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Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-49

STOCHASTIC VOLATILITY MODELS AND THE TAYLOR EFFECT

Alberto Mora-Galán, Ana Pérez and Esther Ruiz*

Abstract

It has been often empirically observed that the sample autocorrelations of absolute financial returns are larger than those of squared returns. This property, know as *Taylor effect,* is analysed in this paper in the Stochastic Volatility (SV) model framework. We show that the stationary autoregressive SV model is able to generate this property for realistic parameter specifications. On the other hand, the *Taylor effect* is shown not to be a sampling phenomena due to estimation biases of the sample autocorrelations. Therefore, financial models that aims to explain the behaviour of financial returns should take account of this property.

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Keywords: autocorrelations, nonlinear transformations, conditional heteroscedasticity, financial returns.

* Mora-Galán, Planificación Operativa y Control de Gestión. Unión FENOSA Gas; Pérez, Departamento de Economía Aplicada, Universidad de Valladolid; Ruiz, Dpt. Estadística. Universidad Carlos III Madrid. E-mail: ortega@est-econ.uc3m.es. The last two authors acknowledge financial support from the Spanish government project BEC2002.03720.

1. INTRODUCTION

It is by now well established in the Financial Econometrics literature that high frequency time series of financial returns are often uncorrelated but not independent because there are non-linear transformations which are positively correlated. Furthermore, Taylor (1986) analyses 40 series of returns and observes that the sample autocorrelations of absolute returns seem to be larger than the sample autocorrelations of squares. A similar phenomena is observed by Ding *et al.* (1993) who examine daily returns of the S&P500 index and conclude that, for this particular series, the autocorrelations of absolute returns raised to the power θ are maximized when θ is around 1, that is, the largest autocorrelations are found in the absolute returns. Granger and Ding (1995) denote this empirical property of financial returns as *Taylor effect.* Therefore, if y_t, $t = 1, ..., T$, is the series of returns and $r_\theta(k)$ denotes the sample autocorrelation of order k of $|y_t|^\theta$, $\theta > 0$, the Taylor effect can be defined as follows

$$r_1(k) > r_\theta(k) \text{ for any } \theta \neq 1. \tag{1}$$

However, Granger and Ding (1994) and Ding and Granger (1996) analyze several series of daily exchange rates and individual stock prices, and conclude that the maximum autocorrelation is not always obtained when $\theta = 1$ but for smaller values of θ. Nevertheless, they point out that the autocorrelations of absolute returns are always larger than the autocorrelations of squares; see also Granger *et al.* (1999). Muller *et al.* (1998) and Dacorogna *et al.* (2001) obtain similar results analyzing tick-by-tick observations of exchange rates. Consequently, Malmsten and Teräsvirta (2004) have recently considered a more restricted alternative definition of the Taylor effect as follows

$$r_1(k) > r_2(k). \tag{2}$$

Anyhow, significant autocorrelations of power transformations of absolute returns are often related with conditional heteroscedasticity and, therefore, with the dynamic evolution of volatilities; Luce (1980) uses axiomatic arguments to show that $|y_t|^\theta$ is an appropriate class of risk measures. Two main types of models have been usually fitted to represent this evolution: Generalized Conditionally Autoregressive Heteroscedasticity (GARCH) models of Engle (1982) and Bollerslev (1986) and Stochastic Volatility (SV) models of Taylor (1982); see Carnero *et al.* (2004a) for the main differences between both alternatives.

In the GARCH framework, the autocorrelation function (acf) of $|y_t|^\theta$ is unknown, except for $\theta = 2$. Therefore, results on whether GARCH-type models are able to represent the Taylor effect are based on simulations. For example, Ding *et al.* (1993) uses Monte Carlo simulations to show that one particular GARCH model with Gaussian disturbances generates the Taylor effect. However, He and Teräsvirta (1999) extend the Monte Carlo design to several Gaussian GARCH(1,1) models and conclude that they do not always generate the Taylor property as defined in (2). They also analyze the Taylor effect in the absolute-value GARCH (AVGARCH) model, where the analytical expressions of the autocorrelations of absolute and square returns are available, and conclude that this model has the Taylor property if the kurtosis is sufficiently large. However the difference between both autocorrelations is, in any case, very small. Finally, Malmsten and Teräsvirta (2004) show, for the exponential-GARCH (EGARCH) model, that the Taylor property holds for high values of the kurtosis. However, looking at their results, it is possible to observe that for empirically relevant values of the kurtosis, the difference between autocorrelations of squares and absolute returns is very small.

The presence of the Taylor effect in conditionally heteroscedastic series can be better analyzed in the context of SV models, because, in this case, the acf of $|y_t|^\theta$ is known for any value of θ. Harvey (1998) derives the expression of this acf for a general SV model and suggests that it is not possible to obtain general results on the value of θ that maximizes this function. On the other hand, Harvey and Streibel (1998) show that, for some particular AutoRegressive SV (ARSV) models, the larger the variance of the volatility, the smaller the value of θ that maximizes the autocorrelations. Another important reason to analyze the Taylor effect in the context of SV models is that they are close to the models often used in Financial Theory; see Ghysels et al. (1996) and Shephard (1996).

As we mentioned before, the Taylor effect is a phenomena empirically observed when comparing sample autocorrelations of different powers of absolute returns. However, in conditionally heteroscedastic models, these autocorrelations may have large negative biases; see, for example, Bollerslev (1988), He and Teräsvirta (1999) and Pérez and Ruiz (2003). If the sample autocorrelations associated with different values of θ have different biases, the Taylor property could turn out to be just a sample effect. Consequently, it is important to distinguish whether this property is a population effect or it is a consequence of the negative biases of the sample autocorrelations of powers of absolute returns. In the former case, the model used to represent the dynamic evolution of returns must be able to generate it while, if the Taylor effect is an estimation problem, the model does not need to have this property.

The objective of this paper is two fold. First, we analyze whether the Taylor property holds in ARSV models. Second, we perform exhaustive Monte Carlo experiments to analyze, in the context of ARSV models, whether the Taylor effect could be attributed to a sampling estimation problem or it is a characteristic of the model that should be represented.

The paper is organized as follows. In section 2, we describe the main statistical properties of ARSV models with special focus on the Taylor property. Section 3 presents the results of several simulation experiments to investigate whether the Taylor effect could be attributed to estimation biases. Section 4 describes the empirical properties of several series of real financial returns in order to determine whether they have the Taylor property. It also examines the influence of outliers on the presence of such property. Finally, section 5 summarizes the main conclusions.

2. THE TAYLOR PROPERTY IN SV MODELS

Taylor (1982) proposed to represent the dynamic evolution of volatility using SV models that specify the volatility as a latent process. One interpretation of the latent volatility is that it represents the random arrival of new information into the market; see, for example, Clark (1973) and Tauchen and Pitts (1983). In the simplest case, the ARSV(1) model assumes that the log-volatility is an AR(1) process. Consequently, the series of returns is given by

$$\begin{aligned} y_t &= \sigma_* \varepsilon_t \sigma_t \\ \log(\sigma_t^2) &= \phi \log(\sigma_{t-1}^2) + \eta_t \end{aligned} \tag{3}$$

where σ_* is a scale parameter that removes the necessity of introducing a constant term in the equation of the log-volatility, ε_t is an independent white noise process with unit variance and symmetric distribution, σ_t^2 is the volatility at time t and η_t is a Gaussian white noise with variance σ_η^2, distributed independently of ε_t. Although the Gaussianity assumption of η_t may seem rather *ad hoc*, Andersen, Bollerslev, Diebold and Ebens (2001) and Andersen, Bollerslev, Diebold and Labys (2001, 2003) show that the empirical distribution of the log-volatility of several exchange rates and index returns could be adequately approximated by the Normal distribution.

The main statistical properties of ARSV models have been reviewed by Ghysels *et al.* (1996) and Shephard (1996). In particular, the series y_t is stationary if the autoregressive parameter, ϕ, satisfies the restriction $|\phi| < 1$. Furthermore, it is well known that ARSV(1) series are leptokurtic even if the noise ε_t is assumed to be

Gaussian. In particular, the kurtosis of y_t, is given by

$$\kappa_y = \kappa_\varepsilon \exp(\sigma_h^2) \tag{4}$$

where $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$ is the variance of the log-volatility process and κ_ε is the kurtosis of the disturbance ε_t. Notice that if κ_ε is finite, the kurtosis of y_t is defined as far as it is stationary, i.e. if $|\phi| < 1$.

The dynamic properties of y_t appear in the acf of $|y_t|^\theta$, derived by Harvey (1998), that is given by

$$\rho_\theta(k) = \frac{\exp\left(\frac{\theta^2}{4}\sigma_h^2\phi^k\right) - 1}{\omega_\theta \exp\left(\frac{\theta^2}{4}\sigma_h^2\right) - 1}, \quad k \geq 1 \tag{5}$$

where $\omega_\theta = \frac{E(|\varepsilon_t|^{2\theta})}{\left\{E(|\varepsilon_t|^\theta)\right\}^2}$. For example, if ε_t is Gaussian, ω_θ is given by $\omega_\theta = \frac{\Gamma(\theta+\frac{1}{2})\Gamma(\frac{1}{2})}{\left\{\Gamma(\frac{\theta}{2}+\frac{1}{2})\right\}^2}$. When $\theta = 2$, ω_2 is the kurtosis of ε_t given by 3 and if $\theta = 1$, $\omega_1 = \frac{\pi}{2}$. On the other hand, if ε_t has a Student-t distribution with $\nu > 5$ degrees of freedom, then $\omega_\theta = \frac{\Gamma(\theta+\frac{1}{2})\Gamma(-\theta+\frac{\nu}{2})\Gamma(\frac{1}{2})\Gamma(\frac{\nu}{2})}{\left\{\Gamma(\frac{\theta}{2}+\frac{1}{2})\Gamma(-\frac{\theta}{2}+\frac{\nu}{2})\right\}^2}$, with $\theta < \nu/2$.[1] In any case, the autocorrelations in (5), whenever defined, are always positive and their rate of convergence towards zero is controlled by the autoregressive parameter ϕ. Consequently, this parameter is often related with the persistence of shocks to the volatility process.

Looking at expression (5), it is rather obvious that the value of θ that maximizes $\rho_\theta(k)$ is a very complicated non-linear function of the lag k, the distribution of the errors ε_t and the parameters that govern the volatility dynamics, i.e. ϕ and σ_η^2. Given that it is not possible to obtain a general analytical expression of the value of θ that maximizes $\rho_\theta(k)$, we simplify the problem by fixing the lag of the autocorrelations to $k = 1$ and analyzing how the distribution of ε_t and the parameters values affect the autocorrelation function. In order to do that, we have maximized numerically $\rho_\theta(1)$

[1] Notice that when the errors have a Student-t_ν distribution, the autocorrelations of $|y_t|^\theta$ are only defined if $\theta < \nu/2$.

with respect to θ, for several ARSV(1) models with two distribution errors, namely Gaussian and Student-7. Table 1 reports the results. This table illustrates that, for a given kurtosis of the returns, κ_y, the value of θ that maximizes $\rho_\theta(1)$ depends on the distribution of the errors. For example, in a model with $\kappa_y \approx 5$, the value of θ that maximizes $\rho_\theta(1)$ is approximately 1.3 when the errors are Gaussian while it is closer to 1 when they are Student-7. In general, it seems that, given the kurtosis, the value of θ that reaches the maximum is larger when the errors are Gaussian than when the errors have a leptokurtic distribution as the Student-7.

On the other hand, Table 1 also shows that for a given distribution, the value of θ that maximizes $\rho_\theta(1)$ decreases as the variance of the log-volatility process, σ_h^2, and, consequently the kurtosis of returns, increases. When the errors are Gaussian and the kurtosis of returns is close to three, i.e. returns are nearly Gaussian and homoscedastic, the autocorrelation of order one is maximum for squares. The value of θ that maximizes $\rho_\theta(1)$ decreases with κ_y and becomes approximately equal to one when the kurtosis is between 5 and 37. When the errors are leptokurtic with a Student-7 distribution and the kurtosis is not unrealistically large, the autocorrelations are always maximized for absolute returns. Another remarkable feature is that, for any of the two distributions considered, the value of θ that maximizes $\rho_\theta(1)$ is only smaller than 1 when the kurtosis of returns is too large as to represent kurtosis of interest from an empirical point of view.

In order to analyze whether the behavior of $\rho_\theta(1)$ keeps the same for other lags, Figures 1 and 2 plot $\rho_\theta(k)$ as a function of θ, for $k = 1, 5$, 10, 20 and 50 and for different ARSV(1) models with Gaussian and Student-7 errors, respectively. In these figures, the maxima of the autocorrelations of a given order are shown by the bullet sign. These figures illustrate that, for a given model, the value of θ that maximizes $\rho_\theta(k)$ is approximately the same for different lags. Therefore, maximization of $\rho_\theta(k)$ will mainly depend on the parameter values of (ϕ, σ_η^2) and the distribution of ε_t.

Regarding the parameter values, it is possible to observe that, for any of the two distributions considered, if σ_η^2 is fixed, increasing the corresponding value of ϕ shifts the peak of the autocorrelation to the left, i.e. the value of θ that maximizes the autocorrelations decreases as ϕ increases; compare with the results in Maslmten and Teräsvirta (2004). On the other hand, for fixed ϕ increasing σ_η^2 also decreases the value of θ. Comparison of both figures confirms our previous result that $\rho_\theta(k)$ reaches its maximum at a smaller value when the distribution error is Student-7 than when it is Gaussian. Moreover, it also confirms that, in both cases, the ARSV(1) model is able to generate Taylor effect for the more realistic parameter specifications. Finally, notice that in the models with less persistence of the volatility ($\phi = 0.9$ or 0.95) and/or smoothest evolution of the volatility ($\sigma_\eta^2 = 0.01$) the curves plotted in Figures 1 and 2 are rather flat. Consequently, the autocorrelations are approximately equal whatever power transformation we consider.

We now focus on the Taylor property as defined in (2). We have tick-marked in Table 1 the models that produce this Taylor effect on the first order autocorrelations, i.e. those where the first order autocorrelation of absolute values is larger than the corresponding autocorrelation of squares. This allow us to highlight that, if κ_y is relatively small, the ARSV(1) model does not have the Taylor effect, while it appears if κ_y is approximately larger than 4, as it is often the case in empirical applications. To further illustrate this result, Figure 3 plots the autocorrelations of order 1 of absolute and squared returns as a function of the parameters ϕ and σ_η^2 when the errors are Gaussian and Student-7. This plot clearly shows that, for the more realistic models, with ϕ close to one and σ_η^2 small, correlations of absolute values are always larger than those of the squared transformation. Moreover, for the same parameter values, the differences between both autocorrelations are larger the larger the kurtosis of the distribution errors. On the other hand, for a given persistence of shocks to volatility, measured by ϕ, this difference is larger the larger the variance σ_η^2. If σ_η^2 is close to zero, i.e. returns are nearly homoscedastic, the autocorrelations of absolute and squared observations are nearly the same. Finally, if σ_η^2 is fixed, the difference between both autocorrelations increases as ϕ approaches one.

3. FINITE SAMPLE TAYLOR EFFECT

In previous section, we have seen that the stationary ARSV(1) model does not always satisfy the Taylor property as defined in (1) or (2). However, it is not clear yet whether it should do it, even if this property is empirically observed. As we mentioned in the Introduction, the sample autocorrelations of powers of absolute observations may have severe biases in series generated by SV models. If the biases associated with different transformations were different, it could be possible to empirically observe the Taylor effect even if it is not a population effect and *viceversa*. Consider, for example, that, as pointed out by Pérez and Ruiz (2003), the biases of the sample autocorrelations of squared returns are negative and larger in magnitude than the biases of absolute returns. In this case, it could be possible that, even if the population autocorrelations of squared and absolute observations were equal, the sample autocorrelations of squares are smaller than the sample autocorrelations of absolute returns.

In order to analyze whether the ARSV(1) model should represent the Taylor effect once it has been empirically observed, we have carried out extensive Monte Carlo experiments, that are summarized in this section. All the results are based on 1000 replicates of series generated by ARSV(1) models with autoregressive parameter $\phi = \{0.9, 0.95, 0.98, 0.99\}$ and variance $\sigma_\eta^2 = \{0.01, 0.05, 0.1\}$. In all cases, the scale parameter has been fixed to one, i.e. $\sigma_* = 1$ and the distribution of the errors is assumed to be Gaussian or Student-7. The sample sizes are $T = 500$, 1000 and 5000. For each series, we have computed the autocorrelations $\rho_\theta(k)$ for $\theta = 0.5$, 1, 1.5 and 2 and $k = 1$, 10, 20 and 50. Tables 2 and 3 report the Monte Carlo results when

$\phi = 0.98$, $\sigma_\eta^2 = \{0.01, 0.05\}$ and the errors are Gaussian and Student-7 respectively[2]. These tables report, for each model, lag and exponent, the sample mean and standard deviation (in parenthesis) of the estimated autocorrelations through the Monte Carlo replicates together with the corresponding population values.

The first conclusion from Table 2 is that, regardless of the transformation parameter θ, the sample autocorrelations are always negatively biased and their biases converge asymptotically to zero. Nevertheless, if we focus on relative biases, important differences arise for different values of θ. For moderate sample sizes, the relative biases are larger the larger is θ. For example, if $T = 500$ and $\sigma_\eta^2 = 0.05$, it can be easily checked that the relative biases of the first order autocorrelations are -19.31%, -21.02%, -23.23% and -23.92% when θ is 0.5, 1, 1.5 and 2, respectively. On the other hand, it is important to notice that, for the two largest sample sizes, the relationship between autocorrelations of a fixed order k for different values of θ is generally the same in the population and in the sample. In Table 2, there is only one exception to this result when $T = 500$ and $\sigma_\eta^2 = 0.01$. In this case, $\rho_2(1) = 0.098$ is slightly larger than $\rho_1(1) = 0.095$, while the Monte Carlo mean of the sample autocorrelations is 0.071 for squares and slightly larger, 0.075, for absolute observations. Therefore, in this particular case, the Taylor effect is not a population effect and it could be attributed to sample biases. However, sample sizes as small as $T = 500$ are not very common in financial applications.

Finally, although it is not a main goal of this paper, Table 2 also shows that the standard deviation of the sample autocorrelations increases with the transformation parameter, specially for small lags. Furthermore, the convergence of the autocorrelations is $\sqrt{T}$ when $\theta = 0.5$ and it is slower as θ increases.

Comparing Tables 2 and 3, we can observe that the theoretical autocorrelations

[2] Results for other values of ϕ and σ_η^2 are very similar and they are not reported to save space. They can be obtained from the authors upon request.

are smaller when the errors are Student-7 than when they are Gaussian and that the relative difference between the autocorrelations in both cases is larger the larger is the transformation parameter. Furthermore, when $\sigma_\eta^2 = 0.05$, the Taylor effect, as defined in (2), is more pronounced in Table 3. However, notice that when the errors are Student-t the autocorrelations could be maximized for values of θ smaller than 1. So, the Taylor effect as defined in (1) does not hold. With respect to the estimated autocorrelations, it turns out that, as in Table 2, they keep the same order relationship observed among the population autocorrelations. Finally, notice that although the sample autocorrelations are smaller than in the Gaussian case, the Monte Carlo standard deviations are similar. Therefore, the relative precision of the estimated autocorrelations is smaller when the errors are Student-7 than when they are Gaussian.

To summarize, we can conclude that the sample properties of the estimated autocorrelations of powers of absolute returns are more appropriate than those of the squares, in the sense that the relative biases and standard deviations are smaller, the smaller the exponent θ. This is important if we take into account that in empirical applications, the usual practice is to chose between $\theta = 1$ or $\theta = 2$. To this respect, the results in this paper agree with previous papers by Harvey and Streibel (1998) and Pérez and Ruiz (2003).

Figure 4 plots, in the top panel, the population acf for an ARSV(1) model with $\left\{\phi = 0.98, \sigma_\eta^2 = 0.05\right\}$ for the four transformations considered and, in the bottom panels, the corresponding mean correlograms for three sample sizes, $T = 500$, 1000 and $T = 5000$. Left hand-side panels correspond to Normal errors while the panels on the right come from Student-7 errors. This figure shows that sample autocorrelations keep the same order, as functions of θ, as the theoretical ones, in spite of being negatively biased. When $T = 500$ or 1000, the bias can be remarkable at some lags, but it never contributes to mask the Taylor effect when this exits.

4. EMPIRICAL APPLICATION

In this section, we describe the empirical properties of several daily series of financial returns with the goal of determining whether they have the Taylor property. We also analyze whether the ARSV(1) model is able to represent the pattern of the sample autocorrelations of these real data. Finally, we examine empirically the effect of outliers on the Taylor property.

4.1. Empirical analysis of Taylor effects on financial returns

The data set we analyze in this paper includes four daily exchange rates against the US Dollar (USD): the Euro (EU), from the 4th of January 1993 to the 31st December 2002, British Pound (BP) and Yen, from the 5th and 15th of January 1979, respectively, to the 31st December 2002 and Canadian Dollar (CAN), from the 4th of January 1971 to the 31st December 2002. We also consider four indexes of stock exchange markets of New York (SP500), Tokyo (Nikkei225), London (FTSE100) and Madrid (IBEX35). These series span from the 6th of June 1960, 4th of January 1984, 2nd of April 1984 and 5th of January 1987, respectively, and end up the 31st December 2002[3]. The sample sizes appear in the first row of Table 4.

The series of daily closing prices, p_t, $t = 1, .., T$, have been transformed into returns as usual, leading to the series $y_t = 100 * \log(p_t/p_{t-1})$, which have been plotted in Figure 5. This figure shows that all the returns move around a zero mean and display volatility clustering, and some of them are affected by very large outliers. For example, the SP500, Nikkei and FTSE100 returns have a large negative observation dated on the Black Monday's crash in October 1987. We have also found that some of the series (BP, CAN, SP500 and IBEX35) have a very small although significant autocorrelation of order one. Therefore, previous to the analysis on conditional second

[3] We are very grateful to C. Chatfield and A. Trapletti for their help to obtain these series.

order moments, we have filtered these series by fitting MA(1) models. Here onwards, when we refer to these four series, we will be working with the residuals from those estimated models. The rest of returns series have been centered with their sample mean.

Table 4 reports, in panel a), several summary statistics describing the main dynamic and distributional properties of the eight series. In this table, we first notice that all the returns have kurtosis significantly larger than 3 and the Jarque-Bera test for Normality always rejects the null. Furthermore, the autocorrelations of squares and absolute returns up to order 10 and 50 are always significant when tested using the Box-Ljung statistics. Therefore, as expected, there is strong evidence favoring the presence of conditional heteroscedasticity. Furthermore, notice that the Box-Ljung statistics for absolute returns, $Q_1(k)$, are always larger than those for squares, $Q_2(k)$, supporting the presence of the Taylor effect as defined in (2). This feature is specially remarkable in the stock indexes. Finally, Table 4 also reports the autocorrelations of order 1 of absolute and squared returns. As postulated by the Taylor property, $r_1(1)$ is larger than $r_2(1)$ for EU, BP, CAN, SP500 and IBEX while this relationship is reversed for the other three series.

Figure 6 plots the correlograms of absolute and squared returns of the eight series up to lag 100. Looking at this figure, it is obvious that, with few exceptions for the very small lags, the autocorrelations of $|y_t|$ are always above those of y_t^2 for all the series considered, in agreement with property (2). The difference is more remarkable in the stock indexes and the CAN/USD exchange rate. Moreover, with the exception of Euro, the autocorrelations of both the squared and absolute returns are all positive even for very long lags.

In order to examine the Taylor property as defined in (1), Figure 7 plots, for the eight series considered, the sample autocorrelations of $|y_t|^\theta$ as a function of θ, for lags $k = 1, 5, 10, 20$ and 50. This figure shows that the behavior of these functions depends on the particular series analyzed. For example, the pattern of the Euro, CAN, SP500 and IBEX35 is similar to the one observed in Figures 1 and 2 for the theoretical ARSV(1) models, with the autocorrelations of all lags maximized at values close to or less than one. However, for the other series, the patterns are slightly different. In the BP exchange rate, the autocorrelation of order 20 is nearly constant for all values of the power transformation parameter, while the other lags are clearly maximized at values of θ close to one. On the other hand, the Nikkei and FTSE100 indexes behave quite similarly, with the first order autocorrelation being maximized for the squares while the others are maximized at values around one. Finally, Figure 7 illustrates the very peculiar behavior of the Yen autocorrelations with the autocorrelation of order 1 maximized at $\theta > 3$ and the autocorrelation of order 20 at $\theta = 2.5$. Later in this paper, we will examine whether these unexpected patterns can be attributed to the presence of outliers.

To analyze whether the ARSV(1) model is able to explain the features observed in Table 4 and Figures 6 and 7, we have fitted such model to each of the eight series of returns considered in this paper. There are several alternative methods proposed in the literature to estimate the parameters of the ARSV(1) model. The results in Broto and Ruiz (2004) suggest that, for large sample sizes, the estimates obtained using the Quasi Maximum Likelihood (QML) estimator of Harvey et al. (1994), although not efficient, are very similar to the ones obtained using the more computationally complicated alternative methods. Therefore, we estimate the parameters by the QML estimator. This estimator is based on linearizing the ARSV(1) in (3) by taking logarithms of squared returns, as follows

$$\begin{aligned} \log y_t^2 &= \mu + \log \sigma_t^2 + \xi_t \\ \log \sigma_t^2 &= \phi \log \sigma_{t-1}^2 + \eta_t \end{aligned} \tag{6}$$

where $\mu = \log \sigma_*^2 + E(\log \varepsilon_t^2)$ and $\xi_t = \varepsilon_t^2 - E(\log \varepsilon_t^2)$. If ε_t has a Student-t distribution with ν degrees of freedom then $E(\log \varepsilon_t^2) \cong -1.27 - \psi(\nu/2) + \log(\nu/2)$ and $\sigma_\xi^2 = \pi^2/2 + \psi'(\nu/2)$ where $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and trigamma functions respectively. When ε_t is a Gaussian process these moments are $E(\log \varepsilon_t^2) \cong -1.27$ and $\sigma_\xi^2 = \pi^2/2$. Expression (6) is a non-Gaussian state space model and the QML estimator is based on obtaining the prediction error decomposition form of the likelihood through the Kalman filter by treating ξ_t as if it were Gaussian.

The estimation results are reported in panel a) of Table 5, where it is possible to observe that all the estimates of σ_η^2 are significant. Therefore, as expected from the results in Table 4 and Figure 6, there is evidence of conditional heteroscedasticity. We can also observe that, as it is often the case in high frequency financial returns, the estimates of the persistence parameter, ϕ, are very close to unity. Table 5 also reports the degrees of freedom of the Student-t_v distribution implied by the estimates of σ_ξ^2. Carrying out a Wald test for the null $H_0 : \sigma_\xi^2 = \pi^2/2$, it is possible to conclude that the assumption of Gaussianity of ε_t seems adequate for Euro, CAN, Nikkei, FTSE and IBEX while BP and Yen are better represented assuming leptokurtic distributions. Finally, the evidence on SP500 is not conclusive.

Figure 8 plots the theoretical autocorrelations of $|y_t|^\theta$ in (5), calculated with the estimated parameters, as a function of θ, for lags $k = 1$, 5, 10, 20 and 50. Comparing Figures 7 and 8 it is possible to conclude that the sample and the theoretical patterns implied by the estimated ARSV(1) models are very similar for the Euro, SP500, BP, CAN and IBEX. In these series, the autocorrelations are maximized in both cases for values of θ closed to one. However, the first order sample autocorrelation of the Nikkei and FTSE in Figure 7 is maximized at $\theta = 2$, while the implied autocorrelations in Figure 8 are maximized for θ around one. With respect to the Yen returns, the sample behavior of their autocorrelations is very peculiar, as we noted before, and does not agree with that of the implied autocorrelations.

Finally, Table 5 reports at the bottom rows of panel a), some diagnostics based on the standardized observations, $\hat{\varepsilon}_t = y_t/\hat{\sigma}_{t/T}$, where $\hat{\sigma}_{t/T}$ is the smoothed estimate of the volatility at time t. The first conclusion from these diagnostics is that the ARSV(1) model explains part of the excess kurtosis observed in the series of returns, with the exception of the IBEX. For this index, it is rather surprising that the kurtosis of the standardized observations (11.489) is larger than the kurtosis of the corresponding original series (7.019). This could be attributed to the presence of outliers. On the other hand, the Box-Pierce statistic for remaining autocorrelation in the squared residuals, $Q_2^{\hat{\varepsilon}}(10)$, is still significant for CAN, SP500, Nikkei, FTSE and IBEX. This could also be due to the presence of outliers, as suggested by the results in Carnero et al. (2004b) about the effects of consecutive outliers on the autocorrelations of squares.

4.2. Sensitivity analysis to the presence of outliers

As there is a concern that outliers would have an undue influence on the presence of Taylor effect and the estimation results, outlier-corrected series have been produced and are forward analyzed in this section. There is not consensus about how to deal with outliers in the context of conditionally heteroscedastic models. In the GARCH framework, Hotta and Tsay (1998), Franses and van Dijk (1999) and Doornik and Ooms (2002) have proposed different alternatives to identify outliers. However, as far as we know, there are not results about the treatment of outliers in the context of SV models. In this paper, following the proposal of Doornik and Ooms (2002), we identify as an outlier any observation that is larger than m times its estimated conditional standard deviation, i.e. $y_t > m\hat{\sigma}_{t/T}$, for some given m. We have chosen three alternative values for m, $m = 5$, 6 and 7. Given that we are modelling the conditional variance of the series, once an observation is identified as an outlier, instead of substituting it by its estimated conditional mean, we substitute y_t^2 by the estimated conditional variance. In particular, it turns out from (6) that y_t^2 is replaced by $\widehat{y}_t^2 = \exp\{\widehat{\mu} + \log \widehat{\sigma}_{t/T}^2\}$.[4]

Table 4 reports, in panels b), c) and d), several summary statistics of the outlier-corrected returns for the three values of m considered. The Euro and BP series do not have any observation larger than 7 or 6 conditional standard deviations, so only correction by outliers greater than $5\widehat{\sigma}_{t/T}$ is performed. The Yen and FTSE100 series do not have observations larger than $7\widehat{\sigma}_{t/T}$ either, and the FTSE100 have the same observations larger than 6 and 5 standard deviations. Therefore, panel d) does not display results for this index.

Comparing the results from the original and the outlier-corrected series, we first notice that, as expected, lower kurtosis are usually achieved if outliers are removed, with values that span from 4.393 for the Euro to 38.743 for the SP500 in the original series to a range that comes from 4.384 to 8.161 for the same returns in the $5\widehat{\sigma}_{t/T}$-outlier-corrected series; see Kim and White (2004) for a Monte Carlo study on the influence of outliers on the estimated coefficients of skewness and kurtosis. The other general conclusion that emerges from the results in Table 4 is that removing outliers steadily increases the correlations in both absolute and squared returns, though the first order autocorrelation sometimes decreases; see Carnero et al. (2004b) for an explanation of these results. The increase in the autocorrelations of absolute returns is usually very low, but the increase in the Box-Ljung statistics for the squares is quite remarkable, specially in those series where a *big* outlier, such as the Black Monday October 1987, has been removed; see, for example, the results on SP500 index. This means that the differences between the autocorrelations of $|y_t|$ and y_t^2 become smaller

[4] Granger *et al.* (1999) correct by outliers replacing any observation outside the interval $\pm 4\widehat{\sigma}$ by $4\widehat{\sigma}$ or $-44\widehat{\sigma}$ as appropriate, where $\widehat{\sigma}$ is the sample standard deviation estimated from the raw data. However, the results in Carnero *et al.* (2004b) show that, using this strategy, it is possible to miss truly outliers and to identify as outliers observations corresponding to periods of high conditional variance.

as the outliers are reduced. This feature is clearly shown in Figure 9, that displays such differences for the original and the three outlier-corrected series for the eight returns considered.

If we look further at the behavior of any particular series, we can conclude the following. In the Euro, BP and Yen exchange rates, the difference between the autocorrelations of absolute and squared returns are very similar for the original and corrected series, and are also very small. Notice also that, for these returns, the values of the Box-Ljung statistics in Table 4 hardly change from the original to the corrected data. As we have seen before, the Taylor effect, as defined in (2), is very weak in these three series. Similar conclusions are obtained when looking at the results for the Canada exchange and IBEX returns, where the differences between the original and the outlier-corrected series are again negligible. However, it is worth noting that, in these two series, the differences displayed in Figure 9, although very similar to one another, are all positive, indicating that absolute returns are more correlated than squares, in agreement with Taylor property (2). If we now focus on the SP500 returns, we first observe that the kurtosis reported in Table 4 decreases from 38.74 in the original data to 8.92 when observations larger than $7\widehat{\sigma}_{t/T}$ are corrected. On the other hand, although the Box-Ljung statistics of absolute returns are similar before and after removing outliers, there is a large increase in the statistics for squares after removing the outliers larger than $7\widehat{\sigma}_{t/T}$. Furthermore, Figure 9 shows that the differences between the autocorrelations of absolute and squared observations are all positive and large in the original series but become smaller once the outliers are taken into account. Therefore, the magnitude of the Taylor effect in the original SP500 returns can be mainly attributed to the very large outliers. Also notice that, in this case, the results obtained for the three outlier-reduced series are again very similar to one another. Similar conclusions can be drawn when looking at the results for the NIKKEI and FTSE100 returns. The only remarkable difference in the FTSE100 index is that, in this case, the first order autocorrelation of the squared original returns is larger than that of the absolute returns. However, after taking into account the outliers, both autocorrelations are similar and so are the autocorrelations at other lags. Notice that the difference between the correlations of $|y_t|$ and y_t^2 in the corrected series, displayed in Figure 9, are negative at first lags and then become positive but very close to zero. Therefore, it seems that the strong Taylor effect observed in the original series can also be due to the presence of outliers.

As a by-product of this analysis, we can also conclude that the sample autocorrelations of absolute observations are more robust against outliers than the sample autocorrelations of squared observations. Furthermore, it should also be remarked that the results for the series considered in this paper are similar regardless of whether we define as outliers observations larger than 7, 6 or 5 conditional standard deviations.

In order to analyze the influence of outliers in the Taylor property as defined in (1), Figure 10 plots, for the $5\widehat{\sigma}_{t/T}$-corrected series, the sample autocorrelations of $|y_t|^\theta$ as a function of θ, for lags $k = 1$, 5, 10, 20 and 50. The results obtained for the other two outliers corrections are very similar and are not displayed here. Comparing this figure with Figure 7, where the autocorrelations were computed for the original observations, we can observe that the plots corresponding to the Euro, BP, CAN, Yen and IBEX are very similar before and after correcting the large observations; this confirms our previous results in Table 4 and Figure 9. With respect to the Nikkei index, it is worth to highlight that the first order autocorrelations of $|y_t|^\theta$, that were maximized for a value of θ over two in the original series (see Figure 7), are maximized at θ close to one after correcting the outliers. Therefore, in this particular case, correcting the outliers has contributed to magnify the Taylor property. Finally, notice that the peculiar behavior of the first order autocorrelation of the Yen shown in Figure 7, still remains in the $5\widehat{\sigma}_{t/T}$-corrected series. Furthermore, after taking into account the outliers, the pattern of the first order autocorrelation of the Yen, SP500 and FTSE100, displayed in Figure 10, is similar and unexpected: this autocorrelation is maximized for values of θ larger than 2 in all cases. We do not have any plausible explanation for this fact, though it could be due to the presence of long-memory or changes in the marginal variance. Anyhow, the effect of these features on the Taylor property is beyond the objectives of this paper. Our results show up that the first order autocorrelation is strongly affected by outliers. Therefore, it seems that it could be very risky to analyze the Taylor effect using only this autocorrelation.

We finally examine the influence of outliers on the estimation results. Table 5 reports, in the panels b), c) and d), the estimated parameter values and some diagnostic statistics for outliers-corrected returns. Looking at these values, it is rather surprising to observe that the estimates of σ_*, ϕ and σ_η^2 are very similar regardless of whether we estimate the ARSV(1) model using the original data (panel a) or any of the three corrected series. The only worth mentioning difference appears in the estimates of the parameter σ_ξ^2, which is related to the distribution of ε_t. Therefore, our results suggest that the dynamics of the underlying volatilities are robustly estimated by QML while the estimated distribution of ε_t depends on whether or not outliers are taken into account; compare this result with those in Carnero et al. (2004b) for GARCH models.

On the other hand, Table 5 shows that correcting outliers clearly improves the diagnostics of the standardized observations. The kurtosis is reduced towards 3 and the Box-Pierce statistics for the squares of the Nikkei, FTSE and IBEX residuals, that were significant in the original series, are no longer significant. However, the statistics of the CAN and SP500 residuals, although smaller, are still significant. This could be suggesting the presence of long-memory in the volatility of these returns, but, as we said before, we do not pursue this issue in this paper.

5. CONCLUSIONS

In this paper we have analyzed the Taylor effect in the SV framework. We have seen that, in stationary ARSV(1) models, the value of θ that maximizes the autocorrelations of $|y_t|^\theta$ depends mainly on the distribution of the errors and the kurtosis of returns. If the errors are Gaussian and the kurtosis of the series is relatively close to 3, the maximum autocorrelations are found in the squares. However, as the kurtosis increases, the value of the exponent that maximizes the autocorrelations decreases and, only for very large and unrealistic values of the kurtosis, θ is smaller than 1. If the distribution of the errors is leptokurtic, for example, a Student-t_ν distribution, the value of θ that maximizes the autocorrelations is never greater than one when $\nu = 7$ and is approximately equal to one in most cases. Once more, if the kurtosis is extremely large, the maximum is reached at values of θ smaller than one. We have also seen that the autocorrelations are maximized approximately at the same value of θ regardless the lag considered.

On the other hand, our Monte Carlo experiments have shown that, for moderately large sample sizes and the more realistic parameter specifications, Taylor effect is not a sample problem due to the biases of the estimated autocorrelations. Therefore, if the sample size is large and the Taylor effect is observed in the sample autocorrelations, the model fitted to the series should be able to generate this effect. However, for relatively small sample sizes and exceptionally low variance and low persistent models, this could be not the case.

Finally, we have illustrated the results with an empirical application to eight series of daily financial returns. Analyzing these series, we have observed that large outliers may have a fundamental influence on whether the Taylor effect holds. This is especially the case when the autocorrelations of order one of squares and absolute returns are compared. We have also illustrated that with the exception of the Yen, SP500 and FTSE100 returns, the ARSV(1) model is able to represent the pattern of the autocorrelations observed in real data.

The results in this paper shows that when the Taylor effect is observed empirically in a financial series, the theoretical model implemented to explain the dynamic behavior of this series should be able to represent such property. However, this requirement is not very strong because, as we have seen, the Taylor effect is rather weak in most cases of empirical interest.

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Table 1. Value of the power parameter, θ, that maximizes the first order autocorrelation of $|y_t|^\theta$ in ARSV(1) models

		Gaussian Errors						Student-7 Errors					
σ_η^2	ϕ	0.80	0.85	0.90	0.95	0.98	0.99	0.80	0.85	0.90	0.95	0.98	0.99
	TE*		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
0.1	θ	1.5	1.4	1.3	1.1	1	0.5	1	0.9	0.9	0.8	0.6	0.5
	σ_h^2	0.28	0.36	0.53	1.03	2.53	5.03	0.28	0.36	0.53	1.03	2.53	5.03
	κ_y	3.96	4.30	5.08	8.37	37.5	457	6.62	7.17	8.49	14.01	62.77	747
	TE				✓	✓	✓	✓	✓	✓	✓	✓	✓
0.05	θ	1.8	1.7	1.5	1.3	1	0.8	1	1	1	0.9	0.7	0.6
	σ_h^2	0.14	0.18	0.26	0.51	1.26	2.51	0.14	0.18	0.26	0.51	1.26	2.51
	κ_y	3.45	3.59	3.90	5.01	10.6	37.0	5.75	5.99	6.48	8.33	17.63	62
	TE					✓	✓	✓	✓	✓	✓	✓	✓
0.01	θ	2	1.9	1.9	1.8	1.6	1.4	1	1	1	1	1	0.9
	σ_h^2	0.03	0.04	0.05	0.10	0.25	0.50	0.03	0.04	0.05	0.10	0.25	0.50
	κ_y	3.08	3.11	3.16	3.32	3.86	4.96	5.15	5.20	5.26	5.53	6.42	8.24

* TE means that the first order autocorrelation of absolute values is larger than that of squares

Table 2. Monte Carlo results on sample autocorrelations of $|y_t|^\theta$ in ARSV(1) models with $\phi = 0.98$ and Gaussian errors

		$\sigma_\eta^2 = 0.01$				$\sigma_\eta^2 = 0.05$			
θ	k	$\rho_\theta(k)$	*500*	*1000*	*5000*	$\rho_\theta(k)$	*500*	*1000*	*5000*
0.5	1	0.078	0.061	0.068	0.076	0.290	0.234	0.256	0.283
			(0.051)	(0.039)	(0.019)		(0.087)	(0.068)	(0.033)
	10	0.065	0.043	0.056	0.064	0.240	0.178	0.204	0.232
			(0.052)	(0.037)	(0.018)		(0.085)	(0.069)	(0.034)
	20	0.053	0.031	0.041	0.052	0.195	0.129	0.157	0.187
			(0.048)	(0.037)	(0.017)		(0.084)	(0.068)	(0.034)
	50	0.029	0.008	0.017	0.027	0.105	0.039	0.067	0.098
			(0.048)	(0.035)	(0.016)		(0.070)	(0.062)	(0.033)
1	1	0.095	0.072	0.081	0.092	0.314	0.248	0.272	0.305
			(0.055)	(0.043)	(0.021)		(0.088)	(0.072)	(0.038)
	10	0.079	0.052	0.067	0.077	0.255	0.183	0.210	0.244
			(0.055)	(0.041)	(0.021)		(0.085)	(0.070)	(0.038)
	20	0.064	0.037	0.049	0.062	0.203	0.129	0.157	0.192
			(0.052)	(0.040)	(0.019)		(0.081)	(0.067)	(0.037)
	50	0.035	0.010	0.020	0.032	0.106	0.035	0.062	0.098
			(0.049)	(0.037)	(0.017)		(0.067)	(0.060)	(0.035)
1.5	1	0.100	0.075	0.085	0.097	0.297	0.228	0.250	0.284
			(0.058)	(0.045)	(0.023)		(0.088)	(0.075)	(0.044)
	10	0.083	0.053	0.069	0.080	0.232	0.160	0.185	0.218
			(0.056)	(0.044)	(0.023)		(0.082)	(0.069)	(0.041)
	20	0.067	0.038	0.050	0.065	0.179	0.109	0.132	0.167
			(0.055)	(0.041)	(0.021)		(0.075)	(0.063)	(0.038)
	50	0.036	0.010	0.020	0.033	0.087	0.026	0.048	0.080
			(0.049)	(0.038)	(0.018)		(0.060)	(0.053)	(0.034)
2	1	0.098	0.071	0.081	0.095	0.255	0.194	0.213	0.242
			(0.061)	(0.048)	(0.026)		(0.091)	(0.081)	(0.054)
	10	0.080	0.050	0.066	0.077	0.188	0.130	0.149	0.176
			(0.057)	(0.046)	(0.025)		(0.080)	(0.070)	(0.048)
	20	0.064	0.036	0.047	0.061	0.138	0.084	0.101	0.128
			(0.056)	(0.042)	(0.024)		(0.070)	(0.059)	(0.041)
	50	0.034	0.009	0.018	0.030	0.061	0.018	0.034	0.058
			(0.048)	(0.038)	(0.019)		(0.053)	(0.046)	(0.032)

* Monte Carlo standard deviations in parenthesis

Table 3. Monte Carlo results on sample autocorrelations of $|y_t|^\theta$ in ARSV(1) models with $\phi = 0.98$ and Student-7 errors

		$\sigma_\eta^2 = 0.01$				$\sigma_\eta^2 = 0.05$			
θ	k	$\rho_\theta(k)$	*500*	*1000*	*5000*	$\rho_\theta(k)$	*500*	*1000*	*5000*
0.5	1	0.069	0.053 (0.051)	0.062 (0.039)	0.067 (0.017)	0.262	0.211 (0.083)	0.235 (0.065)	0.255 (0.031)
	10	0.057	0.040 (0.052)	0.048 (0.037)	0.056 (0.017)	0.217	0.159 (0.084)	0.186 (0.064)	0.211 (0.031)
	20	0.047	0.031 (0.049)	0.038 (0.036)	0.044 (0.017)	0.177	0.118 (0.080)	0.145 (0.063)	0.169 (0.031)
	50	0.025	0.009 (0.045)	0.017 (0.034)	0.023 (0.016)	0.095	0.035 (0.066)	0.062 (0.057)	0.087 (0.030)
1	1	0.075	0.058 (0.054)	0.066 (0.042)	0.072 (0.019)	0.263	0.206 (0.084)	0.228 (0.067)	0.254 (0.037)
	10	0.062	0.042 (0.054)	0.052 (0.039)	0.060 (0.018)	0.213	0.149 (0.082)	0.176 (0.064)	0.205 (0.035)
	20	0.051	0.033 (0.050)	0.041 (0.038)	0.048 (0.018)	0.170	0.108 (0.074)	0.134 (0.061)	0.161 (0.034)
	50	0.027	0.010 (0.045)	0.018 (0.035)	0.025 (0.017)	0.088	0.029 (0.061)	0.054 (0.054)	0.079 (0.031)
1.5	1	0.068	0.052 (0.055)	0.060 (0.043)	0.065 (0.021)	0.217	0.171 (0.086)	0.185 (0.070)	0.209 (0.048)
	10	0.056	0.037 (0.054)	0.047 (0.041)	0.054 (0.019)	0.170	0.117 (0.078)	0.137 (0.063)	0.162 (0.042)
	20	0.045	0.029 (0.049)	0.036 (0.038)	0.043 (0.018)	0.131	0.081 (0.066)	0.101 (0.056)	0.124 (0.036)
	50	0.024	0.009 (0.045)	0.016 (0.035)	0.022 (0.017)	0.064	0.019 (0.053)	0.038 (0.045)	0.057 (0.030)
2	1	0.052	0.043 (0.056)	0.047 (0.044)	0.051 (0.023)	0.147	0.132 (0.089)	0.138 (0.075)	0.153 (0.059)
	10	0.042	0.030 (0.052)	0.037 (0.041)	0.042 (0.021)	0.108	0.085 (0.074)	0.097 (0.063)	0.113 (0.048)
	20	0.034	0.022 (0.048)	0.029 (0.038)	0.033 (0.019)	0.079	0.055 (0.059)	0.069 (0.054)	0.083 (0.038)
	50	0.018	0.007 (0.042)	0.012 (0.034)	0.016 (0.016)	0.035	0.011 (0.047)	0.023 (0.038)	0.035 (0.027)

* Monte Carlo standard deviations in parenthesis

Table 4. Summary descriptive statistics of returns

a) Original series

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
Size	2512	6047	8053	6041	10778	4676	4735	3991
Kurtosis	4.393	6.001	6.988	6.603	38.743	11.106	11.162	7.019
J.-Bera	222.3	2272	5348.1	3532.7	576804	12807	13428	2718.3
$r_1(1)$	0.069	0.136	0.218	0.138	0.254	0.235	0.245	0.229
$Q_1(10)$	144.5	1055.2	2447.9	648.8	4939.8	1846.4	2492.4	2264.3
$Q_1(50)$	458.3	3213.5	6530.4	1578.4	14109	4600.6	5378.1	5956.9
$r_2(1)$	0.051	0.102	0.192	0.188	0.173	0.258	0.381	0.181
$Q_2(10)$	74.9	738.6	1194.9	626.9	1013.5	617.3	2258.9	1555.1
$Q_2(50)$	243.9	2265.9	2469.5	1221.6	1225.9	917.2	3061.6	2734

b) Series corrected by observations larger than $7\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
Kurtosis	-	-	6.835	-	8.924	7.810	-	6.639
J.-Bera	-	-	4938.5	-	15760	4556.9	-	2212.3
$r_1(1)$	-	-	0.217	-	0.224	0.208	-	0.229
$Q_1(10)$	-	-	2492.9	-	5052.3	1874.5	-	2480.2
$Q_1(50)$	-	-	6670.1	-	15993	4928.6	-	6674
$r_2(1)$	-	-	0.196	-	0.263	0.143	-	0.179
$Q_2(10)$	-	-	1289.4	-	2952.8	683.5	-	1958.8
$Q_2(50)$	-	-	2686.2	-	7641.9	1391.3	-	3588.5

c) Series corrected by observations larger than $6\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
Kurtosis	-	-	6.812	6.615	8.899	7.210	6.503	6.637
J.-Bera	-	-	4882	3550.2	15627.3	3472.7	2468.3	2211.7
$r_1(1)$	-	-	0.216	0.139	0.225	0.204	0.195	0.229
$Q_1(10)$	-	-	2511.4	670.9	5112.1	1888.1	2377.1	2521.3
$Q_1(50)$	-	-	6774.8	1653.8	16228	5071.9	5600.4	6814
$r_2(1)$	-	-	0.195	0.188	0.266	0.136	0.239	0.179
$Q_2(10)$	-	-	1314.4	637.7	4039	776.2	3261.8	2000.1
$Q_2(50)$	-	-	2764	1259.2	7822.6	1658.1	5890.5	3684.6

d) Series corrected by observations larger than $5\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
Kurtosis	4.384	5.760	6.499	6.509	8.161	7.192	-	6.639
J.-Bera	218.2	1929.6	4110.8	3313.8	11997.8	3444.2	-	2215.7
$r_1(1)$	0.073	0.134	0.202	0.142	0.214	0.205	-	0.231
$Q_1(10)$	153.7	1058.9	2505.1	716.7	5204.9	1940.5	-	2560.2
$Q_1(50)$	491.9	3229	6964.7	1770.5	16676	5192.1	-	6937.9
$r_2(1)$	0.054	0.097	0.163	0.198	0.246	0.139	-	0.181
$Q_2(10)$	82.5	786.8	1330.9	702.8	4706.0	808.8	-	2038.1
$Q_2(50)$	268.9	2437.3	3033.3	1409.3	9293.9	1726.6	-	3765.1

Table 5. Estimation results

a) Original series

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
σ^*	0.322	0.318	0.049	0.403	0.596	1.336	0.806	1.285
ϕ	0.993 (0.004)	0.988 (0.003)	0.988 (0.004)	0.980 (0.005)	0.994 (0.002)	0.990 (0.004)	0.986 (0.004)	0.991 (0.004)
σ^2_η	0.003 (0.002)	0.012 (0.003)	0.023 (0.002)	0.015 (0.005)	0.009 (0.001)	0.016 (0.003)	0.014 (0.004)	0.015 (0.003)
σ^2_ξ	5.200 (0.247)	5.412 (0.164)	5.203 (0.139)	5.542 (0.165)	5.214 (0.120)	5.289 (0.183)	4.709 (0.179)	4.658 (0.194)
υ	8.5	4.13	8.4	4.2	8.1	6.6	∞	∞
$\kappa_{\widehat{\varepsilon}}$	4.168	4.080	5.151	5.283	7.944	5.888	3.939	11.489
$Q_2^{\widehat{\varepsilon}}(10)$	6.23	16.58	119.58	27.23	484.4	134.70	90.58	91.86

b) Series corrected by observations larger than $7\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
σ^*	-	-	0.049	-	0.572	1.319	-	1.194
ϕ	-	-	0.988 (0.004)	-	0.994 (0.001)	0.991 (0.004)	-	0.991 (0.003)
σ^2_η	-	-	0.023 (0.003)	-	0.009 (0.001)	0.015 (0.003)	-	0.016 (0.004)
σ^2_ξ	-	-	5.260 (0.140)	-	5.172 (0.120)	5.364 (0.184)	-	4.730 (0.195)
υ	-	-	7.1	-	9.5	5.6	-	∞
$\kappa_{\widehat{\varepsilon}}$	-	-	4.713	-	4.322	4.733	-	4.207
$Q_2^{\widehat{\varepsilon}}(10)$	-	-	148.09	-	110.16	38.48	-	40.44

c) Series corrected by observations larger than $6\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
σ^*	-	-	0.048	0.398	0.568	1.307	0.793	1.178
ϕ	-	-	0.988 (0.002)	0.980 (0.005)	0.994 (0.001)	0.991 (0.003)	0.986 (0.004)	0.991 (0.003)
σ^2_η	-	-	0.023 (0.004)	0.015 (0.005)	0.009 (0.002)	0.015 (0.004)	0.013 (0.004)	0.016 (0.004)
σ^2_ξ	-	-	5.223 (0.140)	5.516 (0.166)	5.177 (0.119)	5.177 (0.182)	4.635 (0.178)	4.684 (0.194)
υ	-	-	7.9	3.95	9.25	9.25	∞	∞
$\kappa_{\widehat{\varepsilon}}$	-	-	4.55	4.979	4.061	4.398	3.301	3.728
$Q_2^{\widehat{\varepsilon}}(10)$	-	-	134.18	10.01	98.77	27.57	43.28	29.28

d) Series corrected by observations larger than $5\widehat{\sigma}_{t/T}$

	EU	*BP*	*CAN*	*Yen*	*SP500*	*Nikkei*	*FTSE*	*IBEX*
σ^*	0.318	0.317	0.048	0.388	0.560	1.292	-	1.171
ϕ	0.992 (0.005)	0.988 (0.003)	0.988 (0.002)	0.979 (0.005)	0.994 (0.001)	0.991 (0.003)	-	0.991 (0.003)
σ^2_η	0.003 (0.002)	0.011 (0.003)	0.022 (0.004)	0.017 (0.005)	0.009 (0.002)	0.015 (0.004)	-	0.016 (0.004)
σ^2_ξ	5.247 (0.248)	5.501 (0.164)	5.163 (0.139)	5.471 (0.164)	5.154 (0.119)	5.170 (0.182)	-	4.770 (0.194)
υ	7.35	4.45	9.75	4.65	10.1	9.45	-	∞
$\kappa_{\widehat{\varepsilon}}$	3.890	3.994	4.316	4.560	3.810	4.214	-	3.576
$Q_2^{\widehat{\varepsilon}}(10)$	8.50	17.43	72.03	8.42	73.00	23.89	-	38.90

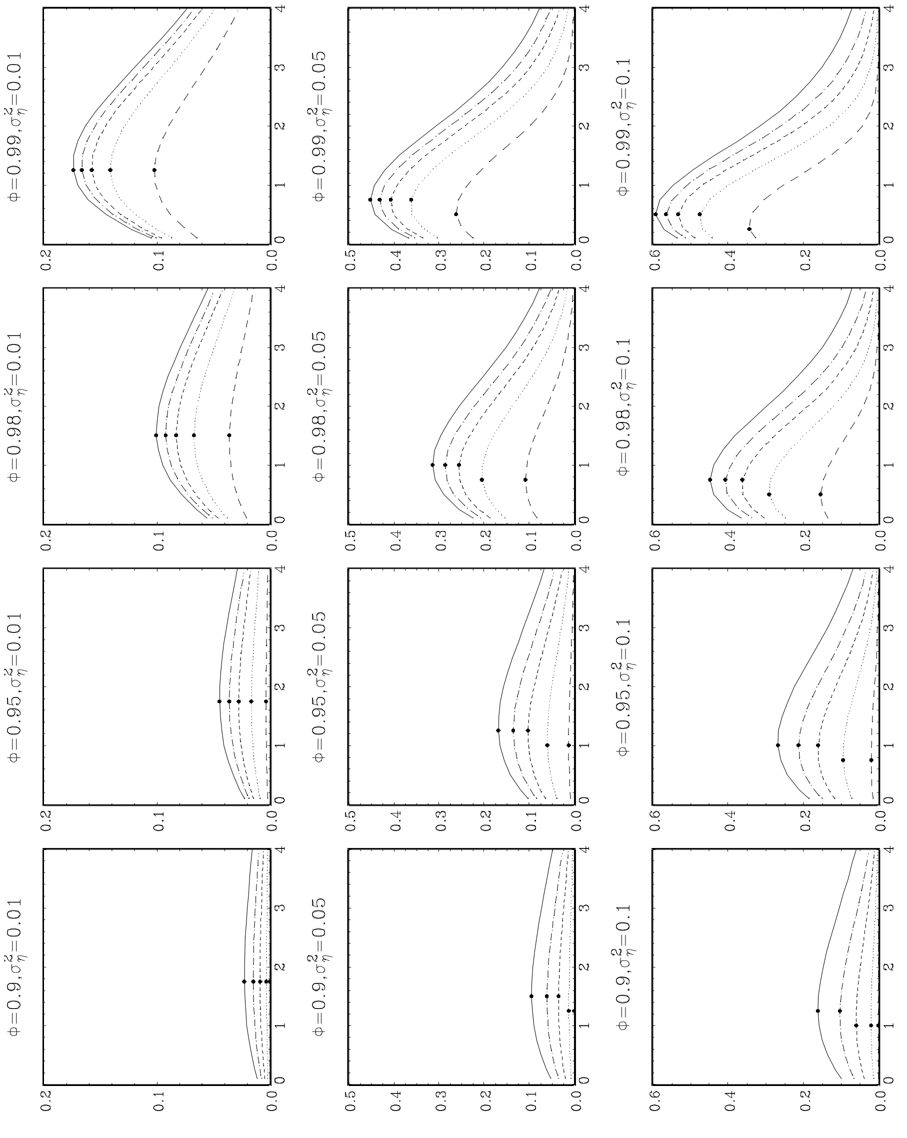


Figure 1: Autocorrelation function of $|y_t|^\theta$ against θ for different lags: $k = 1$ (solid), $k = 5$ (dots and dashes), $k = 10$ (short dashes), $k = 20$ (dots) and $k = 50$ (dashed) and Gaussian errors

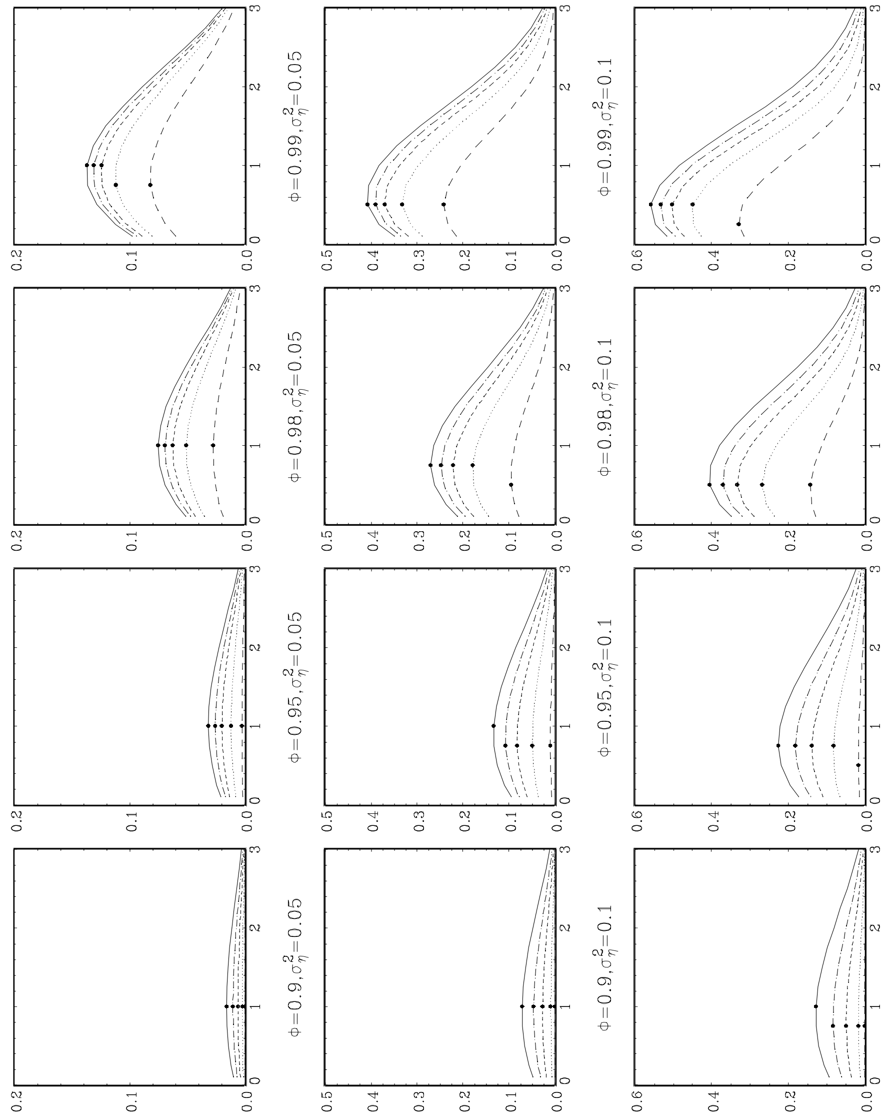
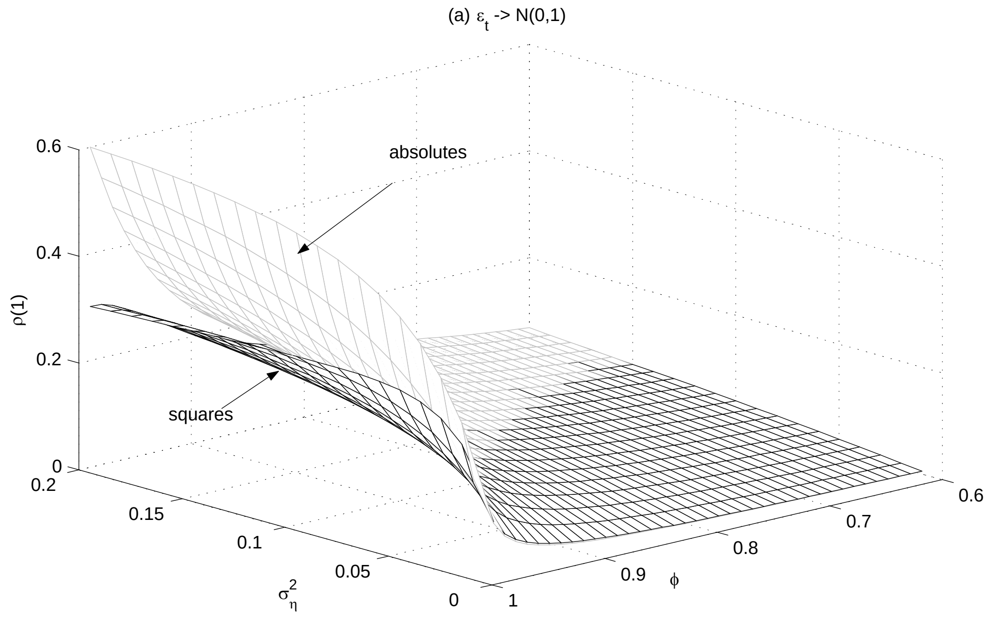


Figure 2: Autocorrelation function of $|y_t|^\theta$ against θ for different lags: $k = 1$ (solid), $k = 5$ (dots and dashes), $k = 10$ (short dashes), $k = 20$ (dots) and $k = 50$ (dashed) and Student-7 errors



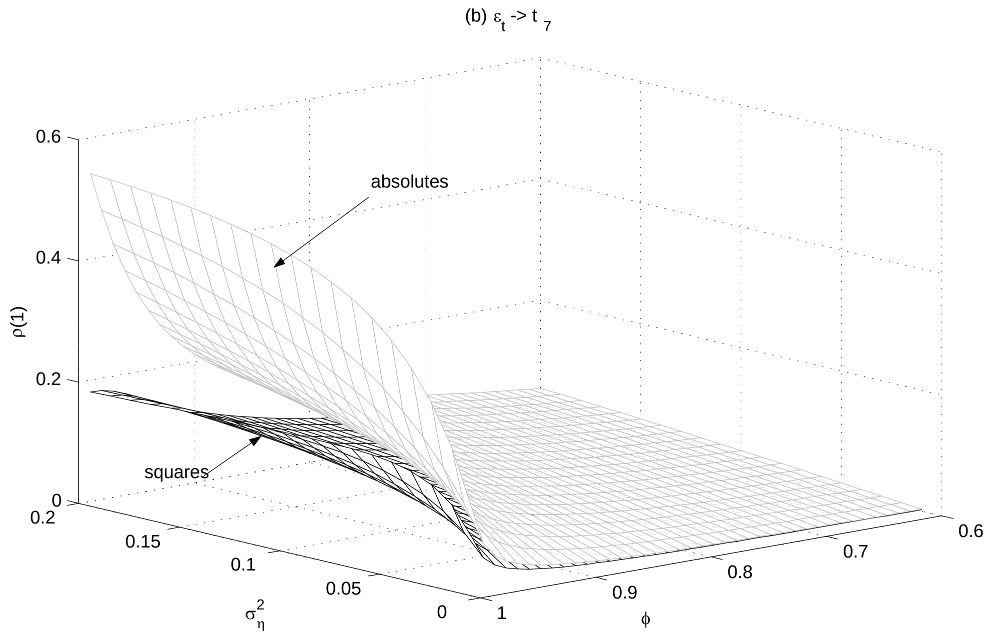


Figure 3: First order autocorrelation of absolute and squared observations against (ϕ, σ_η^2) with Gaussian and Student-7 errors

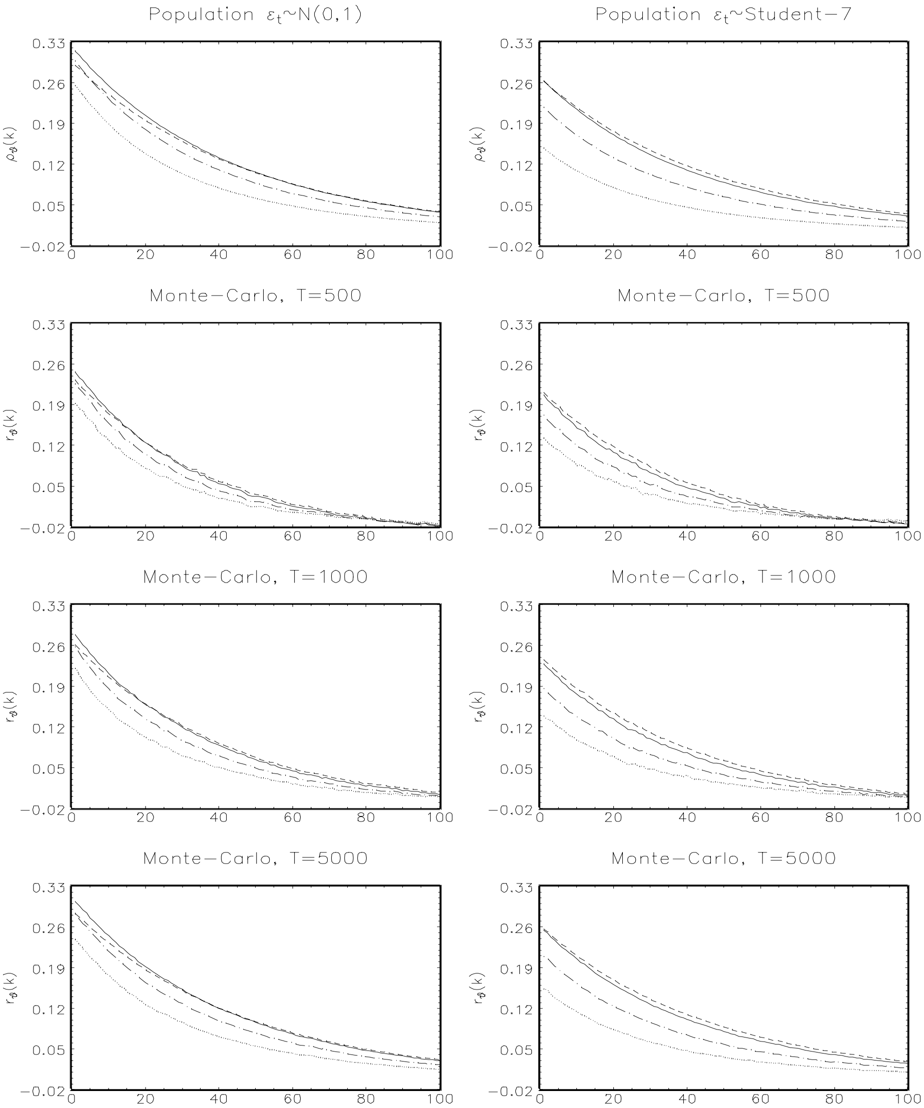


Figure 4: True ACF and mean correlogram of $|y_t|^\theta$ in an ARSV(1) model with $\{\phi = 0.98, \sigma_\eta^2 = 0.05\}$ and $\theta = 0.5$ (dashed), $\theta = 1$ (solid), $\theta = 1.5$ (dots and dashes) and $\theta = 2$ (dots)

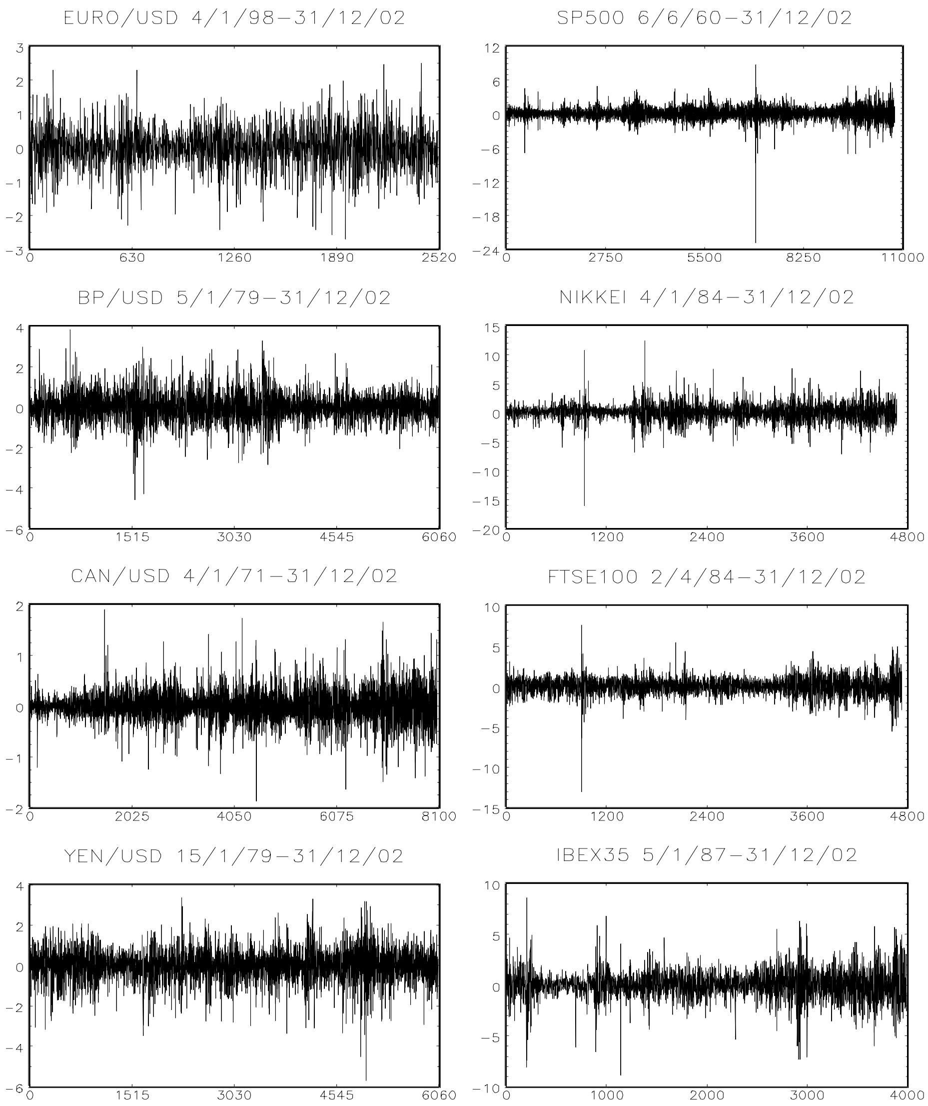


Figure 5: Daily returns

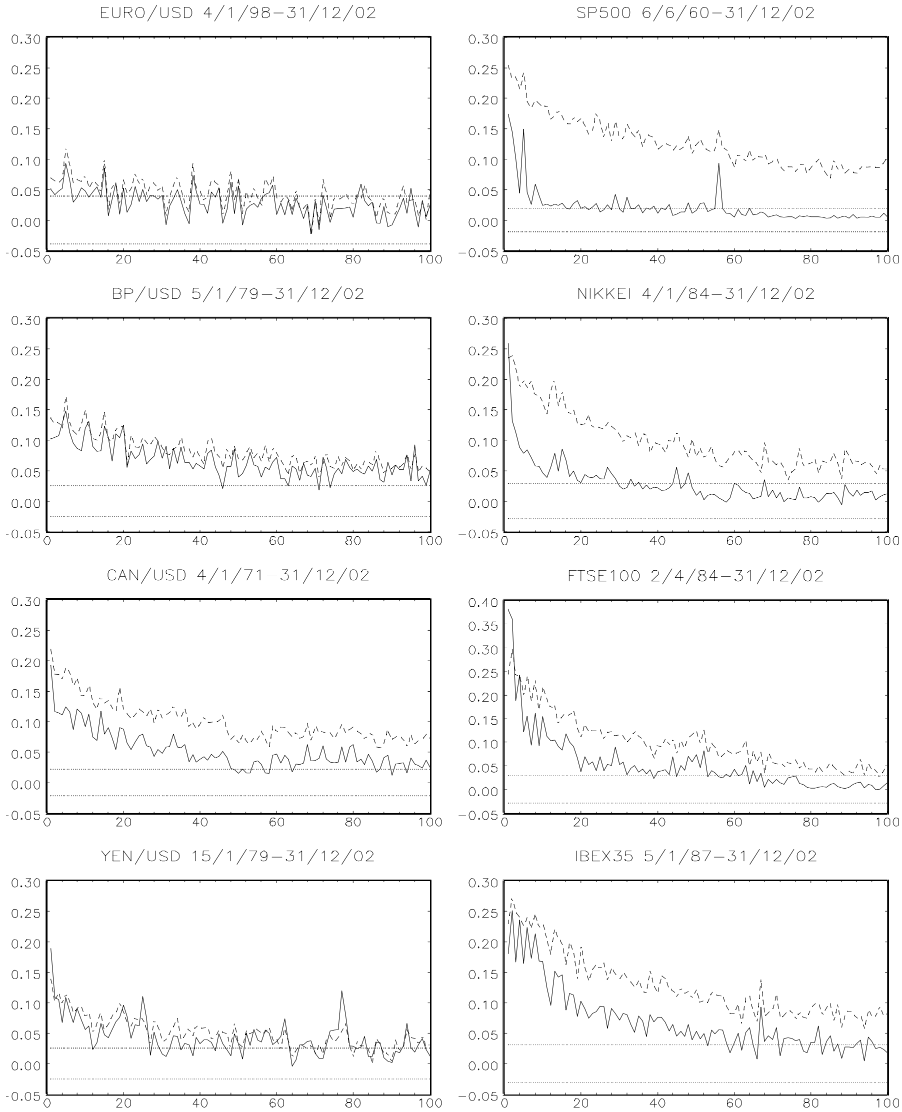


Figure 6: Correlograms of absolute (dashed) and squared (solid) returns

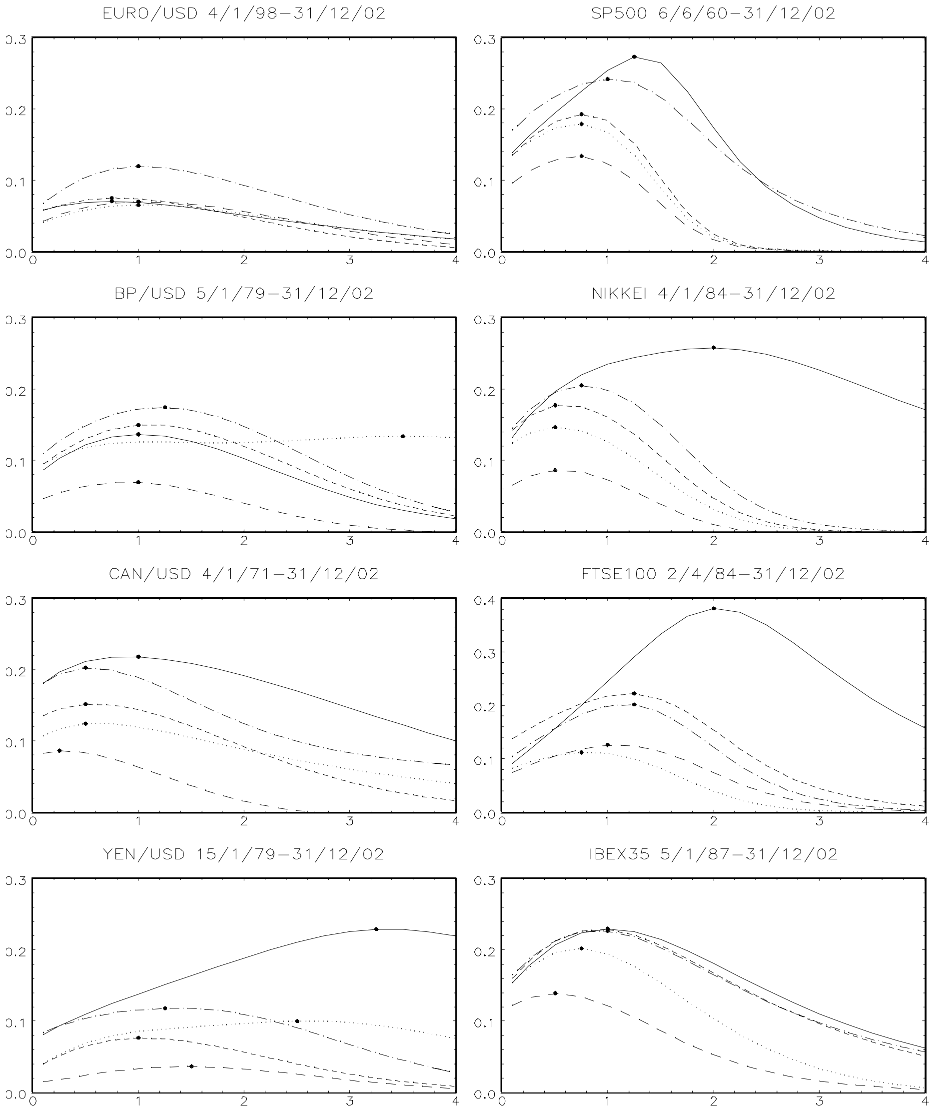


Figure 7: Sample autocorrelations of $|y_t|^\theta$ against θ for different lags: $k = 1$ (solid), $k = 5$ (dots and dashes), $k = 10$ (short dashes), $k = 20$ (dots) and $k = 50$ (dashed)

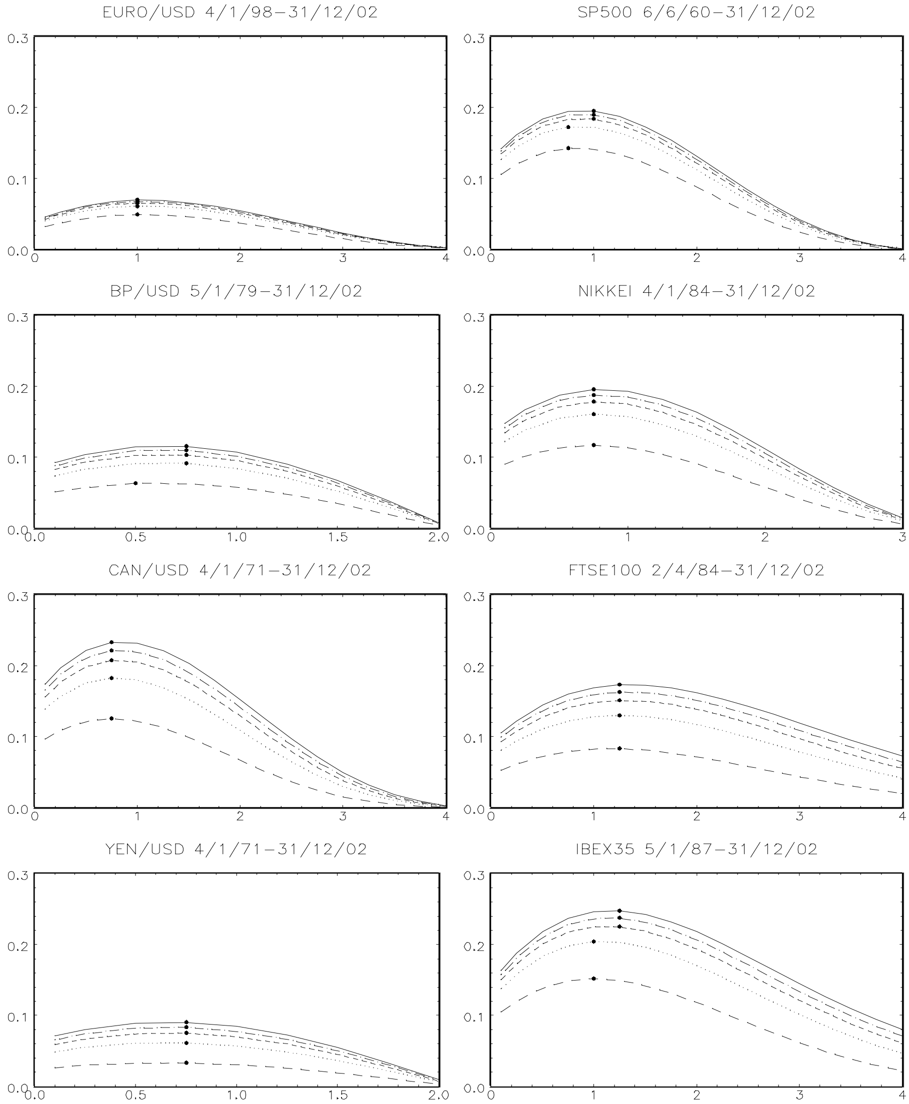


Figure 8: Implied autocorrelations of $|y_t|^\theta$ against θ from estimated ARSV(1) models and for different lags: $k = 1$ (solid), $k = 5$ (dots and dashes), $k = 10$ (short dashes), $k = 20$ (dots) and $k = 50$ (dashed)

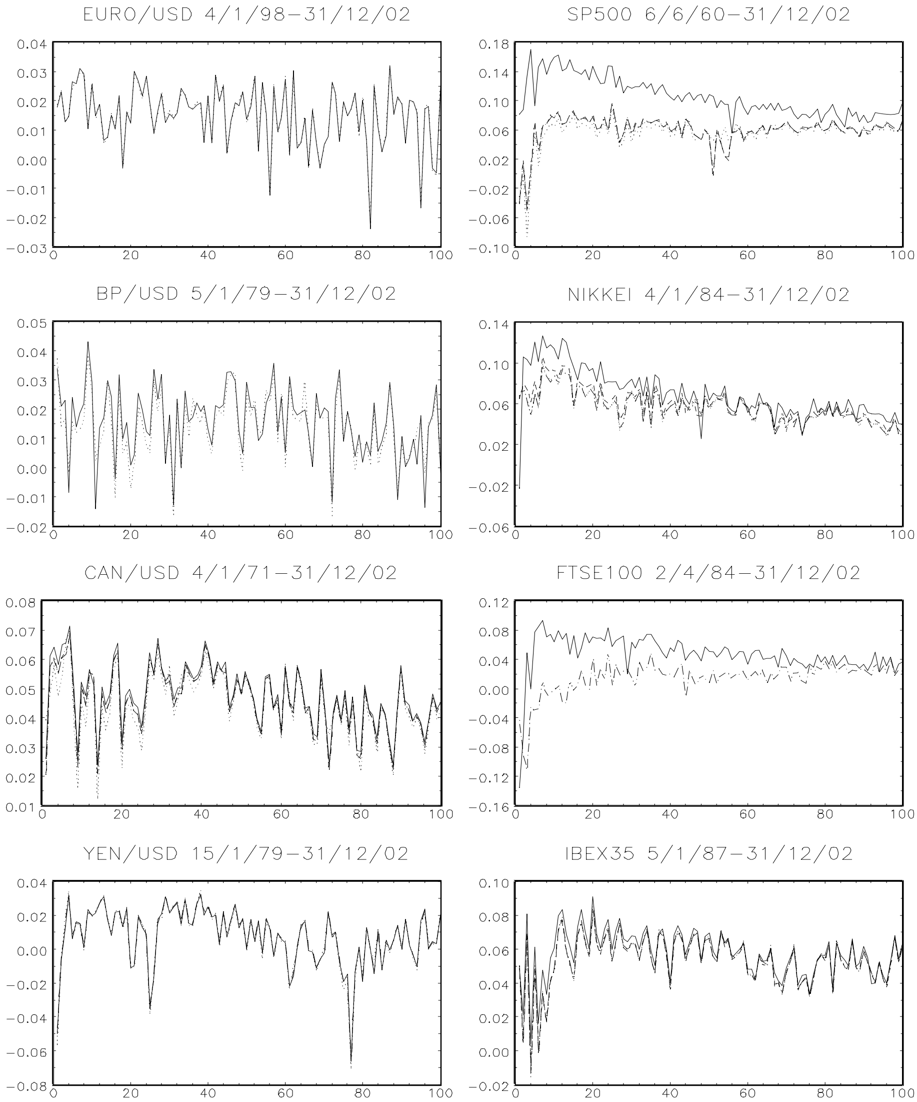


Figure 9: Differences between correlations of $|y_t|$ and y_t^2 for the original (solid) and the $7\widehat{\sigma}_{t/T}$ (dashed), $6\widehat{\sigma}_{t/T}$ (dots and dashes) and $5\widehat{\sigma}_{t/T}$ (dots) outlier-corrected series

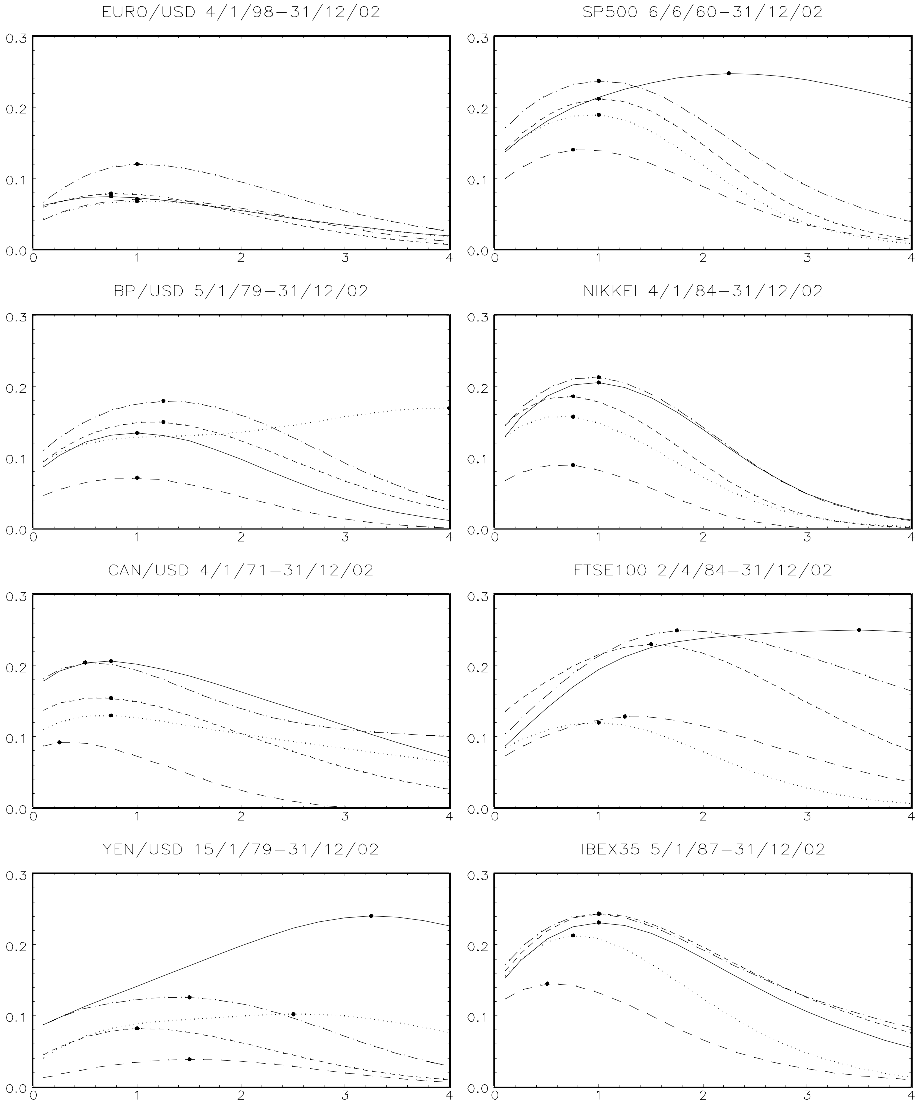


Figure 10: Sample autocorrelations of $|y_t|^\theta$ against θ for the $5\widehat{\sigma}_{t/T}$-outlier-corrected series at different lags: $k = 1$ (solid), $k = 5$ (dots and dashes), $k = 10$ (short dashes), $k = 20$ (dots) and $k = 50$ (dashed)