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On Bidding Markets:The Role of Competition*

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Abstract

This paper analyzes the effects of industrial concentration on bidding behaviour and hence, on the seller´s expected proceeds. These effects are studied under the CIPI model, an affiliated value set-up that nests a variety of valuation and information environments. We formally decompose the revenue effects coming from less competition into four types: a *competition effect*, an *inference effect*, a *winner´s curse effect* and a *sampling effect*. The properties of these effects are discussed and conditions for (non) monotonicity of both the equilibrium bid and revenue are stated. Our results suggest that it is more likely that the seller benefits from less competition in markets with more complete valuation and information structures.

Keywords: auctions, competition, affiliation, inference
JEL Classification: C62, D44, D82, L41

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1 Introduction

The typical concern about any illegal collusion practice (cartels) or legal collusion arrangement (mergers or consortia) is that these practices reduce the number of participants in the market and hence, lessen competition, negatively affecting both the price and the bid-taker's revenue. Nevertheless, in the context of auctions and bidding markets, this conventional wisdom applies only to the case of independent private value settings, as it has been modelled theoretically and empirically supported by abundant literature.[1] The simplicity of the valuation and information environments analyzed by this literature makes collusion practices negatively affect the intensity of competition, what has been called the *competition effect*.[2]

However, under a common value and/or affiliated signals model, the higher concentration provoked by joint bidding leads to other effects that may counteract the competition effect, and induce a more aggressive bidding behavior. These effects can be grouped into three classes: a *winner's curse effect*, an *inference effect* and an *information pooling effect*. First, the reduction in the number of bidders in a common value environment permits alleviation of the winner's curse, because now defeating fewer bidders makes the ex post overoptimism less likely. This implies that a higher industrial concentration increases the expected value of the item conditional on winning the auction, and in consequence, bidders are less conservative.[3] The inference effect may arise from some affiliated information structures, and can be present in both private and common value environments.[4] In this case, the reduction in the number of participants may increase the aggressiveness of the bidding behavior. The reason for this is that, although winning

[1] For theoretical works on (legal) joint bidding under the independent private value setting, see Waehrer [39], Waehrer and Perry [40], Froeb, Tschantz and Crooke [9], [37], and [10], and Dalkir, Logan and Masson [6]. Theoretical analysis on bidding rings with private values are provided by Robinson [35], Mailath and Zemsky [21], McAfee and McMillan [25], Marshall et al. [22], and Pesendorfer [29]. Finally, most empirical literature on illegal collusion derives its estimation models from a theoretical set-up with private values as well. Some papers along these lines are Hewitt, McClave and Sibley [14], Porter and Zona [33], Pesendorfer [29], Lanzillotti [19], Scott [36], Porter and Zona [32], Bajari and Ye [2], and Baldwin, Marshall and Richard [3].

[2] Given some properties of bidding rings (efficiency and the possibility of side payments), illegal collusion and mergers have the same anticompetitive effects on auction markets if values are private (see McAfee [24]).

[3] Theoretical approaches that characterize the winner's curse effect include Bulow and Klemperer [4] and Hendricks, Pinkse and Porter [11]. On the other hand, a number of recent papers provide empirical evidence of this effect in several auction markets such as Hong and Shum [15], Hendricks, Pinkse and Porter [11], and Athias and Nuñez [1].

[4] The previous literature refers to this effect as the *affiliation effect*; see Pinkse and Tan [31], Hong and Shum [15], and Hendricks, Pinkse and Porter [11].

is interpreted as information that the intensity of the competition is lower than before the auction starts, this perception is weakened when the winner faces fewer rivals. Finally, the information pooling effect improves the precision of the bidder's value estimate because a coalition of bidders can observe either a new signal or a larger amount of signals with better stochastic properties than an individual bidder. This effect also allows the winner's curse correction on bids to mitigate, leading to more aggressive bidding behavior.[5]

Therefore, all these effects go in the same direction and encourage more aggressive bids when an auction market becomes more concentrated because of mergers or other joint bidding arrangements. As these effects dominate the competition effect plus the statistic effect produced by the overall reduction in the number of participants (a *sampling effect*), the possibility for increasing the bid-taker's expected revenue remains open. As a result, the standard viewpoint that less competition is always undesirable can clearly become challenged.[6]

All of this underlines the importance of analyzing, in a valuation and information setting which is as complete as possible, the effects of (legal) joint bidding practices. As a starting point for this general objective, this paper studies the effects of a change in the number of bidders on both the equilibrium bid strategy and the seller's proceeds.[7] Consequently, we abstract away from any information pooling type effect. This implies that one can infer the other effects from the hypothetical exercise in which bidders merger but the acquired bidder's information is not used by the acquiring one. We then make this exercise equivalent, from a methodological point of view, to the case in which the number of bidders decrease because some of them do not attend some particular auction or because they leave the industry.

From the previous literature, a good point of departure for our analysis is provided by Pinkse and Tan [31], who examine conditions under which the equilibrium bid is monotonic increasing with respect to the number of bidders n in affiliated private-value models of first-price auctions. In particular, they show the existence of a large class of such models in which the equilibrium bid function is indeed *not* strictly increasing in n. Furthermore, they propose a decomposition of the

[5] See DeBrock and Smith [7], Hendricks and Porter [12], Krishna and Morgan [18]. Mares and Shor [23] show that indeed this information pooling effect works unambiguously for second-price auctions, but for first-price auctions it induces more aggressive bids only for signals that are sufficiently *low*.

[6] In addition, it has been argued that joint bidding has other pros such as facilitating entry of wealth-constrained bidders and improving risk diversification (see DeBrock and Smith [7]).

[7] We do not examine the welfare effects of competition. For an analysis of such issues, see, for instance, Compte and Jehiel [5].

bidding effects into two parts: a competition effect and an affiliation effect. This latter effect is precisely the source of the surprising finding of Pinkse and Tan in a private value environment, and it can also be present in a common value set-up. They illustrate their results with the conditionally independent private value (CIPV) model, a special case of the affiliated private value (APV) model in which bidders' valuations are affiliated through a common random component, but they are independently distributed given a realization of this common component. In this environment , the winner never regrets its winning so that the winner curse effect has no bite.

Accordingly, it is clear that in order to also examine the winner's curse effect, we need to consider a more general framework than that provided by the APV model - and in particular by the CIPV model -, as this effect cannot emerge from the valuation structure characterized by these settings. One way to do this is by means of the conditionally independent private information (CIPI) model, a special class of the general affiliated value (AV) model which encompasses both the CIPV and the pure common value setups as polar cases. In the CIPI model, the bidders' signals (private information) are affiliated through a common variable (which can also be the ex post common value of the object), but they are independently distributed conditional on a realization of this common variable. As a consequence, this framework provides an environment rich enough to evaluate all the revenue effects.

We group the effects on revenue coming from more competition into two classes: (i) those that affect *bidding behavior* and (ii) a pure *sampling effect*. On the one hand, changes in the number of buyers influence the equilibrium bid. As discussed above, in environments with interdependent valuations and dependent information, bidding behavior can become more or less aggressive with more competition. The final sign of these influences on bids, as well as on revenues, is therefore ambiguous, and depends on the relative magnitudes of the bidding-based effects considered. A more in-depth characterization of these bidding effects can then become worthwhile for a seller interested in adopting revenue-enhancing instruments in the face of mergers or any joint bidding practice. Consequently, we propose a decomposition of this bidding effect that allows us to isolate and formally evaluate the *winner's curse*, the *competition* and the *inference* effects. The properties of all these effects are established, and conditions for the (non)monotonicity of the equilibrium bid are stated.

On the other hand, the sampling effect reflects the upward impact on the seller's proceeds due to the fact that more competition implies a winning signal's distribution with better stochastic properties. We then combine both the bidding

effect and the sampling effect, providing conditions for the (non)monotonicity of revenues. In particular, the paper shows that the seller's expected proceeds can be decreasing in the number of buyers as a negative and sufficiently large bidding effect dominates the sampling effect. The main implication is that in the CIPI model, in contrast to the CIPV setting, the conditions that allow the seller to benefit from less competition are less stringent. The rationale of this finding is the presence of the winner's curse effect, absent in affiliated private value environments. In fact, as the winner's curse constitutes an additional force for bids decreasing in the number of buyers, it makes the conditions for nonmontonic revenue to hold more likely. In a broader sense, this work highlights therefore the role played by the valuation and information structure assumed to be satisfied in a particular auction-based market. Accordingly, our results suggest that, by analyzing a more complete auction environment, the traditional idea that more concentration is always undesirable may no longer hold.

Our model accounts for existing empirical evidence that, in auction markets in which the winner's curse seems to be particularly strong, the bid-taker may be better off when the number of bidders decreases. For instance, DeBrock and Smith [7] study offshore oil lease auctions under a framework with values and signals that are log-normally distributed. They show that the joint bidding increases the total social value of the lease offering and, in some cases, *increases* the fraction of this value appropriated by the seller (the government). Similarly, Hong and Shum [15] construct a model of a low-bid procurement auction with common value and affiliated signals. The bidder's cost of completing a project is given by a log-additive formulation that includes both a private (or idiosyncratic) and a common cost component, which are independently log-normal distributed. They find that, for a large subset of construction procurement auction contracts, the median cost *rises* as the number of participants increases.

It is noteworthy that while the evidence presented by these works is derived starting from a framework that assumes specific functional forms for valuations and/or distributions, our model yields these predictions without such restrictive assumptions. What is even more interesting it is likely that the available evidence *against* the nonmonotonicity of revenue with the number of bidders is based largely on these specific assumptions as well. For instance, Mares and Shor [23] develop a model with pure common value and independent signals, where the value of the item is the average of all bidders' signals. Their findings, corroborated by experimental exercises, suggest that the seller's expected revenue decreases with less competition mainly because of the sampling effect.[8] Nevertheless, since their

[8] Since their model assumes independence and symmetry, the revenue equivalence theorem

valuation structure depends precisely upon the number of participants, some of the effects described could be absent if other valuation functions were assumed.

The results of this paper have a scope of applicability that goes beyond a mere academic interest, as they concern antitrust issues which are currently widely discussed. In a recent policy-oriented article, Klemperer [16] analyzes the characteristics that the competition policy on bidding markets should possess. His general conclusion is that, although the markets organized as auctions do have some special features such as common values behavior, a tendency to overemphasize the importance of these features has erroneously lead to positions in favor of a more lenient antitrust policy. In what concerns the role played by the winner's curse, the arguments provided by Klemperer rest on two examples under the pure common value environment in which less competition would unambiguously hurt the seller.[9] However, similar to Mares and Shor's results, the conclusion of Klemperer may strongly depend on the particular valuation structures considered in his examples. In contrast, our main insight, derived without such specific assumptions, suggests the need for an antitrust policy that scrutinizes mergers more carefully or other joint bidding arrangements in bidding markets in which more sophisticated valuation and information environments are present.

This paper is organized as follows. Section 2 summarizes the CIPI model, noting how the CIPV and the pure common value models can be derived from this as a special cases. Section 3 studies the relationship between competition and bidding behavior in a first-price auction under the CIPI setting. As a consequence, we provide conditions for the (non)monotonicity of the equilibrium bid strategy and propose a new three-part decomposition of the bidding-type effects. In Section 4, we examine the conditions that guarantee the (non)monotonicity of the seller's revenue with respect to n. Finally, Section 5 concludes. All the proofs are collected in the Appendix.

2 The CIPI model

Consider a seller who wants to auction off a single object among n bidders, using a first-price auction with a possible reserve price $r \geq 0$. Each bidder observes a signal $x_i \in [\underline{x}, \overline{x}]$, $\underline{x} > 0$, which is private information to him. Bidder i's utility

implies that this result holds for both first-price and second-price auctions.

[9] These examples are the *wallet game* (in which valuation corresponds to the sum of all bidders' signals) and the *maximum game* (in which valuation is the maximum among all bidder's signals).

(valuation) is represented by the function $U(v, x_i)$, where $v \in [\underline{v}, \overline{v}]$, $\underline{v} > 0$, denotes an unknown random variable common to all bidders with c.d.f. F_v and p.d.f. f_v. Let $\mathbf{z} = (x_1, ...x_n, v)$ be a random vector distributed according to the c.d.f. F and the p.d.f. f, with F *affiliated* and symmetric in its first n arguments. All players are risk-neutral.

Whenever the signals x_i's are affiliated through the common random component v, but they are independently and identically distributed given a realization of this common random variable, such a model belongs to the conditionally independent private information family (CIPI, for short). As a consequence, the signals $x_i|v$ are i.i.d. according to the c.d.f. $F_{x|v}(t|s) = \Pr(x_i \leq t|v = s)$ and the p.d.f. $f_{x|v}$ with support $[\underline{x}, \overline{x}]$, $\underline{x} > 0$. Notice that since this statistical structure requires x_i and v to be affiliated, we adopt the equivalent assumption that $F_{x|v}$ satisfies the (strict) MLRP. [10] The CIPI model can then be interpreted as a special case of the more general affiliated value model (AV, for short) described above, as it can be verified that the joint distribution F satisfies affiliation and symmetry in its first n arguments from the following expression:

$$\begin{aligned} f(x_1, ...x_n, v) &= f_v(v) f(x_1, ...x_n|v) \\ &= f_v(v) \prod_{i=1}^{n} f_{x|v}(x_i|v) \end{aligned}$$

Imposing particular functional forms on bidder's valuations (utilities), two polar cases can be derived from the CIPI model.

The CIPV model. Consider the case in which bidder i's utility (valuation) is given by the function $U(v, x_i) = x_i$. Since the valuation to each bidder is given entirely by his own information, we are in the private value setting as each bidder fully knows his valuation ex ante. The only remaining uncertainty is hence about the other bidders' valuations. In particular, since now each bidder's value is equal to his signal, the model corresponds to the conditionally independent private value (CIPV, hereafter) model.[11] An economic interpretation of this model is as follows. While the random variable v is interpreted as the ex post value that the *average* bidder assigns to the object for sale, the difference between each bidder's valuation and this average value, i.e., $(x_i - v)$, represents a bidder's *specific* characteristic

[10] Assuming that the p.d.f. of the signals conditional on v, $f(x_1, ...x_n|v)$, is twice continuously differentiable, affiliation among the signals is equivalent to the following two conditions: (i) $\frac{\partial^2 \log f(x_1,...x_n|v)}{\partial x_i \partial x_j} \geq 0$, and (ii) $\frac{\partial^2 \log f(x_1,...x_n|v)}{\partial x_i \partial v} \geq 0$, for all i, j [see Topkis [38], p. 310]. As de Castro [8] discusses, the conditional independence models only guarantee the first condition. To obtain the second condition, one must assume explicitely that x_i and v are affiliated.

[11] This is the one studied by Pinkse and Tan [31].

such as productive efficiency, opportunity cost or idiosyncratic preference.[12] Note that the CIPV model is a special case of the affiliated private value setting, and also a polar case of the CIPI model.

The CIPI-CV model. Consider now the case in which bidder i's utility (valuation) is given by the function $U(v, x_i) = v$. Since all bidders share the same ex post valuation, and only observe an estimate of this value, we are in the pure common value setting. In consequence, no preference heterogeneity is considered. A traditional economic interpretation of this setting is the so called *mineral rights model*. All bidders exhibit the same ex post value for a tract given by v, derived from its exact mineral content. Nevertheless, at the time of the auction, they only observe a noisy signal of this content, x_i. We will refer to this polar case of the CIPI family as the CIPI-CV model. Finally, notice that this pure common value setting also constitutes a special case of the general affiliated value model.

Model's Choice. As we shall see in the next section, the impact of concentration on bidding behavior can be decomposed into three effects: the *competition effect*, the *inference effect* and *the winner's curse effect*. While the first effect comes from the competitive environment involved in an auction mechanism no matter the valuation and information structure, the last two effects arise in environments with common value and dependence among signals, respectively.

This suggests that a good starting point for our analysis is provided by Pinkse and Tan [31]. They examine conditions under which the equilibrium bid is strictly increasing with respect to the number of bidders in first-price auctions under the CIPV model. From this, a first matter of interest concerning the model's choice is related to the valuation and information structure to be studied. It is clear that in order to examine the winner's curse effect as well, we need to widen our analysis to a more general setting than that provided by affiliated private value environments, and in particular by the CIPV model. We argue that the natural candidate which could have bite is the CIPI setting. As discussed above, this family of models is a special class of the general affiliated value model that encompasses the CIPV and the pure common value (CIPI-CV) setups as polar cases.[13] It is noteworthy that for our purpose, it suffices to focus only on the CIPI-CV case since it constitutes the simplest setting with an environment that is sufficiently rich to evaluate all the effects aforementioned.[14]

[12] This interpreation is taken from Li et al [20].

[13] The CIPI model was first studied by Li et al. [20], who tested their results in OCS wildcat auctions.

[14] The results derived in this paper can be particularly relevant for wildcat lease auctions. For instance, Hendricks, Pinkse and Porter [11] provide evidence that the bidding behavior for oil and gas auctions is consistent with a first-price auction under a symmetric pure common value

A second choice concerning our modelling strategy is given by the auction format to be examined. The bidding trade-off present in the first-price auction implies that the competition effect is more severe in this mechanism than in the second-price auction. Furthermore, as long as we assume any kind of dependence among the signals, the Revenue Equivalence Theorem no longer holds. As the classical linkage principle stated by Milgrom and Weber [26] points out, in such an environment the second-price outperforms the first-price auction. All of this suggests an important reason for preferring the latter format to study the effects of concentration in bidding markets: by analyzing the first-price auction, one does indeed consider the worst scenario for the seller. Hence, if we are able to show that under this mechanism concentration may increase revenues, we can directly extend this conclusion to the second-price auction.[15]

3 Competition and bidding

In this section, we study the relationship between competition and bidding behavior in a first-price auction under the CIPI-CV model. We then provide conditions for (non)monotonicity of the equilibrium bids with respect to the number of buyers and propose a three-effect decomposition of the impact of concentration on bidding.

3.1 (Non)monotonicity of the equilibrium bid

Since our main purpose is to analyze the role played by the number of bidders, in what follows we adopt the (uncommon) notation according to which some functions of the model (bids, distributions, reverse hazard) depend on two arguments: x and n.

Define $y_{1:n-1} = \max_{j=1,..n,\ j\neq i} x_j$, the first-order statistic of all bidders' signals except bidder i's, and denote its c.d.f. and p.d.f. conditional on $x_i = x$ by $F_{y|x}(.|x)$ and $f_{y|x}(.|x)$, respectively. Let $\lambda(x;n) = f_{y|x}(x|x)/F_{y|x}(x|x)$ be its associated reverse hazard rate when the signals of the $(n-1)$ bidder i 's rivals are smaller

environment with conditionally independent private signals, i.e., the CIPI-CV model.

[15] Moreover, by choosing the first-price sealed-bid auction, the conclusions of our work concern an auction format that is more frequently used than the second-price auction in the real world, as stated in Paarsch and Hong [28] (p. 22). In addition, it is likely that first-price sealed-bid auctions account for the bulk of transaction by value since procurements are often conducted via *low-price, sealed-bid tenders.*

than or equal to x, given that its signal realization is x.[16]

We also assume that the seller can set a reserve price $r \geq 0$. Under a symmetric equilibrium, the expected payoff to bidder i when he observes $x_i = x$ and bids b in a first-price auction is then given by

$$\begin{aligned} \pi(b,x) &= E\left[(v-b)1_{\{\max\{B(y_{1:n-1};n),r\}\leq b\}}|x_i = x\right] \\ &= \int_{\underline{x}}^{B^{-1}(b;n)} \left[v(x,s;n) - b\right] f_{y|x}(s|x)ds \end{aligned}$$

where $B(.;n)$ is the equilibrium bidding strategy followed by all bidders except i when facing n rivals and $v(x,y;n) = E(v|x_i = x, y_{1:n-1} = y)$. If B forms part of a symmetric equilibrium, then it must satisfy the following first-order differential equation

$$B_x(x;n) = \left[v(x,x;n) - B(x;n)\right]\lambda(x;n) \tag{1}$$

where $B_x(x;n)$ denotes $\partial B(x;n)/\partial x$;[17] and the appropriate boundary condition given by $B(a;n) = r$, where $a = a(r;n)$ is defined as follows

$$a = \{\inf x | E(v|x_i = x, y_{1:n-1} \leq x) \geq r\}$$

Solving the differential equation, bidder i's equilibrium strategy is given by

$$B(x;n) = rL(a|x) + \int_a^x v(s,s;n)dL(s|x)ds \tag{2}$$

for all $x \in [a, \overline{x}]$, where $L(s|x) = \exp(-\int_s^x \lambda(u;n)du)$.

Pinkse and Tan have shown that, in the CIPV model, if the reverse hazard rate is increasing in n then bids are strictly increasing in the number of buyers. Nevertheless, as the following example illustrates, in the CIPI model properties for the reverse hazard rate no longer suffice for such bid monotonicity.

Example 1 (from Wilson [41]). Consider the pure common value model $U(v, x_i) = v$ for all i. Suppose that v is distributed according to the Pareto distribution such that $F_v(v) = 1 - v^{-\alpha}$ for $v \geq 1$ and $\alpha > 2$. Suppose also that the signals x_i's are i.i.d conditional on v, so that $F_{x|v}(x|v) = (x/v)^{\beta}$ for $0 \leq x \leq v$.

It can be verified that $\lambda(x;n) = (n-1)\beta/x$, and that the equilibrium bid is given by

$$B(x;n) = \left[\frac{(n-1)\beta + \max\{x,1\}^{-(n-1)\beta-1}}{(n-1)\beta+1}\right] v(x,x;n) \tag{3}$$

[16] In other words, $\lambda(x;n)$ corresponds to the reverse hazard rate of the second-order statistic conditional on $x_i = x$ being the first-order stastistic.

[17] For the functions B and v, we use the subscripts x and n throughout the paper to denote their partial derivatives w.r.t. these variables.

Notice that $B(x;n)$ is *not* strictly increasing in n. Figure 1 (see Appendix C) displays the case in which $\alpha = 2.5$ and $\beta = .5$, showing that the equilibrium bidding function is indeed *decreasing* for signals that are sufficiently low when the number of bidders increases from $n = 2$ to $n = 3$. Interestingly, this example shows therefore that a nonmonotonicity of bids can be observed even though the reverse hazard is strictly increasing in n, as $\partial\lambda(x;n)/\partial n = \beta/x > 0$.

We can then conclude that the presence of an additional winner's curse-based effect in the CIPI setting requires more demanding conditions to guarantee the monotonicity of bids in the number of buyers. Equivalently, this also means that the set of conditions under which bids decreasing in n can be observed becomes richer.

We begin characterizing a condition that ensures monotonic equilibrium bids with respect to the number of buyers.

Proposition 3.1 *Let $\overline{b} = \max_n B(\overline{x};n)$. Suppose that for all $x \in (\underline{x}, \overline{x})$,*

$$\frac{v(x,x;n+1) - \overline{b}}{v(x,x;n) - \overline{b}} > \frac{\lambda(x;n)}{\lambda(x;n+1)}$$

Then for all $r < \overline{b}$ and $x \in (a, \overline{x})$, $B(x;n)$ is strictly increasing in n.

A possible interpretation for this result is as follows. Since $\overline{b}$ constitutes the maximum possible bid to be made in the game, let us define $\underline{\pi}(x;n) \equiv v(x,x;n) - \overline{b}$, the *minimum benefits* that a bidder with signal x can get conditional on defeating $(n-1)$ rivals. Thus, $\underline{\pi}$ can be seen as a lower bound of the winning bidder's benefits in the hypothetical case in which he were forced to participate in an auction with a reserve price equal to $\overline{b}$.[18] As is stated in the next section, while the winner's curse effect ensures that $\underline{\pi}(x;n)$ is decreasing in n, the mixed effect coming from more competition and better inference on the degree of this competition may cause $\lambda(x;n)$ to be increasing in n. Consequently, Proposition 3.1 establishes that in the CIPI model, bids will be strictly increasing in the number of buyers as long as the negative effects on the minimum benefits stemming from the winner's curse be overcome (proportionally) by the (possible) positive competition-driven effects.

Since in the CIPV setting we have $v(x,x;n) = x$ for all n, the next result follows directly from Proposition 3.1.

Corollary 3.1 *In the CIPV model, a reverse hazard function strictly increasing in n suffices for Proposition 3.1.*

[18] Of course, $\underline{\pi}$ can be negative.

Therefore, in contrast to the CIPV model, in the CIPI setting the fact that the reverse hazard is strictly increasing in n constitutes only a necessary condition, but *not* a sufficient condition for the equilibrium bid to be strictly increasing as well.[19] The intuition behind this result is the presence of the winner's curse effect in the CIPI model, absent in affiliated private value frameworks such as the CIPV setting. As a consequence, more restrictive conditions are needed for guaranteeing the monotonicity of bidding behavior in environments with interdependent values.

In order to establish conditions for the nonmonotonicity of the equilibrium bid, notice that our assumption of strict MLRP for $F_{x|v}$ guarantees that $\partial v(x,x;n)/\partial n \leq 0$ for all $x \in [\underline{x}, \overline{x}]$ (see Milgrom [27]). This property allows us to characterize the sufficient conditions for bids to be decreasing when signals are sufficiently low as follows.

Proposition 3.2 *Consider the two following situations:*[20]
(1) *Suppose that for some values of n and r, it is verified either (A1) or (A2) with:*

$$\lambda(a(n+1);n+1) < \lambda(a(n+1);n) \qquad (A1)$$

$$\frac{v(a(n+1),a(n+1);n+1)-r}{v(a(n+1),a(n+1);n)-r} < \frac{\lambda(a(n+1);n)}{\lambda(a(n+1);n+1)}. \qquad (A2)$$

Then, $B(x;n+1) < B(x;n)$ must hold for some $x > a(n+1) \geq a(n)$.
(2) *Suppose that there is no reserve price and, for some value of n, it is verified (A3) with:*

$$v(\underline{x},\underline{x};n) > v(\underline{x},\underline{x};n+1). \qquad (A3)$$

Then, $B(x;n+1) < B(x;n)$ must hold for some $x > \underline{x}$.

Condition (A1) emphasizes that the existence of a winner's curse-based effect in the CIPI model means that a reverse hazard decreasing in n for signals that are low enough suffices for the nonmonotonicity of bids. As in the affiliated private value settings the winner's curse phenomenon is absent, the same condition on the reverse hazard also ensures the nonmonotonicity of bids in the CIPV model studied by Pinkse and Tan [31].

At the same time however, the additional presence of the winner's curse effect in the CIPI setting implies that other sufficient conditions for such a nonmonotonicity

[19] This is because such a condition guarantees that $\lambda(x;n)/\lambda(x;n+1) < 1$, which is also satisfied by the ratio $(v(x,x;n+1)-\overline{b})/(v(x,x;n)-\overline{b})$.

[20] Recall that a depends on two arguments so that $a = a(r;n)$. For the sake of presentation, we have omitted r.

can be stated even when the reverse hazard is strictly increasing. One of these conditions is characterized in Proposition 3.2 by (A2), which constitutes a sort of reverse of Proposition 3.1. An interpretation for this condition can be provided following a similar line of reasoning as before. Accordingly, let us define $\overline{\pi}(a(n+1); n+1) \equiv v(a(n+1), a(n+1); n+1) - r$, the *maximum benefits of the marginal bidder* (the one indifferent between participating or not) conditional on defeating n rivals. Then, $\overline{\pi}$ can be thought of as an upper bound of the winning marginal bidder's benefits when participating in an auction with a reserve price r.[21] Note that whereas a decrease in the number of bidders exerts an upward influence on $\overline{\pi}$ due to a reduced winner's curse, it may also induce an downward effect on the reverse hazard. As a result, condition (A2) states that if the first effect dominates (proportionally) the second one for the marginal bidder, then, at least for signals that are sufficiently low, *less* competition will bring *more* aggressive bids.

Furthermore, when there is no reserve price, condition (A3) guarantees the nonmonotonicity of the equilibrium bid irrespective of the properties exhibited by the reverse hazard. Such a sufficient condition is that of v being strictly decreasing in n for the lowest type. Note that Example 1 satisfies this condition, as can be verified that $v(x, x; n) = \max\{x, 1\}(\alpha + n\beta)/(\alpha + n\beta - 1)$ (see details in the Appendix). Hence, we have that $v(x, x; n) = (\alpha + n\beta)/(\alpha + n\beta - 1) > (\alpha + \beta(n+1)) / (\alpha + \beta(n+1) - 1) = v(x, x; n+1)$ for all $0 \leq x \leq 1$ and for all n. Thus, condition (A3) holds and thereby, the nonmonotonicity of the equilibrium bid with respect to n follows.[22]

3.2 The bidding effect: A multiplicative decomposition

The previous subsection characterized the circumstances under which the participation of one more bidder can increase or decrease the bid aggressiveness. The ambiguity of this relationship highlights the importance of studying the sources of this bidding effect. In fact, identifying what forces affect positively or negatively the bidding behavior would allow the seller to improve her decisions on auction formats. Accordingly, in this subsection we propose a decomposition of the bidding effect into three effects, a decomposition that we have named *multiplicative* decomposition.[23]

[21] Notice that $\overline{\pi}$ can be strictly positive as $v(a(n), a(n); n) = E(v|x_i = a(n), y_{1:n-1} = a(n)) \geq E(v|x_i = a(n), y_{1:n-1} \leq a(n)) \geq r$.

[22] See Figure 1 in Appendix C.

[23] This decomposition is refered to as *multiplicative* as an alternative to the *additive* version performed by Pinkse and Tan [31] in the context of the CIPV model. We argue that our decomposition, as opposed to that of Pinkse and Tan, works even when the MLRP assumption

For simplicity, we assume throughout this subsection that there is no reserve price. As a result, the equilibrium bid becomes

$$B(x;n) = v(x,x;n) - \int_{\underline{x}}^{x} v_x(s,s;n)L(s|x)ds \tag{4}$$

Taking derivative on (4) w.r.t. n, we get that

$$B_n(x;n) = \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] - \int_{\underline{x}}^{x} L_n(s|x)v_x(s,s;n)ds \tag{5}$$

where $v_{xn}(x,x;n) = \partial v_x(x,x;n)/\partial n$ and $L_n(s|x) = \partial L(s|x)/\partial n$.

Let $\mathcal{W}$ be the event in which bidder i wins the auction, i.e., $\mathcal{W} \equiv \{x > \max_{j\neq i} x_j\}$. Hence, denote $\rho(v|\mathcal{W},x)$ as the posterior density function of v conditional on a bidder of type x winning the auction.

Then, the reverse hazard can also be written as[24]

$$\lambda(x;n) = \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho(v|\mathcal{W},x)dv \tag{6}$$

where $\lambda(x;n,v) \equiv (n-1)f_{x|v}(x|v)/F_{x|v}(x|v)$ corresponds to the reverse hazard associated to the situation in which $(n-1)$ rivals of a bidder i of type x draw their signals *independently* from the c.d.f. $F_{x|v}(x|v)$. The reverse hazard $\lambda(x;n)$ can thus be written as an average of $\lambda(x;n,v)$ in which the posterior density $\rho(v|\mathcal{W},x)$ are the weights.[25] Then, taking derivative on (6) w.r.t. n, under the assumption that the product inside the integral is twice continuously differentiable, we obtain that

$$\lambda_n(x;n) = \int_{\underline{v}}^{\overline{v}} \lambda_n(x;n,v)\rho(v|\mathcal{W},x)dv + \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho_n(v|\mathcal{W},x)dv \tag{7}$$

only holds weakly (see Appendix B for an example).

[24] See Pinke and Tan [31].

[25] We derive the name the *multiplicative* decomposition proposed in this subsection from the *product* $\lambda(x;n,v)\rho(v|\mathcal{W},x)$.

Substituting (7) into (5), we get the following decomposition

$$\begin{aligned} B_n(x;n) &= \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + \\ & \int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)\left(\int_s^x \int_{\underline{v}}^{\overline{v}} \lambda_n(x;n,v)\rho(v|\mathcal{W},x)dvdu\right)ds + \\ & \int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)\left(\int_s^x \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho_n(v|\mathcal{W},x)dvdu\right)ds \quad (8) \end{aligned}$$

The change in the equilibrium bid strategy due to changes in the number of bidders can then be written as a sum of three components. The first term of the R.H.S. of equation (8) represents the effect coming from the winner's curse phenomenon associated with the common value environment. In fact, as long as we are in the private value setting - the CIPV model, for instance -, the fact that $v(x,x;n) = x$ implies that this effect disappears. Consequently, we refer to this effect as the *winner's curse effect* (WCE).[26]

The second term depends on $\lambda_n(x;n,v)$. Note that by the definition of $\lambda(x;n,v)$, its derivative w.r.t. n is related to how $B(x;n)$ changes with n in a setting with *independence* between the signals. Since under this environment one can associate any change of this class *only* to the traditional bidding trade-off existing in a first-price auction mechanism, this effect corresponds to the so-called *competition effect* (CE).

Finally, the third term depends on the partial derivative $\rho_n(v|\mathcal{W},x)$, which under affiliated information structures is negative (positive) for a large (small) enough v, and for a given x and n.[27] In consequence, this term allows an inverse relationship between the bids and the number of buyers based on an inference-type effect generated by a positive dependency among the signals. Because of this, we will refer to this effect as the *inference effect* (IE).

In sum, we have identified three effects on the equilibrium bid strategy coming from changes in n : the winner's curse effect (WCE), the competition effect (CE) and the inference effect (IE).[28] As we are interested in the nature of these effects, the next proposition formally states their signs.

[26] Krishna and Morgan [18], and Mares and Shor [23] study a similar winner's curse-based effect in the context of consortia, but they call it *inference effect* and *competition effect*, respectively. Notice that we use these terms to name two other effects of a different nature.

[27] See Proof of Proposition 3.3 in the Appendix.

[28] Notice that this decomposition nests indeed a variety of affiliated value models within the CIPI set-up, with the CIPV and the CIPI-CV models as polar cases.

Proposition 3.3 *Suppose that in a CIPI-CV model, it is verified that (A1) $F_{x|v}$ satisfies the MLRP, and (A2) $|v_n(x,x;n)| \geq |v_n(\underline{x},\underline{x};n)|$ for all $x > \underline{x}$. Then, using the Multiplicative Decomposition, for all $x \in (\underline{x},\overline{x})$ and $n \geq 2$, it is verified that:*
(i) The winner's curse effect (WCE) is negative
(ii) The competition effect (CE) is positive
(iii) The inference effect (IE) is ambiguous.

The intuition behind the signs of these effects is as follows. First, the CE comes from the *less* aggressiveness that bidders exhibit when the chances of winning increase because the number of rivals decreases. Thus, this effect can be associated to the traditional negative consequences attributed to the industrial concentration in ordinary markets. Second, more concentration allows the winner's curse to be mitigated because defeating fewer bidders reduces the probability of such an overbidding phenomenon. As a consequence, bidders carry out a lower winner's curse downward correction in bids, and thereby, the WCE takes a negative sign. Finally, the IE stems from the affiliation among signals (or valuations).[29] The affiliation may also cause that a larger concentration results in a lower conservatism in bids. This may occur because, although winning is interpreted by the winner as information of a less degree of competition, this perception is weakened when he faces fewer rivals.[30]

So, whereas the inference effect can exacerbate the negative influence of the winner's curse adjustment, the competition effect always goes in the opposite direction. In consequence, as long as the combination of the first two effects dominate the latter, the equilibrium bid may be decreasing in n as established in the previous subsection.

The signs of these effects are verified for Example 1. For instance, with $\alpha = 2.5$, $\beta = 0.5$, $n = 2$ and $x = 1.4$, the Multiplicative Decomposition yields

Effect	*Magnitude*
WCE	$-8.2408 \times 10^{-2} < 0$
CE	$4.0895 \times 10^{-2} > 0$
IE	$-1.4172 \times 10^{-7} < 0$

[29] Pinkse and Tan [31] also examine an inference-type effect that they call the *affiliation effect*.
[30] A more detailed analysis of the inference effect is provided in the next subsection.

with a final effect given by

$$\begin{aligned} B_n(1.4;2) &= WCE + CE + AE \\ &= -0.041513 < 0 \end{aligned}$$

Note that in this example, although a negative inference exists, its magnitude is smaller than the positive one coming from the competition effect. The winner's curse effect is therefore crucial for the equilibrium bid to be a non-monotonic function in n for signals that are sufficiently low.

3.3 The inference effect: An illustrative example

In order to obtain a better intuition of the inference effect, let us analyze, in the context of Example 1, the source of the non-monotonicity of bids coming from the affiliated structure of the signals.

Similar to $\rho(v|\mathcal{W},x)$, define $p(v|x)$ as the posterior density function of v conditional on a bidder of type x. From Bayes' Theorem, it is easy to check that

$$p(v|x) = \frac{f_{x|v}(x|v)f_v(v)}{\int_{\underline{v}}^{\overline{v}} f_{x|v}(x|v)f_v(v)dv}$$

In Example 1, these two posterior density functions are given by $\rho(v|\mathcal{W},x) = (\alpha+n\beta)\max\{1,x\}^{n\beta+\alpha}/v^{\alpha+n\beta+1}$ and $p(v|x) = 1/(\alpha+\beta)$. We can therefore state that for a given x and n, the two following properties hold:

$$\rho(v|\mathcal{W},x) > p(v|x) \tag{9}$$

$$\rho_n(v|\mathcal{W},x) > 0 \tag{10}$$

for a small enough v, and the reverse inequality is verified otherwise. For instance, with $\alpha = 2.5$, $\beta = 0.5$, $n = 2$ and $x = 1.4$, we get that $\rho(v|\mathcal{W},x) = 11.364/v^{4.5} > p(v|x) = 0.33$ for $v < 1.8201$; otherwise, the reverse inequality is satisfied (see Figure 2). Moreover, notice that

$$\begin{aligned} \rho_n(v|\mathcal{W},x) &= \beta\frac{\max(x,1)^{(\alpha+n\beta)}}{v^{\alpha+n\beta+1}} - \left(\ln v \max(x,1)^{(\alpha+n\beta)}\right)\frac{\alpha\beta+n\beta^2}{v^{\alpha+n\beta+1}} \\ &\quad + \left(\ln(\max(x,1))\max(x,1)^{(\alpha+n\beta)}\right)\frac{\alpha\beta+n\beta^2}{v^{\alpha+n\beta+1}} \end{aligned}$$

Hence, for the same parameter values considered above, we get that $\rho_n(v|\mathcal{W}, x) = (3.535\,1/v^{4.5}) - (5.681\,8/v^{4.5}) \ln v > 0$ for $v < \exp(0.622\,19)$, and the opposite result otherwise (see Figure 3).

The intuition behind these two conditions is the following. The former means that the event of winning the auction indeed represents *bad news* because the probability of small (high) realizations of v increases (decreases) for a type x bidder after knowing that his signal is the largest one. Additionally, the second condition points out that this bad news is reinforced by the increase in the number of bidders, as the posterior probability of small (high) values of v increases (decreases) when n becomes larger.

These conditions then provide a clear source for the non-monotonicity of the reverse hazard and thus, for the non-monotonicity of the equilibrium bid. For instance, in the CIPI-CV model, condition (9) implies that conditional on winning, bidder i of type x will estimate more likely that the ex post common value v is smaller. As a result of the affiliation assumption on $F_{x|v}$, he will estimate more likely that his rivals' signals are smaller as well. Since the symmetric equilibrium bid strategy is increasing in the signals, it will lead finally to a perception of a *less* intense competition from the bidder i's point of view. Given that this analysis is performed fixing the event of winning the auction, bidder i should react following a *less* aggressive bidding behavior, which we call the *inference phenomenon*. However, as condition (10) means that this perception of lower competition is *counteracted* when n decreases, we will eventually observe a lower conservatism in bids when a concentration process takes place. As a result, the *inference effect*, i.e. a possible shade in the inference-based downward adjustement in bids due to the reduction in the number of participants, can finally lead to an inverse relationship between revenue and n.

Note that, in contrast to the winner‘s curse, this inference phenomenon is of a *strategic* nature, as it emerges as a reaction of a rational player who, focusing only on this phenomenon, is able to reduce his bid without decreasing his probability of winning. That is, the winner's curse provokes a decrease in bids because the estimation of his/her *own* object's valuation is shaved. In contrast, in the case of the inference phenomenon, this greater conservatism is caused by a shade in the estimation of the *rivals*' bidding strategies.

4 (Non)monotonicity of revenues

In this section, we examine the conditions that guarantee the (non)monotonicity of the seller's revenue with respect to n under the CIPI-CV model.

In the first-price auction, the expected revenue is given by[31]

$$R(n) = E(B(x_{1:n}; n)) = \int_{\underline{x}}^{\overline{x}} B(\alpha; n) f_{x_1}(\alpha; n) d\alpha \tag{11}$$

where f_{x_1} and F_{x_1} are the p.d.f. and c.d.f. of the maximum signal $x_{1:n} = \max_{i=1,..n} x_i$, respectively. Denote by $G(b; n)$ the distribution function of $B(x_{1:n}; n)$. From (11), it is clear that a first (and natural) condition that guarantees R to be monotonically increasing in n is that $G(b; n+1)$ first-order stochastically dominate $G(b; n)$ for all n. In order to gain an insight into the conditions that allow this stochastic dominance to hold, we need to invest in some additional concepts and notations. Let us define both $MRS_B(x; n)$, the *marginal rate of substitution in bids*, and $MRS_F(x; n)$, the *marginal rate of substitution in the winning signal distribution,* as follows

$$MRS_B(x; n) \equiv \frac{B_n(x; n)}{B_x(x; n)}$$

and

$$MRS_F(x, n) \equiv \frac{F_{x_{1n}}(x; n)}{F_{x_{1x}}(x; n)}$$

where the subscripts x and n in B and F_{x_1} denote the partial derivative of these functions w.r.t. the respective variable.[32] In the context of auctions, the meaning of these marginal rates of substitution is as follows. Suppose that a marginal increase in the number of buyers occurs. In that case, $MRS_B(x; n)$ points out how and how much the change in x needed to keep constant the level of the equilibrium bid is. Similarly, $MRS_F(x, n)$ represents the characteristics of the change in x needed to keep the accumulated probability of the winning signal constant.

On the bid side, this required change in x may be either an increase or a decrease, depending on the sign of $B_n(x; n)$. As discussed in previous sections, this partial derivative can be positive or negative, according to the magnitudes of the winner's curse, the inference and the competition effects. In contrast, the partial derivative $B_x(x; n)$ is always positive, as bids are strictly increasing in signals. All of

[31] For simplicity, we assume throughout this section that there is no reserve price.

[32] For instance, $F_{x_{1n}}(x; n) \equiv \partial F_{x_1}(x; n)/\partial n$.

this implies that, if bids are increasing (decreasing) in n, this larger competition will indeed require a decrease (increase) in signal values to preserve the equilibrium bid's level. As a consequence, the marginal rate of substitution in bids, $MRS_B(x;n)$, may take either a positive or a negative sign.

In contrast, on the side of the winning signal's distribution, the change in x needed to preserve the accumulated probability of $x_{1:n}$ will always be an increase. This non ambiguity follows directly from the fact that $MRS_F(x,n)$ accounts for a fourth effect arising from changes in the number of buyers, which is not present when focusing on bids. This is the so-called *sampling effect*: an additional bidder means an additional draw from the signal distribution. Because of the properties of the first order statistics, the distribution of $x_{1:n+1}$ first-order stochastically dominates the distribution of $x_{1:n}$.[33] As a result, the partial derivative $F_{x_{1n}}(x;n)$ takes a negative sign unambiguously, and thus, the marginal rate of substitution in the winning signal distribution, $MRS_F(x,n)$, is always negative. Furthermore, the first stochastic dominance induced by the sampling effect on F_{x_1} due to more competition translates eventually into higher seller's expected revenue. Notice that from equation (11), this point is very clear as the equilibrium bid is an increasing function in signals.[34]

In sum, when concluding as to the final effect of competition on revenues, we have to examine the properties of both the *bidding effect* and the *sampling effect* by means of $MRS_B(x;n)$ and $MRS_F(x,n)$, respectively. This relationship between both marginal rates of substitution can be summarized defining the following term

$$\Delta(x;n) \equiv MRS_B(x;n) - MRS_F(x,n) \tag{12}$$

According to the previous analysis, three cases can emerge:

Case 1. A positive bidding effect: $MRS_B(x;n) \geq 0$, and hence, $\Delta(x;n) \geq 0$.

Case 2. A negative and dominated bidding effect: $MRS_B(x;n) \leq 0$ and $|MRS_B(x;n)| \leq |MRS_F(x,n)|$. Thus, $\Delta(x;n) \geq 0$.

Case 3. A negative and dominant bidding effect: $MRS_B(x;n) \leq 0$ and $|MRS_B(x;n)| \geq |MRS_F(x,n)|$. Thus, $\Delta(x;n) \leq 0$.

Equipped with these concepts and notation, we can go back to characterize the circumstances under which the seller may be better off or worse off with more

[33] In fact, one additional draw from the signal distribution implies that the highest signal is greater with probability $1/(n+1)$ and equal with probability $n/(n+1)$.

[34] Alternatively, we can interpret the sampling effect as an effect contributing positively to inducing a first stochastic dominance property in the winning bid distribution (see Proposition 4.1).

competition. We start with the next proposition, which provides a sufficient condition to ensure revenues that are strictly increasing in the number of bidders.

Proposition 4.1 *$\Delta(x;n) \geq 0$ for all x if and only if $G_n(b;n) \leq 0$ for all b. Furthermore, $G_n(b;n) \leq 0$ for all b implies that $R(n)$ is increasing in n.*

This result has the following implications. First, it means that as long as the effect of an increase in n on bids is positive for all signals the final effect on revenue will be positive as well. In terms of our previous analysis, this means that as long as the bidding effect is *positive* (i.e. Case 1), the seller will benefit from more competition. This conclusion is true because, as explained before, the sampling effect always induces an increase in proceeds. Consequently, combining the result concerning bids (Proposition 3.1) with Proposition 4.1, we can state the next result.

Corollary 4.1 *Suppose that for all $x \in (\underline{x}, \overline{x})$,*

$$\frac{\underline{\pi}(x;n+1)}{\underline{\pi}(x;n)} > \frac{\lambda(x;n)}{\lambda(x;n+1)}$$

Then for all $r < \overline{b}$, $R(n)$ is strictly increasing in n.

Moreover, Proposition 4.1 also implies that even though more concentration may cause a more aggressive bidding behavior, it may bring a reduction of revenue if the sampling effect is sufficiently large. This can occur when we are in Case 2, i.e. when a negative, but *dominated* bidding effect exists. Nevertheless, and in contrast to the affiliated private value model studied by Pinkse and Tan [31], in a CIPI-CV setting the last property is more difficult to be fulfilled. This is because the winner's curse effect, absent in the private value environments, demands a higher sampling effect to offset the inverse influences arising from bids. A direct consequence of this fact is that in the CIPI-CV model, the family of exponential distributions analyzed by Pinkse and Tan does not necessarily satisfy one of the sufficient conditions for monotonic revenues. This result is formalized in the following statement.

Proposition 4.2 *Consider the CIPI-CV model. Suppose that for some function ψ, we can write $F_{x/v}(x/v) = \exp(\psi(x)v)$ for all x and v. Then, the sign of $\Delta(x;n)$ is ambiguous.*

Note that Pinkse and Tan [31] show that for this class of distributions, $\Delta(x;n) \geq 0$ for all x in the CIPV model. Consequently, Proposition 4.1 allows us to rule out the presence of such a polar case in the CIPI framework as long as an inverse relationship between revenue and number of bidders is observed.

Corollary 4.2 *Consider the CIPI model. Suppose that for some function ψ, we can write $F_{x/v}(x/v) = \exp(\psi(x)v)$ for all x and v. Then, if for some n, $R(n+1) < R(n)$, the valuation environment cannot be that of the CIPV model.*

Finally, note that the reverse of the first part of Proposition 4.1 provides a necessary and sufficient condition for the distribution of the winning bid with n bidders to first-order stochastically dominate the distribution with $n+1$ bidders.

Proposition 4.3 *For a given n, $\Delta(x;n) \leq 0$ for all x if and only if $G(b;n) \leq G(b;n+1)$ for all b. Furthermore, $G(b;n) \leq G(b;n+1)$ for all b implies that $R(n+1) < R(n)$.*

Notice that the last result indeed constitutes a sufficient condition for revenue to be non-monotonically increasing in the number of bidders. Using the analysis performed before, note that such a property of revenues holds as long as we are in Case 3, i.e. when there is a *negative* and *dominant* bidding effect.

Hence, and based on the results stated for bids in the previous section, we can establish the next statement on the nonmonotonicity of the seller's proceeds.

Proposition 4.4 *Consider the two following situations:*
(1) *Suppose that for some values of n and r, it is verified that either (A1) or (A2) hold with:*

$$\lambda(a(n+1);n+1) < \lambda(a(n+1);n) \tag{A1}$$

$$\frac{\overline{\pi}(a(n+1);n+1)}{\overline{\pi}(a(n+1);n)} < \frac{\lambda(a(n+1);n)}{\lambda(a(n+1);n+1)}. \tag{A2}$$

If $|MRS_B(x;n)| > |MRS_F(x;n)|$ then $R(n+1) < R(n)$.
(2) *Suppose that there is no reserve price and it is verified (A3) with:*

$$v(x,x;n) > v(x,x;n+1). \tag{A3}$$

for all n and x. If $|MRS_B(x;n)| > |MRS_F(x;n)|$ then $R(n+1) < R(n)$.

In this statement, both (A1) and (A2) constitute sufficient conditions for the bidding effect to be negative. Additionally, the superiority (in absolute value) of the marginal rate of substitution in bids over that of the winning signal's distribution ensures that the bidding effect dominates the sampling effect.

Moreover, condition (A3) characterizes another situation allowing revenues to be decreasing in the number of bidders, but in a framework without a reserve price. In such a case, this revenue's property requires v to be strictly decreasing in n.

In sum, the last proposition states that as long as bidding behavior becomes more aggressive with concentration (i.e., a negative bidding effect), the seller may indeed benefit from a reduction in competition. This phenomenon could occur if the mixed influence exerted by the winner's curse and the inference effects more than compensates the sampling effect.

This nonmonotonicity of seller's proceeds is illustrated by the following two examples.[35] First, in the case of Example 1, the expected revenue can be analytically computed, and is given by

$$R(n) = \left[1 - \frac{n\beta}{(\alpha + n\beta - \beta)(\alpha + n\beta - 1)}\right] \frac{\alpha}{\alpha - 1}.$$

Notice that

$$R_n(n) = \frac{\left(\alpha - \beta + \alpha\beta - \alpha^2 + n^2\beta^2\right)\alpha\beta}{(\alpha - \beta + n\beta)^2 (\alpha + n\beta - 1)^2 (\alpha - 1)}$$

where $R_n \equiv \partial R(n)/\partial n$. Hence, $sign(R_n(n)) = sign(\ \alpha - \beta + \alpha\beta - \alpha^2 + n^2\beta^2)$. In particular, for $\alpha = 2.5$ and $\beta = 0.5$, we have that

$$sign(R_n(n)) = \begin{cases} < 0 & \text{for all } n \in [0, 3.464\,1] \\ > 0 & \text{otherwise} \end{cases}$$

This case is depicted in Figure 4, showing that in the presence of the winner's curse and the inference effects, the expected revenue may be nonmonotonic with n. In particular, this example illustrates the fact that the seller is better off when a concentration process takes place in a very concentrated market than when it does so in a competitive one.[36]

[35] Pinkse and Tan [30] emphasize that in the CIPV model the dominance of the bidding effects over the sampling effect requires too extreme distributional assumptions. As a result, they are unable to provide an example in which revenue is nonmonotonic with the number of buyers. In contrast, our more general CIPI setting permits us to attain this nonmonotonicity result without these extreme assumptions, as the next two examples show.

[36] A result consistent with this is theoretically stated by Hendricks et al. [13], who analyze the

Second, a seller who benefits from less competition under the CIPI framework is also illustrated by Example 2, which describes a mineral right model that previous literature has showed (numerically) to yield nonmonotonic revenue in the number of bidders.

Example 2 *The lognormal model* (from Reece [34] and DeBrock and Smith [7]). Consider the auction of a single offshore oil tract lease. The gross value of the petroleum reserve is given by v, a random variable distributed according to a *lognormal* probability density function represented by $f_v(v|\mu_v,\ \sigma_v)$, where μ_v and σ_v correspond to the mean and standard deviation of log v, respectively. The net value of the tract is given by $V = v - c$, where c is a known constant that represents the cost of postsale exploratory drilling. Each bidder observes x_i, an estimate of gross tract value that is, conditional on v, drawn from an independent and identically distributed *lognormal* distribution represented by the p.d.f. $f_{x|v}(x|\mu_x(v),\ \sigma_x(v))$ and c.d.f. $F_{x|v}(x|v)$.[37]

Note that in this model it is not possible to obtain an analytical solution for the equilibrium bid strategies starting from the first-order conditions of the bidder's maximization problem. However, DeBrock and Smith [7] find numerical solutions using specified values of parameters (means, standard deviations and number of bidders) consistent with real-world conditions of offshore oil leasing. Interestingly, their results suggest that the share of the social value of the tract captured by the seller (the government) can increase when joint bidding is allowed at a moderate level.

5 Conclusions

This paper has examined the revenue effect of having one more bidder at the auction stage. To this end, we considered the first-price auction format under

mineral rights model. This paper shows that bidders have more incentives to form rings when the number of potential participants in the auction is sufficiently large. Interestingly, they also confirm this prediction empirically for the offshore oil and gas lease auctions run by the U.S. government. However, we do not consider any information pooling effect as Hendricks et al. do. A similar result emerges in both papers notwithstanding, because of the presence of the winner's curse effect: whereas in Hendricks et al. the winner's curse affects the information precision of the bidding ring, in our paper this phenomenon influences directly the individual bidder's behavior.

[37] Notice that $F_{x|v}$ belongs to the normal distribution family, and thus, it satisfies the (strict) MLRP. As discussed before, this property is equivalent to assuming that x_i and v are affiliated. Although irrelevant for fitting the CIPI framework, this class of models additionally assumes that $E(x_i|v) = v$.

the CIPI model, an environment that encompasses a wide variety of valuation and information settings. We decomposed the revenue effect coming from more competition into two general sources: (i) the bidding effect, and (ii) the sampling effect.

The former includes all the effects on bidding behavior, which in turn, we have grouped into three classes: the competition, the inference and the winner's curse effects. The first effect corresponds to the traditional positive consequences on bid aggressiveness due to the fact that higher competition intensity reduces the bidders' probability of winning in environments with independent signals. On the contrary, the inference effect stems from the affiliation among signals. In such environment, the participation of one more bidder induces more bid conservatism as the perception that winning conveys information of less rivalry -the *inference phenomenon*- becomes exacerbated when the number of bidders increases. Finally, the winner's curse effect arises in common value settings, and induces unambiguously less bid aggressiveness since more competition reinforces such an overbidding phenomenon. As a consequence, the sign of the bidding effect on revenue is ambiguous and depends on the relative magnitudes of their three subeffects.

In contrast, as the participation of one more bidder improves the stochastic properties of the winning signal, the sampling effect is always revenue-increasing because of more competition.

Our main result points out that situations exist in which the participation of an additional buyer can lower the seller's expected proceeds. Consequently, from the seller's point of view, more competition is not always desirable, as it may deteriorate revenue. Equivalently, the industrial concentration need not be negative for bid-takers. The results derived in this paper suggest therefore how inconvenient it can be to advise the seller regarding a policy that always either promotes more bidder participation or discourage mergers or any joint bidding practice.[38,39]

This work shows that the situations in which more competition can be revenue-decreasing are characterized by a negative and sufficiently large bidding effect

[38] Note that if the information pooling effect induces more aggressive bids, the situations in which the seller benefits from less competition would constitute a lower bound of the revenue-increasing cases caused by joint bidding arrangements.

[39] Policymakers have de facto adopted a more tolerant position in markets with these characteristics. For instance, in the U.S. offshore oil lease auctions. Before 1976, no restrictions were imposed on joint bidding ventures, and since 1976, these arrangements have been permitted for firms which are small enough. Similarly, bidding consortia in takeover battles is generally accepted as a legal practice; see, for example, the recent bidding takeover processes won by the consortia Enel-Acciona and RBS-Banco Santander-Fortis for the control of Endesa and ABN Amro, respectively.

that dominate the sampling effect. Our analysis identifies two cases in which the last condition is met. First, a bidding effect with these features can emerge if there is a negative and large enough inference effect that overcome the traditional competition effect. This condition, represented by a reverse hazard not strictly increasing in n, can be present in all settings nested by the CIPI environment, including the affiliated private value case given by the CIPV model. Second, we state that, in the CIPI model, an additional condition suffices for a negative and dominant bidding effect, and thereby, for revenue loss in the face of more competition. This extra condition arises from the winner's curse phenomenon, absent in the affiliated private value environments. Accordingly, we show that the seller may also benefit from concentration as long as the winner's curse effect is sufficiently large.[40]

Therefore, we conclude that in the CIPI setting, and thus in the general affiliated value model, the conditions that allow a nonmonotonic revenue in the number of bidders are less stringent than in affiliated private value frameworks. As a result, situations in which the seller is better off with less competition should be more frequent in environments with not only dependent information, but also interdependent valuations. Interestingly, the available empirical evidence supports this prediction, especially that related to bidding markets in which the winner's curse seems to play an important role such as wildcat auctions.

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6 Appendix

Appendix A: Proofs.

Proof of Proposition 3.1. We prove this statement by contradiction. Suppose that, for some n, r and some $\widetilde{x}$, $B(\widetilde{x};n) \geq B(\widetilde{x};n+1)$. From boundary condition, we know that $B(a(n);n) = B(a(n+1);n+1) = r$. Substituting this into the differential equation given by (1), it is verified at $x = a(n)$ that

$$B_x(a(n);n) = [v(a(n),a(n);n) - r]\,\lambda(a(n);n) \tag{13}$$

Since by assumption

$$\frac{v(x,x;n+1)-r}{v(x,x;n)-r} > \frac{v(x,x;n+1)-\overline{b}}{v(x,x;n)-\overline{b}} > \frac{\lambda(x;n)}{\lambda(x;n+1)} \tag{14}$$

it follows from (13) that $B_x(a(n);n+1) > B_x(a(n);n)$. It must therefore be true that for some $x^* \in (a(n),\widetilde{x})$

$$B(x^*;n) = B(x^*;n+1) \equiv b^* \tag{15}$$

and

$$B_x(x^*;n) > B_x(x^*;n+1) \tag{16}$$

Notice however that (16) violates (1) because according to this differential equation it can be established the opposite condition

$$\begin{aligned} B_x(x^*;n) &= [v(x^*,x^*;n) - b^*]\,\lambda(x^*;n) \\ &< [v(x^*,x^*;n+1) - b^*]\,\lambda(x^*;n+1) = B_x(x^*;n+1) \end{aligned} \tag{17}$$

applying the same logic of (14) for b^* instead of r.■

Proof of Proposition 3.2. (1) We prove the first part of this statement by construction. First, because the boundary condition, $B(a(n);n) = B(a(n+1);n+1) = r$ for all n. Hence, evaluating the differential equation given by (1) at $x = a(n+1)$, it follows that

$$\begin{aligned} B_x(a(n+1);n) &= [v(a(n+1),a(n+1);n) - r]\,\lambda(a(n+1);n) \\ &\geq [v(a(n+1),a(n+1);n+1) - r]\,\lambda(a(n+1);n) \\ &> [v(a(n+1),a(n+1);n+1) - r]\,\lambda(a(n+1);n+1) \\ &= B_x(a(n+1);n+1) \end{aligned}$$

where the first inequality holds because $v_n(x,x;n) \leq 0$ for all $x \in [\underline{x},\overline{x}]$ as $F_{x|v}$ satisfies the (strict) MLRP (see Milgrom [27], Proposition 4 and Section 6), and

the second one does since our assumption that the reverse hazard is strictly decreasing at $x = a(n+1)$. All of this implies that $B(x; n+1) < B(x; n)$ for all $x \in (a(n+1), a(n+1)+\delta)$ and $\delta > 0$.

We now show the second part of the first statement. Since boundary condition, $B(a(n); n) = B(a(n+1); n+1) = r$ for all n. Hence, and after evaluating the differential equation (1) at $x = a(n+1)$, it is straightforward to verify that assumption (A2) ensures that, given some n and r, $B_x(a(n+1); n) > B_x(a(n+1); n+1)$. All of this implies finally that the equilibrium bid satisfies the desired property.

(2) Both the boundary condition (without reserve price) and the assumptions of the statement imply that

$$B(\underline{x}; n) = v(\underline{x}, \underline{x}; n) > v(\underline{x}, \underline{x}; n+1) = B(\underline{x}; n+1)$$

for some n. From this, it follows that $B(x; n) > B(x; n+1)$ for some $x \in (\underline{x}, \underline{x}+\delta)$ and $\delta > 0$.■

Proof of Proposition 3.3. (i) First, our assumption of MLRP for $F_{x|v}$ guarantees that $v_n(x, x; n) \leq 0$ for all $x \in [\underline{x}, \overline{x}]$ (see Milgrom [27], Proposition 4 and Section 6). So, if we are able to show that irrespective of the sign of $v_{xn}(s, s; n)$, its magnitude is smaller (in terms of absolute value) than $v_n(x, x; n)$, the desired result is attained. This is indeed true as

$$\begin{aligned} |v_n(x, x; n)| &\geq |v_n(x, x; n)| - |v_n(\underline{x}, \underline{x}; n)| \\ &= \left| \frac{\partial}{\partial n} \left[v(x, x; n) - v(\underline{x}, \underline{x}; n) \right] \right| \\ &= \left| \frac{\partial}{\partial n} \int_{\underline{x}}^{x} v_x(s, s; n) ds \right| \end{aligned}$$

where the first equality follows from assumption (A2). Moreover, since v is twice continuously differentiable, it is verified that

$$\begin{aligned} \left| \frac{\partial}{\partial n} \int_{\underline{x}}^{x} v_x(s, s; n) ds \right| &= \left| \int_{\underline{x}}^{x} v_{xn}(s, s; n) ds \right| \\ &\geq \left| \int_{\underline{x}}^{x} L(s|x) v_x(s, s; n) ds \right| \end{aligned}$$

for all $x \in [\underline{x}, \overline{x}]$ (with the strict inequality for all $x \in (\underline{x}, \overline{x}]$) because of $0 \leq L(s|x) \leq 1$ for all s, $x \in [\underline{x}, \overline{x}]$ as $L(.|.)$ satisfies the properties of a c.d.f.

(ii) Note that $\lambda_n(x,n;v) = f_{x|v}(x|v)/F_{x|v}(x|v) > 0$, as we have assumed that $F_{x|v}(x|v)$ does not depend on n. Furthermore, affiliation ensures that $v_x(s,s;n) > 0$, and $L(s|x)$ and $\rho(v|\mathcal{W},x)$ are positive for all s, $x \in (\underline{x},\overline{x}]$. As a result, the desired property holds.

(iii) First, we need to prove the next auxiliary result.

Lemma 6.1 *In the CIPI model, for a given x and n, it holds that*

$$\rho_n(v|\mathcal{W},x) > 0 \tag{18}$$

for a v small enough, and the reverse inequality is verified otherwise.

Proof of Lemma 6.1. Using the Bayes' Theorem, it is possible to verify that

$$\rho(v|\mathcal{W},x) = \frac{F_{x|v}^{n-1}(x|v)f_{x|v}(x|v)f_v(v)}{\int_{\underline{v}}^{\overline{v}} F_{x|v}^{n-1}(x|v)f_{x|v}(x|v)f_v(v)dv}$$

Hence, it is easy to state that for a given x and n, the desired property holds.■

Then, rewrite $\int_{\underline{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv$ as follows

$$\begin{aligned}\int_{\underline{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv &= \int_{\underline{v}}^{\widehat{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv \\ &\quad + \int_{\widehat{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv\end{aligned}$$

The sign of the last effect depends then on the magnitude of the areas delimited by the cut-off value $\widehat{v}$, from which according to Lemma 6.1, the partial derivative $\rho_n(v|\mathcal{W},x)$ can take a negative sign for a given x and n.■

Proof of Proposition 4.1. Define B^{-1} so that $B^{-1}(B(x;n);n) = x$ for all x and n. Note that since B is increasing in x, it holds that

$$\begin{aligned}G(b;n) &= \Pr(B(x_{1:n};n) \leq b) \\ &= \Pr(x_{1:n} \leq B^{-1}(b;n)) \\ &= F_{x_1}(B^{-1}(b;n);n)\end{aligned}$$

For short, denote $B^{-1}(b;n)$ by t. Then,[41]

$$\begin{aligned} G_n(b;n) &= F_{x_1n}(t;n) + F_{x_1x}(t;n)B_n^{-1}(b;n) \\ &= F_{x_1n}(t;n) - F_{x_1x}(t;n)\frac{B_n(t;n)}{B_x(t;n)} \\ &= -F_{x_1x}(t;n)\Delta(t;n) \end{aligned}$$

which yields the first desired result. Finally, from (11), it is clear that the first-order stochastic dominance induced by an increase in n on $G(b;n)$ guarantees R to be monotonically increasing in n.■

Proof of Proposition 4.2. Recall from (5) that

$$\begin{aligned} B_n(x;n) &= \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] - \int_{\underline{x}}^{x} L_n(s|x)v_x(s,s;n)ds \\ &= \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + \int_{s}^{x} \lambda_n(u;n)du \int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds \end{aligned}$$

where the equality holds because $L(s|x) = \exp(-\int_s^x \lambda(u;n)du)$. Let us define

$$A(x;n) \equiv \lambda(x;n)MRS_F(x,n)$$

Pinkse and Tan [31] shows that

$$\int_{s}^{x} \lambda_n(u;n)du \geq A(x;n) \tag{19}$$

for the CIPV model. Notice however that $A(x;n)$, by definition, considers the source of two effects on revenue coming from more competition: (i) the sampling effect, through $MRS_F(x,n)$, and (ii) the bidding effect, but with the exception of the winner‘s curse effect, through $\lambda(x;n)$. Consequently, the inequality (19) also holds for the CIPI model. All of this implies therefore that

$$B_n(x;n) \geq \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + A(x;n)\int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds \tag{20}$$

Furthermore, using the equilibrium bid function as stated in (2), it is easy to see that

$$B_x(x;n) = \lambda(x;n)\int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds$$

[41] Recall that the subscripts n and x denote the partial derivative of the respective function with respect to these variables.

Hence, and by the definition of $A(x;n)$, the inequality (20) becomes

$$B_n(x;n) \geq \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds \right] + B_x(x;n) MRS_F(x,n)$$

from which, rearranging and using the definition of $\Delta(x;n)$, it follows directly that

$$\Delta(x;n) \geq \frac{v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds}{B_x(x;n)}$$

Note that, according to (21), the numerator of the R.H.S. of the last inequality corresponds to the winner's curse effect. Since this effect is always negative and B is increasing in x, the sign ambiguity of $\Delta(x;n)$ holds.■

Proof of Proposition 4.3. It follows directly from Proposition 4.1 and the properties of the first-order stochastic dominance.

Proof of Proposition 4.4. From Proposition 3.2, either condition (A1) or (A2) implies that $B(x;n+1) < B(x;n)$ for some $x > a(n+1)$. As a result, $MRS_B(x,n)$ is negative, which constitutes a necessary condition for the nonmonotonicity of revenue. This condition and the fact that $|MRS_B(x,n)| > |MRS_F(x,n)|$ ensure then that $\Delta(x;n) \leq 0$, which according to Proposition 4.3, provides a sufficient condition for $R(n+1) < R(n)$.■

Appendix B. The Additive Decomposition: A counter-example.

Following Pinkse and Tan [31], the reverse hazard can be decomposed additively as $\lambda(x;n) = \lambda^Q(x;n) + \Delta\lambda(x;n)$. The first term corresponds to the reverse hazard consistent with the case in which $(n-1)$ bidder i's rivals draw their signals *independently* and identically from the c.d.f. $Q(x|x)$, where $Q(t|x) = \Pr(x_j \leq t | x_i = x)$ and $q(t|x)$ it is its associated p.d.f. Hence, $\lambda^Q(x;n) = (n-1)q(x|x)/Q(x|x)$. The second term, i.e., $\Delta\lambda(x;n)$, is defined residually as it corresponds to the difference $\lambda(x;n) - \lambda^Q(x;n)$.

It is easy to verify that applying this additive decomposition to the CIPI-CV model, we have that

$$\begin{aligned} B_n(x;n) \;=\; & \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds \right] + \\ & \left[\int_{\underline{x}}^{x} L(s|x) v_x(s,s;n) \left(\int_{s}^{x} \lambda_n^Q(u;n) du \right) ds \right] + \\ & \left[\int_{\underline{x}}^{x} L(s|x) v_x(s,s;n) \left(\int_{s}^{x} \Delta\lambda_n(u;n) du \right) ds \right] \end{aligned} \tag{21}$$

By construction, it follows directly that the R.H.S. of equation (21) represents the sum of three bidding-type effects: the *winner's curse effect* (WCE), the *competition effect* (CE), and the *affiliation effect* (AE).

Consider now the pure common value model illustrated by Example 1. First, since

$$v_n(x,x;n) = \frac{-\beta \max\{1,x\}}{(\alpha+n\beta-1)^2} < 0$$

and

$$v_{xn}(x,x;n) = \begin{cases} \frac{-\beta}{(\alpha+n\beta-1)^2} < 0 & \textit{if } x > 1 \\ 0 & \textit{otherwise} \end{cases}$$

the winner's curse effect is then given by

$$WCE = \frac{-\left(\max\{x,1\}^{\beta} + \beta \max\{x,1\}^{1+n\beta}(n-1)\right)\beta}{(\alpha+n\beta-1)^2(n\beta-\beta+1)\left(\max\{x,1\}^{n\beta}\right)} < 0$$

which confirms the sign attributed to this effect.[42] Second, we decompose the competition effect and the affiliation effect based on the Pinkse and Tan's approach. Notice however that given that $F_{x|v}$ does not satisfies the strict MLRP assumption, this decomposition does not work as $\lambda(x;n) = (n-1)\beta/x$ does not depend on v and it is strictly increasing in n.[43] As a result, $\lambda^Q(x;n) = (n-1)\beta/x$ and hence the competition effect is given by

$$CE = \frac{\alpha+n\beta}{\alpha+n\beta-1}\int_1^{\max\{x,1\}} \left((\frac{\max\{x,1\}}{s})^{-(n-1)\beta}\right)\left(\ln(\frac{\max\{x,1\}}{s})^{\beta}\right) ds > 0$$

which also corroborates the expected sign. Nevertheless, since $\Delta\lambda(x;n) = \lambda(x;n) - \lambda^Q(x;n) = 0$, the affiliation effect becomes *null*. Thus, the additive decomposition proposed by Pinke and Tan does not capture in this case the inference-type effect that arises from the statistic structure of the bidders' information assumed in this example. This provides us with the rationale for proposing an alternative *multiplicative* decomposition that identify an inference effect even though the MLRP assumption be weakly satisfied.

[42] In particular, since $v_n(x,x;n) = -\beta \max\{1,x\}/(\alpha+n\beta-1)^2$, the negativeness of the WCE is ensured by $v_n(x,x;n) < 0$ and $|v_n(x,x;n)| \geq |v_n(\underline{x},\underline{x};n)|$ for all $x > \underline{x}$ (see Proposition 3.3)

[43] That is, strict affiliation does not hold as $F_{x|v}$ satisfies only the weak MLRP.

Appendix C: Figures.

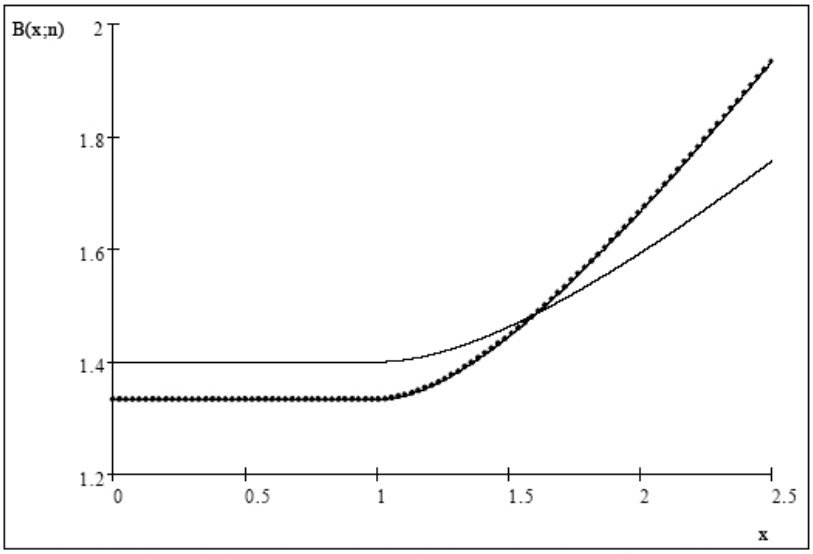

Figure 1. Equilibrium bid of Example 1 with $\alpha = 2.5$ and $\beta = .5$ for $n = 2$ (solid line) and $n = 3$ (dot line).

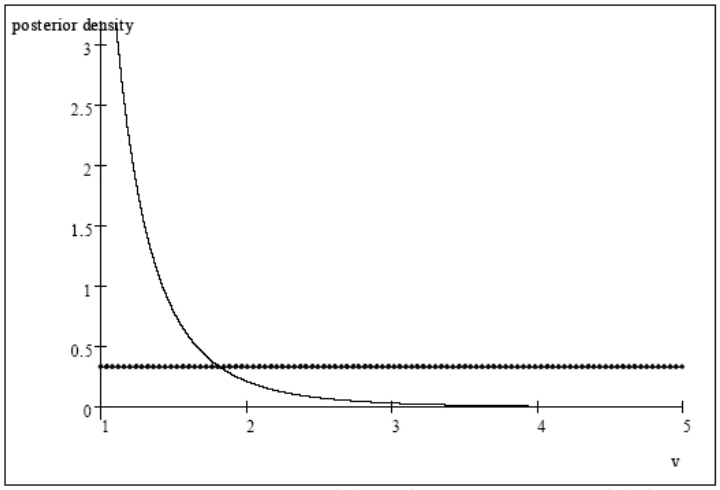

Figure 2. Posterior density functions $\rho(v|W,x)$ (solid line) and $p(v|x)$ (dot line) of type $x = 1.4$ for Example 1 with $\alpha = 2.5$, $\beta = .5$ and $n = 2$.

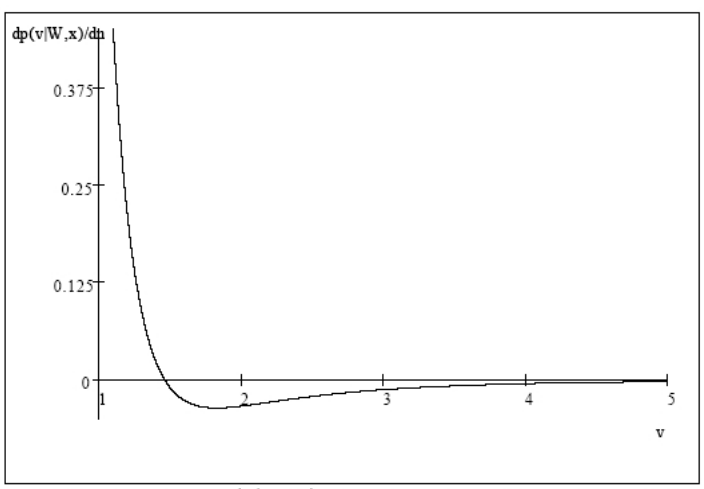

Figure 3. Partial derivative $\rho_n(v|W,x)$ of type $x = 1.4$ for Example 1 with $\alpha = 2.5$, $\beta = .5$ and $n = 2$.

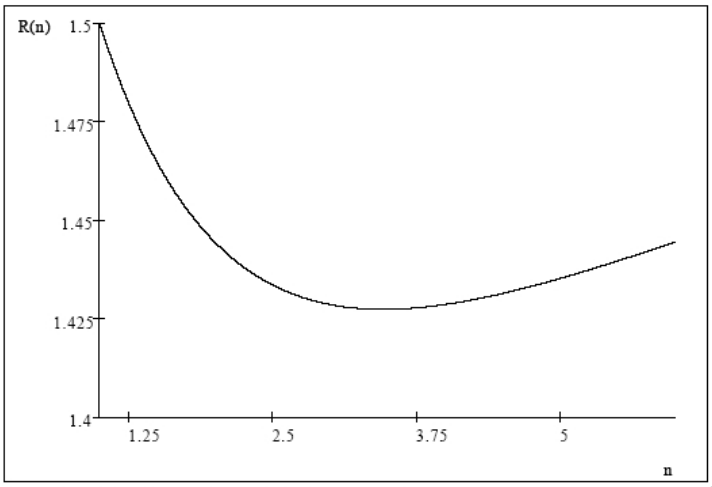

Figure 4. Revenue and number of bidders for Example 1 with $\alpha = 2.5$ and $\beta = .5$.