

# On the competitive effects of divisionalization

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In this paper, we assume that firms can create independent divisions which compete in quantities in a homogeneous good market. Assuming identical firms and constant returns to scale, we prove that the strategic interaction of firms yields Perfect Competition if the number of firms is beyond some critical level. Assuming a fixed cost per firm and an upper bound on the maximum number of divisions, we show that when this upper bound tends to infinity and the fixed cost tends to zero, market equilibrium may yield either Perfect Competition or a Natural Oligopoly. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

One of the reasons for the popularity of the Cournot model is that it provides a justification for the assumption that, only in large economies, perfect competition occurs (see, for example, JET, 1980). This contrasts with the Bertrand model where, with homogeneous product and average costs constant and identical across firms, perfect competition is achieved with two firms, so market structure jumps from monopoly to perfect competition. When firms face an entry cost (no matter how small), monopoly occurs irrespective of the number of potential firms (see Sutton, 1991, p. 32). In both cases, Bertrand equilibrium is not sensitive to an increase in the number of firms beyond

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two. All this suggest that, in despite of the appeal of a model with price-setting firms, it may be preferable to model oligopolistic competition as Cournot did.

However, the performance of the Cournot model depends on the assumption that firms cannot create independent divisions (see Milgrom and Roberts (1998)) for some empirical evidence of divisionalization in oligopolistic markets). When divisionalization is feasible, the Cournot model behaves very much like the Bertrand model: Corchón (1991) showed that with linear or unit-elastic demand functions, Subgame Perfect Nash Equilibrium (SPNE) in pure strategies of a two-stage game (where in the first stage firms decide on the number of divisions competing à la Cournot in the second stage) yields perfect competition even with a small number of firms. Subsequently Polasky (1992) showed a similar result by allowing mixed strategies. In this paper we study the robustness of this result.

In Section 2 we assume zero fixed costs. We show that if the number of firms is large (but finite) in relation to the degree of convexity of the inverse demand function, SPNE yields perfect competition (Proposition 1). If the inverse demand function is concave or the industry profit function is concave and there are more than two firms, SPNE yields perfect competition (Proposition 2). These results generalize those obtained by Corchón (1991).

These results bear some similarity with those obtained in the (polar) case of merger. For instance Salant, Switzer and Reynolds (1983) find that, under linear demand and costs, in some cases, merger is not profitable for the merging firms. Their results have been generalized by Faulí-Oller (1997). He has shown that the degree of convexity of the inverse demand function is the main determinant of merger profitability. However, in despite of the fact that divisionalization is just merger in reverse and the similar role played in both problems by the degree of convexity, results are different. On the one hand, in order to model mergers as a fully fledged non-cooperative game, we have to introduce a bidding stage, as done by Kamien and Zang (1990). This stage does not make sense in the analysis of divisionalization. On the other hand, Kamien and Zang's main insight is that monopolization by merger is unlikely. Our result is that gains from divisionalization are not exhausted until the number of divisions is so large that the output reaches the perfectly competitive level. Thus our Proposition 1 and the results obtained in the merger literature are different.

The results obtained in Propositions 1 and 2 differ also from those obtained from other models of strategic delegation. For instance, in Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), it is assumed the use of incentive contracts between every firm and its (unique) manager, where a non-negative weight is given to sales or output. Under Cournot competition, those contracts yield equilibrium outcomes more competitive than without delegation, but still different from perfect competition. More recently, Kühn (1994) has shown, in the context of a duopoly with vertical integration and uncertainty, that with more general contracts, marginal cost pricing is achieved, in equilibrium, for constant marginal cost. However, in his model, the set of possible demand shocks must be unbounded. This restrictive assumption is not necessary in our model.<sup>1</sup>

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<sup>1</sup>In fact, our Proposition 1 can be proved under the assumption that demand and costs are stochastic (see Section 4)

In Section 3 we perform a sensitivity analysis of the previous results by introducing two frictions. In particular, we assume that each active firm has to pay an entry fixed cost and there is an upper bound on the number of divisions which can be created. We analyze SPNE when frictions disappear, i.e. when the upper bound tends to infinity and the fixed cost tends to zero. We show that depending on the rate at which frictions vanish, the limit may be either perfect competition or a *Natural Oligopoly*, i.e. a situation with a finite number of active firms (see Shaked and Sutton, 1983). This contrasts with the standard theorem with no divisionalization in which convergence to perfect competition always obtains, (see Novshek, 1980).

Finally Section 4 discusses how to extend our results to environments with uncertainty, product differentiation, etc., and the possible significance of our findings.

## 2. The model with no entry costs

We consider a homogeneous good market. There is a given number (possibly infinite) of firms, denoted by  $k (> 1)$ , which can create independent divisions. The inverse demand function is written as  $p = p(z)$ , where  $z$  is total output and  $p$  is the price of the product. The cost function for each division is given by  $cx_i$ , where  $c > 0$  and  $x_i$  is the output of division  $i$ .

If firm  $j$  creates  $m$  divisions producing  $x_i$  each, profits of firm  $j$  are:

$$\Pi_j = m[p(z)x_i - cx_i].$$

All our results hold if divisions pay a fraction of their profits. In this case, the model can be interpreted as each firm holding a patent and deciding on the number of sellers who are allowed to use the patent.

Throughout the paper we will assume that  $p(\cdot)$  is a twice continuously differentiable function satisfying the following conditions:

(i) There is a number  $y$  such that for any  $z > y$ ,  $p(z) < c$  (for large outputs, costs are not covered).

(ii) There is a number  $\gamma < 0$  such that for any  $z \leq y$ ,  $dp/dz \equiv p'(z) \leq \gamma$  (the slope of the inverse demand function is negative and bounded away from zero).

(iii) There is a number  $w > 0$  such that  $p(w) > c$  (feasibility of positive production).

Under assumptions (i) and (iii), aggregate output can be restricted to lie in the closed interval  $\mathcal{F} \equiv [0, y]$ .

Let us define  $\beta \equiv \beta(z) \equiv (p''(z)z)/p'(z)$ . We may interpret  $-\beta$  as a measure of the degree of convexity of the inverse demand function. Under assumption (ii),  $\beta(\cdot)$  is continuous. Since  $\beta(\cdot)$  is defined on a closed interval,  $\exists \beta'$  such that  $\beta(z) \geq \beta' \forall z \in \mathcal{F}$ .

We will now investigate the properties of Subgame Perfect Nash Equilibrium in pure strategies (SPNE) of the following game.

**G.1:** *Stage 1:* Every firm decides (simultaneously) its number of divisions.

*Stage 2:* Every division decides (simultaneously) its output.

Notice that the Cournot Equilibrium at Stage 2 is symmetric: Indeed suppose it is not.

Then, there are two divisions, say,  $i$  and  $r$  with different outputs. W.l.o.g. let us assume that division  $i$  is active. Then

$$p'x_i + p - c = 0, \text{ and } p'x_r + p - c \leq 0.$$

If division  $r$  is not active  $p - c \leq 0$  which is impossible since  $p' < 0$  and division  $i$  is active. Therefore, both divisions are active and first-order conditions hold with equality. Then  $p'x_i = p'x_r$ , and thus  $x_i = x_r$ .

The SPNE of Game G.1 satisfies the following:

**Proposition 1.** *If  $-\beta' \leq (k-2)(k+1)/k-1$  Perfect Competition is the only SPNE of the game G.1.*

**Proof.** First-order conditions in the second stage are given by

$$p(z) - c + x_i p'(z) = 0, \quad i = 1, \dots, n \quad (1)$$

where  $n$  is the total number of divisions. Moreover, as noticed before, the Cournot equilibrium in the second stage is symmetric, which implies

$$z = nx_i. \quad (2)$$

Implicit differentiation in Eq. (1) and Eq. (2) gives,

$$z' = \frac{z}{n(\beta + n + 1)} > 0 \quad \text{and} \quad x'_i = \frac{-z(\beta + n)}{n^2(\beta + n + 1)} < 0 \quad (3)$$

where  $z' \equiv dz/dm$  and  $x'_i \equiv dx_i/dm$ , while the inequalities follow from the fact that  $-\beta' \leq (k-2)(k+1)/(k-1)$  implies  $\beta + n > 0$  (for a justification of the use of derivatives with respect to an integer variable see Seade, 1980, p. 482).

In the first stage, total profits earned by firm  $j$  can be written as

$$\Pi_j = (m/n)(p(z) - c)z = \{m/(m+t)\}\{p(z(m+t)) - c\}z(m+t) \quad (4)$$

where  $m$  is the number of divisions created by firm  $j$ , and  $t$  is the number of divisions created by the rest of the firms. The equilibrium value for  $n$  is determined by backward induction: every firm maximizes its total profits given the number of divisions created by its competitors. Let  $\partial \Pi_j / \partial m(t, m)$  be the partial derivative of profits of firm  $j$  with respect to  $m$ . From Eq. (4) we obtain

$$\frac{\partial \Pi_j}{\partial m} = \frac{n-m}{n^2}(p-c)z + \frac{m}{n}(p-c+zp') \frac{dz}{dm}$$

which, after substitution of Eqs. (1), (2) and (3), gives

$$\begin{aligned} \frac{\partial \Pi_j}{\partial m} &= \frac{n-m}{n^2}(p-c)z + \frac{m}{n}(p-c)(1-n) \frac{dz}{dm} \\ &= \frac{(p-c)z\{(\beta+n+1)n - (\beta+2n)m\}}{n^2(\beta+n+1)}. \end{aligned} \quad (5)$$

If the equilibrium is symmetric,  $n = km$  and  $t = (k - 1)m$ . We will show that in this case this derivative is positive, when all other firms have  $(k - 1)m$  divisions and firm  $j$  has  $m + i$  divisions  $\forall i \in [0, 1]$ . This implies that at any symmetric SPNE with finite  $m$ , firm  $j$  has an incentive to deviate.

If  $\partial \Pi_j / \partial m ((k - 1)m, m + i) \leq 0$  then Eq. (5) must be negative or zero with  $m$  substituted by  $m + i$ , so that

$$\beta + n + i + 1 \leq (m + i)\{2 + \beta/(n + i)\}.$$

If the proposition were false and the SPNE symmetric, the previous inequality, the definition of  $\beta'$  and symmetry would imply

$$\{m(k - 2) + (1 - i)\}(km + i)/m(k - 1) \leq -\beta'.$$

Since the left-hand side of this inequality is non-decreasing on  $m$  and  $m \geq 1$ ,

$$\{(k - 2) + (1 - i)\}(k + i)/(k - 1) \leq -\beta'.$$

Again the left-hand side of this inequality is decreasing on  $i$  so

$$\begin{aligned} \{(k - 2) + (1 - 1)\}(k + 1)/(k - 1) &< (k - 1 - i)(k + i)/(k - 1) \leq -\beta' \\ &\leq (k - 2)(k + 1)/(k - 1). \end{aligned}$$

Therefore, we arrive at a contradiction.

If the equilibrium is not symmetric, then there must be a firm, say  $j'$ , with a number of divisions  $m' < m \equiv n/k$ . Defining  $t'$  as the number of divisions of the firms created by firms other than  $j'$ , Eq. (5) implies that

$$\begin{aligned} \frac{\partial \Pi_{j'}}{\partial m'}(t', m' + i) &= z(p - c)\{(\beta + n + i + 1)(n + i) - (\beta + 2(n + i))(m' + i)\} \\ &> \frac{\partial \Pi_{j'}}{\partial m}(t, m + i) = z(p - c)\{\beta + n + i + 1)(n + i) \\ &\quad - (\beta + 2(n + i))(m + i)\} \geq 0 \end{aligned}$$

where the first inequality comes from the fact that  $m' < m$ , while the second comes from a similar argument to the one used in the symmetric case. Therefore, all firms with a number of divisions equal or smaller than  $m = n/k$  want to set up at least one extra division, which shows that there cannot be a SPNE with a finite number of divisions per firm.  $\square$

Notice that the function  $F(k) \equiv (k - 2)(k + 1)/(k - 1)$  is increasing in  $k$  if  $k \geq 2$  and tends to infinity when  $k$  tends to infinity. Thus Proposition 1 says that perfect competition is achieved if  $k$  is large enough in relation to the degree of convexity of the inverse demand function.

The intuition behind Proposition 1 is that the higher  $-\beta'$ , the more competitive the second stage and therefore the lower the incentive to create divisions in the first stage. Thus, a high value of  $-\beta'$  makes it less likely that perfect competition is a SPNE. Notice that Proposition 1 includes, as special cases, Examples 1 and 2 in Corchón (1991).

The following result identifies three alternative sufficient conditions for  $-\beta' \leq (k-2)(k+1)/k-1$  to hold. These conditions are reasonable and suggest that a small number of firms may suffice for perfect competition to occur.

**Proposition 2.** *Any of the following conditions imply  $-\beta' \leq (k-2)(k+1)/k-1$*

- (a)  $p(z)$  concave for all  $z \in \mathcal{F}$ .
- (b)  $p(z)$  having constant elasticity, denoted by  $\lambda$  – and  $1 - \lambda \leq (k-2)(k+1)/k-1$ .
- (c)  $\Pi(z)$  concave for all  $z \in \mathcal{F}$  and  $k > 2$ .

**Proof.** Parts (a) and (b) are obvious. In order to prove (c), notice that concavity of  $\Pi(z)$  implies that  $\beta(z)z \geq -2z, \forall z \in \mathcal{F}$ . Thus,  $\beta' \geq -2$ , and (c) holds since  $F(3) = 2$  and, as we remarked before,  $F(\cdot)$  is increasing.  $\square$

### 3. The model with an entry cost

In this section we will consider the following game:

**G.2: Stage 1:** Every firm decides (simultaneously) on entry. A fixed cost  $\varepsilon > 0$  is paid by a firm if it decides to enter into the market.

**Stage 2:** Every firm decides (simultaneously) on the number of divisions, this number being less than or equal to a given number  $d$ .<sup>2</sup>

**Stage 3:** Every division decides (simultaneously) on its output.

Now markets contain two kind of frictions,  $d$  and  $\varepsilon$ . Both frictions depend on a parameter  $\rho \in [0,1]$  so we can write  $d(\rho)$  and  $\varepsilon(\rho)$ . Notice that if  $\varepsilon=0$ , at any SPNE of G.2 all firms will enter and, for any given positive  $d$ , if  $-\beta' \leq F(k)$ , Proposition 1 implies that each firm will create  $d$  divisions. The outcome of this game will be close to perfect competition for sufficiently large values of  $d$ . If  $d = \infty$ , for any positive  $\varepsilon$  and  $-\beta' \leq F(k)$ , Proposition 1 implies that were all firms to enter, perfect competition will prevail in the last stage and firms will not recoup the entry cost. Therefore in any SPNE, there is an upper bound (which is independent of the fixed cost) on the number of active divisions. Thus, the outcome is a natural oligopoly.<sup>3</sup>

We will assume that  $d'(\rho) < 0$ ,  $\varepsilon'(\rho) > 0$ ,  $d(0) = \infty$  and  $\varepsilon(0) = 0$ . A frictionless market is one for which  $\rho = 0$ , which is just game G.1. The purpose of this section is to study the limit of SPNE in the game G.2 when  $\rho \rightarrow 0$ . In the rest of this section, we will assume that for any value of  $\rho$  there exists a unique Cournot equilibrium. In relation to this Cournot equilibrium, we will define  $\Pi(n)$  as the industry gross profits (i.e aggregate

<sup>2</sup>An alternative assumption, which would produce identical results, is that there is a fixed cost for each division as in Baye, Crocker and Ju (1996).

<sup>3</sup>In this case, contrary to Schwartz and Thompson (1986), an incumbent firm does not need to create more than one division in order to deter entry.

profits before subtracting the fixed costs), as a function of the total number of divisions. Also we will define  $R(\rho, k)$  as follows,

$$R(\rho, k) = \frac{\Pi [kd(\rho)]}{k\varepsilon(\rho)}$$

where  $k$  is the number of firms entering the market. That is,  $R(\rho, k)$  is the ratio (total industry gross profits)/(total industry fixed costs) for a given degree of friction  $\rho$ .

Let us introduce some more notation. Let  $R_\rho \equiv \partial R / \partial \rho$  and  $R_k \equiv \partial R / \partial k$ . We will denote equilibrium values of the variables by an asterisk. We now present the following result:

**Proposition 3.** *If sign  $\{R_\rho\}$  is constant and  $R_k < 0$ , then*

- (a) *If  $\lim_{\rho \rightarrow 0} R(\rho, k) < 1$  for all  $k$  such that  $F(k) > -\beta'$ , the only SPNE of the game G.2 when  $\rho$  is small enough implies that  $F(k^*) \leq -\beta'$  and  $m^* = 1$ .*
- (b) *If  $\lim_{\rho \rightarrow 0} R(\rho, k) > 1$  for some  $k$  such that  $F(k) > -\beta'$ , when  $\rho$  is small enough the only SPNE of the game G.2 implies that  $F(k^*) > -\beta'$ , and  $m^* = d$ .*

**Proof.** By using a similar argument as that in Proposition 1, if  $F(k) > -\beta'$ , then SPNE of G.1 implies  $m^* = d$ . Now, let us consider parts (a) and (b):

- (a) The assumption made in this part and the monotonicity of  $R(\rho, k)$  with respect to  $\rho$ , implies that profits will be negative, for small enough  $\rho$ , if  $F(k) > -\beta'$ . Thus, SPNE of G.2 is inconsistent with  $k^*$  such that  $F(k^*) > -\beta'$ .
- (b) Under our assumptions on  $R(\rho, k)$ , in this case, if  $\rho$  is small enough, profits are positive for some  $k$  such that  $F(k) > -\beta'$ . Also, the monotonicity of  $R(\rho, k)$  with respect to  $k$  and  $\rho$ , ensures that  $k^*$  is such that  $F(k^*) > -\beta'$  for small enough  $\rho$ , and this completes the proof.  $\square$

In contrast to the standard limit theorem with quantity-setter firms the main conclusion of Proposition 3 is that when frictions are removed, i.e. when  $d \rightarrow \infty$  and  $\varepsilon \rightarrow 0$ , the outcome of a SPNE may be either very close to perfect competition or to natural oligopoly. Therefore, the convergence to perfect competition or to a natural oligopoly depends on how frictions vanish, and this also contrasts with the standard Cournot model.

#### 4. Summary and conclusions

In this paper we have shown that if the number of firms is large enough in relation to the degree of convexity of the inverse demand function, the possibility of divisionalization plus Cournot competition implies perfect competition. In many cases, a small number of firms is enough to obtain perfect competition. However, when there is a positive entry cost, the outcome of divisionalization may be a natural oligopoly. This

implies that, in the context of entry deterrence, the *possibility* of divisionalization might not be desirable from the point of view of social welfare. In both cases, in contrast with standard models, the equilibrium number of independent sellers depends on the shape of the demand function and not on the magnitude of fixed costs.

One may wonder about the robustness of our results. They are still valid under uncertain demand or costs or if firms receive payments from divisions which are based on sales (a proof is available under request). The consideration of product heterogeneity produces more complex results: In the case in which the product is homogeneous across divisions of the same firm but heterogeneous across firms, Ziss (1998) has shown that the result obtained in our Proposition 1 does not hold in the case of linear demand. However, if the product is heterogeneous across divisions of the same firm, the equilibrium number of divisions is not finite in at least three cases: In the Salop model (see Gonzalez-Maestre, 1997)<sup>4</sup>, when the inverse demand function is linear and when the inverse demand function is of the form  $x_i^{\alpha-1}/(\sum_{j=1}^n x_j^\alpha)$ , where  $0 < \alpha < 1$  as in Spence (1976) or Dixit and Stiglitz (1997). The latter result follows from the fact that letting  $y_i = x_i^\alpha$ , profits for division  $i$  become  $y_i/(\sum_{j=1}^n y_j) - cy_i^{1/\alpha}$ . Thus, as noticed by Yarrow (1985), the model is identical to a homogeneous product model with decreasing returns and there is an extra incentive to create new divisions (proofs of the last two results are available on request).

Finally, we might ask about the significance of our results. There are two possible interpretations. A positive interpretation is that our results explain the extreme tendency to divisionalize that occurs in some industries (e.g. fast food). Another interpretation of our results is that they might point out a difficulty the Cournot model has in coping with the possibility that firms become divisionalized, since it produces results that are too extreme. Unfortunately, our analysis does not provide any clue about how to build an alternative model of competition.

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<sup>4</sup>This shows that our result does not depend on the assumption of strategic substitution since in the Salop model there is strategic complementarity



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