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COMPETITION BETWEEN PUBLIC AND PRIVATE UNIVERSITIES: QUALITY, PRICES AND EXAMS *

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Abstract

The rapid growth of private higher education in response to high demand is a recent phenomenon in most European countries. This paper provides a theoretical model of the higher education market in which a public and a private university compete for students in the presence of borrowing constraints. We find that there exists a unique equilibrium in which the public institution is of higher quality than the private institution. This result supports the observation across many European countries that public universities have usually higher quality and admission standards than their private competitors.

JEL Classification: I21; I22; H42

Keywords: Competition between Educational Institutions; Borrowing Constraints

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1 Introduction

Most European universities have, until recently, been controlled by the state, which typically has paid for the costs of higher education out of general taxation. Students pay little or no tuition and public institutions usually determine access to higher education by means of selective exams. Although private universities have long existed, being often founded by the Catholic Church, the rapid growth in the number of private higher education institutions in many Central and Eastern European countries, and also in Greece and Spain, is a recent phenomenon. The expansion of private education has taken place in response to high demand for access to higher education and without a rise in public funding. However, the quality of many of these universities is questionable, and it seems that private colleges and universities are absorbing the demand in fields in which the cost of offering instruction is low.¹

The market for higher education has some distinctive features that differentiate this sector from others.² Firstly, most universities allocate places to students by administrative rationing, using selective exams to determine university admissions. Secondly, the performance or quality of universities depends positively on the ability of their students, which makes higher education provision a case of customer-input technology, as described by Rothschild and White (1995). Moreover, many higher education institutions are non-profit maximizers and their objectives are sometimes difficult to determine.

The special features of this market may help explain why theoretical contributions to the analysis of the higher education system are scarce, despite its importance and the interest of researchers in this topic. In particular, competition among educational institutions has been the object of study of Del Rey (2001) and De Fraja and Iossa (2002), in the case of symmetric universities, and Epple and Romano (1998), in the case of public and private schools. Epple and Romano (1998) obtain that in equilibrium, public schools have lower quality than private schools. This result follows from the interaction between an open-enrolment public system and a competitive private sector. Since public schools are free, students will be willing to attend a private school, which is costly, only if it is of higher quality than the public school. Although their model features the market for

¹See The National Committee of Inquiry into Higher Education (1997) and Altbach (1997) for further evidence.

²Winston (1999) provides an extensive discussion of the main characteristics of the higher education market.

primary or secondary education, provided that all student attend a school, their result may be applied to the US higher education market, in which it is generally the case that private universities are better than public universities.

In many European countries we observe instead that public universities set very low or even zero tuition fees but have higher admission standards than private universities. This paper provides a theoretical framework to explain this phenomenon, focusing in the characteristics of the European higher education market. We also investigate the consequences of the recent entrance of commercially-run institutions into this market and pose the following questions: can self-financed private universities compete with public universities?, why are private universities usually of lower quality than public universities?, and how the presence of private institutions affect public universities' qualities and admission policies?.

The modelling of the university system requires to determine which the higher institutions' objectives. We consider that public universities aim at maximizing public surplus, that is, the sum of the earnings of students attending the public university minus the cost incurred to provide education. Public universities' costs are covered by general taxation. Commercial institutions are profit-maximizers. Universities choose optimally their level of educational quality and use admission requirements and tuition fees to compete for students.³ Students differ in their unobservable ability and in their income endowment, and choose whether to attend a university or remain uneducated in order to maximize their lifetime income. We assume that they cannot borrow against their future income to invest in higher education. Individuals' future earnings are increasing in their own ability and the quality of the university they attend and both inputs are complements in human capital production. Hence, the return of a given level of educational quality is higher for high than for low-ability students.

In order to analyze the impact that competition from private universities may have on public universities' policies, we first consider an economy in which the only providers of higher education are public universities. This benchmark is intended to capture the initial situation in the European higher education market before the expansion of commercial universities. Under the assumption that quality is fairly homogeneous across public universities, we can consider that there is only one public institution. We analyze the optimal

³Fernández (1998) and Fernández and Galí (1999) have investigated the properties of markets and exams as alternative assignment mechanisms under borrowing constraints.

choices of quality, prices and exams of the public monopoly and we obtain that in equilibrium, public educational quality depends positively on the mean ability of the student body. The public institution optimally uses exams to determine admissions and sets a zero tuition fee. Intuitively, exams are preferred to prices as allocation device because they select students according to their ability. In the presence of borrowing constraints, the use of tuition fees limit the admissions of poor and high-ability individuals who are unable to pay university's fees. This, in turn, reduces public surplus, which depends positively on the mean ability of the student body.

Next, we model competition between a public and a private institution. Our main result is the following: the private institution optimally chooses to provide an educational quality lower than the one provided publicly. This result may be explained by the different strategies followed by institutions when competing for students. On the one hand, the public university is able to behave as a monopoly by means of setting admission standards and a zero tuition fee. On the other, the private university's admission policy, based on tuition fees, makes this institution attractive just to those students of lower ability who are not accepted into the public university and can afford to pay the private fee. We show that the presence of a private university in the market involves positive welfare gains compared to the public monopoly. This is because those students attending the public institution under monopoly are not affected by competition, and the presence of a private university allows new students to accede to higher education. This, in turn, increases total income in the economy.

This paper is closely related to Del Rey and Romero (2004) and Oliveira (2004). The first paper investigates the strategic role of prices and exams for public and private institutions competing for students in the presence of borrowing constraints. It also studies if the process of decentralization of public universities in Europe may be the optimal response of public universities to increasing competition from private. Oliveira (2004) analyzes competition between public and private universities in the presence of peer-group effects and perfect capital markets. She finds two types of equilibria depending on the parameters of the model, one in which the public university has higher quality than the private and the other in which the public has lower quality. Our paper differs from both papers in that educational quality is endogenously determined and is another decision variable, apart from exams and prices, in the game between the universities. If universities have an active role in setting tuition fees and exams, then they will be also

active in choosing the quality of their educational services, so we think that a complete analysis of university behavior should take into account the quality choice by universities.

This paper is organized as follows: Section 2 presents a model of the higher education sector, characterizing the behavior of students and universities as well as the alternative allocation mechanisms. In Section 3 we analyze the public monopoly benchmark and the optimal choices made by the public university. Section 4 studies competition between a public and a private university. Section 5 concludes.

2 The Model

2.1 Individuals

The economy consists of a continuum of individuals of measure one. Each individual i is characterized by a different and unobservable ability, a_i , and an initial income endowment, w_i , which are uniformly and independently distributed over the interval $[0, 1]$. An individual i attending university j obtains utility from his total lifetime income, which consists of his initial income endowment, w_i , and his earnings or accumulated human capital, h_i , minus the tuition fee paid to university j , p_j :

$$U_j^i = w_i - p_j + h_i. \quad (1)$$

We assume that earnings are increasing in individual's ability and university's educational quality.⁴ Educational quality and ability are complements in the determination of earnings. For simplicity of computations, we assume that human capital has the following functional form:⁵

$$h_i = a_i Q_i. \quad (2)$$

The human capital of uneducated individuals is normalized to zero, $h_0 = 0$, and hence, a student who does not attend university obtains a utility equal to his initial endowment, $U_0^i = w_i$.

⁴The positive contribution of own ability to educational attainment is well documented in Hanushek (1986).

⁵The specification of the human capital production function does not affect our qualitative results and allows us to obtain analytical solutions.

We assume throughout the paper that individuals cannot borrow to invest in higher education. Although we do not model explicitly the reasons behind this market failure, moral hazard problems in lending to finance education and the impossibility of human capital to act as a collateral for loans may be some reasons to close down capital markets.⁶ This assumption is based on the observation across countries that the majority of students attending university come from high and middle income classes, which is claimed as the evidence of credit constraints in financing higher education. In a recent study, Carneiro and Heckman (2002) conclude that the correlation between income and college attendance is explained by the existence of long-run rather than short-run credit constraints. In our model, the predetermined level of income each individual is endowed with, w_i , may be interpreted as the permanent family income and is the main determinant of long-run credit constraints.

2.2 Universities

We consider that there are two types of universities that provide higher education of quality Q_j , $j = b, v$, where b and v stand for *public* and *private* respectively. Educational quality may be interpreted as the *prestige* of the higher education institution.⁷ Universities compete for students setting qualities, prices and exams. Firstly, they simultaneously choose the level of educational quality, secondly, the tuition fee or price for their educational services and finally, the minimum exam score required to be accepted into the university.

Universities incur in a cost per student equal to $C(Q_j)$, which is increasing and convex in educational quality, $C'(Q_j) > 0$ and $C''(Q_j) > 0$, and has the following functional form:

$$C(Q_j) = \alpha Q_j^k, \quad k > 1, \quad \alpha > 0. \quad (3)$$

Although universities have the same cost technology, they differ in their objectives. The public university aims at maximizing public surplus, that is, the difference between the sum of earnings of students attending the public university, and the costs incurred

⁶The assumption that individuals cannot borrow at all does not affect our qualitative results that would also hold in the presence of less severe imperfections in the credit market. However, this assumption allows us to simplify the model and focus on the effect of borrowing constraints in higher education investments.

⁷In the education literature, school quality is usually measured by per-student expenditures and by indicators of the mean ability of the student body.

to provide education.⁸ The private institution maximizes profits, that is, the difference between the revenue obtained from tuition fees pay by admitted students minus the cost of educating those same students.⁹

2.3 Allocation Mechanisms

2.3.1 Exams

Universities may use an entry exam to select the best students among those who are willing to attend the university.¹⁰ In order to do so, they establish a minimum score such that those who obtain a score equal or higher are accepted into the university. We assume that the exam technology is able to perfectly reveal student's ability, which means that there is a one-to-one relationship between the standard of admission and the ability of the least able student accepted into the university. Let a_j^E be the ability of the less talented student admitted to university j . Then, students who obtain a score higher or equal than the minimum established by the university are those of ability $a_i \geq a_j^E$, $j = b, v$.

2.3.2 Prices

Students may be allocated to schools also by means of prices. This mechanism assigns students according to their willingness and ability to pay. The university price p_j determines the type of students (characterized by their ability and income) who are willing and able to enrol the university, given quality Q_j . Since student's ability and educational quality are complements in the production of human capital, the marginal return from a given level of school quality is higher for high-ability students and hence, they will be willing to outspend low-ability ones. However, in the presence of borrowing constraints only those with a sufficiently high level of income will be able to attend their preferred university.

⁸Public surplus is the sum of the utility of students attending the public university and the utility of the university. Notice that tuition fees are merely a monetary transfer from students to the university and thus, they cancel out when these two utilities are aggregated.

⁹Some private universities in Europe are not profit-maximizers, like religious higher education institutions. However, we focus our analysis on commercially-run universities which are allowed to seek profits.

¹⁰In our model, exams do not involve any wasteful expenditure in contrast to Fernández (1998).

The decision of attending university or remaining uneducated is then affected by university's price. Each individual makes this decision by means of comparing the utility he enjoys if he acquires education and attends university j , which is given by (1), with the utility obtained if he remains uneducated, $U_0^i = w_i$. Let \hat{a}_j be the ability of the student who is indifferent between attending school j and remaining uneducated, i.e., $\hat{U}_0 = \hat{U}_j$:

$$\hat{a}_j = \frac{p_j}{Q_j}, \quad j = b, v. \quad (4)$$

All students with ability $a_i \geq \hat{a}_j$ prefer to attend university j to remain uneducated. Among these students, only those with income $w_i \geq p_j$ are able to do so due to the presence of borrowing constraints.

If an individual i is willing to attend university, he must choose which university to attend comparing his utility at both institutions. In the case in which public school quality is higher or equal than private quality, $Q_b \geq Q_v$, we may define \tilde{a}_b as the ability of the student who is indifferent between attending the public and the private university, i.e., $\tilde{U}_b = \tilde{U}_v$:

$$\tilde{a}_b = \frac{p_b - p_v}{Q_b - Q_v}. \quad (5)$$

Individuals with ability $a_i \geq \tilde{a}_b$ prefer the public to the private university while students with ability $a_i < \tilde{a}_b$ prefer the private to the public institution.¹¹ Alternatively, if private educational quality is higher than public quality, $Q_b < Q_v$, the individual who is indifferent between both universities is an individual with ability

$$\tilde{a}_v = \frac{p_v - p_b}{Q_v - Q_b}. \quad (6)$$

3 The Public Monopoly Benchmark

In order to analyze the impact that the presence of private universities may have on public university's quality and admission policies, we first consider a benchmark economy in which the public university is a monopoly in the higher education market. This benchmark is intended to capture the main features of the higher education market in many European countries before the entrance of commercial universities into this market.

¹¹It is clear that if $p_b < p_v$ then $\tilde{a}_b < 0$, which means that all individuals prefer the public to the private university. This is because the public university is not only of higher quality than the private, but also cheaper.

We assume that the public university has an exogenous budget that allows to cover the costs of educating any chosen number of students. This budget comes from the government and is funded out of general taxation. The fact that the public university is not subject to budgetary constraints means that there are no capacity constraints in the public sector. If we instead consider that the budget at the disposal of the public university is fixed, the choice of the admission standard would be trivial since the number of students that the university can admit is determined by this budget.

Public monopoly's objective is to maximize public surplus, and then, the university is going to choose the level of school quality and the combination of prices and exams to attain this objective.¹² The timing of decisions is the following: in a first stage, the public university chooses the level of educational quality, Q_b . In a second stage, the university selects the price or fee, p_b . In the last stage, the public institution decides whether to use a selective exam to admit only students with ability $a_i \geq a_b^E$, among those students who are willing to attend the university, whose ability is $a_i \geq \hat{a}_b$, or admit all applicants. We solve the surplus-maximization problem by backward induction.

Public university's utility is given by the following expression:

$$U_b = \int_{p_b}^1 \int_{a_b}^1 (a_i Q_b - C(Q_b)) da dw, \quad (7)$$

where a_b is the ability of the least able student admitted to the public university.

In the last stage, the public monopoly decides whether to run an exam or not, taking the price, p_b , and educational quality, Q_b , as given. This institution determines the minimum score to be accepted into the university such that the ability of the least able student, a_b^E , satisfies the following conditions:

$$\begin{aligned} a_b^E &= \arg \max U_b, \\ \text{s.t. } a_b^E &\geq \hat{a}_b. \end{aligned} \quad (8)$$

Notice that the exam should select the more talented students among those who are willing to attend the university, i.e., $a_b^E \geq \hat{a}_b$, otherwise, the exam is useless since it does not restrict admissions.¹³

¹²Notice that public surplus coincides with total surplus when the public university is a monopoly in the higher education market.

¹³The ability of the least able student attending the university must also satisfy $a_b^E < 1$, otherwise, the university is so selective that it does not admit any student. Of course, it is in the interest of the public university to admit a positive measure of students in order to obtain a positive utility and hence, this condition is always satisfied.

Given public quality, Q_b , and public price, p_b , the optimal exam is chosen such that only students with ability $a_i \geq a_b^E$ are accepted into the public university, where a_b^E is the following:

$$a_b^E = \begin{cases} \frac{C(Q_b)}{Q_b} & \text{if } p_b \leq C(Q_b), \\ \hat{a}_b & \text{if } p_b > C(Q_b). \end{cases} \quad (9)$$

Thus, the public university implements a system of exams if the price chosen in the previous stage is smaller or equal than the cost per-student, $C(Q_b)$, and chooses only a system of prices to guide university's admissions otherwise. The optimal exam is determined such that the human capital obtained by the least able student admitted to the public university, $a_b^E Q_b$, is equal to the cost per-student, $C(Q_b)$.

In the second stage, the university chooses the optimal tuition fee, p_b , anticipating the optimal choice of exams in the next stage, and taking educational quality, Q_b , as given. Thus, the optimal public price satisfies

$$p_b = \arg \max \int_{p_b}^1 \int_{a_b}^1 (a_i Q_b - C(Q_b)) da dw,$$

where $a_b = \max \{a_b^E, \hat{a}_b\}$.

We find that the price has a negative effect on public university's utility

$$\frac{dU_b}{dp_b} = - \left(\frac{1 - a_b^2}{2} Q_b - C(Q_b) (1 - a_b) \right) + \frac{da_b}{dp_b} (1 - p_b) (-a_b Q_b + C(Q_b)) < 0,$$

since $-\left(\frac{1 - a_b^2}{2} Q_b - C(Q_b) (1 - a_b)\right) < 0$ and $-a_b Q_b + C(Q_b) \leq 0$ from (9).

Hence, the public university decides to set a zero price, $p_b = 0$, and the selection of students at this university is guided only by means of exams. Since the public institution aims at maximizing surplus, exams are preferred to prices to determine admissions since they are more efficient in allocating students, provided that students' selection is based on ability. A system of prices assigns students not only according to their ability (willingness to attend the university) but also according to their level of income (ability to pay the university's fee). The presence of credit constraints prevents the public university from setting a positive price provided that the tuition fee does restrict the admission of talented and poor students (those with income $w^i < p_b$).

In the first stage, the public institution chooses school quality anticipating the optimal choices of prices and exams in the next stages

$$Q_b = \arg \max \int_0^1 \int_{a_b^E}^1 (a_i Q_b - C(Q_b)) da dw. \quad (10)$$

Public educational quality is optimally determined such that the marginal cost of school quality is equal to the mean ability of students attending the public university

$$C'(Q_b) = \frac{1 + a_b}{2}, \quad (11)$$

where $a_b = a_b^E = \frac{C(Q_b)}{Q_b}$.

Since university's costs are increasing and convex in quality, the optimal level of public educational quality is increasing in the mean ability of its student body, $\frac{1+a_b}{2}$. This result points out the positive contribution of students' abilities to the performance of the university (the customer-input technology) and provides a theoretical explanation for the observation that peer-group effects have an important contribution to educational attainment.¹⁴

We can summarize the results obtained in this section as follows: in the presence of borrowing constraints, the public monopoly uses selective exams and sets a zero price to determine admissions. On the one hand, the public university chooses the optimal exam such that the human capital of the least able student accepted into the public university is equal to the cost per student. On the other hand, the optimal level of educational quality satisfies that the marginal cost of providing this quality level is equal to the mean ability of the student body.

4 Competition between a Public and a Private University

Let assume that a public university competes with a private in the higher education market. We study the following game between institutions: in a first stage, universities simultaneously determine educational quality, in a second stage, they choose prices, and in a third stage, once their demand is determined, they decide whether to run an exam or

¹⁴The principle finding of the Coleman Report (Coleman et al. (1966)) is that a student's educational achievement is strongly and positively correlated to the educational background of his classmates.

accept all applications. We focus on the Subgame Perfect Nash Equilibrium (henceforth, equilibrium) of the game between universities and we solve the game using backward induction.

In order to determine the number of market partitions (combinations of prices, qualities and exams) such that both universities may coexist in the higher education market, we take into account that in the presence of borrowing constraints tuition fees not only affect the willingness to attend university, given educational quality, but also the ability of students to pay university's fees.

Let consider the case in which public quality is higher or equal than private quality, $Q_b \geq Q_v$. In such case, we have two possibilities; either the public price is lower or is higher or equal than private price. If the public institution sets a lower price than the private university, all students prefer to attend the public institution because not only offers higher quality, but is also cheaper than the private. In this case, the only possible market partition in which both institutions are active in the market is the one in which the public university has higher admission standards than the private by means of selective exams, i.e., $a_b^E > a_v = \max\{\hat{a}_v, a_v^E\}$. Then, some students who are not accepted into the public university are willing and can afford to attend the private university. These students are those of ability $a_i \geq a_v$ and initial income endowment $w_i \geq p_v$. The utility of the public university is the same as under monopoly because it is the high-quality institution and then, it is not affected by private university's decisions. Then, public utility is given by (7), where $a_b = a_b^E$, and private university's utility is the following:

$$U_v = \int_{p_v}^1 \int_{a_v}^{a_b^E} (p_v - C(Q_v)) da dw, \quad (12)$$

where $a_v = \max\{\hat{a}_v, a_v^E\}$.

Conversely, if the price of the public university is higher or equal than the price of its private competitor, $p_b \geq p_v$, being public quality higher than private quality, $Q_b \geq Q_v$, we have two possible situations:

- The first case is the one in which the public university has higher or equal admission standards than the private, $a_b \geq a_v$. Since the public university is the high-quality institution, its objective function is the same as under monopoly, and is given by (7), where $a_b = \max\{a_b^E, \hat{a}_b, \tilde{a}_b\}$, and private university's utility is the following:

$$U_v = \int_{p_v}^{p_b} \int_{a_b}^1 (p_v - C(Q_v)) da dw + \int_{p_v}^1 \int_{a_v}^{a_b} (p_v - C(Q_v)) da dw, \quad (13)$$

where $a_v = \max \{a_v^E, \hat{a}_v\}$.

- The other market partition is the one in which the private university is more selective than the public, $a_b < a_v$, when public school quality is higher or equal than private quality.¹⁵ In such partition the utility of the public university is (7), the same as under monopoly, while private university's utility is

$$U_v = \int_{p_v}^{p_b} \int_{a_v}^1 (p_v - C(Q_v)) da dw, \quad (14)$$

where $a_v = \max \{\hat{a}_v, a_v^E\}$.

A similar procedure identifies all possible market partitions when private quality is strictly higher than public quality, $Q_b < Q_v$. However, the total number of market partitions can be further reduced provided that the public university is going to set the public price optimally at zero in the second stage of the game.¹⁶ Hence, students' selection in the public university is guided only by exams and then, $a_b = a_b^E$. This result reduces the scope of our analysis to the following cases:

- The case in which public quality is higher or equal than private quality, $Q_b \geq Q_v$, and the public institution has higher or equal admission standards than the private university, $a_b \geq a_v$, (henceforth, Case 1). The allocation of students corresponding to this case is represented in Figure 1, where the darker area represents enrolments in the public university.

¹⁵Notice that this market partition may appear only if there exist credit constraints and hence, some high-ability students cannot afford to attend the public university but are able to pay private university's fee. Otherwise, high-ability students would outspend low-ability students, independently of their initial endowment, and hence, the low-quality institution could not attract students of higher mean ability than the high-quality institution.

¹⁶This can be verified easily by observing that the public price has a negative effect on public university's utility in all possible market partitions specified above, taking into account that exams are chosen optimally in the following stage.

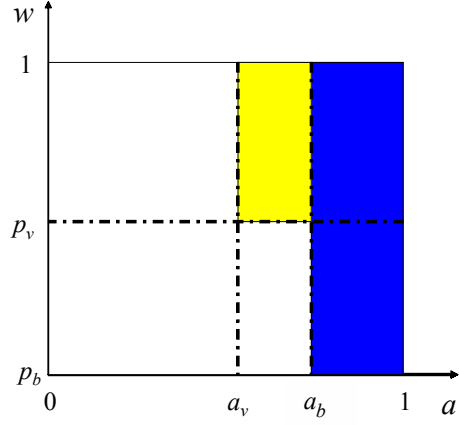


Figure 1: Case 1

- The case in which the private university provides higher quality than the public, $Q_b < Q_v$, and the public institution is more selective than the private, $a_b \geq a_v$ (henceforth, Case 2). The public university may be more selective than the private when its quality is lower due to the presence of credit constraints. The allocation of students to the public and private university in this case is represented in Figure 2, where the darker area also represents enrolments in the public institution.

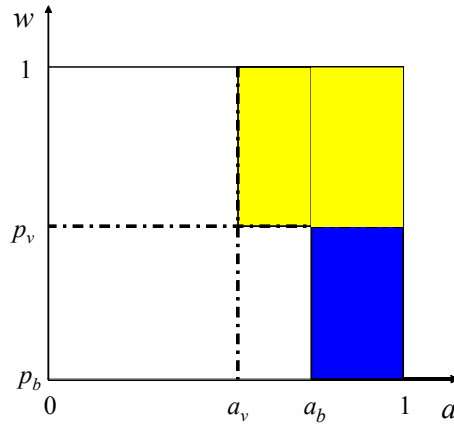


Figure 2: Case 2

- The case in which private quality is higher than public quality, $Q_b < Q_v$, and the private institution is more selective than the public, $a_b < a_v$ (henceforth, Case 3). In this market partition the private university attracts students of higher mean ability than the public institution as represented in Figure 3.

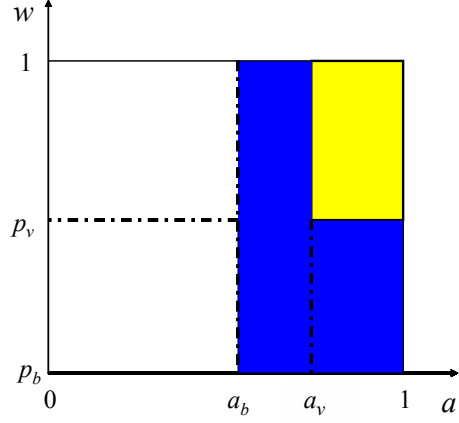


Figure 3: Case 3

Let analyze the game between universities in Case 1. We solve the game by backward induction. We first analyze the last stage of the game, in which universities simultaneously choose their admission standards using selective exams, and taking the admission standard of the competitor as given. Since the public institution is the high-quality university, it is going to choose the admission standard as under monopoly, according to condition (9).

Simultaneously, the private institution selects the requirement for admission, that is, the ability of the least able student accepted into this institution, a_v^E , that maximizes (12), subject to $a_v^E \geq \hat{a}_v$. In doing so, the private university takes the admission standard in the public university, a_b^E , as given. It is easy to see that the private university does not use exams, i.e., $a_v^E = \hat{a}_v$, since private university's utility is strictly decreasing in a_v . Hence, the tuition fee is the only instrument used by the private institution to guide its admission policies. Notice that the price is not only an allocation device, but also the source of funding for the private institution, which helps explain why the private institution prefers prices to exams.

In the second stage of the game, universities simultaneously set prices, taking as given the price of the other institution. As mentioned before, the public institution chooses a zero-price as under monopoly, since public utility is strictly decreasing in p_b . Thus, the public university only uses exams to guide its admission policies and accepts applications of students of ability $a_i \geq a_b^E \equiv \frac{C(Q_b)}{Q_b}$. The private price is chosen to maximize (12), where $a_v = \hat{a}_v$, given the price of the public university. The optimal private price, p_v , satisfies

the following condition:

$$(1 - 2p_v)(a_b^E - \hat{a}_v) + C(Q_v)(a_b^E - \hat{a}_v) + \frac{d\hat{a}_v}{dp_v}(1 - p_v)(-p_v + C(Q_v)) = 0. \quad (15)$$

Strict concavity of private university's utility in private price ensures the existence of a unique interior optimum. Hence, we may rewrite (15) in terms of the ability of the least able student admitted into the private university

$$\hat{a}_v = \frac{(1 - p_v) \left(a_b^E + \frac{C(Q_v)}{Q_v} \right) - a_b^E (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))}. \quad (16)$$

Finally, we solve the first stage of the game, in which universities simultaneously determine educational quality, taking as given the quality of the competitor. It is easy to see that the public university chooses quality as under monopoly while the private institution sets its quality optimally according to the following condition:

$$-C'(Q_v)(a_b^E - \hat{a}_v) + \frac{d\hat{a}_v}{dQ_v}(-p_v + C(Q_v)) = 0. \quad (17)$$

Strict concavity of private university's utility in private educational quality allows us to solve (15) for $a_b^E - \hat{a}_v$, and plugging it into (17) we obtain the following expression for optimal private school quality:

$$C'(Q_v) = \hat{a}_v \left(1 - \frac{p_v - C(Q_v)}{1 - p_v} \right). \quad (18)$$

Thus, private institution's quality is increasing in the ability of the least able student admitted into the private university, \hat{a}_v , since $1 - \frac{p_v - C(Q_v)}{1 - p_v} > 0$ from (15). We obtain that while the public university chooses school quality depending on the mean ability of its students, $\frac{1+a_b^E}{2}$, the private institution takes into account the ability of the least talented student accepted, \hat{a}_v .

After solving the game between universities in Case 1, we may state the following proposition:

Proposition 1 *The market partition in which the public university provides higher educational quality than the private institution is an equilibrium.*

Proof. See the Appendix. ■

In the proof we use the optimality conditions of qualities, prices and exams for both universities together with the properties of the cost technology to show first, that public quality is strictly higher than private, and second, that admissions standards of the public institution are also higher. This proves that the market partition described in Case 1 is an equilibrium.

We proceed to solve the game between universities in Cases 2 and 3, characterized by the private university providing higher quality than the public, and we obtain the following result:

Proposition 2 *There does not exist an equilibrium in which the private university provides higher quality than the public university.*

Proof. See the Appendix. ■

In the first part of the proof, we show that the private institution cannot provide higher quality than the public, being less selective, and then, the market partition described in Case 2 is not an equilibrium. In the second part of the proof, we show that there does not exist an equilibrium in which the private institution has higher quality and admission standards than the public university (Case 3). Intuitively, this last result holds because in the first stage the public institution finds optimal, given private quality, to provide the monopoly level of school quality. The best response of the private university in the next stage is to set a price so high that no student is willing to attend this university. This result may be explained by the admission policy based on tuition fees followed by the private university, which allows the public institution to affect admissions at the private university, just varying public school quality. Moreover, the use of exams, combined with a zero-price policy by the public university allows this institution to preserve its monopolistic position in the market. The public university is able to attract the best students while the private can only absorb the residual demand.

From the result stated in Proposition 2 it follows immediately that there exists a unique equilibrium in the game between universities, and in such equilibrium, the public university has higher quality than the private. We now turn to investigate the impact that the presence of a private university has on total welfare compared to public monopoly.

Proposition 3 *Competition raises total welfare compared to public monopoly.*

Proof. See the Appendix. ■

This result follows from the fact that students attending the public university under monopoly are the same students attending this institution under competition, and they obtain the same school quality in both cases. The presence of a private university in the higher education market raises total welfare because some students who did not attend university in the public monopoly can now educate in the private university. This, in turn, increases total welfare.

Summarizing, to solve the game between universities we first identify all possible market partitions in which a public and a private university may compete in the same market. Since the public university chooses a zero price in all market partitions, the number of cases of analysis is reduced to just three. Then, we proceed to solve the game in these cases, and we find that there exists a unique equilibrium in which the public university has higher quality than the private. We may find a simple explanation for this result: in the absence of budgetary constraints, the public university chooses an admission policy, based exclusively on exams, that provides this institution with a competitive advantage over the private in attracting the best students.

5 Concluding Remarks

In this paper we investigate how the recent expansion of private universities affects the European higher education market, focusing on the optimal reaction of public universities to the presence of private institutions, and the potential welfare effects derived from the creation of new universities. In our analysis, public and private universities have different objectives; while the public university maximizes public surplus, the private maximizes profits.

After characterizing all possible configurations of quality, prices and exams in which both institutions are competing for students in the higher education market, we find that there exists a unique equilibrium. In such an equilibrium, the public university always provides higher educational quality than the private institution. This result may be explained by the admission policy chosen by the public, based on exams and no fees, versus the price policy followed by the private institution. Our results point out that the use of exams not only allows the public university to behave as a monopoly but also prevents the private institution from providing a higher quality than the public university.

The fact that public universities are not subject to budgetary constraints is crucial

in explaining the inability of private institutions to attract the best students. In this situation, the public sector can preserve its monopolistic position in the market. We believe that our results fit quite well the main features of the higher education market in many European countries, characterized by the presence of public universities using primarily exams to allocate their students, and providing higher quality than private institutions.

However, we do observe that in European countries there are also some private universities with higher admission standards than public institutions. The existence of objectives different from profit maximization in some private institutions (such as Catholic universities) may help explain this observation. In the context of our model, the relaxation of the hypothesis on the absence of budgetary constraints in funding public institutions may lead to an equilibrium in which private institutions provide higher quality than public ones.

Finally, we cannot ignore that the European higher education market may change dramatically in the following years. Sluggish economic performance and high unemployment in Europe have restricted the funding available for higher education and some countries have initiated, or plan to initiate, processes of “privatization” of public higher education. These tendencies are expected to affect competition between public and private universities. We leave these issues for further research.

6 Appendix

Proof of Proposition 1. We prove the existence of this equilibrium. Firstly, we show that $Q_b > Q_v$. From (9) and (11), we obtain that public educational quality is given by the following expression:

$$C'(Q_b) = \frac{1 + \frac{C(Q_b)}{Q_b}}{2}. \quad (19)$$

Since the cost function is increasing and convex in quality, then $Q_b > Q_v$ if and only if $C'(Q_b) > C'(Q_v)$. From the condition for optimal private quality, given by (18), we obtain that $C'(Q_v) < a_v$ and then, a sufficient condition for $Q_b > Q_v$ is

$$a_v < \frac{1 + \frac{C(Q_v)}{Q_v}}{2}.$$

From (15) and the existence of an interior optimal private price, we obtain that $1 - 2p_v + C(Q_v) > 0$, and then, the private price is increasing in selectivity at the public university, given by a_b^E . Since $a_b^E < 1$, the following inequality holds:

$$a_v \leq \frac{(1 - p_v) \left(1 + \frac{C(Q_v)}{Q_v}\right) - (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))} < \frac{1 + \frac{C(Q_v)}{Q_v}}{2}.$$

Hence, $C'(Q_b) > C'(Q_v)$ which implies that $Q_b > Q_v$.

Secondly, we can show that the public institution is more selective than the private, i.e., $a_b^E > a_v$, where a_v is given by (16), if the following condition is satisfied:

$$a_b^E > \frac{(1 - p_v) \left(a_b^E + \frac{C(Q_v)}{Q_v}\right) - a_b^E (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))} \Leftrightarrow a_b^E > \frac{C(Q_v)}{Q_v}.$$

The above inequality holds since we know from (9) that $a_b^E = \frac{C(Q_b)}{Q_b}$, and $Q_b > Q_v$ implies that $\frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}$ provided that $\frac{C(Q)}{Q}$ is strictly increasing in Q . Hence, $a_b^E > a_v$. ■

Proof of Proposition 2. We prove that in equilibrium, the private university does not provide a quality higher than the public university. We start showing that $Q_b < Q_v$, $p_b < p_v$ and $a_b \geq a_v$ (Case 2) is not an equilibrium. Let assume that $Q_b < Q_v$, $p_b < p_v$ and $a_b \geq a_v$. Hence, the objective functions of the public and the private university are respectively:

$$U_b = \int_{p_b}^{p_v} \int_{a_b}^1 (a_i Q_b - C(Q_b)) da dw, \quad (20)$$

$$U_v = \int_{p_v}^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw, \quad (21)$$

where $a_b = \max\{a_b^E, \hat{a}_b\}$ and $a_v = \max\{a_v^E, \hat{a}_v, \tilde{a}_v\}$.

We solve the game backwards; in the last stage of the game, it is easy to see that the public institution is going to choose exams as under monopoly: $a_b^E = \frac{C(Q_b)}{Q_b}$. The private institution does not use exams because its utility is strictly decreasing in a_v . In the second stage of the game, the public institution finds optimal to set a public price equal to zero, while the private university chooses its price as follows:

$$(1 - a_v)(1 - 2p_v + C(Q_v)) - \frac{da_v}{dp_v}(1 - p_v)(p_v - C(Q_v)) = 0, \quad (22)$$

where $a_v = \tilde{a}_v$ since $p_b = 0$ and then, $\tilde{a}_v > \hat{a}_v$.

In the first stage of the game, both universities choose educational quality simultaneously. The private university behaves as a monopoly and private quality is determined according to this condition:

$$C'(Q_v) = \frac{a_v}{1 - a_v} \left(a_v - \frac{C(Q_v)}{Q_v - Q_b} \right), \quad (23)$$

Optimal quality satisfies $C'(Q_v) > 0$ if and only if $a_v - \frac{C(Q_v)}{Q_v - Q_b} > 0$. Notice that $\frac{C(Q_v)}{Q_v - Q_b} > \frac{C(Q_v)}{Q_v}$ and $\frac{C(Q)}{Q}$ is increasing in Q . Thus, we obtain that if $Q_b < Q_v$ then $\frac{C(Q_b)}{Q_b} < \frac{C(Q_v)}{Q_v}$, which implies that $a_b < a_v$, provided that $a_b = a_b^E = \frac{C(Q_b)}{Q_b}$. This contradicts $a_b \geq a_v$ and then, the market partition described in Case 2 is not an equilibrium.

Now we turn to show that the market partition characterized by $Q_b < Q_v$, $p_b < p_v$ and $a_v > a_b$ (Case 3) is not an equilibrium either. The utility of the public university is the following:

$$U_b = \int_{p_b}^{p_v} \int_{a_v}^1 (a_i Q_b - C(Q_b)) da dw + \int_{p_b}^1 \int_{a_b}^{a_v} (a_i Q_b - C(Q_b)) da dw, \quad (24)$$

where $a_b = \max\{a_b^E, \hat{a}_b\}$ and $a_v = \max\{a_v^E, \tilde{a}_v, \hat{a}_v\}$.

Private university's utility is the following:

$$U_v = \int_{p_v}^1 \int_{a_v}^1 (p_v - C(Q_v)) da dw. \quad (25)$$

The public institution chooses exams optimally as under monopoly, according to (9). The private institution does not use exams since private utility is strictly decreasing in a_v . Optimal public prices are zero since public utility is strictly decreasing in p_b :

$$\begin{aligned} \frac{dU_b}{dQ_b} &= -\frac{(1 - a_v^2)}{2} Q_b - \frac{(a_v^2 - a_b)}{2} Q_b + C(Q_b) (1 - a_b) \\ &+ \frac{da_b}{dp_b} (1 - p_b) (-a_b Q_b + C(Q_b)) \\ &+ \frac{da_v}{dp_b} (1 - p_v) (a_v Q_b - C(Q_b)) < 0, \end{aligned} \quad (26)$$

since $-\frac{(1 - a_v^2)}{2} Q_b - \frac{a_v^2 - a_b}{2} Q_b + C(Q_b) (1 - a_b) < 0$, $\frac{da_b}{dp_b} \geq 0$, $-a_b Q_b + C(Q_b) \leq 0$, $\frac{da_v}{dp_b} < 0$ and $(a_v Q_b - C(Q_b)) > 0$, since $a_v > a_b^E = \frac{C(Q_b)}{Q_b}$.

The optimal private price is determined by (22), where $a_v = \tilde{a}_v = \frac{p_v}{Q_v - Q_b}$. The ability of the least able student accepted into the private university may be written as follows:

$$\hat{a}_v = \frac{(1 - p_v) \left(1 + \frac{C(Q_v)}{Q_v - Q_b} \right) - (p_v - C(Q_v))}{2(1 - p_v) - (p_v - C(Q_v))}. \quad (27)$$

From (27), we observe that some students are willing to attend the private university, i.e., $\hat{a}_v < 1$, if and only if public quality is sufficiently low compared to private quality: $Q_b \leq Q_v - C(Q_v)$.

Educational quality at the private university is chosen according to the following condition:

$$C'(Q_v) = \tilde{a}_v \left(1 - \frac{p_v - C(Q_v)}{1 - p_v} \right).$$

Public university's utility may be written as follows:

$$U_b = \begin{cases} \int_{p_b}^{p_v} \int_{\tilde{a}_v}^1 (a_i Q_b - C(Q_b)) da dw \\ + \int_{p_b}^1 \int_{a_b^E}^{\tilde{a}_v} (a_i Q_b - C(Q_b)) da dw & \text{if } Q_b < Q_v - C(Q_v), \\ \int_{p_b}^1 \int_{a_b^E}^1 (a_i Q_b - C(Q_b)) da dw & \text{if } Q_b \geq Q_v - C(Q_v). \end{cases} \quad (28)$$

Note that if $Q_b < Q_v - C(Q_v)$, then, $\tilde{a}_v < 1$ from (27). On the contrary, if public quality is sufficiently high, $Q_b \geq Q_v - C(Q_v)$, then $\tilde{a}_v = 1$ and thus, the public university is a monopoly in the market. We identify this threshold level of public quality as $\bar{Q}_b \equiv Q_v - C(Q_v)$.

Note first that public utility under monopoly is higher than under competition:

$$\int_{p_b}^1 \int_{a_b^E}^1 (a_i Q_b - C(Q_b)) da dw \geq \int_{p_b}^{p_v} \int_{\tilde{a}_v}^1 (a_i Q_b - C(Q_b)) da dw + \int_{p_b}^1 \int_{a_b^E}^{\tilde{a}_v} (a_i Q_b - C(Q_b)) da dw, \forall Q_b \in (0, \bar{Q}_b],$$

since $\tilde{a}_v \leq 1$ and the utility of the public university under competition is increasing in the ability of the least able student in the private university, \tilde{a}_v .

We now turn to show that it is optimal for the public university, given private quality, to choose a level of quality that kicks the private institution out of the market. The argument of the proof is the following: if the minimum level of quality required to expel the private institution from the market (and become a monopoly), \bar{Q}_b , is smaller or equal than optimal quality at monopoly, Q_b^m , we can prove that there does not exist an equilibrium with the private institution providing higher quality than the public. Given private quality, it is optimal for the public university to provide the monopoly level of school quality, Q_b^m , and this level is sufficiently high to expel the private university from the market.

We proceed to show that $Q_b^m \geq \bar{Q}_b$ for all possible levels of private quality, Q_v . Optimal quality under monopoly, Q_b^m , is determined optimally by the following condition:

$$C''(Q_b) = \frac{1 + \frac{C(Q_b)}{Q_b}}{2}, \quad (29)$$

where $C(Q_b) = \alpha Q_b^k$, $k > 1$, $\alpha > 0$. Therefore, substituting $C(Q_b)$ and $\frac{dC(Q_b)}{dQ_b}$, we solve (29) for Q_b^m :

$$Q_b^m = \left(\frac{1}{\alpha(2k-1)} \right)^{\frac{1}{k-1}}. \quad (30)$$

The minimum level of public quality required for the public university to become a monopoly is $\bar{Q}_b = Q_v - C(Q_v)$, and this level is maximum when private quality, Q_v , satisfies, $C'(Q_v) = 1$. In this case, \bar{Q}_b is the following:

$$\bar{Q}_b = \left(\frac{1}{\alpha k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right). \quad (31)$$

We prove that $Q_b^m \geq \bar{Q}_b$ if the following condition holds: $\left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right) \leq 1$. The above inequality holds since $\left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right)$ is strictly increasing in k and $\lim_{k \rightarrow \infty} \left(\frac{2k-1}{k} \right)^{\frac{1}{k-1}} \left(1 - \frac{1}{k} \right) = 1$, and thus, $Q_b^m \geq \bar{Q}_b$. Therefore, this equilibrium does not exist. ■

Proof of Proposition 3. Total welfare raises with respect to the public monopoly if the surplus generated by the private university, S_v , is positive:

$$S_v = \int_{p_v}^1 \int_{\hat{a}_v}^{a_b^E} (a_i Q_v - C(Q_v)) da > 0. \quad (32)$$

Notice that $S_v > 0$ if and only if $\frac{a_b^E + \hat{a}_v}{2} > \frac{C(Q_v)}{Q_v}$. Using the expression for \hat{a}_v given by (16), we obtain that $\frac{a_b^E + \hat{a}_v}{2} > \frac{C(Q_v)}{Q_v}$ if and only if $a_b^E > \frac{C(Q_v)}{Q_v}$. The properties of the cost function ensures that $a_b^E = \frac{C(Q_b)}{Q_b} > \frac{C(Q_v)}{Q_v}$ provided that $Q_b > Q_v$ and hence, $S_v > 0$. ■

References

- [1] Altbach, P.G. (1997). "Let the Buyer Pay: International Trends in Funding for Higher Education," *International Higher Education* Number 9, Fall 1997.

- [2] Carneiro, P. and Heckman, J.J. (2002). "The Evidence on Credit Constraints in Post-Secondary Schooling," *The Economic Journal* 112, pp. 705-734.
- [3] Coleman, J.S., Campbell, E.Q., Hobson, C.J., McPartland, J., Mood, A.M., Weinfeld, F.D. and York, R.L. (1966). *Equality of Educational Opportunity* (Washington: U.S. Department of Health, Education, and Welfare, National Center Educational Statistics).
- [4] De Fraja, G. and Iossa, E. (2002). "Competition among Institutions and the Emergence of the Elite University," *Bulletin of Economic Research* 54 (3), pp. 275-293.
- [5] Del Rey, E. (2001). "Teaching versus Research: a Model of State University Competition," *Journal of Urban Economics*, 49 (2), pp. 356-373.
- [6] Del Rey, E. and Romero, L. (2004). "Prices versus Exams as Strategic Instruments for Competing Universities," *Working Paper N12 Department of Economics, Universitat de Girona*.
- [7] Epple, D. and Romano, R. (1998). "Competition between Private and Public Schools, Vouchers, and Peer-Group Effects," *American Economic Review*, 88, pp. 33-62.
- [8] Fernández, R. (1998). "Education and Borrowing Constraints: Tests versus Prices", *NBER Working Paper* No 6588.
- [9] Fernández, R. and Galí, J. (1999). "To Each According to...? Tournaments, Markets and the Matching Problem under Borrowing Constraints," *Review of Economic Studies* 66 (4), pp.799-824.
- [10] Hanushek, E.A. (1986). "The Economics of Schooling: Production and Efficiency in Public Schools," *Journal of Economic Literature* 49(3), pp. 1141-1177.
- [11] Oliveira, T. (2004). "How public and private universities compete for students", mimeo, University of York.
- [12] Rothschild, M. and White, L. (1995). "The Analytics of the Pricing of Higher Education and Other Services in which Customers are Inputs", *Journal of Political Economy* Vol. 103.

- [13] The National Commity Inquiry into Higher Education (1997). *Higher Education in the learning society*, Annex A.
- [14] Whinston, G. (1999). “Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education”, *Journal of Economic Perspectives* Vol. 13, No 1, pp. 13-36.