NOTES, COMMENTS, AND LETTERS TO THE EDITOR

A Note on the Optimal Structure of Production[1]

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We analyze the advantages of centralization and decentralization in industries in which production takes place in several stages and the costs are privately observed by the agents in charge of production. We demonstrate that "informational diseconomies" arise when uncorrelated information is concentrated in the hands of a single agent. These diseconomies arise when the stages of production are different activities with different cost supports. *Journal of Economic Literature* Classification Numbers: D82, L22, L51.

1. INTRODUCTION

The organization of production is especially important in industries like the electric power industry, where production takes place in several stages (distribution, transmission, and generation), and/or the final good is composed of several components. In such industries, a single firm may be responsible for the different stages or components. Alternatively, multiple firms may operate, each undertaking a different activity. We will refer to the former

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possibility as "centralized structure," and to the latter as "decentralized structure." This paper studies the optimal organization of production when the costs of its different stages are uncorrelated private information, observable only by those who undertake it.

When costs are correlated across activities, the principal may find it optimal to choose a decentralized structure and rely on relative performance schemes to lower the cost of the contract. Demski and Sappington [5, 6] have shown that whenever it is possible to divide production into similar activities whose costs are correlated, it is better to decentralize production. Dana [3] establishes that informational economies of scope arise when one firm produces both goods. The regulator should weigh these economies against the benefits of relative performance schemes.

If costs are uncorrelated, the centralized structure enables the regulator to avoid the informational externalities that would arise with multiple independent firms. This argument is similar to those used in a moral hazard framework suggesting that the formation of teams tends to increase welfare by eliminating risk and commitment externalities.[2] In this vain, Baron and Besanko [2] and Gilbert and Riordan [7] have shown that the optimal regulation of complementary products may involve a centralized structure.[3] For this to be the case the support of the cost distributions in the different stages of production must be symmetric.

Our main contribution in this paper is to provide a theoretical basis for the choice of a decentralized structure when costs are uncorrelated and the products are complementary. More specifically, we show that when the supports of the cost distributions in the different stages of production are sufficiently different, activities are optimally separated. With different supports, two different effects arise under centralized production. The first effect favors centralization: the cost of the contracts is reduced when agents internalize the externalities that arise in decentralized structures. The second effect favors decentralization: when production is centralized, the firm obtains greater informational rent because of its ability to coordinate messages. That is, centralization increases the size of the agents' message space, and this generates new informational rents. This effect, which we will call *Concentration Effect*, increases the cost of the contract. We will show that under the assumption of equal cost supports, the concentration effect does not arise but under more general circumstances it may even prevail.

[2] See Itoh [8] and Macho-Stadler and Pérez-Castrillo [9]. On hierarchies and collusion among agents see Tirole [12].

[3] Baron and Besanko, [2] also study *informational delegation* or subcontracting. Given strict complementarities between the two suppliers, they find that decentralized and delegated contracting can be performance equivalent. Melumad *et al.* [10] have shown that delegated contracting may be inferior, in the absence of complementarities.

The paper is organized as follows. In Section 2 we introduce the model and characterizes the optimal contract under each structure. Section 3 compares the two structures. Finally, Section 4 concludes.

2. THE MODEL

We will consider a good which is produced in two stages ($i=1, 2$) (or is assembled from two components). Production can be undertaken separately by different agents, or jointly by a single one. We will assume a constant marginal cost at each stage of production, θ^i. Hence, the total cost of q units of output is given by $C(q, \theta^1, \theta^2)=(\theta^1+\theta^2)\,q$.

Cost realizations at each stage are independently distributed over two-point supports. Formally, $\theta^i \in \{\theta^i_L, \theta^i_H\}$, with $\Delta\theta^i=\theta^i_H-\theta^i_L>0$, $\forall i=1, 2$. The probability of the low cost realization, γ, is common to both activities and public knowledge. We allow the supports of the cost distributions to be different. Without loss of generality, we will assume that $\Delta\theta^1 \geqslant \Delta\theta^2$.[4]

The principal chooses the way in which production is to be organized. In particular, he contracts production out to either a single agent or to several, each of which is put in charge of a different activity.[5] We will refer to the first situation as *centralization*, and to the second as *decentralization*. In both cases, the principal designs contracts so as to maximize his net expected payoff W, which is the expected value of the difference between an increasing and concave function of output $V(q)$ and the expected payments to the agent.[6] Both the principal and the agents are risk neutral.

2.1. *Decentralization*

Under decentralization, the marginal cost of each producer is private information, and the principal induces truthful revelation in a Bayesian Nash equilibrium. That is, he designs a menu of contracts, $\{T^1(\hat{\theta}^1, \hat{\theta}^2), T^2(\hat{\theta}^1, \hat{\theta}^2), q(\hat{\theta}^1, \hat{\theta}^2)\}$ specifying the payments to each agent and the required output level as a function of announced costs, $(\hat{\theta}^i, \hat{\theta}^{-i})$. We will write q_{jk} to denote the required output when $\hat{\theta}^1=\theta^1_j$ and $\hat{\theta}^2=\theta^2_j$, where $j, k \in \{L, H\}$. The

[4] A possible interpretation is that the realization of the marginal costs for each of the stages may be the sum of an expected cost common to all of them, $\bar{\theta}$ defined by $\gamma\theta^i_L+(1-\gamma)\,\theta^i_H=E\theta^i=\bar{\theta}$, $\forall i=1, 2$, plus the realization of a random variable ε^i, which captures the specific effects which affect each stage of production, and whose impact on the marginal cost can be different in each stage. Formally, $\theta^i=\bar{\theta}+\varepsilon^i$, $\varepsilon^i \in \{-\omega^i, \omega^i\}$ $\forall i=1, 2$, where $\omega^1 \geqslant \omega^2$.

[5] In contrast to our model, in Riordan and Sappington [11] the principal can choose between undertaking second stage production himself or contracting out the second stage production to the agent who performs the first stage.

[6] $V(q)$ can be interpreted as the consumer's surplus, if we think of the principal as a benevolent regulator, or as a firm's revenues.

optimal contract maximizes the principal's expected payoff subject to appropriate incentive compatibility and participation constraints. We will require the "participation" constraint to hold *ex post*. One possible interpretation which seems realistic in a regulation context is that firms have limited liability.

It is easy to show that if the monotonicity constraint on the expected production levels holds, the principal will ensure that the ex post participation constraints for the less efficient agents and the incentive compatibility constraints for the more efficient agents hold with equality. A low cost agent will always participate if a high cost type does (for he can always sign the other type's contract and obtain a payoff above the reservation level). On the other hand, given that the low-cost agents expected output is no lower than that of the high cost agents, the latter do not find it attractive to pose as low cost types. Hence, optimal contracts satisfy:

$$\begin{aligned}
&\gamma(T^1_{LL}-\theta^1_L q_{LL})+(1-\gamma)(T^1_{LH}-\theta^1_L q_{LH})\\
&\qquad =\gamma(T^1_{HL}-\theta^1_L q_{HL})+(1-\gamma)(T^1_{HH}-\theta^1_L q_{HH}),\\
&\gamma(T^2_{LL}-\theta^2_L q_{LL})+(1-\gamma)(T^2_{HL}-\theta^2_L q_{HL})\\
&\qquad =\gamma(T^2_{LH}-\theta^2_L q_{LH})+(1-\gamma)(T^2_{HH}-\theta^2_L q_{HH}),
\end{aligned}$$

$$T^1_{Hk}-\theta^1_H q_{Hk}=0, \qquad \text{for all} \quad k\in\{L,H\},$$

$$T^2_{jH}-\theta^2_H q_{jH}=0, \qquad \text{for all} \quad j\in\{LH\}$$

Using equations above, we compute the informational rents earned by each agent in each state. Substituting the informational rents in the principal's objective function, the decentralized optimal contract satisfies

$$V'(q_{LL})=\theta^1_L+\theta^2_L \tag{1}$$

$$V'(q_{LH})=\theta^1_L+\theta^2_H+\frac{\gamma}{1-\gamma}\Delta\theta^2 \tag{2}$$

$$V'(q_{HL})=\theta^1_H+\theta^2_L+\frac{\gamma}{1-\gamma}\Delta\theta^1 \tag{3}$$

$$V'(q_{HH})=\theta^1_H+\theta^2_H+\frac{\gamma}{1-\gamma}[\Delta\theta^1+\Delta\theta^2]. \tag{4}$$

As is usually the case in adverse selection problems, the principal distorts the choice of output levels by high-cost types in order to reduce the informational rents accruing to low cost agents. Moreover, only low cost agents earn positive informational rents.

2.2. *Centralization*

Under centralization, a single agent simultaneously observes the realization of the costs at both stages of production. It is straightforward to prove that the principal must base contracts on the sum of the cost of the two activities,[7] given by $\Theta=(\theta^1+\theta^2)$, in

$$A=\{\Theta_{LL},\Theta_{LH},\Theta_{HL},\Theta_{HH}\}=\{\theta^1_L+\theta^2_L,\theta^1_L+\theta^2_H,\theta^1_H+\theta^2_L,\theta^1_H+\theta^2_H\}.$$

The principal designs a menu of contracts, $\{T_{\hat{\Theta}},q_{\hat{\Theta}}\}$, specifying the payment to the firm and the required output level as a function of announced costs, $\hat{\Theta}$.

To characterize the optimal contract, we begin by observing that the global incentive constraints are equivalent to

$$T_{LL}-\Theta_{LL}q_{LL}\geqslant T_{LH}-\Theta_{LL}q_{LH} \tag{5}$$

$$T_{LH}-\Theta_{LH}q_{LH}\geqslant T_{LL}-\Theta_{LH}q_{LL} \tag{6}$$

$$T_{LH}-\Theta_{LH}q_{LH}\geqslant T_{HL}-\Theta_{LH}q_{HL} \tag{7}$$

$$T_{HL}-\Theta_{HL}q_{HL}\geqslant T_{LH}-\Theta_{HL}q_{LH} \tag{8}$$

$$T_{HL}-\Theta_{HL}q_{HL}\geqslant T_{HH}-\Theta_{HL}q_{HH} \tag{9}$$

$$T_{HH}-\Theta_{HH}q_{HH}\geqslant T_{HL}-\Theta_{HH}q_{HL} \tag{10}$$

together with the monotonicity restriction on output levels[8]

$$q_{LL}\geqslant q_{LH}\geqslant q_{HL}\geqslant q_{HH}.$$

Note that if (5), (7), and (9) are binding then (6), (8), and (10) also hold since $q_{LL}\geqslant q_{LH}\geqslant q_{HL}\geqslant q_{HH}$. Given the binding constraints, we can write

$$T_{LL}-\Theta_{LH}q_{LL}=T_{LH}-\Theta_{LH}q_{LH}+\Delta\theta^2(q_{LH}-q_{LL})$$

$$T_{LH}-\Theta_{HL}q_{LH}=T_{HL}-\Theta_{HL}q_{HL}+[\Delta\theta^1-\Delta\theta^2](q_{HL}-q_{LH})$$

$$T_{HL}-\Theta_{HH}q_{HL}=T_{HH}-\Theta_{HH}q_{HH}+\Delta\theta^2(q_{HH}-q_{HL}).$$

[7] Notice that for any pair of contracts $(T(\theta^1,\theta^2),q(\theta^1,\theta^2))$ and $(T(\hat{\theta}^1,\hat{\theta}^2),q(\hat{\theta}^1,\hat{\theta}^2))$ such that $\hat{\theta}_1+\hat{\theta}_2=\theta^1+\theta^2$, the incentive compatibility contraints imply $T(\theta^1,\theta^2)=T(\hat{\theta}^1,\hat{\theta}^2)$ and $q(\theta^1,\theta^2)=q(\hat{\theta}^1,\hat{\theta}^2)$. Whenever $\Delta\theta^1>\Delta\theta^2$, the principal's information is the same no matter he receives announcements about the combined marginal cost, Θ, or about the marginal cost at each stage, (θ^1,θ^2).

[8] As in the case of decentralization, the assumption on the size of the supports determines the ordering of the output levels. Had we assumed $\Delta\theta^2\geqslant\Delta\theta^1$, then the monotonicity condition would be written $q_{LL}\geqslant q_{HL}\geqslant q_{LH}\geqslant q_{HH}$.

Finally, if the participation constraint for the agent with high costs in both stages of production holds with equality, the informational rents accruing to any other agent are given by

$$\begin{aligned}
\Pi_{LL} &= \Delta\theta^2 q_{LH} + \Pi_{LH} \\
\Pi_{LH} &= [\Delta\theta^1 - \Delta\theta^2]\, q_{HL} + \Pi_{HL} \\
\Pi_{HL} &= \Delta\theta^2 q_{HH} + \Pi_{HH} \\
\Pi_{HH} &= \underline{\Pi} \equiv 0.
\end{aligned}$$

The principal's problem is therefore equivalent to

$$P^C \equiv \begin{cases}
\max_{\{q_{ij}\}} \gamma^2[V(q_{LL}) - \Theta_{LL}q_{LL}] + \gamma(1-\gamma)[V(q_{LH}) - \Theta_{LH}q_{LH}] \\
\quad + \gamma(1-\gamma)[V(q_{HL}) - \Theta_{HL}q_{HL}] \\
\quad + (1-\gamma)^2 [V(q_{HH}) - \Theta_{HH}q_{HH}] \\
\quad - \gamma^2 \Delta\theta^2 q_{LH} - \gamma[\Delta\theta^1 - \Delta\theta^2]\, q_{HL} - (1-(1-\gamma)^2)\, \Delta\theta^2 q_{HH} \\
\quad \text{s.t.} \begin{cases} q_{LL} \geqslant q_{LH} \\ q_{LH} \geqslant q_{HL} \\ q_{HL} \geqslant q_{HH}. \end{cases}
\end{cases}$$

In the absence of distortions on the production level of the agent with low costs in both stages of production, q_{LL} will always be larger than q_{LH}. We can therefore ignore the first constraint and then check that the solution of the resulting problem does indeed satisfy it. This procedure will not work for the remaining (monotonicity) constraints. However, the incentive compatibility constraints $q_{LH} \geqslant q_{HL}$ and $q_{HL} \geqslant q_{HH}$ are never simultaneously binding. This allows us to reduce the number of possible cases to three. Next lemma establishes the three possible regimes and the ranking between them.

LEMMA. *Which regime prevails in each case depends on the relationship between $\Delta\theta^1$ and $\Delta\theta^2$. In particular, the solution to the principal's problem yields*

Regime I $(q_{LL} > q_{LH} = q_{HL} > q_{HH})$ *if* $\Delta\theta^2 \leqslant \Delta\theta^1 \leqslant \frac{2}{2-\gamma}\Delta\theta^2$

Regime II $(q_{LL} > q_{LH} > q_{HL} > q_{HH})$ *if* $\frac{2}{2-\gamma}\Delta\theta^2 < \Delta\theta^1 < \frac{2-\gamma}{1-\gamma}\Delta\theta^2$

Regime III $(q_{LL} > q_{LH} > q_{HL} = q_{HH})$ *if* $\frac{2-\gamma}{1-\gamma}\Delta\theta^2 \leqslant \Delta\theta^1$.

Proof. See the Appendix. ▮

The intuition behind this result is the following. In adverse selection problems, the principal distorts the choice of output levels (or sets inefficient output levels) in order to reduce informational rents. The larger the marginal informational cost associated with a level of output, the larger is the optimal distortion from the point of view of the principal. When $\Delta\theta^1$ is not "very different" from $\Delta\theta^2$ (regime I), the marginal rents associated with q_{LH} are larger than those associated with q_{HL}. Hence, the principal would like to distort q_{LH} more than the incentive constraints allow him to. Hence, the principal "bunches" the production levels associated with the states "LH" and "HL." In regime III, the principal cannot distort q_{HL} as much as he would like and, in order for the contracts to satisfy the incentive restrictions, the constraint $q_{HL} - q_{HH} \geqslant 0$ must be binding.

3. CENTRALIZATION VERSUS DECENTRALIZATION

Baron and Besanko [2] and Gilbert and Riordan [7] have shown that when the supports of the cost distributions are identical the principal prefers to centralize production because he finds it less costly to induce honest revelation when a single agent has all the information. We now show that the assumption of identical supports is far from innocuous.

PROPOSITION. *If* $\Delta\theta^1 < (1/1-\gamma)\,\Delta\theta^2$ *the principal strictly prefers to centralize production, but if* $\Delta\theta^1 > (1/1-\gamma)\,\Delta\theta^2$ *he strictly prefers to decentralize production. When* $\Delta\theta^1 = (1/1-\gamma)\,\Delta\theta^2$, *centralization and decentralization are equivalent.*

Proof. For any value $q \equiv (q_{HH}, q_{HL}, q_{LH}, q_{LL})$, the expected rents under decentralization ($E\Pi^D$) and under centralization ($E\Pi^C$), are given by

$$E\Pi^D(q) = \gamma^2(\Delta\theta^2 q_{LH} + \Delta\theta^1 q_{HL}) + \gamma(1-\gamma)[\Delta\theta^1 + \Delta\theta^2]\, q_{HH},$$

$$E\Pi^C(q) = \gamma^2\Delta\theta^2 q_{LH} + \gamma[\Delta\theta^1 - \Delta\theta^2]\, q_{HL} + (1-(1-\gamma)^2)\,\Delta\theta^2 q_{HH}.$$

Hence, the difference in rents across structure types is equal to

$$E\Pi^D(q) - E\Pi^C(q) = \gamma[\Delta\theta^2 - (1-\gamma)\,\Delta\theta^1](q_{HL} - q_{HH}). \tag{11}$$

Denoting by q^S to the optimal output under the structure of production S, $S = C, D$, optimality of output choices and (11) imply $W^C(q^C) \geqslant W^C(q^D) > W^D(q^D)$ if

$$E\Pi^D(q^D) - E\Pi^C(q^D) = \gamma[\Delta\theta^2 - (1-\gamma)\,\Delta\theta^1](q^D_{HL} - q^D_{HH}) > 0,$$

and, similarly, $W^D(q^D) \geqslant W^D(q^C) > W^C(q^C)$ if

$$E\Pi^C(q^C) - E\Pi^D(q^C) = \gamma[(1-\gamma)\,\Delta\theta^1 - \Delta\theta^2](q^C_{HL} - q^C_{HH}) > 0.$$

Thus,

(i) If $\Delta\theta^1 < (1/1-\gamma)\,\Delta\theta^2$, we have $W^C(q^C) > W^D(q^D)$ since $q^D_{HL} > q^D_{HH}$.

(ii) If $\Delta\theta^1 > (1/1-\gamma)\,\Delta\theta^2$, then $W^D(q^D) > W^C(q^C)$. To see this, notice that if $\Delta\theta^1 \in ((1/1-\gamma)\,\Delta\theta^2, (2-\gamma/1-\gamma)\,\Delta\theta^2)$, we have $\Pi^D(q^C) - \Pi^C(q^C) = \gamma[(1-\gamma)\,\Delta\theta^1 - \Delta\theta^2](q^C_{HL} - q^C_{HH}) > 0$. And if $\Delta\theta^1 \geqslant (2-\gamma/1-\gamma)\,\Delta\theta^2$, $\Pi^D(q^C) = \Pi^C(q^C)$ due to $q^C_{HL} = q^C_{HH}$, and $q^D_{HL} \neq q^D_{HH}$ imply $W^D(q^D) > W^C(q^C)$.

Finally, it is straightforward to see that $\Delta\theta^1 = (1/1-\gamma)\,\Delta\theta^2$, implies $\Pi^D(q) = \Pi^C(q)$ for all q, and, consequently, $W^C(q^C) = W^D(q^D)$. ▌

Each structure has advantages and disadvantages when it comes to extracting information. To see this, let Π^C_{jk} and Π^D_{jk} denote the informational rents associated with (θ^1_j, θ^2_k), $j, k \in \{L, H\}$, under centralization and decentralization, respectively, where $\Pi^D_{jk} = \sum_{i=1,2} \Pi^{D_i}_{jk}$. We can write

$$\begin{aligned}
\Pi^C_{LL} &= \Pi^D_{LL} - \Delta\theta^2(q_{HL} - q_{HH}) \\
\Pi^C_{LH} &= \Pi^D_{LH} + [\Delta\theta^1 - \Delta\theta^2](q_{HL} - q_{HH}) \\
\Pi^C_{HL} &= \Pi^D_{HL} \\
\Pi^C_{HH} &= \Pi^D_{HH}.
\end{aligned}$$

Note that the centralization of production has two different effects:

• *Efficiency Effect.* The informational rents associated with the state "LL" are smaller when production is centralized, i.e., $\Pi^C_{LL} < \Pi^D_{LL}$. The reason is that with decentralized production each agent does not take into account the fact that when he announces high costs, he reduces the informational rents accruing to the other agent (when the latter's costs are low). When production is centralized, agents internalize that externality and the expected cost of the contract is lowered by $\gamma^2\Delta\theta^2(q_{HL} - q_{HH})$.

• *Concentration Effect.* The informational rents associated with state "LH" are larger when production is centralized, $\Pi^C_{LH} > \Pi^D_{LH}$. The reason is that centralization increases the set of possible announcements, and this increases the cost of inducing honest revelation. If we decentralized production, the set of possible announcements becomes smaller, and informational rents shrink (the expected cost of the contract falls by $\gamma(1-\gamma)[\Delta\theta^1 - \Delta\theta^2](q_{HL} - q_{HH})$).

Hence, if $\Delta\theta^1 < (1/1-\gamma)\,\Delta\theta^2$, the *efficiency effect* prevails over the *concentration effect* and it is better to centralize production. In particular, when $\Delta\theta^1 = \Delta\theta^2$ there is no concentration effect and therefore centralization always dominates. When $\Delta\theta^1 = (1/1-\gamma)\,\Delta\theta^2$ both effects offset each other and therefore centralization and decentralization are equally efficient. Finally, if $\Delta\theta^1 > (1/1-\gamma)\,\Delta\theta^2$, it is better to decentralize production. Note that decentralization is now the optimal choice even when the cost of the optimal contract is the same for both structures since decentralization allows the principal to loose the incentive constraints he faces.[9]

4. CONCLUSIONS

In this paper we have analyzed the advantages of centralization and decentralization in industries in which production takes place in several stages (or the final good is compose of several components) and the costs at each stage are uncorrelated private information.

Our main contribution has been to identify the existence of "informational diseconomies" associated with the concentration of information in the hands of a single agent. Centralizing production generates two different effects on the cost of the contracts: it makes for the internalization of informational externalities (*efficiency effect*), but it enables the coordination of the announcements of both costs (*concentration effect*). When the supports of the cost distributions are sufficiently different, the second effect dominates the first, thus the principal is better off when he decentralizes. In De Frutos and Da Rocha [4] it is shown that this insight generalizes when more general discrete distributions are considered.[10] In particular, it is shown that, allowing the probability of obtaining the low cost realization to vary across the two stages of production does not alter the preference for decentralized production.

Organizational structure is particularly important in regulated industries like electric utilities. In this line, when Dana Jr. [3] concludes that regulators must centralize production when costs are sufficiently correlated, he writes "... *the break-up of electric utilities along distribution, transmission, and generation lines would be inconsistent with the theory.*" This paper

[9] When $\Delta\theta^1 \geqslant (2-\gamma/1-\gamma)\,\Delta\theta^2$ both contracts are equally costly but the principal prefers to decentralize since it gives him a larger margin of choice: under decentralization contracts must satisfy $q_{HL} \geqslant q_{HH}$, whereas under centralization the only allocations which can be implemented are those for which $q_{HL} = q_{HH}$.

[10] The interested reader will also find there an example for the continuous case for which decentralization is the optimal regulator's choice. The crucial feature of that example is that the distribution function of $\theta^1(F_1(\theta^1))$ stochastically dominates the distribution function of $\theta^2(F_2(\theta^2))$.

provides theoretical support for the separation of electricity generation from its distribution and commercialization even when costs are uncorrelated, a practice which has been adopted in a number of countries. This choice may be seen as an attempt to reduce the informational advantages derived from the concentration of all the relevant information in the hands of a single firm.

APPENDIX

Given that we can ignore the first constraint, $q_{LL} \geqslant q_{LH}$, the optimal contract satisfies

$$\frac{\partial L}{\partial q_{LL}} = \gamma^2[V'(q_{LL}) - \Theta_{LL}] = 0 \tag{12}$$

$$\frac{\partial L}{\partial q_{LH}} = \gamma(1-\gamma)[V'(q_{LH}) - \Theta_{LH}] - \gamma^2 \Delta\theta^2 + \mu_1 = 0 \tag{13}$$

$$\frac{\partial L}{\partial q_{HL}} = (1-\gamma)\,\gamma[V'(q_{HL}) - \Theta_{HL}] - \gamma[\Delta\theta^1 - \Delta\theta^2] - \mu_1 + \mu_2 = 0 \tag{14}$$

$$\frac{\partial L}{\partial q_{HH}} = (1-\gamma)^2\,[V'(q_{HH}) - \Theta_{HH}] - (1-(1-\gamma)^2)\,\Delta\theta^2 - \mu_2 = 0, \tag{15}$$

where μ_1 is the multiplier associated with $q_{LH} - q_{HL} \geqslant 0$ and μ_2 that of $q_{HL} - q_{HH} \geqslant 0$.

First we will show that the incentive compatibility constraints $q_{LH} - q_{HL} \geqslant 0$ and $q_{HL} - q_{HH} \geqslant 0$ are never simultaneously binding. Suppose the contrary, i.e. let $q_{LH} = q_{HL} = q_{HH} = q$ and therefore $\mu_1 \geqslant 0$ and $\mu_2 \geqslant 0$. By adding (13) and (14), and from (15), it is easy to see that we must have $(2\gamma - 1/\gamma)\,\Delta\theta^2 \geqslant \Delta\theta^1 \geqslant (2/\gamma(1-\gamma))\,\Delta\theta^2$, which is impossible since $2/\gamma(1-\gamma) > 2\gamma - 1/\gamma$. Hence, there are only three possible regimes. In order to compare them, we will write q_{jk}^R to denote the optimal quantity in regime R, $R = I, II, III$, when the cost announcement is $\hat{\Theta}_{jk}$, with $j, k \in \{L, H\}$.

- Regime I. $\mu_1 > 0$, $\mu_2 = 0$ ($q_{LH} = q_{HL}$, $q_{HL} > q_{HH}$),

$$V'(q_{LL}^I) = \Theta_{LL}$$

$$V'(q_{LH}^I) = V'(q_{HL}^I) = \Theta_{LH} + \frac{2-\gamma}{2(1-\gamma)}\,\Delta\theta^1 - \Delta\theta^2$$

$$V'(q_{HH}^I) = \Theta_{HH} + \frac{1-(1-\gamma)^2}{(1-\gamma)^2}\,\Delta\theta^2.$$

- Regime II. $\mu_1 = \mu_2 = 0$ $(q_{LH} > q_{HL},\ q_{HL} > q_{HH})$,

$$V'(q^{II}_{LL}) = \Theta_{LL}$$

$$V'(q^{II}_{LH}) = \Theta_{LH} + \frac{\gamma}{1-\gamma}\,\Delta\theta^2$$

$$V'(q^{II}_{HL}) = \Theta_{HL} + \frac{1}{1-\gamma}\,[\Delta\theta^1 - \Delta\theta^2]$$

$$V'(q^{II}_{HH}) = \Theta_{HH} + \frac{1-(1-\gamma)^2}{(1-\gamma)^2}\,\Delta\theta^2.$$

- Regime III. $\mu_1 = 0,\ \mu_2 > 0$ $(q_{LH} > q_{HL},\ q_{HL} = q_{HH})$,

$$V'(q^{III}_{LL}) = \Theta_{LL}$$

$$V'(q^{III}_{LH}) = \Theta_{LH} + \frac{\gamma}{1-\gamma}\,\Delta\theta^2$$

$$V'(q^{III}_{HL}) = V'(q^{III}_{HH}) = \Theta_{HH} + \frac{\gamma}{1-\gamma}\,\Delta\theta^1.$$

Notice that we have $V'(q_{LL}) < V'(q_{LH})$ in all regimes. Observe too that the solution corresponding to regime II is at least as efficient as those for other regimes, since it corresponds to a free maximum of the objective function. Moreover,

$$V'(q^{II}_{HL}) - V'(q^{II}_{LH}) = \frac{1}{1-\gamma}\,[(2-\gamma)\,\Delta\theta^1 - 2\Delta\theta^2]$$

$$V'(q^{II}_{HH}) - V'(q^{II}_{HL}) = \frac{1}{(1-\gamma)^2}\,[(2-\gamma)\,\Delta\theta^2 - (1-\gamma)\,\Delta\theta^1].$$

Thus, if $\Delta\theta^1 \in ((2/2-\gamma)\,\Delta\theta^2, (2-\gamma/1-\gamma)\,\Delta\theta^2)$ then

$$V'(q^{II}_{HL}) - V'(q^{II}_{LH}) > 0 \quad \text{implies} \quad q^{II}_{LH} > q^{II}_{HL},$$

$$V'(q^{II}_{HH}) - V'(q^{II}_{HL}) > 0 \quad \text{implies} \quad q^{II}_{HL} > q^{II}_{HH},$$

and the optimal solution to the principal's problem yields $q^{II}_{LL} > q^{II}_{LH} > q^{II}_{HL} > q^{II}_{HH}$.

If $\Delta\theta^1 \leqslant (2/2-\gamma)\,\Delta\theta^2$, or if $\Delta\theta^1 \geqslant (2-\gamma/1-\gamma)\,\Delta\theta^2$, the regulator must choose between regimes I and III. To see which regime prevails, notice that from a comparison of the first order conditions corresponding to these regimes, we obtain:

(1) For all $\Delta\theta^2$ and $\Delta\theta^1$, we have $q^I_{LL} = q^{III}_{LL}$ and $q^I_{HL} > q^{III}_{HL}$.

(2) If $\Delta\theta^1 \leqslant (2/2-\gamma)\, \Delta\theta^2$, then $q^I_{LH} \geqslant q^{III}_{LH}$ and $q^I_{HH} < q^{III}_{HH}$.

(3) If $\Delta\theta^2 \geqslant (2-\gamma/1-\gamma)\, \Delta\theta^2$, then $q^I_{LH} < q^{III}_{LH}$ and $q^I_{HH} \geqslant q^{III}_{HH}$.

Consider first (2). The concavity of $V(q)$ implies

$$\begin{aligned} &W^C(q^I) - W^C(q^{III}) \\ &\quad \geqslant \gamma(1-\gamma)[V'(q^I_{LH}) - \Theta_{LH}](q^I_{LH} - q^{III}_{LH}) \\ &\qquad + \gamma(1-\gamma)[V'(q^I_{HL}) - \Theta_{HL}](q^I_{HL} - q^{III}_{HL}) \\ &\qquad + (1-\gamma)^2 [\Theta_{HL} - V'(q^I_{HH})](q^{III}_{HH} - q^I_{HH}) - \gamma^2 \Delta\theta^2 (q^I_{LH} - q^{III}_{LH}) \\ &\qquad - \gamma[\Delta\theta^1 - \Delta\theta^2](q^I_{HL} - q^{III}_{HL}) + (1-(1-\gamma)^2)\, \Delta\theta^2 (q^{III}_{HH} - q^I_{HH}). \end{aligned}$$

Since $q^I_{LH} = q^I_{HL}$ and $q^{III}_{LH} > q^{III}_{HL}$, straightforward computations yield

$$\begin{aligned} &W^C(q^I) - W^C(q^{III}) \\ &\quad \geqslant \left[\gamma\Delta\theta^2 - \frac{\gamma(2-\gamma)}{2}\Delta\theta^1\right] [q^{III}_{LH} - q^I_{LH} + q^I_{HL} - q^{III}_{HL}], \end{aligned}$$

implying that regime I prevails over regime III since both expressions between brackets are non-negative.

Finally if (3) holds, i.e., if $\Delta\theta^1 \geqslant (2-\gamma/1-\gamma)\, \Delta\theta^2$, then it is easy to see, by using arguments similar to those in the previous case, that regime III dominates regime I.

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