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HOW TO GET THE BLANCHARD-KAHN FORM FROM A GENERAL LINEAR RATIONAL EXPECTATIONS MODEL

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Abstract

In this paper, we prove that every linear model with rational expectations can be transformed by the means of an one-to-one mapping into another model which has one of the following properties:

i) it is degenerated,

ii) it is backward,

ii) it has the Blanchard-Kahn form.

In addition to some simple illustrations, we provide two applications on two nonlinear forward-looking economic models in order to show how to use our theoretical analysis for local stability assessment.

Key Words

Rational Expectations; Blanchard-Kahn Form; Reduction Algorithms.

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1. Introduction

In their seminal paper (1980), Blanchard and Kahn give the solution of linear difference models under rational expectations together with the conditions for existence and uniqueness. These conditions are usually called Blanchard- Kahn conditions. But they are obtained under the crucial assumption that the linear difference model with rational expectations may be arranged in the Blanchard-Kahn form which is the following one:

$$(1) \qquad \begin{bmatrix} E_t P_{t+1} \\ X_t \end{bmatrix} = A \begin{bmatrix} P_t \\ X_{t-1} \end{bmatrix} + Z_t$$

Where X is an n-vector of variables predetermined at date t, P is an m-vector of variables non predetermined at t and Z is a (n+m)-vector of exogenous variables, E_t being the mathematical expectation operator conditionally to the information set available at t. A is a (m+n)-square matrix. Form (1) is very important as it allows to check the so-called Blanchard-Kahn saddlepoint conditions: the model has a unique solution if and only if the number of eigenvalues of A outside the unit circle is equal to m, the number of non-predetermined variables.

Unfortunately, form (1) is hardly ever directly obtainable on the structural linear or linearized rational expectations models, which disables the direct verification of the saddlepoint conditions mentioned above. Therefore, an important theoretical question turns out to be the following: how to obtain the Blanchard-Kahn form from any linear(ized) rational expectations models and is it always possible? [1] To answer this question, we start our analysis on a reduced form that is always directly obtainable on every difference model with rational expectations (see Broze, Gourieroux and Szafarz (1989) or Laffargue (1990)):

$$(2) \qquad C_1\, y^1_{t-1} + C_0\, y_t + C_2\, E_t y^2_{t+1} = z_t$$

[1] Numerical procedures can be also used to check saddlepoint conditions without an explicit Blanchard-Kahn form, see Boucekkine and Le Van (1995). However, to find out some theoretical foundations for their procedure, these authors assume that the models can be transformed into the Blanchard-Kahn form. One contribution of our paper is to show that this assumption actually ensures that the considered forward-looking models are well specified.

where y^1 is an n_1 -vector of lagged variables, y^2 is an n_2 - vector of non-predetermined variables and $y = (y^1, y^2, y^3)$, y^3 being an n_3 -vector of "static" variables. We set $n = n_1 + n_2 + n_3$. C_1 is a nxn_1 matrix, C_2 is a nxn_2 matrix, and C_0 an n-square matrix. Note that if C_0 is singular the model does not make sense: we need to ensure existence and uniqueness of contemporaneous variables' solutions given the past and the future. Hereafter, we normalize C_0 to the identity matrix. Now, the question is to establish if model (2) admits a Blanchard- Kahn form in order to check the Blanchard-Kahn conditions of existence and uniqueness of solutions. In this paper, we prove that model (2) may always be transformed by means of an one-to-one mapping into another form which has one of the three following properties :

i) it is degenerated,

ii) it is backward,

iii) it has a Blanchard-Kahn form.

The paper is organized as follows : in section 2 we state and prove the main result in addition to an illustration on three examples of Blanchard-Kahn's paper. Section 3 applies the presented algorithm on a Real Business Cycle model and on the german country model of the IMF multicountry model, MULTIMOD. Section 4 concludes.

2. The main result

Let us consider again model (2). Partition C_1 and C_2 as follows:

$$C_1 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} \qquad \text{and, } C_2 = \begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \end{bmatrix}.$$

Model (2) splits in three systems:

$$C_{11}\ y^1_{t-1} + y^1_t + C_{21}\ E_t y^2_{t+1} = z^1_t$$

$$C_{12}\ y^1_{t-1} + y^2_t + C_{22}\ E_t y^2_{t+1} = z^2_t$$

$$C_{13}\ y^1_{t-1} + y^3_t + C_{23}\ E_t y^2_{t+1} = z^3_t.$$

Obviously, one can eliminate the third system involving y^3_t and just consider the first two systems. For simplicity, we assume that $z_t = 0$, or in other words, that the model is stationary and the variables are in fact the differences between their transitory and stationary values.

From now on, we consider model (3):

$$(3) \qquad \begin{cases} y_t^2 = P_1\ E_t y_{t+1}^2 \ + Q_1\ y_{t-1}^1 \\ y_t^1 = P_2\ E_t y_{t+1}^2 \ + Q_2\ y_{t-1}^1 \end{cases}$$

Matrices P_i and Q_i, $i = 1, 2$, are trivially computed from the submatrices of C_i defined in the partition above. In particular, P_1, being equal to $-C_{22}$, is a square n_2-matrix and P_2 a $(n_1 x n_2)$ matrix. Before stating our main result, we mention the following lemma which will be used later.

Lemma

Let A be a square n-matrix with rank m, $m < n$. There exists an invertible square matrix M, a square m-matrix P, an $mx(n-m)$ matrix Q such that:

$$A = M^{-1} \begin{bmatrix} P & Q \\ 0 & 0 \end{bmatrix} M$$

with rank $(P, Q) = m$.

Proof: see e.g. Horn and Johnson(1985).

We are now able to state and prove the main result of the paper. We develop a reduction algorithm allowing to get the Blanchard-Kahn form from form (3).

2.1. The reduction algorithm

The following proposition states the main result of the paper, the reduction algorithm being exposed in the proof of the proposition:

Proposition

There exists an one-to-one linear mapping $T : \eta = T \begin{bmatrix} y^2 \\ y^1 \end{bmatrix}$

which transforms model (3) in another one which has one of the three following properties :

i) it is degenerated,

ii) it is backward,

iii) it has a Blanchard-Kahn form.

Proof: Obviously, if P_1 is invertible, the Blanchard-Kahn form is immediate:

$$\begin{bmatrix} E_t y_{t+1}^2 \\ y_t^1 \end{bmatrix} = \begin{bmatrix} P_1^{-1} & -P_1^{-1}\, Q_1 \\ P_2\, P_1^{-1} & -P_2\, P_1^{-1}\, Q_1 + Q_2 \end{bmatrix} \begin{bmatrix} y_t^2 \\ y_{t-1}^1 \end{bmatrix}$$

We assume that rank $P_1 = m < n_2$, which is indeed the most frequent case in practice. From our lemma, there exists matrices M, R_1, R_2 such that

$$P_1 = M^{-1} \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} M$$

where R_1 is a square m-matrix. System (3) becomes

$$\begin{cases} M\; y_t^2 = \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} M\; E_t y_{t+1}^2 + M\; Q_1\; y_{t-1}^1 \\ y_t^1 = (P_2\; M^{-1})\; M\; E_t y_{t+1}^2 \; + Q_2\; y_{t-1}^1 \end{cases}$$

Set $w_t = (w_t^2, w_t^1) = M y_t^2$, with w^2, w^1 respectively m and $(n_2 - m)$ vectors. We have:

$$\begin{cases} w_t^2 = R_1\; E_t w_{t+1}^2 \; + R_2\; E_t w_{t+1}^1 \; + Q_1^1\; y_{t-1}^1 \\ w_t^1 = Q_1^2\; y_{t-1}^1 \\ y_t^1 = S_1\; E_t w_{t+1}^2 \; + S_2\; E_t w_{t+1}^1 \; + Q_2\; y_{t-1}^1 \end{cases}$$

where the matrices Q_1^1, Q_1^2, S_1, S_2 are trivially computed from Q_1, P_2 and M.

Replacing $E_t w_{t+1}^1$ by $Q_1^2\; y_t^1$, we get:

$$\text{(4)} \qquad \begin{cases} w_t^2 - R_2\; Q_1^2\; y_t^1 = R_1\; E_t w_{t+1}^2 \; + Q_1^1\; y_{t-1}^1 \\ (I - S_2\; Q_1^2)\; y_t^1 = S_1\; E_t w_{t+1}^2 \; + Q_2\; y_{t-1}^1 \\ w_t^1 = Q_1^2\; y_{t-1}^1 \end{cases}$$

We have, at this step, two cases :

i) $I - S_2\; Q_1^2$ is not invertible : the system is degenerated.

ii) $I - S_2\, Q_1^2$ is invertible. We obtain:

$$(5)\qquad \begin{cases} w_t^2 = R_1'\; E_t w_{t+1}^2 \;+ R_2'\; y_{t-1}^1 \\ y_t^1 = R_3'\; E_t w_{t+1}^2 \;+ R_4'\; y_{t-1}^1 \\ \qquad w_t^1 = Q_1^2\; y_{t-1}^1 \end{cases}$$

The matrices R_k', $k = 1$ to 4, are trivially obtained from the matrices of system (4). In particular, R_1' is a square m-matrix. It is to be noted here that the intermediate variables w_t^1 are residual in the sense that they can be computed residually once the dynamic system corresponding to the first two equations of system (5) is solved. So the dynamic properties of system (5) are entirely determined by its first two equations and we can omit variables w_t^1. In this sense, our algorithm is a reduction one as it allows to locate and eliminate the superfluous dynamics or the so-called **redundancies**.

Now, define

$$(6)\qquad \eta_t^1 = y_t^1 \qquad , \eta_t^2 = w_t^2$$

The first two equations of system (5) yield the compact form (3):

$$\begin{cases} \eta_t^2 = P_1'\; E_t \eta_{t+1}^2 \;+ Q_1'\; \eta_{t-1}^1 \\ \eta_t^1 = P_2'\; E_t \eta_{t+1}^2 \;+ Q_2'\; \eta_{t-1}^1 \end{cases}$$

with $P_1' = R_1'$, the dimension of P_1' being smaller than the dimension of P_1, the corresponding matrix in the initial form (3). If P_1' is invertible, we obtain a Blanchard-Kahn form. If not, we conduct another reduction step exactly as before. This algorithm must stop at some step i, because:

i) either the model is degenerated,

ii) or $P_1^{(i)} = 0$, and in that case we have:

$$\begin{cases} \qquad \eta_t^2 = Q_1^{(i)}\; \eta_{t-1}^1 \\ \eta_t^1 = P_2^{(i)}\; Q_1^{(i)}\; \eta_t^1 \;+ Q_2^{(i)}\; \eta_{t-1}^1 \end{cases}$$

If $I - P_2^{(i)} Q_1^{(i)}$ is invertible, the model is backward. If not, it is degenerated.

iii) $P_1^{(i)}$ is invertible, and we get a Blanchard-Kahn form.

The proof is now complete

Q.E.D.

2.2. Some theoretical illustrations

Example 1: Example D of Blanchard-Kahn's paper

$Y_t = C_t + I_t$

$C_t = \alpha (Y_t + E_t Y_{t+1}),\ \alpha > 0$

$I_t = \beta (E_t Y_{t+1} - E_{t-1} Y_t),\ \beta > 0.$

Solving the equilibrium condition, one gets:

$(1-\alpha)\, Y_t = (\alpha + \beta)\, E_t Y_{t+1} - \beta\, E_{t-1} Y_t.$

Define $X_t = E_t Y_{t+1}$.

If $\alpha = 1$, the model is degenerated.

If $\alpha \neq 1$, then we get the system

$Y_t = \frac{(\alpha+\beta)}{1-\alpha}\ E_t Y_{t+1}$ - $\frac{\beta}{1-\alpha}\ E_{t-1} Y_t$

and $X_t = E_t Y_{t+1}$.

Since $\frac{(\alpha+\beta)}{1-\alpha} \neq 0$, we have a Blanchard-Kahn form.

Example 2: Example B of Blanchard-Kahn's paper

The considered model is: $y_t + \alpha\, y_{t-2} + \beta\, E_t y_{t+2} = 0$, with $\beta \neq 0$. First, we find out the reduced form *à la* Broze, Gourieroux and Szafarz (1989).

Define $z_t = (y_t, E_t y_{t+1})$ and $z_t^2 = (z_t, z_{t-1})$. We obtain:

$$A\, z_t^2 + B\, z_{t-1}^2 + C\, E_t z_{t+1}^2 = 0$$

where $A = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix}$, I being the identity matrix of dimension 2, $B = \begin{bmatrix} 0 & \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix} \\ -I & 0 \end{bmatrix}$ and $C = \begin{bmatrix} \begin{pmatrix} 0 & \beta \\ -1 & 0 \end{pmatrix} & 0 \\ 0 & I \end{bmatrix}$.

Since $\beta \neq 0$, C is invertible. Therefore, there exists a Blanchard-Kahn form.

Example 3: Example C in Blanchard-Kahn's paper

Consider the model: $y_t - a\ E_{t-1} y_t$. Define $z_t = (y_t, E_t y_{t+1})$.

We have $z_t + B\ z_{t-1} + C\ E_t z_{t+1} = 0$ with:

$B = \begin{bmatrix} 0 & -a \\ 0 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$.

One can check that $C = M^{-1} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} M$ with $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Define $\eta_t = M\ z_t = (\eta_t^1, \eta_t^2)$. One obtains:

$$\begin{cases} \eta_t^2 = -a\ \eta_{t-1}^1 \\ \eta_t^1 = -E_t \eta_{t+1}^2 \end{cases}$$

If $a = 1$, the model is degenerated. If not, it is backward.

3. Numerical applications

In this section, we provide two applications on two nonlinear forward-looking economic models in order to show how to use our theoretical analysis for local stability assessment. First, we show how our reduction algorithm can be advantageously used in the context of the traditional Real Business Cycles (RBC) analysis methodology. Then, an application is provided on a medium-scale model, the german model of the IMF country model MULTIMOD (46 equations per period) in order to test our algorithm when the number of equations is relatively high. Also, the latter application shows how useful can be the algorithm in the analysis of the models used for economic policy design, in which (local) saddlepoint stability is a minimal requirement.

3.1. Application on an RBC model

As an RBC example, we use a simple model with indivisible labour and a "depreciation in use" assumption (as in Greenwood-Hercowitz-Huffman (1988)). We consider a perfectly competitive economy with indivisible labour in which individuals have identical preferences and are covered by a full unemployment insurance. At any date t, each individual maximizes the utility function:

$$E_t \sum_{s=0}^{\infty} \beta^s \, [\log(c_{t+s}) \; - \; B\,(1 - n_{t+s})]$$

where c_t and n_t represent consumption and labour at date t (the total time endowment has been normalized to 1.). β $(0 < \beta < 1)$ is the time preference parameter. The productive capital stock at date t (k_{t-1}) is predetermined but can be used with a variable intensity $u_t > 0$. The production function of the representative firm at each date $t = 0, 1, 2, ...$ is Cobb-Douglas:

$$y_t \;=\; A_t\,(k_{t-1}\,u_t)^{1-\alpha}\,n_t^{\alpha} \tag{1}$$

where y_t is the output level at date t and n_t is the labour input. A_t is the exogenous total productivity variable.

To compute the competitive allocation in the above described economy, it is sufficient to analyse the central planner's decision problem. At each date t, he chooses c_t, n_t, u_t and k_t in order to maximize the utility function subject to the macroeconomic ressource constraint:

$$c_t \;+\; k_t \;-\; (1 - \bar{\delta}\,u_t^{\phi})\,k_{t-1} \;= y_t \tag{2}$$

and given the technology (1). The parameter δ $(0 < \delta < 1)$ is a depreciation constant. The parameter ϕ $(\phi > 0)$ reflects the sensitivity of the depreciation rate to the capital utilization rate u_t. The first-order conditions of this maximization program are:

$$B = c_t^{-1}\,\alpha\,\frac{y_t}{n_t} \tag{3}$$

$$c_t^{-1} \;=\; E_t\left[\beta\,c_{t+1}^{-1}\,\left((1-\alpha)\,\frac{y_{t+1}}{k_t} \;+\; 1 \;-\; \bar{\delta}\,u_{t+1}^{\phi}\right)\right] \tag{4}$$

$$\bar{\delta}\,\phi\,u_t^{\phi-1} = (1-\alpha)\,\frac{y_t}{k_{t-1}} \tag{5}$$

From now on, we will refer to model (M) as the system of equations (1) to (5). Obviously the model is nonlinear, however Blanchard-Kahn conditions (and so the Blanchard-Kahn form) are still required to check locally for saddlepoint conditions (see e.g. Woodford (1986)). As RBC authors study the small fluctuations around (deterministic) steady states, saddlepoint conditions are checked on the linearized versions of the models around their steady states. A quick look at model (M) is sufficient to conclude that this structural models includes many redundancies: for example equation (5) can be used to eliminate variable u_t. In fact, model (M) can be reduced easily to a system of two equations with one forward variable and one predetermined variable. RBC authors do eliminate "manually" these redundancies to get the Blanchard-Kahn form, which is used ultimately to derive the solutions paths of the (linearized) models (see e.g King, Plosser et Rebelo (1988)). One can guess therefore how useful may reveal our reduction algorithm in the context of this methodology.

To demonstrate that, we first apply our algorithm on a linearized structural model (M). The model is calibrated in order to obtain a steady-state equilibrium consistent with a list of stylized facts or available estimations for the US economy: $\alpha = 0.64$, $\beta = 0.992$, $\bar{\delta} = 0.02$, $\phi = 1.44$ and $B = 2.5$. Linearizing the model (M) around the computed steady state gives directly the reduced form of Broze-Gourieroux-Szafarz. Applying our reduction algorithm on this form [2], we get the following results:

i) Among the initial three forward-looking variables (c, y and u), two are redundant.[3]

ii) One reduction step is needed to eliminate the redundant forward variables, the obtained Blanchard-Kahn transition matrix (i.e matrix A is form (1)) being:

$$\begin{bmatrix} 1.0081589 & -0.00310276 \\ -0.79435342 & 1.0023509 \end{bmatrix}$$

which gives the following eigenvalues: 1.0549854, 0.95552436. The saddlepoint conditions are locally checked.

To check the goodness of our results, we have conducted another experiment. By a series of elementary but tedious substitutions, we have eliminated the rédundancies of

[2] The reduction algorithm has been written in Gauss and is available upon request. To test the singularity of a matrix, we use the rank test included in Gauss.

[3] The term redundancies has to be taken in a wide sense. By the statement:"two variables are redundant", we mean that, given the structure of the linearized model, the three forward variables may be substituted by a single linear combination of these variables.

model (M) and obtained a nonlinear model (M') without redundancies. Of course, as we have two redundant forward variables, form (M') is not unique. We check that the results regarding the eigenvalues values[4] are independent of the choice of the variables to be eliminated. As an example, if we eliminate variables c and u (in addition to the static variable n), and apply the reduction algorithm on the corresponding (linearized) model (M'), we get the following results:

i) Of course, no reduction is needed and we obtain directly the transition matrix:

$$\begin{bmatrix} 1.0199607 & 0.00256780 \\ 0.87890622 & 0.99054937 \end{bmatrix}$$

ii) The corresponding eigenvalues are: 1.0549855, 0.95552459.

The comparison between the eigenvalues values obtained on the redundant model (M) using our reduction algorithm and those obtained directly on the non-redundant model (M') is highly meaningful regarding to the accuracy of our algorithm. We also check the tractability of the algorithm when the models under consideration include a relatively high number of variables, as it is reported in the following section.

3.2. Application on a medium scale model

For the purpose mentioned just above, we have also applied our reduction algorithm on a stationary version of the german country model of MULTIMOD (referred to as Multigr hereafter), the IMF multicountry model. A detailed exposition of the specification of this model can be found in Masson *et alii* (1990). The stationarization is taken from Loufir and Malgrange (1995) who also computed the long run of this whole multi-country model. Multigr includes 46 equations and shares the same characteristics as the other industrial country models of MULTIMOD, namely a Mundell-Fleming structure. Consumption specification is an adaptation of the Blanchard model (1985). The demand for capital relies on Tobin's q theory. Wealth is the sum of human wealth (ie. the present value of all future labor income), non-human wealth (ie. the present value of future profits) and the real value of money balances, of government bonds and of net foreign

[4] But obviously not regarding the Blanchard-Kahn transition matrix.

assets. Foreign trade in manufactured goods is formalized with conventional export and import functions. The real effective exchange rate is defined through "the ratio of the home country's export price to a foreign price index". The LM curve is described by a conventional demand for money balances and a money supply consisting in a reaction function of the short run interest rate to a nominal money target. The aggregate supply side is given by a reduced-form inflation equation summarizing demand and supply in the labour market, the augmented Phillips curve and a mark-up on unit labor costs depending on the the degree of capacity utilization. The treatment of the government sector is quite conventional, with endogenous taxes to allow for progressive adjustment to nominal debt targets.

Multigr is a good example of macroeconomic models used for economic policy design. By "shocking" some precise exogenous variables (like public expenditures or money supply), the users of such models try to evaluate the effects of different economic policies on the aggregate variables of the economy (like GDP, unemployment or prices). As the magnitudes of the shocks involved in such exercises is in general small, local stability assessment is also useful. In the case of forward-looking models as Multigr, this assessment is necessary as the users have to ensure that their economic policy prescriptions are derived from unique solution paths of the models under consideration. So saddle-point conditions have to be checked locally. We expose here some of the problems that can face the practitioners dealing with this issue.

i) First, on models like Multigr, even the Broze-Gourieroux-Szafarz reduced form is not directly available: many variables exhibit leads and lags greater than one period. For example, the price variable (p) in Multigr appears with a lead equal to 5 periods in the real long term interest rate equation. Of course, this problem is very easy to solve by just adding some artificial variables[5], but these elementary operations increase markedly the dimension of the models. In the case of Multigr, these operations increase the dimension of the model from 46 to 62, the number of forward-looking variables rising from 6 to 13.

[5] For example, the price variable p quoted in the text gives rise to 4 additional forward variables with a one-period lead if one has to write down the Broze-Gourieroux-Szafarz form.

ii) Once the Broze-Gourieroux-Szafarz form obtained, we can apply our reduction algorithm on the linearized model around its long run [6]. However, unlike the RBC model application above, we have faced an additional numerical problem. Depending on the parameters of the rank test to be conducted along the reduction algorithm in order to check for the invertibility of some matrices (see our footnote 2), we get quite different results. This test is based on the singular value decomposition. In our initial program, a singular value is considered zero if its modulus is less than or equal to $\tau = 10^{-10}$. If we use a more strict bound [7], exactly $\tau = 10^{-7}$, a value that can be considered even more acceptable from an economic point of view, the results are quite different:

v) For $\tau = 10^{-10}$ or $\tau = 10^{-8}$, three reduction steps are needed and three forward-variables are found redundant. Blanchard-Kahn saddlepoint conditions are checked, the unstable eigenvalues being (in modulus)[8]: 269098.55, 8.1558005, 4.7515067, 4.7515067, 3.1211009, 3.1211009, 1.8892543, 1.5731517, 1.1536862, and 1.0215632.

vv) For $\tau = 10^{-7}$, four reduction steps are needed and four forward variables are found redundant. The Blanchard-Kahn conditions are also checked with the following corresponding unstable eigenvalues (in modulus): 8.1558858, 4.7515440, 4.7515440, 3.1211043, 3.1211043, 1.8892402, 1.5731584, 1.1536863, and 1.0215631.

Of course, the saddlepoint conditions are checked in both cases. Therefore, the local stability diagnostic is the same. Moreover, the unique relevant difference between the two computed spectra consists in the huge eigenvalue 269098.55 that is obtained in case v) but not in case vv). Probably, an economist will find case vv) more acceptable and consider the other not more than a numerical peculiarity. In any case, as explained in this section, our algorithm is designed to address all the issues related to spectral computations using simple experimental parameters (essentially the parameter τ) allowing for a clear interpretation.

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[6] We use the long run values derived in Loufir and Malgrange (1995).

[7] The default bound being 10^{-13} in Gauss, version 3.2.13.

[8] The complete results are available upon request. Also note given the results just below that the linearized model displays complex and conjugate eigenvalues.

4. Conclusion

In this paper, we have presented a reduction algorithm that allows:

i) to conclude if a given linear or linearized rational expectations model admits the Blanchard-Kahn form,

ii) to compute explicitly this form by the means of a theoretically founded reduction algorithm.

Using two example models, we have also indicated how to take advantage of our theoretical analysis in the context of the RBC methodology and in relation with the traditional empirical frameworks designed to economic policy evaluations. Of course, given the ultimate goal of our analysis, namely saddlepoint stability assessment, and as rational expectations models especially require the latter assessment, we do think that the presented algorithm can be highly useful for economic practitioners. As explained in the numerical section of this paper, our algorithm does not require any particular computational expertise, and can be used with confidence to investigate the local stability of medium-large scale models, the numerical control and the interpretability of the outcomes of the algorithm being straightforward.

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