \in I(Q)} \mathcal{F}_i) = u_t(\cdot, \mathbf{x}(\cdot, t)),$$

and

$$E_t(u_t(\cdot, \mathbf{z}(\cdot, t)) \mid \bigvee_{i \in I(Q)} \mathcal{F}_i) = u_t(\cdot, \mathbf{z}(\cdot, t)).$$

Now for all $\omega \in A(\omega_0)$ we have

$$\begin{aligned} E_t(u_t(\cdot, \mathbf{z}(\cdot, t)) \mid \textstyle\bigvee_{i \in I(Q)} \mathcal{F}_i)(\omega) &= u_t(\omega, \mathbf{z}(\omega, t)) = u_t(\omega, \hat{\mathbf{y}}(t)) = u_t(\omega_0, \hat{\mathbf{y}}(t)) \\ &> u_t(\omega_0, \hat{\mathbf{x}}(t)) = u_t(\omega_0, \mathbf{x}(\omega_0, t)) = u_t(\omega, \mathbf{x}(\omega, t)) \\ &= E_t(u_t(\cdot, \mathbf{x}(\cdot, t)) \mid \textstyle\bigvee_{i \in I(Q)} \mathcal{F}_i)(\omega). \end{aligned}$$

Thus, $\mathbf{z}$ is an interim fine improvement of Q upon $\mathbf{x}$ on $A(\omega_0)$, which contradicts the fact that $\mathbf{x}$ is a interim fine core allocation of $\mathcal{E}$. □

The following example shows that in atomless economies the interim fine core may be a proper subset of the ex-post core. In fact, in this example the interim fine core is empty, while the ex-post core is not.

Example 3.1. Consider an economy $\mathcal{E}$ in which the commodity space is $\Re^2_+$, and the set of traders is $([0,3], \mathcal{B}, \mu)$, where $\mathcal{B}$ is the σ-field of Borel subsets of $[0,3]$ and μ is the Lebesgue measure. The space of states of nature is $\Omega = \{\omega_1, \omega_2\}$, and $\mathcal{F} = 2^\Omega$. Let $T_1 = [0,1], T_2 = (1,2]$, and $T_3 = (2,3]$. The information partition of a trader $t \in T_1$ is $\Pi_1 = \{\{\omega_1, \omega_2\}\}$, his prior is $q_1 = (\frac{2}{3}, \frac{1}{3})$, and his initial endowments are $\mathbf{e}(\omega_1, t) = \mathbf{e}(\omega_2, t) = (4,0)$. The information partition of a trader $t \in T_2$ is $\Pi_2 = \{\{\omega_1, \omega_2\}\}$, his prior is $q_2 = (\frac{1}{3}, \frac{2}{3})$, and his initial endowments are $\mathbf{e}(\omega_1, t) = \mathbf{e}(\omega_2, t) = (0,4)$. Finally, the information partition of a trader $t \in T_3$ is $\Pi_3 = \{\{\omega_1\}, \{\omega_2\}\}$, his prior is $q_3 = (\frac{1}{2}, \frac{1}{2})$, and his initial endowments are $\mathbf{e}(\omega_1, t) = (4,0)$ and $\mathbf{e}(\omega_2, t) = (0,4)$. All traders have the same utility function, given for $(\omega, (x,y)) \in \Omega \times \Re^2_+$ by

$$u(\omega, (x,y)) = \sqrt{xy}.$$

Let $\mathbf{x}$ be the allocation in $\mathcal{E}$ given by

$$\mathbf{x}(\omega_1, t) = \begin{cases} (2,1) & t \in T_1 \cup T_3 \\ (4,2) & t \in T_2, \end{cases}$$

and

$$\mathbf{x}(\omega_2,t)=\begin{cases}(2,4) & t\in T_1\\ (1,2) & t\in T_2\cup T_3.\end{cases}$$

It is easy to see that $\mathbf{x}$ is the unique ex-post core allocation of the (atomless) economy $\mathcal{E}$. Moreover, $\mathbf{x}$ is not an interim fine core allocation of $\mathcal{E}$. Indeed, define the assignment $\mathbf{y}$ in $\mathcal{E}$ given by

$$\mathbf{y}(\omega,t)=\begin{cases}(2,2) & (\omega,t)\in\Omega\times(T_1\cup T_2)\\ \mathbf{e}(\omega,t) & \text{otherwise.}\end{cases}$$

Let $S=T_1\cup T_2$. Then $\bigvee_{i\in I(S)}\mathcal{F}_i=\{\emptyset,\{\omega_1,\omega_2\}\}$. For all $t\in S$ we have

$$E_t(u(\cdot,\mathbf{y}(\cdot,t))\mid\bigvee_{i\in I(S)}\mathcal{F}_i)=2$$

whereas

$$E_t(u(\cdot,\mathbf{x}(\cdot,t))\mid\bigvee_{i\in I(S)}\mathcal{F}_i)=\frac{4\sqrt{2}}{3}<2.$$

Therefore $\mathbf{y}$ is an interim fine improvement of S upon $\mathbf{x}$ on Ω. Thus, $\mathbf{x}$ is not an interim fine core allocation of $\mathcal{E}$. By Theorem 1, the interim fine core is empty. Note also that since the Wilson's fine core is a subset of our interim fine core, it is also empty in this example.

The allocation $\mathbf{x}$ in Example 3.1 is a rational expectations equilibrium allocation of $\mathcal{E}$ (see Radner (1979) and Allen (1981)), which corresponds to a "completely revealing" equilibrium price system p, where $p(\omega_1)=(\frac{1}{3},\frac{2}{3})$ and $p(\omega_2)=(\frac{2}{3},\frac{1}{3})$.

Our next example shows that Theorem 1 does not hold for finite economies.

Example 3.2. Consider a finite economy $\mathcal{E}$ in which the commodity space is $\Re^3_+$, and the set of traders is $T=\{1,2,3\}$. The space of states of nature is $\Omega=\{\omega_1,\omega_2\}$, and $\mathcal{F}=2^\Omega$. Traders' initial endowments are $\mathbf{e}(\omega_1,1)=\mathbf{e}(\omega_2,1)=(60,0,0)$, $\mathbf{e}(\omega_1,2)=\mathbf{e}(\omega_2,2)=(0,60,0)$, and $\mathbf{e}(\omega_1,3)=\mathbf{e}(\omega_2,3)=(0,0,60)$. The information partition of traders 1 and 2 is $\Pi_1=\{\{\omega_1,\omega_2\}\}$, and that of Trader 3 is $\Pi_3=\{\{\omega_1\},\{\omega_2\}\}$. All traders have the same prior, $q=(\frac{1}{2},\frac{1}{2})$, and the same utility function, given for $(\omega,(x,y,z))\in\Omega\times\Re^3_+$ by

$$u(\omega,(x,y,z))=\sqrt{x}+\sqrt{y}+\sqrt{z}.$$

Define the allocation $\mathbf{x}$ in $\mathcal{E}$ by

$$\mathbf{x}(\omega_1, t) = \begin{cases} (19,19,19) & t \in \{1,2\} \\ (22,22,22) & t = 3, \end{cases}$$

and

$$\mathbf{x}(\omega_2, t) = \begin{cases} (13,13,13) & t \in \{1,2\} \\ (34,34,34) & t = 3. \end{cases}$$

We show that $\mathbf{x}$ is an interim fine core allocation of $\mathcal{E}$, but it is not an ex-post core allocation of $\mathcal{E}$. For $t \in \{1,2\}$ and $\omega \in \Omega$ we have

$$E_t(u(\cdot, \mathbf{y}(\cdot, t)) \mid \mathcal{F}_1 \bigvee \mathcal{F}_2)(\omega) = \frac{3}{2}(\sqrt{19} + \sqrt{13}).$$

Now, if $\mathbf{y}$ is an assignment in $\mathcal{E}$ which is feasible for {1,2} at each state of nature, then for all $\omega \in \Omega$ we have

$$\begin{aligned} \min\{E_1(u(\cdot, \mathbf{y}(\cdot, 1)) \mid \mathcal{F}_1 \vee \mathcal{F}_2)(\omega), E_2(u(\cdot, \mathbf{y}(\cdot, 2)) \mid \mathcal{F}_1 \vee \mathcal{F}_2)(\omega)\} &\leq 2\sqrt{30} \\ &< \tfrac{3}{2}(\sqrt{19} + \sqrt{13}). \end{aligned}$$

Therefore $\mathbf{y}$ cannot be an interim fine improvement of $\{1,2\}$ upon $\mathbf{x}$ on Ω. It is clear that if $S \neq \{1,2\}$ is a coalition, then it does not have an interim fine improvement upon $\mathbf{x}$ on a non-empty event in $\bigvee_{i \in I(S)} \mathcal{F}_i$. Thus, $\mathbf{x}$ is an interim fine core allocation of $\mathcal{E}$. Define an assignment $\mathbf{y} : \Omega \times T \to \Re^3_+$ by

$$\mathbf{y}(\omega, t) = \begin{cases} (30,30,30) & (\omega, t) \in \Omega \times \{1,2\} \\ \mathbf{e}(\omega, 3) & \text{otherwise}. \end{cases}$$

Then $\mathbf{y}$ is an ex-post improvement of $\{1,2\}$ upon $\mathbf{x}$ at the state of nature ω_2, and therefore $\mathbf{x}$ is not an ex-post core allocation in $\mathcal{E}$.

Note that in this argument we have not assumed measurability of the blocking assignment $\mathbf{y}$ with respect to the joint partition of the blocking coalition; hence in this example Wilson's fine core is not a subset of the ex-post core either.

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