

# Income taxation, uncertainty and stability

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## Abstract

This paper develops a political model to analyze the stability of income tax schedules. It is assumed that agents perceive any proposed alternative tax policy as more uncertain than the status quo. A tax policy is stable if it is a Condorcet winner. It is well known that in a model without uncertainty the existence of such a policy is very rare. We show, however, that in real cases this might not be a serious problem since small amounts of uncertainty can bring stability to the status quo. It is also shown that linear tax functions can only be stable in economies with very egalitarian income distributions and high taxation levels. © 1998 Elsevier Science S.A.

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## 1. Introduction

It is supposed that in democratic societies public opinion is quite relevant when deciding which tax policies should be adopted. It may be thought that the implemented tax schedule is the outcome of some voting process. For example, both the incumbent and the opposition party propose alternative tax policies and the one with the most votes is implemented. An agent, who is usually assumed to

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be self-interested, would vote for the proposed alternative she prefers. This approach may be criticized on the grounds that in real economies this type of voting never takes place. However, as Roberts (1977) notes the essential point is that parties behave and choices are made as if a vote on proposals were taken. That is, when designing their tax proposals parties try to maximize the popular support for their alternatives. A proposal is a majority winner (or a Condorcet winner) if it obtains a majority of votes in pairwise confrontations with all the other alternatives. Such a tax policy would be stable and we would expect it to arise as the equilibrium outcome in the parties competition game.

There is an important problem associated with this approach: the existence of such a majority winner is not, in general, guaranteed (see Bucovetsky, 1991). This is a serious difficulty due to the fact that, given a tax schedule, it is always possible to design an alternative one that hurts a minority of agents and, at the same benefits the rest. One way to overcome the problem is to impose restrictions on the set of admissible tax schedules and on the preferences of agents. Thus, most of this literature assumes that voting only takes place over the set of flat rate tax policies (see for example Roberts (1977), Romer (1975, 1977) and Foley (1967)). In these cases, and under some natural additional restrictions, the median income voter's ideal tax policy is a majority winner.

In Cukierman and Meltzer (1991), Snyder and Kramer (1988), Roemer (1994), and Gans and Smart (1996) different tax functions are considered but some new, also quite restrictive, assumptions are needed. In Snyder and Kramer (1988) only tax functions which are individually optimal for some voter are allowed. Cukierman and Meltzer (1991) analyze a model in which the tax functions are quadratic in income. They provide sufficient conditions for the median voter's most preferred tax policy to be a majority winner. They also show that under additional, rather strong conditions, such a tax function is progressive. Roemer (1994) provides some simulated equilibria in a model with constituency-representing parties and uncertainty. But the admissible tax functions are also quadratic in income. Gans and Smart (1996) show that the majority preference relation over convex tax schedules satisfying a 'single-crossing' property is quasi-transitive.<sup>1</sup>

Thus, unless we impose very strong restrictions, there exists no majority winner. Therefore, this political approach to tax determination predicts a lot of instability on the tax schedules. The reason being that '...voting over redistributive tax schedules is much like a majority-rule divide-the-dollar game, and there is no hope to obtain the existence of a static equilibrium in such a game' (Piketty, 1992, page 8).

<sup>1</sup>This is an interesting result. It however rules out the important comparisons of functions that cross each other at more than one point.

But in most democratic countries tax schedules are quite stable.<sup>2</sup> It appears, then, that the theory should be modified to capture this fact (see Piketty (1992) for a discussion of all these issues and critical comments on some alternative approaches found in the literature).

In this paper we analyze the existence of a majority winner in a political model of tax determination in which the income distribution is fixed — so many incentive considerations are left aside. In order to achieve positive results the analysis will concentrate on models with the following two properties: (i) the set of admissible income tax functions will consist of all concave and all convex increasing functions (including linear functions) which collect an exogenously given tax amount. (ii) It will be assumed that there is a status quo tax schedule and agents compare it with new proposals. Furthermore, when making these comparisons agents are certain about the tax liabilities associated with the status quo but attach some uncertainty to the tax liability of the alternative proposal.

Property (i) imposes two important restrictions on our model. Firstly, we analyze problems in which the total tax amount to be collected has been already decided so that the only political issue is how to collect it. Secondly, tax functions showing, for example, increasing marginal rates in some income intervals and decreasing marginal rates in others are not allowed. However compared with the previous literature these are weak restrictions. Moreover, most income tax schedules in western countries are convex (increasing marginal tax rate).

Considering only convex and concave functions is not enough of a restriction to obtain a majority winner. Consider for example a strictly convex income tax function. It is possible to find a different one which is also convex, collects the same amount and renders a majority of agents better off. The basic trick to obtain such a function is the following: increase the taxes for people belonging to a very small income interval around, for example, the median (people in that interval have to be a minority of agents). Then, reduce the taxes for all agents with income above that interval. A simple picture could help the reader to see that this can be done in such a way that the new function is convex, collects the same amount as the first one and a majority of agents pay equal or less taxes. For convex functions with linear parts a very similar trick will do it. In the case of a concave or linear function, it can be proved that any convex one is preferred by a majority of agents (see Marhuenda and Ortuño-Ortín, 1995).

Thus, in spite of this restriction on the set of admissible tax schedules, our problem still remains similar to the ‘divide-the-dollar’ problem. In other words,

<sup>2</sup>This is a relative concept and one might believe that tax schedules are not stable. Moreover, recent tax reforms like the one that took place in the USA in the 1980s could be seen as a proof of such instability. However the net effect of those tax reforms is not clear yet and many authors believe that the effective rates remain quite the same (see Kasten et al. (1994), Musgrave (1987)).

there is still enough flexibility to beat any given tax schedule by a tax reform involving a redistribution among taxpayers which benefits a majority of them.

Property (ii) captures the most innovative part of the model. It assumes that agents have better information on the status quo than on proposals which have not been implemented yet.

One may see this property as a very ad hoc feature to achieve positive results. We believe that, on the contrary, it is a very natural feature and it is worth exploiting it in this type of environment.<sup>3</sup> In real modern economies tax policies are usually very complex. Such policies have to describe the amount due by each income base level. This part is, in most cases, very simple. But the calculation of this income base depends on personal exemptions, standard deductions, two-earner deductions, income averaging, etc. A tax reform will normally involve changes in these features and we should not expect agents to be well informed on all these complexities. Things can become even more complicated if we consider income and corporate taxation (Sheffrin (1994) explores the way people perceive tax policies. It is shown that agents have a very imperfect knowledge on these issues. See also Musgrave, 1987).

An additional justification of our approach could be given by the fact that policy designers cannot accurately estimate the tax revenue that would be obtained under an alternative tax schedule. Thus, agents would believe that, should a new policy be implemented, it may well be the case that taxes collected were short of the target level (or exceeded it) and consequently the tax rates (or the income base) will have to be modified. The status quo, on the contrary, has already been implemented and each agent knows how much she paid.<sup>4</sup>

The 1986 Tax Reform Act (TRA) provides a nice example of this assumption on agent's uncertainty about tax reforms. According to Minarik (1989, page 293), during the Congressional debate on TRA '...one senator argued against the bill on the ground that 75% of the respondents to a poll said they expected their taxes to be higher — while the Joint Committee on Taxation estimated that 79% of those whose taxes would change would receive a tax cut once the law was fully phased in. Presumably, at least 54% of the population would receive a tax cut, but did not believe it'. The author also analyzes the reasons why taxpayers are misinformed and might expect the worst from a tax reform.

It is clear that the type of uncertainty we consider here makes things harder for alternative tax schedules. Assuming that all agents are risk averse, a proposal that, under no uncertainty beats the status quo, might well be opposed by a majority of

<sup>3</sup>A similar idea has been used in Bernhardt and Ingberman (1985). They consider an abstract model in which voters do not believe the winning party will always implement the proposed policy. Also, voters perceive the incumbent as less risky than challengers. They show that the equilibrium policy does not coincide with the median voter's ideal policy.

<sup>4</sup>An alternative approach would assume that agents are uncertain about their future income. This possibility is not exploited in this paper.

agents. It is clear then that we need to impose some structure and restrictions on the way uncertainty is modeled. Otherwise, we could render the whole approach trivial and useless. Thus, it will be assumed that each agent believes that her expected tax under an alternative tax schedule coincides with the announced tax liability. So we do not consider agents to expect the 'worst' from a tax reform. They are uncertain but not pessimistic about it.

For a tax function to obtain more popular support than the status quo it is not enough that the proposed tax liability is lower for a majority of agents. It is also needed that this tax cut is big enough to compensate agents for the risk associated with it. We think that this may capture very real situations. Consider for example a tax reform that announces a 0.5% reduction in taxes for 90% of taxpayers and a 5% raise for the rest. Even though, the majority of agents are expected to be better off under this reform it seems clear that (ex-ante) it would get little popular support.

Which concave or convex tax functions may arise as majority winner status quo for reasonable levels of uncertainty and risk aversion? This, unfortunately, turns out to be a hard question. Our analysis, then, will concentrate on a very specific but important case: when can a flat-rate tax schedule be a majority winner status quo?

Without uncertainty a linear tax function is always defeated by any convex one. Thus we now want to know whether incorporating uncertainty may prevent this from happening. The main result in this paper shows that a flat-rate tax policy is a majority winner status quo depending on a reduced number of parameters. In particular, only three statistics of the income distribution are relevant: the mean, the median and the percentage of total income owned by the richest 50%. It will be shown that economies with very egalitarian distributions of income (in the sense of small percentage of total income held by the richest 50%) and high taxation levels are the best candidates to have such a linear status quo. That is, these economies require smaller amounts of uncertainty and risk aversion for a linear status quo to become a majority winner.<sup>5</sup> The insight behind this is simple. In an economy with a very unequal income distribution and low taxation levels a linear tax function can be defeated by a convex one which brings large (in percentage) tax cuts for the majority. In the case of an egalitarian distribution it is only possible to beat the linear tax with convex functions that provide small tax cuts for the majority.

The mentioned result only implies that it is relatively easier for a linear tax schedule to be a majority winner in egalitarian economies. But the needed level of uncertainty may be too high even in those most favorable cases. This is, of course, a matter of empirical estimation. We provide some rough calibrations to realistic parameter values and conclude that at least for some egalitarian societies (in which

<sup>5</sup>This seems paradoxical since those egalitarian economies usually have very progressive tax schedules.

the richest 50% owns around 60% of the total income) moderate levels of uncertainty can make a linear tax a majority winner.

It should be remarked that even though we do not provide precise conditions for a convex tax function to be a stable status quo our analysis gives important hints on it. It is intuitive that, in general, it is easier to ‘beat’ a linear function than a convex one. We can believe, then, that it is more probable to observe a convex function than a linear one as a majority winner status quo. Moreover, many different convex functions could all be stable. Thus our approach would allow for the possibility that economies with similar income distributions present different convex tax schedules.

The main message in the paper is that, in order to beat the status quo, a tax proposal should not only reduce taxes for a majority of agents. It is also needed that such reduction is large enough. There is also an alternative interpretation of our model which does not need to consider uncertainty on the alternative tax policy. Under such an alternative interpretation, the question would be how to defeat the status quo, not by the largest number of votes, but by proposing the ‘maximum’ improvement for a majority of agents. This is an important question which arises in many political models (see for example Hinich et al., 1972) where a voter’s probability of voting diminishes as the utility differential between alternatives decreases). Suppose, for example, that voters consider two issues when deciding for which party to vote. The first one is the tax policy and the second the position of the parties on some non-economic issue. It might be the case that a majority of agents prefer the position of the incumbent party on the non-economic issue. In this case the challenger party may want to propose a tax policy which is also preferred by a majority of agents. However, for this party to obtain a majority of votes this may not be enough. Perhaps the improvement in utility experienced by the agents associated with such an alternative tax policy has to be quite large to compensate for the position on the other issue. The question, then, is to know the tax policy which, among those preferred to the status quo by at least 50% of the agents, reduces the taxes of this majority by the largest amount. Thus, the model presented in this paper may also be useful in answering this question. We will, however, stick to the first interpretation, that exploits the fact that agents perceive the status quo as less risky than new proposals, to obtain our results on the stability of the tax policy.

## 2. The model

We consider a population of agents whose income is distributed according to the fixed density function  $f$  defined on the interval  $[0,1]$  with  $\int_0^1 f(x)dx = 1$ . We identify an agent with her income  $x \in [0,1]$ .

**A1.** *All the admissible income tax functions  $t$  must satisfy the following requirements*

1. For each  $x \in [0, 1]$ ,  $0 \leq t(x) \leq x$ .
2. The tax function  $t(x)$  is continuous and non-decreasing.
3.  $\int_0^1 t(x) f(x) dx = R$  where  $R$  is some fixed real positive number.

The first requirement says that tax liabilities cannot exceed income and that redistribution of income is not allowed. In Section 3 we will comment on a case in which redistribution is possible. The second requirement is a very natural condition. Note that we do not require  $t$  to be differentiable at all points. Requirement 3 states that the total tax collected must meet the target  $R$ . Through this paper, Assumption 1 will be assumed.

We will distinguish between the tax function that is already in place and any alternative tax function that can be proposed. We denote the status quo tax function by  $q(x)$ , and a new proposal by  $t(x)$ . At this point we do not need to be very specific about how the status quo came to place and the way a proposal is announced. Our only objective will be to compare the popular support obtained by a given status quo versus any alternative tax function. It is important to note that both functions  $q(x)$  and  $t(x)$  must satisfy Assumption 1. In particular, the proposed function  $t(x)$  must be such that, were it implemented, the total tax collected would coincide with the total tax collected under the status quo, i.e. the amount  $R$ . Thus we are assuming that the only differences between alternative tax functions are about how to collect the already fixed amount  $R$ .

**A2.** All agents have the same (ex-post) utility function on money  $u(x)$ , with  $u' > 0$  and  $u'' < 0$ . Furthermore, this function has constant relative risk aversion, i.e.  $-x(u''(x))/u'(x) = \sigma$  for all  $x \in [0, 1]$ .

Given the status quo  $q$  and a proposal  $t$  we write  $q \succ_x t$  (resp.  $q \sim_x t$ ) whenever  $u(x - q(x)) > u(x - t(x))$ , (resp.  $u(x - q(x)) = u(x - t(x))$ ). Since  $u$  is increasing,  $q \succ_x t$  ( $q \sim t$ ) is equivalent to  $q(x) < t(x)$ , ( $q(x) = t(x)$ ). Let  $\Omega \equiv \{x: q \succ_x t\}$  and  $\Omega' \equiv \{x: q \sim_x t\}$ . We suppose that when indifferent between  $t$  and  $q$ , agents 'prefer'  $q$  with probability  $1/2$ . Let  $N(q, t) \equiv \int_{\Omega} f(x) dx + 0.5 \int_{\Omega'} f(x) dx$ . (We will suppose that  $\Omega$  and  $\Omega'$  are measurable sets). Thus  $N(q, t)$  indicates the percentage of agents who prefer  $q$  better than  $t$ . This is a measure of the popular support for the status quo when it faces alternative  $t$ . Agents might express their support for a tax policy by means of an election or by polls, but here we do not need to specify it. All the information required to determine  $N(q, t)$  can be deduced from the preferences of the agents in pairwise comparisons between the status quo and alternative tax functions. However, we will say that agent  $x$  votes for  $q$  whenever  $q \succ_x t$ .

**Definition 1.** The status quo  $q$  is a Condorcet winner if  $N(q, t) \geq 1/2$ , for all  $t$ .

We now introduce the basic assumption of the approach taken in this paper: it is assumed that when comparing  $q$  and  $t$  agents have better information on the first

one. Thus, we will impose an asymmetry in the way agents compare the status quo and the proposal. The status quo has been already implemented and agents are sure about its consequences, i.e. agent  $x$  knows exactly how much she has to pay under the tax scheme  $q(x)$ . It will be assumed, however, that even though the nominal tax under  $t$  is  $t(x)$ , agent  $x$  is uncertain about how much she will end up paying should this alternative tax scheme be implemented.<sup>6</sup> We will suppose that agent  $x$  believes that her ex-ante expected tax burden under  $t$  will coincide with the nominal tax  $t(x)$ , but the actual amount paid, ex-post, could be different. Since it will also be assumed that agents are risk averse this uncertainty about  $t$  would give an advantage to the status quo. One question to study, then, will be whether small degrees of uncertainty are enough to guarantee now the existence of a Condorcet winner.

The uncertainty about the function  $t$  can be modeled in many different ways. In this paper it will be assumed that agent's  $x$  beliefs on the tax she will end up paying depend only on  $t(x)$ . An alternative approach would make those beliefs depend on  $x$ . As long as tax functions take always positive values, the two approaches yield the same type of results. We will comment more on it in Section 4. The following assumption specifies the way we incorporate such uncertainty.

**A3.** *Given the announced tax function  $t$ , any agent  $x \in [0,1]$  believes that if  $t$  is implemented she will pay either the amount  $t(x) + \epsilon t(x)$  or the amount  $t(x) - \epsilon t(x)$ , where  $1 > \epsilon \geq 0$ . The probability of each of these alternatives is  $1/2$ . The constant  $\epsilon$  will be referred as the degree of uncertainty.*

Note that the expected tax payment coincides with the announced tax liability  $t(x)$ . Thus, it is assumed that agents believe in the proposal in expectation terms.<sup>7</sup>

A controversial aspect of Assumption 3 is that the degree of uncertainty  $\epsilon$  is a constant independent of the level  $t(x)$ . Thus all agents believe that the tax paid will deviate from the announced nominal level in the same percentage. Even though the paper does not provide a justification of this assumption we believe this is a natural way to tackle the uncertainty problem. Moreover our basic results would also hold if the degree of uncertainty were an increasing function of  $t(x)$ .

Since we will be interested in the existence of a Condorcet winner for not very large degrees of uncertainty it will be assumed that for all income levels  $x - t(x) - \epsilon t(x) \geq 0$ .

Assumption 3 changes the way agents compare  $q$  and  $t$  since the proposal  $t$  is now an uncertain alternative. Let  $U(t,x,\epsilon)$ , denote agent's  $x$  expected utility from

<sup>6</sup>We could also assume that tax payers are uncertain about the final tax paid under both  $t$  and  $q$ . The qualitative results would not change if the uncertainty on  $t$  is greater than the uncertainty on  $q$ .

<sup>7</sup>In many real situations there may exist an additional uncertainty about the expected tax. For example, some agents could consider that, in order to obtain the public approval, the announced  $t$  underestimates the expected tax.



the proposed tax  $t$ , i.e.  $U(t,x,\epsilon) \equiv \frac{1}{2}u(x-t(x) - \epsilon t(x)) + \frac{1}{2}u(x-t(x) + \epsilon t(x))$ . We also write  $U(q,x) \equiv u(x-q(x))$ .

**A4.** For all  $x \in [0,1]$  we have  $q >_x t \Leftrightarrow U(q,x) > U(t,x,\epsilon)$ ,  $(q \sim_x t \Leftrightarrow U(q,x) = U(t,x,\epsilon))$ .

Thus, agents behave as self-interested expected utility maximizers. Let  $\Phi \equiv \{x: q >_x t\}$ , and  $\Phi' \equiv \{x: q \sim_x t\}$ . It is also supposed (as we did in the certainty case) that whenever  $q \sim_x t$ , agent  $x$  ‘votes’ for the status quo with probability  $1/2$ . Let  $N(q,t,\epsilon) = \int_{\Phi} f(x)dx + 0.5 \int_{\Phi'} f(x)dx$ . Thus  $N(q,t,\epsilon)$  denotes the mass of agents who prefer  $q$  over  $t$  when the degree of uncertainty is  $\epsilon$ .

The next definition is the natural extension of Definition 1 to this environment with uncertainty on  $t$ .

**Definition 2.** The status quo tax function  $q$  is stable for the degree of uncertainty<sup>8</sup>  $\epsilon$  if for all  $t$  we have  $N(q,t,\epsilon) \geq 1/2$ .

Since  $N(q,t,0) = N(q,t)$  it is clear that there exists  $t$  such that  $N(q,t,0) < 1/2$ . For extremely large degrees of uncertainty the opposite would be true, i.e. for  $\epsilon'$  large enough  $N(q,t,\epsilon') > 1/2$ , for all  $t$ . This follows from the fact that agents are risk averse. Obviously this extreme case is of no relevance for our analysis. What we would like to find is the minimum  $\epsilon$  that guarantees the stability of  $q$ . This is however a quite difficult question. Thus, our strategy now will be to impose additional restrictions that allow us to obtain positive results.

### 3. The main results

In this section we give conditions under which a linear tax function  $q(x) = \alpha x$  is stable for a given degree of uncertainty. If the only restrictions imposed on the admissible proposals are given by Assumption 1 the problem would be quite intractable. The reason for this difficulty is that there are many possible ways in which the linear function  $q$  and a non-decreasing  $t$ , can cross each other. Thus, additional restrictions on the set of admissible tax functions are needed.

**A5.** The proposed tax function  $t$  must be concave or convex.

This restriction simplifies the analysis since a convex (concave) function, satisfying also Assumption 1, must cross the linear status quo  $q$  at only two points.

In many cases it can be interesting to allow for an exogenously given maximum

<sup>8</sup>Whenever it does not create confusion we will just say ‘stable’.

marginal tax rate  $\gamma \leq 1$ . This can capture perhaps some incentive concerns and since its value can be 1 it will not reduce the generality of our results.

Let  $\mu = \int_0^1 xf(x)dx$  stand for both the mean and the total income. Let  $m$  be the median of the income distribution, i.e.  $0.5 = \int_0^m f(x)dx$ .

**A6.** *The density function  $f$  is such that  $m \leq \mu$ .*

This is a very weak and natural restriction on the income distribution. It has important consequences however. In Marhuenda and Ortuño-Ortín (1995) it is shown that, for distribution functions satisfying the above assumption, if  $q$  is linear and  $t$  concave then  $N(q,t) \geq 1/2$ . Since  $N(q,t) = N(q,t,0) \geq 1/2$  then  $N(q,t,\epsilon) \geq 1/2$ , for all  $\epsilon \geq 0$ , and the mentioned result implies that we do not need to consider concave functions as successful proposals, i.e. if  $t$  is concave then  $N(q,t,\epsilon) \geq 1/2$ , for all  $\epsilon$ . Therefore only convex functions will have to be analyzed.

When uncertainty is allowed the existence of a stable linear status quo,  $q$ , will depend on the degree of uncertainty  $\epsilon$  and on the degree of risk aversion  $\sigma$ . Clearly higher values of  $\sigma$  increase the possibility of such a status quo.

The proof of our main result will make use of the function which minimizes the tax burden of the agent with median income. We define next such a function and provide some technical results.

Denote by  $\zeta$  the set of tax functions satisfying the above assumptions and consisting of only two marginal tax rates, i.e. if  $t \in \zeta$  then,

$$t(x) = \begin{cases} bx & \text{if } x \leq x' \\ cx + k & \text{if } x \geq x' \end{cases}.$$

where  $x' \in [0,1]$ ,  $b, c \in [0, \gamma]$ ,  $b < c$ ,  $k \leq 0$ . Let  $t_m \in \zeta$  be such that  $t_m(m) \leq t(m)$  for all  $t \in \zeta$ . Thus, the tax function  $t_m$  minimizes, among all the two marginal rate functions, the tax liability of the agent with median income. In principle this tax function does not need to be unique. For example, when  $R$  is small enough it is possible to have many tax functions in  $\zeta$  all of them requiring zero taxes from the median income. It is straightforward to see that there exists a  $t_m$  such that

$$t_m(x) = \begin{cases} \beta x & \text{if } x \leq d \\ \gamma x + k & \text{if } x > d \end{cases}.$$

where  $m \leq d < 1$ . Remark that  $\gamma$  is the maximum marginal tax rate allowed and that, by continuity,  $\beta m = \gamma m + k$ . Whenever the tax function  $t_m$  is not unique, we pick one of the form just described. Nevertheless, the only indeterminacy about  $t_m$  is on  $\gamma$  and  $d$ . The value of  $\beta$  is uniquely defined by the revenue requirement. A simple computation shows that

$$\beta = \max \left\{ 0, \frac{\alpha\mu - \gamma r + \frac{m\gamma}{2}}{\mu - r + \frac{m}{2}} \right\}. \quad (1)$$

where  $r = \frac{1}{m} \int x f(x) dx$ ,  $\alpha = R/\mu$ . Thus,  $\beta$  depends only on two statistics of the density function  $f$ : the mean income and the total income held by the richest 50% group of agents.

Note that a convex function  $t(x)$  which collects the same amount as the linear status quo,  $q(x) = \alpha x$  and satisfies  $t(0) \leq q(0)$  must intersect the graph of  $q$  at exactly one point (other than 0). Furthermore, at that point,  $t(x)$  must intersect  $q$  from below. This single-intersection property means that  $q$  is stable if and only if there is no convex tax policy which is preferred by all the agents whose income is below the median. The next proposition shows that to analyze the stability of the status quo one only needs to compare  $t_m$  with the status quo  $q$ .

**Proposition 1.** *Suppose that the hypotheses 1 through 6 are satisfied. Let  $q(x) = \alpha x$ . Then  $q$  is a stable status quo if  $N(q, t_m, \epsilon) \geq 1/2$ .*

**Proof.**<sup>9</sup> The only part is clear. To see the other implication note that it follows from Assumption 2 that the utility functions of the agents are of the form  $u(x) = K_1 + K_2 (x^{1-\sigma})/1-\sigma$  for  $\sigma \neq 1$ , or  $u(x) = K_1 + K_2 \log(x)$  for  $\sigma = 1$ , with  $K_1$  and  $K_2$  constant. Let  $t \in \zeta$  be an admissible tax function and assume  $\sigma \neq 1$  (the case  $\sigma = 1$  follows from a similar argument). A straightforward computation shows that, under Assumption 3, we have that  $t$  is preferred by agent  $x$  to the status quo  $q$  if and only if

$$\frac{1}{2} \left( \left( 1 - (1 + \epsilon) \frac{t(x)}{x} \right)^{1-\sigma} + \left( 1 - (1 - \epsilon) \frac{t(x)}{x} \right)^{1-\sigma} \right) > (1 - \alpha)^{1-\sigma}$$

The left-hand side of the above inequality decreases with  $t(x)/x$ . Since the tax function is convex, the average tax,  $t(x)/x$ , must increase with  $x$ . Hence, the set of people who prefer  $t(x)$  to the status quo is some interval  $[0, X]$  of the income distribution. It follows that the convex tax policy  $t$  will win the election if and only if the median voter  $m$  prefers  $t$  over the status quo  $q$ . We construct now a tax policy  $s(x)$  consisting of the two line segments connecting  $(0, 0)$  with  $(m, t(m))$  and  $(m, t(m))$  with  $(1, t(1))$ . Clearly,  $t_m(m) \leq s(m) \leq t(m)$ . Therefore, if  $t$  defeats the status quo, so does  $t_m$ .

The consequence of the above proposition is clear: whenever we want to study conditions to guarantee that the linear tax function is a Condorcet winner, it will be enough to compare it with the one which minimizes the tax liability of the agent

<sup>9</sup>We thank an anonymous referee for providing this more intuitive version of our original proof.

with median income. It follows that a necessary and sufficient condition for  $q$  to be a Condorcet winner is given by the following inequality

$$U(t_m, m, \epsilon) \equiv \frac{1}{2}(u(m - \beta m - \epsilon \beta m) + u(m - \beta m + \epsilon \beta m)) < U(q, m). \quad (2)$$

The closer  $\beta$  is to  $\alpha$  the lower the value of  $\epsilon$  needs to be for the above inequality to hold. Note that a necessary condition for Eq. (2) to be satisfied is that  $\alpha < \beta + \epsilon \beta$ . Hence the lowest possible value for the degree of uncertainty to guarantee Eq. (2) is  $\hat{\epsilon} = \frac{\alpha}{\beta} - 1$ .

Differentiating (see Appendix A) in Eq. (1) we have that, for  $\beta > 0$ ,

$$\frac{\partial \beta}{\partial r} < 0, \quad \frac{\partial \beta}{\partial \gamma} < 0, \quad \frac{\partial(\alpha/\beta)}{\partial \alpha} < 0 \quad (3)$$

Eq. (3) shows that the higher the concentration of income on the richest 50% group the lower  $\beta$  is and therefore, other things the same, the more difficult it is for  $q$  to be a Condorcet winner. This suggests the interesting result that it is in economies with an equal distribution of income (in the sense of a low value of  $r$ ) in which a linear tax scheme can find enough popular support. It is also important to remark on the nice fact that  $\beta$  depends only on the statistics  $m$  and  $r$ .

The derivatives in Eq. (3) also imply that allowing for a higher maximum marginal tax rate reduces the possibilities for  $q$  of arising as a stable status quo. This is less surprising than the former result. Greater values of  $\gamma$  allow for lower levels of  $\beta$  which makes it harder for  $q$ . The last derivative in Eq. (3) suggests that higher values of  $\alpha$  increase the chances of  $q$  being a Condorcet winner.

It would appear, then, that societies with very high taxation levels<sup>10</sup> and egalitarian distribution of income are best candidates for adopting a linear tax schedule. Although, at the same time it is in this type of societies where we observe high marginal tax rates and, since,  $\partial \beta / \partial \gamma < 0$  and  $\partial(\alpha/\beta) / \partial \alpha < 0$ , this makes more difficult for  $q$  to be a Condorcet winner.

The analysis above shows how simple it is now to check for stability of a linear tax function. Namely, if  $u(x) = (x^{1-\sigma}) / (1-\sigma)$ , then it is immediately obvious that  $N(q, t, \epsilon) > 1/2$  if and only if

$$(1 - \alpha)^{1-\sigma} > 0.5[(1 - \beta - \epsilon \beta)^{1-\sigma} + (1 - \beta + \epsilon \beta)^{1-\sigma}].$$

Many authors have defended a negative tax function, i.e. a function of the form  $\hat{q}(x) = \alpha x - k$ , as an alternative to the tax policies observed in most countries. It might be said, then, that the relevant question to analyze is the possible stability of  $\hat{q}$  (instead of  $q = \alpha x$ ). Note that  $\hat{q}$  implies some income redistribution among agents and Assumption 1 eliminated that possibility in our analysis. However, our model can be easily extended to this important case with the additional restriction that all the allowed functions  $t$  satisfy the inequality  $\hat{q}(0) \geq t(0)$ . The results of

<sup>10</sup>Remember that  $\alpha$  also measures the percentage of total income collected as taxes

such extension would be qualitatively identical to the ones we have provided here. Thus, our analysis offers also some insights on the stability of a negative income tax schedule.

#### 4. Simulations

This section provides some numerical examples offering an approximate idea of the possible magnitudes of the parameters. Table 1 shows values of  $\beta$  for several realistic distributions of income and values of  $\alpha$  (the percentage of total income collected as taxes) and  $\gamma$  (the maximum marginal tax rate). The mean and median income are expressed in thousand of dollars and are close to the Spanish values for 1989 (see Melis and Díaz, 1993). Since the only other statistic necessary is  $r$  we do not need to specify the whole income range. The parameter  $\hat{r}$  stands for the percentage of total income held by the richest 50% of the population, i.e.  $\hat{r} \equiv r/\mu$ .

Remember that the closer  $\beta$  and  $\alpha$  are to each other the lower the degree of uncertainty needed for the linear tax function to be a Condorcet winner. Thus, for example, in the economy of the first row in the table we have  $\alpha=0.25$ ,  $\beta=0.14$ . This implies that to make  $q$  a Condorcet winner the degree of uncertainty must be at least  $\hat{\epsilon}=0.78$ . That is, agents must believe that the final tax paid will be  $t(x) \pm 0.78t(x)$ . Of course, the specific  $\epsilon$  that makes  $q$  a Condorcet winner depends on the degree of risk aversion. In many cases  $\epsilon$  may turn out to be much larger than  $\hat{\epsilon}$ . Thus, in this example the linear tax seems clearly unstable. It is important to remember, however, that the way uncertainty has been modeled here is really unfavorable for the status quo: an agent believes that her tax liability will be either  $(1+\epsilon)t(x)$  or  $(1-\epsilon)t(x)$ . In reality, taxpayers might be more ‘pessimistic’ about the implementation of alternative tax policies. For example, all our analysis can be repeated assuming that agents believe that the tax liability will be  $(1+\epsilon)t(x)$  with probability  $p$  and  $(1-\epsilon)t(x)$  with probability  $1-p$ , where  $p>1/2$ . With this bias towards  $(1+\epsilon)t(x)$  the stability of  $q$  would be guaranteed for lower values of  $\epsilon$ .

The second row provides an example that roughly fits the Spanish case (the maximum marginal rate in Spain in 1989 was 56%, but things would not change much if we take higher values of  $\gamma$ ). Here  $\beta$  takes the value zero and consequently there is no  $\epsilon$  that makes  $q$  a Condorcet winner. However this is due to the fact that uncertainty depends on the nominal tax  $t(x)$ . As was already mentioned in Section

Table 1  
Simulations

$\alpha$	$\gamma$	$\mu$	m	$r$	$\beta$	$\epsilon$
0.25	0.56	1.4	1.3	0.7	0.14	0.78
0.18	0.56	1.4	1.25	0.75	0	$\infty$
0.4	0.8	1.4	1.38	0.6	0.35	0.14
0.4	0.8	1.4	1.38	0.85	0.17	1.35

2, under an alternative approach uncertainty would depend on the tax base  $x$ . In that case, agent  $x$  believes that her tax liability will be  $t(x \pm \epsilon x)$ . All the previous results would also hold here. The only difference is that under this alternative approach the degree of uncertainty that makes  $q$  a Condorcet winner would take different values and, in particular, the minimum possible uncertainty level would be  $\hat{\epsilon} = \alpha - \beta$ . The advantage of this alternative is that in principle  $q$  can be a Condorcet winner even though  $\beta$  is zero. This suggests that the essential parameters to analyze when deciding on the possibility of a stable status quo are  $\alpha$  and  $\beta$ . In the example we are commenting on, and regardless of the way we model uncertainty, it seems that the linear tax function can hardly be a Condorcet winner. The intuition is clear: a majority of the population would gain a lot by moving from  $q$  to  $t_m$ .<sup>11</sup>

The third row in the table shows an economy with a higher taxation level (40%), high maximum marginal tax rate (80%) and a very egalitarian distribution of income: the mean remains as in the second row but the median now is higher and the richest 50% only own 60% of the total income. As a consequence  $\beta$  is relatively close to  $\alpha$  suggesting that here the linear tax could be a Condorcet winner.

The economy in the last row is identical to the third one in all the parameters but the percentage  $\hat{r}$ . The corresponding value of  $\beta$  is quite far from  $\alpha$  showing the critical effect of  $r$  on the stability of the status quo.

The conclusion that arises after trying many other examples is that the distance between  $\alpha$  and  $\beta$  is quite sensitive to changes in the values of  $r$  and  $\alpha$ .

## 5. Final comments

In this paper a model to analyze the stability of tax schedules has been developed. The main characteristic of the model consists in the asymmetric way agents assess the currently implemented tax policy versus a proposed new alternative.

A tax policy is a majority winner if a majority of the population prefers it to any other admissible policy in pairwise comparisons. It is well known that in a model without uncertainty the existence of such a policy is very rare. We have argued, however, that in some cases this might not be a real problem since small amounts of uncertainty can make the status quo a Condorcet winner.

We have mainly concentrated the analysis on the linear tax function. This is a very specific, although quite important, case. The basic insight obtained is that it is mostly in very egalitarian economies with high taxation levels that the linear function might be stable.

<sup>11</sup>This coincides with the empirical estimations on the Spanish case provided in Castañer (1991) Approximately the poorest 53% of the taxpayers would see their taxes substantially increased if instead of the 1988 tax schedule a linear one were implemented

It is important to note again that uncertainty has been modeled in a ‘fair’ way. Under other, perhaps more realistic alternatives, the status quo would enjoy additional advantages. Furthermore, the linear tax scheme does not seem to be the best candidate for stability. Thus, if our objective is to show that small degrees of uncertainty about new alternatives can render the status quo policy stable, we suspect that we have chosen the difficult candidate. For example, it seems intuitive that it is easier for a strictly convex tax function (as the ones implemented in most democratic economies) to be stable than for a linear one. It is also easy to find numerical simulations in which the linear function cannot be stable for any reasonable degree of uncertainty but a convex function can. However, we do not have yet any general analytical result here. This is left for future research.

Even though we are far from providing a complete positive model of tax determination — such a model should contain many other elements as, for example, voting on multidimensional issues, multiparty elections, consideration of the distributional effects of government expenditure, etc. — our result may be useful to understand the observed stability of different tax policies in most western countries.

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## Appendix A

Differentiating in the expression of  $\beta$  with respect to  $r$  we obtain

$$\frac{\partial \beta}{\partial r} = \frac{(\alpha - \gamma)\mu}{\left(\mu - r + \frac{m}{2}\right)^2}$$

since  $\gamma$  is the maximum marginal tax rate, we have that  $\gamma > \alpha$  and the sign of this derivative is negative. Differentiating now  $\beta$  with respect to  $\gamma$  we get

$$\frac{\partial \beta}{\partial \gamma} = \frac{-r + \frac{m}{2}}{\mu - r + \frac{m}{2}}. \quad (4)$$

Since  $1/2 = \int_m^1 f(z)dz$  and  $m/2 = m \int_m^1 f(z)dz < \int_m^1 zf(z)dz = r$ , it follows that the

numerator in Eq. (4) is negative. The denominator is clearly positive. Therefore the sign in Eq. (4) is negative. Finally, the derivative of  $\alpha/\beta$  with respect to  $\alpha$  is

$$\frac{\partial\left(\frac{\alpha}{\beta}\right)}{\partial\alpha} = \frac{\beta - \frac{\mu\alpha}{\mu - r + \frac{m}{2}}}{\beta^2} \quad (5)$$

and the same argument as above yields

$$-r + \frac{m}{2} < 0, \text{ so } \frac{\mu}{\mu - r + \frac{m}{2}} > 1,$$

and given that  $\beta < \alpha$ , the sign of Eq. (5) is negative.

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