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Departamento de Estadística y Econometría  
Universidad Carlos III de Madrid  
Calle Madrid, 126  
28903 Getafe (Spain)  
Fax (341) 624-9849

## NON-EXACT PRESENT VALUE RELATIONS

Jesús Gonzalo and Martín González Rozada\*

### Abstract

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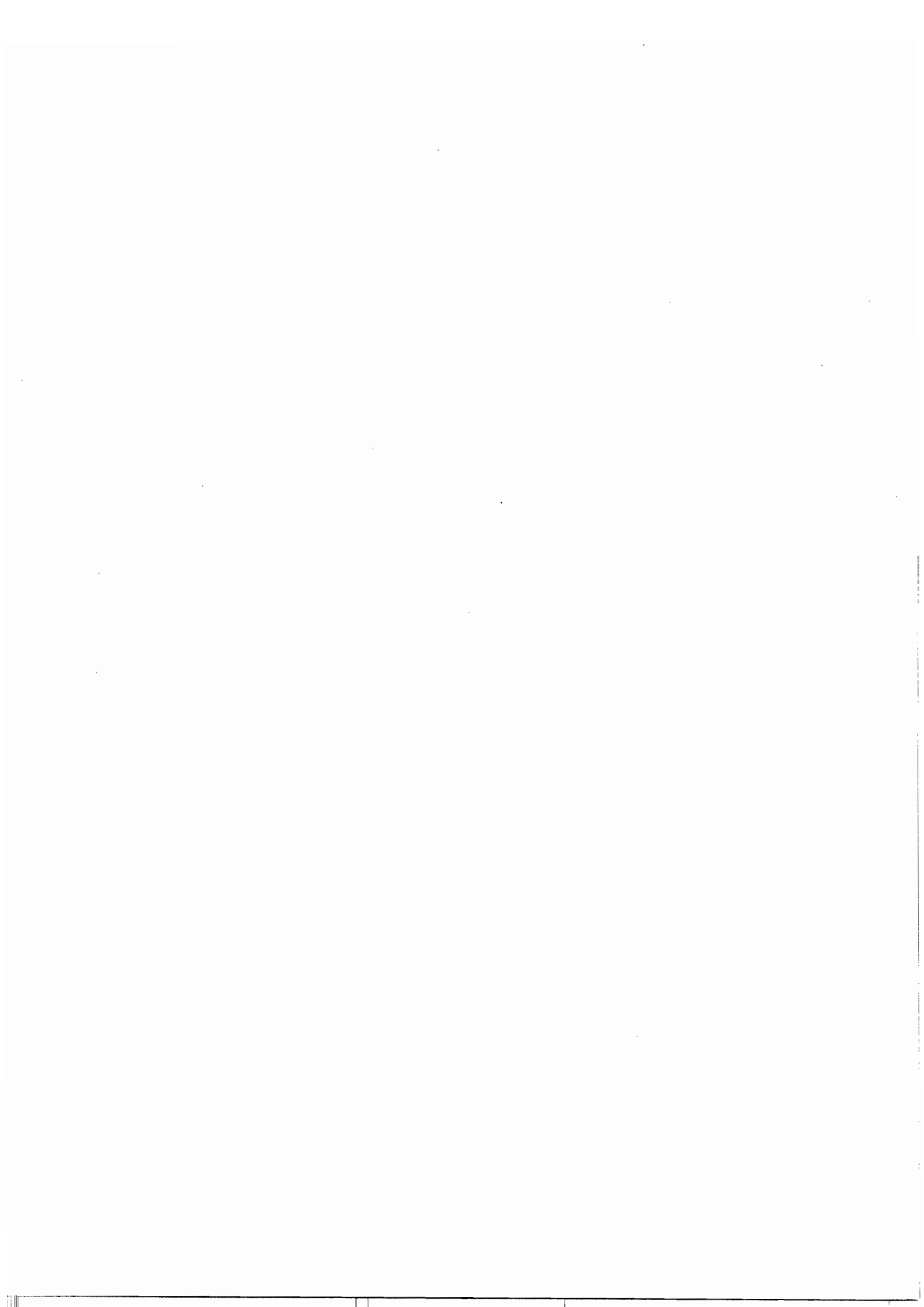
One of the most commonly used and, at the same time, rejected models in finance and macroeconomics is the exact present value model (PVM), where a variable  $Y_t$  is expressed as the expected value at time  $t$  of the sum of discounted future values of another variable  $X_t$ . This paper generalizes the PVM by making it non-exact (NEPVM) in a simple way, allowing us to study situations with time varying discount factors, transitory deviations from the exact PVM, as well as situations with correlated market returns. The proposed NEPVM satisfies all the equilibrium conditions the exact PVM does, but at the same time it is more robust in the sense that rejections produced by the standard volatility and cross-equation restriction tests are not enough to reject the NEPVM. The paper presents the new variance bounds and cross-equation restrictions implied by the NEPVM and it shows how to test them. This paper also shows how to discriminate between the exact PVM and the NEPVM by testing for a deeper level of cointegration: multicointegration. The paper finished by analyzing empirically the cases of stock prices and dividends, short- and long-term interest rates, and farmland prices. Although the exact PVM is rejected in the first two examples, as the literature has largely reported, we are unable to reject the NEPVM. This fact, together with the theoretical results contained in the paper, suggests that the proposed NEPVM could be compatible with some of the empirical findings in the literature.

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### Keywords:

Multicointegration; present value model; VAR; volatility tests.

\*Gonzalo, Boston University and U. Carlos III de Madrid.; González, Boston University and Universidad de Buenos Aries. Jesús Gonzalo gratefully acknowledges financial aid from the Fundación Caja de Madrid. Corresponding address: Jesús Gonzalo, Departamento de Estadística y Econometría, U. Carlos III de Madrid, 28903 Getafe, Madrid (Spain).



## NON-EXACT PRESENT VALUE RELATIONS

Martín González Rozada

Boston University and

Universidad de Buenos Aires

Email: che@cedes.edu.ar

fax: (541) 862-0805

Jesús Gonzalo\*

Boston University and

U. Carlos III de Madrid

Email: jgonzalo@elrond.uc3m.es

fax: (341) 624-9849

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# 1 Introduction

Present value models have been extensively used to interpret the behavior of financial and macroeconomic time series. A present value relationship between two variables states that one of the variables (an endogenous variable) can be written as a linear function of the summed discounted value of expected future values of the other variable (a forcing variable). Let  $Y_t$  and  $X_t$  be an endogenous and a forcing variable, respectively. Then,

$$Y_t = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_t X_{t+i}, \quad (1)$$

where  $\theta$  is the coefficient of proportionality and  $\delta < 1$  is the discount factor.  $E_t$  denotes mathematical expectation conditional on the full public information set  $I_t$ , which includes  $Y_t$ ,  $X_t$  and their lagged values. For simplicity we do not add a constant term in the right hand side.

In finance, dynamic stochastic models like (1) have been used, for instance, to describe the expectations theory of the term structure, where  $Y_t$  is the long-term yield and  $X_t$  is the short-term yield (see e.g. Campbell and Shiller 1987a, 1987b, and Matthey and Meese 1986); and to explain the behavior of stock prices and dividend payments (see e.g. Campbell and Shiller 1987a, 1989, Bong-Soo Lee 1991 and West 1987, 1988).

In macroeconomics, the present value model (PVM) (1) has been applied in such situations as the following: testing the validity of Cagan's model of hyperinflation (see Engsted, 1993); analyzing whether the conduct of US fiscal policy has been influenced by constraints on the accumulated stock of outstanding Federal debt (see Kremers, 1989 and Hamilton and Flavin, 1986); and representing the permanent income theory of consumption. In the third case, equation (1) can be rearranged so that it becomes a statement about savings, by writing saving equals the expected present value of future declines in labor income (see e.g. Campbell 1987, Campbell and Deaton 1989 and Flavin 1981, 1993).

Despite the simplicity of its structure, or maybe as a consequence of it, there exists a high degree of controversy about the validity of this *exact* PVM (EPVM). In fact, the EPVM has been rejected very often in the applications reported above.

The principal goal of this paper is to present and analyze a model that maintains the fundamental aspects of the standard PVM but, at the same time, is more difficult to reject.

The new model is obtained by incorporating an error term into equation (1). This non-exact present value model (NEPVM) maintains all the the essential features of the EPVM and is derived from the same type of equilibrium conditions. Formally, it is given by the following expression,

$$Y_t = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_t X_{t+i} + \epsilon_t, \quad (2)$$

where the additive component  $\epsilon_t$  is an error term, which has a different economic interpretation depending on the application under study. In general,  $\epsilon_t$  will represent transitory deviations from the equilibrium conditions that generate a PVM like (1).

In the expectation theory of the term structure  $\epsilon_t$  could represent a time-varying term premium. In the dividends-stock prices models  $\epsilon_t$  could capture the influence of noise traders. In the permanent income theory of consumption, this disturbance term describes the transitory consumption component. Three additional sources for this error, valid under any theory, are: measurement error in the observed variables, the possibility that equation (2) represents an approximation of a more complex non-linear relationship, and the consequence of considering a constant discount factor when, in fact, it is time varying.

The generalization of the EPVM by adding an error term changes drastically most of the standard conditions (cross-equation restrictions and volatility conditions) used to test for the EPVM. This paper shows the new conditions and identifies the cases under which cointegration is the only testable econometric implication of the NEPVM.

The paper is organized as follows. In Section 2 we derive an NEPVM from an arbitrage condition and we show how to discriminate statistically between the EPVM and the NEPVM by testing for a deeper level of cointegration: multicointegration. Section 3 introduces the new set of cross-equation restrictions, implied by the NEPVM, and shows how to test them for different structures of the error term. Section 4 shows the volatility tests implied by both models. Section 5 empirically analyzes the cases of stock prices and dividends, short- and long-term interest rates and farmland prices. Although the EPVM is rejected in the first two examples, as the literature has largely reported, we are unable to reject the NEPVM. This fact, together with the theoretical results contained in the paper, suggests that the proposed NEPVM could be compatible with some of the empirical findings in the literature. The conclusions are found in Section 6. Proofs are provided in the appendix.

Throughout the paper, the variables involved in the PVM will be assumed to contain a random walk component (to be I(1)).

## 2 Cointegration and Multicointegration

This section generalizes the EPVM by allowing for an error term, like in expression (2). As we mentioned in the introduction, the error term has a different meaning depending on the application under study. One of the standard excuses given in the literature, when the EPVM is rejected, is that the discount factor may not be constant through time. Here, we consider the error,  $\epsilon_t$ , as a way to model time varying discount factors. There are many forms to model that, and in this paper we choose to do it in a very simple way to avoid possible non-linearities.

For concreteness, the discussion is centered on the model for stock prices. In that model, if risk neutral agents arbitrage between a risky asset,  $Y_t$ , and a riskless asset, the expected rate of return of  $Y_t$ , which is equal to the expected rate of capital gain plus the dividend-price ratio, must equal the riskless rate,

$$E_t R_t \equiv \frac{E_t Y_{t+1} - Y_t}{Y_t} + \frac{X_t}{Y_t} = r_t = r + \frac{\nu_t}{Y_t}, \quad (3)$$

where  $r_t = r + \frac{\nu_t}{Y_t}$  is the interest rate, assumed to be variable, but known. That is, the time varying interest rate has two components, a constant rate,  $r$ , plus a variable term, represented by an error  $\nu_t$  in terms of the stock price. Both, the constant rate and the variable term are observed by private agents, so that there is no room to make any profit by arbitraging between the two assets. Equation (3) can be rearranged as

$$Y_t = \delta E_t(Y_{t+1} + X_t) - \delta \nu_t. \quad (4)$$

where  $\delta = (1 + r)^{-1}$ .

From (4), and assuming the transversality condition of the EPVM,  $\lim_{i \rightarrow \infty} \delta_j^i E_t Y_{t+i} = 0$ , is satisfied, we can express  $Y_t$  as the expected discounted value of current and future values of  $X_t$ . This is expression (2) with  $\epsilon_t = -\delta \nu_t$ .

Throughout the paper we allow for stationary ARMA structures in the error term. In particular,  $\epsilon_t$  could be an invertible  $MA(q)$ ,  $q = 0, 1, 2, \dots$  or a stationary  $AR(p)$ ,  $p = 1, 2, \dots$  process. The  $MA(0)$  case, corresponds to an error term uncorrelated with all lagged information, and

this is usually the assumption made for the transitory consumption when working with the permanent income theory. The  $MA(q)$ , with  $q \neq 0$ , and the  $AR(p)$  processes, for instance, are consistent with correlated market returns, in the stock prices and dividends case. If  $\epsilon_t$  is equal to zero, the model collapses into an EPVM. Notice that with this structure, the error term cannot represent a bubble term, since the bubble is explosive by construction. That is, if  $\epsilon_t$  is a bubble term, it must satisfy  $\epsilon_t = \delta E_t \epsilon_{t+1}$  in order to be another solution to (1), but to be a solution to (4) we require that  $E_t \epsilon_{t+1} = 0$ , which contradicts the definition of a bubble term.

Since implications of exact and non-exact models are different, it is important to learn how to tell between the EPVM and NEPVM. Following Engsted, Gonzalo and Haldrup (1995) (EGH hereafter), the following proposition establishes how to do it.

**Proposition 1.** *Let  $Y_t$  and  $X_t$  satisfy the present value relationship (2). Then,*

1.  $Y_t$  and  $X_t$  are cointegrated, with cointegrating vector  $(1, -\theta)$ , even when  $\epsilon_t = 0$ .
2. If  $\epsilon_t = 0$ , the PVM implies that  $Y_t$  and  $X_t$  are multicointegrated time series.

What Proposition 1 states, is that both, exact and non-exact PVMs, imply cointegration, but only the EPVM implies a deeper level of cointegration: multicointegration. It also shows another reason why  $\epsilon_t$  can not be a bubble, otherwise the variables would not be cointegrated.

**Definition of Multicointegration.** *(Granger and Lee (1988)) Assume that  $Y_t$  and  $X_t$  are cointegrated time series such that  $S_t = Y_t - \theta X_t$  is stationary. If the integral  $I(1)$  - variable  $\Delta^{-1} S_t = \sum_{j=1}^t S_j$  cointegrates with  $X_t$  such that a parameter  $\lambda$  exists whereby  $\Delta^{-1} S_t - \lambda X_t$  is also a stationary relationship, then  $Y_t$  and  $X_t$  are said to be multicointegrated.*

The basic characteristic of multicointegrated time series is that the integral of equilibrium errors at one level of cointegration, will cointegrate with the level of the variables  $Y_t$  and  $X_t$ . Therefore, knowing that the statistical concept of multicointegration delivers a necessary condition for the model to be an EPVM, we can check empirically these kind of models by testing for multicointegration.

There are two methods to contrast multicointegration, a two-step procedure and a one-step procedure. In the two-step procedure, cointegration is first tested by using standard techniques

such as a unit root test on the residuals of a regression of  $Y_t$  over  $X_t$ . In the second step, these residuals are cumulated, and this new variable is regressed on  $X_t$ , a constant and a trend. Subsequently, the integration order of the residuals from the second step regression is tested. If the residuals are  $I(0)$  the series are multicointegrated.

The one-step procedure simultaneously tests both levels of cointegration and, as EGH have argued, it has several statistical advantages compared to the two-step procedure. So, following these authors, we use the one-step procedure.

The idea is to contrast the integration order of the residuals,  $\hat{u}_t$ , from the integral regression,

$$\Delta^{-1}Y_t = \alpha_0 + \alpha_1 t + \alpha_2 \Delta^{-1}X_t + \alpha_3 X_t + u_t. \quad (5)$$

If  $\hat{u}_t$  follows an  $I(0)$  process, then  $Y_t$  and  $X_t$  are multicointegrated time series; on the other hand, if the residuals follow an  $I(1)$  process, there is cointegration but no multicointegration between  $Y_t$  and  $X_t$ ; and finally if  $\hat{u}_t$  follows an  $I(2)$  process, there is neither cointegration nor multicointegration between the variables of the model. The inclusion of a time trend, in the regression above, obeys the fact that if the single series  $Y_t$  and  $X_t$  have a non-zero mean, then cumulated series will have a trend.

Hence, testing for multicointegration can be done by using a residual based test, such as the Dickey-Fuller unit root test, applied to the regression residuals  $\hat{u}_t$ . The limiting distribution of this Dickey-Fuller test, is a non standard one, and it is tabulated in EGH.

### 3 Cross-Equation Constraints

Since both models imply cointegration, by the Granger Representation Theorem (Engle and Granger, 1987),  $Y_t$  and  $X_t$  obey an error-correction model. Therefore, they have a vector autoregressive representation. We approximate this representation by using a finite vector error correction model of order  $p$ :

$$W_t = C + \gamma \alpha' Z_{t-1} + \sum_{j=1}^p \Gamma_j W_{t-j} + \eta_t, \quad (6)$$

where  $W_t = (\Delta Y_t, \Delta X_t)'$ ,  $Z_{t-1} = (Y_{t-1}, X_{t-1})'$ , and  $\eta_t$  is a vector white noise.

Following Campbell and Shiller, we define a limited information set  $H_t$ , observable to the econometrician, that includes current and lagged values of  $X_t$  and  $Y_t$ . The difference with



respect to the full market information set,  $I_t$ , comes through the amount of knowledge of the disturbance term. Private agents generally have more information than econometricians ( $H_t \subseteq I_t$ ). This paper reflects this fact by assuming that,

$$E(\epsilon_t|H_t) = \mu_t. \quad (7)$$

With this specification, we allow for both, the possibility that the econometrician observe the time varying discount factor, in which case,  $\mu_t = \epsilon_t$ ; and the opposite case, where the econometrician does not have any information about it (i.e.  $\mu_t = 0$ ). The results obtained in this section are not modified in any of these events.

Using (6), it is possible to write a VAR model for the stationary variables  $\Delta X_t$  and  $S_t$ ,

$$\begin{bmatrix} \Delta X_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}, \quad (8)$$

where  $S_t = Y_t - \theta X_t$ ,  $\Delta = 1 - L$  and the polynomials in the lag operator  $a(L), b(L), c(L)$  and  $d(L)$  are all of order  $p$ . To simplify notation, (8) can be written in first order form as  $z_t = Az_{t-1} + v_t$ , where  $z_t$  is the vector  $[\Delta X_t, \dots, \Delta X_{t-p+1}, S_t, \dots, S_{t-p+1}]$  and the matrix  $A$  is called the companion matrix of the VAR. Then for all  $i$ ,  $E(z_{t+i}|H_t) = A^i z_t$ .

The cross-equation constraints implied by the EPVM (NEPVM) are given by subtracting  $\theta X_t$  from expression (1) (expression (2)) and projecting both sides of the resulting equation onto  $H_t$ . To do this, we define two vectors of  $2p$  elements,  $g'$  and  $h'$ , such that  $g'z_t = S_t$  and  $h'z_t = \Delta X_t$ . These constraints are given, for the exact and non exact PVM, in the following two propositions,

**Proposition 2.** (EPVM) *Let  $Y_t$  and  $X_t$  satisfy the exact present value relationship (1). Then, the true innovation at time  $t$  in  $Y_t$ ,  $\xi_t \equiv Y_t - \frac{1}{\delta}[Y_{t-1} - \theta(1 - \delta)X_{t-1}]$ , is unpredictable given information available at time  $t-1$ . This implication can be contrasted by regressing  $S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t$  on information available at time  $t-1$  or by testing the following cross-equation restrictions derived from the VAR model (8)*

$$g'(I - \delta A) = \theta h' \delta A$$

**Proposition 3. (NEPVM)** Let  $Y_t$  and  $X_t$  satisfy the non exact present value relationship (2).

**Case 1:  $\epsilon_t$  is an  $MA(q)$  error term.**

$\xi_{t+q+1}$ , is unpredictable given information available at time  $t-1$ . This implication can be contrasted by regressing  $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$  onto information available at time  $t-1$  or by testing the following cross-equation restrictions derived from the VAR model (8)

$$g'(I - \delta A)A^{q+1} = \theta\delta h'A^{q+1}.$$

**Case 2:  $\epsilon_t$  is an  $AR(p)$  error term.**

$\xi_{t+i}, i = 0, 1, 2, \dots$ , is predictable given information available at time  $t-1$ . No cross-equation restrictions can be derived from a VAR model for  $\Delta X_t$  and  $S_t$ .

Proposition 2 presents the usual implications of the EPVM analyzed in the literature (see Campbell and Shiller 1987a), and Proposition 3 shows the implications of the more general model. In the last case, when the error term follows an  $MA(q)$  process, the cross-equation constraints posed by the NEPVM on the coefficients of the VAR change with respect to the constraints imposed by the EPVM (see Proposition 2), while when  $\epsilon_t$  is an  $AR(p)$  process, it is not possible to derive any of the standard cross-equation restrictions on the VAR model. It should be clear that one of the consequences from Proposition 3 is that rejection of the cross equation restrictions, given in Proposition 2, do not imply that the NEPVM is false. Moreover, rejection of the cross-equation constraints of Proposition 3 is compatible with an NEPVM with autocorrelated errors. Generalization of last proposition's results to an  $ARMA(p, q)$  process for  $\epsilon_t$  is straightforward.

It is worth mentioning that the presence of the error term, even in the case of a white noise process, produces autocorrelation in the errors of the regression used to test the cross equation constraints. Therefore, in order to contrast properly the hypothesis that  $\xi_{t+q+1}$  is unpredictable given information available at time  $t-1$ , the test has to be robust against autocorrelation of the perturbation term. To see the existence of this correlation, consider the case of  $q = 0$ , and write the VAR model (8) for period  $t+1$ . Multiply that by the linear combination  $(\theta, 1)$  and subtract  $\frac{1}{\delta}S_t$  from it. Writing the result in  $(z_t, v_t)$  notation, we have

$$g'z_{t+1} + \theta h'z_{t+1} - \frac{1}{\delta}g'z_t = (g'A + \theta h'A - \frac{1}{\delta}g')z_t + (g' + \theta h')v_{t+1} \implies$$

$$\begin{aligned}
g'z_{t+1} + \theta h'z_{t+1} - \frac{1}{\delta}g'z_t &= \frac{1}{\delta}(\theta\delta h' - g'(I - \delta A))Az_{t-1} \\
&+ \frac{1}{\delta}(\theta\delta h' - g'(I - \delta A))v_t + (g' + \theta h')v_{t+1}.
\end{aligned} \tag{9}$$

Using the last expression and the definition of  $\xi_t$  it is possible to write  $\xi_{t+1} = g'z_{t+1} + \theta h'z_{t+1} - \frac{1}{\delta}g'z_t$  (see Appendix). Therefore,  $\xi_{t+1}$  is a function of lagged values of  $S_t$  and  $\Delta X_t$  plus an autocorrelated error term. It is clear now that the testing of  $E_{t-1}\xi_{t+1} = 0$  has to be robust against autocorrelation.

An additional fact arising from equation (9) is that, in order to get  $E_{t-1}\xi_{t+1} = 0$  we need  $(\theta\delta h' - g'(I - \delta A))A = 0$ . These are the cross-equation constraints given in Proposition 3 for  $q = 0$ . Therefore, running a regression of  $S_{t+1} - \frac{1}{\delta}S_t + \theta\Delta X_{t+1}$  on information available at time  $t-1$  and then testing that the coefficients of the variables reflecting this information are jointly zero is equivalent to construct a Wald test for the cross-equation restrictions in the VAR model (8).

## 4 Volatility Tests

The most common rejection of the PVM comes from the so called volatility tests. These tests are designed to examine if it is possible that stock prices can be explained by the present discount value of dividends. In the following proposition we present the principal variance bound tests.

**Proposition 4. (EPVM)** *Let  $Y_t$  and  $X_t$  satisfy the present value relationship (1). Define  $Y_t^*$  as the “perfect foresight” or “ex-post rational price”*

$$Y_t^* = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i X_{t+i},$$

and let  $Y_t^0$  be some “naive forecast” of  $Y_t$ :

$$Y_t^0 = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i F_t X_{t+i},$$

where  $F_t X_{t+i}$  denotes a naive forecast of  $X_{t+i}$  made at time  $t$ . Rational agents at time  $t$  have access to this naive forecast. Then,

1. The variance of the ex-post rational price provides an upper bound to the variance of the observed  $Y_t$ ,

$$\text{Var}(Y_t^*) \geq \text{Var}(Y_t).$$

2. The market price is a better forecast of the ex-post rational price, in terms of the mean square error, than is the naive forecast price,

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2,$$

3. The ex-post rational price is more volatile around  $Y_t^0$  than the market price.

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2.$$

4. The variance ratio between the actual spread,  $S_t$ , and the theoretical spread,  $S_t'$ , defined as the unrestricted VAR forecast given  $H_t$ , of the present value of all future changes in  $X$ , should be one. Also, the innovations variance ratio between the true innovation at time  $t$  in  $Y_t$ ,  $\xi_t$ , and the innovation at time  $t$  in  $Y_t$  given  $H_t$ ,  $\xi_t'$ , should be one. That is,

$$\text{Var}(S_t)/\text{Var}(S_t') = 1,$$

and,

$$\text{Var}(\xi_t)/\text{Var}(\xi_t') = 1,$$

The first statement of Proposition 4, was presented by Shiller (1981) in his seminal work on the volatility of stock markets. This statement is valid when the variables are stationary. The second and third tests were developed by Mankiw, Romer and Shapiro (1985) and they showed

that these statements are immune to the problems of the Shiller's test. The bias problem generated by the use of sample variances instead of population variances, and the fact that when the forcing variable does not follow a stationary process, Shiller's test may not be appropriated. The last statement of Proposition 4, presents two volatility tests under the VAR framework explained in the precedent section. These tests are also consistent with variables integrated of order one.

**Proposition 5. (NEPVM)** *Let  $Y_t$  and  $X_t$  satisfy the present value relationship (2). Assume that  $E(\epsilon_t|H_t) = \mu_t \neq 0$ . Define  $Y_t^*$  as the "perfect foresight" or "ex-post rational price" and let  $Y_t^0$  be some "naive forecast" of  $Y_t$  as in Proposition 4. Then,*

1.

$$\text{Var}(Y_t^*) \geq \text{Var}(Y_t) \iff \text{Var}(w_t) + \text{Var}(\epsilon_t) \geq 2\text{Cov}(Y_t, \epsilon_t),$$

where  $w_t$  is a forecast error equal to  $Y_t^* - (Y_t - \epsilon_t)$ .

2.

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2 \iff E(Y_t - Y_t^0)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]$$

3.

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2 \iff E(Y_t^* - Y_t)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)].$$

4. *The variance ratio between the actual spread,  $S_t$ , and the theoretical spread,  $S'_t$ , defined in the previous proposition, could be different from one. Also, the innovations variance ratio between  $\xi_t$ , and  $\xi'_t$ , could be different from one.*

Proposition 5, shows the same tests of Proposition 4 but for the NEPVM. Again, the first statement is valid when variables are stationary, while the last three tests are valid even when the variables are integrated of order one.

From the last proposition, it is clear that the variance bound test procedures valid for the exact PVM produce inconclusive results under the more general NEPVM. In other words, rejection of the EPVM by the standard volatility tests do not imply rejection of the NEPVM.

Implications of Proposition 5 do not change if the econometrician observe the time varying discount factor,  $\mu_t = \epsilon_t$ . If the econometrician does not have any information about it,  $\mu_t = 0$ , the only modification is that statement 4 in Proposition 5 is equivalent to the last statement in Proposition 4, that is, the variance ratios are equal to unity.

## 5 Applications

In this section we contrast the validity of the present value model by applying the tests developed in this paper, to stock prices and dividends, short- and long-term interest rates and farmland prices series. Since, there is clear empirical evidence in the literature that the individual series are  $I(1)$ <sup>1</sup> we do not present these results here.

The rest of the section is organized as follows. For each case, first, we test for cointegration using Johanssen's LR test. Then, we test for multicointegration to check if the model is an exact PVM or an NEPVM. Finally, we contrast the implications of the specific model.

If the EPVM is rejected in favor of an NEPVM, then to test that  $E_{t-1}\xi_{t+q+1} = 0$ , we use the following test procedure. cross-equation constraints are nested, in the sense that, if they are satisfied for an NEPVM with  $MA(q)$  errors, they are also satisfied for models with  $MA$  errors of order greater than  $q$ . Therefore, we choose a maximum value for  $q$  and perform the test, if the cross-equation restrictions are satisfied, we go down, and perform the test for  $q - 1$ , and so on. We stop when we find a value of  $q$ , let say  $q = k$ , for which the cross-equation restrictions are not satisfied. Then, we select the model as a NEPVM with  $MA(k + 1)$  errors.

For a given  $q$ , in order to test the cross-equation constraints derived in Proposition 3, we use a regression test instead of a Wald test. The main reason is that the cross-equation constraints are nonlinear, and as Gregory and Veall (1985) pointed out, in finite samples, changing the form of a nonlinear restriction to a form which is algebraically equivalent under the null hypothesis will change the numerical value of the Wald test statistic. Therefore, we perform the equivalent test (robustized against autocorrelation) given by regressing  $\xi_{t+q+1}$  on information at  $t - 1$  and then testing that the coefficients of the variables reflecting this information are jointly zero.

If the cross-equation constraints are not satisfied for all  $q$ , we can interpret this result as

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<sup>1</sup>See Campbell and Shiller 1987a, for stock prices and dividends and for short and long term interest rates; and Tegene and Kuchler 1991, for farmland prices.

consistent with an NEPVM with autocorrelated errors or as evidence against the validity of the NEPVM.

As we already mentioned in the introduction, in practice, the EPVM has been rejected very often because the volatility conditions of Proposition 4 are violated or because the cross-equation restrictions of Proposition 2 are not satisfied by the data<sup>2</sup>.

## 5.1 Stock Prices and Dividends

The data we use here is the same data used by Campbell and Shiller (1987a)<sup>3</sup>. That is, the stock prices series is the Standard and Poor's composite index for January, divided by the January producer price index (1967=100), while the dividends series is a combination of series taken from Cowles (1939) up to 1925, and the dividends per share for the Standard and Poor's composite index from 1926 on. The total sample goes from 1871 to 1985. In what follows, we use the dividends of  $t-1$  as a proxy for period  $t$ <sup>4</sup>.

Figure 1, displays stock prices and dividends series.

FIGURE 1 ABOUT HERE

Table 1, shows Johansen's LR and EGH multicointegration tests results. Assuming the possibility of a linear trend in the data, the likelihood ratio test indicates cointegration between the variables at 5% significance level. The EGH test shows no evidence of multicointegration. Therefore, since there is cointegration but not multicointegration, we conclude that stock prices and dividends do not obey an exact PVM and they could follow a NEPVM.

TABLE 1 ABOUT HERE

Next, we check the cross-equation restrictions using a maximum value of  $q = 4$ . We construct robust tests against heteroskedasticity and autocorrelation using the correction suggested by Newey and West (1987). Table 2 presents the results of these regressions. These results are

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<sup>2</sup>In particular, for the three cases analyzed here, see e.g. Shiller 1989, Chapter 4, and all the references quoted there; Campbell and Shiller 1987a, and Engsted 1994.

<sup>3</sup>We thanks John Campbell for providing us with the data

<sup>4</sup>This approximation is usually made in the literature due to the fact that stock prices are observed at the beginning of the period but dividends are paid some time during the same period.

consistent with a NEPVM with an  $MA(1)$  error term, since the cross-equation constraints are accepted at standard significance levels. This fact implies correlated market returns, but not necessarily evidence against the hypothesis of market efficiency, as many papers have already reported (see Fama, 1991, for a review on efficient capital markets).

TABLE 2 ABOUT HERE

## 5.2 Short- and Long-Term Interest Rates

We use the zero-coupon yield data set from McCulloch (1990). The same data Campbell and Shiller (1991) used to reject the exact version of the expectations hypothesis of the term structure of interest rates. The data is monthly and covers the period 1952:1 - 1991:2 of a 1-month yield and a 5-year yield. We found similar results for the total period than for the 1952-1978 subperiod (the longest possible subsample which avoids the 1979 monetary policy regime shift) and the 1979-1991 subsample. Therefore, we only report here results using the entire sample.

In figure 2, the interest rates series are displayed.

FIGURE 2 ABOUT HERE

Johansen's cointegration test assuming a linear trend in the data indicates cointegration at 5% significance level (Table 3). The cointegration vector is  $(1, -1.007)$ . The cointegration ADF-test allowing for variables integrated of order two indicates no multicointegration at standard significance levels. Therefore, we conclude that interest rates do not obey an EPVM and they could be represented by an NEPVM.

TABLE 3 ABOUT HERE

Table 4, shows the cross-equation restrictions regression tests for a fixed value of  $q = 8$ . It is clear from the table, that the cross-equation constraints are not satisfied for any value of  $q$ . Therefore, this could be interpreted as evidence against the validity of an NEPVM or as consistent with an NEPVM with autocorrelated errors.

TABLE 4 ABOUT HERE



### 5.3 Farmland Prices

The rational expectations version of the PVM is also used to explain the behavior of farmland prices, see e.g. Falk (1991) and Engsted (1994). The PVM explains the real price per acre of farmland in terms of the expected future discounted sum of the real rent paid per acre of farmland.

We use annual farmland prices and rent data for the Corn-Belt agricultural region in the US. The data span the period 1921 to 1989 and it is the same used by Tegene and Kuchler (1994).

Figure 3 displays the farmland price and rent series. Johansen's LR test, assuming a linear trend in the data, indicates cointegration at standard significance levels.

FIGURE 3 ABOUT HERE

The ADF-test of the residuals of the regression of  $\Delta^{-1}Y_t$  on a constant, a trend,  $X_t$  and  $\Delta^{-1}X_t$  implies multicointegration at 5% significance level (Table 5). Since the time series are multicointegrated, they could be represented by an exact PVM. These results are equivalent to those founded by EGH. Engsted (1994) presents and rejects the cross-equation constraints and volatility tests implied by the exact model. Since our results are similar to his, we don't show them here.

TABLE 5 ABOUT HERE

## 6 Conclusion

The standard Present Value Model holds much theoretical attraction but it has been empirically rejected very often, as the literature of finance and macroeconomics has reported.

This paper shows that what has been rejected in most cases is the *exact* version of the PVM. A very simple generalization of it, the NEPVM, while maintaining all the fundamental characteristics of the EPVM, is more difficult to reject. In fact, there are situations where cointegration is the only testable econometric implication from the non-exact PVM.

Further research, we believe, should analyze the testable implications of the NEPVM with different models for the error term, especially GARCH and non-linear models.

## Appendix

**PROOF: Proposition 1.** For Statement 1,  $\epsilon_t = 0$ , see Campbell and Shiller (1987), and for Statement 1,  $\epsilon_t \neq 0$ , see EGH. For Statement 2. see EGH.

**PROOF: Proposition 2.** See Campbell and Shiller (1987a).

**PROOF: Proposition 3, Case 1.** Assume,  $\epsilon_t = \sum_{k=0}^q \alpha_k u_{t-k}$ ,  $\alpha_0 = 1$ . Define the variable,

$$\xi_t \equiv Y_t - \frac{1}{\delta} [Y_{t-1} - \theta(1 - \delta)X_{t-1}]. \quad (10)$$

Replacing  $Y_{t-1}$  into (10) and rearranging we get,

$$\xi_t = Y_t - \frac{1}{\delta} [\theta(1 - \delta) \sum_{i=1}^{\infty} \delta^i E_{t-1} X_{t+i-1} + \epsilon_{t-1}], \quad (11)$$

using the last expression we obtain,

$$\xi_t = Y_t - \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_{t-1} X_{t+i} - \frac{1}{\delta} \epsilon_{t-1}, \quad (12)$$

or,

$$\xi_t = Y_t - E_{t-1} Y_t + \sum_{k=1}^q \alpha_k u_{t-k} - \frac{1}{\delta} \epsilon_{t-1}. \quad (13)$$

Writing (13) for period  $t+q+1$  and taking expectations conditional on information available at time  $t-1$ ,  $E_{t-1} \xi_{t+q+1} = 0$ . This means that  $\xi_{t+q+1}$  is unpredictable using information at time  $t-1$ . To show that this implication can be contrasted by using a regression of  $S_{t+q+1} - \frac{1}{\delta} S_{t+q} + \theta X_{t+q+1}$  on information at time  $t-1$ , consider the following,

$$\begin{aligned} S_{t+q+1} - \frac{1}{\delta} S_{t+q} + \theta \Delta X_{t+q+1} &= \theta \sum_{i=1}^{\infty} \delta^i E_{t+q+1} \Delta X_{t+i+q+1} + \epsilon_{t+q+1} \\ &\quad - \frac{1}{\delta} (\theta \sum_{i=1}^{\infty} \delta^i E_{t+q} \Delta X_{t+i+q} + \epsilon_{t+q}) + \theta \Delta X_{t+q+1}, \implies \\ S_{t+q+1} - \frac{1}{\delta} S_{t+q} + \theta \Delta X_{t+q+1} &= Y_{t+q+1} - \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_{t+q} X_{t+i+q+1} - \frac{1}{\delta} \epsilon_{t+q} \implies \\ S_{t+q+1} - \frac{1}{\delta} S_{t+q} + \theta \Delta X_{t+q+1} &= Y_{t+q+1} - E_{t+q} Y_{t+q+1} + \sum_{k=1}^q \alpha_k u_{t-k+q+1} - \frac{1}{\delta} \epsilon_{t+q} = \xi_{t+q+1}. \quad (14) \end{aligned}$$

Since  $\xi_{t+q+1} = S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$ , one can contrast the hypothesis  $E_{t-1}\xi_{t+q+1} = 0$ , by regressing  $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$  on information available at time  $t-1$ , and then testing the hypothesis that the coefficients on the variables reflecting information at  $t-1$  are jointly equal to zero. Alternatively, one can construct cross-equation restrictions from a VAR model for the spread and the change in  $X_t$ . To see this, subtract  $\theta X_t$  from both sides of (2); write the resulting expression for period  $t+q+1$  and take expectations conditional on information at time  $t$ ,

$$E_t S_{t+q+1} = E_t \left[ \theta \sum_{i=1}^{\infty} \delta^i E_{t+q+1} \Delta X_{t+i+q+1} + \epsilon_{t+q+1} \right]. \quad (15)$$

Rearranging equation (15),

$$E_t S_{t+q+1} = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i+q+1}, \quad (16)$$

and projecting both sides of (16) onto  $H_t$ , we get

$$g' A^{q+1} = \theta \sum_{i=1}^{\infty} \delta^i h' A^{i+q+1} = \theta \delta h' (I - \delta A)^{-1} A^{q+1}. \quad (17)$$

We can rearrange (17) such that the cross-equation constraints are given by,

$$(\theta \delta h' - g'(I - \delta A)) A^{q+1} = 0. \quad (18)$$

To interpret these restrictions, write the VAR model (8) for period  $t+q+1$ , multiply the  $\Delta X_{t+q+1}$  equation by  $\theta$ , add it to the  $S_{t+q+1}$  equation and subtract from it  $\frac{1}{\delta}S_{t+q}$ ,

$$g' z_{t+q+1} + \theta h' z_{t+q+1} - \frac{1}{\delta} g' z_{t+q} = (g' A + \theta h' A - \frac{1}{\delta} g') z_{t+q} + (g' + \theta h') v_{t+q+1} \implies$$

$$\begin{aligned} \xi_{t+q+1} = g' z_{t+q+1} + \theta h' z_{t+q+1} - \frac{1}{\delta} g' z_{t+q} &= \frac{1}{\delta} (\theta \delta h' - g'(I - \delta A)) A^{q+1} z_{t-1} \\ &+ \frac{1}{\delta} (\theta \delta h' - g'(I - \delta A)) \sum_{j=0}^q A^j v_{t+q-j} + (g' + \theta h') v_{t+q+1}. \end{aligned}$$

The left hand side is equal to  $\xi_{t+q+1}$ , then, in order to get  $E_{t-1}\xi_{t+q+1} = 0$  we need that  $(\theta \delta h' - g'(I - \delta A)) A^{q+1} = 0$ , but these are the cross equation constraints given by expression

(18). Therefore, testing  $E_{t-1}\xi_{t+q+1} = 0$  by imposing the restrictions given by (18) in a VAR model for  $\Delta X_t$  and  $S_t$  is equivalent to contrast it by regressing  $S_{t+q+1} - \frac{1}{\delta}S_{t+q} + \theta\Delta X_{t+q+1}$  on information available at time  $t-1$ , and test the null hypothesis that the coefficients of the variables reflecting information at time  $t-1$  are jointly equal to zero.

**Case 2.** Assume  $\epsilon_t = \sum_{k=1}^p \rho_k \epsilon_{t-k} + v_t$ . From (11) we have,

$$\xi_t = Y_t - \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_{t-1} X_{t+i} - \frac{1}{\delta} \epsilon_{t-1} = Y_t - E_{t-1} Y_t + \sum_{k=1}^p \rho_k \epsilon_{t-k} - \frac{1}{\delta} \epsilon_{t-1}. \quad (19)$$

From (19) is clear that,  $E_{t-1}\xi_{t+j} \neq 0 \quad \forall j$ , since  $E_{t-1}\epsilon_{t+j} \neq 0 \quad \forall j$ . In this case  $\xi_{t+j} \quad \forall j$  is predictable using information at time  $t-1$ . Consistent with this finding, it is not possible to get cross-equation restrictions from a VAR model for  $\Delta X_t$  and  $S_t$  due to the presence of the autoregressive error term. Formally,

$$E_t S_{t+j} = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i+j} + E_t \epsilon_{t+j}. \quad (20)$$

Since  $E_t \epsilon_{t+j} \neq 0 \quad \forall j$  in last expression, we cannot derive any of the standard cross-equation restrictions from (20).

**PROOF: Proposition 4.** For Statement 1. see Shiller, (1981). For Statements 2. and 3. see Mankiw, Romer and Shapiro, (1985). For Statement 4. see Campbell and Shiller (1987a).

**PROOF: Proposition 5. Statement 1.** From the NEPVM we have

$$Y_t = E_t Y_t^* + \epsilon_t, \quad (21)$$

hence

$$Y_t^* = (Y_t - \epsilon_t) + w_t, \quad (22)$$

where  $w_t$  is a forecast error. As a rational forecast error it is uncorrelated with information available at time  $t$ . From (22),

$$Var(Y_t^*) = Var(Y_t) + Var(\epsilon_t) + Var(w_t) - 2Cov(Y_t, \epsilon_t), \quad (23)$$

therefore it follows from (23) that

$$\text{Var}(Y_t^*) \geq \text{Var}(Y_t) \iff \text{Var}(w_t) + \text{Var}(\epsilon_t) \geq 2\text{Cov}(Y_t, \epsilon_t). \quad (24)$$

**Statements 2. and 3.** Consider the following identity,

$$Y_t^* - Y_t^0 = (Y_t^* - Y_t) + (Y_t - Y_t^0), \quad (25)$$

notice that,  $Y_t^* - Y_t = w_t - \epsilon_t$ , then

$$E_t[(Y_t^* - Y_t)(Y_t - Y_t^0)] = E_t[(w_t - \epsilon_t)(Y_t - Y_t^0)]. \quad (26)$$

Since  $E_t[\epsilon_t(Y_t - Y_t^0)] \neq 0$  expression (26) is different from zero. Therefore,

$$E_t(Y_t^* - Y_t^0)^2 = E_t(Y_t^* - Y_t)^2 + E_t(Y_t - Y_t^0)^2 - 2E_t[\epsilon_t(Y_t - Y_t^0)]. \quad (27)$$

From (27) is clear that only if  $E_t(Y_t - Y_t^0)^2 \geq 2E_t[\epsilon_t(Y_t - Y_t^0)]$ , then

$$E_t(Y_t^* - Y_t^0)^2 \geq E_t(Y_t^* - Y_t)^2. \quad (28)$$

Similarly, if  $E_t(Y_t^* - Y_t)^2 \geq 2E_t[\epsilon_t(Y_t - Y_t^0)]$ , expression (27) implies

$$E_t(Y_t^* - Y_t^0)^2 \geq E_t(Y_t - Y_t^0)^2. \quad (29)$$

Finally, the law of iterated projections allows us to replace expectations conditional on information available at time  $t$  with expectations conditional on information available prior to the beginning of the sample period. That is, letting  $E$  denote the expectation conditional on the initial conditions, we have

$$E(Y_t^* - Y_t^0)^2 = E(Y_t^* - Y_t)^2 + E(Y_t - Y_t^0)^2 - 2E[\epsilon_t(Y_t - Y_t^0)]. \quad (30)$$

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t^* - Y_t)^2 \iff E(Y_t - Y_t^0)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]. \quad (31)$$

$$E(Y_t^* - Y_t^0)^2 \geq E(Y_t - Y_t^0)^2 \iff E(Y_t^* - Y_t)^2 \geq 2E[\epsilon_t(Y_t - Y_t^0)]. \quad (32)$$

**Statement 4.** Consider the non-exact present value model (2), adding and subtracting  $\theta X_t$  we get,

$$S_t = Y_t - \theta X_t = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t. \quad (33)$$

Now, define the “theoretical spread”,  $S'_t$ , as the optimal forecast, given the econometrician information set,  $H_t$ , of the present value of all future changes in  $X$ ,

$$S'_t = \theta \sum_{i=1}^{\infty} \delta^i E (\Delta X_{t+i} | H_t). \quad (34)$$

Subtracting (34) from (33),

$$S_t - S'_t = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t - \theta \sum_{i=1}^{\infty} \delta^i E (\Delta X_{t+i} | H_t) \implies$$

$$S_t - S'_t = \theta \sum_{i=1}^{\infty} \delta^i E_t X_{t+i} - \theta \sum_{i=1}^{\infty} \delta^i E_t X_{t+i-1} + \epsilon_t - \theta \sum_{i=1}^{\infty} \delta^i E (\Delta X_{t+i} | H_t),$$

adding and subtracting  $\theta X_t$  we get,

$$S_t - S'_t = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_t X_{t+i} + \epsilon_t - \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E (X_{t+i} | H_t).$$

Projecting both sides of equation (2) onto  $H_t$  we have,

$$E(Y_t | H_t) = Y_t = \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E (X_{t+i} | H_t) + \mu_t, \quad (35)$$

where  $\mu_t = E(\epsilon_t | H_t)$ . Then,

$$S_t - S'_t = Y_t - E(Y_t | H_t) + \mu_t = \mu_t,$$

therefore,

$$\text{Var}(S_t) = \text{Var}(S'_t) + \text{Var}(\mu_t) + 2\text{Cov}(S'_t, \mu_t),$$

or,

$$\frac{\text{Var}(S_t)}{\text{Var}(S'_t)} = 1 + \frac{\text{Var}(\mu_t) + 2\text{Cov}(S'_t, \mu_t)}{\text{Var}(S'_t)}. \quad (36)$$

From equation (36), is clear that the volatility test given by  $Var(S_t)/Var(S'_t) = 1$  is not an implication of the NEPVM.

For the second volatility test, we have,

$$S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t = \theta \sum_{i=1}^{\infty} \delta^i E_t \Delta X_{t+i} + \epsilon_t - \frac{1}{\delta}(\theta \sum_{i=1}^{\infty} \delta^i E_{t-1} \Delta X_{t+i-1} + \epsilon_{t-1}) + \theta\Delta X_t \implies$$

$$S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t = Y_t - \theta \sum_{i=0}^{\infty} \delta^i E_{t-1} \Delta X_{t+i} + \theta \sum_{i=2}^{\infty} \delta^{i-1} E_{t-1} \Delta X_{t+i-2} - \frac{1}{\delta}\epsilon_{t-1} \implies$$

$$S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t = Y_t - \theta(1 - \delta) \sum_{i=0}^{\infty} \delta^i E_{t-1} \Delta X_{t+i} - \frac{1}{\delta}\epsilon_{t-1} \implies$$

$$S_t - \frac{1}{\delta}S_{t-1} + \theta\Delta X_t = Y_t - E_{t-1}Y_t + \sum_{k=1}^p \rho_k \epsilon_{t-k} - \frac{1}{\delta}\epsilon_{t-1} = \xi_t. \quad (37)$$

Now, consider

$$\xi'_t = S'_t - \frac{1}{\delta}S'_{t-1} + \theta\Delta X_t = \theta \sum_{i=1}^{\infty} \delta^i E(\Delta X_{t+i}|H_t) + \theta\Delta X_t - \frac{1}{\delta}\theta \sum_{i=1}^{\infty} \delta^i E(\Delta X_{t+i-1}|H_{t-1}) \implies$$

$$\xi'_t = Y_t - E(Y_t|H_{t-1}) + E(\epsilon_t|H_{t-1}) - \mu_t.$$

Then

$$\xi_t - \xi'_t = E(Y_t|H_{t-1}) - E_{t-1}Y_t + \mu_t - E(\epsilon_t|H_{t-1}) + \sum_{k=1}^p \rho_k \epsilon_{t-k} - \frac{1}{\delta}\epsilon_{t-1} = \phi_t,$$

therefore

$$Var(\xi_t) = Var(\xi'_t) + Var(\phi_t) + 2Cov(\xi'_t, \phi_t),$$

or

$$\frac{Var(\xi_t)}{Var(\xi'_t)} = 1 + \frac{Var(\phi_t) + 2Cov(\xi'_t, \phi_t)}{Var(\xi'_t)}. \quad (38)$$

Again, is clear from equation (38) that the second volatility test given by,  $Var(\xi_t)/Var(\xi'_t) = 1$ , is not an implication of the NEPVM.

**Table 1: Cointegration and Multicointegration Tests  
Stock Prices and Dividends**

Johansen's Cointegration Test		
Eigenvalue	Likelihood Ratio	5 Percent Critical value
0.118260	16.18557	15.41
0.018483	2.089499	3.76

Augmented Dickey Fuller Unit Root Test on $\hat{u}_t$		
ADF Statistic Test	5% Critical Value	1% Critical Value
-1.919165	-4.25	-4.84

Note: Johansen's LR testing of  $r=1$  vs  $r=2$  and  $r=0$  vs  $r=2$  was performed on the following model,  $W_t = C + \Pi X_{t-1} + \Gamma W_{t-1} + \eta_t$ , where  $W_t = (\Delta Y_t, \Delta X_t)'$ , and  $X_{t-1} = (Y_{t-1}, X_{t-1})'$ .  $\hat{u}_t$  are the residuals of regressing  $\Delta^{-1}Y_t$  on a constant, a time trend,  $\Delta^{-1}X_t$  and  $X_t$ .

Lags used in both tests were selected using Akaike information criterion.

**Table 2: cross-equation Constraints Regression Tests  
Stock Prices and Dividends**

Dependent Variable	$\xi_{t+1}$	$\xi_{t+2}$	$\xi_{t+3}$	$\xi_{t+4}$	$\xi_{t+5}$
F-statistic	4.621	0.388	1.490	1.281	1.073
p-Value	0.071	0.958	0.387	0.459	0.551

Note: All test procedures are robust to both heteroskedasticity and autocorrelation. The dependent variable is regressed in all cases on information available at time  $t-1$ .



**Table 3: Cointegration and Multicointegration Tests  
Short and Long Term Interest Rates**

Johansen's Cointegration Test		
Test Assumption: Linear deterministic trend in the data		
Eigenvalue	Likelihood Ratio	5 Percent Critical value
0.050290	26.98210	15.41
0.005679	2.676611	3.76

Augmented Dickey Fuller Unit Root Test on $\hat{u}_t$		
ADF Statistic Test	5% Critical Value	1% Critical Value
-2.460397	-4.14	-4.73

Note: Johansen's LR testing of  $r=1$  vs  $r=2$  and  $r=0$  vs  $r=2$  was performed on the following model  $W_t = C + \Pi X_{t-1} + \sum_{j=1}^8 \Gamma_j W_{t-j} + \eta_t$ , where  $W_t = (\Delta Y_t, \Delta X_t)'$ , and  $X_{t-1} = (Y_{t-1}, X_{t-1})'$ .  $\hat{u}_t$  are the residuals of regressing  $\Delta^{-1}Y_t$  on a constant, a time trend,  $\Delta^{-1}X_t$  and  $X_t$ .

Lags used in both tests were selected using Akaike information criterion.

**Table 4: cross-equation Constraints Regression Tests  
Short and Long Term Interest Rates**

Dependent Variable	$\xi_{t+1}$	$\xi_{t+2}$	$\xi_{t+3}$	$\xi_{t+4}$	$\xi_{t+5}$	$\xi_{t+6}$	$\xi_{t+7}$	$\xi_{t+8}$
F-statistic	73.48	32.64	17.77	8.742	6.022	5.032	6.028	4.911
p-Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: All test procedures are robust to both heteroskedasticity and autocorrelation. The dependent variable is regressed in all cases on information available at time  $t-1$ .

**Table 5: Cointegration and Multicointegration Tests  
Farmland Prices and Rents**

<b>Johansen's Cointegration Test</b>		
Test Assumption: Linear deterministic trend in the data		
Eigenvalue	Likelihood Ratio	5 Percent Critical value
0.266418	22.21452	15.41
0.026413	1.766680	3.76

<b>Augmented Dickey Fuller Unit Root Test on <math>\hat{u}_t</math></b>		
ADF Statistic Test	5% Critical Value	1% Critical Value
-4.603667	-4.37	-5.03

Note: Johansen's LR testing of  $r=1$  vs  $r=2$  and  $r=0$  vs  $r=2$  was performed on the following model  $W_t = C + \Pi X_{t-1} + \Gamma W_{t-1} + \eta_t$ , where  $W_t = (\Delta Y_t, \Delta X_t)'$ , and  $X_{t-1} = (Y_{t-1}, X_{t-1})'$ .  $\hat{u}_t$  are the residuals of regressing  $\Delta^{-1}Y_t$  on a constant, a time trend,  $\Delta^{-1}X_t$  and  $X_t$ .

Lags used in both tests were selected using Akaike information criterion.

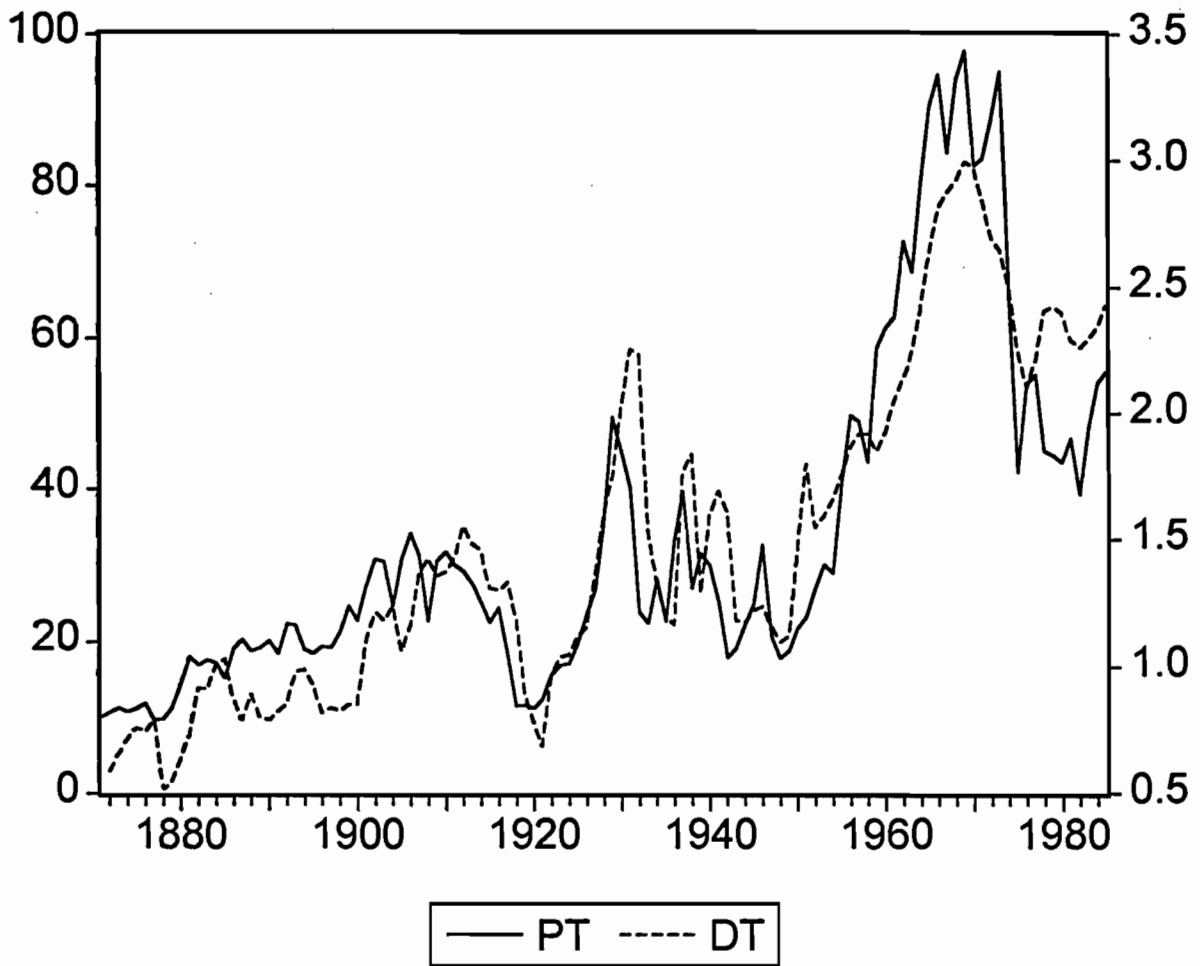


Figure 1

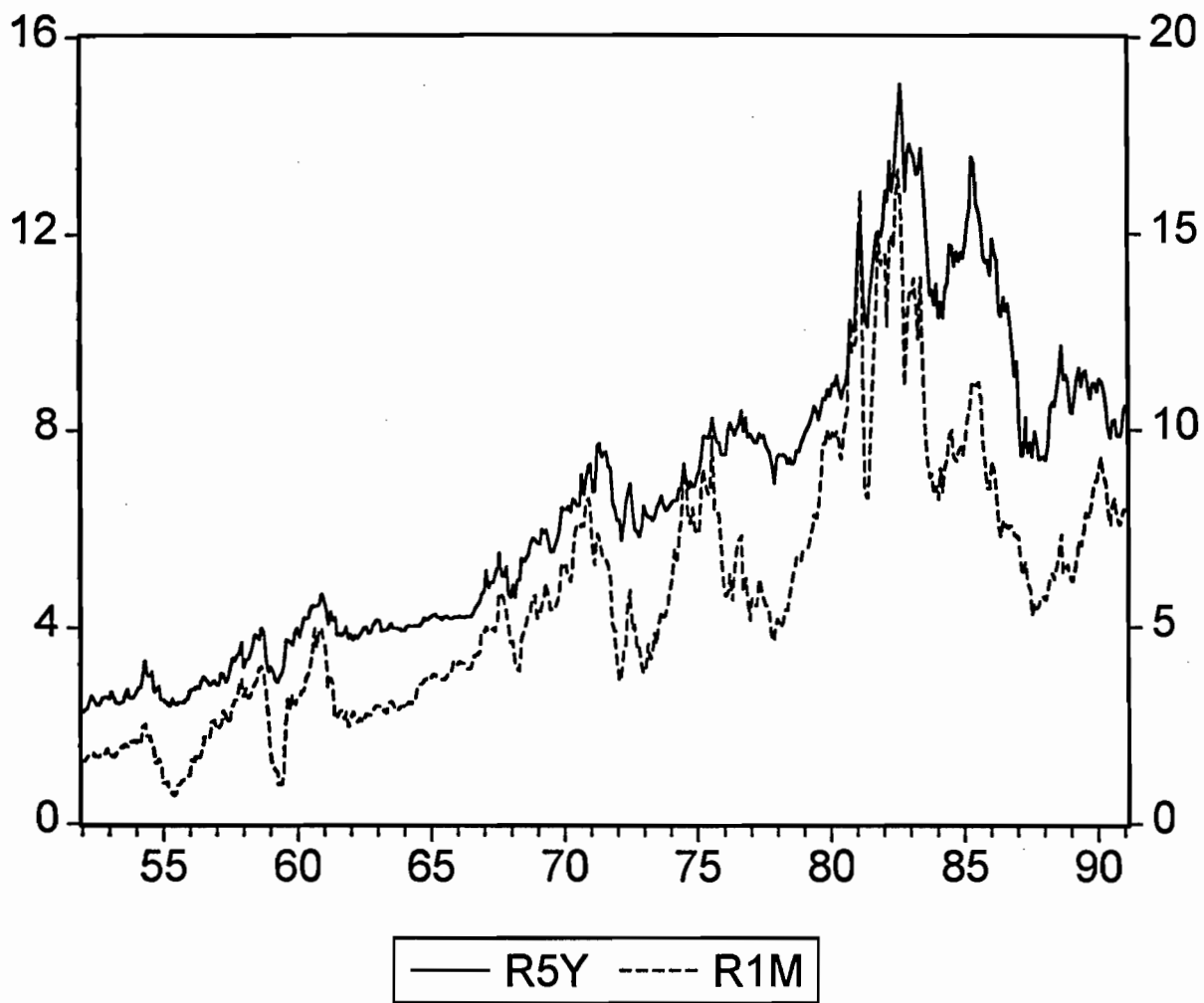


Figure 2

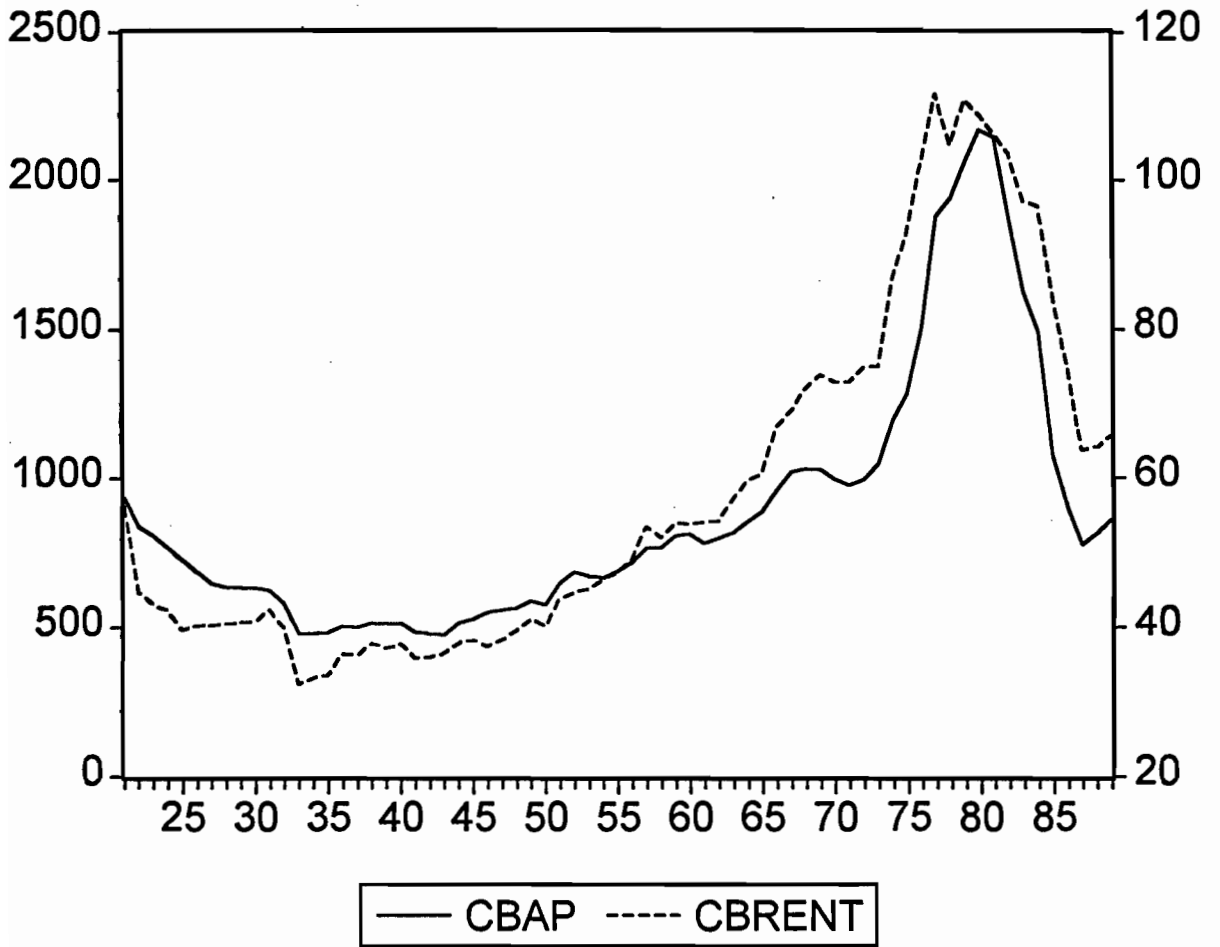


Figure 3

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