Working Paper 98-26
Economics Series 07
February 1998

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-98-75

MORE ON PREFERENCE AND FREEDOM

Antonio Romero-Medina*

Abstract

The paper seeks to formalize the notion of effective freedom or freedom to realize meaningful choices. The definition of meaningful choice used in this paper is based on the preference orderings of a reasonable person in a society. I argue that only the alternatives that can be selected by a reasonable person from the set of all possible alternatives provide a meaningful choice. I discuss this approach and provide an axiomatization of the cardinal rule and two lexicographic versions of this rule in this context.

Keywords: Ranking Sets; Freedom of Choice.

Journal of Economic Literature Classification Number: D71

* Romero-Medina, Departamento de Economía, Universidad Carlos III de Madrid. E-mail: aromero@eco.uc3m.es.

I am grateful to Prasanta K. Pattanaik for many stimulating discussions and helpful suggestions. Several improvements are due to the comments of Salvador Barberá, Iñigo Iturbe-Ormaetxe and an anonymous referee. Financial support form M.E.C./Fullbright and DGCYT project PB 92-0590 is gratefully acknowledged.

More On Preference and Freedom *

Antonio Romero-Medina
Departamento de Economia,
Universidad Carlos III de Madrid
C/ Madrid, 126. 28903 Getafe, Madrid, Spain
Email: aromero@eco.uc3m.es

First Version: February 1996
This version: February 18, 1998

Abstract

The paper seeks to formalize the notion of effective freedom or freedom to realize meaningful choices. The definition of meaningful choice used in this paper is based on the preference orderings of a reasonable person in a society. I argue that only the alternatives that can be selected by a reasonable person from the set of all possible alternatives provide a meaningful choice. I discuss this approach and provide an axiomatization of the cardinal rule and two lexicographic versions of this rule in this context.

Keywords: Ranking Sets, Freedom of Choice

Journal of Economic Literature Classification Number: D71

* I am grateful to Prasanta K. Pattanaik for many stimulating discussions and helpful suggestions. Several improvements are due to the comments of Salvador Barberá, Iñigo Iturbe-Ormaetxe and an anonymous referee. Financial support from M.E.C./Fulbright and DGCYT project PB 92-0590 is gratefully acknowledged.

1. Introduction

The purpose of this paper is to explore some issues about the intrinsic value of freedom of choice. The paper considers an agent who faces alternative feasible sets of alternatives A, B, etc. Each one of these sets is a non-empty subset of some given universal set of alternatives X. The agent has to choose exactly one alternative from one of these sets. The intuition behind this paper is that freedom of choice is valuable in itself. The problem considered is how to rank different sets according to the freedom that they offer. The paper focus on the capacity of the sets to provide meaningful choices. I will discriminate between the alternatives that constitute meaningful choices and those that do not, using as reference the set of preferences that a reasonable person may have and considering the alternatives that they may choose in the universal set of alternatives.

The need for introducing preferences into the analysis of the freedom was pointed out by Sen (1991). In his idea of effective freedom an individual is free if she has access to alternatives that she regards as valuable in terms of some criteria. This criteria may be her preferences or, as in Pattanaik and Xu (1997), the preferences that a reasonable person in her place may have.

The use of the preferences of a reasonable person as a reference point for the evaluation of the freedom that a set of alternatives offers was suggested by Jones and Sugden (1982) and considered by Pattanaik and Xu (1997). According to Jones and Sugden (1982) "if any reasonable person would be indifferent between two particular alternatives, then offering choice contributes to little diversity." Pattanaik and Xu (1997) takes the role of preferences a step forward. According to them, the intrinsic value of freedom of choice should be judged "not [in terms of] the preferences that the agents actually have, nor the preferences ordering as his future preference ordering, but the preference orderings that a reasonable person in the agent's situation can possibly have." The model they propose has the virtue of capturing effective freedom without collapsing into an indirect utility ranking.

In comparing two opportunity sets, A and B, Pattanaik and Xu (1997) concentrate on $\overline{A}$ and $\overline{B}$, where $\overline{A}$ is the set of all alternatives in A which reasonable persons may choose from the feasible set A and similarly for $\overline{B}$. However, in this paper I follow an approach that differs from this. I consider first the set $\overline{X}$ of all alternatives in the universal set X that reasonable people will choose if the universal set were the feasible set and then concentrate on $\overline{X} \cap A$ and $\overline{X} \cap B$ in comparing A and B. The intuitive difference between the procedure of Pattanaik and Xu (1997) and the procedure discussed here may be illustrated with an example. Consider the case where the universal set of alternatives X contain four alternatives: life in prison, to be beheaded, to be hanged and to be killed in the electric chair. Suppose that there are two sets of alternatives A and B to be ranked. The set A contains three alternatives: to be beheaded, to be hanged and to be killed in the electric chair. The set B contains two alternatives: to be hanged and to be killed in the electric chair. It is not unthinkable that any reasonable person in a society will prefer life to any of the other alternatives. It is also reasonable to think that facing the possibility of death, we can find a reasonable person who may prefer any of the possible methods in X or may be indifferent between them.

The first rule characterized by Pattanaik and Xu (1997) considers relevant alternatives in the comparison of A and B those in $\overline{A}$ and $\overline{B}$. A set offers more freedom than other if it contains more of the relevant alternatives. In that case, A is considered offering more freedom than B. In a second characterization the rule takes into account the number of alternatives that are contained in the intersection of each of the sets A or B and the choices of a reasonable person in $A \cup B$, i.e. $\overline{A \cup B}$ $\cap A$ and $\overline{A \cup B}$ $\cap B$, again A is considered offering more freedom than B. Both characterizations give importance in terms of freedom of choice to alternatives that are irrelevant for the agent. This is in the sense that these alternatives will never be chosen by reasonable person if there were no feasibility constraints over the alternatives. The alternatives in A or B are unable to fulfill the vital project of a reasonable person because no reasonable person will ever choose them. A reasonable person will never choose to be beheaded at dawn as the alternative that helps her "shaping his [her] life in accordance with some overall plan" or "giving meaning to his [her] life" or "being with the capacity so to shape his life can have or strive for a meaningful life" (Nozick 1974 p. 50). The idea to consider only those options which are capable of "shaping ones own life" is captured formally in this paper using $\overline{X} \cap A$ and $\overline{X} \cap B$ in comparing A and B. In the example both sets A, B are declared indifferent in terms of the freedom they offer because no reasonable person will choose an alternative in these sets if this choice is not constrained.

Choosing for oneself and shaping ones own life is essential for a meaningful human life. The alternatives that can be chosen by a reasonable person in a society where no constraints exists are the ones that allow the agents to shape their lives and provide them with full control over themselves. They allow the agents to formulate plans and choosing among these plans an agent is considering alternative steps in her vital project. If what is relevant to measure the freedom contained in a set depends on the composition of the particular set then a person that has to choose between three different ways to die can have more freedom than other who has to choose between life and death. Also, a slave who has to choose among hundreds escape plans can have more freedom than her master who decides among lesser options, even when the masters options are more real and desirable to the eyes of any reasonable person than the freedom dreams of the slave.

To use as reference the set of alternatives a reasonable person may choose generate situations that may seem puzzling. This is because alternatives that are not feasible are having a role in evaluating the freedom that a set offers. Let's consider a situation where all the reasonable persons in a society believe that the woman a is the most desirable mate. These preferences may seem inadequate when dealing with the perspective of a reasonable mate but it is close to the set of reasonable preferences that kids might have about a particular toy as the best Christmas present. In that case the sets $A = \{c, d, b\}$ and $B = \{d, b\}$ are indifferent in terms of the number of alternatives that intersect with $\overline{X}$. Since that a being unanimously considered the best possible choice seems hardly enough reason to claim that A or B contains as much freedom as the empty set. It can be argued that, if the social consensus so described then only a can fulfill their vital project, in that case the election of other person when a is not available will fulfill only utilitarian purposes.

The example is, somehow, pathological. A reasonable person may have any preference that is not contradictory or illogical. In the example it is not unthinkable that any woman can be a reasonable mate and be a choice of some reasonable person. The freedom attributed to a set depends on the preferences that can be conceived as reasonable by a person in her situation. These preferences will be paradoxical only if the preferences attributed to a reasonable people in her situation are too.

The paper is organized as follows. The next section introduces some notation and definitions. Section 3 contains the main characterization result. The rule characterized provide a complete order of the sets of alternatives. In Section 4 two lexicographic versions of the previous rule are considered. The relation between the original approach and its lexicographic versions is also discussed . The paper finishes with some final remarks in Section 5.

2. Notation and definitions

Let X be the universal set of alternatives, assumed to be finite. At any given time, the set of alternatives available to the individual will be a non-empty subset of X, and she has to choose exactly one alternative from this set of available alternatives.

Let Z be the set of all non-empty subsets of X. The elements of Z are the feasible sets that the agent may face. Let $\succsim$ be a binary relation defined over Z. For all $A, B \in Z$, $[A \succsim B]$ means that A offers at least as much freedom as B. For all $A, B \in Z$, $[A \succ B$ iff $A \succsim B$ and $\neg(B \succsim A)]$ and $[A \sim B$ iff $A \succsim B$ and $\neg(B \succsim A)]$.

The reference set of preference orderings over X is denoted as $\wp = (R_1, ..., R_n)$. A preference ordering over X is a reflexive, complete and transitive weak preference relation, at least as good as, over X. $\wp$ will be interpreted as the set of all possible preference orderings over X that a reasonable person may have. I denote by $\max(A)$ the set of all alternatives x in A such that x is a best alternative in X for some ordering in $\wp$ and let $\mathcal{P}(\wp) = \max(X)$. I will call $\mathcal{P}(\wp)$ the set of relevant alternatives in X.

Now I consider a number of properties that the binary relation $\succsim$ over Z may satisfy.

Definition 2.1. *A binary relation $\succsim$ over Z satisfies:*

1. *indifference of no-choice situations (INS) iff, for all $x, y \in X, \{x\} \sim \{y\}$;*
2. *simple non-dominance (SND) iff, for all $x, y \in X$, if $\#\max(\{x\}) = \#\max(\{y\})$, then $\{x\} \sim \{y\}$;*
3. *inclusion monotonicity (IMON) iff, for all $A, B \in Z$ if $A \supseteq B$ and $\max(A \setminus B) \neq \emptyset$, then $A \succ B$. If $A \supseteq B$ and $\max(A \setminus B) = \emptyset$, then $A \sim B$;*
4. *composition (COM) iff, for all $A, B, C, D \in Z$, such that $A \cap C = B \cap D = \emptyset$ and $A, B, C, D \subseteq \mathcal{P}(\wp)$,*

$$[A \succsim B \text{ and } C \succsim D] \rightarrow [A \cup C \succsim B \cup D], \text{ and,}$$
$$[A \succsim B \text{ and } C \succ D] \rightarrow [A \cup C \succ B \cup D].$$

The preceding axioms are versions of traditional ones adapted to the context of the paper. This provides them with new meaning. The INS axiom is a classical axiom. We can think in situations where INS fail to capture the idea of effective freedom. For example consider two situations, such that in each case you must read a book. In the first case the book considered is a telephone directory. In the second, the book is one that a reasonable person may have chosen from the set of all the available books ever written. INS will rank both sets as indifferent. In terms of freedom both the situations are different. The second set contains a book to read. No reasonable person will consider the telephone directory a reading book, this set is empty from a reader's point of view and thus less preferred than the one containing a book that a reasonable person may choose. INS cannot capture this situation where we are comparing between a set containing a book and an empty set. In the model the spirit of INS is captured by SND when the alternatives compared are both relevant. The cardinality rule characterized in the next section do not satisfy INS but satisfy SND.

IMON adapts a preference independent axiom, called by Sen (1991) weak dominance. IMON requires that inclusion in terms of relevant alternatives implies preference. It also restrains the role of no relevant alternative and exclude them from being considered when a set is compared with one of its subsets.

The axiom of COM was originally defined by Sen (1991). He requires only that $A \cap C = B \cap D = \emptyset$. As Pattanaik and Xu (1997) remarks, there may be differences in the contributions that sets C and D give to A and B. To avoid this problem Pattanaik and Xu (1997) reduces the sets to be considered those where all $A \cup C$ and $B \cup D$ are relevant alternatives. In line with the same approach. I simplify the axiom in the context defined in this paper. It is enough that the sets considered are subsets of the set of relevant alternatives $\mathcal{P}(\wp)$ and it is not necessary to establish conditions over their union.

3. The result

In this section I characterize the binary relation defined by the cardinality rule in terms of relevant alternatives. This is, for all $A, B \in Z$,

$$A \succsim^* B \leftrightarrow \# \max(A) \geqq \# \max(B)$$

a set A will be declared preferred B if and only if the number of relevant alternatives A contains is bigger that the number of relevant alternatives contained in B.

Proposition 3.1. $\succsim$ *satisfies SND, COM and IMON if and only if* $\succsim = \succsim^*$.

Proof.

The necessity part of the proposition is obvious; I prove only the sufficiency part. Let $\succsim$ satisfy SND, COM and IMON. First, I show:

$$\text{for all } A, B \in Z, \text{ if } \#\max(A) = \#\max(B)\,, \text{ then } A \sim B. \tag{3.1}$$

Suppose $A, B \in Z$ and $\#\max(A) = \#\max(B) = g$. Let $\max(A) = \{a_1, ..., a_g\}$ and $\max(B) = \{b_1, ..., b_g\}$. By SND,

$$\{a_1\} \sim \{b_1\} \tag{3.2}$$

and

$$\{a_2\} \sim \{b_2\}\,. \tag{3.3}$$

$\{a_1\} \cap \{b_1\} = \{a_2\} \cap \{b_2\} = \emptyset$ and, further $\max(\{a_1\}) = \{a_1\}, \max(\{a_2\}) = \{a_2\}$ and $\max(\{b_1\}) = \{b_1\}, \max(\{b_2\}) = \{b_2\}$, since $a_1, a_2 \in \max(A)$ and $b_1, b_2 \in \max(B)$. Hence by 3.2, 3.3 and COM, I have

$$\{a_1, a_2\} \sim \{b_1, b_2\}\,. \tag{3.4}$$

By SND, again,

$$\{a_3\} \sim \{b_3\}\,. \tag{3.5}$$

By 3.4, 3.5 and COM,

$$\{a_1, a_2, a_3\} \sim \{b_1, b_2, b_3\}\,. \tag{3.6}$$

Proceeding in this way, I finally have $\{a_1, ..., a_g\} \sim \{b_1, ..., b_g\}$, that is $\max(A) \sim \max(B)$. If $A = \max(A)$, then $A \sim \max(B)$. Suppose $\{A \backslash \max(A)\} \neq \emptyset$. Let $\{A \backslash \max(A)\} = \{\bar{a}_1, ..., \bar{a}_m\} \neq \emptyset$. It is clear that $T_1 = \max(A) \cup \{\bar{a}_1, ..., \bar{a}_m\}$ is such that $T_1 \subseteq A$ and $\max(A \setminus T_1) = \emptyset$. Then by IMON $T_1 \sim \max(B)$. Hence I have

$$A \sim \max(B)\,. \tag{3.7}$$

Similarly, by the use of IMON, from 3.7, $A \sim B$, which proves 3.1.

Next, I show:

$$\text{for all } A, B \in Z, \text{ if } \#\max(A) > \#\max(B)\,, \text{ then } A \succ B. \tag{3.8}$$

Suppose $A, B \in Z$ and $\#\max(A) > \#\max(B)$. Let $\#\max(B) = g$ and $\#\max(A) = g+t$ (where $t > 0$). further, let $\max(B) = \{b_1, ..., b_g\}$ and $\max(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$. Note that $\max\{a_1, ..., a_g\} = \{a_1, ..., a_g\}$. Hence, by 3.1,

$$\{a_1, ..., a_g\} \sim B \tag{3.9}$$

since $\max(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$ it is clear that $T_{g+1} = \max\{a_1, ..., a_g\} \cup \{a_{g+1}\}$ is such that $\{a_1, ..., a_g\} \subseteq T_{g+1}$ and $\max(T_{g+1} \setminus \{a_1, ..., a_g\}) \neq \emptyset$. Then by IMON and 3.9, it follows that

$$T_{g+1} \succ \{a_1, ..., a_g\}$$

and by 3.9

$$T_{g+1} \succ \max(B) \tag{3.10}$$

Taking 3.10, adding $a_{g+2}, ..., a_{g+t}$ on the left hand side, and using IMON repeatedly, I have

$$\{a_1, ..., a_{g+t}\} \succ B \tag{3.11}$$

Taking 3.11 and using similar argument for the proof of 3.7, by IMON, I have $A \succ B$, which proves 3.8. 3.1 and 3.8 complete the proof of the sufficiency part of the proposition. ▮

The binary relation $\succsim^*$ is transitive. In this relation indifferences between two set may arise because the sets considered have empty intersection with $\mathcal{P}(\wp)$ or because the intersection has the same number of elements. In both cases, particularly in the first one, a lexicographic version of the original rule may enrich the ranking and allows to determine the indifferences.

4. A lexicographic approach

I have pointed out in the introduction that the set of preferences that a reasonable person may have should be rich enough and take into account any valuable alternative in terms of freedom. Nevertheless, there may be situations where the aspirations of a reasonable person in X cannot be satisfied by the sets to be compared to. Let's consider the example of a country where free press is not available but there is a consensus that free newspapers are the unique relevant choice. Even in that case it can be claimed that different sets of newspapers may provide alternatives that deserves some value in terms of freedom.

Admitting this possibility, a way to adapt the previous approach to be used in this situation is to remove the first element in all the reasonable person's preferences and compare the available sets of newspapers according with this new set of reasonable preferences and proceed in this way if possible till the indifference is determined. This enriches the discrimination of the proposed approach making it according to the alternatives available in the sets to be considered. Clearly the sequentially considered alternatives as reference are not representing the absolute idea of freedom proposed in the first section of this paper but a compromised idea of freedom that can only be justified when the set of reasonable preferences cannot distinguish among the sets of alternatives.

Two different lexicographic versions are studied in this section. The first one eliminates the most preferred elements in a reasonable person's preferences until the preferences are able to discriminate between both sets. A second lexicographic ranking proposes a stronger criterion. This second criterion will declare A preferred over B only if the intersection of A with the set of relevant alternatives generated from the sequential elimination of the most preferred elements in the set of reasonable persons' preferences is always equal or bigger than B's.

Let R_j^1 be the preference where best elements have been removed and I denote the set of alternatives in X that are in R_j^1 as $\widetilde{R_j^1}$. in general

$$R_j^i \equiv \left\{ R_j^{i-1} - \{\Theta\} \mid \forall y, x \in \widetilde{R_j^{i-1}}, x \in \Theta \leftrightarrow x R_j^{i-1} y \quad \right\}$$

where $R_j^i - \{\Theta\}$ denotes the preference R_j^i where the alternatives in Θ have been removed. I denote by $\wp^i = (R_1^i, ..., R_n^i)$ the set of preference orderings over X where the best elements have been removed i times from the set of preferences that a reasonable person may have. Thus $\max^i(A)$ is the set of all alternatives x in A such that x is a best alternative in X for some ordering in $\wp^i$. Let $\mathcal{P}(\wp^i)$ be the set of alternatives in $\max^i(X)$.

For all $A, B \in Z$, $[A \succsim^i B]$ will be interpreted as "A offers at least as much freedom as B according with $\wp^i$." For all $A, B \in Z$, $[A \succ^i B$ iff $A \succsim^i B$ and $\neg(B \succsim^i A)]$ and $[A \sim^i B$ iff $A \succsim^i B$ and $\neg(B \succsim^i A)]$.

Now I consider a number of properties of the binary relation $\succsim$ over Z.

Definition 4.1. *A binary relation $\succsim$ over Z satisfies;*

1. *simple non-dominance by levels (SND^i) iff, for all $x, y \in X$, [if $\#\max^i(x) = \#\max^i(y)$, then $\{x\} \sim^i \{y\}$];*
2. *inclusion monotonicity by levels (IMON^i) iff, for all $A, B \in Z$ if $A \supseteq B$ and $\max^i(A \setminus B) \neq \emptyset$, then $A \succ^i B$. If $A \supseteq B$ and $\max^i(A \setminus B) = \emptyset$, then $A \sim^i B$;*

3. *property 1 (P-1) iff, for all* $A, B \in Z$ *if* $\#\max^i(A) = \#\max^i(B)$ *for all* i, *then* $A \sim B$;

4. *property 2 (P-2) iff, for all* $A, B \in Z$ *if* $\#\max^i(A) > \#\max^i(B)$ *and* $\#\max^j(A) = \#\max^j(B)$ *for all* $j < i$, *then* $A \succ B$;

5. *property 3 (P-3) iff, for all* $A, B \in Z$ *if there is a level* i *such that* $\#\max^i(A) > \#\max^i(B)$, *then* $\neg(B \succsim A)$.

The first two axioms are an adaptation of SND and IMON defined in Section 1. They apply to the different levels in which the set of relevant alternatives may be defined. The axioms P-1, P-2 and P-3 are three preference dependent axioms. Property 1 establishs that if two sets have the same number of relevant alternatives for any $\wp^i$ both the sets are indifferent. The axiom P-2 establishs that A is preferred to B in terms of freedom if the number of relevant alternatives in A is bigger that in B according to $\mathcal{P}(\wp^i)$ and no other $\mathcal{P}(\wp^j)$ $j < i$ gives different number of relevant alternatives for one of the sets. Axiom P-3 implies that if A has more relevant elements than B according with some $\mathcal{P}(\wp^i)$ then B is not going to be declared prefer to A.

The binary relation $\succsim^{\text{i}}$ that represents a lexicographic version of $\succsim^*$ is such that,

$$\text{for all } A, B \in Z, \left[A \succsim^{\text{i}} B \text{ iff } \begin{cases} \#\max^j(A) = \#\max^j(B) \text{ for all } j < i \\ \exists i \text{ such that } \#\max^i(A) > \#\max^i(B). \end{cases} \right] \quad (4.1)$$

Proposition 4.2. $\succsim$ *satisfies SND*^{i}, *COM, P-1, P-2 and IMON*^{i} *if and only if* $\succsim = \succsim^{\text{i}}$.

Proof.

The necessity part of the proposition is obvious; I prove only the sufficiency part. Let $\succsim$ satisfy SND^i, COM, P-1, P-2 and IMON^i. First, I show:

$$\text{for all } A, B \in Z, \text{ if } \#\max^i(A) = \#\max^i(B) \text{ for all } i, \text{ then } A \sim B. \quad (4.2)$$

Suppose $A, B \in Z$ and $\#\max^i(A) = \#\max^i(B) = g$. Let $\max^i(A) = \{a_1, ..., a_g\}$ and $\max^i(B) = \{b_1, ..., b_g\}$. By SND^i,

$$\{a_1\} \sim^i \{b_1\} \quad (4.3)$$

and

$$\{a_2\} \sim^i \{b_2\}. \tag{4.4}$$

$\{a_1\} \cap \{b_1\} = \{a_2\} \cap \{b_2\} = \emptyset$ and, further $\max^i(\{a_1\}) = \{a_1\}, \max^i(\{a_2\}) = \{a_2\}$ and $\max^i(\{b_1\}) = \{b_1\}, \max^i(\{b_2\}) = \{b_2\}$ (since $a_1, a_2 \in \max^i(A)$ and $b_1, b_2 \in \max^i(B)$). Hence by 4.3, 4.4 and COM, I have

$$\{a_1, a_2\} \sim^i \{b_1, b_2\}. \tag{4.5}$$

By SND^i, again,

$$\{a_3\} \sim^i \{b_3\}. \tag{4.6}$$

By 4.5, 4.6 and COM,

$$\{a_1, a_2, a_3\} \sim^i \{b_1, b_2, b_3\}. \tag{4.7}$$

Proceeding in this way I finally have $\{a_1, ..., a_g\} \sim^i \{b_1, ..., b_g\}$, that is $\max^i(A) \sim^i \max^i(B)$. If $A = \max^i(A)$, then $A \sim^i \max^i(B)$. Suppose $\{A \setminus \max^i(A)\} \neq \emptyset$. Let $\{A \setminus \max^i(A)\} = \{\bar{a}_1, ..., \bar{a}_m\} \neq \emptyset$. It is clear that $T_1 = \max^i(A) \cup \{\bar{a}_1\}$ is such that $T_1 \subseteq A$ and $\max^i(A \setminus T_1) = \emptyset$. Then by IMON^i and 4.7, I have $A \sim^i \max^i(B)$ in this case. Thus, in all cases,

$$A \sim^i \max^i(B) \tag{4.8}$$

Similarly using IMON^i in 4.8, $A \sim^i B$ for all i. By P-1 $A \sim B$,which proves 4.2.

Next, I show:

$$\text{for all } A, B \in Z, \text{ if } \begin{cases} \#\max^j(A) = \#\max^j(B) \text{ for all } j < i \\ \exists i \text{ such that } \#\max^i(A) > \#\max^i(B) \end{cases} \text{ then } A \succ B. \tag{4.9}$$

Suppose $A, B \in Z$ and $\#\max^i(A) > \#\max^i(B)$. Let $\#\max^i(B) = g$ and $\#\max^i(A) = g + t$ (where $t > 0$). Further, let $\max^i(B) = \{b_1, ..., b_g\}$ and $\max^i(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$. Note that $\max^i\{a_1, ..., a_g\} = \{a_1, ..., a_g\}$. Hence, by 4.2,

$$\{a_1, ..., a_g\} \sim^i B \tag{4.10}$$

since $\max^i(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$ it is clear that $T_{g+1} = \max^i\{a_1, ..., a_g\} \cup \{a_{g+1}\}$ is such that $\{a_1, ..., a_g\} \subseteq T_{g+1}$ and $\max^i(T_{g+1} \setminus \{a_1, ..., a_g\}) \neq \emptyset$. Then by IMON^i and 4.10, it follows that

$$T_{g+1} \succ^i \{a_1, ..., a_g\}$$

and by 4.10

$$T_{g+1} \succ^i \max^i (B) \tag{4.11}$$

Taking 4.11, adding $a_{g+2}, ..., a_{g+t}$ on the left hand side, and using IMON^i, I have

$$\{a_1, ..., a_{g+t}\} \succ^i B \tag{4.12}$$

Taking 4.12 and using similar argument for the proof of 4.8, by IMON^i, I have $A \succ^i B$. It is also known that A$\sim^j B$ by 4.2. Then By P-2 $A \succ B$ which proves 4.9. 4.2 and 4.9 complete the proof of the sufficiency part of the proposition. ▮

I have already commented on the differences between the rule just characterized and the relation $\succsim^*$. There are situations where there may be some interest in strengthening the requirements to declare a set preferred to another in terms of freedom. In the previous example about free press we were dealing with an issue of fundamental rights. Once any reasonable person agree that no available journal represents a reasonable choice we may need to impose a stronger criterion to discriminate between two sets despite the previous indifference. A way to increase the requirements to determine the indifference between two sets is to use full lexicographic domination.

The binary relation $\succsim^{\mathrm{I}}$ that represents a lexicographic version of $\succsim^*$ is such that,

$$\text{for all } A, B \in Z, \left[A \succsim^{\mathrm{I}} B \text{ iff } \# \max^i (A) \geq \# \max^i (B) \text{ for all } i\right] \tag{4.13}$$

Proposition 4.3. *$\succsim$ satisfies SND^i, COM, P-1, P-3 and IMON^i if and only if $\succsim = \succsim^{\mathrm{I}}$.*

Proof.

The necessity part of the proposition is obvious; I prove only the sufficiency part. Let $\succsim$ satisfy SND^i, COM, P-1, P-3 and IMON^i. First, I show:

$$\text{for all } A, B \in Z, \text{ if } \# \max^i (A) = \# \max^i (B) \text{ for all } i, \text{ then } A \sim B. \tag{4.14}$$

Suppose $A, B \in Z$ and $\# \max^i (A) = \# \max^i (B) = g$. Let $\max^i (A) = \{a_1, ..., a_g\}$ and $\max^i (B) = \{b_1, ..., b_g\}$. By SND^i,

$$\{a_1\} \sim^i \{b_1\} \tag{4.15}$$

and

$$\{a_2\} \sim^i \{b_2\}. \tag{4.16}$$

$\{a_1\} \cap \{b_1\} = \{a_2\} \cap \{b_2\} = \emptyset$ and, further $\max^i(\{a_1\}) = \{a_1\}, \max^i(\{a_2\}) = \{a_2\}$ and $\max^i(\{b_1\}) = \{b_1\}, \max^i(\{b_2\}) = \{b_2\}$ (since $a_1, a_2 \in \max^i(A)$ and $b_1, b_2 \in \max^i(B)$). Hence by 4.15, 4.16 and COM, I have

$$\{a_1, a_2\} \sim^i \{b_1, b_2\}. \tag{4.17}$$

By SND^i, again,

$$\{a_3\} \sim^i \{b_3\}. \tag{4.18}$$

By 4.17, 4.18 and COM,

$$\{a_1, a_2, a_3\} \sim^i \{b_1, b_2, b_3\}. \tag{4.19}$$

Proceeding in this way, finally I have $\{a_1, ..., a_g\} \sim^i \{b_1, ..., b_g\}$, that is $\max^i(A) \sim^i \max^i(B)$. If $A = \max^i(A)$, then $A \sim^i \max^i(B)$. Suppose $\{A \setminus \max^i(A)\} \neq \emptyset$. Let $\{A \setminus \max^i(A)\} = \{\bar{a}_1, ..., \bar{a}_m\} \neq \emptyset$. It is clear that $T_1 = \max^i(A) \cup \{\bar{a}_1\}$ is such that $T_1 \subseteq A$ and $\max^i(A \setminus T_1) = \emptyset$. Then by IMON^i and 4.19, I have $A \sim^i \max^i(B)$ in this case. Thus, in all cases,

$$A \sim^i \max^i(B) \tag{4.20}$$

Similarly, using IMON^i in 4.20, $A \sim^i B$ for all i. By P-1 $A \sim B$ which proves 4.14.

Next, I show:

$$\text{for all } A, B \in Z, \text{ if } \#\max^i(A) > \#\max^i(B) \text{ for all i, then } A \succ B. \tag{4.21}$$

Suppose $A, B \in Z$ and $\#\max^i(A) > \#\max^i(B)$. Let $\#\max^i(B) = g$ and $\#\max^i(A) = g + t$ (where $t > 0$). Further, let $\max^i(B) = \{b_1, ..., b_g\}$ and $\max^i(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$. Note that $\max^i\{a_1, ..., a_g\} = \{a_1, ..., a_g\}$. Hence, by 4.14,

$$\{a_1, ..., a_g\} \sim^i B \tag{4.22}$$

since $\max^i(A) = \{a_1, ..., a_g, ..., a_{g+t}\}$ it is clear that $T_{g+1} = \max^i\{a_1, ..., a_g\} \cup \{a_{g+1}\}$ is such that $\{a_1, ..., a_g\} \subseteq T_{g+1}$ and $\max^i(T_{g+1} \setminus \{a_1, ..., a_g\}) \neq \emptyset$. Then by IMON^i and 4.22, it follows that

$$T_{g+1} \succ^i \{a_1, ..., a_g\}$$

and by 4.22

$$T_{g+1} \succ^i \max^i(B) \tag{4.23}$$

Taking 4.23, adding $a_{g+2}, ..., a_{g+t}$ on the left hand side, and using IMON^i, I have

$$\{a_1, ..., a_{g+t}\} \succ^i B \tag{4.24}$$

Taking 4.24 and using similar argument for the proof of 4.20, by IMON^i, I have $A \succ^i B$,and this for all i. By P-3 $\neg(B \succsim A)$ which proves 4.21.

Next, I show:

$$\text{for all } A, B \in Z, \text{ if} \begin{cases} \#\max^i(A) > \#\max^i(B) \\ \#\max^j(A) < \#\max^j(B) \end{cases} \text{ then } A \text{ is non comparable with } B \tag{4.25}$$

Using the previous argument I have the conclusion that $A \succ^i B$ and by P-3 $\neg(B \succsim A)$ and $B \succ^j A$ and $\neg(A \succsim B)$. Then A and B are non comparable.

4.14, 4.21 and 4.25 complete the proof of the sufficiency part of the proposition. ∎

Both lexicographic rules turn out to be very similar in terms of the axioms that characterized them. They share in their characterizations the axioms SND^i, COM, IMON^i (that link them with $\succsim^*$) and P-1. The difference between $\succsim^{\mathrm{i}}$ and $\succsim^{\mathrm{I}}$ relies in axioms P-2 and P-3. While axiom P-2 gives decisive power over the ranking of A and B to the first $\mathcal{P}(\wp^i)$ that discriminates between both sets. P-3 only to guarantees that once a decisive set of preferences $\mathcal{P}(\wp^i)$ is found the ranking of the sets in terms of freedom will not contradict this result. As we have seen this is the difference that determine which lexicographic rule is characterized.

An example may clarify the differences between $\succsim^{\mathrm{i}}$ and $\succsim^{\mathrm{I}}$. Let $A = \{x, z, u\}$ and $B = \{w, u\}$ be two sets of alternatives. Let's consider that the preferences that a reasonable person may have in a society are in the following list:

u	z	u	z	w
x	u	y	w	z
v	v	z	x	u
w	x	x	v	x
z	y	w	y	v
y	w	v	u	y

In each ranking shown above, the elements are arranged in strictly descending order of preference. We can easily see that $A \sim^0 B$, $A \succ^1 B$, $A \succ^2 B$, $A \sim^3 B$,

$A \sim^4 B$, $A \prec^5 B$. It means that according to the rules defined, $A \succ^{\text{i}} B$. However, A and B are declared non comparable by $\succsim^{\text{I}}$. Finally, it is easy to check that both $\succsim^*$ and $\succsim^{\text{I}}$ are independent and implies $\succsim^{\text{i}}$.

5. Concluding remarks

The problem studied in this paper has been previously considered by several authors. One of the main questions that has come up in this context is the role of preferences over the various alternatives in assessing an agent's freedom. Pattanaik and Xu (1990) use a set of axioms that do not refer to preferences at all. Their axioms imply a ranking of opportunity sets based exclusively on their cardinality. On the other hand, Sen (1991, 1993), Foster (1992) and Puppe (1996) have argued in favor of introducing preferences as the basis for the evaluation of opportunity sets in terms of freedom.

This paper extend the initial approach of Pattanaik and Xu (1997) based on the notion of preference orderings that a reasonable person may have. It uses these orderings and introduces the idea that the sets of alternatives offer freedom only if they are able to provide meaningful choices to the persons in the society.

The alternatives no considered in the feasible sets are relevant for assessing the freedom that the feasible sets offer. This aspect of freedom is captured by the formulation proposed in this paper. The specification of the set of relevant alternatives formulate in the paper takes into account the fact that expectations have a role to play in considering freedom. For example, let's consider the alternative to go for a business trip to America from Europe sailing in a XV century ship. This is an alternative even in the XX century, however it is remotely a relevant alternative even if I cannot afford a plane ticket. If suddenly the world ran out of oil and there are no longer intercontinental airplanes or modern ships then the alternative to sail in a caravel becomes relevant. Any change in the set of universal alternatives can change the perception over the freedom that a set of alternatives can give.

The perception that a particular agent has of her effective freedom depends not only on the feasible alternatives but also on the set of unfeasible alternatives that contribute to shape her aspirations and focus her achievements in the construction of vital projects capable to provide a meaningful life.

References

[1] Foster, J. (1992) "Notes on Effective Freedom", *mimeograph.*

[2] Jones, P. and R. Sugden (1982) "Evaluating Choice," International Review of Law and Economics **2,** 47-65.

[3] Nozick, R. (1974) *Anarchy, State and Utopia.* Oxford: Blackwell.

[4] Pattanaik, P.K. and Y. Xu (1990) "On Ranking Opportunity Sets in terms of Freedom of Choice," *Recherches Economiques de Louvain* **56**, 383-390.

[5] Pattanaik, P.K. and Y. Xu (1997) "On Preference and Freedom" forthcoming in *Theory and Decision.*

[6] Puppe, C. (1996) "An axiomatic Approach to 'Preference for Freedom of Choice'," *Journal of Economic Theory* **68**, 174-199.

[7] Sen, A.K. (1991) "Welfare, Preference and Freedom," *Journal of Econometrics* **50**, 15-29.

[8] Sen, A.K. (1993) "Markets and Freedoms," *Oxford Economic Papers* **45**, 519-541.