Working Paper 95-07
Economics Series 03
March 1995

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-9875

"INTERPERSONAL WELFARE COMPARISONS, REDISTRIBUTIVE EFFECTS, AND HORIZONTAL INEQUITIES IN THE INCOME TAX SYSTEM"

Javier Ruiz-Castillo[1]

Abstract

Vertical and horizontal equity principles occupy the core of income tax systems evaluation. Vertical inequality is measured in terms of relative or absolute income inequality indices, as in the analysis of the redistributive effects (RE) of progressive tax systems. Horizontal equity measurement has been more controversial. Classical horizontal inequities (HI), undestood as unequal treatment of equals or close similars, should be distinguished from reranking (RKG) caused by equity and non equity tax breaks, in actual tax schemes. We propose to integrate the measurement of RE, HI, and RKG in a social welfare framework where tax units non-income differences in needs are recognized. Additively decomposable measurement instruments by population subgroup are found essential to clarify the issues involved.

Key words: income tax systems; vertical inequality; horizontal inequality; social welfare; additive decomposability by population subgroup

[1] Departamento de Economía, Universidad Carlos III de Madrid.
This paper is the result of many conversations with Carmen Vargas about her dissertation work. It was also sparked by Lambert and Ramos (1994), presented at the Conference on "Income Distribution and Economic Development", held at the Universidad Carlos III of Madrid in December 1994. The paper is part of the research programme of a network financed by the European Communities (Contract #ERBCHRXCT940647). Financial help from the Fundación Caja de Madrid is also gratefully acknowledged. Only the author is responsible for all shortcomings.

INTRODUCTION

Income tax systems have been often evaluated in the light of the classical principles of horizontal and vertical equity which demand, respectively, equal treatment of equals and different treatment of unequals. The satisfaction of vertical equity, has been generally identified with an improvement in either relative or absolute inequality in an appropiate income space. However, the application of the principle of horizontal equity has been more controversial. The first difficulty is that, in practice, few people are found in identical circumstances.

Two solutions have been proposed. Some authors have been content with widening the notion of "exact equals" to "close similars" in empirical work. Admitedly, to implement any definition of close similars is a thorny issue. But it can be done(1). Therefore, we reserve the term horizontal inequities (HI for short) to denote situations in which identical (or similar) individuals pay different taxes.

Other authors have prefered to substitute for the original idea by identifying the maintenance of horizontal equity with the preservation of the ranking in the before tax distribution. The extent of horizontal inequity is then measured as a function of the number of rank reversals(2). Independently of whether the reranking caused by a tax scheme (RKG for short) should be considered a horizontal or a vertical phenomenon, we agree with Plotnik (1982) that the measurement of RKG has an interest of its own. In any case, it is clear that HI are neither necessary nor sufficient for RKG. Hence, both phenomena should be conceptually distinguished and subject to independent measurement.

The source of the second difficulty, lies in the notion of equals in a heterogenous world, the need for interpersonal welfare comparisons, and the ways in which tax systems are actually designed. To see this, assume for a moment that, from a social point of view, all individuals have the same needs. It is well known(3) that relative (absolute) income inequality is unambigously reduced if, and only if, average tax liability (tax liability) is increasing with income. We refer to this as the redistributive effect (RE for short) of a progressive tax system in the relative or the absolute case.

In this homogeneous world, individuals with the same income will be taxed equally, so that no HI are possible. If, in addition, marginal rates are less than one, there can be no RKG either. But actual tax systems allow for a variety of tax exemptions on account of income source, housing tenure, financial or cultural investments, etc. However legitimate their respective aims from other points of view, these exemptions will have a twofold impact: (i) they will affect the RE of a progressive income tax in a manner that cannot be signed *a priori*; (ii) and they may very well create HI, RKG or both.

Let us now recognize that households and families are typically heterogeneous: they differ not only in income, but in non-income characteristics which give rise to differences in needs. In an income tax context, we will take as our primitive notion the tax unit, consisting of the taxpayer together with the non-earning dependents (children, handicapped, or the aged) claimed to be under his/her responsability. Tax units of the same size are assumed to have identical needs and, therefore, welfare comparisons through their incomes can be directly performed. Since this is not the case for differently sized units, some notion of adjusted or equivalent income may be introduced to perform welfare comparisons between them.

Intuitively, tax deductions for dependents are not meant to create HI. On the contrary, they arise from the realization that to establish who are the equals we must go beyond income to include a demographic dimension. Implicit in any tax code, there is what we will call a fiscal equivalence scale, usually independent on income: all tax units with identical ethically relevant characteristics are granted the same tax break.

However, rather than applying progressive tax rates to income adjusted for dependents according to such fiscal scale, actual systems apply progressive tax rates to unadjusted incomes and then allow a dependents exemption. This practice, as well as discrepancies between the fiscal scale and other alternative scales an analyst may care to use, create HI which must be taken into account when assessing the tax system's impact on social welfare.

Such HI is an avoidable feature of actual tax systems. We will show how it disappears as soon as we do away with tax allowances for dependents but apply the progressive tariff on adjusted income. The only question left, will be the empirical one of searching for a range of new fiscal scales which permit to achieve at least the same social welfare as the standard tax system.

We propose to integrate the measurement of HI, RKG and RE in a social welfare framework where all the normative aspects are made explicit. We will remain within the tradition where social or aggregate welfare is summarized by two statistics of the adjusted income distribution: the mean, and an index of vertical inequality. After tax social welfare will be compared to the before tax situation where all income is in tax units hands. We will address the following three issues:

(1) As far as the measurement of the RE effect, we simply keep separate the impact on social welfare of: i) the progressive tariff; ii) dependent allowances justified on equity reasons; and iii) other fiscal exemptions justified on other grounds. Lacking evidence in favor of a particular standard of comparison, we follow Coulter et al (1992a, 1992b) and parametrize the value judgements implicit in the definition of adjusted income(4). Combining such parametrization with additively

separable social evaluation functions, we will be able to keep track of within- and between-groups RE effects in the ethically relevant partition by tax unit size.

(2) We do not introduce explicit value judgements on the deleterious effects of classical HI. Instead, we follow Aronson, Johnson and Lambert (1994) and, above all, Lambert and Ramos (1994), applying decomposable vertical measures to the partition of exact equals (or close similars in empirical applications) in adjusted income space[5]. We distinguish between HI created by equity deductions for dependents, and HI caused by non equity based exemptions.

(3) Contrary to King (1983) or Plotnik (1982, 1985), we do not introduce explicit value judgements on the bad aspects of RKG. Consider, for instance, the impact of non equity deductions. If they cause RKG, it must be because some poor tax units are allowed greater tax breaks than richer ones. This increases the overall progressivity of the tax system. To assess the RKG importance, we suggest a novel method to estimate the "progressivity savings" which will lead to the same social welfare and tax revenue without reranking.

We believe that the additive separability property of the measurement instrument is essential to clarify the distinction between welfare comparisons across households with different needs, and the impact of a progressive tariff and deductions of all sorts on both RE, RKG and HI. Fortunately, as reviewed in Ruiz-Castillo (1995), when we add this property to the usual ones and require that welfare within each group be weighted by population shares, we end up with the Kolm-Pollak family of social evaluation functions in the absolute case, and a single member of the general entropy family in the relative case. Although in this context we prefer an absolute notion of vertical inequality, we treat the relative case as well.

A final word is in order. Like any other measurement approach, this has to be tested in empirical work. This is currently the subject of independent research.

The rest of the paper is organized in three sections. The first one presents notation and a statement of the social evaluation problem. The second section, which is central to the paper, is devoted to the absolute case. In the first place, we study RE, HI and RKG resulting from progressive tax rates on original incomes, equity deductions, and tax exemptions on grounds other than equity. In the second place, actual practice is compared to the dismissal of dependent allowances and the application of tax rates on adjusted rather than original income: as in the homogeneous case, no HI or RKG is then possible. Finally, we study the consequences of eliminating HI caused by tax breaks from non equity deductions. The third section briefly reviews these topics in the relative case.

I. THE CONCEPTUAL FRAMEWORK

I.1. Welfare comparisons between heterogeneous tax units

Assume we have a heterogeneous population of $i = 1,\ldots,N$ tax units, which may differ in their income before taxes x^i and/or the number of dependents d^i. Since all tax units are headed by a person known as the taxpayer, tax unit size is equal to $d^i + 1$.

As in Pollak and Wales (1979), tax units are endowed with a common unconditional utility function defined on commodities *and* ethically relevant characteristics, tax unit size in our context. Omiting prices, which remain constant in the sequel, indirect utility is given by

$$u^i = \varphi(x^i, d^i + 1).$$

Naturally, original incomes x^i and x^j are non comparable unless $d^i = d^j$. Otherwise, one can define adjusted or equivalent income as

$$x^i(\Theta) = x^i/(d^i + 1)^{\Theta}, \Theta\in[0,1],$$

where $(d^i + 1)^{\Theta}$ is interpreted as the number of equivalent taxpayers in the tax unit, and Θ is a parameter indicating the importance we are prepared to give to economies of scale in consumption: the greater Θ is, the smaller the economies of scale, and the closer adjusted income becomes to income *per capita*. Thus, $x^i(\Theta)$ is the income necessary for a reference taxpayer to enjoy the utility level $u^i = \varphi(x^i, d^i + 1)$.

Alternatively, adjusted income can be defined as

$$x^i(\lambda) = x^i - \lambda d^i, \lambda\in[0, \lambda^{\#}],$$

where the parameter λ can be interpreted as the cost of a reference taxpayer, so that λd^i is the income we can subtract from a tax unit of size $d^i + 1$ for a reference taxpayer to enjoy the utility level $u^i = \varphi(x^i, d^i + 1)$ with the remaining income.

Under certain conditions on tax unit preferences reviewed in Ruiz-Castillo (1995),

$$x^i(\Theta) \geq x^j(\Theta) \Leftrightarrow u^i \geq u^j$$

or

$$x^i(\lambda) \geq x^j(\lambda) \iff u^i \geq u^j .$$

Therefore income adjusted for non-income needs in either of these two polar forms provides a comparable indicator of tax unit welfare.

I.2. Social Evaluation Functions(6)

A Social Evaluation Function (SEF) is a real valued function W defined in the space R^N of adjusted incomes, with the interpretation that for each income distribution, say $r = (r^1,\dots,r^n)$, W(r) provides the "social" or, simply, the aggregate welfare from a normative point of view.

Consider the following axiom set on W covering both a relative and an absolute concept of vertical inequality: A.1 S-concavity; A.2 continuity; A.3 population replication invariance; plus A.4R weak-homotheticity and A.5R monotonicity along rays from the origin, in the relative case; or A.4A weak-translability and A.5A monotonicity along rays parallel to the line of equality, in the absolute case. Under these conditions, there exists a unique function H such that

$$W(r) = H(\mu(r), V(r)),$$

where μ is the function giving the mean, V an index of *vertical* relative or absolute inequality, and H is increasing in its first argument and decreasing in the second.

We are interested in complete quantitative assessments of welfare change and its decomposition into changes in the mean and changes in either relative or absolute inequality. For that purpose, we have to be more specific about the trade-off between efficiency and distributional considerations. We will consider the multiplicative case

$$W(r) = \mu(r)\,[1 - I(r))],$$

where I is an index of relative inequality, and the additive case

$$W(r) = \mu(r) - A(r),$$

where A stands for an index of absolute inequality.

On the other hand, for any partition of the population, we are also interested in welfare measures capable of distinguishing -in a convenient additive way- between two components: welfare within the subgroups, and the loss of welfare due to inequality between the subgroups. Without loss of generality, let us choose the partition by tax unit size into $m = 1,\dots,M$ subgroups.

Consider first the relative case and define between-group inequality as the inequality remaining after removing all within-group inequality by assigning each tax unit her subgroup mean, that is, the inequality of the distribution $\mu^* = (\mu^1,\ldots,\mu^M)$ where, for each m,

$$\mu^m = (\mu(r^m).1^{N^m}), \quad 1^{N^m} = (1,\ldots,1) \in R^{N^m},$$

and $\Sigma_m N^m = N$. Then, one investigates under what conditions overall inequality can be expressed as

$$I(r) = \Sigma_m \alpha^m I(r^m) + I(\mu^*), \qquad (1)$$

where the weights α^m are functions only of the set of subgroup means and sizes. If these weights are subgroup shares in total income, then we have

$$W(r) = \Sigma_m [N^m/N] W(r^m) - \mu(r) I(\mu^*).$$

This is a useful expression, indicating that aggregate welfare can be expressed as a weighted average of the welfare within each subgroup, with weights equal to population shares, minus the between-group inequality weighted by the population mean. Although all the members of the generalized entropy family of relative inequality indices admit the decomposition of equation (1), only the first Theil index

$$I_1(r) = [1/N] \Sigma_i [r^i/\mu(r)] \ln [r^i/\mu(r)]$$

has the required properties, so that

$$W_1(r) = \mu(r)[1 - I_1(r)] = \Sigma_m [N^m/N] W_1(r^m) - \mu(r) I_1(\mu^*).$$

Blackorby, Donaldson and Auersperg (1981) define between-group inequality as the inequality that would result if each household received her subgroup's equally-distributed-equivalent-income (EDEI) ξ^m. The separability conditions required to estimate the EDEI of any subgroup in any partition independently of the rest of the distribution, combined with assumptions A.1, A.2, and A.5A for a translatable W, lead to the Kolm-Pollak family:

$$W_\gamma(r) = -[1/\gamma] \ln[(1/N) \Sigma_i e^{-\gamma r^i}], \quad \gamma > 0,$$

where γ is interpreted as an aversion to inequality parameter: as γ increases, the social indifference curves show increasing curvature until only the income of the poorest person matters. An index of absolute inequality consistent with W_γ is

$$A_\gamma(r) = [1/\gamma] \ln [(1/N)\Sigma_i e^{\gamma(\mu(r) - r^i)}], \quad \gamma > 0.$$

Since

$$A_\gamma(r) = \Sigma_m [N^m/N] A_\gamma(r^m) + A_\gamma(\xi^*),$$

where

$$\xi^* = (\xi^1,\ldots,\xi^M), \; \xi^m = (\xi(r^m).1^{N^m}), \; m = 1,\ldots,M,$$

we have

$$W_\gamma(r) = \mu(r) - A_\gamma(r) = \Sigma_m [N^m/N] W_\gamma(r^m) - A_\gamma(\xi^*).$$

This is an appealing decomposition, in which social welfare is seen to be equal to the weighted average of the aggregate welfare within each of the subgroups, with weights equal to population shares, minus the inequality between the subgroups.

We must recall that, in our definition of adjusted income, we have parametrized the weight we are prepared to give to the economies of scale in consumption achieved by tax units. Aggregate welfare in the relative case will be

$$W_1(x(\Theta)) = \Sigma_m [N^m/N][W_1(x^m)/m^\Theta] - \mu(x(\Theta)) I_1(\mu^*(\Theta)),$$

where for each m

$$\mu^m(\Theta) = [\mu(x^m)/m^\Theta] \, 1^{N^m} \text{ and } I_1(x^m(\Theta)) = I_1(x^m).$$

In the absolute case,

$$W_\gamma(x(\lambda)) = \Sigma_m [N^m/N] W_\gamma(x^m) - \lambda(d/N) - A_\gamma(\xi^*(\lambda)),$$

where $d = \Sigma_i d^i$ is the total number of dependents and, for each m,

$$\xi^m(\lambda) = [\xi(x^m) - \lambda(m-1)] \, 1^{N^m} \text{ and } A_\gamma(x^m(\lambda)) = A_\gamma(x^m).$$

I.3. Tax systems

A tax system is a triple < P, D_1, D_2 >, where P is an income tax function defined on the positive real numbers, D_1 is a rule which gives a general deduction to all taxpayers and a tax allowance for each dependent, and D_2 is a rule which gives all other tax exemptions as a function of the tax unit status in regard to income source, housing tenure and other characteristics.

We are given an actual tax system where the function P is said to be progressive because average and absolute tax rates are increasing, and where marginal tax rates are less than one so that there can be no rerankings from the operation of the tariff. Moreover, we will assume that D_1 is a linear function

$$D_1^i = \alpha\,(d^i + 1),\ i = 1,\dots,N,$$

according to which all tax unit members are treated equally. We will refer to the parameter α as the implicit fiscal scale.

Denote by T^i income taxes actually paid. Since tax rates are assumed to apply to original incomes,

$$T^i = P(x^i) - D_1^i - D_2^i.$$

Define after tax income as

$$v^i = x^i - T^i.$$

It will be useful to define also income before deductions

$$y^i = x^i - P(x^i),$$

and income z^i and taxes T_1^i after equity related deductions

$$z^i = y^i + D_1^i\,,\ T_1^i = P(x^i) - D_1^i\,,$$

so that

$$v^i = z^i + D_2^i \text{ and } T^i = T_1^i - D_2^i.$$

We will denote the corresponding vectors by

$$\mathbf{x} = (x^1,\dots,x^n),\quad \mathbf{y} = (y^1,\dots,y^n),\quad \mathbf{z} = (z^1,\dots,z^n),\quad \mathbf{v} = (v^1,\dots,v^n),$$

$$\mathbf{T} = (T^1,\dots,T^n),\quad \mathbf{P(x)} = (P(x^1),\dots,P(x^n)),\quad \mathbf{D_1} = (D_1^1,\dots,D_1^n),\quad \mathbf{D_2} = (D_2^1,\dots,D_2^n),$$

where

$$\mathbf{y} = \mathbf{x} - \mathbf{P(x)}\,,\quad \mathbf{z} = \mathbf{y} + \mathbf{D_1}\,,\quad \mathbf{v} = \mathbf{z} + \mathbf{D_2}\,,\quad \mathbf{T} = \mathbf{P(x)} - \mathbf{D_1} - \mathbf{D_2}\,.$$

For any vector $\mathbf{r} = (r^1,\dots,r^n)$, we will use the notation

$$\sigma(\mathbf{r}) = \Sigma_i\, r^i,$$

so that, for instance, $\sigma(\mathbf{P(x)})$ would be total tax revenue accruing from the tax on original income, while $\sigma(\mathbf{T}) < \sigma(\mathbf{P(x)})$ is actual tax revenue after allowances of all types.

I.4. The social evaluation problem

Given a SEF W, social welfare before taxes is $W(\mathbf{x}(\cdot))$, while welfare after taxes is $W(\mathbf{v}(\cdot))$. Therefore, the welfare change induced by the tax system in the relative case will be

$$\Delta RW(\Theta) = W_1(\mathbf{v}(\Theta)) - W_1(\mathbf{x}(\Theta)).$$

Taking into account that

$$W_1(\cdot) = \mu(\cdot)[1 - I_1(\cdot)],$$

we have

$$\Delta RW(\Theta) = RE(\Theta) - \mu(T(\Theta))[1 - I_1(\mathbf{v}(\Theta))]$$

where

$$RE(\Theta) = \mu(\mathbf{x}(\Theta))[I_1(\mathbf{x}(\Theta)) - I_1(\mathbf{v}(\Theta))]$$

is the RE of an actual income tax system, measured by the change in an index of vertical inequality before and after taxes, weighted in this context by mean income before taxes.

Similarly, in the absolute case the welfare change induced by the tax system will be

$$\Delta AW(\lambda) = W_\gamma(\mathbf{v}(\lambda)) - W_\gamma(\mathbf{x}(\lambda)).$$

Since

$$W_\gamma(\cdot) = \mu(\cdot) - A_\gamma(\cdot),$$

we have

$$\Delta AW(\lambda) = RE(\lambda) - \mu(T)$$

where

$$RE(\lambda) = A_\gamma(\mathbf{x}(\lambda)) - A_\gamma(\mathbf{v}(\lambda))$$

is the RE of an actual income tax system.

In both cases, there appears a negative term which captures tax revenues or, what is the same, the disposable income reduction in tax units hands. This welfare loss may or may not be offset by the benefits -and its distribution across tax units- which the public sector might give rise to via transfers or public expenditure, an activity beyond this paper's scope.

II. THE ABSOLUTE CASE

II.1. RE of an actual tax system

In the first place, we investigate the impact of a progressive tax system on original income. For that purpose, we apply the absolute inequality index's additive separability property to the partition by tax unit size in

$$RE_{I1}(\lambda) = A_\gamma(x(\lambda)) - A_\gamma(y(\lambda))$$

$$= \Sigma_m [N^m/N] [A_\gamma(x^m) - A_\gamma(y^m)] + [A_\gamma(\overset{*}{\xi}_{xm}(\lambda)) - A_\gamma(\overset{*}{\xi}_{ym}(\lambda))]$$

where, for example,

$$\overset{*}{\xi}_{xm}(\lambda) = (\xi_x^1(\lambda),...,\xi_x^M(\lambda)),\ \xi_x^m(\lambda) = (\xi(x^m(\lambda)).1^{N^m}),\ \text{and}\ 1^{N^m} = (1,...,1) \in R^{N^m}.$$

Notice that $RE_{I1}(\lambda)$ has been broken down into two terms. The first is the sum of redistributive effects within each subgroup m, weighted by population shares. Vertical inequality improves within each subgroup. Therefore, the first term as a whole, which is independent of λ, must be positive. The second term is the change in inequality between groups. Whether or not a progressive tariff imposed on original income improves absolute inequality between groups is an empirical question. In any case, the magnitude of the change depends on the value judgements summarized by λ.

In the second place, we investigate the impact of tax allowances for dependents and taxpayers. Consider the expression

$$RE_{I2}(\lambda) = A_\gamma(y(\lambda)) - A_\gamma(z(\lambda)) =$$

$$\Sigma_m [N^m/N] [A_\gamma(y^m) - A_\gamma(z^m)] + [A_\gamma(\overset{*}{\xi}_{ym}(\lambda)) - A_\gamma(\overset{*}{\xi}_{zm}(\lambda))].$$

For any i with tax unit size equal to m,

$$z^i = y^i + \alpha m.$$

Therefore, for each m,

$$A_\gamma(y^m) = A_\gamma(z^m).$$

Hence

$$RE_{I2}(\lambda) = A_\gamma(\overset{*}{\xi}_{ym}(\lambda)) - A_\gamma(\overset{*}{\xi}_{zm}(\lambda)).$$

That is, the redistributive effect of equity deductions is only the change in between-groups inequality when we go from distribution $\mathbf{y}(\lambda)$ to distribution $\mathbf{z}(\lambda)$. The sign, of course, is unknown *a priori.*

In the third place, we investigate the impact of other exemptions. Consider the expression

$$RE_{II}(\lambda) = A_\gamma(\mathbf{z}(\lambda)) - A_\gamma(\mathbf{v}(\lambda)) =$$

$$\Sigma_m [N^m/N]\,[A_\gamma(\mathbf{z}^m) - A_\gamma(\mathbf{v}^m)] + [A_\gamma(\xi^*_{\mathbf{z}m}(\lambda)) - A_\gamma(\xi^*_{\mathbf{v}m}(\lambda))].$$

None of the two terms of this decomposition can be signed *a priori.*

Collecting terms, the welfare change induced by an actual tax system in the absolute case is seen to be

$$\Delta AW(\lambda) = RE_{I1}(\lambda) + RE_{I2}(\lambda) + RE_{II}(\lambda) - \mu(T)$$

$$= \Sigma_m [N^m/N]\,[A_\gamma(\mathbf{x}^m) - A_\gamma(\mathbf{v}^m)] + [A_\gamma(\xi^*_{\mathbf{x}m}(\lambda)) - A_\gamma(\xi^*_{\mathbf{v}m}(\lambda)) - \mu(T).$$

II.2. HI of an actual tax system

Consider now the partition by exact equals, $e = 1,\ldots,E$ in the distribution by before tax adjusted income. We will apply the additive separability property to the three RE expressions already studied, beginning with

$$RE_{I1}(\lambda) = A_\gamma(\mathbf{x}(\lambda)) - A_\gamma(\mathbf{y}(\lambda)) =$$

$$\Sigma_e [N^e/N]\,[A_\gamma(\mathbf{x}^e(\lambda)) - A_\gamma(\mathbf{y}^e(\lambda))] + [A_\gamma(\xi^*_{\mathbf{x}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{y}e}(\lambda))]$$

where, for example,

$$\xi^*_{\mathbf{x}e}(\lambda) = (\xi^1_{\mathbf{x}}(\lambda),\ldots,\xi^E_{\mathbf{x}}(\lambda)),\ \xi^e_{\mathbf{x}}(\lambda) = (\xi(\mathbf{x}^e(\lambda)).\mathbf{1}^{N^e}),\ \mathbf{1}^{N^e} = (1,\ldots,1) \in R^{N^e},$$

and $\Sigma_e N^e = N$. Since all tax units belonging to subgroup e are exact equals, we have that $A_\gamma(\mathbf{x}^e(\lambda)) = 0$. Therefore,

$$RE_{I1}(\lambda) = HI_{I1}(\lambda) + V_{I1}(\lambda),$$

where

$$HI_{I1}(\lambda) = -\Sigma_e [N^e/N]\, A_\gamma(y^e(\lambda)) < 0$$

measures HI created because the tariff is applied to unadjusted income, while

$$V_{I1}(\lambda) = A_\gamma(\xi^*_{\mathbf{x}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{y}e}(\lambda))$$

is the pure vertical effect induced by the tax. We expect $V_{I1}(\lambda) > 0$ for all λ.

In the second place,

$$RE_{I2}(\lambda) = \Sigma_e [N^e/N]\, [A_\gamma(y^e(\lambda)) - A_\gamma(\mathbf{z}^e(\lambda))] + [A_\gamma(\xi^*_{\mathbf{y}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{z}e}(\lambda))]$$

so that

$$RE_I(\lambda) = RE_{I1}(\lambda) + RE_{I2}(\lambda) = HI_I(\lambda) + V_I(\lambda),$$

where

$$HI_I(\lambda) = -\Sigma_e [N^e/N]\, A_\gamma(\mathbf{z}^e(\lambda))]$$

and

$$V_I(\lambda) = [A_\gamma(\xi^*_{\mathbf{x}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{z}e}(\lambda))].$$

The negative term $HI_I(\lambda)$ measures HI created by the double feature of actual tax systems: tariff exaction on original incomes, plus compensatory tax dependent allowances. In absolute value, we expect $HI_I(\lambda) < HI_{I1}(\lambda)$. The term $V_I(\lambda)$ measures the combined vertical effect induced by these two features.

In a similar vein, the second round of deductions gives rise to the following decomposition:

$$RE_{II}(\lambda) = \Sigma_e [N^e/N]\, [A_\gamma(\mathbf{z}^e(\lambda)) - A_\gamma(\mathbf{v}^e(\lambda))] + [A_\gamma(\xi^*_{\mathbf{z}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{v}e}(\lambda))].$$

Notice that, for any e, nothing can be said *a priori* about the sign of $A_\gamma(\mathbf{z}^e(\lambda)) - A_\gamma(\mathbf{v}^e(\lambda))]$. Therefore, in the expression

$$RE_{II}(\lambda) = HI_{II}(\lambda) + V_{II}(\lambda),$$

where

$$HI_{II}(\lambda) = \Sigma_e [N^e/N]\, [A_\gamma(\mathbf{z}^e(\lambda)) - A_\gamma(\mathbf{v}^e(\lambda))]$$

and

$$V_{II}(\lambda) = A_\gamma(\xi^*_{\mathbf{z}e}(\lambda)) - A_\gamma(\xi^*_{\mathbf{v}e}(\lambda))$$

neither of the two terms can be signed *a priori*. This simply reflects the fact that whether this type of HI generates a progressive or a regressive impact is an empirical question.

Collecting terms, we have

$$\Delta AW(\lambda) = RE(\lambda) - \mu(T) = HI(\lambda) + V(\lambda) - \mu(T)$$

where

$$HI(\lambda) = HI_I(\lambda) + HI_{II}(\lambda) = -\Sigma_e [N^e/N]\, A_\gamma(v^e(\lambda))] < 0$$

and

$$V(\lambda) = V_I(\lambda) + V_{II}(\lambda) = A_\gamma(\xi^*_{xe}(\lambda)) - A_\gamma(\xi^*_{ve}(\lambda)).$$

II.3. RKG of an actual tax system

Suppose that $\mathbf{v}(\lambda)$ and $\mathbf{x}(\lambda)$ are not equally ordered. Consider the permutation $\mathbf{v}'(\lambda)$ of $\mathbf{v}(\lambda)$ which orders all tax units as in $\mathbf{x}(\lambda)$. For every i and every λ, define total taxes

$$T^{i'}(\lambda) = v^{i'}(\lambda) - x^i(\lambda).$$

Clearly, for every λ

$$W(\mathbf{v}'(\lambda)) = W(\mathbf{v}(\lambda))$$

and

$$\sigma(\mathbf{T}'(\lambda)) = \sigma(\mathbf{T}).$$

That is, for every λ the tax vector $\mathbf{T}'(\lambda)$ raises the same revenue as $\mathbf{T}$, and leads to the same after tax welfare as $\mathbf{v}(\lambda)$.

Our index of RKG will be:

$$RKG(\lambda) = A_\gamma(\mathbf{T}) - A_\gamma(\mathbf{T}'(\lambda)).$$

II.4. Empirical questions

(1) We are interested in studying the robustness of our estimates to variations in the generosity of the scale used to define adjusted income. Thus, in the first place, we want to study reranking, horizontal and vertical effects, redistributive effects of different sorts, and the change in absolute welfare as a function of λ.

(2) In particular, we want to study the consequences of adopting the fiscal scale in the valuation exercise, that is, we want to study the case

in which $\lambda = \alpha$. When we agree to make all evaluations at the fiscal scale, we have for every i

$$z^i(\alpha) = y^i(\alpha) + \alpha(d^i + 1) = x^i - P(x^i) - \alpha d^i + \alpha(d^i + 1) = x^i - P(x^i) + \alpha = y^i + \alpha,$$

so that, for every e we will have

$$A_\gamma(\mathbf{z}^e(\alpha)) = A_\gamma(\mathbf{y}^e(\alpha)) > 0.$$

Therefore,

$$HI_{I1}(\alpha) = HI_I(\alpha) = - \Sigma_e [N^e/N]\, A_\gamma(\mathbf{y}^e(\alpha)) < 0.$$

That is, there remains only the HI created by the application of the tariff to original rather than adjusted income.

(3) Presumably, everyone would agree that, *ceteris paribus*, supressing HI is better. In our framework this must mean that supressing HI will lead to greater social welfare.

We investigate this question in two steps. In the first place, by discarding all dependents allowances, maintaining the general exemption on taxpayers, and applying the given progressive tax function P to adjusted income. In the second place, we will study HI created by non-equity deductions.

For each i and λ define taxes

$$T_1^i(\lambda) = P(x^i(\lambda)) - \lambda$$

and after tax income

$$Y^i(\lambda) = x^i(\lambda) - P(x^i(\lambda)) + \lambda.$$

Welfare change before the second round of deductions will be

$$\Delta A\, W_I^*(\lambda) = W_\gamma(\mathbf{Y}(\lambda)) - W_\gamma(\mathbf{x}(\lambda)) = RE_I^*(\lambda) - \mu(\mathbf{T_1}(\lambda))$$

where

$$RE_I^*(\lambda) = A_\gamma(\mathbf{x}(\lambda)) - A_\gamma(\mathbf{Y}(\lambda)).$$

Clearly, tax units in income-dependents space pay identical taxes, so that there is no HI and all redistributive effect is of the vertical sort. Such effect can be broken down into

$$RE_I^*(\lambda) = \Sigma_m [N^m/N]\, [A_\gamma(\mathbf{x}^m(\lambda)) - A_\gamma(\mathbf{Y}^m(\lambda))] + [A_\gamma(\overset{*}{\xi}_{\mathbf{x}m}(\lambda)) - A_\gamma(\overset{*}{\xi}_{\mathbf{Y}m}(\lambda))]$$

But the interesting question is: does there exist a set of values of λ for which

$$\Delta A\, W_I^*(\lambda) = RE_I^*(\lambda) - \mu(T_1(\lambda)) > \Delta A\, W_I(\lambda) = RE_I(\lambda) - \mu(T_1)$$

where

$$RE_I(\lambda) = RE_{I1}(\lambda) + RE_{I2}(\lambda) = A_\gamma(y(\lambda)) - A_\gamma(x(\lambda))?$$

If so, is α in this set? Moreover, are there values for which

$$RE_I^*(\lambda) > RE_1(\lambda) \text{ and } \sigma(T_1(\lambda)) = \sigma(T_1).$$

Otherwise, if we want to supress this type of HI and at least maintain social welfare, we will be forced to change the tax funtion P.

(4) To complete the exercise, we must add the second round of deductions. Define total taxes $T^i(\lambda)$ and after tax adjusted income $f^i(\lambda)$ by

$$T^i(\lambda) = T_1^i(\lambda) + D_2^i,$$

and

$$f^i(\lambda) = Y^i(\lambda) + D_2^i,$$

respectively. Then

$$\Delta A\, W_{II}^*(\lambda) = W_\gamma(f(\lambda)) - W_\gamma(Y(\lambda)) = RE_{II}^*(\lambda) - \mu(D_2)$$

where

$$RE_{II}^*(\lambda) = A_\gamma(Y(\lambda)) - A_\gamma(f(\lambda)).$$

Non-equity deductions will surely give rise to HI. Taking into account the partition of $x(\lambda)$ by exact equals in $e = 1,\ldots,E$ subgroups, we have

$$RE_{II}^*(\lambda) = HI_{II}^*(\lambda) + V_{II}^*(\lambda)$$

where

$$HI_{II}^*(\lambda) = -\Sigma_e[N^e/N]\, A_\gamma(f^e(\lambda))]$$

and

$$V_{II}^*(\lambda) = [A_\gamma(\xi_{Ye}^*(\lambda)) - A_\gamma(\xi_{fe}^*(\lambda))].$$

The question is: which will be the consequences of making $HI_{II}^*(\lambda) = 0$? Suppose we redistribute $\sigma(D_2^e)$ equally among tax units in e. That is, define after tax income for each i in e by

$$F^i(\lambda) = Y^i(\lambda) + \mu(D_2^e).$$

Then for each e

$$A_\gamma(F^e(\lambda)) = A_\gamma(Y^e(\lambda)) = 0,$$

so that HI are eliminated. Thus, if we define

$$RE_{II}^{**}(\lambda) = A_\gamma(Y(\lambda)) - A_\gamma(F(\lambda)),$$

then

$$HI_{II}^{**}(\lambda) = \Sigma_e[N^e/N]\,[A_\gamma(Y^e(\lambda) - A_\gamma(F^e(\lambda))] = 0$$

and

$$V_{II}^{**}(\lambda) = A_\gamma(\overset{*}{\xi}_{Y^e}(\lambda)) - A_\gamma(\overset{*}{\xi}_{F^e}(\lambda))$$

so that

$$RE_{II}^{**}(\lambda) = V_{II}^{**}(\lambda).$$

But the fact that $HI_{II}^{**}(\lambda) = 0$, does not guarantee that $RE_{II}^{**}(\lambda) > RE_{II}^{*}(\lambda)$. The reason is that if deductions had a regressive impact, i. e. $RE_{II}^{*}(\lambda) < 0$, then the supression of HI in this manner will typically lead to an even greater worsening of vertical inequality.

II. THE RELATIVE CASE

III.1. RE of an actual tax system

In the first place, consider the impact of a progressive tax system on original incomes:

$$W_1(y(\Theta)) - W_1(x(\Theta)) = RE_{I1}(\Theta) - L_1(\Theta),$$

where

$$L_1(\Theta) = \Sigma_m[N^m/N]\,[\mu(P(x^m(\Theta))]\,[1 - I_1(y^m) - I_1(\mu^*_{ym}(\Theta))],$$

$$RE_{I1}(\Theta) = \Sigma_m[N^m/N]\,[\mu(x^m)/m^{\Theta}]\,[I_1(x^m) - I_1(y^m)]$$

$$+ [\mu(x^m)/m^{\Theta}][I_1(\mu^*_{xm}(\Theta)) - I_1(\mu^*_{ym}(\Theta))]$$

and, for example,

$$\mu^*_{xm}(\Theta) = (\mu^1_x(\Theta),\ldots,\mu^M_x(\Theta)),\ \ \mu^m_x(\Theta) = [\mu(x^m)/m^{\Theta}].1^{N^m}),\ \ 1^{N^m} = (1,\ldots,1)\in R^{N^m}.$$

The first expression is the welfare loss caused by the reduction in tax units disposable income. The second expression is the RE of the tax system, which is the sum of two terms: the weighted sum of redistributive effects within each subgroup m, plus the change in between-group inequality before and after tax, weighted by the before tax mean. Since $I_1(x^m) > I_1(y^m)$ for every m, the first term must be positive, but the sign and magnitude of the second term is not known *a priori.*

In the second place, consider the impact of equity deductions:

$$W_1(z(\Theta)) - W_1(y(\Theta)) = RE_{I2}(\Theta) + G_1(\Theta),$$

where

$$G_1(\Theta)) = \Sigma_m[N^m/N]\,[\mu(D^m_1)/m^{\Theta}]\,[1 - I_1(z^m) - I_1(\mu^*_{zm}(\Theta))],$$

and

$$RE_{I2}(\Theta) = \Sigma_m[N^m/N]\,[\mu(y^m)/m^{\Theta}]\,[I_1(y^m) - I_1(z^m)]$$

$$+ [\mu(y^m)/m^{\Theta}][I_1(\mu^*_{ym}(\Theta)) - I_1(\mu^*_{zm}(\Theta))].$$

The first expression is the welfare gain caused by the tax break accruing to tax units. The second expression is the RE caused by equity deductions. In the absolute case, equal equity deductions to all tax units of a given size, did not change within-group inequality. Now we have

$$I_1(y^m) > I_1(z^m)$$

for each m. Therefore, the first term in this expression is positive, although the second term cannot be signed *a priori.*

In the third place, consider the impact of non equity deductions:

$$W_1(v(\Theta)) - W_1(z(\Theta)) = RE_{II}(\Theta) + G_2(\Theta),$$

where

$$G_2(\Theta)) = \Sigma_m[N^m/N]\,[\mu(D_2^m)/m^\Theta]\,[1 - I_1(v^m) - I_1(\mu^*_{vm}(\Theta))],$$

and

$$RE_{II}(\Theta) = \Sigma_m[N^m/N]\,[\mu(z^m)/m^\Theta]\,[I_1(z^m) - I_1(v^m(\Theta))]$$

$$+ [\mu(z^m)/m^\Theta][I_1(\mu^*_{zm}(\Theta)) - I_1(\mu^*_{vm}(\Theta))].$$

The first expression is the welfare gain caused by the second tax break accruing to tax units. The second expression is the RE caused by non equity deductions. In this case, neither of the two terms can be signed *a priori.*

Finally, the aggregate welfare change can be expressed as

$$\Delta RW(\Theta) = W_1(v(\Theta)) - W_1(x(\Theta)) = RE(\Theta) - L(\Theta)$$

where

$$L_1(\Theta) = L_1(\Theta) + G_1(\Theta) + G_2(\Theta)$$

and

$$RE(\Theta) = RE_{I1}(\Theta) + RE_{I2}(\Theta) + RE_{II}(\Theta)$$

$$= \Sigma_m[N^m/N][\mu(x^m)/m^\Theta][I_1(x^m) - I_1(v^m(\Theta))]$$

$$+ [\mu(x^m)/m^\Theta][I_1(\mu^*_{xm}(\Theta)) - I_1(\mu^*_{vm}(\Theta))].$$

The first expression is the net welfare loss caused by the reduction in tax units disposable income. The second expression is the RE of the actual tax system as a whole, which is the sum of two terms neither of which can be signed *a priori.*

III.2. HI of an actual tax system

Consider now the partition by exact equals, e = 1,...,E in the distribution by before tax adjusted income. We will apply the additive separability property to the three expressions for welfare change already studied, beginning with

$$W_1(y(\Theta)) - W_1(x(\Theta)) = HI_{I1}(\Theta) + V_{I1}(\lambda) - L_1^H(\Theta),$$

where

$$L_1^H(\Theta) = \Sigma_e [N^e/N] [\mu(P(x^e(\Theta))] [1 - I_1(y^e) - I_1(\mu^*_{ye}(\Theta))],$$

$$HI_{I1}(\Theta) = - \Sigma_e [N^e/N] [\mu(x^e(\Theta))] I_1(y^e(\Theta)),$$

$$V_{I1}(\Theta) = [\mu(x^e(\Theta))][I_1(\mu^*_{xe}(\Theta)) - I_1(\mu^*_{ye}(\Theta))]$$

and, for example,

$$\mu^*_{xe}(\Theta) = (\mu^1_x(\Theta),...,\mu^E_x(\Theta)),\ \mu^e_x(\Theta) = [\mu(x^e(\Theta))].1^{N^e}),\ 1^{N^e} = (1,...,1)\in R^{N^e}.$$

The expression $HI_{I1}(\Theta) < 0$, measures HI created because the tariff is applied to unadjusted income, while $V_{I1}(\Theta)$ is the pure vertical effect induced by the tax. We expect $V_{I1}(\Theta) > 0$ for all Θ.

In the second place,

$$W_1(z(\Theta)) - W_1(y(\Theta)) = HI_{I2}(\Theta) + V_{I2}(\Theta) - L_2^H(\Theta),$$

where

$$L_2^H(\Theta) = \Sigma_e [N^e/N] [(D_2^e(\Theta))] [1 - I_1(z^e) - I_1(\mu^*_{ze}(\Theta))],$$

$$HI_{I2}(\Theta) = \Sigma_e [N^e/N] [\mu(y^e(\Theta))] [I_1(y^e(\Theta)) - I_1(z^e(\Theta))],$$

and

$$V_{I2}(\Theta) = [\mu(y^e(\Theta))][I_1(\mu^*_{ye}(\Theta)) - I_1(\mu^*_{ze}(\Theta))],$$

so that

$$W_1(z(\Theta)) - W_1(x(\Theta)) = HI_I(\Theta) + V_I(\Theta) - L_{12}^H(\Theta)$$

where

$$L_{12}^H(\Theta) = \Sigma_e [N^e/N] [(T_1^e(\Theta))] [1 - I_1(z^e) - I_1(\mu^*_{ze}(\Theta))],$$

$$HI_I(\Theta) = - \Sigma_e [N^e/N] I_1(z^e(\lambda))],$$

and

$$V_I(\Theta) = [\mu(x^e(\Theta))][I_1(\mu^*_{x^e}(\Theta)) - I_1(\mu^*_{z^e}(\Theta))].$$

The negative term $HI_I(\Theta)$ measures HI created by the double feature of actual tax systems: tariff exaction on original incomes, plus compensatory tax dependent allowances. In absolute value, we expect $HI_I(\Theta) < HI_{I1}(\Theta)$. The term $V_I(\Theta)$ measures the combined vertical effect induced by these two features.

Similarly, non equity deductions give rise to the following decomposition:

$$W_1(\mathbf{v}(\Theta)) - W_1(\mathbf{z}(\Theta)) = HI_{II}(\Theta) + V_{II}(\Theta) - L_3^H(\Theta),$$

where

$$L_3^H(\Theta) = \Sigma_e [N^e/N] [(D_2^e(\Theta))] [1 - I_1(v^e) - I_1(\mu^*_{v^e}(\Theta))],$$

$$HI_{II}(\Theta) = \Sigma_e [N^e/N] [\mu(\mathbf{z}^e(\Theta))] [I_1(\mathbf{z}^e(\Theta)) - I_1(\mathbf{v}^e(\Theta))],$$

and

$$V_{II}(\Theta) = [\mu(\mathbf{v}^e(\Theta))][I_1(\mu^*_{z^e}(\Theta)) - I_1(\mu^*_{v^e}(\Theta))].$$

Notice that neither of these two terms can be signed *a priori.*

Collecting terms, we have

$$\Delta RW(\Theta) = W_1(\mathbf{v}(\Theta)) - W_1(\mathbf{x}(\Theta))$$

$$= HI_I(\Theta) + HI_{II}(\Theta) + V_I(\Theta) + V_{II}(\Theta) - [L_{12}^H(\Theta) + L_3^H(\Theta)]$$

$$= HI(\Theta) + V(\Theta) - L^H(\Theta)$$

where

$$L^H(\Theta) = \Sigma_e [N^e/N] [(T_1^e(\Theta))] [1 - I_1(v^e) - I_1(\mu^*_{v^e}(\Theta))]$$

$$HI(\Theta) = - \Sigma_e [N^e/N] I_1(\mathbf{v}^e(\Theta)),$$

$$V(\Theta) = [\mu(\mathbf{x}^e(\Theta))][I_1(\mu^*_{x^e}(\Theta)) - I_1(\mu^*_{v^e}(\Theta))].$$

II.3. RKG of an actual tax system

Suppose that $\mathbf{v}(\Theta)$ and $\mathbf{x}(\Theta)$ are not equally ordered. Consider the permutation $\mathbf{v}'(\Theta)$ of $\mathbf{v}(\Theta)$ which orders all tax units as in $\mathbf{x}(\lambda)$. For every i and every Θ, define total taxes

$$T^{i'}(\Theta) = v^{i'}(\Theta) - x^{i}(\Theta).$$

Clearly, for every λ

$$W(\mathbf{v}'(\Theta)) = W(\mathbf{v}(\Theta))$$

and

$$\sigma(\mathbf{T}'(\Theta)) = \sigma(\mathbf{T}).$$

That is, for every λ the tax vector $\mathbf{T}'(\Theta)$ raises the same revenue as $\mathbf{T}$, and leads to the same after welfare as $\mathbf{v}(\Theta)$.

Our index of RKG will be:

$$RKG(\Theta) = A_{\gamma}(\mathbf{T}) - A_{\gamma}(\mathbf{T}'(\Theta)).$$

NOTES

(1) See, for instance, Berliant and Strauss (1985), Aronson et al (1994), Lambert and Ramos (1994), Camarero et al (1993), and Pazos et al (1994).

(2) See, for instance, Feldstein (1976), Atkinson (1980), Plotnik (1982, 1985), and King (1983).

(3) See, for instance, Jakobsson (1976) and Eichhorn et al (1984) for the relative case, and Moyes (1988) for the absolute case.

(4) For other applications, see Ruiz-Castillo (1994a, 1994b) and, in an income tax context, Aronson et al (1994).

(5) In this we differ from those who use indices of distributional change studied by Cowell (1980, 1985): Jenkins (1988) applies them to the original income in the ethically relevant partition (tax unit size in our case), while Camarero et al (1993) and Pazos et al (1994) apply them to income adjusted by a single equivalence scale in the partition by close similars.

(6) See Ruiz-Castillo (1995) for a review of the material covered in this subsection.

REFERENCES

Aronson, J. R., P. Johnson and P. Lambert (1994), "Redistributive Effect and Unequal Tax Treatment," **104**: 262-270.

Atkinson, A. (1980), "Horizontal Equity and the Distribution of the Tax Burden," in H. Aaron and M. Boskin (ed), *The Economics of Taxation*, Washington D.C.: The Brookings Institution.

Berliant, M. C. and R. C. Strauss (1985), "The Horizontal and Vertical Equity Characteristics of the Federal Individual Income Tax, 1966-1977," in M. David and T. Smeeding (ed), *Horizontal Equity, Uncertainty, and Economic Wellbeing* , Chicago: NBER Studies in Income and Wealth, vol 50, Chicago University Press.

Blackorby, C., D. Donaldson and M. Auersperg (1981), "A New Procedure for the Measurement of Inequality Within and Among Population Subgroups," *Canadian Journal of Economics*, **14**: 665-685.

Camarero, R., O. Herrero and I. Zubiri (1993), "La medición de la inequidad horizontal: teoría y una aplicación al caso de Vizcaya," *Investigaciones Económicas*: **XVII**: 333-362.

Coulter, F., F. Cowell and S. Jenkins (1992a), "Differences in Needs and Assessment of Income Distributions," *Bulletin of Economic Research*, **44**: 77-124.

Coulter, F., F. Cowell and S. Jenkins (1992b), "Equivalence Scale Relativities and the Extent of Inequality and Poverty," *Economic Journal*, **102**: 1067-1082.

Cowell, F. A. (1980), "Generalized Entropy and the Measurement of Distributional Change," *European Economic Review*, **13**: 147-159.

Cowell, F. A. (1985), "Measures of Distributional Change: An Axiomatic Approach," *Review of Economic Studies*, **52**: 135-151.

Eichhorn, W., H. Funke and W. F. Richter (1984), "Tax Progression and Inequality of Income Distribution," **13**: 127-131.

Feldstein, M. (1976), "On the Theory of Tax Reform,"*Journal of Public Economics* **6**: 51-71.

Jacobsson, U. (1976), "On the Measurement of the Degree of Progression," **5**: 161-168.

Jenkins, S. (1988), "Empirical Measurement of Horizontal Inequity," *Journal of Public Economics*, **37**: 305-329.

King, M. (1983), "An Index of Inequality: With Applications to Horizontal Equity and Social Mobility," *Econometrica* **51**: 99-115.

Lambert, P. and J. R. Aronson (1993), "Inequality Decomposition Analysis and the Gini Coefficient Revisited," **103**: 1221-1227.

Lambert, P. and X. Ramos (1994), "Vertical Redistribution and Horizontal Inequity," Mimeo,

Moyes, P. (1988), "A Note on Minimally Progressive Taxation and Absolute Income Inequality," **5**: 227-234.

Pazos, M., I. Rabadán and R. Salas (1994), "Medición de la desigualdad horizontal en España en el IRPF," Instituto de Estudios Fiscales, Papeles de Trabajo, P.T. Nº 8/94.

Plotnick, R. (1982), "The Concept and Measurement of Horizontal Inequity," *Journal of Public Economics*, **17**: 3730-391.

Plotnick, R. (1985), "A Comparison of Measures of Horizontal Inequity," in M. Davia and T. Smeeding (ed), *Horizontal Equity, Uncertainty, and Economic Wellbeing* , Chicago: NBER Studies in Income and Wealth, vol. 50, Chicago University Press.

Ruiz-Castillo, J. (1994a), "The Distribution of Expenditure in Spain: 1973-74 to 1980-81," *Journal of Income Distribution.*

Ruiz-Castillo, J. (1994b), "The Evolution of the Standard of Living in Spain, 1973-74 to 1980-81,"Working Paper 94-10, Economic Series 04, Universidad Carlos III de Madrid.

Ruiz-Castillo, J. (1995), "Income Distribution and Social Welfare," *Investigaciones Económicas.*