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Essays in Two-Sided Platforms

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Resumen extendido

Introducción. El tema de estudio se ha enfocado hacia lo que se conoce como Plataformas y Mercados Bilaterales. Estos son mercados con dos o más grupos de agentes que necesitan interactuar para conseguir un beneficio que, en cada grupo, depende del número de personas que hay en el otro grupo. El punto de partida de la teoría de los mercados bilaterales es que miembros del grupo A no internalizan el impacto que su participación tiene sobre los miembros del grupo B. En este contexto, la plataforma es una firma (o institución) que posibilita o facilita la interacción entre los grupos e internaliza las externalidades que pudieran surgir entre ellos a través de los precios. Son ejemplos de plataformas las tarjetas de crédito, los centros comerciales, los portales web de subastas, las consolas de videojuegos, los sistemas operativos, los canales de televisión y los periódicos. Si bien algunos de estos mercados existen desde hace mucho tiempo, solo recientemente la literatura económica los ha reconocido como "diferentes". En particular, las primeras publicaciones académicas no tienen una antigüedad superior a los cinco años.

En relación a estos mercados, la tesis que aquí se presenta se compone de cuatro capítulos. El primero incluye un resumen crítico y recopilación de la literatura en mercados bilaterales. El segundo capítulo analiza los efectos de demanda que surgen al permitir que el lado vendedor (con mayor o menor poder de mercado) fije un precio por su producto al lado comprador. El estudio permite realizar una comparación entre los mercados de sistemas operativos para ordenadores y las consolas de video juegos, mercados que representan ejemplos típicos de plataformas tecnológicas. El tercer capítulo introduce un modelo de dos firmas asimétricas para analizar los incentivos de las plataformas tecnológicas para hacer compatibles sus respectivas aplicaciones. Se observa también el efecto que esta compatibilidad podría tener en los incentivos de las firmas a innovar, en el bienestar de los usuarios y en el bienestar total de la economía. El cuarto y último capítulo introduce consideraciones de calidad entre los agentes en mercados donde la literatura sólo ha tenido en cuenta los efectos de red existentes entre ellos.

Capítulo primero: Una revisión introductoria a las plataformas bilaterales. El objetivo de este capítulo es presentar a los lectores no familiarizados con la literatura en mercados bilaterales los modelos canónicos presentes en Armstrong (2006) y Rochet-Tirole (2003 y 2006). Se realiza también un análisis crítico de los supuestos en estos modelos y una revisión de los mercados que han sido estudiados con este marco teórico. Finalmente, se pretende además ubicar las contribuciones de esta tesis en el mapa de la literatura existente.

Capítulo segundo: Estrategias de precios en plataformas de software. Un supuesto importante presente en los modelos canónicos de mercados bilaterales, y mantenido en la mayoría de los artículos en la literatura, es que los miembros de un grupo son indiferentes en relación al número de miembros del propio grupo. Este supuesto excluye la posibilidad de competencia entre los agentes que participan en la plataforma (ausencia de efecto rivalidad) e implica que vendedores y compradores actúan como monopolistas.

El modelo que se presenta en este capítulo permite la competencia en el lado de los vendedores y considera además los precios que el lado vendedor aplica a los compradores a diferencia de los modelos canónicos donde los únicos precios que existen son los que la plataforma fija a cada lado del mercado.

Se asume la presencia de una plataforma tecnológica (un sistema operativo o consola de video juegos) que fija un precio a los desarrolladores de aplicaciones para la plataforma y otro a los usuarios. Cuando los desarrolladores establecen el precio que cobrarán a los usuarios por las aplicaciones, surgen dos escenarios alternativos de estudio, que siguiendo la terminología empleada por Lerner y Tirole (2004), se denominan "margen de demanda" y "margen de competencia". La diferencia fundamental entre uno y otro margen radica en que en el primero los desarrolladores fijan precios teniendo en cuenta el efecto que estos ejercen sobre la demanda total del sistema (plataforma más aplicaciones) mientras que en el segundo fijan sus precios en función de la valoración marginal por las aplicaciones de los usuarios. El margen que en equilibrio estará activo viene determinado por el grado de sustitución entre las aplicaciones y el número de vendedores. En la medida en que las aplicaciones sean sustitutas cercanas es más probable que el margen de competencia esté activo. Lo mismo ocurre si el número de vendedores es muy alto. Por el contrario, si las aplicaciones no son muy sustitutas y/o el número de vendedores es muy bajo el margen de demanda tenderá a estar activo.

El capítulo está relacionado a los artículos de Nocke, Peitz y Stahl (2007) y Hagiu (2006a y 2006b), en el sentido de que el modelo permite capturar los efectos de demanda que surgen cuando los vendedores/desarrolladores fijan precios. Sin embargo, en los trabajos citados el margen de competencia está siempre activo, por lo que la contribución general de este capítulo es identificar los nuevos resultados que se generan en un contexto en el que el margen de demanda es el que está activo.

El estudio de los dos márgenes permite comparar la eficiencia de una plataforma abierta o de "open source" (que no fija precios) versus una plataforma cerrada en ambos escenarios. También se analiza la eficiencia de la integración vertical entre la plataforma y el lado de los vendedores.

El modelo también permite analizar y explicar algunas características de los mercados de sistemas operativos de ordenadores y consolas de video juegos. En particular, ciertos hechos estilizados observados llevan a pensar que los desarrolladores de aplicaciones para sistemas operativos están restringidos por el margen de demanda, mientras que los desarrolladores de videojuegos lo están por el margen de competencia (por ejemplo, las aplicaciones de sistemas operativos son menos sustitutas entre sí que lo que parecen ser los video juegos; por otra parte, los consumidores de video consolas confiesan poseer y comprar importantes cantidades de video juegos y lo hacen de manera regular, lo que no es el caso en lo que a aplicaciones de sistemas operativos se refiere).

En línea con estas interpretaciones, los resultados que arroja el modelo sugieren que los gobiernos deberían promover el "open source" en el mercado de sistemas operativos, pero no necesariamente en el de las consolas de videojuegos. El modelo también explica la tendencia observada en las firmas que operan plataformas tecnológicas a estar integradas con las que se conoce como las "killer applications" (las aplicaciones más demandadas o de mayor peso para los usuarios).

Capítulo tercero: Compatibilidad e innovación en mercados con una firma dominante. Este capítulo pretende explicar los incentivos que tienen las firmas que producen plataformas tecnológicas para hacer compatibles sus productos. También se intenta analizar el efecto que la compatibilidad puede tener en los incentivos de las firmas para continuar innovando y particularmente, el efecto que la compatibilidad puede ejercer en el bienestar de los consumidores y en el bienestar total de la economía.

Los problemas que los temas de compatibilidad despiertan han sido largamente analizados en la literatura económica y de organización industrial, véase por ejemplo Economides (1989), Farrell and Saloner (1985), Katz and Shapiro (1985), Matutes and Regibeau (1988, 1989, 1992).

Uno de los aportes de este capítulo es introducir dos conceptos diferentes de compatibilidad. En particular, se analiza lo que se denomina "compatibilidad en las aplicaciones", que implica que las aplicaciones puedan ser utilizadas en diferentes plataformas. Por otra parte, se reconoce la "compatibilidad entre redes", que implica que usuarios de diferentes plataformas puedan "conectarse" o intercambiar información, por ejemplo, mantener correspondencia por e-mail. En este último tipo de compatibilidad la presencia de efectos directos de red entre los usuarios es fundamental. El modelo muestra que los resultados relativos a los incentivos de las firmas y el efecto sobre el bienestar de promover un tipo de compatibilidad o la otra pueden ser diferentes.

Se plantea un modelo de mercado con dos firmas de plataformas tecnológicas donde cada una tiene asociada una aplicación completamente compatible, en el sentido de que puede ser usada perfectamente con la plataforma. Al mismo tiempo, esta aplicación no es compatible, o lo es en un grado menor, con la otra plataforma. Las firmas son asimétricas en el valor que los usuarios otorgan a la aplicación completamente compatible. Esto determina entonces que existe una firma dominante y una débil en el mercado.

Se intenta estudiar entonces cómo cada tipo de plataforma reacciona ante de la posibilidad de aumentar el grado de compatibilidad de sus aplicaciones y como afectan estas decisiones a sus beneficios y al bienestar de los usuarios.

En primer lugar, se observa que la firma dominante nunca está interesada en promover la compatibilidad en las aplicaciones. Este resultado se explica por el hecho de que la compatibilidad principalmente lleva a una disminución del poder de mercado de la dominante al reducir su ventaja de diferenciación de su producto. Por su parte, la firma débil siempre demandará esta compatibilidad. Sin embargo, en relación a la compatibilidad entre redes, se muestra que ambas firmas encuentran beneficioso promoverla.

Se remarca también que la compatibilidad no es siempre beneficiosa para los consumidores. Además, en muchas circunstancias el interés de la firma dominante en relación a la compatibilidad está en línea con el de los usuarios y opuesto al de la firma débil, que por su parte siempre demandará compatibilidad. En este caso, imponer un mayor grado de compatibilidad dañaría el bienestar total y el de los consumidores.

Asimismo, se muestra que el efecto de la compatibilidad en el bienestar depende esencialmente del nivel de complementariedad/sustitución de las aplicaciones. En particular, la compatibilidad impacta negativamente en el bienestar y los consumidores si las aplicaciones son sustitutas cercanas.

El nivel de las asimetrías en las aplicaciones y/o en el valor de la plataforma puede influir también en el efecto de la compatibilidad en aplicaciones sobre el bienestar y los consumidores. En particular tiende a ser negativo cuando las asimetrías son importantes. Se muestra también que la presencia de fuertes efectos directos de red refuerza la potencial incidencia negativa de un creciente grado de compatibilidad en aplicaciones. Además, los resultados exhiben que un cambio marginal en la compatibilidad entre-redes tiene siempre un efecto negativo para los consumidores y pueden también ser negativo para el bienestar si las asimetrías son muy fuertes. Finalmente, se demuestra que cuando los sistemas son independientes o complementarios la compatibilidad en aplicaciones debería ser promovida por cualquier sector de la sociedad.

En relación a los efectos sobre los incentivos a innovar, los incentivos marginales a invertir en el valor de la plataforma son crecientes en el grado de compatibilidad para la plataforma débil, y también lo son los relativos a los incentivos a invertir en la aplicación siempre y cuando las asimetrías sean importantes. Por el contrario, para la firma dominante los incentivos marginales a invertir en la plataforma como así también en el valor de la aplicación son decrecientes en el grado de compatibilidad. Por su parte, los cambios en el nivel de compatibilidad entre-redes afectan positivamente los incentivos marginales a invertir de la plataforma débil pero negativamente los correspondientes a la plataforma dominante.

En pocas palabras, el capítulo destaca dos aspectos sobresalientes de la relación entre compatibilidad y bienestar. Primero, los bienes sustitutos son "malos" para la compatibilidad. Si los productos son sustitutos, la compatibilidad tiende a ser negativa para el bienestar. Segundo, las potenciales consecuencias negativas de la compatibilidad son más probables cuando las asimetrías son fuertes. Estos son nuevos resultados en el tema, como así también el estudio que se hace de la compatibilidad desde la perspectiva de la complementariedad/sustitución entre los bienes a los que se les impondría la compatibilidad.

El análisis reúne varias implicaciones que permiten evaluar decisiones de política del "mundo real". En particular, los resultados son un llamado de atención para algunos puntos relacionados con el caso entre la Comisión Europea y Microsoft.[1]

En esta investigación se muestra que a la hora de decidir sobre la necesidad de forzar la compatibilidad, la existencia de una firma dominante no es argumento suficiente para sostener que es más deseable la compatibilidad. Además, los efectos de red, generalmente presentes en este tipo de industrias, pueden ser también una razón para no forzar la compatibilidad en aplicaciones. En relación a los incentivos a innovar, cuando la Comisión tomó la decisión de forzar la compatibilidad argumentó que el remedio sería bueno para la innovación. En esta investigación se muestra que la compatibilidad genera un efecto "free-rider" que lleva a que la compatibilidad desincentive la innovación tanto de la firma dominante como de la débil.

Capítulo cuarto: Plataformas bilaterales con diferenciación en calidad. La literatura en mercados bilaterales se ha concentrado preferentemente en los efectos de red existentes entre los agentes, ignorando que en ciertos mercados

[1] En marzo 2004 la Comisión ordenó a Microsoft a abrir a los competidores códigos confidenciales que permitirían la compatibilidad entre los productos de los últimos y los de Microsoft. La decisión fue confirmada en septiembre 2007 por la Corte Europea de Primera Instancia.

la calidad o el tipo de los agentes que participan en la plataforma puede afectar también la utilidad del resto de los usuarios. Por ejemplo, en los centros comerciales o "shopping malls" se observa que algunas marcas (lado de los vendedores) están presentes en todos ellos mientras que otras no. Al mismo tiempo, los compradores visitan un centro comercial en función de las marcas que esperan encontrar. También suele observarse que marcas caras eligen estar agrupadas a pesar de que esto refuerza la competencia entre ellas. Un ejemplo de este fenómeno son los Village Outlets (Fidenza Village en Milan, Bicester Village en Londres, Las Rozas Village en Madrid, etc.) que reúnen sólo marcas muy caras como Loewe, Versace o Dior.

En este capítulo se presenta un modelo simple de plataformas que compiten en precios con dos nuevas características. Primero, las plataformas al fijar precios deciden endógenamente la calidad del servicio que ofrecen. Segundo, cada grupo exhibe preferencias no sólo sobre el número de agentes en el otro lado del mercado, sino también sobre su tipo o calidad. Además, los vendedores podrían también interesarse en el tipo de agentes presentes en su mismo lado.

El modelo propuesto considera agentes heterogéneos y en particular, vendedores de dos tipos: el tipo alto que recibe beneficios esencialmente de la presencia de compradores de "alta calidad" mientras que los vendedores de tipo bajo se benefician de la masa (cantidad) de participación de los compradores. El modelo se extiende luego al caso donde los vendedores de tipo alto también atienden a la calidad de los otros vendedores Esta característica que se denomina "efecto reputación" implica que los vendedores de tipo alto prefieren no compartir plataforma con los de tipo bajo.

El interés del capítulo es examinar el conjunto de condiciones bajo las cuales, más allá de la presencia de los efectos de red, en el mercado coexistirá más de una plataforma.

Un resultado central del estudio es que aunque las plataformas son ex-ante simétricas, en equilibrio pueden surgir dos plataformas asimétricas en calidad y/o beneficios. Para obtener este resultado es esencial la presencia del efecto reputación. Los resultados muestran además que los compradores maximizan su bienestar con un resultado de mercado en el cual están presentes dos plataformas con diferente calidad.

Abstract

In the first chapter we present a critical survey of the literature on platforms and two-sided markets. First, we introduce two general models, called the "Armstrong's model" and the "Rochet-Tirole's model", to understand the main issues that arise in the literature. Second, we analyze some of the main assumptions of the above models. Finally, we review some articles related to specific markets and to competition policy issues.

In the second chapter we study software platforms for which the total amount that users spend depends on the two-sided pricing strategy of the platform firm, and on the pricing strategy of application developers. When setting prices, developers may be constrained by one of two margins: the demand margin and the competition margin. By analyzing how these margins affect pricing strategies we find some conditions which explain features of the market of operating systems and its differences with the one corresponding to the video consoles. The problem that arises when the platform does not set prices (as an open platform) is considered. We show that policy makers should promote open source in operating systems platforms but not necessarily in video consoles. We also analyze the incentives for a platform to integrate with applications as a function of the extent of substitutability among them and provide a possible explanation for the observed fact of vertical disintegration in these industries.

Third chapter analyzes the effect of firm dominance on the incentives to become compatible and how compatibility decisions affect investment incentives. We will consider compatibility in two dimensions: compatibility of the complementary good and inter-network compatibility. We show that if products are substitutes, compatibility tends to be welfare decreasing with the potential negative consequences of increasing compatibility being more likely when asymmetries are strong. We also find that in many instances the dominant firm's interests regarding compatibility are in line with those of users, and are opposite to those of the weak firm, which will always demand more compatibility to be enforced. Finally we show that compatibility may harm innovation, particularly for the dominant firm.

In the fourth chapter we construct a simple model of platform price competition with two main novel features. First, platforms endogenously decide the quality of their 'access service' and second, each group exhibits preferences not only about the number of agents in the other side of the market, but also about their type or quality. Additionally, sellers may also care about the type of agents in their own side. Our interest is to examine the set of conditions under which, in spite of the presence of network effects, more than one platform coexist in the market. We show that although there are two ex-ante identical platforms, in equilibrium they could be asymmetric in quality and profits. In particular, buyers prefer a market outcome in which two different quality platforms are present.

Chapter 1. An Introductory Survey to Two-Sided Platforms

1. Introduction

Think of a market where two groups of agents, B and S, need to interact among them to get a surplus that, in each group, depends on the number of agents in the other group. The starting point to the theory of two-sided markets is that members of group i do not internalize the welfare impact of his participation on members of group j. In this context, a platform is a firm (or institution) that enables or facilitates the interaction between the two sides and internalizes the externalities through prices.

In section 2, we present two general models of monopoly platforms that encompass several special cases. In the first one, that we call the "Armstrong's model", we assume that the platform incurs a fixed cost of serving a participant of group i. Each customer has to pay a fixed charge to participate in the platform and her demand depends on the charge and the number of participants in the other group.

In the second model, the "Rochet-Tirole's model", the surplus of customers i arises when a transaction takes place and depends on the number of member in side j. Participants pay a per transaction price and the platform affords a variable per transaction cost. In this case, the demand of a member just depends on the per transaction charge.

In the framework of their model, Rochet and Tirole (2006) set two features that characterizes two-sided markets, namely the presence of indirect network externalities and the impact of price structure on transaction volume. Furthermore, they set a definition of two-sidedness: *"Consider a platform charging per-interaction charges a^B and a^S to the buyer and seller sides. The market for interactions between the two sides is one-sided if the volume V of transactions realized on the platform depends only on the aggregate price level $a = a^B + a^S$. If by contrast V varies with a^B while a is kept constant, the market is said to be two-sided".* It says that in a two-sided market the price structure is non neutral. They also state that it implies the failure of the Coase Theorem: in a Coasian world the gains from trade between two participants depend on the price level but not its structure. In particular, the failure of the Coase Theorem is a necessary condition for non neutrality of prices although not sufficient.

A key problem of the platform is "getting both types of customers on board" in order to have a product to offer. Indeed, the good or service is consumed jointly by members of both sides and a wrong pricing structure could imply the non existence of the product at all. The importance that the optimal pricing structure has is the main difference with industries based on one-sided markets.

We will see in the Armstrong's model that the profit maximizing prices are adjusted, relative to the cost, by the externality effect. In the Rochet-Tirole's model the total price charged to users on both sides has a relationship with the marginal cost incurred by the platform per transaction. However, the particular price charged to one side does not have generally any relationship either to that marginal cost or to costs specific to that side and prices can be set below costs as a consequence of a competitive practise. Furthermore, we can find new strong implications relative to antitrust policy once we take into account the two-sided nature of these markets.

In section 3 we present a non-exhaustive list of papers that work with particular markets and in section 4 we present some papers dealing with specific issues in competition policy.

2. The canonical models

This section presents the two canonical models of two-sided markets one introduced by Armstrong (2006) and another one by Rochet and Tirole (2003). Both models share some common characteristics: 1) There is a monopoly platform; 2) There are two sides of the market or two groups that interact denoted by B and S, with a continuum of members on each side; 3) A member of group i cares about the number of members of the other group j who go to the platform but does not care about the number of members of group i; 4) The platform sets a price on each side of the market. The main difference between them relies on the fact that in Armstrong platform charges are levied as a lump-sum fee, whereas in Rochet and Tirole agents pay a per-transaction fee to the platform.

We will first explain both models and will present the main results that they yield. We will then discuss some of their modeling assumptions, in particular those regarding platform competition and sellers competition within a platform.

2.1 The Armstrong's model. In a monopoly platform buyers and sellers interact. An end-user of side i, $i \in \{B, S\}$, has to pay a fixed platform charge A^i to participate in the platform. If N^j denotes the number of side-j end-users who decide to join the platform, then the net utility of an end-user on side i is given by

$$U^i = \beta^i N^j - A^i,$$

where the parameter β^i measures how a group i participant cares about the number of participants in the other side.

Assume that the number of participants of each side is determined by $N^i = \phi^i(U^i)$, for some increasing function ϕ^i, and that the platform incurs in a fixed cost C^i for serving a group i participant. Then, platform's profit equals

$$\pi = (A^B - C^B)N^B + (A^S - C^S)N^S,$$

which can be expressed in terms of the offered utilities (by setting implicit prices $A^i = \beta^i N^j - U^i$) as

$$\pi = (\beta^B N^S - U^B - C^B)N^B + (\beta^S N^B - U^S - C^S)N^S.$$

The profit maximizing prices become

$$A^i = C^i - \beta^j N^j + \frac{N^i}{\frac{\partial N^i}{\partial U^i}} = C^i - \beta^j N^j + \frac{1}{\sigma^i},$$

or, alternatively, using the "Lerner" formula

$$A^i - \big(C^i - \beta^j N^j\big) \over A^i = \frac{1}{\eta^i(A^i/N^j)}, \tag{1}$$

where $\eta^i(A^i/N^j) = \frac{\partial N^i}{\partial U^i}\frac{A^i}{N^i}$ is the price elasticity of demand for group i, for a given level of participation by the group j.

In a two sided-market the profit maximizing prices are adjusted, relative to the cost, by the externality effect $\beta^j N^j$ and the price structure is determined by the relative externalities on each side, β^i and β^j. The side with the larger β is more likely to receive larger charges than the side with lower indirect network externalities. Furthermore, for "getting both sides on board", the platform may optimally set a zero or even negative price on the side that generates the largest externality.

The socially optimal prices are

$$A^i = C^i - \beta^j N^j.$$

The optimal price for group i equals the fixed cost of serving this group adjusted downward by the external benefit that an extra group-i agent brings to the agents of the other group on the platform. In particular, optimal prices will be below cost as long as β^B and β^S are positive.[2]

2.2 The Rochet and Tirole' model. Consider a model where the surplus is created by "transactions" between pairs of end-users (think of a credit card market or of matchmaker services). Assume that there are no fixed costs but that there is a variable cost c which is incurred when a transaction takes place and is therefore not attributable to a single side alone. Each member on side i enjoys a per transaction benefit b^i and has to pay a per-transaction price a^i. End-users are heterogeneous over the transaction benefit and transactions between end-users involve no payment. The net utility of a member on side i is defined by

$$U^i = (b^i - a^i)N^j,$$

where the network externalities are reflected by the fact that the surplus of a customer with b^i depends on the number of customers j, N^j. However, the demand function

$$N^i = P(U^i \geq 0) = N^i(a^i) \quad i \in \{B, S\},$$

is independent of that number. Platform members use the platform if and only if $b^i \geq a^i$, thus the participation decision is determined by the per transaction surplus and does not depend on the number of agents who will join the platform on the other side.

Let the volume of transactions be $N^B N^S$, a multiplicative demand that captures the interaction between the two market sides.[3] The platform chooses a^B and

[2] The prices that maximizes the total welfare subject to the zero profits constraint for the platform are

$$A^i = C^i - \beta^j N^j + \frac{(\lambda - 1)}{\lambda}\frac{N^i}{\frac{\partial N^i}{\partial U^i}}, \qquad i \in \{B, S\}.$$

[3] See Evans (2003a) p. 341 for a discussion of this assumption.

a^S to maximize total profit

$$\pi = (a^B + a^S - c)N^B N^S,$$

and the optimal price structure is characterized by

$$a^B + a^S - c = \frac{1}{\sigma^B} = \frac{1}{\sigma^S},$$

where $\sigma^i = \frac{\partial N^i}{\partial a^i}\frac{1}{N^i}$ is a semi-elasticity.

In particular, prices can be characterized by formulae that are reminiscent of Lerner's formula:

$$\frac{a^B + a^S - c}{a^B} = \frac{1}{\eta^B},$$
$$\frac{a^B + a^S - c}{a^S} = \frac{1}{\eta^S}$$

where $\eta^i = \frac{\partial N^i}{\partial a^i}\frac{a^i}{N^i}$. Moreover, the total price $a = a^B + a^S$ chosen by the private monopoly is given by the classical Lerner formula

$$\frac{a - c}{a} = \frac{1}{\eta},$$

where $\eta = \eta^B + \eta^S$, and the price structure is given by the ratio of elasticities

$$\frac{a^B}{\eta^B} = \frac{a^S}{\eta^S}.$$

The condition to set the total price $a = a^B + a^S$ in two-sided markets is analogous to the Lerner condition for monopoly pricing in one-sided markets. The key point is that, in equilibrium, the ratio of the prices charged by the platform is proportional to the ratio of the elasticities of demand on the two sides. It is important to notice that the price structure does not depend on c. Thus, the particular price charged to one side may not have any relationship either with its marginal cost or with any cost specific to that side. Furthermore, prices can be set below costs as a consequence of a competitive practice.[4]

2.3 The integrated model. Rochet and Tirole (2006) develop a model that integrates the two models presented above.[5] Each member on side i enjoys a benefit per transaction b^i and a fixed membership benefit β^i (positive or negative). On each side i, end-users are heterogeneous over both benefits. Members on side i pay to the platform A^i for membership and a usage fee a^i. The platform's costs include a fixed cost C^i per member on side i and a marginal cost c per transaction between two members of different sides. Transactions between end-users involve no payment.

The net utility of an end-user on side i is

$$U^i = (b^i - a^i)N^j + \beta^i - A^i.$$

[4]The prices that maximize social welfare subject to the constraint $a^B + a^S = c$ satisfy

$$\frac{a^B}{\eta^B}[\frac{V^B}{N^B}] = \frac{a^S}{\eta^S}[\frac{V^S}{N^S}],$$

where V^i is the net surplus of side i from an average transaction.

[5]Authors consider a "Pure-membership model" the Armstrong model and a "Pure-usage model" the Rochet and Tirole model.

The number of members of side i is thus

$$N^i = Pr(U^i \geq 0),$$

which generates "quasi-demand functions" defined by

$$N^i = N^i(A^i, a^i, N^j) \quad i \in \{B, S\}. \tag{2}$$

Platform's profit are equal to

$$\pi = (A^B - C^B)N^B + (A^S - C^S)N^S + (a^B + a^S - c)N^B N^S. \tag{3}$$

Under regularity conditions, system (2) has a unique solution characterizing memberships N^B and N^S as functions of (A^i, a^i). In particular,

$$N^B = n^B(A^B, a^B, A^S, a^S) \text{ and } N^S = n^S(A^B, a^B, A^S, a^S).$$

If we define the price that the platform sets as

$$p^i = a^i + \frac{A^i - C^i}{N^j},$$

then equation (3) can be transformed into

$$\Pi = (p^B + p^S - c)n^B(p^B, p^S)n^S(p^B, p^S).$$

The optimal price structure is hence determined by

$$\frac{1}{p^B + p^S - c} = \frac{\partial n^B}{\partial p^B}\frac{1}{n^B} + \frac{\partial n^S}{\partial p^B}\frac{1}{n^S} = \frac{\partial n^B}{\partial p^S}\frac{1}{n^B} + \frac{\partial n^S}{\partial p^S}\frac{1}{n^S}. \tag{4}$$

Note that it takes into account the direct effect of the price of side i on the quasi-demand of side i and the indirect effect of this price on the quasi-demand of side j.[6]

2.4 Modeling assumptions.

2.4.1 Platform competition. The two canonical models assume that there is just one platform.[7] In this subsection we present some extensions of the canonical models that allow for platform competition.

Whenever there is more than one platform it is necessary to define if agents on each side patronize only one platform (single-home) or more than one platform (multi-home). In particular, we have to set if this feature is exogenously imposed or if it is an equilibrium result. There are three cases to consider: (i) both sides single-home, (ii) one side single-homes while the other multi-homes, and (iii) both sides multi-home.

In what follows we first present the main results derived under platform competition under the proviso that each agent can only join a single platform. We will then allow for multihoming and endogenous patronizing.

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[6] In terms of elasticities expression (4) can be rewriten as

$$\frac{p^B + p^S - c}{p^B} = \frac{1}{\eta^B + \eta^{SB}}$$

and

$$\frac{p^B + p^S - c}{p^S} = \frac{1}{\eta^S + \eta^{BS}}.$$

where $\eta^i$and η^{ij} are the price elasticity of demand and the cross elasticity of demand, respectively.

[7] This is a restrictive assumption given that empirically it is difficult to find a real-life market organized around a single platform.

Exogenous two-sided single-homing. Armstrong (2006) extends its "pure-membership pricing" model to study competition between two platforms. He assumes that platforms are horizontally differentiated and that, for exogenous reasons, each participant chooses to only participate in one platform. The number of members of group i who goes to platform 1 is given by

$$N_1^i = \frac{1}{2} + \frac{U_1^i - U_2^i}{2t_i}$$

where t_i is the differentiation parameter on side i, with $t_i > \beta^i$.

Symmetric equilibrium prices are given by

$$\begin{aligned} A^B &= C^B + t^B - \frac{\beta^S}{t^S}(\beta^B + A^S - C^S), \\ A^S &= C^S + t^S - \frac{\beta^B}{t^B}(\beta^S + A^B - C^B). \end{aligned} \tag{5}$$

In a Hotelling model without network effects, the equilibrium price for group i would be $A^i = C^i + t^i$. However, in a two-sided market the price is adjusted downwards by the factor $\frac{\beta^j}{t^j}(\beta^i + A^j - C^j)$. The term $(\beta^i + A^j - C^j)$ represents the benefit to a platform of having an additional customer of group j and $\frac{\beta^j}{t^j}$ measures the additional j customers attracted by an extra group i customer. Solving the simultaneous equations in (5), the resulting expressions for equilibrium prices are

$$\begin{aligned} A^B &= C^B + t^B - \beta^S, \\ A^S &= C^S + t^S - \beta^B. \end{aligned} \tag{6}$$

The platform will charge more to the group that is less competitive and that yields smaller benefits to the other group. Thus, a price in (6) can be negative for the side of the market which has lower fixed and transport costs and/or which generates a larger external benefit to the other side. These prices can be expressed as

$$\frac{A^i - \left(C^i - 2\beta^j N^j\right)}{A^i} = \frac{1}{\eta^i}, \quad i \in \{B, S\} \tag{7}$$

where $\eta^i = p_i/t_i$ is the platform's own-price elasticity of demand. Note that, when a duopoly platform increases the price inducing an agent from a group to leave, this agent goes to the rival platform, making it harder to attract agents from the other group. This is the reason behind the term $2\beta^j N^j$ in formula (7) when compared with the monopoly expression in (1) for which the corresponding term is only $\beta^j N^j$.
Multi-homing and endogenous patronizing. The presence of multihoming on at least one market side influences the degree of competition. The competitive pressure will be stronger wherever a platform can get rid of its competitors, which occurs more easily where singlehoming prevails. A "competitive bottleneck" situation will emerge whenever one side, say group B, continues to deal with a single platform whereas the other side wishes to deal with both. The outcome is then that the platform ignores the interests of the multihoming group while strongly attends those of the singlehoming group.

Caillaud and Jullien (2003) propose an imperfect-competition Bertrand game between two matchmakers in the presence of indirect network externalities. They analyze both the case of exogenous two-sided single-homing and the case of multi-homing. They show that the relevant pricing strategies are of a "divide-and-conquer" nature, subsidizing the participation of one side (divide) and recovering the loss on the other side (conquer). It implies a highly contestable market structure when single-homing is imposed. In contrast, when users wish to engage in multi-homing, it is more difficult to "conquer", and platforms can avoid fierce competition.

Armstrong and Wright (2007) discuss the conditions under which agents singlehome or multihome under the assumption of non-negative prices. In the context of a standard Hotelling model of price competition between two platforms, agents make a subscription decision (a pure-membership model). They prove that a unique equilibrium may emerge in which all users single-home, the two platforms offer the same pair of prices, and half of the agents from each group join each platform. In particular,

if $C^B + t^B \geq \beta^S$ and $C^S + t^S \geq \beta^B$ equilibrium prices are like (6);
if $C^B + t^B < \beta^S$ equilibrium prices are

$$\begin{aligned} A^B &= 0 \\ A^S &= C^S + t^S - \frac{\beta^B(\beta^S - C^B)}{t^B}; \end{aligned}$$

if $C^S + t^S < \beta^B$ equilibrium prices are

$$\begin{aligned} A^B &= C^B + t^B - \frac{\beta^S(\beta^B - C^S)}{t^S} \\ A^S &= 0. \end{aligned}$$

The prices of the first case have been analyzed in the previous subsection. Consider the second case (the third one is analogous). When the asymmetry in the size of the network effect between the two groups is sufficiently large, i.e., when $C^B + t^B < \beta^S$ and $C^S + t^S \geq \beta^B$, platforms would pay buyers to join, given the resulting increase in demand from sellers. Under the assumption of non-negative prices, this results in platforms offering buyers a zero price, which is a usual strategy in two-sided markets. Furthermore, platforms will compete very aggressively for sellers in order to attract more buyers, and hence, due to the positive indirect network effects, to attract more sellers.

Under product differentiation on one side, for instance $t^S = 0$ and $t^B > \beta^B$, sellers view the competing platforms as homogenous (ignoring the number of buyers on the platforms), while buyers prefer using a particular platform over the other. In this case, buyers will tend to singlehome, what is an incentive for sellers to multihome in order to maximize network benefits. If there are no network effects for one side, i.e., β^i for some i, the equilibrium is unique and symmetric, group i singlehomes and group j multihomes.

Rochet and Tirole (2003) provide an extension of their "pure-usage pricing" model in which there are two competing platforms, 1 and 2. The buyer's gross surplus is denoted by b_i^B when the transaction takes place on platform i. In contrast, the sellers gross surplus is b^S no matter the platform they join. It is further assumed that (b_1^B, b_2^B, b^S) are private information and are drawn from continuous distributions. The charges set by platform i are a_i^B to the buyer side and a_i^S to the seller side. Under a symmetric distribution of (b_1^B, b_2^B) there is a symmetric equilibrium ($a_1^B = a_2^B = a^B$ and $a_1^S = a_2^S = a^S$ with) where the sellers multihome, and such that

$$a^B + a^S - c = \frac{a^B}{\eta_0^B} = \frac{a^S}{(\eta^S/\sigma)}, \tag{8}$$

where $\eta_0^B = -\frac{p^B \partial d_i^B / \partial p_i^B}{d^B}$ is the "own-brand elasticity of demand for buyers", d_i^B is the proportion of buyers that trade in platform i, and σ_i measures the "loyalty" of consumers of platform i with $d_1^B = d_2^B = d^B$ and $\sigma_1 = \sigma_2 = \sigma$ in the symmetric equilibrium pricing of equation (8).[8] Note that (8) resembles the analogous formula in the monopoly case. On the buyer side, η_0^B plays now the role of η^B with $\eta_0^B > \eta^B$. On the seller side η^S is adjusted by σ, and coincides with the result in the monopoly case when all buyers singlehome so that $\sigma = 1$. In contrast, if buyers multihome ($\sigma \to 0$) the charge to sellers tends to decrease as they have now more options.

Gabszewicz and Wauthy (2004) model the competition between two platforms within a vertical differentiation framework where quality in one platform is endogenously determined by the size of the network in this platform. In other words, the platform with the larger number of members on side i, is seen by members on side j as a good of higher quality than the other platform. Assuming heterogeneous agents, authors show that when multhoming is allowed there is a unique equilibrium exhibiting positive profits for both firms. Monopoly prices are set on the side that multihomes and products are given for free on the side that singlehomes. This equilibrium, which is the unique equilibrium outcome, has similar features to the equilibrium in Armstrong (2006) when he *assumes* single-homing on one side and multihoming in the other.

Viecens (2008a) constructs a simple model where platforms endogenously decide the quality of their 'access service', the buyers singlehome and sellers are allowed to multihome. The sellers are of two types, high and low, and the quality of a platform depends on the number of high type sellers relative to the total number of sellers in the platform. It is shown that ex-ante symmetric platforms may become asymmetric in equilibrium. Moreover, depending on parameter values, different type of sellers may follow different strategies, i.e., an equilibrium where high type sellers multihome but low type sellers join only one platform may arise.

2.4.2 End user's market: no rivalry effects vs. competition. An important assumption introduced in the canonical models, and kept in many follow-up papers, is that a member of group i does not care about the number of members of her own group. This assumption excludes competition between subjects within the same platform (no rivalry effects) and implies that retailers and consumers act as monopolists.

Nocke, Peitz and Stahl (2007) discuss platform ownership in a model where competition between sellers exists. They consider a continuum of sellers and denote a seller's (gross) variable profit by

$$\Pi(m_s) = z(m_s)\pi(m_s),$$

where m_s is the measure of entering sellers (it is also the number of goods bought from the sellers since consumers buy one good from each of them). The term $\pi(m_s)$ is the seller's variable profit per unit mass of buyers and $z(m_s)$ is the mass of buyers visiting the market place, which is increasing in m_s. They assume $\pi(m_s)$

[8] The σ is equal to 1 if platform i buyers stop trading when sellers leave that platform.

will decrease with m_s by two reasons: first, there is a *market share effect*: for given prices, if the amount of sellers increases buyers buy less from each seller, and second, there is a *price effect*: as the amount of sellers increases, prices fall as competition increases. Regarding the impact of m_s on variable profits, it will depend on the interplay between two contrasting effects: on one hand, a positive indirect network effect that makes sellers' profit to increase as the number of firms on its own side increases by the impact of $z(m_s)$ on profits, and, on the other hand, a negative competition effect which makes firm's profit to decrease as the number of firms on its own side increases via $\pi(m_s)$. They show that the platform ownership structure and the total welfare will depend on the strength of these two aforementioned effects. In particular, if the positive indirect network effect is the strongest, then equilibrium platform size under monopoly ownership is larger than under dispersed ownership and a monopoly is the socially preferred ownership structure. In contrast, if the competition effect is the strongest then monopoly ownership induces a smaller platform than an open ownership structure.[9] In particular, under closed ownership and weak indirect network effect allowing sellers to integrate downwards onto the platform may be socially beneficial.

Hagiu (2006) explicitly considers the price that sellers set for the good that they sell to consumers. He assumes that sellers act competitively, thus,for a given gross consumers' surplus $V(m_s)$, sellers set a price equal to the marginal contribution of a new variety to the surplus, in a symmetric equilibrium, i.e., $p = V'(m_s)$. The author shows that one platform yields a higher social welfare than two platforms. It mainly occurs because the monopoly platform offers more product diversity to its users than any of the two competing platforms given that it is able to internalize a larger share of user benefits.

Viecens (2007) works with a model similar to that in Hagiu (2006b) and considers the case where sellers have some market power. It follows that they can set a price $\hat{p}$ that maximizes their profits, provided that $\hat{p} < V'(m_s)$.[10] This scenario allows to consider the reduction in the demand for the platform when contemplating a price increase in the product of the sellers, an effect that has been largely ignored by the literature. Furthermore, we show that $\pi(m_s)$ is not necessarily decreasing. The model also permits to analyze the incentives for a platform to integrate with the seller side as a function of the extent of substitutability among their goods.

2.4.3 Overview. The table below summarizes the main modelling assumptions adopted in the literature discussed so far.

	Platform competition	End user's market
Armstrong (2006)	Monopoly and Hotelling	No rivalry effects
Armstrong and Wright (2007)	Hotelling	No rivalry effects
Caillaud and Jullien (2003)	Duopoly	No rivalry effects
Gabszewicz and Wauthy (2004)	Vertical differentiation	No rivalry effects
Hagiu (2006)	Monopoly and Hotelling	Competition
Nocke, et al. (2007)	Monopoly	Competition
Rochet and Tirole (2003)	Monopoly and duopoly	No rivalry effects

[9]The authors define a closed platform or club, as one in which access can be restricted by the incumbent intermediaries. An open platform in their setting is not one which sets zero charges but one with unrestricted access.

[10]If $\hat{p} > V'(m_s)$ holds, sellers are forced to set $p = V'(m_s)$.

2.5 An alternative model. Economides and Katsamakas (2006) propose a very different model to the one presented in sections 2.1 and 2.2 in which firms do also follow a two sided strategy. Their setting consists of one platform firm selling platform A_0 and an independent firm selling good B_1, which is complementary to the platform. The platform firm sells the platform to end-users at price p_0. The independent application provider sells the application to end-users at price p_1 and also pays a per unit access fee s to the platform firm (where s can be negative).

They assume a linear demand structure such that the demand function of the platform A_0 is given by

$$q_0 = a_0 - b_0 p_0 - dp_1,$$

and the demand of the application B_1 is set at

$$q_1 = a_1 - b_1 p_1 - dp.$$

They find that following a two-sided strategy yields an additional value for the platform (as compared to the one-sided strategy, i.e., when $s = 0$ is given). Note that this value vanishes when the platform and the application are perfect complements, i.e., when $d = b_0 = b_1 = b$ and $a_0 = a_1 = a$. They also find that the platform sets a negative s in equilibrium (subsidizes the independent application provider) as long as the ratio a_0/a_1 is large.

3. Literature on particular markets

In this section we present a non-exhaustive summary of some of the markets that have been analyzed by using the framework of two-sided markets.

3.1 Card Payment Systems. A platform that has been widely studied is the card payment system where on the one hand card holders (buyers) and on the other hand merchant affiliated to the platform (sellers) are both served by service providers, issuing and acquiring banks, that set their prices for issuing and acquiring.

Wright (2003) presents a monopoly platform model with two different types of merchant pricing, monopoly pricing and Bertrand.[11] For some of the analysis consumers make a separate subscription decision (facing a membership fee) and then a usage decision (earning rebates).

Let the gross benefit of each purchase to a consumer be v per good and the gross cost to each merchant be d whith $v > d > 0$. Using a card for a transaction generates a benefit of b^B to cardholders and a benefit of b^S to merchants, where b^B is continuously distributed on the interval $[\underline{b}^B, \bar{b}^B]$, according to the distribution function $H(b^B)$. All merchants have the same value of b^S. A transaction done using cards costs the issuer c^I and the acquirer c^A. The fee set by symmetric issuers for issuing is denoted by f, the interchange fee that the acquirers pay to the issuers is a and the fee that merchants pay is m. Acquirers are assumed to be perfectly competitive setting the equilibrium merchant fee $m = c^A + a$. Further, the timing of decisions is as follows: 1. Payment system rules are set. In particular, a rule is set whereby merchants are either allowed to set a surcharge for card payment, or not. Also, the centralized interchange fee a is set. 2. Issuing and acquiring banks set their prices for issuing and acquiring respectively (f and m). 3. Consumers and

[11] Ses also Rochet and Tirole (2002). Guthrie and Wright (2007) extend the model in order to discuss two competing payment schemes.

merchants decide whether to join the payment network. 4. Merchants set prices for goods (p_{card} and p_{cash}). 5. Consumers decide which merchant to purchase from and what payment method to use.

Two cases are analyzed: "merchant surcharging", where merchants can set a surcharge on goods purchased with cards, and the "no-surcharge rule" case.

Merchant surcharging. When surcharging is allowed, Wright (2003) finds a unique equilibrium where monopolistic merchants engage in excessive surcharging. The marginal cardholder, defined by b^B_m, satisfies the equation

$$b^B_m = f(b^S - c^A) + \frac{1 - H(b^B_m)}{h(b^B_m)}, \tag{9}$$

where $p_{cash} = v$ and p_{card} is set to maximize the merchant's profits which are given by

$$\pi = H(b^B_m)(v - d) + (1 - H(b^B_m))(p_{card} - d + b^S - m).$$

Inspection of equation 9 shows that the marginal cardholder can be defined without reference to the interchange fee (neutrality of intercharge fees).

Under Bertrand competition the prices are $p_{cash} = d$ and $p_{card} = d + m - b^S$ and the marginal cardholder is defined by $b^B_m = f(b^S - c^A)$.

No-Surcharge rule. When a no-surcharge rule is in place, monopolistic merchants set a uniform price $p = v$, the card association sets an interchange fee $a^* = b^S - c^A$ and consumers will get and use cards if and only if $b^B \geq f(b^S - c^A)$.

Under Bertrand competition results will depend on the relation ship between mercahnt benefit b^S and merchants fee m. In particular, if $b^S \leq m$ then merchants will either accept only cash sales at a price $p_{cash} = d$ or will acept both card sales and cash sales at a common price $p^* = d + m - b^S$. If $b^S > m$, in a competitive equilibrium merchants will discount for card purchases, setting $p_{cash} = d$ and $p_{card} = d + m - b^S$, with at least one firm accepting cards at these prices. In either case, the marginar cardholder is defined by $b^B_m = f(b^S - c^A)$. Note that if merchants are forced to price the same for both card and cash, then customers will also induce merchants to "separate" into those that accept cards and those that do not.

The author finds that both monopolistic pricing and perfect competition constrain the ability of card schemes to use interchange fees and the no-surcharge rule in anticompetitive ways. Merchants with market power will not accept cards unless merchant fees are at or below the cost savings which card acceptance provides them. On the other hand, competitive merchants will not be able to sustain any cross-subsidy between cash and card customers under the no-surcharge rule. The paper highlights the positive role of the no-surcharge rule in preventing excessive merchant surcharging. In either case, the no-surcharge rule and privately set interchange fees cannot reduce welfare, and in the case of monopolistic merchants, it will be welfare enhancing.

3.2 Technology/Software markets. Evans et.al (2006) present an exploration of the two-sided economics of software platforms. They offer detailed studies of the personal computer, video game console, personal digital assistant, smart mobile phone, and digital media software platform industries, focusing on the business decisions made by industry players. The authors argue that in order to understand the successes of software platforms, it is necessary to understand their role as a technological ground where application developers and end users meet (i.e., their role as a platform).

Hagiu (2006) and Economides and Katsamakas (2006) focus on markets organized around a software platform allowing users to access the large number of applications supplied by independent developers, who must gain access to the platform. Both papers compare industry structures based on a proprietary platform (such as Windows) with those based on an open source platform (such as Linux), and analyze industry implications in terms of social welfare. Hagiu (2006) identifies a economic welfare trade-off between two-sided open platforms and two-sided proprietary platforms. The latters create two-sided dead weight losses through monopoly pricing but at the same time, when setting prices they partially internalize the two-sided positive indirect network effects and the direct competitive effects on the producers side. In contrast, open platforms internalize neither of these effects. Because of this, the author finds that proprietary platforms can be socially more desirable than open ones. Economides and Katsamakas (2006) find that the variety of applications is larger when the platform is open source.

Viecens (2007) provides an explanation for some features of the market of operating systems and for its differences with the video consoles market. Her results suggest that policy makers should promote open source in operating systems but not necessarily in platforms as video consoles. She also provides a possible explanation for the observed vertical disintegration in these industries (an empirical fact reported in Evans, et. al. (2006)).

Clements and Ohashi (2005) have estimated network effects in the home video game industry and the effectiveness of console price and software variety as two alternative ways of stimulating hardware demand.

3.3 Media industries. A media firm (the platform) works between two markets: the media market in which it sells magazines, newspapers, TV channels, web-sites, etc. to a population of viewers, listeners and readers and the advertising market in which it sells spaces to advertisers. The acceptance of media consumers towards advertising is not clear, some of them dislike ads while others like them.[12]

Newspapers. Gabszewicz et.al. (2001) show in a very simple model of two editorials that newspapers may moderate their political message in order to get a large number of readers ranging from the extreme left to the extreme right in political opinions. Readers are indifferent about the level of advertising implying that the network effects go in just one direction.

TV. In Anderson and Coate (2005) two media platforms compete in a framework where the content of tv programming is given. They analyze the conditions under which there exist over- or underprovision of advertising. In the work of Peitz and Valletti (2004) the platforms set the content and the level of advertising in a TV market under pay and free-to-air schemes. Authors compare the welfare properties of both systems.

Yellow pages. Rysman (2004) estimates two demand curves simultaneously in order to measure the network effects between consumers of directories and advertisers in yellow pages. He considers the inverse demand for advertising as a function of the amount of advertising and the number of uses per consumer. Consumer demand for usage is a function of the amount of advertising. He finds that the effect

[12]Authors consider advertisement as a nuisance and so it generates a negative externality. It could be argued that advertisments in magazines and newspapers are not as much of a nuisace as they are in TV, radio or web-pages (a point made in Anderson and Gabszewicz (2006)). See also Reisinger (2004) for a study of negative externalities in two-sided markets.

of advertising on usage and the effect of usage on advertising are both positive and significant, showing that network effects exist. The effect of the amount of advertising on the price of advertising is negative and significant. He also examines the welfare trade-off between competition and a monopoly structure and finds that, despite the positive network effects, a competitive market structure is preferable.

3.4 Call termination on mobile telephone networks. Call termination refers to the service whereby a network completes a call made to one of its subscribers by a caller on another network. There are two types of call termination on mobile telephone networks: termination of calls made from other mobile networks (mobile to mobile, MTM), and termination of calls made by callers on the fixed-line telephone network (fixed to mobile, FTM).

Armstrong and Wright (2007b) argue that call termination on mobile telephone networks is a leading example of a competitive bottleneck.[13] Because of this, in the case of FTM termination the existing literature predicted termination charges being set at the monopoly level. In the case of MTM call termination, the study of two-way interconnection has led the literature to focus on whether mobile networks can use a negotiated termination charge to relax competition for subscribers and on showing that the predicted termination charge is below the efficient level (see Laffont, Rey and Tirole (1998)).

Armstrong and Wright (2007b) combine a model of FTM calls with a model of MTM calls what allows them to consider the impact of wholesale arbitrage and demand-side substitution. They show that whenever a large number of mobile networks exist, the market failure associated with MTM termination is negligible, in contrast to the situation with FTM termination.

3.5 Overview. Next table summarizes the industry specific literature on two-sided markets.

Card Payment Systems	Guthrie and Wright(2007) Rochet and Tirole (2002) Wright (2003)
Technology/Software markets	Clements and Ohashi(2005) Economides and Katsamakas(2006) Evans et.al (2006) Hagiu (2006) Viecens (2007)
Media industries	Anderson and Coate (2005) Anderson and Gabszewicz (2006) Gabszewicz et.al. (2001) Peitz and Valletti (2004) Rysman (2004)
Mobile telephone networks	Armstrong and Wright (2007b)

[13]When a competitive bottleneck takes place firms compete to attract the set of consumers that wishes to deal with just one firm. The other set of consumers wishes to interact with the first group so that firm can charge the second group higher prices for access to its captive customers.

4. Literature on competition policy issues

Evans (2003b) remarks that the antitrust policy in these industries should consider the particular economic principles that characterize them. A lack of understanding of the typical nature of two-sided markets, especially about the interdependence of agents' decisions, may lead to erroneous conclusions. In particular, he notes that prices will not follow marginal costs on each side of the market, since price levels, price structures, and investment strategies must optimize output taking into account the indirect network effects available on both sides.[14] Economides (2007) discusses how antitrust law and regulatory rules should be applied to industries where network effects are present. He finds that significant differences may appear between the effects of application of antitrust law in network and non-network industries.

In a static one-sided context more competition always increases social welfare because the deadweight losses due to pricing by firms with market power are reduced. In a two-sided context however, an economic efficiency tradeoff arises between internalizing two-sided indirect network effects and creating two-sided deadweight loss. Thus, the desirability of competition between two-sided platforms may be compromised.

Choi (2006) analyzes the effects of tying arrangements on market competition and social welfare in two-sided markets when economic agents can engage in multi-homing. He finds that tying can be welfare-enhancing if multi-homing is allowed, even in cases where its welfare impacts are negative in the absence of multi-homing. Amelio and Jullien (2007) show that a multi-product monopoly platform uses bundles to raise participation on both sides, which benefit consumers, whereas tying may not be optimal for a contested platform in a duopoly context. The impact on consumers surplus and total welfare depends on the extent of asymmetry in externalities between the two sides.

Armstrong and Wright (2007) explore the role of exclusive dealing and show that exclusive contracts will be used by platforms to prevent multihoming, allowing one of them to attract one side exclusively and capture the network benefits generated to the other side. If we compare to multi-homing, such contracts are inefficient since many buyers are forced to visit a platform they do not like, and the remaining buyers no longer enjoy any network benefits. Doganoglu and Wright (2007) analyze the ability of an incumbent to use exclusive deals or introductory offers to dominate a market under the threat of entry when network effects are present. They find that when consumers can only join one firm, the incumbent will make discriminatory offers that are anticompetitive and inefficient. In contrast, if consumers are allowed to multihome, they find offers that only require consumers to commit to purchase from the incumbent are not anticompetitive. Corts and Lederman (2007) estimate a model of hardware demand and software supply to investigate the scope of indirect network effects in the home video game industry. They argue that the increasing presence of non-exclusive software gives rise to indirect network effects that exist between users of competing and incompatible hardware platforms.

Viecens (2008b) considers a model with two asymmetric platforms, each of them with an associated application produced by competitive third party developers. She analyzes the effect of making compatible the applications between the platforms.

[14] See also Rochet, J.C. and Tirole J. (2007).

Under incompatibility the situation is like an ex-ante single-homing or exclusivity of the developers, where compatibility leads to partial (or full) multihoming or non-exclusivity. Users are assumed that single-home, a reasonable assumption for industries organized as software platforms. The weak platform always promotes compatibility. In contrast, the dominant platform never does when the market is covered. If the market can be expanded, the higher value that compatibility yields (a larger network, ver según Economides) may compensate the reducing differentiation effect due to compatibility (that generates a loss of market power of the dominant one, it experiments a reduction of its dominant position) and then the dominant platform may also promote compatibility.

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Chapter 2. Pricing Strategies in Software Platforms

1. Introduction

Many modern industries work around software platforms. Typical examples are operating systems for computers, personal digital assistants, smart mobile phones or video-game consoles. The usual feature is that they connect or attend different types of customers that benefit from the interaction among them, characterizing what is known in the literature as two (multi)-sided platforms. On the one side, developers write the applications or software that improve the value of the platform for the users. On the other side, users derive utility from consuming the system (the platform and the applications). Because of this, users are concerned about the system price, i.e., the total amount spent in the platform and the software. The system price will hence depend on the two-sided pricing strategy of the platform firm which in turn affects the market of complementary applications, and on the pricing strategy in the developers' market. This paper offers a model of a monopolist two-sided platform that allows us to analyze the pricing strategies it will adopt, the level of entry it will induce in the applications' market and the welfare it will generate. Furthermore, by considering that it can become either an open platform or a proprietary one, we will study the implications of having one or the other. Finally, issues related to the vertical structure of the platform and to the role of outside options will also be analyzed.

Two well known and widely used software platforms are video consoles and computer operating systems. In both, users care for the total charge of the system (platform and applications). Nevertheless they have followed quite different pricing strategies. Operating system platforms charge high prices to the users and subsidize developers. However, video console firms charge low prices to users and make profits on the developers' side.[15] We provide here a possible explanation for the difference based on the margin at which developers compete. When setting prices, developers may be constrained by one of two margins, the demand margin and the competition margin. As long as the demand margin binds, prices of developers affect the overall demand of the system and they set the price that maximizes their profits, a price that is lower than their marginal contribution to the users utility. In contrast, if competition margin binds, developers can not affect overall demand of the system and they are forced to set a price equal to their contribution to the users surplus.[16] What margin is binding depends on the number of applications in the market and on the level of substitutability among them. In particular, the competition margin

[15] This issue is largely analyzed by Hagiu (2006b).

[16] Lerner and Tirole (2004) introduce the two margins to analyze pricing strategies in patent pools.

is more likely to bind as long as users prefer a system with many applications and these are near substitutes. In the market of video console gamers state that price is very important in deciding what game to buy. Some of them report having a huge number of games and, for instance, among the ten top rated PlayStation 2 games, 3 of them belong to the adventure genre and 3 to the role-playing genre.[17] These facts allow us to presume that developers writing for the video console are constrained by the competition margin. However, users of operating systems need a lower number of applications that indeed are far substitutes, like a text processor, a spreadsheet or a browser, so that we suspect the developers in this market are constrained by the demand margin. By analyzing how these margins affect the pricing strategies and the profits of the platform, we find some conditions that may help to explain features of the market of operating systems and its differences with that corresponding to the video consoles, and shed some light on the different pricing routes they have followed. We observe that the platform price for users is higher when demand margin binds than when competition margin binds, and this is consistent with the observed fact that operating systems charge high prices to users, whereas video console firms charge low prices to them.

When considering the problem that arises if the platform does not set prices (as an open platform), our model allows us to contribute to the current enthusiastic discussion on whether governments should promote (as some of them do) open source platforms. Nowadays, 50% of European public administrations declare that they use some open source software and the figure is 35% for the USA. In addition, some large companies are also using open source programs.[18] The literature is not conclusive about recommendations. Hagiu (2006b) shows that there is a trade-off between the extent to which proprietary platforms internalize indirect network effects through profit-maximizing pricing and the two-sided deadweight loss they create. He shows that a proprietary platform may generate a higher level of product variety and welfare than an open platform. In contrast, Economides and Katsamakas (2006a) find that the variety of applications and social welfare is always larger when the platform is open source. We here show that outcomes may depend on the margin that binds. We find some results that suggest that policy makers should promote open source platforms where demand margin binds (as operating systems) but not necessarily in platforms where competition margin binds (as video consoles). In particular, we prove that if demand margin binds, a proprietary platform and an open platform will provide the same level of applications, so that the latter will generate more welfare for users. However, if competition margin binds a proprietary platform may generate a larger number of applications and higher welfare to users than an open platform.

In a book about empirical business and economics aspects of software based platforms, Evans, et. al. (2006) document that almost all the successful firms in these industries started being one-sided, producing applications at home, and later

[17]See *Game Daily: June 22, 2004* at http://www.gamedaily.com, *Video Game Culture: Leisure and Play Preferences of B.C. Teens - Summary of Findings* at http://www.media-awareness.ca, and www.gamespot.com, September 2006.

[18]In a sample of 600 large companies in USA, 35% use one or more "free" software and 39% of 300 European large firms do so. Forrester Consulting, in El Mundo Digital 22/11/2006. In Spain, for instance, some "Comunidades Autónomas" are supporting open source. In 2007, the public administration of Extremadura will start to work with Linux. Andalucía and the Basque Country are also heading in the same direction (El País Digital, 16/11/2006).

they disintegrated becoming in firms producing only the platform and supported by independent developers.[19] We here try to provide a possible explanation for this observed fact based again on the margin that binds for developers. We analyze the incentives of a platform to integrate with applications (becoming one-sided) as a function of the extent of substitutability among them. We derive some conditions about the relationship between the welfare effects of a merger and the degree of substitution of the applications. We also offer an explanation for partial integration and we show that in the long run the platform will be partially integrated with the killer applications for which demand margin will bind and will allow free entry for developers of other applications.[20]

Finally, we study the effects on incumbent platform strategies for facing the threat of an outside option that offers a surplus for developers or users. Examples of outside options for users of the video game consoles are those games that can be played in the computer or online in the internet.[21] Writing these games is the outside option that developers have to the video console. Outside options for a proprietary operating system are the open platforms such as Linux. It is developing quickly in terms of number, variety and quality of applications and availability of support and other complementary services. In this sense, Linux is now an outside option to Windows and nowadays it is considered a serious threat to the latter.[22] Thus, we can interpret the analyses as an option that competes or threatens the incumbent platform. Questions we try to answer with this analyses are, for instance, given Windows being the incumbent firm, is it the grow importance of Linux in the users' benefit? What about developers of software?. If Linux becomes more important so that the value of writing applications for it increases, is this profitable for them? We find that it would not be in the interest of the users to promote the outside options (i.e., online games or computer games) to the video game console since, whenever competition margin binds, a higher outside option value for the users may lead to a decrease in their surplus. However, an increase in the value for developers of writing for an open platform such as Linux or Google has a positive impact in the users' surplus. This is the case because if demand margin binds, an increase in the outside option of the developers will always increase the users surplus.[23]

Since the model includes the pricing decision of developers, it captures the demand effects that arise from the price set by the developers. In this respect the paper is closely connected to Nocke, Peitz, and Stahl (2007) and Hagiu (2006a, 2006b). In particular, Nocke, Peitz and Stahl (2007) discuss platform ownership in a model where competition between sellers exists. They consider a continuum of

[19] Several facts that we cite along the article are documented by Evans, et. al. (2006).

[20] For instance, Microsoft produces operating system Windows and Office package. Nintendo wrote Mario Brothers, its killer game.

[21] Gamers report an average of 6,65 of hours spent per week on online-games and the home PC use of time explains 25% of children's and adult's games. http://www.cybersurvey.com/reports

[22] See www.cnn.com, World Business, "Reclusive Linux founder opens up", 19/05/2006, and "Microsoft vs. Open Source: Who Will Win?- HBS Working Knowledge, June 2005.

[23] In November 2006 Microsoft and Novell have signed a deal so that Linux programs can operate with Windows. Rivals will collaborate on technical development and marketing programs (The New York Times, 3/11/2006). A priori it seems the deal would benefit users and developers, but it warrants further analyses.

sellers and denote a seller's (gross) variable profit by

$$\Pi(m_s) = z(m_s)\pi(m_s),$$

where m_s is the measure of entering sellers. The term $\pi(m_s)$ is the seller's variable profit per unit mass of buyers and $z(m_s)$ is the mass of buyers visiting the market place, which is increasing in m_s. They assume that $\pi(m_s)$ will decrease with m_s by two reasons. First, there is a market share effect: for given prices, if the amount of sellers increases buyers buy less from each seller; and second, there is a price effect: as the amount of sellers increases, prices fall as competition increases. Regarding the impact of m_s on variable profits, it will depend on the interplay between two contrasting effects: on one hand, a positive indirect network effect that makes sellers' profit to increase as the number of firms on its own side increases by the impact of $z(m_s)$ on profits, and, on the other hand, a negative competition effect which makes firm's profit to decrease as the number of firms on its own side increases via $\pi(m_s)$. Our explicit model allow us to study in depth the driving forces of these effects. And, in particular, we show that when the demand margin binds $\pi(m_s)$ may not necessarily decrease as more sellers enter.

The structure of the paper is as follows. We present the model of a monopoly platform in section 2, and in section 3 we analyze the developers problem. In section 4 we solve the problem of a profit platform and compare its performance to that of an open platform in section 5. In section 6 we analyze incentives for integration and partial integration of the platform with applications. In section 7 we introduce outside options to the monopoly platform for developers and users. Finally, section 8 concludes.

2. A monopoly platform model

We assume that there is a monopoly platform and preferences of users are defined over the platform, its applications and an outside good. Unless stated otherwise applications are assumed symmetric**.** There is a measure one of users with a preference for software variety and whose tastes for the platform are uniformly distributed along the unit interval. The utility of a user located at distance t from the platform is

$$U = V(M) + x - kt,$$

where M is the number of software varieties or applications, x is the numeraire good and k measures the degree of platform differentiation,and $V(M)$ is assumed concave and increasing in M.[24] Unless stated otherwise, applications are considered symmetric and such that there is a certain degree of substitutability among them.

Every user who purchases the platform consumes at most one unit of each application and maximizes her utility by choosing applications and consumption of the outside good subject to the constraint

$$\Sigma_{j=1}^{M} p_j + x + P^U = y,$$

where p_j is the price of a unit of application variety j, P^U is the charge that platform sets to the users and y is their income. A user's decision can be decomposed into

[24]Similar utility functions are used by Church and Gandal, (1992, 1993, 2000) and Church et.al. (2003).

two decision problems. First, the user sets her optimal basket of applications among the total number in the market,

$$G\left(M,\Sigma_{j=1}^{M}p_j,P^U\right)=\max_{M\leq N}\{V\left(M\right)-\left(\Sigma_{j=1}^{M}p_j\right)\}-P^U, \tag{10}$$

where N is the number of applications in the market.[25] Then, the user buys the platform if and only if

$$G\left(M,\Sigma_{j=1}^{M}p_j,P^U\right)-kt\geq 0.$$

The users demand for the system (size of the network) is hence determined by

$$t^d=\frac{G\left(M,\Sigma_{j=1}^{M}p_j,P^U\right)}{k}\epsilon\left[0,1\right].$$

Note that demand depends on the price that platform sets for the users, but also on the number and prices of applications.

On the other side there are $\bar{N}$ potential developers of applications, each of them providing a single different application. Profits of developer of application i are given by

$$\pi_i=p_it^d-F-P^D,$$

where F is a fixed cost of production, and P^D is the price that platform charges developers to allow them to write platform compatible applications.

Costs of the platform are assumed zero, so that platform profits are given by,

$$\Pi=P^Ut^d+P^DN.$$

In this set-up we study the pricing strategies of the platform and developers. To do so we consider a game whose timing is as follows: in the first stage, the platform sets the charge to developers and these decide upon entry. In the second stage, the platform sets the price to the buyers. In the third stage, developers compete and set the prices for their applications to the buyers, then finally buyers decide if they buy the platform and the number of applications. Timing above takes into account that developers of applications join platforms before buyers do, a common feature in the software and video-game markets. Since the development of application is a costly activity, platform firms often deal with developers before selling their product in order to ensure that enough applications will be available to be used with the platform.[26] An alternative timing is the one in Hagiu (2006b) in which users decide about the developers products once they have purchased the platform. This alternative timing can be embedded in our model as it is equivalent to assume that developers can not affect the overall demand of the system when setting prices, i.e., the competition margin does always bind.

3. Application prices, users payments and system effects

When a user considers buying the platform, her decision will depend upon the prices set by developers. No user will purchase a video console without buying some video games, nor an operating system without buying the application software. Because of this we first study how developers set prices which will be a key point in our analysis. We then solve the second stage of the game at which the platform

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[25] With this formulation we are implicitly assuming that products are sorted by increasing price.

[26] See Hagiu (2006a) for more details about the reasonability of this timing.

sets the price for users, taking N as given. Before that let us define two elasticities that will be used throughout the paper.

Ignoring the integer problem we define the elasticity of $V(N)$, a measure of the degree of substitutability of applications for the users,[27] as follows,

$$e_v(N) = \frac{V'(N)\,N}{V(N)}.$$

Since $V(N)$ is increasing and concave, it lies in the interval $(0,1)$. For a given N, we consider that applications to be near substitutes if $e_v(N)$ is sufficiently low.[28]

Similarly, let us define the elasticity of $V'(N)$,

$$\varepsilon_v(N) = \frac{V''(N)\,N}{V'(N)}.$$

Given that $V(N)$ is concave, it follows that $\varepsilon_v(N)$ is negative. The relationship between these two elasticities is the content of next lemma

Lemma 1 $e_v(N)$ is increasing in N as long as

$$e_v(N) < 1 + \varepsilon_v(N), \tag{11}$$

and is decreasing if the other inequality holds.[29]

3.1 Equilibrium application prices. The problem faced by developers is similar to the problem faced by a licensor in a patent pool. In the context of patents, the licensor problem has been studied by Lerner and Tirole (2004). In their model, the surplus derived from using N patents is also a function $V(N)$, strictly increasing in N. They show that, when setting a licensing fee, an individual licensor may be constrained by either of two margins that they call the *competition margin* and the *demand margin*. In our context, developers are constrained in a similar way. If the developer can not increase her price without, because of this, being excluded from the set of applications selected by the users, (in user's problem (10)) then the competition margin binds. In contrast, demand margin is said to bind for developer i, if she can individually raise her price without being excluded but leading to a reduction in the overall demand for the system (effect on t^d). In particular, if the demand margin binds, a developer chooses a price $p_i = \widehat{p}$ such that

$$\widehat{p} = \arg\max_{p_i} \; \{p_i \frac{V(N) - P^U - (N-1)\widehat{p} - p_i}{k}\}. \tag{12}$$

Consequently,

$$\widehat{p} = \frac{V(N) - P^U}{(N+1)}.$$

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[27] It has also been interpreted as a measure of "degree of preference for variety" (see Kühn and Vives (1999) and Hagiu (2005)).

[28] Our interpretation here is similar to the one in Lerner and Tirole (2004): given N patents and two surplus functions $V_1(\cdot)$ and $V_2(\cdot)$, such that $V_1(N) = V_2(N)$, applications are more substitutable for surplus function $V_1(\cdot)$ than for $V_2(\cdot)$ if $V_1'(\cdot) < V_2'(\cdot)$.

[29] For instance, functions $V(N) = \log(1+N)$ and $V(N) = (1-\exp(-N))$ have $e_v(N)$ decreasing for all $N > 0$ and it is easy to show that they satisfy the reverse of (11) in all the relevant range of N. Function $V(N) = N^{\beta}$, with $\beta < 1$, presents constant elasticities, $e_v(N) = \beta$ and $\varepsilon_v(N) = \beta - 1$, then $e_v(N) = 1 + \varepsilon_v(N)$.

In contrast, if the competition margin binds, the price that a developer sets is its marginal contribution to the users utility, i.e.,

$$\widetilde{p} = V(N) - V(N-1).$$

Note that $\widetilde{p}$ depends on $V(N)$ but neither on the demand of the system t^d nor on P^U.[30] Besides, $\widetilde{p}$ is always positive, whereas $\widehat{p}$ is not necessarily so, as it will depend on the value of P^U.

Next lemma follows immediately from propositions 1 and 4 in Lerner and Tirole (2004).

Lemma 2 *There exists a unique and symmetric equilibrium such that, if $\widetilde{p} < \widehat{p}$, developers are constrained by the competition margin and charge equilibrium price $\widetilde{p}$, whereas if $\widetilde{p} > \widehat{p}$, developers are constrained by the demand margin and charge equilibrium price $\widehat{p}$.*

As long as demand margin binds, developers set the price that maximizes their profits and this price is lower than their marginal contribution to the users' utility. In contrast, if the competition margin binds, the price that maximizes profits, as defined in (12), is higher than the marginal contribution to users surplus and then developers are forced to set a price equal to this contribution.

The consideration of both scenarios allows us to include in the analysis situations where the developers set the price that maximizes their profits and consider the reduction in the overall demand for the system when contemplating an application price increase (i.e. when demand margin is binding). Other papers in the literature, such as Hagiu (2006b) and Church et. al. (2003), implicitly restrict their analyses to an scenario where the competition margin is always binding. In particular, Hagiu (2006b) assumes that developers set prices for applications once users have bought the platform. Similarly, Church et. al. (2003) derive the equilibrium prices set by developers under the proviso that platform sales are invariant to application pricing.[31] Our contribution here will not only be to study the case in which the demand margin binds, but also the comparisons that will follow. Clearly, some of our results when the competition margin is the one that binds are similar to those found in these previous papers.

3.2 What is the binding margin? We now try to establish what the conditions are that determine the margin that will bind, by using lemma 3.1, and the equilibrium values of prices $\widehat{p}$ and $\widetilde{p}$.

Lemma 3 *Developers are constrained by the competition margin if the platform sets a price to the buyers such that*

$$P^U < V(N) - \widetilde{p}(N+1). \tag{13}$$

If the opposite inequality holds, developers are constrained by the demand margin.

[30] If we ignore the integer problem, $\widetilde{p} = V'(N)$. Then, $\varepsilon_v(N)$ also represents the applications price elasticity to N when competition margin binds.

[31] In Church et. al. (2003), $V(N) = N^{\beta}$. For this utility function they show that the Nash equilibrium in developers' prices is given by $p(N) = V'(N)$ when $N > 1$ and $\beta \leq \frac{1}{2}$, so that, in our terminology, the competition margin binds.

A closer look at (13) allows us to determine the binding margin as a function of the primitives in the model.

Proposition 1 *If*

$$e_v(N) < \left[1 - \frac{1}{\sqrt{N+1}}\right], \tag{14}$$

the competition margin will bind. If the opposite inequality holds, the demand margin will bind.

Proof: See Appendix A.

The proposition above shows that the degree of substitution among applications and the number of developers determine the margin that binds. As long as applications are near substitutes the competition margin is more likely to bind. The same occurs when N is large, as the following corollary shows.

Corollary 1 *If $e_v(N)$ is non-increasing there exists N^* such that if $N < N^*$ the demand margin binds and if $N > N^*$ the competition margin binds. However, if $e_v(N)$ is strictly increasing, N^* may fail to exist, so that the demand margin always binds.*[32]

Proof See Appendix A.

From proposition 1 we deduce that those developers that write applications which are not near substitutes or are indeed complements will tend to compete in the demand margin. Similarly, those systems composed by a very high number of applications are more likely to have developers competing in the competition margin.

Using the results above, if one looks at the observed facts in the video game industry discussed in the introduction,
1) 76% of gamers state that price is very/somewhat important in deciding what game to buy,
2) From a survey of over 1,000 game consumers it is known that around 19.10% of them purchase 1 or 2 games per month, 26.50% purchase 1 every two month and 6.90% 3 or more per month,[33]
3) Some players report having more than 50 games,
4) Among the ten top rated PlayStation 2 games, 3 of them belong to the adventure genre and 3 to the role-playing genre. Among the ten top rated Xbox 360 games, 2 of them belong to the Ice Hockey genre.
Facts 1 and 4 suggest that there exists a near substitution between the games. Facts 2 and 3 show that consumers usually own a system of console and video games composed of many applications.

If we compare these facts with those observed for systems of operating systems and applications (i.e. Windows) we find that it is not easy to find a consumer using a huge number of applications.[34] Moreover, applications are far substitutes (and

[32] This is the case for instance for $V(N) = N + \sqrt{N}$ for which demand margin always binds. Note that if N^* exists, it is defined by $N^* = \left(\frac{1}{1-e_v(N^*)}\right)^2 - 1$.

[33] Zelos Group Survey: What Do Gamers Want? Everything. Electronic Gaming Business, Nov 19, 2003. http://www.findarticles.com

[34] Evans et.al. (2006) point out that, as opposed to the case of video consoles, "there's probably not much correlation between the number of applications that someone uses on a computer and the value that person places on that computer".

sometimes complements). A user may need a text processor and a spreadsheet and also a browser. Then, we presume that developers writing for an operating system are constrained by the demand margin whereas those writing for the video console are constrained by the competition margin.

3.3 Users prices and system effects. In the second stage of the game, the platform sets the price for users, taking N as given. When the demand margin binds, the platform will set a price to the users as[35]

$$\widehat{P}^U = \frac{V(N)}{2}. \tag{15}$$

It then follows that the price set by developers will be $\widehat{p} = \frac{V(N)}{2(N+1)}$.

Meanwhile, if the competition margin binds, the optimal price that platform chooses for the users is

$$\widetilde{P}^U = \frac{V(N) - \widetilde{p}N}{2}. \tag{16}$$

Equations (15) and (16) put in evidence the existence of "system effects" in the industry. These effects arise when the value of one component depends on complementary components in the system.[36] The presence of system effects is reflected in the price that the platform sets to users which increases with the number of applications. In addition, when competition margin binds $\widetilde{p}$ affects $\widetilde{P}^U$ because of the complementarity between the applications and the platform. In particular, for a given N, when the price of the applications increases, the benefit that the platform makes per user decreases.[37]

When the competition margin binds, the relative charges paid by users can be expressed as a function of $e_v(N)$,

$$\frac{\widetilde{P}^U}{\widetilde{P}^U + \widetilde{p}N} = \frac{1 - e_v(N)}{1 + e_v(N)}.$$

Lemma 4 *As long as applications are more substitutes, applications will be relatively less expensive, and the platform can charge users more.*

When substitution is strong on the developers' side, prices in this market are very low and the platform takes advantage of this situation setting a higher price for the platform. Lemma 4 implies that it is profitable for the firm selling the console to accept games that compete among them or are near substitutes, which is consistent with the observed practice in the video game industry as stated in fact 4.

The relative charge paid by users for the platform when demand margin binds is given by

$$\frac{\widehat{P}^U}{\widehat{P}^U + \widehat{p}N} = \frac{N+1}{2N+1}.$$

[35] See proof of proposition 1.

[36] See Evans and Schmalensee (2001). System effects are a clear feature of software platforms where the user buys a system (platform and applications) and cares for the total charge of the system.

[37] Note also that for the same N, $\widehat{P}^U > \widetilde{P}^U$, it is consistent with the observed fact that operating systems charge high prices to users whereas video console firms charge low prices to them.

Note that when demand margin binds, relative charges depend only on N whereas it depends on both, N and $V(N)$, when competition margin binds. The next proposition presents how relative charges vary with N.

Proposition 2 *If demand margin binds, the relative payment made by users to the platform is decreasing in N. However, if the competition margin binds, the relative payment is increasing in N whenever $e_v(N)$ is decreasing in N.*

As N increases, users tend to spend more on the bulk of applications when demand margin is binding. The same occurs, whenever the competition margin binds provided that $e_v(N)$ is increasing in N. However, $\frac{N+1}{2N+1} > \frac{1}{2}$, meaning that more than one half of the money that users spend in the system goes to the platform when demand margin binds. Meanwhile, it may occur that $\frac{1-e_v(N)}{1+e_v(N)} < \frac{1}{2}$ if $e_v(N) > \frac{1}{3}$.

When setting the price to users, the platform should optimally preserve this ratio, if not, a competitor with a better pricing strategy may easily overcome the incumbent's advantages.[38]

Let the users demand elasticity with respect to the price by the platform be $E_p = \frac{\partial t^D}{\partial P^U}\frac{P^U}{t^D} = -1$ and the elasticity of demand with respect to the number of applications be $E_s = \frac{\partial t^D}{\partial N}\frac{N}{t^D} = -\frac{\epsilon_v \widetilde{\varepsilon}_v}{1-\epsilon_v}$. The ratio $-\frac{E_s}{E_p}$ measures the effect of platform price equivalent to a 1% increase in N.[39] In the users' interest, a 1% increase in the number of applications is equivalent to a $\frac{\epsilon_v \widetilde{\varepsilon}_v}{1-\epsilon_v}$% price cut.[40] This ratio is increasing in ϵ_v and $\widetilde{\varepsilon}_v$. That is to say that an increase in N is more valued as long as it conveys a reduction in developers applications prices and applications are near complements.

4. Developers entry and welfare: profit platform versus open platform

In the first stage a proprietary platform sets a price to the developers that then decide upon entering the market. If the platform is open, this price is zero.[41] One could think that the platform, through the choice of prices for developers, determines the number of applications. However, this assertion may not always be true. In particular, if developers' gross profits (i.e., $p(N)\, t^d(N)$) are increasing in the number of applications, then the platform can not affect entry which will equal $\bar{N}$. This is the case when the positive indirect network effect more than compensates the direct negative effect of competition. An additional developer exerts a positive effect on other developer's profits, explained by the fact that more participation by one side (i.e., developers) induces more participation by the other side (i.e.,

[38] For instance, in the market for video players, VHS overcame Beta after six years of higher installed base by Beta. The strategy of the winner was a widespread licensing of VHS and a low- priced VHS player, compared with a high-priced Beta player and restricted licensing (See Economides 2006).

[39] Note that if $V(N) = N^\beta$ the ratio is $\frac{-E_s}{E_p} = 1$.

[40] Clements and Ohashi (2005) have computed this ratio for the USA video game industry. They find that a 1% increase in game titles is equivalent (in average) to a 2.3% price cut of the console price.

[41] An open platform will charge zero to both users and developers. Nevertheless, we will assume that developers set positive prices to users for their applications. Applications for open platforms like Linux are often free for consumers. However, there are also several applications that are not free that are offered for Linux operating system (Economides and Katsamakas (2005a)).

users), which benefits customers and makes them more willing to participate.[42] Consequently, whenever developers' gross profits are increasing at $\bar{N}$, the platform would charge a price $P^D\left(\bar{N}\right) = p\left(\bar{N}\right) t^d\left(\bar{N}\right) - F$ and its profits will be

$$\Pi = P^U\left(\bar{N}\right) t^d\left(\bar{N}\right) + \left(p\left(\bar{N}\right) t^d\left(\bar{N}\right) - F\right)\bar{N}.$$

In contrast, if developers' gross profits are decreasing at $\bar{N}$, because the positive effect on the demand is compensated by the negative effect on the price, then there is a one-to-one relation between N and P^D, so that the platform rather than maximizing profits over P^D can do so directly over N. The platform will hence optimally choose N to maximize its profits given by

$$\Pi = P^U(N)\, t^d(N) + \left(p(N)\, t^d(N) - F\right) N. \tag{17}$$

From the expression above it is clear that an increase in N affects the profits of the platform in two ways, through the profits made on users (first term in (17)) and through the profits made on the developers (second term in (17)). How these effects depend on the degree of substitution between the applications that developers offer is quite clear when looking at the profits made on the developers' side. If substitution is strong, their profits, gross of P^D, are lower, then the surplus that the platform may extract from them is also lower (or even negative if it is optimal for the platform to subsidize the developers, i.e., $P^D < 0$). Regarding the profits made on the users' side, recall that both $\tilde{P}^U$ and $\hat{P}^U$ are increasing in N. In addition, the positive effect of entry on $\tilde{P}^U$and $\tilde{t}^d = \frac{\tilde{P}^U}{k}$ is higher when substitution between developers is higher (whenever $\frac{\partial \widetilde{p}}{\partial N} = V''(N)$ is high). When N increases $\widetilde{p}$ decreases, and this additional effect is taken into account by the platform when allowing access to the developers, becoming an additional incentive to promote entry. The optimal level of entry will depend on the margin that binds.

If demand margin binds, the platform will optimally choose $\hat{N}$ such that it solves

$$\frac{V(N)\, V'(N)\,(2N+1) - (V(N))^2 \frac{N}{N+1}}{2k\,(N+1)^2} = F, \tag{18}$$

whereas if competition margin binds, it will choose $\tilde{N}$ such that

$$\frac{V\left(\tilde{N}\right) V'\left(\tilde{N}\right) - \left(V'\left(\tilde{N}\right)\right)^2 \tilde{N}\left[1+\varepsilon_v\right]}{2k} = F. \tag{19}$$

The discussion above is the content of next lemma.

Lemma 5 *Let $\pi^{DM}(N)$ ($\pi^{CM}(N)$) stand for the developers' gross profits when demand (competition) margin binds, and let N^* be such that if $N < N^*$ the demand margin binds and if $N > N^*$ the competition margin binds. Assume $\pi^{CM}(N^*) > \pi^{DM}(N)$ for all N.*[43] *The patterns of equilibrium entry in a proprietary platform will depend on the binding margin and the size of $\bar{N}$. In particular:*

[42] Farrell and Klemperer (2004) state that an indirect network effect arises whenever the indirect benefit outweighs any direct loss from more participation by one's own side. Thus, following this definition, there is an indirect network effect among developers as long as profits are increasing in N.

[43] This is not a restrictive assumption, all the surplus functions that we are considering here satisfy it.

i) If $\pi^{CM}(N^*) > \pi^{DM}(N)$ *for all N and* $\bar{N} > N^*$*, in any stable equilibrium of developers' entry the competition margin will always bind and the level of entry will be*

$$N = \begin{cases} \bar{N} & \text{if} \quad \pi'^{CM}\left(\bar{N}\right) > 0 \\ \min\left(\tilde{N}, \bar{N}\right) & \text{if} \quad \pi'^{CM}\left(\bar{N}\right) < 0, \end{cases}$$

where $\tilde{N}$ *solves (19).*

ii) If $\bar{N} < N^*$*, the level of entry will be*

$$N = \begin{cases} \bar{N} & \text{if} \quad \pi'^{DM}\left(\bar{N}\right) > 0 \\ \min\left(\hat{N}, \bar{N}\right) & \text{if} \quad \pi'^{DM}\left(\bar{N}\right) < 0, \end{cases}$$

where $\hat{N}$ *solves (18).*

Proof: See Appendix A.

When the platform is open there are no platform prices to affect agents decisions (recall that now $P^U = P^D = 0$), so that developers will enter until their profits are zero, i.e.,

$$p(N)\, t^d(N) - F = 0.$$

Lemma 6 *Let* $\pi^{DMo}(N)$ $(\pi^{CMo}(N))$ *stand for the developers' gross profits when demand (competition) margin binds in an open platform and let* N^{o*} *be the N that determines the binding margin. Then,*

i) $\pi'^{DMo}\left(\hat{N}\right) = \pi'^{DMo}\left(\hat{N}\right) = 0$

ii) $\hat{N} = N^{o*} < N^*$

iii) $\pi^{DMo}\left(\hat{N}\right) = \pi^{CMo}\left(\hat{N}\right)$

Proof: See Appendix A.

Point *i)* implies that the maximum in gross profits when demand margin binds occurs at the same N in both types of platforms. Point *ii)* implies that if demand margin binds, gross developers profits are increasing. Whereas, if competition margin binds, profits may be increasing or not. Note that a comparison of outcomes under open and proprietary platforms is not direct for the range of $N \;\epsilon\; \left(\hat{N}, N^*\right)$ as competition margin will bind under an open platform whereas the demand margin binds under a proprietary platform. Finally, point *iii)* shows that developers' profits are continuous at the point where the change from a margin to the other occurs.

If gross profits are increasing at $\bar{N}$, then $\bar{N}$ developers will entry. If not, the number of developers is determined by

$$\text{(20)} \qquad \frac{V\left(\tilde{N}^o\right) V'\left(\tilde{N}^o\right) - \left[V'\left(\tilde{N}^o\right)\right]^2 \tilde{N}^o}{k} = F.$$

The next proposition compares the levels of entry that occur in each case and the effect on users' welfare.

Proposition 3

i) If demand margin binds, a proprietary platform and an open platform will provide the same level of N, so that the latter will generate more welfare for users.

ii) If competition margin binds a proprietary platform may generate a larger number of applications and higher welfare to users than an open platform.

Proof See Appendix A.

For comparison purposes, consider now the problem solved by a benevolent social planner. She would choose the optimal number of applications, N^{FB}, to maximize social welfare given by

$$W^* = \int_0^t V(N)\,dz - \int_0^t kzdz - FN,$$

where $t^{FB} = \frac{V(N^{FB})}{k}$.

The first order necessary condition yields the first best allocation,

$$\frac{V\left(N^{FB}\right)V'\left(N^{FB}\right)}{k} = F. \tag{21}$$

Condition (21) that determines the first best level of N equalizes the marginal benefit with the marginal cost of an additional application. The former is the marginal utility enjoyed by users ($V'\left(N^{FB}\right)$ times the size of the market t^{FB}), whereas the latter is the fixed cost of producing one more application. Then if $\bar{N} < N^{FB}$ social planner chooses $\bar{N}$ and chooses N^{FB} otherwise.

As long as $\bar{N} < \hat{N}$ entry is $\bar{N}$ and equals N^{FB}. The same occurs when competition margin binds and $\bar{N} < \tilde{N}$. Then, when the effect of N on platform profits is strong (and this is more likely when e_v is high) the platform will tend to generate the same level of entry as the social planner.

Proposition 4 *Assume* $\bar{N} > \max\left(N^{FB},\ \hat{N},\ \tilde{N}\right)$. *If demand margin binds, a proprietary platform chooses a level of N smaller than the first best. However, if competition margin binds the comparison is not conclusive.*

Proof: See Appendix A.

Proposition 3 and 4 yield some insights into policies regarding the emergence of open source platforms competing with platforms such as Windows (Linux is the classic one, but there are also some others like Google which offer programs for free). In contrast, we do not observe the emergence of open platforms in the market of video consoles. The propositions above suggest that policy makers should promote open source in platforms like operating systems but not necessarily in those like video consoles.

5. Integration and the margin

Assume now that the platform firm can also develop its applications at zero marginal cost and at a fixed cost F per application. Then, if the platform is integrated, meaning that one firm produces the platform and the N applications, its system price will be

$$P^I = \frac{V(N)}{2}$$

and profits will be

$$\Pi^I = \left(\frac{V(N)}{2}\right)^2 \frac{1}{k} - FN.$$

We have shown that when integration is absent and demand margin binds, the resulting system price is $\frac{V(N)}{2}\left(1+\frac{N}{N+1}\right)$ which is larger than P^I. The rational behind the result is clear: under separation there is a double marginalization as neither the platform nor the developers take into account the reduction of sales of the others when raising the price so that an inefficiently large price arises.

However, if integration is absent and competition margin binds, the resulting system price is $\frac{V(N)+V'(N)N}{2}$ which gets close to P^I as $V'(N)$ gets close to zero, which is the case when applications are very substitutes.

Proposition 5 *Inefficiencies of disintegration tend to disappear as long as competition margin binds and applications are near substitutes.*

Consider total profits of the firm. If demand margin binds, these are

$$\Pi^{DMB}=\left(\frac{V\left(N\right)}{2}\right)^2\frac{1}{k}\left(\frac{2N+1}{N^2+2N+1}\right)-FN<\Pi^I,$$

so that the platform will always prefer being integrated in order to get developers to aware of the impact of their pricing strategies on the other developers and on the platform profits. Note that under separation even if the platform can control N through P^D, it can not control the price developers set.

If competition margin binds, profits are

$$\Pi^{CMB}=\left(\frac{V\left(N\right)-\widetilde{p}N}{2}\right)^2\frac{1}{k}+\widetilde{p}\left(\frac{V\left(N\right)-\widetilde{p}N}{2}\right)N\frac{1}{k}-FN.$$

Again, as long as $\widetilde{p}=V'(N)$ tends to zero (because the extent of substitutability among applications is great or N is very high), profits tend to Π^I.[44]

The results above are consistent with the observed phenomena that initially platforms are vertically integrated and later disintegrate. Recall from corollary 1 that there exists N^* which determines the margin that is going to be binding. When the industry is less developed (initial steps of the industry with N low) the platform strictly prefers being integrated. As the industry evolves and the number of developers available in the market increases, the competition margin is likely to bind, prices of applications will be $V'(N)$, decreasing in N, and at this stage of the industry, the platform will be more willing to disintegrate.[45] As the market of developers matures and becomes more competitive, the firm can concentrate on producing only the platform. Note that other alternative explanations are offered in the literature for the phenomena of vertical disintegration that not can be explained within this model. For instance, Stigler notes that firms need to arise vertically integrated since technology is not familiar in the market. When the industry grows, production process are well known and scale of the market allows specialization, such that disintegrating is profitable. [46] Another different explanation for no integration is given by Gawer and Henderson (2005), when discussing Intel's strategy. They suggest that managers were aware of how important the generation

[44]In particular, the necessary condition is that $V'(N)\,N$ be decreasing, i.e., $\varepsilon_v\left(N\right)>1$.

[45]PDA's were born as "smart agendas" offering a limited number of applications. Then, they evolved to become "small computers". Something similar has occurred in the mobile phone industry. In addition to the traditional communication service, today they allow for hundreds of applications. See "What is a Window Mobile" in www.microsoft.com.

[46]George Stigler, "The division of labor is limited by the extent of the market", Journal of Political Economy 59 (June 1951), quoted by Evans, et. al.(2006).

of complements was to the success of Intel's business; however, although it is in the interest of the platform to enter complementary markets, the platform knows that this could discourage entry by new firms.[47] A more trivial explanation comes from the fact that the platform does not always possess the requisite capabilities to produce some of the complementary goods.[48]

5.1 Partial integration. A widely observed fact in software industries is that some computer software are clearly more useful or more commonly used than others, Office software and Messenger are illustrative examples. At the same time, some video games are the most popular (killer games) in the market, so that applications' contributions to total surplus may be different. To incorporate this feature into our model, in what follows we allow applications to be heterogeneous.

Assume that each application i has a contribution $N_i \in [0, N]$, with the normalization

$$\sum_{i=1}^{N} N_i = N.$$

Note that $N_i = 1$ will bring back the homogeneity we have considered so far. Let us further assume that $\frac{\partial N_i}{\partial i} > 0$ and let us define $V(\cdot)$ by $V\left(\sum_{i=1}^{N} x_i N_i\right)$, where $x_i = 1$ if user buys application i and $x_i = 0$ otherwise. The next lemma is inspired in proposition 6 in Lerner and Tirole (2004).

Lemma 7 *Assume that gross surplus of users by applications is* $V\left(\sum_{i=1}^{N} x_i N_i\right)$, *where* $x_i = 1$ *if users buy application i and* $x_i = 0$ *otherwise, with* $\frac{\partial N_i}{\partial i} > 0$. *Then, there is a mass* $0 \leq n \leq N$ *of developers that are constrained by the competition margin and charge a price* $\widetilde{p}_i = V_i'$, *their marginal contribution to the total surplus. The rest of the developers are constrained by the demand margin and all of them set the same price* $\widehat{p} = \frac{V(N) - P^U - \int_0^n \widetilde{p}_i d_i}{N - n + 1}$. *Finally, the platform sets* $P^U = \frac{V(N) - \int_0^n \widetilde{p}_i d_i}{2}$.

When the platform decides P^U, it defines the value n, i.e., the mass of developers that will be constrained by the competition margin. For every $i \in [0, n]$ it must hold that $\widetilde{p}_i = V_i' < \widehat{p}$ and that V_i' is increasing in i. If $n = 0$, we have that every developer is constrained by the demand margin. Analogously, if $n = N$ every developer is constrained by the competition margin.

Proposition 6 *In the long run the platform will be partially integrated with the killer applications for which demand margin will bind, and will allow free entry for developers of other applications.*

This proposition may help us to explain why platforms are often partially integrated, most of them with the core application. Microsoft produces operating systems and some of the applications (i.e. Office package). Nintendo wrote Mario Brothers, the killer game of one of its consoles. In the US the proportion of games

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[47] Dave Johnson, a director of Intel, explained: "The market segment gets hurt if third parties think: "Intel, the big guys, are there, so I do not want to be there..."... it is not what we want, because we are trying to encourage people to do these complementary things". Gawer and Henderson (2005), pp. 18.

[48] Claude Leglise, director of the Developer Relation Group, responded: "Intel has no corporate competence in entertainment software. We do not know how to do video games, so forget it". Gawer and Henderson (2005), pp. 13.

developed in house is about 10% for GambeCube and 8% for PlayStation and Xbox.

6. Platform competition: the role of outside options

Up to now we have assumed that a monopolist platform (either proprietary or open) provides a good with no competition at all. Nevertheless in many industries, either open and proprietary platforms coexist, or there are several for-profit platforms competing to attract both users and developers. We now extend our basic framework by assuming that a proprietary and an open platform operate in the same industry.[49] Our aim here is to analyze how a firm that offers a proprietary solution will respond to changes in the value of an outside option that provides a positive surplus or profit to their clients (i.e., to users and developers). We analyze how the monopoly reacts in terms of prices and we abstract from other strategies such as investment.[50]

A user who purchases the open platform gets a net surplus $v = V(Z) - h$, where $V(Z)$ measures the utility users derive given the applications written for the open platform and h is an exogenous cost (interpreted as a transportation cost or a cost of learning to use this outside good). Consequently, users will purchase the proprietary platform as long as

$$V(\mathcal{N}) - kt \geq v > 0.$$[51]

In what follows we provide some comparative statics analyses to changes in v in order to study its impact on users welfare. We start assuming that the competition margin binds. Then, we move to an scenario where the demand binds. We restrict the analyses to values of N for which developers' profits are decreasing so that the proprietary platform can affect entry.

Consider the impact of a change in v on developers' profits and on the number of applications. The condition that arises when the platform at the first stage maximizes with respect to N is

$$\frac{V(N)V'(N) - (V'(N))^2 N \ [1 + \ \varepsilon_v]}{2k} - \frac{V'(N)v}{2k} = F. \tag{22}$$

and from the comparison with equation (19) it follows that the monopolist will reduce entry due to the term $\frac{V'(N)v}{2k}$. This term is decreasing in N and smaller as long as applications are very substitutes.

It means that developers of video consoles may not have incentives to increase the value of v (i.e., writing applications for computers or online games) because the monopolist may react reducing the level of entry and thus the incentives for them. However, this response will not be important whenever the games are near substitutes.

By taking into account its impact on entry, the next proposition provides results on the impact of outside options on users surplus.

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[49]Since the open platform is considered non-profit, we will assume that it behaves myopically and hence does not play a best response against the pricing strategies by the proprietary platform. In contrast, the proprietary platform will take into account the presence of the open platform when deciding upon its pricing strategies.

[50]Economides and Katsamakas (2006b) study investment incentives of platforms and developers in a proprietary system and in an open source one.

[51]Note that v is used to proxy for the extent of product market competition.

Proposition 7 *Whenever the competition margin binds, a higher outside option value for the users may lead to a decrease in their surplus. In contrast, if the demand margin binds, the impact on users' surplus of a higher outside option value will generally be positive.*
Proof. See Appendix A.

Regarding the other side of the market, we now assume that developers can obtain a profit of w when writing applications for the open platform. Note that nothing changes if developers are allowed to write for both platforms (i.e., to multihome). In that case developers get a higher total profit but the strategies of the proprietary platform do not change. Results are different if we assume that developers are forced to choose one of the platforms (i.e., to singlehome) due, for instance, to contractual arrangements. Thus, developers will enter the market of the proprietary platform as long as

$$\pi_i = p_i t^d - F - P^D \geq w.$$

The effect of an increase in w is analogous to an increase in the fixed cost, so it clearly leads to a reduction in the level of N.

Proposition 8 *If the competition margin binds, an increase in the outside option of the developers will always reduce the users' surplus. However, if the demand margin binds, an increase in the outside option of the developers will always increase the users' surplus.*
Proof. See Appendix A.

We have shown that reinforcing competition pressure for developers when competition margin binds leads to a reduction in the users welfare. Results are quite different when demand margin binds. Promoting the benefits that writing for Linux has for the developers (sometimes interpreted as a "reputation effect"[52]) would be in favour of the users.

Let us provide an illustrative example. Consider $V(N) = N^{\beta}$ where $\beta = 0.45$ and a fixed cost $F = 0.14$. A value $\beta = 0.45$ determines that competition margin is binding as long as $N > 2.3$ and we restrict the analyses to this range of N. For a value $v = 0.1$, the surplus of the users is 0.71 whereas for an increase $\Delta v = 0.05$, the new users surplus is 0.59. It represents in terms of elasticities that a 1% increase in the users outside option implies a 19% decrease in the users surplus.[53]

To compare the effects of w and v, consider now $\beta = 0.25$ (so that competition margin binds as long as $N > 0.8$) and a fixed cost $F = 0.075$. Given the initial values $w = v = 0.1$, we find that a change in v (i.e., $\Delta v = 0.05$) exerts a direct impact on P^U equal to $\frac{\partial P^U}{\partial v} = -\frac{1}{2}$, whereas there is no direct impact when w changes (i.e., $\Delta w = 0.05$). However, when we compute the total effect, considering the indirect one by the effect on N, we find that $\frac{\partial \widetilde{P}^U}{\partial v}\frac{v}{\widetilde{P}^U} = -0.01$ and $\frac{\partial \widetilde{P}^U}{\partial w}\frac{w}{\widetilde{P}^U} = -0.03$, meaning that, under these parameters, the monopolist decides to reduce the price more for users when there is an outside option for the developers than when there is one for the users themselves.

[52]Economides and Katsamakas (2005.b). Other motivations are explained in "Microsoft vs. Open Source: Who Will Win?- HBS Working Knowledge, June 2005.

[53]The exercise has been computed assuming $k = 1$.

7. Conclusions

We have solved a model that provides some results for a better understanding of the two-sided pricing strategies of a platform that sells a good whose value depends on the applications sold in a market of developers. We note that when setting prices the developers are constrained by two margins: the demand margin and the competition margin. What margin is binding depends on the number of applications in the market and on the level of substitutability among them.

We find that if the demand margin binds, policy makers should promote open source platforms. However this is not necessarily the case when competition margin binds.

We consider the case where applications are asymmetric in the users' surplus and we find that in the long run the platform will remain integrated with the applications for which demand margin binds and will leave for third-party developers the production of applications for which competition margin binds.

Finally, we find that it would not be in the interest of the users to promote the value of outside options for the platform when competition margin binds. However, an increase in the value of the outside option for developers would have a positive impact on the users surplus if demand margin binds.

Appendix

Proof of proposition 1

To show the result we compute the profits that each situation generates for the platform, then we compare them and deduce the optimal strategy for the platform. If the platform sets a price that satisfies $P^U < V(N) - \widetilde{p}(N+1)$, then the competition margin will bind for the developers and platform profits will be

$$\widetilde{\Pi}^{PU} = P^U \left[\frac{V(N) - \widetilde{p}N - P^U}{k} \right].$$

The price that maximizes profits, given the constraint, is

$$\begin{aligned} P^U &= \frac{V(N) - \widetilde{p}N}{2} \text{ if } \widetilde{p} < \frac{V(N)}{N+2}, \text{ and} \\ P^U &= V(N) - \widetilde{p}(N+1) \text{ if } \widetilde{p} > \frac{V(N)}{N+2}. \end{aligned}$$

If the platform sets a price such that $P^U > V(N) - \widetilde{p}(N+1)$, so that demand margin will bind for the developers, platform profits will be

$$\widehat{\Pi}^{PU} = P^U \left[\frac{V(N) - P^U}{k(N+1)} \right].$$

The price that maximizes profits, given the constraint, is

$$\begin{aligned} P^U &= \frac{V(N)}{2} \text{ if } \widetilde{p} > \frac{V(N)}{2(N+1)}, \text{ and} \\ P^U &= V(N) - \widetilde{p}(N+1) \text{ if } \widetilde{p} < \frac{V(N)}{2(N+1)}. \end{aligned}$$

Comparing above the profits we observe that if $\widetilde{p} < \frac{V(N)}{2(N+1)}$ the price that generates highest profits for the platform is $\widetilde{P}^U = \frac{V(N) - \widetilde{p}N}{2}$. If $\widetilde{p} > \frac{V(N)}{N+2}$, the platform will optimally choose $\widehat{P}^U = \frac{V(N)}{2}$. Finally, whenever the relevant interval is $\frac{V(N)}{2(N+1)} < \widetilde{p} < \frac{V(N)}{N+2}$, if $\widetilde{p} < \frac{V(N)}{N}\left[1 - \frac{1}{\sqrt{N+1}}\right]$ the platform will set $\widetilde{P}^U = \frac{V(N)-\widetilde{p}N}{2}$ and will set $\widehat{P}^U = \frac{V(N)}{2}$ otherwise. It follows that the competition margin will bind if $\widetilde{p} < \frac{V(N)}{N}\left[1 - \frac{1}{\sqrt{N+1}}\right]$, and this occurs whenever $e_v(N) < \left[1 - \frac{1}{\sqrt{N+1}}\right]$, as claimed.

Proof of corollary 1

Note that the function $\left[1 - \frac{1}{\sqrt{N+1}}\right]$ is increasing in N, equals zero at $N = 0$, and goes to one as N goes to infinity. Since $e_v(N) \, \epsilon \, (0,1)$, if $e_v(N)$ is a non increasing function, it will necessarily cross $\left[1 - \frac{1}{\sqrt{N+1}}\right]$. However, if $e_v(N)$ is an increasing function, a crossing point may not exist.

Proof of lemma 5

When the demand margin binds, developers will enter until profits are zero so that it is satisfied

$$\left(\frac{V(N)}{2(N+1)}\right)^2 \frac{1}{k} - F - P^D = 0.$$

If competition margin binds, the developers zero profit condition will be

$$V'(N)\left(\frac{V(N) - V'(N)N}{2k}\right) - F - P^D = 0.$$

Consequently, let $\pi^{DM}(N) = \left(\frac{V(N)}{2(N+1)}\right)^2 \frac{1}{k}$ and $\pi^{CM}(N) = V'(N)\left(\frac{V(N)-V'(N)N}{2k}\right)$.

i) In a stable equilibrium, profits are zero and decreasing. Consider now an equilibrium such that $\pi^{DM} = F + P^D$ (so that demand margin binds). Since $\pi^{DM} < \pi^{CM}(N^*)$, when $\bar{N}$ is sufficiently large a coalition of developers will enter to obtain (at least) profits $\pi^{CM}(N^*)$, and the result follows.

Then,

1) if $\pi'^{CM}\left(\bar{N}\right) > 0$ gross developers profits are strictly increasing so that entry is $\bar{N}$.

2) if $\pi'^{CM}\left(\bar{N}\right) < 0$ gross developers profits are strictly decreasing so that the platform will choose $N = \min\left(\tilde{N}, \bar{N}\right)$, and the result follows.

ii) We must distinguish two cases. 1) If $\pi'^{DM}\left(\bar{N}\right) > 0$ gross developers profits are increasing and entry is $\bar{N}$.

2) If $\pi'^{DM}\left(\bar{N}\right) < 0$ gross developers profits are decreasing so that the platform will choose $N = \min\left(\hat{N}, \bar{N}\right)$, and the result follows.

Figure 1 below, although does not encompass all the possible cases, may help to clarify each of the previous points.

Proof of lemma 6

Note that $\pi^{DMo}(N) = \left(\frac{V(N)}{(N+1)}\right)^2 \frac{1}{k}$ and $\pi^{CMo}(N) = V'(N)\left(\frac{V(N)-V'(N)N}{k}\right)$. Result i) follows trivially. Note that the concavity of V ensures that $\hat{N}$ always exists. ii) Note that $\hat{N}$ solves $V'\left(\hat{N}\right) = \frac{V(\hat{N})}{\hat{N}+1}$. The equality $\hat{N} = N^{o*}$ follows from the fact that in an open platform competition margin binds as long

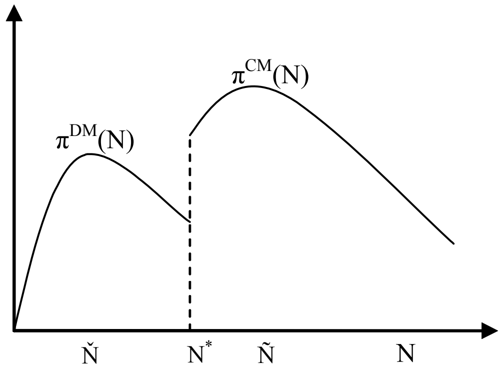

FIGURE 1. The entry problem

as $V'(N) < \frac{V(N)}{(N+1)}$. To prove that $\hat{N} < N^*$ recall from corollary 1 that N^* satisfies $V'(N^*) = \frac{V(N^*)}{N^*}\left(1 - \frac{1}{\sqrt{(N^*+1)}}\right)$. Since $V'(N)$ is decreasing and $\frac{V(N)}{(N+1)} > \frac{V(N)}{N}\left(1 - \frac{1}{\sqrt{(N+1)}}\right)$ for all N, it follows that $\hat{N} < N^*$. Part iii) follows from straightforward computations.

Proof of Proposition 3

i) The first statement follows from point i) in lemma 6 (profits of developers are increasing under both regimes for the same range of N) then in both cases entry will equal $\overline{N}$. If demand margin binds, with a proprietary platform the system price is $P^U + pN = \frac{V(N)}{2} + \frac{V(N)}{2(N+1)}N$, that is higher than $\frac{V(N)}{N+1}N$, the system price with an open platform, so that the second statement follows.

ii) From the comparison between (19) and (20), it follows that as long as $e_v > \frac{1}{1-\varepsilon_v}$ (i.e.$V'(N) - V''(N)N > \frac{V(N)}{N}$), a profit platform yields a higher N than the open platform. The second statement is proven by the fact that when competition margin binds, the users' surplus (net of kt) is increasing in N. The condition $e_v > \frac{1}{1-\varepsilon_v}$ imposes that $\varepsilon_v < -1$ since $e_v < 1$. An example for which a proprietary platform yields a higher N than an open platform is given by

$$V(N) = \begin{cases} (1 - \exp(-0.05N)) & \textit{if} \quad N \leq 7 \\ (0.8 - \exp(-0.1N)) & \textit{if} \quad N > 7, \end{cases}$$

with $F = 0.0045$.[54] The proprietary platform chooses $N \simeq 25$ whereas the open chooses $N \simeq 24$. The competition margin binds for all $N > 13$.

Proof of Proposition 4

The first statement follows from the comparison between (18) and (21). The second statement follows from the comparison between (19) and (21) and the fact that as long as $-e_v(1+\varepsilon_v) > 1$ (i.e.$-V''(N)N > \frac{V(N)}{N} + V'(N)$) the proprietary platform may generate excess of entry. As in the previous proof, the condition

[54]The equilibrium occurs at $N > 7$ so that $V(N) = (0.8 - \exp(-0.1N))$. Note that $V(N) = (1 - \exp(-0.05N))$ if $N \leq 7$ ensusres that $V(0) = 0$.

$-e_v(1+\varepsilon_v) > 1$ requires $\varepsilon_v < -1$ and it is more stringent than the condition in the proof of proposition 3. It does not contradict the condition to be in the competition margin $e_v < 1 - \frac{1}{\sqrt{N+1}}$, nor the condition for a maximum in the social planner problem, $e_v <\mid \varepsilon_v \mid$, nor the fact that $V(N)$ is concave.

Proof proposition 7

Given a user t, if competition margin binds, her surplus gross of kt is equal to $V(N) - pN - P^U$. We observe that this surplus will be increasing (decreasing) in v as long as $\left[1 - V''(N) N \frac{\partial N}{\partial v}\right] \lesseqgtr 0$, and the first statement follows. To prove the second statement note that if demand margin binds, users' prices are: $\widehat{p} = \frac{V(N)-v}{2(N+1)}$ and $\widehat{P}^U = \frac{V(N)-v}{2}$. The platform optimally chooses the N that maximizes profits

$$\frac{1}{k(N+1)}\left(\frac{V(N)-v}{2}\right)^2 + \left(\left(\frac{V(N)-v}{2(N+1)}\right)^2 \frac{1}{k} - F\right)N.$$

Note that expression (18) can also be written as

$$\frac{V(N)}{2k(N+1)^2}\left[V'(N)(2N+1) - V(N)\frac{N}{N+1}\right] = F,$$

and when the outside option appears it transforms in

$$\frac{V(N)}{2k(N+1)^2}\left[V'(N)(2N+1) - V(N)\frac{N}{N+1}\right] -$$

$$\frac{v}{2k(N+1)^2}\left[V'(N)(2N+1) - (2V(N)-v)\frac{N}{N+1}\right] = F.$$

So, the effect on N of v will depend on the second term. If this is positive, the monopolist will reduce N whereas if this is negative the impact on N will be positive. Both situations may occur; however since platform profits are lower for each N, the most likely case is that the monopolist will reduce N.

Now, note that whenever demand margin binds and there is an outside option v, the users surplus, gross of the cost kt, equals

$$V(N) - \widehat{p}N - \widehat{P}^U = V(N) - \frac{[V(N)-v]N}{2(N+1)} - \frac{[V(N)-v]}{2}$$

The first derivative of this surplus with respect to v is going to be positive as long as $\frac{\partial N}{\partial v}\left[\frac{V'(N)}{N+1} - \frac{V(N)}{(N+1)^2} + \frac{v}{(N+1)^2}\right] + \frac{2N+1}{N+1} > 0$. The second term of the left hand side of the inequality is always positive. However, the term in brackets is negative as long as $\epsilon_V < \left[1 - \frac{v}{V(N)}\right]\frac{N}{N+1}$, and this is the case along the relevant range of N (when gross developers profits are decreasing). The first term will be positive if $\frac{\partial N}{\partial v} < 0$ (the most likely case) and negative otherwise, so that the result follows.

Proof proposition 8

If the competition margin binds the effect of an increase in w on users surplus is equal to $-\frac{V'''(N)N\frac{\partial N}{\partial W}}{2} < 0$ and the first statement follows. To prove the second statement, note that the surplus is decreasing in N if $e_v < \frac{N}{N+1}$ and this occurs for the relevant range of N. Given that $\frac{\partial N}{\partial w} < 0$, the second statement follows.

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Chapter 3. Compatibility and Innovation with Firm Dominance

1 Introduction

"*Customers have mandated that the companies, long arch-rivals, must ensure that their infrastructure work together well. But beyond that, they will continue to compete tooth and nail" (Ron Hovsepian, Novell's president, BrainShare 2007).*[55]

"We're evolving to a point where there are a couple of platforms. You can't standardize everything ... They (customers) pushed on us a lot about interoperability, but also about continuing innovation" (Craig Mundie, Microsoft's chief research and strategy officer, BrainShare 2007).[56]

Several dominant firms are being scrutinized by the European Commission in cases where interoperability/compatibility issues have been one of the main arguments in the authorities' concerns. For instance, Sun Mycrosystems charged that Microsoft was refusing to share information that would allow interoperability between its servers and the equipment produced by the software giant. In March 2004 the Commission ordered Microsoft to disclose confidential computer code to competitors and in September 2007, Europe's Second-Highest Court reaffirmed the decision. In January 2008, the European Commission began two new antitrust investigations into Microsoft focussed on the compatibility of its Office package with other companies' software. Similarly, in March 2007 the European Commissioner for Consumers complained about the fact that iPod (Apple's device) was the only portable device that will play iTunes.[57] Given the importance of these antitrust cases and of these industries in modern economies, compatibility/ interoperability appears as a relevant issue for competition policy and regulation.

Discussions about compatibility and market power are certainly nothing new. However, the recent evolution of technological industries has brought forth a new set of issues, in particular the implications of market dominance in terms of strategic decisions on compatibility/interoperability and the impact of the latter on innovation incentives. After the EU resolution on the Microsoft case, some voices believe that the Commission will now go after other technology firms with large market shares, which will force companies to give up intellectual property and will curb

[55]See http://www.crn.com/software/198100037

[56]Ibidem.

[57]Norway has threatened to take action against Apple if it does not open up its digital rights management (DRM) system to other companies. France proposed a law requiring digital music retailers to make all downloaded songs compatible with various MP3 players. Other music download sites use a variety of DRM technologies that are incompatible with the iPod. (see Reuters, April 3, 2007 and http://www.crmbuyer.com/story/ipod/56255.html).

the incentives to innovate.[58] Among these voices, Thomas O. Barnett, the assistant attorney general for the Justice Department's Antitrust Division has declared that "*rather than helping consumers, it may have the unfortunate consequence of harming consumers by chilling innovation and discouraging competition*".[59]

The impact of firms' decisions on compatibility issues and/or on innovation goes beyond antitrust analysis. On one hand, the extent to which a firm will be compatible with other firms is one of its key strategic decisions. Note that vertical arrangements lead to a situation of incompatibility. In contrast, it may be in the firms' interest to promote compatibility, as the agreement signed by Microsoft and Novell to enhance interoperability between Linux and Windows illustrates.[60] On the other hand, consumers will clearly be affected by firms decisions. A console firm that impedes its video-game developers to provide their products to its competitors is de facto refusing compatibility, and it is hence affecting to consumers of the multi-billion video game industry.[61]

In this paper we will analyze firms' attitudes towards compatibility and how compatibility decisions affect incentives on investments in producing a product improvement. In particular, we study how different levels of firm dominance, measured by a premium in consumer valuations, influence these incentives. Thus, the questions we address include (a) is there a relationship between market dominance and compatibility incentives?, (b) does compatibility enhance or discourage innovation?, (c) is a policy of mandatory compatibility socially desirable?.

To address these issues we propose a model of platform competition. We assume that users buy a platform and its compatible applications. We allow for applications to be substitutes, complements or independent. We consider compatibility in two dimensions. First, compatibility of the complementary good, to which we will refer as compatibility in applications. Typical examples are software that can be either run or not with different hardware. Second, we consider inter-network compatibility. In this case, direct network externalities are present in the sense that one user's value for a good is higher when another user buys the compatible good, as in the case of personal computers that allow users to exchange e-mails. Literature has largely ignored the difference between both types of compatibility, however our model yields different results for each of them (see Section 1.2 for more details about these two forms of compatibility and about related literature).

We find that the dominant firm will never promote compatibility in applications. In contrast, both firms find inter-network compatibility profitable.

Compatibility in applications is not always beneficial for consumers. Moreover, we find that the dominant firm's incentives to be compatible may coincide with

[58] See www.economist.com, September 20, 2007.

[59] The New York Times, September 18, 2007. See also Nicholas Economides in his "Commentary of the EU Microsoft Antitrust Case" (September 2007, www.NETInst.org) where he remarks: "By requiring full disclosure at a nominal price, the EU decision in effect reduces the value of intellectual property for dominant firms. Additionally, in the particular case, full understanding of internal Windows functions is valuable to Sun beyond interoperability (...). That is, the Commission's vertical remedy gives an advantage to Sun in horizontal competition".

[60] The Novell and Microsoft Collaborate relates to both interoperability and innovation. Note that the companies expressed the intention of creating a joint research facility to pursue new software solutions for virtualization, management, and document format compatibility (for details, see http://www.novel.com/linux/microsoft/faq.html).

[61] See Lee (2007) for an empirical analysis on the impact of vertical integration and exclusive contracting in the US video game industry during the period 2000-2005.

those of users while being against those of the weak firm. When this is the case, imposing compatibility would harm both consumer surplus and total welfare. In particular, compatibility in applications decreases both consumer surplus and welfare if applications are close substitutes. Furthermore, the effect of compatibility on welfare and on consumer surplus tends to be negative when firm dominance is important. We also find that the presence of strong direct network effects strengthen the potential negative incidence of compatibility in applications. Regarding inter-network compatibility, it is always consumers' surplus reducing and has a negative impact on total welfare as long as the market is very asymmetric.

We also show that any type of compatibility often reduces the incentives of the firms to invest, particularly those of the dominant one. When compatibility in applications is present a free-riding problem arises: an investment made by one firm that adds value to the compatible good is shared by the other firm, thus compatibility may induce lower incentives to invest. Additional reasons for the lack of interest in compatibility (besides the free-riding problem) arise from the substitutability between the applications.

The main antitrust implication of our results is to advise that decisions on compatibility and/or on exclusivity must be tailored to each particular case as the degree of substitutability, direct network effects, and firm dominance may alter the desirability or not of enforcing compatibility.

The paper is organized as follows. The rest of this Section presents the related literature and a discussion on compatibility and its possible modeling strategies. Section 2 presents the basic framework. In Section 3 we show the results for compatibility in applications and inter-network compatibility. Section 4 analyses investment incentives related to compatibility. Section 5 studies the robustness of the model proposed in Section 2. Section 6 concludes. Finally, proofs are relegated to the Appendix.

1.1 Related literature. This paper borrows modelling strategies from, and contributes to, two strands of literature. First, to the literature on compatibility in industries with network effects. The seminal papers of Katz and Shapiro (1985) and Farrell and Saloner (1985) predict that incentives for incompatibility will differ across firms and will be greater for firms with larger networks, since under compatibility these firms will lose the competitive advantage that their networks confer. Adopting the Katz and Shapiro-model, Crémer, Rey and Tirole (2000) analyze the competition between Internet backbone providers with asymmetric installed bases. They show that a firm with a large installed base may have incentives to reduce the degree of compatibility towards its smaller rivals. In the same vein, Malueg and Schwartz (2006) analyze the conditions under which the firm with the largest market share of installed-base customers will prefer incompatibility with smaller rivals that are compatible among themselves. They find that the largest firm is more likely to prefer incompatibility over compatibility if with the latter its market share rises above fifty percent or if the potential to add consumers falls. Chen, Doraszelski and Harrington (2007) consider product compatibility with market dominance in a dynamic setting. They find that if firms have similar installed bases they make their products compatible. But, if a firm gets a larger installed base then it may make its product incompatible.

Similar to these papers, we find that the dominant firm is against compatibility related to applications. However, this firm would indeed promote inter-network compatibility.

In the absence of consumption externalities, some papers have analyzed firms' incentives to make components of different systems compatible, in what is known as the *mix-and-match literature*.[62] Economides (1989) finds that profits are higher under compatibility, so that a fully compatible regime is the unique perfect equilibrium. However, symmetry of system demands is crucial for compatibility to occur.

Economides (2006) argues that it is socially efficient to move towards compatibility. In the same vein, Katz and Shapiro (1985) show that a move to complete compatibility would raise consumers' surplus. In contrast, these results do not always hold in our framework. In particular, we conclude that a move towards a larger degree of compatibility in the applications may be harmful for users and social welfare, particularly when asymmetries are strong. Moreover, in the case of inter-network compatibility our model predicts that it should not be promoted by consumers.

The second strand of the literature which this paper relates to, is devoted to studying incentives to innovate when firms are asymmetric. Cabral and Polak (2007) analyze the effect of firm dominance on the incentives for R&D. They find that an increase in firm dominance increases the dominant firm's incentives while decreasing the other firm's incentives for R&D. They also show that total research effort decreases when firm dominance increases, and that firm dominance is good for innovation only when property rights are strong.

Few papers combine the two aforementioned strands of the literature. Cabral and Salant (2007), in a model of R&D competition and cooperative standards setting, argue that standardization leads to a free riding problem and thus to a decrease in marginal incentives for R&D investment. Our model also identifies the free-riding problem and a decreasing marginal incentive for the dominant firm, but an increasing one for the weak firm. Hannan and Borzekowski (2006) investigate whether incompatibility across rival systems may influence firms' incentives to invest in product changes that are beneficial to the consumer in the case of bank ATM networks. They consider the number of ATM locations as the measure of product quality and the surcharge fees as an index of incompatibility. They find that an increase in incompatibility for Iowa banks caused a substantial increase in the number of ATM locations offered to customers. Besides, the effect is greater for larger banks than for smaller ones. In a symmetric setting Choi (2003) analyzes the effect of compatibility on R&D incentives. He finds that a firm that makes two components incompatible increases its R&D level and outside firms reduce theirs in a linear demand model with quadratic R&D cost functions.

1.2 On the modeling of compatibility in the literature. Standards specify properties that a product must have in order to work (physically or functionally) with complementary products within a product or service system. Compatibility refers to "the possibility of costlessly combining various links and nodes on the network to produce demanded goods. Two complementary components A and B are

[62] The mix-and-match literature does not assume a priori network externalities; however, it is clear that demand in mix-and-match models exhibits network externalities. The mix-and-match approach was originated by Matutes and Regibeau (1988), and has been used in several papers including Economides (1989), Economides and Salop (1992), Matutes and Regibeau (1989, 1992).

compatible when they can be combined to produce a composite good or service" (see Economides (2006)). Interoperability refers to "the ability of two or more systems or components to exchange information and to use the information that has been exchanged" (as defined by the IEEE Standard Computing Dictionary (IEEE, 1990)). Both compatibility and interoperability are hence related to the possibility of sharing standards.

The economic literature has modelled compatibility (or interoperability) in two ways. First, as in the seminal paper of Katz and Shapiro (1985), compatibility affects the consumers' surplus of buying one unit of a good, making it depend on the number of other agents who join the network associated with that product. If x_i^e denotes the number of customers that a consumer expects firm i to have and y_i^e denotes the consumers' prediction of the size of the network with which firm i is associated, then brands are incompatible whenever $y_i^e = x_i^e$. In contrast, if m firms products are compatible then

$$y_i^e = \sum_{j=1}^{m} x_j^e \text{ for } i = 1, 2, ..., m.$$

Crémer, Rey and Tirole (2000) adapted Katz and Shapiro (1985) to incorporate installed-base customers where firms differ in their locked-in, installed bases of customers.[63] Without compatibility, the size of the network of each firm is its installed base and the mass of new customers. With compatibility, the networks include all new customers and the installed bases of all the firms. In all of these papers network effects are direct since the main purpose of the consumers is to contact other users.

There is a second set of papers that also studies compatibility issues in a mix-and match framework (Economides (1989), Matutes and Regibeau, (1988, 1992), Choi (2003)). They consider systems that are composed of two components (e.g. hardware and software) produced by two firms A and B (Economides (1989) generalizes to n firms). If the components sold by the two firms are not compatible, only two systems are available for consumers, systems A_1A_2 and B_1B_2. If the components are compatible, consumers have the additional options of A_1B_2 and B_1A_2. In these papers consumers derive utility from the system itself.

We will analyze here platforms that together with a set of applications compose a system so that, in the context of the hardware-software paradigm, we relate compatibility to the number of applications that can be used with the platform.[64] Because of this, we say that there is compatibility in applications if the users have the possibility to construct a wider variety of systems (e.g. $A_1A_2B_2$ and $B_1B_2A_2$).

In contrast, we talk about inter-network compatibility when consumers are concerned with exchanging information and files with other consumers. Specifically, consumers may exchange information with consumers of other platforms and not only with consumers that bought the same platform. The modeling of compatibility

[63]Other papers that consider similar models include Malueg and Schwartz (2006) or Chen, Doraszelski and Harrington (2007).

[64]Corts and Lederman (2007) consider the existence of both exclusive (non-compatible) and non-exclusive (compatible) software in an empirical model of indirect network effects. They argue that over the last 20 years - over succesive technological generations - software has become less likely to be exclusive to a particular platform, and that this trend and its consequences have largely been ignored.

in Katz and Shapiro (1985) seems to us the appropriate model to address this type of compatibility as direct network effects are more implicated.[65]

2. The model

We consider a market in which consumers derive utility from consuming a platform and its compatible applications. There are two platforms l, $l = A, B$ that compete à la Hotelling and are located at the two extremes of the unit line. We assume that they cover the market.

Each platform has a fully compatible application i, $i = 1, 2$. Application 1 is fully compatible with platform A and its value is W_1. This application is compatible with platform B in a degree δ, so that its value for a user of platform B is δW_1.[66] Analogously, application 2 is fully compatible with platform B with value W_2 (δW_2) for users of platform B (A). Platforms have also a stand-alone value to consumers so that the value of platform l is either V_A or V_B.

Applications are produced by third party developers. The consumers pay a price for the platform and for each application that they buy from the developers. We consider that platform firms follow a two-sided strategy, setting a fee to developers per unit of application sold to be used with the platforms. We allow platforms to discriminate by type of application. Developers behave competitively, consequently, the price they charge to consumers for their applications is their marginal cost given here by the per unit fee set by the platforms.[67] Note that the model resembles the traditional models of telephony (Laffont, Rey and Tirole (1998)).

We assume that the market structure is asymmetric with a dominant and a weak firm. There are two potential sources of asymmetry: an asymmetry in the stand-alone value of the platform and/or an asymmetry in the value of the fully compatible application. Note that under full compatibility the latter asymmetry disappears.[68] We assume that $W_1 > W_2$ and initially, we also assume that $V_A = V_B = V$.
Applications side. A representative consumer of platform A maximizes the following net utility from consuming the applications,

$$u_A = W_1 q_{A1} + \delta W_2 q_{A2} - \frac{1}{2} b \left(q_{A1}^2 + 2\sigma q_{A1} q_{A2} + q_{A2}^2 \right) - p_{A1} q_{A1} - p_{A2} q_{A2},$$

where q_{li} is the quantity of application i consumed by a consumer of platform l, p_{li} is its price, and $b > 1$. Negative values of σ would make the model one of demand for complementary applications. If $\sigma = 0$ the applications are independent in demand. As σ approaches 1, the applications become closer substitutes.[69] The representative

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[65] In some sense our interpretation of the difference between "compatibility in applications" and "inter-network compatibility" is similar to the one employed by Clements (2004) to distinguish between direct and indirect network effects. Similarly, Economides and White (1994), differentiates between two-way networks and one-way network.

[66] Alternatively, we may interpret the application fully compatible for platform l as a set of applications and δ as a measure of the subset of applications that can be used by a user of the other platform.

[67] This assumption is not innocuous. We introduce it for simplicity to concentrate on the consequences of changes in compatibility. However, we conjecture that, although quantitative results will change if we introduce some market power on the sellers' side (i.e., there is a double marginalization in the price), they will not change qualitatively.

[68] The way we introduce asymmetries in the platforms is similar to Carter and Wright (2003).

[69] Examples of substitute applications are text processors written for different operating systems, or football games written for different video consoles. Complementary applications are

consumer of platform B maximizes an analogous net utility u_B. These utilities yield demands

$$\begin{aligned}
q_{A1} &= \frac{1}{b\left(1-\sigma^2\right)}\left(W_1-\sigma\delta W_2-p_{A1}+\sigma p_{A2}\right),\\
q_{B2} &= \frac{1}{b\left(1-\sigma^2\right)}\left(W_2-\sigma\delta W_1-p_{B2}+\sigma p_{B1}\right),\\
q_{A2} &= \begin{cases} 0 & \text{if } \delta=0 \\ \frac{1}{b(1-\sigma^2)}\left(\delta W_2-\sigma W_1-p_{A2}+\sigma p_{A1}\right) & \text{if } \delta>0 \end{cases},\\
q_{B1} &= \begin{cases} 0 & \text{if } \delta=0 \\ \frac{1}{b(1-\sigma^2)}\left(\delta W_1-\sigma W_2-p_{B1}+\sigma p_{B2}\right) & \text{if } \delta>0 \end{cases}.
\end{aligned}$$

Note that although the market for the platforms is fixed and covered, the size of the applications market is increasing in both δ and the value of the applications.
Users side. Consumers are uniformly distributed on the unit line with respect to their preferences for the platforms. A consumer with a most preferred platform type (or location) x derives utility from the use of the platform she buys and from its compatible applications. In particular, the net utility derived by a consumer who buys platform l located at the beginning of the unit line is given by

$$U_l = w_l\left(\delta\right)+V-tx-s_l. \tag{23}$$

The first and second term reflect the value of owning platform l. The term $w_l\left(\delta\right)$ is the indirect utility that the applications compatible with platform l generate. Thus, $w_A\left(\delta\right)$ is a function of W_1 and δW_2, where the level of δ is exogenously determined, for instance, by a regulator. The third term is the disutility stemming from not consuming the most preferred platform type (the transportation cost or the degree of differentiation in the standard Hotelling model). Finally, s_l is the price charged by platform l. Note that in (23) users' utility do not depend on the number of platform users, i.e., there are no direct network effects.

If direct network effect exist then (23) becomes

$$U_l = w_l\left(\delta\right)+V+\eta\theta_l-tx-s_l, \tag{24}$$

where η measures the extent of the network effect. We will model consumers' net utility as in (24) to analyze inter-network compatibility where direct network effects among users are present. These effects could be important in markets where users want to exchange or share the applications (for instance users of Word can share files, gamers of the same video console can exchange the games, etc).
The problem of the platforms. Each platform has two sources of income, one from the consumers and another from the developers of applications. Thus, under the proviso that costs are null, platform profits are given by

$$\Pi_l\left(\delta\right)=\left(s_l\left(\delta\right)+\pi_l\left(\delta\right)\right)\theta_l\left(\delta\right),$$

where $\pi_l\left(\delta\right)$ represents the profit that the platform gets, per consumer, from the developers side.[70]

The timing of the game is the following: first, the platforms set prices to consumers who then decide which platform to buy. Then, the platforms set fees to

a text processor and a spreadsheet, while independent applications are a role-play game and a boxing game.

[70]Our model is quite similar to the model in Church and Gandal 1992a, 1992b and 2000. However, they only consider platforms that follow a one-sided strategy with profits $\Pi_l = s_l\left(\delta\right)\theta_l\left(\delta\right)$.

developers. Finally, the developers set competitive prices to the consumers for the applications they sell. We compute the subgame perfect equilibria, which implies that consumers form rational expectations to determine both the size of each network and the prices that developers will set for the applications, while knowing the prices set by the platforms.

3. Compatibility in applications

To study the impact of compatibility on platform competition we first analyze the market for applications, then analyze the stage at which platforms set prices. Since developers behave competitively, the prices that they set coincide with the fees that platforms charged to them so that we can rewrite the problem faced by platform l in the second stage as follows

$$\max_{p_{l1},p_{l2}} \pi_l = p_{l1}q_{l1} + p_{l2}q_{l2}. \tag{25}$$

Note that at this stage each platform's market share is given so that each platform sets prices for applications as if it were a monopolist. Lemma 1 summarizes some properties of the indirect utilities and profits which arise from the applications.

Lemma 1

i) If applications are complementary or independent, then $\frac{\partial w_l(\delta)}{\partial \delta} > 0$ *and* $\frac{\partial \pi_l(\delta)}{\partial \delta} > 0$ *for all* l.

ii) If applications are substitutes, there exist $\bar{\sigma}_1 = \frac{\delta W_1}{W_2}$ *and* $\bar{\sigma}_2 = \frac{\delta W_2}{W_1} < \bar{\sigma}_1$ *such that*

if $\sigma < \bar{\sigma}_2$ *then* $\frac{\partial w_l(\delta)}{\partial \delta} > 0, \frac{\partial \pi_l(\delta)}{\partial \delta} > 0$ *for all* l,

if $\sigma > \bar{\sigma}_1$ *then* $\frac{\partial w_i(\delta)}{\partial \delta} < 0, \frac{\partial \pi_i(\delta)}{\partial \delta} < 0$ *for all* l,

if $\bar{\sigma}_2 < \sigma < \bar{\sigma}_1$ *then* $\frac{\partial w_A(\delta)}{\partial \delta} < 0, \frac{\partial \pi_A(\delta)}{\partial \delta} < 0, \frac{\partial w_B(\delta)}{\partial \delta} > 0, \frac{\partial \pi_B(\delta)}{\partial \delta} > 0.$

Proof. see the Appendix.

Results in Lemma 1 resemble the standard results in the literature on multiproduct monopolist (see Tirole (1987)). The price and the demanded quantity of the non-fully compatible applications are increasing in δ for each platform and for any value of σ. However, how the quantity of the fully compatible applications reacts to changes in δ depends on the sign of σ (the price is neutral). As expected, this quantity increases with δ if applications are complementary, it decreases if applications are substitutes and it is neutral if they are independent.

Let us define $w(\delta)$ $(\pi(\delta))$ as the difference in consumer surplus (profits from developers' side) generated by the two platforms, i.e.,

$$w(\delta) = w_A(\delta) - w_B(\delta) = \frac{1}{4(1-\sigma^2)b}\left(W_1^2 - W_2^2\right)\left(1-\delta^2\right)\left(1-\frac{1}{b}\right), \tag{26}$$

$$\pi(\delta) = \pi_A(\delta) - \pi_B(\delta) = \frac{1}{4(1-\sigma^2)b}\left(W_1^2 - W_2^2\right)\left(1-\delta^2\right). \tag{27}$$

Two facts that trivially follow from (26) and (27) are next stated:

Fact 1. $\frac{\partial w(\delta)}{\partial \delta} < 0$ and $\frac{\partial \pi(\delta)}{\partial \delta} < 0$.

Fact 2. $w(\delta) > 0$ and $\pi(\delta) > 0$ if $\delta < 1$ and $w(\delta) = \pi(\delta) = 0$ if $\delta = 1$.

The rationale behind the above facts has an intuitive reasoning. The difference between the two platforms in the indirect utility that consumers derive from the applications decreases with the degree of compatibility. In the limit, when $\delta = 1$, this difference is zero since both platforms will provide both applications with values W_1 and W_2. Similarly, the platform with the most valuable application receives higher profits from the developers side, a difference that disappears if there is full compatibility $(\delta = 1)$.

To avoid platform A tipping the market, an additional assumption is introduced:

Assumption 1. $t > \frac{w(\delta)+\pi(\delta)}{3}$.

This implies that the transportation cost parameter is greater than the term that reflects the asymmetries between the platforms. Note that if there is no asymmetry in the applications or if there is full compatibility, i.e., $w(\delta) = \pi(\delta) = 0$, Assumption 1 is trivially satisfied.

Platform competition. At the first stage, platforms compete by setting prices to consumers. We assume that consumers have beliefs about network sizes which lead them to make purchase decisions that in equilibrium confirm their beliefs. Moreover, they anticipate that monopoly prices for applications will be set at the last stage. Since platform profits are given by

$$\Pi_l = (s_l + \pi_l(\delta))\,\theta_l(\delta),$$

the degree of compatibility affects price competition as it has an impact on platforms' market shares. Moreover, compatibility makes platforms softer or tougher competitors depending on whether applications are complementary or substitutes. The problem faced by platforms at this stage is similar to that of a multimarket oligopoly.[71] An increase in the level of compatibility acts as a positive (negative) shock in the markets of applications when $\sigma < \bar{\sigma}_2$ $(\sigma > \bar{\sigma}_1)$, as Lemma 1 shows. Consequently, a change in δ has a direct effect on the profits made from the applications side and an indirect effect on the platforms market share.

Consider first the case of independent or complementary applications. An increase in δ moves platforms' reaction curves, i.e., $s_A(s_B)$ and $s_B(s_A)$, inwards. Furthermore, this move is larger for the platform A which is stronger in the applications market as compatibility "reduces differentiation" which affects this platform negatively. Recall that as compatibility increases the platforms tend to offer the same surplus in terms of applications. Consequently, both firms will be more aggressive in the platform market. This is explained by two facts: first, the complementarity between the applications and the platform, and second, by the fact that the reaction curves $s_A(s_B)$ and $s_B(s_A)$ are upward sloping as prices are strategic complements. Assume now that applications are close substitutes. In this case both firms will be less aggressive in the platform's market since an increase in δ moves platforms' reaction curves outwards.

These results are the content of the next lemma.

Lemma 2

i) If applications are complementary or independent, platform price competition is more aggressive the higher the degree of compatibility between the firms.

ii) If applications are close substitutes, platform price competition is less aggressive the higher is the degree of compatibility.

[71]See Bulow, Geanakoplos and Klemperer (1985).

iii) There is a degree of substitution for which the platform with the dominant application becomes less aggressive whereas the platform with the weak application becomes more aggressive as the degree of compatibility increases.

Proof. see the Appendix.

In our framework, incompatibility generates a similar situation to the one in which developers single-home. In contrast, when compatibility takes place, it leads to partial (or full) multihoming.[72] Lemma 2 shows that if applications are complementary or independent, platform price competition (referred to the price set to users) is more aggressive the higher the degree of compatibility, a result that is consistent with a competitive bottleneck equilibrium. In contrast, platform price competition is less aggressive, as compatibility increases, when applications are close substitutes. It follows that the typical outcome related to a competitive bottleneck equilibrium (that the platforms strongly attend the interests of the single-homing group) may not be robust to introducing some substitution on the sellers side.

3.1 Welfare analysis. We have seen that the degree of compatibility affects the prices that platforms charge to users. We next analyze its impact on platforms' profits. We will say that firm l has incentives to promote compatibility if its profits increase when the degree of compatibility increases, otherwise we will say that firm l does not have incentives to promote compatibility.

A priory, one may think that both platforms would earn more as the degree of compatibility increases since the market for applications expands and at the same time both platforms will offer a better product. However this is only true for the platform that is weaker for which an increase in compatibility positively affects its total profits. Regarding the other platform, compatibility has a positive and a negative impact on its profits. Consider complement or independent applications. Thus, on the one hand, platform A increases its profits in the applications market. But, on the other hand, the strong competition for the users in the first stage in a covered and fixed market, leads to a price reduction. Platform A uses the gains obtained on the applications side to offer lower prices to consumers in the first stage. The reduction in $s_A(\delta)$ overtakes the gains in $\pi_A(\delta)$ due to the fight to maintain the market share. In contrast, for platform B the reduction in price $s_B(\delta)$ is indeed compensated by the gains in profits from the applications, $\pi_B(\delta)$.[73] In this case, platforms are less aggressive as compatibility increases but since the "reducing differentiation" effect remains, the profits of the strong platform in the market of applications still decrease whereas the profits of the weaker one increase. Finally, note that if we consider platforms with symmetric applications, their profits would not depend on the degree of compatibility. The following lemma summarizes the above discussion.

Lemma 3

[72] Incompatibility can also be interpreted as exclusive developers and compatibility as non-exclusive ones. Although we focus on compatibility, decisions on standards and/or on exclusivity arrangements can be considered forms of compatibility.

[73] Note that if applications are close substitutes, profits in the applications market are decreasing in δ. Under these conditions platforms charge more to the users side and less to the applications side when compatibility increases. More precisely, there is a range of σ for which the weak platform charges more to the applications side whereas the dominant charges more to users for the platform as a response to a higher compatibility (see lemmas 1 and 2).

i) If $W_1 = W_2$ both firms are indifferent about compatibility
ii) If $W_1 \neq W_2$ the dominant firm in the applications market does not benefit from an increase in the degree of compatibility whereas the weaker one does

Proof. see the Appendix.

Lemma 3 shows that whenever applications are symmetric, the higher profits that the platforms can make on one side due to a greater compatibility get discounted in the price of the other side in such a way that in equilibrium total profits are not affected by δ. The lemma also shows that, with asymmetric applications, the dominant firm has lower profits under compatibility but the weak firm has larger ones. Consequently, the strong platform has no incentive to promote compatibility whereas the weak one would like compatibility to be enforced. This implies that if compatibility were an outcome of a coordinated decision between firms, it would never arise.

Compatibility, consumer surplus and welfare

From (23) it follows that total consumer surplus denoted by $TS(\delta)$ is given by

$$\int_0^{\theta_A(\delta)} (w_A(\delta) + V - tx - s_A(\delta))\,dx + \int_{\theta_A(\delta)}^1 (w_B(\delta) + V - t(1-x) - s_B(\delta))\,dx,$$

which equals

$$V + \theta_A(\delta)(w_A(\delta) - s_A(\delta)) + (1 - \theta_A(\delta))(w_B(\delta) - s_B(\delta)) - \frac{1}{2}t\left(\theta_A^2 + (1-\theta_A)^2\right).$$

Total welfare denoted by $TW(\delta)$ can also be written using a similar decomposition as above as

$$V + \theta_A(\delta)(w_A(\delta) + \pi_A(\delta)) + (1 - \theta_A(\delta))(w_B(\delta) + \pi_B(\delta)) - \frac{1}{2}t\left(\theta_A^2 + (1-\theta_A)^2\right).$$

Next proposition shows that the overall effect on consumer surplus and on welfare of an increase in compatibility may turn negative. In particular, it shows that there is a critical degree of substitutability among applications above (below) which compatibility is bad (good) for consumers.

Proposition 1
There exist σ^* and σ^{**}, $0 < \sigma^{**} < \sigma^*$, which are decreasing in $W_1 - W_2$ and increasing in δ, such that
if $\sigma > \sigma^*$ then $\frac{\partial TS(\delta)}{\partial \delta} < 0$ and $\frac{\partial TW(\delta)}{\partial \delta} < 0$,
if $\sigma^{**} < \sigma < \sigma^*$ then $\frac{\partial TS(\delta)}{\partial \delta} > 0$ and $\frac{\partial TW(\delta)}{\partial \delta} < 0$,
if $\sigma < \sigma^{**}$ then $\frac{\partial TS(\delta)}{\partial \delta} > 0$ and $\frac{\partial TW(\delta)}{\partial \delta} > 0$.

Proof. see the Appendix.

Note that the impact of an increase in δ depends on both the level of substitution between the applications and on their asymmetry. In particular, an increase in the degree of compatibility is more likely to yield a reduction in the consumers surplus and in the total welfare if there is a high level of substitution and a low value of δ. The fact that welfare is more likely to decrease than consumers surplus is explained by the fact that the total profits of the industry are decreasing in the degree of compatibility, i.e., $\frac{\partial(\Pi_A(\delta)+\Pi_B(\delta))}{\partial\delta} = \frac{2}{9t}\left((w(\delta) + \pi(\delta))\right)\frac{\partial(w(\delta)+\pi(\delta))}{\partial\delta} < 0$. Moreover, based on Lemma 3 and Proposition 1 we can conclude that a move towards a larger degree of compatibility in the applications may be harmful for users and social welfare, particularly when asymmetries are strong. Consequently, it may be undesirable to force a larger degree of compatibility not only because it is against the dominant firm interests but also because it may lower consumers' surplus. Based on these arguments, next corollary follows.

Corollary 1

There is a range of parameter values for which the dominant firm, the users and the social planner share the same interests regarding compatibility that are against those of the weak firm. Furthermore, this range is higher the larger the dominance of the strong firm.

Note finally that firms can differ in another dimension, namely in the stand alone values of their platforms. Denoting by Δ to $V_A - V_B$, it is easy to see that Δ has the same impact on equilibrium market outcomes than $w(\delta)$ has. Consequently, all the potential negative effects on consumers' surplus and on welfare of increasing compatibility get reinforced when this second source of asymmetry is present, i.e., the critical values σ^* and σ^{**} in Proposition 1 are decreasing in Δ.

3.2 Direct network effects and Inter-network compatibility. We have assumed so far that there are no direct network effects which may be considered as a limiting assumption. We next introduce direct network effects among users with a twofold objective: on one hand, we want to study if they make compatibility in applications more or less desirable, on the other hand, by introducing direct network effects we can analyze inter-network compatibility. Note that since consumers derive value from the existence of consumers using the other platform, inter-network compatibility might turn desirable. Furthermore, it may be in the interest of firms to enforce it even though they might be against compatibility in applications.

When there are direct network effects users' utility is given by (24), i.e., by

$$U_l = w_l(\delta) + V + \eta\theta_l - tx - s_l,$$

where η measures the extent of the network effects. Since compatibility makes the platforms more symmetric, it reduces the positive direct network externalities. Thus, in markets where direct network effects are important, the potential harming effect that an increase in compatibility has on consumers surplus and on welfare is more likely to occur. In other words, strong direct network effects make more likely the negative consequence of forcing compatibility in applications. It hence follows that the presence of network effects make compatibility in applications less desirable.[74]

Regarding inter-network compatibility, it allows consumers who buy platform l to get an extra net-utility of $\eta\beta(1-\theta_l)$ from the existence of users of the other platform, so that consumers overall utility becomes

$$U_l = w_l + V + \eta[\theta_l + \beta(1-\theta_l)] - tx - s_l$$

where $\beta \epsilon [0,1]$ measures the level of inter-network compatibility.

[74]More precisely, the critical values σ^* and σ^{**} in Proposition 1 are decreasing in η, as the market share of the dominant firm is now given by

$$\theta_A(\delta) = \frac{1}{2} + \frac{1}{6(t-\eta)}(w(\delta) + \pi(\delta)),$$

that is increasing in η.

We find that inter-network compatibility would indeed be promoted by the strong firm. Although an increase in β reduces the platform differentiation, it also allows platforms to set higher prices to consumers. This last positive effect compensates the negative impact on the dominant firm's market share of the reduced differentiation. Furthermore, inter-network compatibility reduces consumers surplus and its effect on total welfare might turn negative if the level of asymmetries are large. Firms' incentives to become inter-network compatible are opposite to those of consumers. Furthermore, as the market dominance of the strong platform (either in platforms or applications values) grows larger it is more likely that an increase in inter-network compatibility will decrease welfare. These results are the content of next Proposition.

Proposition 2
i) Both firms benefit from an increase in inter-network compatibility.
(ii) Inter-network compatibility has a negative impact on consumers' surplus, which is larger the stronger the asymmetries are.
(iii) If asymmetries are strong, inter-network compatibility has a negative impact on welfare.

Proof. see the Appendix.

4. Incentives to innovate

A platform can invest in the stand-alone value of the platform itself and/or in the value of the fully compatible application. Think of the video-game industry, a sector characterized by huge levels of investment. We observe that firms invest in developing new applications for the platform (console),[75] but also invest in producing more advanced technology.[76]

We will consider here that the result of an innovation is an increase in a firm's value, so that the steeper the slope of the firm's profit function with respect to its own value level, the larger the firm's incentive to increase this value.[77] We will determine the effect of compatibility on a firm's incentives to undertake an innovation by the sign of $\frac{\partial|\frac{\partial \Pi_l(\delta)}{\partial V_l}|}{\partial \delta}$ and $\frac{\partial|\frac{\partial \Pi_l(\delta)}{\partial W_i}|}{\partial \delta}$. Based on the nature (positive or negative) of the incentives to undertake these innovations we will propose a four type taxonomy of firms:

	$\frac{\partial\|\frac{\partial \Pi_l(\delta)}{\partial W_i}\|}{\partial \delta} > 0$	$\frac{\partial\|\frac{\partial \Pi_l(\delta)}{\partial W_i}\|}{\partial \delta} \leq 0$
$\frac{\partial\|\frac{\partial \Pi_l(\delta)}{\partial V_l}\|}{\partial \delta} > 0$	Type I	Type II
$\frac{\partial\|\frac{\partial \Pi_l(\delta)}{\partial V_l}\|}{\partial \delta} \leq 0$	Type III	Type IV

Table 1: Firm taxonomy

[75] Satoru Iwata, the president of Nintendo declared "*By designing products for existing gamers and neglecting non-gamers, it undermines the prospects to future growth*"...."*by providing non-gaming functions such as news and weather too, the aim is to overcome non-gamers' aversion to consoles, so that eventually they might flick to the "games" channel and give something a try*" (see The Economist, 26/10/2006).

[76] Sony is pushing the PS3 as the most advanced console, with a powerful new processor chip and a high definition "Blu-ray" optical drive (see The Economist 26/10/2006 and 10/11/2006).

[77] We introduce investment incentives as in Valletti and Cambini (2005).

We will say that a firm is of type I if compatibility increases the marginal incentive to invest in both values. In contrast, a firm is of type IV if compatibility decreases both marginal incentives. Finally, a firm is of type II or III if the marginal incentive to invest is increasing in compatibility for one value but decreasing for the other one. By analyzing firm types we will state the relationship between increasing firms' compatibility and getting more valuable products in the market.

We next study the effect of compatibility in applications on the incentives to invest. The next proposition presents the results regarding the typology introduced in Table 1 and the effect on total marginal incentives.

Proposition 3

When we consider an increase in δ,

i) The weak firm is type I if the asymmetries (any of them) are very strong, otherwise it is type II.

ii) The dominant firm in the market of applications is type IV.

iii) The total marginal incentive to invest in the stand-alone value does not change, however the corresponding to the value of the fully compatible applications decreases.

Proof. see the Appendix.

As the degree of compatibility increases, the strong platform in the market of applications has lower marginal incentives to invest in either type of investment. In contrast, compatibility positively affects the marginal incentive of the weak platform to invest in its stand-alone value. The effect is also positive regarding its incentive to invest in the fully compatible application when existing asymmetries are high.

The rationale is as follows. Consider first the incentives to invest in the applications. When a firm invests in the value of its fully compatible application, this investment is shared with the other firm, through δ, which free-rides from this increase in its value. Consequently, the marginal benefit of this investment is decreasing in δ. In the case of the dominant platform, this effect is reinforced by the fact that an increase in δ also leads to a loss in its relative advantage, so that ii) follows. Consider now the incentives of the weak platform. When asymmetries are very strong, the benefit due to its gain in the relative advantage compensates the loss due to the free-rider effect, which explains i). The opposite occurs when asymmetries are weak.

In the case of investment in the stand-alone value, the free-rider effect does not exist but the effect of δ on the relative advantages between firms remains. It reduces the incentives of the strong firm, while increasing those of the weak firm. These effects balance each other out and this explains first part in point (iii). The free-rider effect explains the second part of the last point.

Consider now the effect on the incentives to invest that inter-network compatibility yields.

Proposition 4

When internetwork compatibility increases, i.e., when β increases,

i) The dominant firm is type IV

ii) The weak firm is type I

iii) The total marginal incentive to invest in the stand-alone value does not change, however the corresponding to the value of the fully compatible applications decreases.

Proof. see the Appendix.

When considering the effect of β on the incentives to invest the problem of the free-rider problem disappears, and only the effect on the relative advantage is present. It explains that the weak firm is always type I, whereas the rest of the results are similar to those under compatibility in the applications, stated in proposition 4.

5. Robustness

We have shown that whenever the applications to be made compatible are close substitutes, users' surplus and welfare are decreasing in the degree of compatibility. Moreover, the dominant firm will never promote compatibility in applications as both its profits and its incentives to invest will decrease.

Throughout the analysis we have assumed that for a given compatibility level the two platforms compete à la Hotelling in attracting customers. Note that two assumptions are hidden behind this mode of competition. First, that the market size is fixed, and second, that although the model allows for applications to be substitutes, complements or independent, the systems as a whole are assumed substitutes.[78] Either of these assumptions is not innocuous for the aforementioned results. On the one hand, the incentives of the strong platform may change if we allow for a non-covered market. An increase in the value of its product due to compatibility would expand the market and the profits generated by this expansion may compensate the reduction in the advantage of the dominant firm. If this were the case, the dominant firm might also be interested in making applications compatible. On the other hand, complementarities among systems do exist (there are complementarities among computer operating systems and mobile phones, as well as between video consoles and mobile phones),[79] and they may alter firms' attitudes towards compatibility. Our purpose here is to study the robustness of the results by examining the extent to which they hold true for a potential expandable market and for systems that are not necessarily substitutes.

To do so we analyze a market in which two systems provide a service to consumers.[80] System i, $i = 1, 2$, is composed of platform i, application i which is fully compatible for this system and application j which is fully compatible for system j but compatible in a degree δ for system i. Consumers' valuations are given by

$$\begin{aligned}\alpha_1 &= V_H + W_1 + \delta W_2,\\ \alpha_2 &= V_L + W_2 + \delta W_1.\end{aligned}$$

where V_H, V_L are the stand alone values of the platforms, W_1 is the value of the application that is fully compatible for system 1, W_2 the value of the application that is fully compatible for system 2 and $0 \leq \delta \leq 1$ measures the degree of compatibility. We assume $W_1 > W_2$ and $V_H \geq V_L$.

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[78] A system is the good composed by the platform plus the applications.

[79] For instance, the Wii of Nintendo has a game that is played with photographs, so that it can be played with a camera or a mobile phone.

[80] We here present the main results and their intuition. All the details are relegated to Appendix B.

The price that consumers pay for the system can be decomposed in the following way:

$$\begin{aligned} p_i &= s_i + x_{ii} && \text{if } \delta = 0 \\ p_i &= s_i + x_{ii} + x_{ij} && \text{if } 0 < \delta \leq 1, \end{aligned}$$

where s_i is the price that the platform of the system i sets to users, and x_{ij} is the price that users pay for an application j to be used in system i. Firms compete in prices and the representative consumer maximizes a net utility based on Bowley's (1924) model and given by

$$U = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2} b \left(q_1^2 + 2\sigma q_1 q_2 + q_2^2 \right) - p_1 q_1 - p_2 q_2, \tag{28}$$

where q_i is the quantity of system i consumed, $b > 0$ and $-1 \leq \sigma \leq 1$. Negative values of σ make the model one of demand for complementary systems. If $\sigma = 0$ the two systems are independent in demand. As σ approaches 1, the systems become closer substitutes.

Since the market is two-sided, the profits of platform i depend on the price set to consumers, s_i, as well as on the prices that it sets to the developers so that

$$\Pi_i = \left(s_i + d_{ii} + d_{ij} \right) q_i$$

where d_{ii} (d_{ij}) is the price that platform i sets to the developer of its fully compatible application (of application j).

The timing of decisions is as follows. First, platforms set prices d_{ii}, d_{ij}, and s_i. Then developers of the complementary product set their prices competitively, i.e., $x_{ij} = d_{ij}$. Because of this and since the platform and the applications are perfect complements, the model becomes equivalent to that of a one-sided market in which firms compete by setting the prices for their systems p_1, p_2.

The best response of firm i to a price p_j by its rival is given by

$$p_i \left(p_j \right) = \frac{1}{2} \left(\alpha_i - \sigma \alpha_j + \sigma p_j \right). \tag{29}$$

Note that price systems are strategic complements, with $\frac{\partial p_i}{\partial p_j} = \frac{\sigma}{2} > 0$, if systems are substitutes. If they are complementary ($\sigma < 0$), prices are strategic substitutes. From (29) equilibrium prices are derived, with

$$\begin{aligned} p_1^* &= \frac{1}{4 - \sigma^2} \left(\left(2 - \sigma^2 \right) \alpha_1 - \sigma \alpha_2 \right), \\ p_2^* &= \frac{1}{4 - \sigma^2} \left(\left(2 - \sigma^2 \right) \alpha_2 - \sigma \alpha_1 \right). \end{aligned} \tag{30}$$

The impact of an increase in the degree of compatibility on the equilibrium prices depends, on one hand, on the nature of the strategic interaction between firms' choices, and on the other hand on the interplay between two effects which are brought about by compatibility: a market expansion effect and a reducing differentiation effect.

When prices are strategic substitutes, so that $\sigma < 0$, both firms respond to an increase in the degree of compatibility by increasing their price. Demanded quantities also increase, thus when systems are complementary or independent, both firms benefit from compatibility. When they are strategic complements, so that $\sigma > 0$, the price of system 2 also increases and the effect is always positive for the weak firm's profits. Regarding system 1 two outcomes can occur: i). The equilibrium price and quantity of system 1 increase as compatibility increases. This outcome emerges in either of the following two circumstances. First, if the market expansion effect dominates the reducing differentiation effect, i.e., if $\frac{\partial \alpha_1}{\partial \delta} > \sigma \frac{\partial \alpha_2}{\partial \delta}$, which happens when $W_1 < \frac{1}{\sigma} W_2$, then, in equilibrium, the positive effect from $W_1 < \frac{1}{\sigma} W_2$ is reinforced by the price strategic complementarity (since $\frac{\partial p_2^*}{\partial \delta} > 0$). Compatibility shifts system 1's reaction function outwards and in equilibrium its price and quantity sold are higher (see Figure 1 (a)). Second, if the reducing differentiation effect dominates the market expansion effect, but this negative effect is compensated by the price increase of the weak platform, the strategic complementarity still leads to $\frac{\partial p_1^*}{\partial \delta} > 0$ in equilibrium. This is the case when $\frac{1}{\sigma} W_2 < W_1 < \frac{2-\sigma^2}{\sigma} W_2$ and the final effect looks as depicted in Figure 1 (b). ii). The equilibrium price and quantity of system 1 decrease as compatibility increases. This outcome emerges whenever $W_1 > \frac{(2-\sigma^2)}{\sigma} W_2$ so that the negative impact brought by the reducing differentiation effect dominates overall and in equilibrium $\frac{\partial p_1^*}{\partial \delta} < 0$ (see Figure 1 (c)).

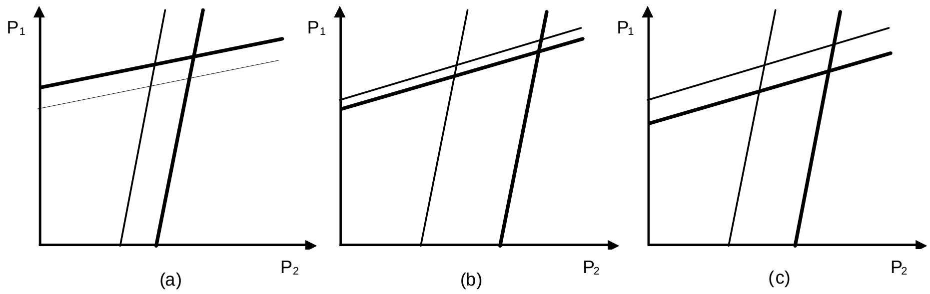

Figure 1

Based on the above discussion it follows that compatibility will hurt the dominant platform as long as $W_1 > \frac{(2-\sigma^2)}{\sigma} W_2$. The reducing differentiation effect tends to dominate as long as σ and the difference between the values of the complements are high. In contrast to the Hotelling case, the dominant platform might now be interested in inducing compatibility with the weak platform. However, a strong market expansion is crucial for this result.

Regarding users's surplus and welfare, Proposition B1 in Appendix B shows that whenever systems are complements or independent, they are both increasing in the degree of compatibility in applications. However, if the systems are substitutes the opposite result can hold.

Finally, when considering the marginal incentives to invest, the net effect will depend on whether or not the gains coming from offering a higher product value compensate the losses coming from the free-rider effect and from the reduction in the relative advantage the dominant system enjoys (the formal result is presented in Proposition B2). If this happens, the dominant firm can be of type I (i.e., compatibility encourages investment incentive in the platform and the fully compatible application), something that never occurs under the Hotelling price competition. Again this will only occur if a very important market expansion takes place.

The incentives to invest in the stand-alone value only depend on the difference $(W_1 - W_2)$. The change, due to δ, in the dominant platform's marginal incentive to invest is positive as long as $W_1 < \frac{2-\sigma^2}{\sigma} W_2$. Note that this is the condition under which p_1 and q_1 are increasing in δ. Thus, the marginal benefit of investing in V_H when δ increases is negative if the effect of δ on profits is also negative. Since this effect is always positive for the weak platform, its marginal incentive to invest in V_L is always positive.

The incentives to invest in the value of the applications depend on both sources of asymmetries $W_1 - W_2$ and $V_H - V_L$. When asymmetries are strong, compatibility increases the weak platform's incentives to invest. For the dominant firm, the effect of δ on incentives goes in the opposite direction, and when asymmetries are important it is negative (the losses coming from the free-rider effect and from decreasing its relative advantage are larger than the gains from offering a higher product value).

In sum, even though some of our basic findings could change if a larger degree of compatibilityo brings about the possibility of a market expansion, we have shown that a large expansion is needed to compensate the prevailing (negative) forces identified in the basic model.

6. Final remarks

We have developed a model where firms are horizontally differentiated á la Hotelling and are asymmetric in the value of their fully compatible application. We have analyzed firms' compatibility decisions and investment incentives to assess the benefits to users and to welfare of imposing a larger degree of compatibility among firms. We have stressed that compatibility is not always beneficial for consumers. Furthermore, in many instances the dominant firm's interests regarding compatibility decisions are in line with those of users, and are opposite to those of the weak firm, which will always demand more compatibility to be enforced. When this is the case, imposing a higher degree of compatibility will damage total welfare and consumer surplus.

Let us first recap the driving forces that underlie our main results. We have shown that the gap or the level of asymmetry given by $(W_1 - W_2)$ determines firms attitudes towards compatibility in applications. In particular, if $W_1 = W_2$ they are indifferent about the degree of compatibility that is enforced. When applications have different values so that $W_1 \neq W_2$ then the weak firm is always interested in promoting compatibility in applications. In contrast, the dominant one is never interested in doing so. The reason behind this result is that compatibility mainly decreases the market power of the dominant firm by reducing its advantage in product differentiation. However, regarding inter-network compatibility we have shown that both firms find profitable to promote it.

We have also shown that the welfare results strongly depend on the level of complementarity/substitutability of the applications, with compatibility decreasing both consumer surplus and welfare if applications are close substitutes. The level of the asymmetries in applications and/or in platform value, may also shape the effect of compatibility on welfare and consumer surplus. In particular, it tends to be negative when the asymmetries are important. We have also found that strong direct network effects strengthen the potential negative incidence of increasing the degree of compatibility in the applications. Moreover, our results show that a marginal increase in inter-network compatibility is always consumers' surplus reducing and it may have a negative impact in total welfare if asymmetries are strong. Finally, when systems are independent or complements, compatibility should indeed be promoted by any sector of the society.

Regarding the effects on the incentives to innovate, we have found that the marginal incentives to invest in the stand-alone value for the weak platform is increasing in δ, and so are the marginal incentives to invest in the application provided that asymmetries are important. For the dominant one, the marginal incentives to invest in either the stand-alone value or in the value of the fully compatible application are decreasing in the degree of compatibility. Changes in the level of inter-network compatibility positively affect the weak platform's marginal incentives to invest and negatively affect those of the dominant one.[81]

In sum, two salient aspects of the compatibility-welfare puzzle follow. First, substitutes goods are "bad" for compatibility. If products are substitutes, even if the market is not covered, compatibility tends to be welfare decreasing. Second, the negative potential consequences of compatibility are more likely when asymmetries are strong. To the best of our knowledge these are two novel results in the literature, as we offer a study of compatibility issues from the perspective of complementarities/substitutabilities among the goods to be made compatible.

Our analysis has several implications for evaluating "real world" policy decisions. In particular, we believe that these results shed light on some issues related to the EC vs. Microsoft case that we mentioned in the introduction. We find that when deciding about forcing compatibility/interoperability, the existence of a dominant firm does not provide enough arguments for more compatibility to be desirable. Moreover, network effects, almost always present in this kind of industries, may act as a countervailing reason to not force more compatibility in applications. Related to the incentives to innovate, when the Commission took the decision of imposing interoperability, it stated that the remedy would be good for innovation. We have shown that interoperability generates a free-rider effect, so that more interoperability may not encourage innovation either for the dominant firm or for the weak firm.

Appendix A

Proof of Lemma 1

Solving the maximization problem in (25) gives equilibrium prices

$$\begin{aligned} p_{A1} &= \frac{1}{2}W_1,\ p_{A2} = \frac{1}{2}\delta W_2, \\ p_{B1} &= \frac{1}{2}\delta W_1,\ p_{B2} = \frac{1}{2}W_2, \end{aligned}$$

[81] We have considered how compatibility affects the marginal incentives to invest of both platforms. We acknowledge that the study of the effect of compatibility on innovation demands analysing its impact on equilibrium investment levels. We consider this as a future extension.

so that the corresponding indirect utilities and the profits from applications are given by

$$\begin{aligned}
w_A(\delta) &= \frac{1}{4(1-\sigma^2)b}\left(W_1^2 + (\delta W_2)^2 - 2\sigma\delta W_1 W_2\right)\left(1-\frac{1}{b}\right), && (31)\\
w_B(\delta) &= \frac{1}{4(1-\sigma^2)b}\left(W_2^2 + (\delta W_1)^2 - 2\sigma\delta W_2 W_1\right)\left(1-\frac{1}{b}\right), && (32)\\
\pi_A(\delta) &= \frac{1}{4(1-\sigma^2)b}\left(W_1^2 + (\delta W_2)^2 - 2\sigma\delta W_1 W_2\right), \text{ and} && (33)\\
\pi_B(\delta) &= \frac{1}{4(1-\sigma^2)b}\left(W_2^2 + (\delta W_1)^2 - 2\sigma\delta W_2 W_1\right). && (34)
\end{aligned}$$

By inspecting expressions above it trivially follows that

$$\begin{aligned}
\frac{\partial w_A(\delta)}{\partial\delta} &> 0, \quad \frac{\partial \pi_A(\delta)}{\partial\delta} > 0 \Leftrightarrow \delta W_2 - \sigma W_1 > 0,\\
\frac{\partial w_B(\delta)}{\partial\delta} &> 0, \quad \frac{\partial \pi_B(\delta)}{\partial\delta} > 0 \Leftrightarrow \delta W_1 - \sigma W_2 > 0,
\end{aligned}$$

inequalities that always hold if applications are complementary or independent, so that i) trivially follows. In contrast, if applications are substitutes ($0 < \sigma < 1$) the sign of $\frac{\partial w_A(\delta)}{\partial\delta}$ equals the sign of $\frac{\partial \pi_A(\delta)}{\partial\delta}$ and it depends on the relationship between σ and the ratio $\frac{\delta W_2}{W_1}$ so that ii) follows. Similar arguments apply to the other platform.

Proof of Lemma 2

By looking for the indifferent consumer as is usual in the Hotelling model, the market share of platform A is given by the following expression,

$$\theta_A = \frac{1}{2} + \frac{w(\delta) - (s_A - s_B)}{2t},$$

and the market share of B by $\theta_B = 1 - \theta_A$. Note that Assumption 1 is sufficient to ensure that the market share equation above is well behaved.

Each platform will set a price s_l to maximize

$$\Pi_l = (s_l + \pi_l(\delta))\,\theta_l.$$

By solving maximization problem above we derive the platform's reaction functions which are given by

$$\begin{aligned}
s_A(s_B) &= \frac{1}{2}\left(t + w(\delta) + s_B - \pi_A(\delta)\right)\\
s_B(s_A) &= \frac{1}{2}\left(t - w(\delta) + s_A - \pi_B(\delta)\right).
\end{aligned}$$

Since they are increasing in the choice of the rival, prices are strategic complements. From reaction functions above it is straightforward to derive the equilibrium platform prices, which are given by

$$\begin{aligned}
s_A(\delta) &= t + \frac{1}{3}w(\delta) - \frac{2}{3}\pi_A(\delta) - \frac{1}{3}\pi_B(\delta) && (35)\\
s_B(\delta) &= t - \frac{1}{3}w(\delta) - \frac{2}{3}\pi_B(\delta) - \frac{1}{3}\pi_A(\delta).
\end{aligned}$$

If applications are either complements, independent or substitutes with $\sigma < \bar{\sigma}_2$, we observe that $\frac{\partial s_A(\delta)}{\partial\delta} < 0$. Note that for platform B, an increase in compatibility has a positive effect on $s_B(\delta)$ via a reduction in $w(\delta)$. However, this is compensated by the increase in $\pi_A(\delta)$ and $\pi_B(\delta)$, so that it also holds for platform B that $\frac{\partial s_B(\delta)}{\partial \delta} < 0$.

If applications are substitutes and $\sigma > \bar{\sigma}_1$, then $\frac{\partial s_i(\delta)}{\partial \delta} > 0$ holds for both platforms. Finally, if $\bar{\sigma}_2 < \sigma < \bar{\sigma}_1$ then $\frac{\partial s_A(\delta)}{\partial \delta} > 0$ and $\frac{\partial s_B(\delta)}{\partial \delta} < 0$ and the statements in the lemma follow.

Proof of Lemma 3

Platform profits are given by

$$\Pi_l(\delta) = (s_l(\delta) + \pi_l(\delta))\,\theta_l(\delta),$$

where, using (35), (33) and (34), platform's revenues per consumer are given by

$$\begin{aligned} s_A(\delta) + \pi_A(\delta) &= t + \frac{1}{3}(w(\delta) + \pi(\delta)) \\ s_B(\delta) + \pi_B(\delta) &= t - \frac{1}{3}(w(\delta) + \pi(\delta)), \end{aligned} \tag{36}$$

and equilibrium market shares by

$$\begin{aligned} \theta_A(\delta) &= \frac{1}{2} + \frac{1}{6t}(w(\delta) + \pi(\delta)) \\ \theta_B(\delta) &= \frac{1}{2} - \frac{1}{6t}(w(\delta) + \pi(\delta)). \end{aligned}$$

If $W_1 = W_2$ occurs, then it happens that $w(\delta) = \pi(\delta) = 0$ and point *i*) in the lemma follows. To prove *ii*) note that because of lemma 1 we have that $s_A(\delta) + \pi_A(\delta)$ and $\theta_A(\delta)$ are decreasing in δ, so that $\Pi_A(\delta)$ is strictly decreasing in δ. In contrast, $s_B(\delta) + \pi_B(\delta)$ and $\theta_B(\delta)$ are all increasing in δ functions so that $\Pi_B(\delta)$ is strictly increasing in δ.

Proof of Proposition 1

Consider first the analysis for the consumers' surplus. For expositional simplicity, let us define A and B as

$$\begin{aligned} A &= w_A(\delta) + V - s_A(\delta) = \frac{2}{3}(w_A(\delta) + \pi_A(\delta)) + \frac{1}{3}(w_B(\delta) + \pi_B(\delta)) + V, \\ B &= w_B(\delta) + V - s_B(\delta) = \frac{2}{3}(w_B(\delta) + \pi_B(\delta)) + \frac{1}{3}(w_A(\delta) + \pi_A(\delta)) + V. \end{aligned}$$

To study the effect of compatibility on total consumers surplus we need to compute

$$\frac{\partial\left(\theta_A(\delta)\,A + (1-\theta_A(\delta))\,B - \frac{1}{2}t\left(\theta_A(\delta)^2 + (1-\theta_A(\delta))^2\right)\right)}{\partial \delta}$$

that is

$$\theta_A\frac{\partial A}{\partial \delta} + (1-\theta_A)\frac{\partial B}{\partial \delta} + (A - t\theta_A)\frac{\partial \theta_A}{\partial \delta} + (B - t(1-\theta_A))\frac{\partial(1-\theta_A)}{\partial \delta}.$$

By definition of the indifferent consumer in equilibrium we have that

$$(A - t\theta_A)\frac{\partial \theta_A}{\partial \delta} + (B - t(1-\theta_A))\frac{\partial(1-\theta_A)}{\partial \delta} = 0,$$

Consequently, the effect of compatibility on total consumers surplus does only depend on the sign of

$$F(\delta,\sigma) = \theta_A\frac{\partial A}{\partial \delta} + (1-\theta_A)\frac{\partial B}{\partial \delta}. \tag{37}$$

We next show $F(\delta,\sigma)$ that is decreasing in σ. First, note that from the expressions for w_A, w_B, π_A, and π_B given respectively in (31), (32), (33) and (34) it follows that their cross partial derivatives $\frac{\partial^2}{\partial\delta\partial\sigma}$ are negative. Consequently, $\frac{\partial^2 A}{\partial\delta\partial\sigma}$ is negative iff

$$\frac{2\sigma}{(1-\sigma^2)}(\bar{\sigma}_2-\sigma)<1 \Leftrightarrow \bar{\sigma}_2<\frac{1+\sigma^2}{2\sigma},$$

and $\frac{\partial^2 B}{\partial\delta\partial\sigma}$ is negative iff

$$\frac{2\sigma}{(1-\sigma^2)}(\bar{\sigma}_1-\sigma)<1 \Leftrightarrow \bar{\sigma}_1<\frac{1+\sigma^2}{2\sigma},$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are defined in lemma 1 and they both belong to the interval $(0,1)$. Since $\frac{1+\sigma^2}{2\sigma}>1$ we can conclude that both $\frac{\partial^2 A}{\partial\delta\partial\sigma}$ and $\frac{\partial^2 B}{\partial\delta\partial\sigma}$ are negative. Since $\frac{\partial\theta_A}{\partial\sigma}>0$, $\frac{\partial(A-B)}{\partial\delta}<0$ and $\frac{\partial(A-B)}{\partial\delta\partial\sigma}<0$ it follows straightforwardly that $F(\delta,\sigma)$ stated in (37) is decreasing in σ.

By Lemma 1 we have that for all $\sigma>\bar{\sigma}_1$ it holds that $\frac{\partial TS(\delta)}{\partial\delta}<0$ and for all $\sigma<\bar{\sigma}_2$ it holds that $\frac{\partial TS(\delta)}{\partial\delta}>0$. Appealing to the mean value theorem there is a critical degree of substitutability among applications $\sigma^*\in(\bar{\sigma}_2,\bar{\sigma}_1)$ that makes (37) zero. Furthermore since $F(\delta,\sigma)$ is decreasing in σ, this value is unique which shows our claim.

Consider now total welfare. We first show $\frac{\partial^2 TW}{\partial\delta\partial\sigma}<0$. To do so we only need to show that the cross derivative of the total profits of the industry with respect to δ and σ is also negative, given that we already shown that this is the case for consumers' surplus. Total industry profits are given by

$$\Pi_A+\Pi_B=\left(t+\frac{1}{3}(w(\delta)+\pi(\delta))\right)\theta_A+\left(t-\frac{1}{3}(w(\delta)+\pi(\delta))\right)(1-\theta_A).$$

Since $\theta_A=\left(\frac{1}{2}+\frac{1}{6t}(w(\delta)+\pi(\delta))\right)$ profits can be rewritten as

$$\Pi_A+\Pi_B=t+\frac{1}{9t}(w(\delta)+\pi(\delta))^2$$

so that

$$\frac{\partial(\Pi_A+\Pi_B)}{\partial\delta}=\frac{2}{9t}(w(\delta)+\pi(\delta))\frac{\partial(w(\delta)+\pi(\delta))}{\partial\delta}<0,$$

and

$$\begin{aligned}\frac{\partial^2(\Pi_A+\Pi_B)}{\partial\delta\partial\sigma} &= \frac{2}{9t}\frac{\partial(w(\delta)+\pi(\delta))}{\partial\sigma}\frac{\partial(w(\delta)+\pi(\delta))}{\partial\delta}+\\ &\quad \frac{2}{9t}(w(\delta)+\pi(\delta))\frac{\partial^2(w(\delta)+\pi(\delta))}{\partial\delta\partial\sigma}\end{aligned}$$

Since $\frac{\partial(w(\delta)+\pi(\delta))}{\partial\sigma}>0$, $\frac{\partial(w(\delta)+\pi(\delta))}{\partial\delta}<0$ and $\frac{\partial^2(w(\delta)+\pi(\delta))}{\partial\delta\partial\sigma}<0$ it follows that $\frac{\partial(\Pi_A(\delta)+\Pi_B(\delta))}{\partial\delta\partial\sigma}<0$ holds.

Now, to set the effect of a change in δ on welfare we need to sign

$$(38)\quad \theta_A\frac{\partial A'}{\partial\delta}+(1-\theta_A)\frac{\partial B'}{\partial\delta}+\left[(A'-t\theta_A)\frac{\partial\theta_A}{\partial\delta}+(B'-t(1-\theta_A))\frac{\partial(1-\theta_A)}{\partial\delta}\right],$$

where $A'=w_A(\delta)+\pi_A(\delta)+V$ and $B'=w_B(\delta)+\pi_B(\delta)+V$. Since $A'>A$ and $B'<B$ we have that $(A'-t\theta_A)>(B'-t(1-\theta_A))$. Moreover, since $\frac{\partial\theta_A}{\partial\delta}=-\frac{\partial(1-\theta_A)}{\partial\delta}$, we can conclude that the term in brackets in (38) is always negative and it is decreasing in the level of asymmetry measured by W_1-W_2.

Appealing again to Lemma 1 it follows that and if $\sigma > \bar{\sigma}_1$ then $\frac{\partial TW(\delta)}{\partial \delta} < 0$. If $\sigma < \bar{\sigma}_2$ the expression $\frac{\partial TW(\delta)}{\partial \delta}$ can be positive or negative. However, at $\sigma = 0$ we have that $\frac{\partial TW(\delta)}{\partial \delta} > 0$. Consequently, there is $\sigma^{**} \in (0, \bar{\sigma}_1)$ that makes (38) zero.

Given that,

$$\theta_A \left(\frac{\partial A}{\partial \delta} - \frac{\partial A'}{\partial \delta} \right) + (1 - \theta_A) \left(\frac{\partial B}{\partial \delta} - \frac{\partial B'}{\partial \delta} \right) < 0$$

it follows that $\sigma^{**} < \sigma^*$. Moreover, by Lemma 1, we also have that $\frac{\partial A}{\partial \delta}(\sigma^*) < \frac{\partial B}{\partial \delta}(\sigma^*)$ and $\frac{\partial A'}{\partial \delta}(\sigma^{**}) < \frac{\partial B'}{\partial \delta}(\sigma^{**})$. Thus, the critical values are decreasing in

$$\theta_A = \frac{1}{2} + \frac{1}{6t}\left(w(\delta) + \pi(\delta)\right),$$

and consequently they are decreasing in $W_1 - W_2$ and increasing in the level of δ as the proposition states.

Proof of Proposition 2

With inter-network compatibility, equilibrium prices, market share and profits are given by

$$\begin{aligned}
s_A(\delta,\beta) &= t + \frac{1}{3}w(\delta) - \frac{2}{3}\pi_A(\delta) - \frac{1}{3}\pi_B(\delta) - \eta(1-\beta), \\
s_B(\delta,\beta) &= t - \frac{1}{3}w(\delta) - \frac{2}{3}\pi_B(\delta) - \frac{1}{3}\pi_A(\delta) - \eta(1-\beta), \\
\theta_A(\delta,\beta) &= \frac{1}{2} + \frac{1}{6(t-\eta(1-\beta))}\left(w(\delta)+\pi(\delta)\right) \\
\theta_B(\delta,\beta) &= \frac{1}{2} - \frac{1}{6(t-\eta(1-\beta))}\left(w(\delta)+\pi(\delta)\right) \\
\Pi_A(\delta,\beta) &= \frac{1}{2(t-\eta(1-\beta))}\left(t + \frac{1}{3}\left(w(\delta)+\pi(\delta)\right) - \eta(1-\beta)\right)^2, \\
\Pi_B(\delta,\beta) &= \frac{1}{2(t-\eta(1-\beta))}\left(t - \frac{1}{3}\left(w(\delta)+\pi(\delta)\right) - \eta(1-\beta)\right)^2.
\end{aligned}$$

To prove statement i) note that $\frac{\partial \Pi_A}{\partial \beta}$ and $\frac{\partial \Pi_B}{\partial \beta}$ are positive if

$$t > \eta(1-\beta) + \frac{1}{3}\left(w(\delta)+\pi(\delta)\right),$$

a condition which is implied by the analogous condition to Assumption 1, namely *Assumption 1': * $t > \eta + \frac{w(\delta)+\pi(\delta)}{3}$.

We next study its impact on consumers' surplus and on welfare. To do so, note that total surplus $TS(\delta,\beta)$ is given by

$$V + \int_0^{\theta_A(\delta,\beta)} \left(w_A(\delta) + V - tx + \eta\left[\theta_A(\delta,\beta) + \beta\left(1-\theta_A(\delta,\beta)\right)\right] - s_A(\delta,\beta)\right)dx +$$
$$\int_{\theta_A(\delta,\beta)}^1 \left(w_B(\delta) + V - t(1-x) + \eta\left[\left(1-\theta_A(\delta,\beta)\right) + \beta\theta_A(\delta,\beta)\right] - s_B(\delta,\beta)\right)dx.$$

To study how changes in β affect $TS(\delta,\beta)$ note that

$$\begin{aligned}\frac{\partial TS(\delta,\beta)}{\partial\beta} &= \theta_A\frac{\partial A(\beta)}{\partial\beta}+(1-\theta_A)\frac{\partial B(\beta)}{\partial\beta}+\\ &\qquad (A(\beta)-t\theta_A)\frac{\partial\theta_A}{\partial\delta}+(B(\beta)-t(1-\theta_A))\frac{\partial(1-\theta_A)}{\partial\delta},\end{aligned}$$

where

$$\begin{aligned}A(\beta) &= (w_A(\delta)+V+\eta[\theta_A+\beta(1-\theta_A)]-s_A(\delta,\beta)),\ \text{and}\\ B(\beta) &= (w_B(\delta)+V+\eta[(1-\theta_A)+\beta\theta_A]-s_B(\delta,\beta))\end{aligned}$$

By definition of indifferent user we know that

$$(A(\beta)-t\theta_A)\frac{\partial\theta_A}{\partial\delta}+(B(\beta)-t(1-\theta_A))\frac{\partial(1-\theta_A)}{\partial\delta}=0.$$

Therefore to study how β affects $TS(\delta,\beta)$ we only need to analyze the sign of

$$\theta_A\frac{\partial(\eta[\theta_A+\beta(1-\theta_A)]-s_A)}{\partial\beta}+(1-\theta_A)\frac{\partial(\eta[(1-\theta_A)+\beta\theta_A]-s_B)}{\partial\beta}. \tag{39}$$

Given that $\frac{\partial s_A}{\partial\beta}=\frac{\partial s_B}{\partial\beta}=\eta$, it follows

$$\eta\left(\theta_A\frac{\partial[\theta_A+\beta(1-\theta_A)]}{\partial\beta}+(1-\theta_A)\frac{\partial[(1-\theta_A)+\beta\theta_A]}{\partial\beta}\right)-\eta.$$

Taking into account that $\frac{\partial(1-\theta_A)}{\partial\beta}=-\frac{\partial\theta_A}{\partial\beta}$, computing the derivatives and rearranging terms lead to

$$\frac{\partial TS(\delta,\beta)}{\partial\beta}=\eta\left(2\theta_A(1-\theta_A)+(2\theta_A-1)\frac{\partial\theta_A}{\partial\beta}(1-\beta)\right)-\eta.$$

Since $\frac{\partial\theta_A}{\partial\beta}<0$ and $2\theta_A(1-\theta_A)<1$ it follows that the impact of β on $TS(\delta,\beta)$ is negative as claimed. Furthermore, he first term is decreasing in θ_A so that the negative impact of β is higher, the higher the asymmetries as stated in ii).

We can obtain the effect on total welfare $TW(\delta,\beta)$ by signing

$$\frac{\partial TW(\delta,\beta)}{\partial\beta}=\frac{\partial TS(\delta,\beta)}{\partial\beta}+\frac{\partial\Pi_A(\delta,\beta)}{\partial\beta}+\frac{\partial\Pi_B(\delta,\beta)}{\partial\beta}.$$

Straightforward computations yield that

$$\frac{\partial\Pi_A(\delta,\beta)}{\partial\beta}+\frac{\partial\Pi_B(\delta,\beta)}{\partial\beta}=\eta\left(1-\frac{(w(\delta)+\pi(\delta))^2}{(3(t-\eta(1-\beta)))^2}\right)<\eta,$$

Note that $\frac{\partial\Pi_A(\delta,\beta)}{\partial\beta}+\frac{\partial\Pi_B(\delta,\beta)}{\partial\beta}$ is decreasing in the size of the asymmetries. With no asymmetries the total gain of the industry would be η. However, given the existence of asymmetries, this gain is lower. Moreover, the higher the asymmetries in applications, the lower the gain of the industry due to β. Thus, given that both components of total welfare are decreasing in the asymmetries, point iii) follows.

Proof of Proposition 3

To prove the first statement note that

$$\frac{\partial\left(\frac{\partial\Pi_A(\delta)}{\partial V_A}\right)}{\partial\delta}=\frac{1}{9(t-\eta)}\frac{\partial(\pi(\delta)+\upsilon(\delta))}{\partial\delta}<0.$$

If we consider investment in the fully compatible application, straightforward computation shows that

$$\frac{\partial \Pi_A(\delta)}{\partial W_1} = \left(\frac{\left(t + \frac{1}{3}\left(\Delta + w(\delta) + \pi(\delta)\right) - \eta\right)}{(t-\eta)} \right) \left(\frac{\left(2 - \frac{1}{b}\right)\left(1-\delta^2\right) W_1}{6\left(1-\sigma^2\right) b} \right).$$

Since $\frac{\partial \Pi_A(\delta)}{\partial W_1}$ is the product of two decreasing and positive functions, it trivially follows that

$$\frac{\partial \left(\frac{\partial (\Pi_A(\delta))}{\partial W_1} \right)}{\partial \delta} < 0,$$

and the first statement follows.

To prove the second statement we note that

$$\frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial V_B} \right)}{\partial \delta} = -\frac{1}{9(t-\eta)} \frac{\partial\left(\pi(\delta) + \upsilon(\delta)\right)}{\partial \delta} > 0.$$

To analyze the marginal incentive of the weak firm to invest in the fully compatible application we observe that

$$\frac{\partial \Pi_B(\delta)}{\partial W_2} = \frac{\left(t - \frac{1}{3}\left(\Delta + w(\delta) + \pi(\delta)\right) - \eta\right)}{(t-\eta)} \left(\frac{\left(2 - \frac{1}{b}\right)\left(1-\delta^2\right) W_2}{6\left(1-\sigma^2\right) b} \right),$$

so that, differentiating expression above and rearranging gives

$$\frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial W_2} \right)}{\partial \delta} = \frac{2\left(1-\sigma^2\right)(2b-1)\,\delta W_2}{9(t-\eta)(\sigma+1)^2(1-\sigma)^2 b^2} \left(w(\delta) + \pi(\delta) - \frac{1}{2}\left(3(t-\eta) - \Delta\right) \right).$$

Note that

$$\frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial W_2} \right)}{\partial \delta} \lesseqgtr 0$$

as long as

$$w(\delta) + \pi(\delta) - \frac{1}{2}\left(3(t-\eta) - \Delta\right) \lesseqgtr 0,$$

or equivalently if

$$\frac{1}{3}\left(2\left(w(\delta) + \pi(\delta)\right) + \Delta\right) + \eta \lesseqgtr t.$$

Then, $\frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial W_2} \right)}{\partial \delta} > 0$ and the weak firm is type I, if the asymmetries are very strong, i.e., if either Δ or $w(\delta) + \pi(\delta)$ are large enough. Otherwise this firm is type II.

Finally, note that $\frac{\partial \left(\frac{\partial \Pi_A(\delta)}{\partial V_A} \right)}{\partial \delta} = -\frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial V_B} \right)}{\partial \delta}$ and that $\frac{\partial \left(\frac{\partial (\Pi_A(\delta))}{\partial W_1} \right)}{\partial \delta} + \frac{\partial \left(\frac{\partial \Pi_B(\delta)}{\partial W_2} \right)}{\partial \delta} =$

$$\frac{G}{3(t-\eta)} \frac{\partial\left(w(\delta) + \pi(\delta)\right)}{\partial \delta} \left(1-\delta^2\right)(W_1 - W_2)$$
$$-\frac{2\delta G}{(t-\eta)} \left(t(W_1 + W_2) + \left(\frac{1}{3}\left(\Delta + w(\delta) + \pi(\delta)\right) - \eta \right)(W_1 - W_2) \right)$$

where $G = \frac{\left(2 - \frac{1}{b}\right)}{6(1-\sigma^2)b}$. Since $\frac{\partial(w(\delta)+\pi(\delta))}{\partial \delta} < 0$, and $\frac{1}{3}\left(\Delta + w(\delta) + \pi(\delta)\right) - \eta < t - \eta < t$ by Assumption 1, the last statement of the proposition follows.

Proof of Proposition 4

The changes in the marginal incentives to invest in the stand-alone value of the dominant platform are given by

$$\frac{\partial\left(\frac{\partial(\Pi_A(\delta,\beta))}{\partial V_A}\right)}{\partial\beta}=-\frac{\eta}{9\left(t-\eta\left(1-\beta\right)\right)^2}\left(\pi\left(\delta\right)+w\left(\delta\right)+\Delta\right)<0,$$

and the changes in the marginal incentives to invest in its fully-compatible application are given by

$$\frac{\partial\left(\frac{\partial(\Pi_A(\delta,\beta))}{\partial W_1}\right)}{\partial\beta}=-\frac{2\eta W_1}{9\left(t-\eta\left(1-\beta\right)\right)^2}H\left(H\left(W_1^2-W_2^2\right)+\Delta+\pi\left(\delta\right)\right)<0,$$

where $H=\frac{1}{4(1-\sigma^2)b}\left(1-\frac{1}{b}\right)\left(1-\delta^2\right)>0$, so that i) follows. Regarding the weak firm, the corresponding expressions are given by

$$\frac{\partial\left(\frac{\partial(\Pi_B(\delta,\beta))}{\partial V_B}\right)}{\partial\beta}=\frac{\eta}{9\left(t-\eta\left(1-\beta\right)\right)^2}\left(\pi\left(\delta\right)+w\left(\delta\right)+\Delta\right)>0,$$

and

$$\frac{\partial\left(\frac{\partial(\Pi_B(\delta,\beta))}{\partial W_2}\right)}{\partial\beta}=\frac{2\eta W_2}{9\left(t-\eta\left(1-\beta\right)\right)^2}H\left(H\left(W_1^2-W_2^2\right)+\Delta+\pi\left(\delta\right)\right)>0,$$

consequently ii) follows.

Finally, since $\frac{\partial\left(\frac{\partial(\Pi_A(\delta,\beta))}{\partial V_A}\right)}{\partial\beta}=-\frac{\partial\left(\frac{\partial(\Pi_B(\delta,\beta))}{\partial V_B}\right)}{\partial\beta}$ and the total incentive $\frac{\partial\left(\frac{\partial(\Pi_A(\delta,\beta))}{\partial W_1}\right)}{\partial\beta}+\frac{\partial\left(\frac{\partial(\Pi_B(\delta,\beta))}{\partial W_2}\right)}{\partial\beta}$ becomes

$$-\left(W_1-W_2\right)\frac{2\eta H\left(H\left(W_1^2-W_2^2\right)+\Delta+\pi\left(\delta\right)\right)}{9\left(t-\eta\left(1-\beta\right)\right)^2}<0,$$

part iii) follows.

Appendix B

We first introduce an assumption to ensure that system 2 is active, namely,
Assumption 2: $\left(\left(2-\sigma^2\right)\alpha_2-\sigma\alpha_1\right)>0$ for all δ.[82]
From the utility (28) in the text, the demand function for system i becomes

$$q_i=\frac{(\alpha_i-\sigma\alpha_j)-p_i+\sigma p_j}{b\left(1-\sigma^2\right)}.$$

Assuming marginal costs are null firm i obtains profits of

$$\Pi_i=p_i\left(\frac{(\alpha_i-\sigma\alpha_j)-p_i+\sigma p_j}{b\left(1-\sigma^2\right)}\right).$$

Next lemma formalizes the discussion in the main text.

Lemma B1

If the systems are complementary or independent the dominant platform in the market of applications benefits from an increase in the degree of compatibility. If the

[82] The assumption is trivially satisfied if systems are independent or complementary, i.e., $\sigma\leq 0$.

systems are substitutes, it benefits if the reducing differentiation effect induced by compatibility is not too strong. The weak platform always benefits from compatibility an increase in the degree of compatibility.

Proof: From the expressions for the equilibrium prices given in (30), equilibrium quantities and profits are respectively given by

$$\begin{aligned} q_1 &= \frac{\left(2-\sigma^2\right)\alpha_1 - \sigma\alpha_2}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)} \\ q_2 &= \frac{\left(2-\sigma^2\right)\alpha_2 - \sigma\alpha_1}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)} \\ \Pi_1 &= \frac{1}{b\left(1-\sigma^2\right)}\left(\frac{\left(2-\sigma^2\right)\alpha_1 - \sigma\alpha_2}{4-\sigma^2}\right)^2 \\ \Pi_2 &= \frac{1}{b\left(1-\sigma^2\right)}\left(\frac{\left(2-\sigma^2\right)\alpha_2 - \sigma\alpha_1}{4-\sigma^2}\right)^2. \end{aligned}$$

We now show that if $\sigma \leq 0$ then $\frac{\partial \Pi_i}{\partial \delta} > 0$ for all i, whereas if $\sigma > 0$ then $\frac{\partial \Pi_2}{\partial \delta} > 0$, but $\frac{\partial \Pi_1}{\partial \delta} > 0$ if and only if $\left(2-\sigma^2\right)W_2 - \sigma W_1 > 0$. To see this note that

$$\begin{aligned} \frac{\partial \Pi_1}{\partial \delta} &= \frac{2}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)^2}\left(\left(2-\sigma^2\right)\alpha_1 - \sigma\alpha_2\right)\left(\left(2-\sigma^2\right)\frac{\partial \alpha_1}{\partial \delta} - \sigma\frac{\partial \alpha_2}{\partial \delta}\right) \\ &= \frac{2}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)^2}\left(\left(2-\sigma^2\right)\alpha_1 - \sigma\alpha_2\right)\left(\left(2-\sigma^2\right)W_2 - \sigma W_1\right), \text{ and} \\ \frac{\partial \Pi_2}{\partial \delta} &= \frac{2}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)^2}\left(\left(2-\sigma^2\right)\alpha_2 - \sigma\alpha_1\right)\left(\left(2-\sigma^2\right)W_1 - \sigma W_2\right). \end{aligned}$$

Since $\left(\left(2-\sigma^2\right)\alpha_2 - \sigma\alpha_1\right) > 0$ because of Assumption 2 and $\left(2-\sigma^2\right)\alpha_1 - \sigma\alpha_2 > 0$ (recall that $\alpha_1 > \alpha_2$ and $2-\sigma^2 > \sigma$) then $\frac{\partial \Pi_2}{\partial \delta} > 0$, and $sign\left(\frac{\partial \Pi_1}{\partial(\delta)}\right) = sign\left(\left(2-\sigma^2\right)W_2 - \sigma W_1\right)$. Consequently, the dominant platform in the market of applications benefits with compatibility iff $\left(2-\sigma^2\right)W_2 - \sigma W_1 > 0$, an inequality that always holds if $\sigma \leq 0$.

We next study how changes in the degree of compatibility as measured by δ affect consumers surplus and total welfare.

Proposition B1

If $\sigma \leq 0$ both consumer surplus and total welfare are increasing in δ, otherwise they may be increasing or decreasing.

Proof: Consumer surplus and total welfare are is given by

$$\begin{aligned} CS &= \left(\alpha_1 - p_1\right)q_1 + \left(\alpha_2 - p_2\right)q_2 - \frac{1}{2}b\left(q_1^2 + 2\sigma q_1 q_2 + q_2^2\right), \text{ and} \\ W &: \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}b\left(q_1^2 + 2\sigma q_1 q_2 + q_2^2\right). \end{aligned}$$

Substituting prices and quantities for their equilibrium prices expressions above can be rewritten as

$$\begin{aligned} CS &: B\left(\left(4-3\sigma^2\right)\left(\alpha_1^2+\alpha_2^2\right) - 2\sigma^3\alpha_1\alpha_2\right), \text{ and} \\ W &= B\left(\left(12+2\sigma^4-9\sigma^2\right)\left(\alpha_1^2+\alpha_2^2\right) - 2\sigma\left(8-3\sigma^2\right)\alpha_1\alpha_2\right), \end{aligned}$$

where $B = \frac{1}{2b(1-\sigma^2)(4-\sigma^2)^2} > 0$. Thus,

$$\frac{\partial CS}{\partial \delta} = B\left(\left(4-3\sigma^2\right) \frac{\partial\left(\alpha_1^2+\alpha_2^2\right)}{\partial \delta} - 2\sigma^3 \frac{\partial\left(\alpha_1\alpha_2\right)}{\partial \delta} \right),$$

$$\frac{\partial W}{\partial \delta} = B\left(\left(12+2\sigma^4-9\sigma^2\right) \frac{\partial\left(\alpha_1^2+\alpha_2^2\right)}{\partial \delta} - 2\sigma\left(8-3\sigma^2\right) \frac{\partial\left(\alpha_1\alpha_2\right)}{\partial \delta} \right)$$

where

$$\frac{\partial\left(\alpha_1^2+\alpha_2^2\right)}{\partial \delta} = 2\delta\left(W_1^2+W_2^2\right) + 2\left(2W_1W_2 + V_LW_1 + V_HW_2\right) > 0$$

and

$$\frac{\partial\left(\alpha_1\alpha_2\right)}{\partial \delta} = \left(W_1^2+W_2^2\right) + 2\delta W_1W_2 + V_HW_1 + V_LW_2 > 0.$$

If $\sigma \leq 0$ it follows that $\frac{\partial CS}{\partial \delta} > 0$ and $\frac{\partial W}{\partial \delta} > 0$. If $0 < \sigma < 1$, consumer surplus and welfare can decrease or increase as the following example shows. Let $W_1 = 24$, $W_2 = 4$, $V_H = 8$, $V_L = 2$ and $\delta = 0.25$. Figure A1 depicts $\frac{\partial CS}{\partial \delta}$ (solid line) and $\frac{\partial W}{\partial \delta}$ (dotted line) as a function of σ.

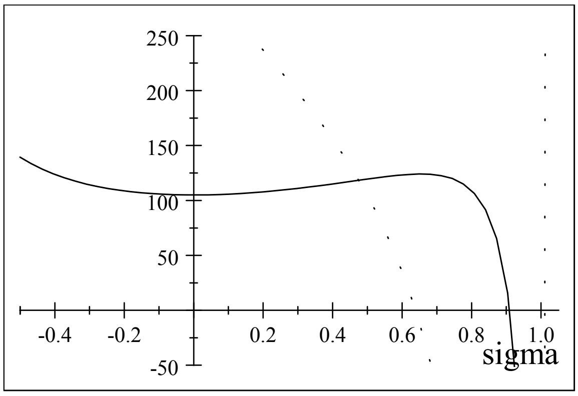

Figure A1

Note that there are values of σ for which both $\frac{\partial CS}{\partial \delta}$ and $\frac{\partial W}{\partial \delta}$ can be either negative or positive, which shows our claim. The picture suggests that high values of σ make $\frac{\partial CS}{\partial \delta}$ and $\frac{\partial W}{\partial \delta}$ negative.

We next analyze the incentives to invest

Proposition B2

(i) There exist $\bar{\sigma}, \widetilde{\sigma} \in (0,1)$, $\bar{\sigma} < \widetilde{\sigma}$, such that

if $\sigma < \bar{\sigma}$ the dominant firm is type I

if $\sigma \in (\bar{\sigma}, \widetilde{\sigma})$ the dominant firm is type II

if $\sigma > \widetilde{\sigma}$ the dominant firm is type IV

(ii) The weak firm is either type I or type II. It is more likely type I if asymmetries in either stand-alone values or in application values are large.

Proof.

Consider first the dominant system. The effect of δ on the marginal profit from a higher stand-alone value is given by

$$\frac{\partial\left(\frac{\partial\Pi_1(\delta)}{\partial V_H}\right)}{\partial\delta}=\frac{2\left(2-\sigma^2\right)}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)^2}\left(\left(2-\sigma^2\right)W_2-\sigma W_1\right),$$

which is increasing if the inequality

$$\left(2-\sigma^2\right)W_2-\sigma W_1>0$$

holds. Let $w=W_1/W_2$. Then $\frac{\partial\left(\frac{\partial\Pi_1(\delta)}{\partial V_H}\right)}{\partial\delta}>0$ if $\sigma<\widetilde{\sigma}=\frac{1}{2}\left(\sqrt{w^2+8}-w\right)$. Note that $\widetilde{\sigma}<1$ as it is strictly decreasing in w.

Regarding its marginal incentive to invest in the application, we have

$$\frac{\partial\Pi_1(\delta)}{\partial W_1}=4B\left(2-\sigma\left(\delta+\sigma\right)\right)\left(\left(2-\sigma^2\right)\left(V_H+W_1+\delta W_2\right)-\sigma\left(V_L+W_2+\delta W_1\right)\right)$$

and

$$\frac{\partial\left(\frac{\partial\Pi_1(\delta)}{\partial W_1}\right)}{\partial\delta}=4B\left(\begin{array}{c}\left(4\left(1-\sigma\delta\right)-\sigma^2\left(3-\sigma\left(\sigma+2\delta\right)\right)\right)W_2-2\sigma\left(2-\sigma\left(\delta+\sigma\right)\right)W_1\\-\sigma\left(\left(2-\sigma^2\right)V_H-\sigma V_L\right)\end{array}\right)$$

Depending on the parameters, this expression can be negative or positive. The first term evaluated at $\widetilde{\sigma}$ is $-\frac{1}{2}\left(w^2-1\right)\left(\frac{W_1^2}{W_2}+4W_2-W_1\sqrt{w^2+8}\right)$. Since we only care about its sign we divide it by W_2 and we find that it is negative as it equals $\left(-\frac{\left(w^2-1\right)}{2}\right)\left(w^2+4-w\sqrt{w^2+8}\right)\leq 0$ for all $w\geq 1$. It is positive at $\sigma=0$, while negative and increasing at $\sigma=1$. Consequently, there is $\sigma^*<\widetilde{\sigma}$ such that the first term equals zero at $\sigma=\sigma^*$. Since the second term is always negative we can conclude that $\frac{\partial\left(\frac{\partial\Pi_1(\delta)}{\partial W_1}\right)}{\partial\delta}<0$ for all $\sigma>\sigma^*$. Furthermore since at $\sigma=0$ the second term equals zero, there is $0<\bar{\sigma}<\sigma^*$ such that $\frac{\partial\left(\frac{\partial\Pi_1(\delta)}{\partial W_1}\right)}{\partial\delta}\geq 0$ for any $\sigma<\bar{\sigma}$. The dominant firm type does hence depend on σ. If $\sigma<\bar{\sigma}$ it is type I. If $\sigma\in(\bar{\sigma},\widetilde{\sigma})$ it is type II. And if $\sigma>\widetilde{\sigma}$ it is type IV, then (i) follows.

Consider now the weak system. The effect of δ on the marginal profit from a higher stand-alone value is given by

$$\frac{\partial\left(\frac{\partial\Pi_2(\delta)}{\partial V_L}\right)}{\partial\delta}=\frac{2\left(2-\sigma^2\right)}{b\left(1-\sigma^2\right)\left(4-\sigma^2\right)^2}\left(\left(2-\sigma^2\right)W_1-\sigma W_2\right)$$

which is always positive. When analyzing the incentives to invest in the application value we have that

$$\frac{\partial\Pi_2(\delta)}{\partial W_2}=4B\left(2-\sigma\left(\delta+\sigma\right)\right)\left(\begin{array}{c}2\left(V_L+W_2+\delta W_1\right)-\\\sigma\left(W_1+V_H+\delta W_2\right)-\sigma^2\left(W_2+V_L+\delta W_1\right)\end{array}\right)$$

and

$$\frac{\partial\left(\frac{\partial\Pi_2(\delta)}{\partial W_2}\right)}{\partial\delta}=4B\left(\begin{array}{c}\left(4\left(1-\sigma\delta\right)-\sigma^2\left(3-\sigma\left(\sigma+2\delta\right)\right)\right)W_1-2\sigma\left(2-\sigma\delta-\sigma^2\right)W_2\\+\sigma\left(\sigma V_H-\left(2-\sigma^2\right)V_L\right)\end{array}\right).$$

Depending on the parameters expression above can be negative or positive. It is strictly positive at both $\sigma=0$ and $\sigma=1$. Furthermore, it is strictly increasing in both W_1 and V_H and strictly decreasing in W_2 and V_L. Similarly, it is increasing in both w and v ($v=V_H/V_L$) and decreasing in δ. To analyze the impact of asymmetries on the sign of $\frac{\partial\left(\frac{\partial \Pi_2(\delta)}{\partial W_2}\right)}{\partial \delta}$, consider first a full symmetric scenario with $V_H = V_L = W_1 = W_2$. In this case $\frac{\partial\left(\frac{\partial \Pi_2(\delta)}{\partial W_2}\right)}{\partial \delta}$ evaluated at $\delta = 1$ is positive if $\sigma \neq 1$, and $\sigma \leq 0.44$. If we introduce an asymmetry in the stand alone value, for instance by setting $V_H = 1.5 > V_L = 1$, while keeping $W_1 = W_2 = W$, then $\frac{\partial\left(\frac{\partial \Pi_2(\delta)}{\partial W_2}\right)}{\partial \delta}$ is positive iff $\sigma \notin (\sigma_1(W), \sigma_2(W))$ where both $\sigma_1(W)$ and $\sigma_2(W)$ increase with W with $\sigma_1(1) > 0.44$ and $\sigma_2(W) < 1$. Finally, if we introduce an asymmetry in the applications value by setting $W_1 = 1.5 > W_2 = 1$, while keeping $V_H = V_L = V$, then $\frac{\partial\left(\frac{\partial \Pi_2(\delta)}{\partial W_2}\right)}{\partial \delta}$ is positive iff $\sigma \notin (\sigma_1(V), 1)$ where $\sigma_1(1) > 0.44$. Thus, asymmetries and/or lower values of δ increase the probability of $\frac{\partial\left(\frac{\partial \Pi_2(\delta)}{\partial W_2}\right)}{\partial \delta}$ being positive. The weak system is hence either type I or type II. It is more likely type I if asymmetries either in stand-alone values or in application values are large, and statement in (ii) follows.

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Chapter 4. Two Sided Platforms with Quality Differentiation

1. Introduction

Nowadays two-sided platforms are present in many aspects of our life. When we pay with a debit or credit card in the gas station, when we search for a flight ticket or a hotel in the web, when we buy a newspaper or simply go to the shopping mall, we are having access or using a platform that allows us to connect in a particular way with agents on the other side of the market. In general, the more numerous these agents are the larger is our interest on the platform.[83] However, in many cases the choice of the platform is also conditioned by other circumstances like the identity of the agents that we are going to meet, the brands that are offered inside the platform or our level of income. Think of malls in a big city and sellers and buyers visiting them. On the sellers' side, we observe that some brands are present in all of them, whereas others are not. On the buyers' side, buyers choose the mall they visit according to the sellers they are intending to buy from. At the same time, expensive brands have preferences about the type of buyers and locate in malls visited by high income people. Finally, sellers belonging to the group of expensive brands choose to group together in the same malls, although it makes competition between them stronger. A clear example of this phenomena are the Village Outlets (Fidenza Village in Milan, Bicester Village in London, Las Rozas Village in Madrid, etc.) that group only expensive brands as Loewe, Versace or Dior, and present themselves as the "Chic Outlet Shopping".[84] Another example is observed in online travel platforms. In some of them low-cost airlines as "EasyJet", "GermanWings" or "Ryanair" participate (see for instance, Edreams, Rumbo), whereas there is a different group of platforms where only big or high-cost airlines participate (CheapTickets).

Consequently, we may think that heterogeneity of the customers on each side of the market may play an important role in the formation of the platforms, the type of platform that may arise and/or the prices that platforms can set.

The recent and growing literature on two-sided platforms have largely considered models in which members of each customer group benefit if more members of the other customer group are on the same platform.[85] In particular, this literature has focused on the network effects assuming that agents' choice of platform is independent of their type. However, in many markets, agents' decisions also affect the

[83] Note what the advertisement of Mastercard says: "There are some things money can not buy, for everything else there is Mastercard".

[84] The webpage of the chain store is www.chicoutletshopping.com, where the expresion "luxury" is present everywhere.

[85] See the classical papers of Armstrong (2006) and Rochet and Tirole (2003, 2006).

level of quality offered by the platform and then other customers' utilities, so that, a quality externality takes place. The existence of this externality may help us to explain why, in spite of the presence of strong network effects, it is not common to observe concentration in industries based on two-sided platforms. As Evans et al. (2005) remark, product differentiation may be an important countervailing force working against concentration.

This paper proposes a tractable framework that allows for both network and quality effects. There are two sides of the market, buyers and sellers. Sellers are of two types, the sellers that offer a product of high quality and the sellers that offer a low quality product. For the low type sellers we maintain the traditional assumption that they only care about the mass of buyers that participate in their platform. In contrast, we assume that high type sellers care about the mass of buyers of a particular type. Buyers are heterogeneous and care about the mass and type of sellers.

Working with a model where ex-ante platforms are equal, the goal is to determine the conditions under which concentration prevails (i.e., there is a single active platform in equilibrium) and, in particular, the conditions by which market equilibria can be characterized by two platforms having different qualities, prices and profits.[86]

The platforms' quality is endogenously determined by the type of sellers that they house, so that the level of quality is increasing in the proportion of sellers of high quality.

We find that quality concerns about agents on the other side of the market are not enough to obtain more than one active platform with sellers playing pure strategies. However, once we consider that high type sellers care about their partners in the platform,[87] any equilibrium market configuration is characterized by the coexistence of two active platforms. In particular, it occurs when we assume that high type sellers value in a positive way sharing the platform with sellers of their type but not with low type ones.[88] We find that depending on parameter values there are equilibria where sellers separate by type, equilibria where every seller multihomes, and equilibria where low types singlehome whereas high types multihome.

In any equilibrium where sellers singlehome and separate by type, the profits of the platform that houses the low quality sellers are no lower than the profits of the platform housing high quality sellers. In equilibria where low type sellers singlehome while high type ones multihome, profits of both platforms must be equal, although their qualities are different. Finally, we also find that there are equilibria with two

[86] A common feature in the literature is the presence of symmetric equilibria with identical competing platforms setting the same prices, see Rochet and Tirole (2003), and Armstrong and Wright, (2007), Gabszewicz and Wauthy (2004). An exception is Ambrus and Argenziano (2007). They find equilibria where one network is cheaper and larger on one side, while the other network is cheaper and larger on the other side. In their model, agents differ in the willingness to pay for participating in a larger network.

[87] Pashigian and Gould (1998) analyze empirically the demand externalities existing among stores in a shopping mall. They argue that there exist "anchor stores" (well known stores) that create external economies to other stores.

[88] Any own side effect has been largely ignored in the literature. An exception is Nocke, Peitz and Stahl (2007) who assume competition between sellers.

identical platforms where the prices that they set and the profits they get could be different.

Buyers prefer the equilibria where high type sellers multihome and low type ones singlehome, so that in the market there is a platform with high quality and the other one with medium quality. Under some conditions, this kind of equilibrium also generates the highest level of welfare.

A paper close to this one is Damiano and Li (2007). Their model considers duopoly price competition in a matching environment. Participants care about the identities of other participants and the quality of the matching is endogenously determined. However, there are no network effects between customers, and agents are assumed to not care about members on their own side. The main consequence of the differences between theirs and our model is that, whenever platforms compete in prices simultaneously, only equilibria in mixed strategies will exist, whereas we find conditions under which pure strategies equilibria indeed exist. In addition, in the Damiano and Li's model an equilibrium with a single active platform never arises. In contrast, this is our unique equilibrium with sellers playing pure strategies whenever they do not care about who are their partners in the platform.

The paper is organized as follows. In the next section, we present the model where agents are concerned with quality of members on the other side. In section 3, we solve the game. In section 4, we search for the equilibria when sellers also care about the quality of the other sellers in the platform. Finally, Section 5 discusses the main results we obtained and the adopted modelling assumptions.

2. The model

We study platform price competition in an environment with endogenous vertical differentiation. There are two ex-ante identical platforms operating in a two-sided market. One side has a measure one of sellers and the other side has a measure one of buyers. Platforms offer an access service that provides each side with the possibility of connecting with agents on the other side. This service conveys two characteristics for each side of the market: the quality of the platform and the number (mass) of agents on the other market's side participating in the platform. The platforms set a charge to permit the access and then, endogenously, determine the characteristics of the service offered.

Buyers and sellers' must decide whether to access to the platforms or not. In particular, buyers are allowed to access only one platform (singlehome) while sellers can access both of them simultaneously (multihome). We think of platforms in such a way that at a given point in time a seller or a brand can be present in more than one platform while a buyer has to choose one of them to visit (malls are a good example).

Buyers

The surplus that a buyer derives from access to a platform depends on the number of sellers who join the platform and on its quality. Buyers are heterogeneous in the value they assign to the platform's quality and homogeneous in their valuations of the network. A consumer of type θ derives a utility

$$u_i^\theta = \theta q^i + \gamma N_i^S$$

of a platform with quality q^i and a mass of sellers N_i^S, where γ is the network parameter, which can be interpreted as the benefit buyers enjoy from interacting with each seller.[89] Note that our representation of buyers' population encompasses as a particular case Armstrong's model with membership fees. A buyer of type θ chooses the platform for which $\theta q^i + \gamma N_i^S$ is the largest. If this is negative for both platforms, then a type θ buyer stays out of the market.

We assume that θ is distributed according to a Burr type XII distribution[90] with parameter λ:

$$\theta \sim F(\theta) = 1 - \left[1 - \frac{\theta - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right]^{\frac{1}{\lambda}} \quad ; \lambda \geq 0, \quad \theta \,\epsilon\, \Theta = \left[\underline{\theta}, \overline{\theta}\right], \ \overline{\theta} - \underline{\theta} = 1 \text{ and } \underline{\theta} \geq 0.$$

The value of λ identifies the level of concentration around high or low values of θ. In particular, if $\lambda = 1$, θ is uniformly distributed. If $\lambda > 1$, high valuation consumers are more numerous than low valuation consumers and the opposite occurs if $\lambda < 1$. If $\lambda = 0$, distribution becomes degenerate at $\theta = \underline{\theta}$.

Note that quality of the platform and mass of sellers are substitutes in the surplus of the buyer. Heterogenity determines differences in the weights to each surplus component.[91] Beyond their heterogeneity, all the buyers are more attracted by both, the platform that houses the largest number of sellers and the platform with the highest quality. Buyers act to maximize their surplus and they do not pay to accesing the platform, i.e., platforms charge zero to the buyers side.[92]

Sellers

There are two type of sellers, the high (H) type, with measure x, and the low (L) type with measure $1 - x$ (assume that $x < \frac{1}{2}$). The quality of a platform depends on the number of high type sellers relative to the total of sellers in the platform, so that its value belongs to the interval $q^i \epsilon [0, 1]$. In particular, it takes value $q^L = 0$ when the platform houses only low type sellers and value $q^H = 1$ when the platform accounts only for high type sellers. If the platform houses all sellers its quality is $q^M = x$.

We define the mass of sellers (sellers' demand) on each platform according to

$$N_i^S = N_i^H + N_i^L$$

where N_i^H and N_i^L are the mass of H type and L type sellers in platform i, respectively. We denote the mass of buyers (buyers' demand) that visit the platform i by N_i^B.

The net utility of each type of seller when singlehoming in platform i is equal to

$$U_i^H = N_i^B \left(\beta^H\right) - P_i^S \tag{40}$$

[89] Alternatively, the buyers have probability N_i^S to find the product they need.

[90] Burr type XII distribution has been used by Basaluzzo et al. (2005).

[91] It may be better understood if it is interpreted as a heterogeneity in income instead of preferences. A priori every buyer values the high quality products but before visiting the platform they anticipate the purchases they can make, so that θ is the result of a problem previously solved by the buyer (as an indirect utility function).

[92] Nocke. et. al. (2007) also assume a zero access price for consumers. They note that this applies whenever it is not feasible to charge buyers or whenever the platform would like to subsidize them but is not allowed to set negative access prices. Furthermore, they argue that the latter situation is likely to occur in models with two platforms provided that sellers multi-home and buyers single-home. Both of these features are present in our model.

and

$$U_i^L = N_i^B - P_i^S, \tag{41}$$

and the utility of sellers that multihome is equal to $U_i^s + U_j^s$, $s = H, L$. The parameter β^H determines the type of buyers that high type sellers are interested in, where $N_i^B\left(\beta^H\right)$ is a function defined by

$$N_i^B\left(\beta^H\right) = P\left(\theta \geq \beta^H\right) \text{ for all } \theta \text{ s.t. } U_i^{B_\theta} \geq \max\{U_j^{B_\theta}, 0\}.$$

It follows that $N_i^B\left(\beta^H\right) \leq N_i^B$, so that high type sellers only perceive utility from the buyers in the platform whose types are in the interval $\left[\beta^H, \overline{\theta}\right]$. In other words, $N_i^B\left(\underline{\theta}\right) = N_i^B$ is the complete mass of buyers visiting platform i and $N_i^B(\beta^H)$ is the mass of buyers visiting platform i of the type that high type sellers are interested in. The intuition is that sellers only derive utility from high type buyers (i.e., high income buyers) to which they expect to sell. Implicitly it is assumed that $\beta^L = \underline{\theta}$, i.e., low type sellers derive utility from any kind of buyer. Finally, note that inside each group sellers are homogeneous.[93]

Platforms

Platforms face no cost, they can not discriminate in prices within sellers and they can not set positive prices for the buyers.[94] The profits of platform i are given by

$$\Pi_i = P_i^S N_i^S.$$

The reasoning behind our modelling strategy is similar to that in Gabszewicz and Wauthy (2004). From the viewpoint of a seller, the willigness to pay to access a platform depends on her own type and the number of additional sales this seller expects to realize in the platform. All of them are conditioned by the number and type of buyers and sellers participating in the platform. From the viewpoint of the buyers, the willigness to visit the platform depends on the buyers' type and on the number of purchases that they expect to make in the platform, which in turn depends on the number and types of sellers housed by it.

The timing of the game is the following: in the first stage platforms set prices, in the second stage sellers observe prices and decide their locations. Finally, buyers observe sellers' locations, infer platforms' quality and choose the one they visit. We search for subgame perfect equilibria of this game.

3. The game

We solve the game by backward induction. First we solve the buyers' problem, then we solve the sellers' subgame and finally the platforms' problem. In the third stage, each buyer takes the decision that maximizes her utility given her type. In the second stage, we search for equilibrium locations of the sellers. In the first stage, platforms choose the prices they will charge to sellers.

As there is a continuum of sellers, there are multiple locations by the sellers that can constitute a Nash equilibrium. Given this plethora of equilibria, in order

[93]The utility function of the sellers follows Armstrong's model.

[94]Pashigian and Gould (1988) note that shopping malls charge nothing for access to buyers whereas they collect rent from retailers. Hagiu (2006) finds that the sellers side pays relatively more when the "intensity" of buyers' preferences for variety is higher.

to obtain sharp predictions, in what follows we will concentrate on Nash equilibria that are robust to coalitional deviations so that they satisfy the Strong Nash refinement. We will search for choices by platforms for which no subgroup of sellers can deviate by changing strategies jointly in a manner that increases payoffs to all of its members, given that non-members stick to their original choice.

3.1 The buyers' problem. At the third stage of the game, each buyer decides on visiting platform 1 or platform 2. We assume that the strategy of not participating in any platform yields a zero payoff to any buyer. Sellers have already been located in stage 2 and qualities of the platforms are known in the last stage. Buyers may hence face one of the three following possible situations: 1) Two active platforms with different qualities, 2) Two active platforms with the same quality, and 3) A single active platform.

Two active platforms with $q^i > q^j$. For each $\left(N_i^S, N_j^S\right)$ we define θ^* as the buyer who is indifferent between visiting platform i and platform j, i.e.,

$$\theta^* = \min\{\overline{\theta}, \gamma\frac{\left(N_j^S - N_i^S\right)}{(q^i - q^j)}\}. \tag{42}$$

Consequently, the mass of buyers in platform i is given by $N_i^B = 1 - F(\theta^*)$ and the mass in platform j is given by $N_j^B = 1 - N_i^B = F(\theta^*)$.

We will here further assume that $\gamma < \overline{\theta}$. If this assumption is not satisfied all the buyers will visit the same platform and they will never separate as the network effect is too strong. Two particular cases will be of importance in the analysis that follows. If there is one platform with quality $q^H = 1$ and the other one with quality $q^L = 0$ then $\theta^* = \gamma(1-2x)$. However, if the last platform has quality q^M then $\theta^* = \gamma$.

Two active platforms with $q^i = q^j$. Two cases may arise. If $N_i^S > N_j^S$, then $N_i^B = 1$ and $N_j^B = 0$, whereas if $N_i^S = N_j^S$, then $N_i^B = N_j^B = \frac{1}{2}$.

A single active platform j with q^j. Every buyer will visit this platform, so that $N_j^B = 1$.

Among all the configurations that may arise in the market when all sellers participate, the particular cases that will be relevant in the analysis that follows are:

Configuration 1: a single platform with quality q^M
Configuration 2: two platforms with the same quality q^M
Configuration 3: two platforms, one with quality q^M and the other with quality q^H
Configuration 4: two platforms, one with quality q^L and the other with quality q^H
Configuration 5: two platforms, one with quality q^L and the other with quality q^M

Before continuing to solve the rest of the game, let us see which of the five cases is the one that buyers would prefer. We find that any buyer is indifferent between Configuration 1 and Configuration 2 (recall that buyers are not charged by accessing to the platform). In either of them, they enjoy a medium quality platform and they will face the total mass of sellers. Buyers types in the interval $[\underline{\theta}, \gamma]$ are also indifferent between any of the two former configurations and Configuration 3, while they prefer any of those to Configuration 4 where both, platform quality and mass of sellers, would be lower than in any of the three aforementioned configurations. In contrast, buyers types in the interval $\left[\gamma, \overline{\theta}\right]$ are indifferent between configurations 3 and 4. In either of them they enjoy a high quality platform and face a mass of sellers of size x. Note that they are better off under this situation than in the one where they get a medium quality platform and have access to all the sellers. Finally, Configuration 5 is dominated by any other one. Thus, the next proposition follows.

Proposition 1 *A market with two platforms, one with quality q^M and the other with quality q^H (Configuration 3) is the preferred configuration by any buyer type.*

The most elitist buyers can visit a platform where only the high type sellers are present. Since they are interested in this type of sellers, the network effect of the rest of the sellers is not important for them. Other buyers, who may be looking for a greater variety of sellers, will be more satisfied when visiting the medium quality platform.

3.2 The sellers' problem. At the second stage sellers decide where to locate: at one of the two platforms, at both platforms or at none of them, once the prices have been already set in the first stage. We assume that the strategy of not participating in any platform yields a zero payoff to both types of sellers.

A seller of type s will go to platform i (singlehome in i) if and only if

$$U_i^s \geq \max\{U_j^s, U_i^s + U_j^s,\ 0\}, \quad s = H,\ L$$

and she will multihome if and only if

$$U_i^s + U_j^s \geq \max\{U_i^s, U_j^s,\ 0\}, \quad s = H,\ L.$$

Recall that the presence of a continuum of sellers yields multiplicity of Nash equilibria. In order to avoid this situation, our equilibrium concept is the Strong Nash Equilibrium. It is defined as a strategic profile for which no subset of players has a joint deviation that strictly benefits all of them.

Definition *(Aumann 1959): Let N be the number of players. A strategic profile σ^* is a Strong Nash Equilibrium if for all $M \subset N$ there is not any $(\sigma_k)_{k\in M}$ such that for all $k \in M$*

$$U_k\left((\sigma_k)_{k\in M}, (\sigma_l^*)_{l\in N\setminus M}\right) > U_k(\sigma^*).$$

The definition immediately implies that any Strong Equilibrium is a Nash equilibrium. Throughout the paper, whenever we write that a strategy profile is a subgame perfect equilibrium we refer to a strategy that represents a Strong Nash equilibrium in the subgame where sellers play.

3.3 The subgame perfect equilibrium. In a symmetric equilibrium in which all sellers of a given type follow the same strategy, sellers' location decisions can give rise to three possible type of equilibria: 1) a single active platform in which both types of sellers singlehome (Configuration 1); 2) singlehoming with separation by type (Configuration 4); 3) some form of multihoming, in particular, the equilibrium at which both types multihome (Configuration 2), the one at which only low types multihome and high types singlehome (Configuration 5), and the equilibrium at which only high types multihome and low types singlehome (Configuration 3).

Note that the preferred configuration by the buyers requires of multihoming to be allowed and, in particular, requires that high type sellers multihome.[95]

We will restrict the attention to situations where $\beta^H > \gamma(1-2x)$. We do this to emphasize the importance that quality relative to network size has for high sellers. Recall that $1-F(\beta^H)$ is the value that the high type sellers attach to the network, that is, the utility that the mass of buyers with θ's higher than β^H yield to the high type sellers. Similarly, $1-F(\gamma(1-2x))$ is the mass of buyers that go to the platform with the highest quality when sellers separate by type. Thus, whenever $\beta^H > \gamma(1-2x)$ holds, the mass of buyers that high sellers value is smaller than the mass of buyers that visit the high quality platform when sellers separate. The intuition of a sufficiently large β^H is that high type sellers are sufficiently exigent about the buyers type, so that, the network size is small for them.

We next show that when platforms set prices a unique subgame perfect equilibrium arises when only pure strategies are allowed. In this equilibrium, network effects dominate quality differentiation so that only one platform can survive in the market. There is also an equilibrium where sellers play mixed strategies. In particular, they randomize with probability $\frac{1}{2}$ between the strategies of visiting one platform or the other.

Proposition 2 *A single active platform with quality q^M (Configuration 1) is the unique subgame perfect equilibrium configuration in pure strategies. At this equilibrium, prices are $P_1^S = P_2^S = 0$.*

Proof: see Appendix A.

4. Sellers reputation effect

In the previous section we have seen that quality considerations about agents on the other side of the market are not enough to obtain an equilibrium with more than one active platform. In this section we will introduce a second source of heterogeneity between the two types of sellers. We will assume that high type sellers care about their partners in the platform, an effect that we interpret as a reputation effect. In particular, high type sellers value sharing the platform with sellers of their type but not with low type ones.[96] The intuition is that for H type sellers is more profitable to participate alone conforming an elitist platform since they can charge higher prices for their products.[97] Furthermore, in a platform where low sellers participate, H type sellers may be forced to set lower prices due to competition of the other type of sellers that offer cheaper products.

Consider now the following net utility of joining platform i for the high type sellers

$$U_i^H = V(q^i, N_i^B) + N_i^B(\beta^H) - P_i^S, \tag{43}$$

[95] Exclusive dealings between a platform and high sellers (not low types) would avoid this configuration to be an equilibrium outcome.

[96] Although the effect of sharing the platform with sellers of the same type might result in a fiercer competition among them, we assume that the positive reputation effect overcomes any potential negative effect that clustering may generate.

[97] For instance, a shirt in Christian Dior's shop in Piazza Spagna in Rome is much more expensive than in others Christian Dior's shops located in more ordinary places.

where $V(q^i, N_i^B)$ measures the value that these sellers assign to platform's quality. This is defined in the following way

$$V(q^i, N_i^B) = \begin{cases} V(q^i) & if \quad N_i^B > 0 \\ 0 & if \quad N_i^B = 0 \end{cases}$$

and we further assume $V(q^H) > V(q^M) \geq V(q^L) = 0$.

Equation (43) embeds the assumption that in the high sellers utility function the concern for quality and the concern for network size are additively separable, so that quality and network are substitutes. This assumption allows us to characterize sellers' equilibrium strategies in a convenient way. We will comment on the role played by this assumption in the last section.[98]

Since the buyers' problem remains like in section 3.1, we solve now the sellers' stage considering their new utility. The following lemma shows that, if sellers are very exigent about their partners in the platform, the buyers' most preferred configuration is still absent as an equilibrium outcome,

Lemma 1 *If* $V(q^M) = 0$ *there is no equilibria involving multihoming by any type of seller.*

Consider the strategies of low type sellers. If H type sellers are in platform i, going to this platform generates a benefit $1 - P_i^s$ for L types while multihoming yields $1 - P_i^s - P_j^s$. So, given that H type sellers are located in one platform, the strategy of multihoming by the low sellers will never be a best response. Also, if L type sellers decide to multihome, platform i will have a level of quality q^M, while platform j with only L types will have quality q^L. No buyer will visit platform j given that it will have a lower quality and a lower number of sellers.

Consider now the high type sellers. If low types are in platform i, going to platform j generates a benefit

$$V(q^H) + \left[1 - F(\beta^H)\right] - P_j^s$$

for H type, while multihoming yields

$$V(q^H) + \left[1 - F(\beta^H)\right] - P_i^s - P_j^s.$$

So, given that L type sellers are located in one platform, the strategy of multihoming by the high type will never be a best response.

Let us stress that, as we did in subsection 3.3, we are here also restricting the analysis to the case where $\beta^H > \gamma(1-2x)$, a condition that implies $\left[1 - F(\beta^H)\right] < [1 - F(\gamma(1-2x))]$. This condition is indeed relevant for the result in Lemma 1. If it does not hold, platform j would generate a benefit

$$V(q^H) + [1 - F(\gamma(1-2x))] - P_j^s \tag{44}$$

for the high sellers. And, in particular, they may be interested in multihoming given that this strategy yields

$$V(q^H) + \left[1 - F(\beta^H)\right] - P_i^s - P_j^s,$$

what implies a higher network than in (44).

[98] However, we conjecture that a utility function for the sellers that would allow for some complementarity between quality of the platform and network size would not change our results dramatically.

Thus, as long as $V(q^M) = 0$ and $\beta^H > \gamma(1-2x)$ hold, there is to say, whenever the quality concern of the high type sellers, regarding buyers and sellers, is sufficiently strong they will never multihome. In the analysis that follows we will see that multihoming may arise if $V(q^M) > 0$.

Let us introduce some new pieces of notation that will simplify the presentation that follows. Let $T_1 = [1 - F(\gamma(1-2x))]$, $T_2 = 1 - F(\gamma)$ and $T_3 = 1 - F(\beta^H)$. In particular, T_2 is the mass of buyers that go to the platform with the highest quality when low type sellers go to one platform while high type sellers multihome. Let us also detone by D_q to $V(q^H) - V(q^M)$, i.e., to the extra benefit that high sellers enjoy in terms of reputation from sharing the platform only with sellers of their type, instead of also sharing the platform with the low type sellers.

To simplify the exposition we write the locations of sellers as "LiHj" to refer to a location where the L type sellers follow the strategy i and H type sellers follow the strategy j, where $i, j \in \{0, 1, 2, M\}$, i.e., sellers either do not go to any platform, or go to platform 1, or to platform 2 or multihome. So, for instance, L1H1, means that both type of sellers only go to platform 1; L1HM means that L type sellers go to platform 1 and H type multihome. Note that, most of the configurations can be achieved by more than one location of sellers. In particular, L1H1 and L2H2 locations lead to Configuration 1, L1HM and L2HM lead to Configuration 3, finally L1H2 and L2H1 lead to Configuration 4. Finally, we will call platform 2 the platform that sets the higher price, i.e., $P_2^S \geq P_1^S$.

Figure 1 illustrates the equilibrium configurations that may arise. In particular, it provides the map of equilibria at the sellers' stage that will be formally presented in proposition 3 below.[99]

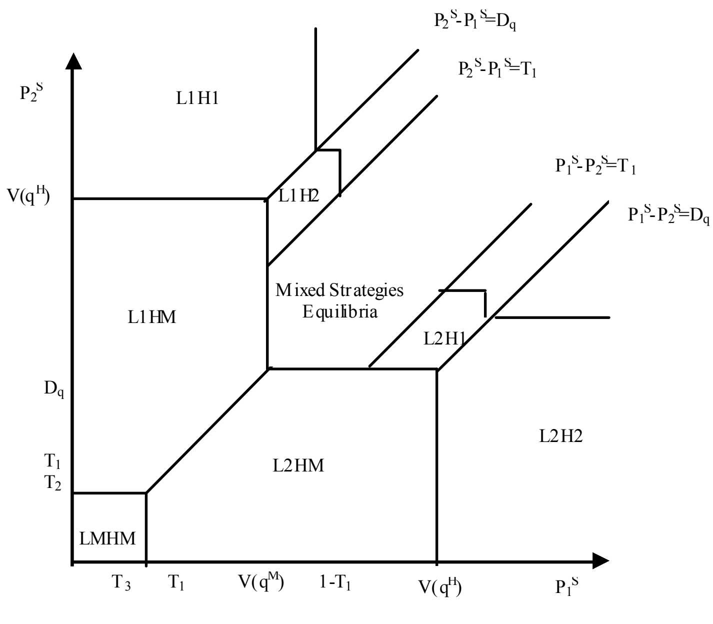

Figure 1: The Nash Equilibria at the sellers'stage

[99]To construct figure 1 we have further assumed $V(q^M) > T_1$.

From the figure it follows that Configuration 5, where high type sellers singlehome and low type multihome (locations LMH1 or LMH2) never arises. The explanation relies on the same arguments used to show Lemma 1.

Let us explain now the rationale behind conditions in Figure 1 which will be formally stated in next proposition. Consider location L1H2. At this location sellers' profits are given by $U_2^H = V(q^H) + T_3 - P_2^S$ and $U_1^L = (1 - T_1) - P_1^S$. High type sellers will not deviate to location L1H1 whenever

$$V(q^H) + T_3 - P_2^S \geq V(q^M) + T_3 - P_1^S,$$

which requires $P_2^S - P_1^S \leq D_q$ to hold, and they will not deviate to location L1HM as long as

$$V(q^H) + T_3 - P_2^S \geq V(q^H) + V(q^M) + T_3 - P_1^S - P_2^S$$

is satisfied. This is the case whenever $P_1^S \geq V(q^M)$ holds. Similarly, low type sellers will not deviate to L1H1 if

$$(1 - T_1) - P_1^S \geq 1 - P_2^S,$$

which requires $P_2^S - P_1^S \geq T_1$. Participation constraints impose $P_1^S \leq 1 - T_1$ and $P_2^S \leq V(q^H) + T_3$. In sum, an equilibrium as *(L1H2)* requires prices such that $T_1 \leq P_2^S - P_1^S < D_q$, $V(q^M) \leq P_1^S \leq 1 - T_1$ *and* $P_2^S \leq V(q^H) + T_3$ (see Figure 1). Symmetric prices determine location L2H1.

The general conditions for the existence of other type of equilibria at the sellers' stage is the content of next Proposition.[100]

Proposition 3
i) There is an equilibrium where sellers separate by type (L1H2) if and only if $T_1 \leq P_2^S - P_1^S < D_q$, $V(q^M) \leq P_1^S \leq 1 - T_1$ *and* $P_2^S \leq V(q^H) + T_3$
ii) There is an equilibrium where all the sellers multihome (LMHM) if and only if $P_1^S + P_2^S \leq 1$ *and* $P_2^S \leq \min\{T_2, V(q^M)\}$
iii) There is an equilibrium where high type sellers multihome and low type singlehome (L1HM) if and only if $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ *and* $T_2 \leq P_2^S \leq V(q^H)$
iv) There is an equilibrium where all the sellers singlehome in the same platform (L1H1) if and only if $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + T_3\}$ *and* $P_2^S \geq V(q^H)$.
Proof. See Appendix A.

We find that a necessary condition for separation to exist is that the extra benefit that high type sellers obtain when they separate, as compared with a situation where both types of sellers are together, measured by D_q, is larger than the cost that separation has on low type sellers in terms of network size, measured by T_1. The latter is the difference in potential clientele between this kind of equilibrium and a situation where all sellers are together. Thus, the reputation effect has to be sufficiently higher when they do not share the platform with the low type sellers than when they do. As D_q and T_1 tend to get close to each other, separation becomes unlikely. The same occurs when $1 - T_1$ and $V\left(q^M\right)$ get close (see Figure 1).

Whenever $\lambda > 1$, the distribution function of θ stochastically dominates the uniform distribution so that T_1 is larger than in the case of $\lambda = 1$. The contrary

[100]Proposition 3 focus on pure strategies equilibria. The mixed strategies equilibria are discussed in appendix B. Note also that it ignores the potential strategies where any type of sellers do not participate, i.e., locations L0Hi and the LiH0, for any $i\ \epsilon\{0, 1, 2, M\}$.

occurs if $\lambda < 1$. Then, the more concentrated the buyers' distribution is around the higher types, the less likely it is that separation emerges.

Note that the mass of high type sellers, x, affects T_1 directly, so that a smaller mass of high type sellers facilitates separation. On the contrary, changes in γ affect T_1 negatively and make separation more likely. It may appear counterintuitive that a higher buyers' valuation for the network facilitates buyers separation. This is due to the fact that the low quality platform has a higher mass of buyers ($1 - x > x$ by assumption) . Thus, if γ increases, the utility that this platform generates for every buyer is higher, and more buyers will decide to visit this platform. Consequently T_1 decreases, and the cost for the low type sellers of being separated gets smaller. Buyers' heterogeneity is also crucial for this equilibrium to exist, as otherwise buyers would never have an incentive to separate.

As long as T_2 is large, the equilibrium where all the sellers multihome becomes more likely. Recall that T_2 is the cost for the low type sellers of staying in only one platform, compared with the benefit of the multihoming strategy, when high type multihome. The impact of λ and γ on T_2 are analogous to those explained for T_1.

In a L1HM equilibrium the quality of platform 1 is q^M while the quality of platform 2 is q^H. To attract the low type buyers, the price in platform 1 must be lower than the one in platform 2, and P_2^S must be larger than the extra benefit (T_2) that would accrue to the low type sellers by multihoming.

Finally, consider the equilibrium with a single platform. For high type sellers to stay in the active platform, the difference in prices must compensate for the difference in terms of reputation between strategies H1 and H2.

4.1 The subgame perfect equilibrium with sellers reputation effect. We next focus on the first stage of the game when platforms set prices. We first show that when reputation is a concern an equilibrium with a single platform (as the one described in Proposition 2) will never arise.

Proposition 4 *If $V(q^H) > V(q^M) \geq 0$ there is no subgame perfect equilibrium in which only one platform is active in the market (Configuration 1 never arises).*

Proof. See Appendix C

Corollary 1 *If sellers reputation effect is small there is no a subgame perfect equilibrium with sellers playing pure strategies.*

Under conditions $V(q^M) = 0$ and $V(q^H) < T_1$, Configuration 1 is the only candidate to be an equilibrium where sellers play pure strategies. Since proposition 4 indicates that this configuration will never exist, the result of the corollary follows. We next show that depending on parameter values, equilibrium configurations with the two active platforms and sellers playing pure strategies may arise.

Proposition 5 *There exist parameter values under which Configurations 2, 3 and 4 may arise as a subgame perfect equilibrium outcome.*

Proof. See Appendix C

To illustrate the likelihood of the different configurations in Proposition 5, let us analyze the case of Configuration 3, the preferred one by the buyers. Let us fix platform prices at $P_1^{S*} = V(q^M)$ and $P_2^{S*} = V(q^H)$. At these prices, provided that $V(q^M) < 1 - T_2$ (see Proposition 3), there is a candidate equilibrium with Configuration 3, location of sellers L1HM and platforms' profits $\Pi_1^* = V(q^M)$ and

$\Pi_2^* = V(q^H)x$. To ease the exposition, we here restrict the attention to the parameter values under which Figure 1 follows. In particular, we here assume $V(q^M) > T_2$ and $V(q^H) > 1$.

To analyze under what conditions location L1HM is in fact an equilibrium outcome we must study deviations of each platform given the other's price.

Deviations to prices that guarantee a location of sellers L1HM are not profitable as P_1^{S*} and P_2^{S*} are the best prices that platforms can set among those that yield this location. Because of this, deviations by platform 1 to a lower price $P_1^S < V(q^M)$ and deviations by platform 2 to prices in the interval $(V(q^M), V(q^H)]$ are never profitable.

Deviations by platform 2 to prices $P_2^S > V(q^H)$ are never profitable given that these prices would lead to a location of sellers L1H1 that would imply $\Pi_2 = 0$. Deviations by platform 2 to prices lower than $V(q^M)$ would make this platform to attract the low type sellers while the high types will multihome (location L2HM will emerge). This deviation will not be profitable whenever

$$V(q^M) \leq V(q^H)x. \tag{45}$$

Consider now deviations by platform 1. As mentioned above we only need to consider deviations to higher prices. The first possibility is to set $P_1^S \leq 1 - T_1$. It would induce a location of sellers L1H2, so that platform 1 will not deviate if and only if

$$V\left(q^M\right) > (1 - T_1)(1 - x)\,. \tag{46}$$

There are two other intervals to be considered. Interval of prices $1 - T_1 < P_1^S < 1$ and $1 \leq P_1^S \leq V\left(q^H\right)$. Prices in the second interval would lead to a location L0H1 and platform 1 would not deviate if

$$V(q^M) \geq V(q^H)x. \tag{47}$$

At prices in the interval $1 - T_1 < P_1^S < 1$ sellers play mixed strategies in equilibrium and we present the analysis of this deviation in lemma C3, in the appendix. However, the condition for such deviations to be unprofitable is guaranteed to hold when (47) holds. Consequently, if (45), (46) and (47) are satisfied then Configuration 3 can be sustained as a subgame perfect equilibrium.

Regarding platforms profits, note that conditions (45) and (47) imply $V(q^M) = V(q^H)x$, so that equilibrium profits under Configuration 3 are the same for both platforms. The rationale is that each platform has the possibility of replicating the profits of its rival by setting its price. The result holds true for any set of parameters that allow for configuration 3 to be an equilibrium outcome. Under Configuration 3, although platforms are not symmetric, the profits that they get are the same. It follows that the price set by the high quality platform is higher than the price set by the medium quality one. High quality sellers pay more to participate in a platform where they are alone than in a platform shared with low type ones.

Under Configuration 4, the low quality platform (say platform 1) has always the possibility of getting the profits of the high type one. Setting a price $P_1^S = P_2^S - \varepsilon$, platform 1 attracts H type sellers, loses the L types and gets the profits of the high quality platform. In contrast, the high quality platform (say platform 2) can not replicate the situation of platform 1. Note that in an equilibrium in which Configuration 4 is attained, there is the largest differentiation. Platforms enjoy more market power and they can extract all the surplus of at least one type of sellers. The explanation is the following: from Proposition 3 we know that the best response prices are: $P_1^S = \min\left(1 - T_1,\ P_2^S - T_1\right)$ and $P_2^S = \min\left(V\left(q^H\right) + T_3,\ P_1^S + D_q\right)$. Since $D_q > T_1$ at least one of the sellers type is at its participation level in equilibrium, $P_1^S = 1 - T_1$ or $P_2^S = V\left(q^H\right) + T_3$.

Finally, under Configuration 2 platforms can enjoy different profits. These results are summarized in next Proposition.

Proposition 6

i) Symmetric platforms in a equilibrium with Configuration 2 may have different profits.

ii) In any equilibrium with Configuration 3, although platforms are asymmetric, profits are equal.

iii) In any equilibrium with Configuration 4, profits of the low quality platform are higher than or equal to profits of the high quality platform.[101]

Proof. See Appendix C.

5. Discussion

We have studied competition between two-sided platforms with heterogeneous buyers and sellers when each side of the market cares not only about the size of the other side, but also about the type of its members. When buyers and sellers interact through the platforms there are network and quality effects operating from one side of the market to the other. In this set-up we have shown that due to the presence of network effects, the market tends to be concentrated in spite of the quality effect being present.

The purpose of this section is to discuss the market outcomes that we have identified, their robustness with respect to changes in the modelling assumptions and their welfare.

Market outcomes. The unique equilibrium that may emerge, with sellers playing pure strategies, involves a single active platform and all the sellers visiting it, conforming a medium quality platform. At this equilibrium sellers pay prices equal to the platforms' marginal cost.

Given that sellers are allowed to multihome, this result contradicts, in some sense, the well known "competitive bottleneck equilibrium" of Armstrong (2006) at which the multihoming side is charged with nearly monopoly prices. The timing of the game explains the different outcomes. Here buyers decide their location after observing sellers' choices so that sellers anticipate that if they group together, they will meet all the buyers. Because of this, the strategy of multihoming will always be a dominated one. When this occurs, platforms have incentives for "undercut" prices to "steal" sellers. It follows that the competitive bottleneck equilibrium may not be robust to introducing sequential moves so that the two sides of the market do not play simultaneously.

Once we introduce an own side effect on the sellers' side, in particular, a kind of reputation effect for the high type sellers, we find that a unique platform configuration will never arise as an equilibrium outcome. Despite the network effects and the ex-ante symmetric platforms, in any equilibria we find more than a single active platform. The sufficient condition for this result is that $V(q^H) > 0$. The platforms

[101] Damiano and Li (2007) show that in a sequential-move game where platforms compete in prices, the platform that moves first chooses prices such that this platform becomes the low quality one.

have the possibility of offering the extra reputation effect to the high sellers in order to attract them and so making impossible the existence of an equilibrium with Configuration 1.

In particular, whenever the reputation effect is present, platforms have two types of pricing strategies. First, they can "lowering the own price" to attract sellers. Second, they can "increase the own price" to provide a higher level of quality.[102] For instance, given a location LMHM, platform 2 would achieve a location L1HM by setting a price higher than $\min\{T_2, V\left(q^M\right)\}$ and lower than $V\left(q^H\right)$ (see Proposition 3). By doing so, platform 2 would offer a service of quality q^H, higher than the initial quality q^M, and would attract the buyers that care more about quality. Something similar occurs in locations L1HM and L1H2. In both cases, the low quality platform could increase its price to expel the low type sellers while keeping the high types. By this way, this platform becomes of the same quality as its rival. Once this is the case, location L0H1 might emerge.

Modelling assumptions. From lemma 1 it follows that in order to have multihoming a necessary condition is that $V(q^M) > 0$. In our model it means that sharing the platform with low sellers still yields a positive extra utility. However, in some equilibria the condition implies that, for exogenous reasons not related to the network, participating in both platforms yields an extra profit to the sellers, compared with the singlehoming strategy. This is a situation that we accept may be hard to justify.

Thus, the additive separability in quality and $N_i^B(\beta^H)$ in the sellers utility function can be called into question. A multiplicative form would be, for instance, a more attractive assumption, but instead, a more difficult to work with. Moreover, results would not change qualitatively, given that the restrictions on platform prices for each type of equilibrium to arise would go in the same direction.

Throughout the paper we have assumed $\beta^H > \gamma(1-2x)$. Note that if we consider a lower value of β^H, so that high sellers care more for the network, we also may find equilibria where they multihome, with no necessity of $V(q^M) > 0$. However, it would imply less heterogeneous sellers and less emphasis on quality by the high quality sellers.

We consider that a merit of our model is that it provides a tractable framework to combine both networks and quality effects, where the former effect is the most important for low sellers and the latter is for high sellers.

Welfare. We have found that Configuration 3 is the preferred one by the buyers: those buyers that are only interested in high sellers can have access to them, while those buyers more interested in the number of sellers may visit the rival medium quality platform.

Note that taking total welfare as the sum of the buyers' surplus, plus sellers' surplus and platforms' profits, this configuration also yields, in general, the highest level of welfare.

Denote with Λ the aggregate of the sellers' surplus and the platforms' profits. Since buyers access the platform for free, we have that

$$\Lambda = \left[V\left(q^i\right) + N_i^B\left(\beta^H\right)\right] N_i^H + N_i^B N_i^L + \left[V\left(q^j\right) + N_j^B\left(\beta^H\right)\right] N_j^H + N_j^B N_j^L.$$

[102] Note that this strategy is similar to the "overtaking strategy" in Damiano and Li (2007).

The value that Λ takes under the different equilibrium configurations is as follows

$$\begin{aligned}
\Lambda^{LiHi} &= \left[V\left(q^{M}\right)+M\right]x+(1-x)\,,\ i\neq M,\ \text{for Configuration 1,}\\
\Lambda^{LMHM} &= \left[2V\left(q^{M}\right)+M\right]x+(1-x)\ \text{ for 2,}\\
\Lambda^{LiHM} &= \left[V\left(q^{H}\right)+V\left(q^{M}\right)+M\right]x+(1-x)\left(1-T_{2}\right),\ i\neq M,\ \text{for 3, and}\\
\Lambda^{LiHj} &= \left[V\left(q^{H}\right)+M\right]x+(1-x)\left(1-T_{1}\right),\ i\neq j\neq M\ \text{for 4.}
\end{aligned}$$

Since $\Lambda^{LiHM}(i\neq M)>\Lambda^{LiHj}\,(i\neq j\neq M)$, Configuration 4 is always dominated by Configuration 3. Similarly, platforms and sellers aggregate profits in an equilibrium with location LMHM (Configuration 2) are always higher than in a Configuration 1 equilibrium, that is, $\Lambda^{LMHM}>\Lambda^{LiHi}\,(i\neq M)$. Finally, note that whenever $\frac{D_q}{T_2}>\frac{1-x}{x}$ it follows that $\Lambda^{LiHM}(i\neq M)>\Lambda^{LMHM}$. Thus, if D_q is sufficiently high, Configuration 3 generates the highest level of Λ.

Appendix A

Proof of proposition 2

The statement follows from Bertrand's arguments. Both plaforms yield a benefit (gross of prices) to the high type sellers of $T_3=\left[1-F\left(\beta^{H}\right)\right]$. The best reply of low type sellers is participating in the same platform than H type sellers (i.e., in platform 1) as

$$1-P_1^S>1-T_1-P_2^S.$$

Since platforms compete to attract the sellers and equilibrium prices are equal to the marginal cost of the platforms, there is no profitable deviation to higher prices. Due to the network effects, even though prices are zero, sellers locate all together in one platform as claimed.■

Proof of Proposition 3

i) In any equilibrium configuration in which sellers separate by type as L1H2, prices satisfy $T_1\leq P_2^S-P_1^S<D_q$, $V(q^M)\leq P_1^S\leq 1-T_1$ and $P_2^S\leq V(q^H)+T_3$.

We next show that prices that satisfy these restrictions imply an equilibrium with location L1H2, by using iterated elimination of dominated strategies. Under prices $P_1^S\leq 1-T_1$ and $P_2^S\leq V(q^H)+T_3$, strategies L0 and H0 are eliminated. Given that $P_2^S-P_1^S\geq T_1$, L1 dominates L2 and LM. Finally, the best response to L1 by high type sellers is H2. Thus, the conditions $T_1\leq P_2^S-P_1^S<D_q$, $V(q^M)\leq P_1^S\leq 1-T_1$ and $P_2^S\leq V(q^H)+T_3$, are necessary and sufficient to ensure the existence of an equilibrium with location L1H2.

ii) We first prove that in any LMHM equilibrium location, prices satisfy $\min\{T_2,V(q^M)\}\geq\max\{P_1^S,P_2^S\}$ and $P_1^S+P_2^S\leq 1$. The profits of the sellers are $U_1^H+U_2^H=2V(q^M)+T_3-P_1^S-P_2^S$ and $U_1^L+U_2^L=1-P_1^S-P_2^S$, where $P_1^S+P_2^S\leq 1$ and $P_1^S+P_2^S\leq 2V(q^M)+T_3$ must hold to ensure sellers' participation. High type sellers will not deviate whenever

$$2V(q^M)+T_3-P_1^S-P_2^S\geq\max\{V(q^M)+T_3-P_1^S,V(q^M)+T_3-P_2^S\}$$

which requires $P_1^S\leq V(q^M)$ and $P_2^S\leq V(q^M)$ to hold. Similarly, low type sellers will not deviate whenever

$$1-P_1^S-P_2^S\geq\max\{(1-T_2)-P_1^S,(1-T_2)-P_2^S\}$$

which requires $P_1^S\leq T_2$ and $P_2^S\leq T_2$. Thus, the first implication follows.

Now, by iterated elimination of dominated strategies, we show that prices such that $\min\{T_2, V(q^M)\} \geq \max\{P_1^S, P_2^S\}$ and $P_1^S + P_2^S \leq 1$ ensure the existence of a equilibrium location LMHM as claimed. Trivially, strategies L0 and H0 are eliminated given that participation conditions are fullfilled. Given that $\max\{P_1^S, P_2^S\} \leq V(q^M)\}$, HM dominates H1 and H2. Finally, the best reply of L type sellers to the HM strategy is LM under prices $\max\{P_1^S, P_2^S\} \leq T_2$.

iii) We first show that in any L1HM location prices satisfy $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$.

In a location as L1HM, sellers' profits are given by $U_M^H = V(q^H) + V(q^M) + T_3 - P_1^S - P_2^S$ and $U_1^L = (1 - T_2) - P_1^S$. Participation constraints require $P_1^S \leq 1 - T_2$ and $P_1^S + P_2^S \leq V(q^H) + V(q^M) + T_3$ to hold. High type sellers will not deviate whenever $P_1^S \leq V(q^M)$ and $P_2^S \leq V(q^H)$ hold. Similarly, low type sellers will not deviate whenever $P_2^S - P_1^S \geq 0$ and $P_2^S \geq T_2$ are satisfied.

We next show that under prices that satisfy $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$, only a L1HM equilibrium can arise. As in previous parts of this proof we obtain the result by iterated elimination of dominated strategies. The strategies L0 and H0 are trivially eliminated. Given that $P_1^S \leq V(q^M)$, HM dominates H2. And then, L1 dominates L2 and LM. Finally, the best response of high type sellers to L1 is HM.

Thus, the conditions $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$ are necessary and sufficient to ensure the existence of an equilibrium location L1HM, as claimed.

iv) First, we prove that in any L1H1 equilibrium prices satisfy $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + T_3\}$ and $P_2^S \geq V(q^H)$.

In a location as L1H1, sellers' profits are given by $U_1^H = V(q^M) + T_3 - P_1^S$ and $U_1^L = 1 - P_1^S$, where $P_1^S \leq \min\{1, V(q^M) + T_3\}$ ensure that profits above are positive. High type sellers will not deviate to platform 2 whenever $P_2^S - P_1^S \geq D_q$ and will not deviate to a multihome strategy whenever $P_2^S \geq V(q^H)$ is satisfied. Similarly, low type sellers will not deviate whenever $P_2^S - P_1^S \geq -T_1$. Note that this condition is implied by $P_2^S - P_1^S \geq D_q$.

We next show that under prices that satisfy $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + T_3\}$ and $P_2^S \geq V(q^H)$ only a L1H1 equilibrium will arise. The implication is proved by iterated elimination of dominated strategies. Strategies L0 and H0 are trivially eliminated. Under prices $P_2^S - P_1^S \geq D_q$, strategy H1 dominates H2 and under condition $P_2^S \geq V(q^H)$, HM is dominated by H1. Finally, given strategy H1, the best reply of low type sellers is L1. Consequently, the necessary and sufficient conditions for a L1H1 equilibrium location are $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + T_3\}$ and $P_2^S \geq V(q^H)$ as claimed.■

Appendix B

NE in mixed strategies of the sellers' subgame. **B1)** We present the set of Nash Equilibria in mixed strategy of the sellers' subgame when $P_2^S = V\left(q^H\right)$ and the price of platform 1 belongs to the interval $(1 - T_1, 1)$.

Under these prices and assuming $V\left(q^{H}\right)>1$, iterative elimination of dominated strategies[103] shows that low type sellers can only randomize between strategies L1 and L0 and high type sellers can only randomize between strategies H1 and H2.

Denoting by a the probability of playing L1 and by y the probability of playing H1, high sellers' expected utility is given by

$$U^{H}(a;y)=y\left(V(q^{H})-aD_{q}-P_{1}^{S}\right).$$

The best reply by H type sellers involves

$$BR^{H}(a;y)=\begin{cases} y=1 & \text{if } V(q^{H})-aD_{q}-P_{1}^{S}>0 \\ y=0 & \text{if } V(q^{H})-aD_{q}-P_{1}^{S}<0 \\ y\,\epsilon\,[0,1] & \text{if } V(q^{H})-aD_{q}-P_{1}^{S}=0. \end{cases}$$

And the low type sellers' utility is given by

$$U^{L}(a;y)=a\left(1-P_{1}^{S}-T_{1}+T_{1}y\right),$$

so that their best reply is

$$BR^{L}(a;y)=\begin{cases} a=1 & \text{if } \left(1-P_{1}^{S}-T_{1}+T_{1}y\right)>0 \\ a=0 & \text{if } \left(1-P_{1}^{S}-T_{1}+T_{1}y\right)<0 \\ a\in[0,1] & \text{if } \left(1-P_{1}^{S}-T_{1}+T_{1}y\right)=0. \end{cases}$$

Note that, given $P_{2}^{S}=V\left(q^{H}\right)$, whenever P_{1}^{S} belongs to the interval $\left[1,V\left(q^{H}\right)\right]$, the equilibrium location is L0H1. Similarly, if P_{1}^{S} belongs to the interval $\left[V\left(q^{M}\right),1-T_{1}\right]$, the equilibrium is L1H2. Consequently, we only need to derive the set of NE in mixed strategies for the interval of prices $1-T_{1}<P_{1}^{S}<1$.

Lemma B1 *If $1-T_{1}<P_{1}^{S}<1$, high type sellers randomize between strategies H1 and H2 with probabilities $y=\frac{1}{T_{1}}\left(P_{1}^{S}-(1-T_{1})\right)$ and $1-y$ respectively, whereas low type sellers randomize between strategies L1 and L0 with probabilities $a=\frac{V\left(q^{H}\right)-P_{1}^{S}}{D_{q}}$ and $1-a$. The size of the market of platform 1 is given by*

$$\left(\frac{V\left(q^{H}\right)-P_{1}^{S}}{D_{q}}\right)(1-x)+\left(\frac{1}{T_{1}}\left(P_{1}^{S}-(1-T_{1})\right)\right)x \tag{48}$$

Proof. The value $a=\frac{V\left(q^{H}\right)-P_{1}^{S}}{D_{q}}$ leaves the high type sellers indifferent between strategies H1 and H2. Analogously, the value $y=\frac{1}{T_{1}}\left(P_{1}^{S}-(1-T_{1})\right)$ leaves the low type sellers indifferent between strategies L1 and L0.■

B2) We present the set of Nash Equilibria in mixed strategy of the sellers' subgame when $P_{1}^{S}=1-T_{1}$ and the price of platform 2 belongs to the interval $V\left(q^{M}\right)\leq P_{2}^{S}\leq 1$.

The set of equilibria is computed assuming that LM is a dominated strategy (it occurs if $1-T_{1}>T_{1}$ or if $V(q^{M})>T_{1}$).

In addition, this set of equilibria exists when parameters satisfy also the following conditions: $2\left(1-T_{1}\right)<V(q^{H})+V(q^{M})<2-T_{1}$.

[103]We find that L1 dominates LM, then H1 weakly dominates HM, L0 dominates L2 and finally, H2 dominates H0.

We denote by a the probability of the low type sellers playing strategy L1 and consequently $(1-a)$ the probability of L2. We denote by y, b and $c = 1-y-b$ the probabilities of the high type sellers playing strategies H1, H2 and HM, respectively.

Given $P_1^S = 1 - T_1$, the high type sellers' expected utility is given by

$$\begin{aligned} U^H(a;(y,b)) &= T_3 + y\left(T_1 + V(q^H)(1-a) + aV\left(q^M\right) - 1\right) + \\ &\quad b\left(V\left(q^M\right)(1-a) + aV(q^H) - p_2\right) + c\left(V\left(q^M\right) + V(q^H) + T_1 - 1 - p_2\right) \\ &= T_3 + yz_1 + bz_2 + cz_3. \end{aligned}$$

The best reply by H involves

$$BR^H(a;(y,b,c)) = \begin{cases} y = 1 & \text{if } z_1 > \max(0, z_2, z_3) \\ b = 1 & \text{if } z_2 > \max(0, z_1, z_3) \\ c = 1 & \text{if } z_3 > \max(0, z_1, z_2). \end{cases}$$

Given $P_1^S = 1 - T_1$, the high type sellers' expected utility is given by

$$U^L(a;(y,b,c)) = a\left(yT_1 + T_1 - bT_1 - 1 + p_2\right) - yT_1 + 1 - T_2 + T_2 y + T_2 b - p_2,$$

and the corresponding best reply by low type sellers is

$$BR^L(a;(y,b,c)) = \begin{cases} a = 1 & \text{if } (1+y-b)T_1 - 1 + p_2 > 0 \\ a = 0 & \text{if } (1+y-b)T_1 - 1 + p_2 < 0 \\ a \in [0,1] & \text{if } (1+y-b)T_1 - 1 + p_2 = 0. \end{cases}$$

Using the best reply functions above, we next show the set of Nash equilibria in mixed strategies.

Lemma B2 *Along the interval $V(q^H)+V(q^M)-(1-T_1) < P_2^S \leq 1$ there exists a set of mixed strategy Nash Equilibria where low type sellers randomize between strategies L1 and L2 and high type sellers randomize between strategies H1 and H2 with probabilities $a = \left(\frac{1}{2}\frac{D_q + P_2^S - (1-T_1)}{D_q}\right)$, $y = \left(\frac{1}{2}\frac{1-P_2^S}{T_1}\right)$ and $b = 1-y$. The platform 2's market in this interval is equal to*

$$\left(\frac{1}{2}\frac{D_q - P_2^S + (1-T_1)}{D_q}\right)(1-x) + \left(\frac{1}{2}\frac{T_1 - (1-T_1) + P_2^S}{T_1}\right)x \tag{49}$$

Proof. If $V(q^H) + V(q^M) - (1-T_1) < P_2^S$, then $z_3 < 0$ so that strategy HM is dominated. At $a = \left(\frac{1}{2}\frac{D_q + P_2^S - (1-T_1)}{D_q}\right)$ type H sellers are indifferent between H1 and H2. Similarly, $y = \left(\frac{1}{2}\frac{1-P_2^S}{T_1}\right)$ is the probability that makes low type sellers indifferent between L1 and L2. The size of the market follows trivially from the probabilities above.■

Lemma B3 *Along the interval $(1-T_1) < P_2^S < V(q^H) + V(q^M) - (1-T_1)$, L type sellers randomize between strategies L1 and L2 whereas H type sellers randomize between strategies H2 and HM with probabilities $a = \frac{\left(V\left(q^H\right) - (1-T_1)\right)}{D_q}$, $b = \left(\frac{1}{T_1}\left(P_2^S - (1-T_1)\right)\right)$ and $c = 1-b$.*[104] *Platform 2's market in this interval is equal to*

$$\left(\frac{(1-T_1) - V\left(q^M\right)}{D_q}\right)(1-x) + x \tag{50}$$

[104] If the price is $P_2^S = V(q^H) + V(q^M) - (1-T_1)$, L type sellers randomize between L1 and L2 whereas H type sellers randomize by putting positive weights in their three strategies.

Proof. At $a = \frac{\left(V\left(q^H\right)-(1-T_1)\right)}{D_q}$, H type sellers are indifferent between their three possible strategies as $z_1 = z_2 = z_3$. The values that leave L type sellers indifferent between L1 and L2 are $b = \left(\frac{1}{T_1}\left(P_2^S - (1-T_1)\right)\right)$ *and* $c = 1-b$, which trivially follows from $BR^L(a;(y,b))$.■

Lemma B4 *If* $V^M < P_2^S < (1-T_1)$ *low type sellers randomize between strategies L1 and L2 with* $a = \frac{\left(P_2^S - V\left(q^M\right)\right)}{D_q}$*, and high type sellers randomize between strategies H1 and HM with probabilities* $y = \frac{\left(1-T_1-P_2^S\right)}{T_1}$ *and* $c = 1-y$.[105] *The size of the market of platform 2 in this interval of prices is*

$$\left(\frac{\left(V\left(q^H\right)-P_2^S\right)}{D_q}\right)(1-x)+\left(\frac{T_1-(1-T_1)+P_2^S}{T_1}\right)x \tag{51}$$

Proof. Whenever the value of a is smaller than $\frac{V\left(q^H\right)-(1-T_1)}{D_q}$, the best response of the high type sellers is $b = 0$. The value $y = \frac{\left(1-T_1-P_2^S\right)}{T_1}$ leaves low sellers indifferent between strategies L1 and L2. Platform 2's market trivially follows.■

Note that, whenever $\frac{D_q}{T_1} \geq \frac{1-x}{x}$, markets defined by equation (48) , (49), (50) and (51) are non-decreasing in prices.

Appendix C

Proof of Proposition 4

Assume first $V(q^M) > 0$. From proposition 3 we know that there is an equilibrium where all the sellers locate in platform 1 if the following conditions are satisfied: $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M)+T_3\}$ and $P_2^S \geq V(q^H)$. (Symmetric prices lead to an equilibrium with all the sellers in platform 2).

Assume by way of contradiction that there are equilibrium prices $\left(P_1^S, P_2^S\right)$ such that sellers locate all together in platform 1 so that $\Pi_2 = 0$. As $P_1^S \in \left[0, V(q^M)+T_3\right]$, consider the following deviation by platform 2 $P_2' = P_1^S + V(q^H) - \varepsilon > 0$. Since $V(q^H) > 0$, this deviation never leads to a configuration in which all the sellers go to platform 2 as that would require $P_1^S - P_2' \geq D_q > 0$ to hold while $P_1^S - P_2' < 0$.

At the new prices, and depending on the parameter values, the following continuation equilibria at the sellers' stage game may arise:

1) If $V(q^H) > T_1$ and $V(q^M) \leq P_1^S \leq 1-T_1$ hold, then sellers will separate by type, low type sellers will remain in platform 1 whereas high type sellers will go to platform 2. The deviation will hence result in $\Pi_2 > 0$.

2) If $P_1^S < \min\{V(q^M), 1-T_2\}$ and $P_2' > T_2$, the new location is L1HM and the deviation will yield $\Pi_2 > 0$.

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[105]If $P_2^S = (1-T_1)$ there is a set of NE in mixed strategies where L type sellers randomize between L1 and L2 with probabilities $a \in \left(\frac{(1-T_1)-V\left(q^M\right)}{D_q}, \frac{V\left(q^H\right)-(1-T_1)}{D_q}\right)$ whereas H type sellers play the strategy HM.

If $P_2^S = V\left(q^M\right)$ L type sellers play strategy L2 and H type sellers randomize between strategies H1 and HM with probabilities $y \in [0, \frac{\left(1-T_2-P_2^S\right)}{T_1})$.

3) If $P_1^S < \min\{V(q^M), T_2\}$ and $P_1^S + P_2' \leq 1$, the new location is LMHM and the deviation will also yield $\Pi_2 > 0$.

4) In any other case, sellers will play mixed strategies. The strategy of not participating in any platform is a dominated strategy for high sellers, thus, they will randomize between going to platform 1 and going to platform 2. Consequently, $\Pi_2 > 0$.

Since the deviation will be profitable, all the sellers singlehoming in one platform can not be an equilibrium, as claimed.

Consider now the case $V(q^M) = 0$. Assume again by contradiction that there are equilibrium prices $\left(P_1^S, P_2^S\right)$ such that sellers locate all together in platform 1 so that $\Pi_2 = 0$. For this to be the case prices must satisfy the following two conditions (analogous to the ones in proposition 3) i) $P_2^S - P_1^S \geq V(q^H)$; ii) $P_1^S \leq T_3$. Since $P_1^S \in [0, T_3]$, consider the following deviation by platform 2: $P_2' = P_1^S + V(q^H) - \varepsilon > 0$. Since $V(q^H) > 0$, this deviation never leads to a configuration in which all the sellers go to platform 2 as that would require $P_1^S - P_2' \geq V(q^H) > 0$ to hold and $P_1^S - P_2' < 0$.

At the new prices, and depending on parameters, the following continuation equilibria at the sellers' stage game may arise:

1) If $V(q^H) > T_1$ and $P_1^S \leq 1 - T_1$ hold then sellers will separate by type, low type sellerswill remain in platform 1 whereas high type sellers will go to platform 2. The deviation will hence result in $\Pi_2 > 0$.

2) If at least one of the two conditions in 1) is not satisfied, sellers will play mixed strategies. The strategy of not participating in any platform is a dominated strategy for high sellers, thus, they will randomize between going to platform 1 and going to platform 2. Consequently, $\Pi_2 > 0$.

Since the deviation will be profitable, all the sellers singlehoming in one platform can not be an equilibrium, as claimed.■

Proof. of Proposition 5

The proof of this proposition is organized around three lemmas: lemma C1 shows that Configuration 2 may arise as a subgame perfect equilibrium, lemma C2 shows it for Configuration 4, and lemma C3 complements the proof that is presented in the paper for conditions that imply a subgame perfect equilibrium with Configuration 3.

Lemma C1 *If $V(q^H)x < \min\{T_2, V(q^M)\} \leq \frac{1}{2}$ Configuration 2 arises as a subgame perfect equilibrium with prices $P_1^{S*} = P_2^{S*} = \min\{T_2, V(q^M)\}$. If $\min\{T_2, V(q^M)\} > \frac{1}{2}$, there is a set of subgame perfect equilibria with Configuration 2 if $P_1^S + P_2^S = 1$ and $xV(q^H) < \min\{1 - P_2^S, 1 - P_1^S\}$ are satisfied.*

Proof. We first show that there is no profitable deviation by platform 2 given $P_1^{S*} = \min\{T_2, V(q^M)\}$.

At the candidate equilibrium platform 2's profits are $\Pi_2^* = \min\{T_2, V(q^M)\}$ provided that $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$, which ensures low type sellers participation.

As shown in proposition 3 no other price by platform 2 will yield a higher profit among the prices that induce Configuration 2. Consequently, we only need to check for deviations to a higher price that would induce sellers' location L1HM provided that $P_2^S < V(q^H)$. Such a deviation is not profitable if and only if

$$P_2^S x < \min\{T_2, V(q^M)\}.$$

As $P_2^S < V(q^H)$, a sufficient condition to deter this deviation is given by $V(q^H)x < \min\{T_2, V(q^M)\}$. If $\min\{T_2, V(q^M)\} > \frac{1}{2}$ holds, participation of low type sellers is ensured by condition $P_1^S + P_2^S = 1$. The last condition deter any deviation by platform 2. Similar arguments apply to platform 1 deviations, which show our claim.■

Lemma C2 *If x is sufficiently lower than $(1 - T_1)$, a Configuration 4 with location of sellers L1H2 and prices $P_1^{S*} = (1 - T_1)$ and $P_2^{S*} = V(q^H) + T_3$ may exist as an equilibrium.*

Proof. At the candidate equilibrium platform's profits are $\Pi_1^* = (1 - T_1)(1 - x)$ and $\Pi_2^* = \left(V(q^H) + T_3\ \right) x$.

We first analyze deviations by platform 2 given $P_1^{S*} = 1 - T_1$.These deviations can be divided into three groups:

1) Deviations to prices that guarantee sellers' separation, i.e., to prices in the interval $[1, D_q + 1 - T_1]$. These deviations are not profitable as P_2^{S*} is the monopoly price.

2) Platform 2 can deviate to a lower price $P_2^S \leq 1$ to attract more sellers and obtain higher benefits. There are three price intervals to be considered. In all of them sellers play mixed strategies in equilibrium (see B2 in Appendix B):

Interval 1: $V(q^H) + V(q^M) - (1 - T_1) < P_2^S \leq 1$

As shown in lemma B1 of appendix B, at these prices low type sellers go to platform 2 with probability $(1 - a)$ and high type sellers with probability b, hence the market of platform 2 in this interval is (49).

Interval 2: $(1 - T_1) < P_2^S \leq V(q^H) + V(q^M) - (1 - T_1)$

At these prices lemma B2 shows that platform 2 gets all the high type sellers and gets the low type with probability $(1 - a)$, so that the market of platform 2 is (50). Note that (50) is higher than (49).

A sufficient condition for no deviation to any price in the two previous intervals by platform 2 is to evaluate profits at $P_2^S = 1$. The condition that arises is

$$\left(V(q^H) + T_3\right) x \geq \left(\frac{(1 - T_1) - V\left(q^M\right)}{D_q}\right)(1 - x) + x \tag{52}$$

Interval 3: $V(q^M) < P_2^S \leq (1 - T_1)$

At these prices from lemma B3 we know that platform 2 gets the low type sellers with probability $(1 - a)$ and the high type sellers with probability $(1 - y)$.The relevant market of platform 2 for this interval is given by (51). If the market in this interval is non-decreasing in P_2^S, i.e., condition $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ holds, a sufficient condition for no deviation arises. If $\Pi_2^* \geq \left[Market\left(P_2^S = 1 - T_1\right)\right](1 - T_1)$, i.e.,

$$\left(V(q^H) + T_3\right) x \geq \left[\left(\frac{V\left(q^H\right) - (1 - T_1)}{D_q}\right)(1 - x) + x\right](1 - T_1) \tag{53}$$

If $P_2^S \leq V(q^M)$ the condition that avoids any deviation to a location L2HM is

$$\left(V(q^H) + T_3\right) x \geq V(q^M) \tag{54}$$

3) Deviations to a higher price $P_2^S \geq D_q + 1 - T_1$. These deviations are not profitable deviations given that they would lead to a location of sellers $L1H1$ that implies $\Pi_2 = 0$.

Now, consider deviations by platform 1 given $P_2^{S*} = V(q^H) + T_3$. These deviations can be divided into three groups:

1) Deviations to prices that guarantee sellers' separation, i.e., to prices in the interval $\left[V(q^H) + T_3 - D_q, V(q^H) + T_3 - T_1\right]$. These deviations are not profitable as P_2^{S*} is the monopoly price.

2) Platform 1 can deviate to a lower price $P_1^S \leq V(q^H)+T_3-D_q = V(q^M)+T_3$. These prices would lead to a location of sellers L1H1 that implies $\Pi_1 = V(q^M)+T_3$. This deviation is not profitable whenever

$$V(q^M) + T_3 < (1 - T_1)(1 - x) \tag{55}$$

3) Deviations to a higher price $P_1^S \geq V(q^H)+T_3-T_1$. The best deviation would imply attrating H type sellers while L type are lost (location L0H1). The maximum price that ensures to attain L0H1 is $P_1^S = V(q^H) + T_3 - \varepsilon$, so that platform 1 will not deviate if and only if

$$\left(V(q^H) + T_3\right) x \leq (1 - T_1)(1 - x) \tag{56}$$

Note that (53) and(56)are compatible if and only if $\frac{(1-T_1)-V(q^M)}{D_q} > \frac{x}{1-x}$

Note that (52) and(56)are compatible if and only if $\frac{(1-T_1)(D_q-1)+V(q^M)}{D_q} > \frac{x}{1-x}$.

Condition (56) is compatible with $V(q^H) > 1$ if and only if $\frac{(1-T_1)}{(1+T_3)} > \frac{x}{1-x}$.

The common feature of the conditions are that x must be sufficiently low and $(1 - T_1)$ sufficiently high to hold.■

Lemma C3 *Consider $V(q^H) > 1$ and $V(q^M) > T_2$. If $xV(q^H) = V(q^M) > (1 - T_1)(1 - x)$, Configuration 3 arises as a subgame perfect equilibrium with prices $P_1^{S*} = V(q^M)$ and $P_2^{S*} = V(q^H)$ and location of sellers* L1HM.

Proof. Since for $P_1^S = V(q^M)$ to be an equilibrium it is needed that $(1 - T_2) > V(q^M)$ (see proposition 3), we further assume that this condition holds. The result is obtained for the case where $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ satisfies.[106]

The proof of this lemma is presented in the paper after the statement in proposition 5. There is only one deviation that rest to be analyzed, the one in the interval $1 - T_1 < P_1^S < 1$ at which sellers play mixed strategies in equilibrium (see B2 in appendix B).

In particular, platform 1 gets the low type sellers with probability a and gets the high type sellers with probability y. Platform 1's market is given by (48) that under the condition $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ is non-decreasing in P_1^S. Consequently profits at these prices are bounded above by the profits at $P_1^S = 1$. No deviation will take place if

$$V\left(q^M\right) > \left(\frac{V\left(q^H\right) - 1}{D_q}\right)(1 - x) + x. \tag{57}$$

Note that (57) is implied by (47) as $V\left(q^H\right) > 1$.■

Proof. of Proposition 6

i) From lemma C1 it follows that if $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$, platforms' profits are $\Pi_1^* = \Pi_2^* = \min\{T_2, V(q^M)\}$. If the contrary occurs, the profits are $\Pi_1 = 1 - P_2^S$

[106]We have shown in the Appendix B that under this condition the market of platform 1 when sellers play mixed strategies is non-decreasing in its own price.

and $\Pi_2 = 1 - P_1^S$, in which case there is a set of possible equilibria and profits of both platforms will be equal if and only if $P_1^S = P_2^S = \frac{1}{2}$.

ii) From proposition 3 we know that prices are going to be such that $P_1^S \leq \min\{1 - T_2, V(q^M), P_2^S\}$ and $P_2^S \leq V(q^H)$. Note that in equilibrium platforms will optimally charge prices $P_1^S = \min\{1 - T_2, V(q^M)\}$ and $P_2^S = V(q^H)$. At any other price there exists a profitable deviation for at least one of the platforms. Moreover, both platforms have the possibility of getting the other platform's profits by setting its price. Setting a price $P_2^S = \min\{1 - T_2, V(q^M)\} - \varepsilon$ platform 2 attracts low type sellers and gets the profits that platform 1 obtains in the equilibrium. With a price $P_1^S = V(q^H) - \varepsilon$ platform 1 attracts high type sellers, loses low type sellers and gets the profits of platform 2 in equilibrium. These two deviations will not be profitable whenever profits in equilibrium are equal, which shows the statement.

iii) Let platform 1 be the low quality platform and platform 2 be the high quality one. The statement follows from two facts. On the one hand, the low quality platform always has the possibility of getting the profits of the high type one (whenever $P_2^S > 1 - T_1$). Setting a price $P_1^S = P_2^S - \varepsilon$, platform 1 attracts H type sellers, loses L type sellers and gets the profits of the high quality platform. It would be a profitable deviation if the high quality platform had higher profits than the low quality one. On the other hand, given P_1^S, platform 2 can not always replicate the situation of platform 1. This fact explains the asymmetry between profits.■

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