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FORECASTING MONTHLY US CONSUMER PRICE INDEXES THROUGH A DISAGGREGATED I(2) ANALYSIS

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Abstract

In this paper we carry a disaggregated study of the monthly US Consumer Price Index (CPI). We consider a breakdown of US CPI in four subindexes, corresponding to four groups of markets: energy, food, rest of commodities and rest of services. This is seen as a relevant way to increase information in forecasting US CPI because the supplies and demands in those markets have very different characteristics. Consumer prices in the last three components show I(2) behavior, while the energy subindex shows a lower order of integration, but with segmentation in the growth rate. Even restricting the analysis to the series that show the same order of integration, the trending behavior of prices in these markets can be very different. An I(2) cointegration analysis on the mentioned last three components shows that there are several sources of nonstationarity in the US CPI components. A common trend analysis based on dynamic factor models confirms these results.

The different trending behavior in the market prices suggests that theories for price determinations could differ through markets. In this context, disaggregation could help to improve forecasting accuracy. To show that this conjecture is valid for the non-energy US CPI, we have performed a forecasting exercise of each component, computed afterwards the aggregated value of the non energy US CPI and compared it with the forecasts obtained directly from a model for the aggregate. The improvement in one year ahead forecasts with the disaggregated approach is more than 20%, where the root mean squared error is employed as a measure of forecasting performance.

Keywords: CPI, cointegration, disaggregation, factor models, forecasting, I(2) analysis, univariate time series models

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1 Introduction

Consumer Price Index (CPI) is, perhaps, the most intensively price indicator used by economic analysts. Financial markets continuously asses expectations on CPI and react to the innovations contained in new published data. Inflation is also a key issue in monetary policy. Forecasting monthly inflation is nowadays a necessity for analysts, and they highly demand immediate updates as soon as new information is available. Authors use a great variety of techniques to forecast inflation. For instance, Stock and Watson (1999) use a type of generalized Phillips curve based on measures of real aggregate activity and build a new activity index based on a great number of economic indicators, Jacobson et al (2001) build a VAR model, Bidarkota (2001) uses regime switching models and Moshiri and Cameron (2000) apply neural network techniques.

In this paper we disaggregate the monthly US CPI in food, rest of non-energy commodities, services and energy components as a way to increase the information used in forecasting CPI. We confirm that all the components but energy are I(2), while the behavior of the energy component is I(1), probably with segmented means. For this reason we perform a joint analysis of the other three components. A methodology for I(2) cointegration analysis was developed by Johansen (1995, 1997), Rahbek et al. (1999) and Paruolo (1996) amongst others. We have used this technique to show that there is not a unique source of nonstationarity in the data. Some other applied works dealing with the I(2) cointegration analysis of the CPI are Juselius (1999) that studies price convergence of several quarterly price indices, and Banerjee et al. (2001) that relates quarterly Australian CPI to several macroeconomic variables, among others. In this paper we study convergence through markets which have quite different demand and supply properties. In particular, the incorporation of technological innovations and the effects of changes in the consumer habits can differ substantially and persistently through these markets.

Factor analysis provides complementary information to the cointegration analysis. It explicitly models the different sources of non-stationarity present in data. Escribano and Peña (1994) showed the equivalence between cointegration and common factors. Dynamic factor analysis was first introduced by Geweke (1979) and Geweke and Singleton (1981) in the frequency domain, Peña and Box (1987) studied stationary dynamic factor models in time domain and Stock and Watson (1988) and Peña and Poncela (2000) analyzed the nonstationary case. In this paper we confirm coincident results from both approaches.

Our disaggregated analysis shows different trending behavior for the several CPI components, which suggests that theories for price determination could differ through markets. This is in line with the results in Hendry (2001) where a valid econometric model for UK inflation must include variables referring to different theoretical explanations. In this context, disaggregation could also help to improve the forecasting results. To show that the previous conjecture is valid for the Non-Energy US CPI, we have performed a disaggregated forecasting exercise of each component and computed afterwards the aggregated value of the Non-Energy US CPI. Then, we have compared it with the forecasts obtained directly without disaggregating the Non-Energy US CPI. This exercise shows that the bottom-up approach reduces the prediction root mean squared error by 23% in one year ahead forecast. To test if the improvement of the disaggregated approach over the aggregated one was statistically significant, we have used the Diebold and Mariano (1995) test. Our disaggregated approach builds on the results of Espasa et al (1987) and Lorenzo (1997) for the Spanish inflation. For annual GDP data and using Bayesian techniques, disaggregation has also been successfully applied in forecasting by Zellner and Tobias (2000) and Zellner and Chen (2000).

Although in order to detect the existence of different sources of nonstationarity we have used dynamic factor analysis and cointegration techniques, multivariate forecasts did not outperform the forecasts from univariate disaggregated models. There is mixed evidence in the literature about the improvement in forecasting accuracy using univariate or multivariate time series models in the case of cointegration. Although Engle and Yoo (1987) advocate that taking into account the presence of cointegration improves the long run predictions, other authors like Christoffersen and Diebold (1998), García-Ferrer and Novales (1998) and Lin and Tsay (1996), among others, provide evidence on the opposite direction. In our case,

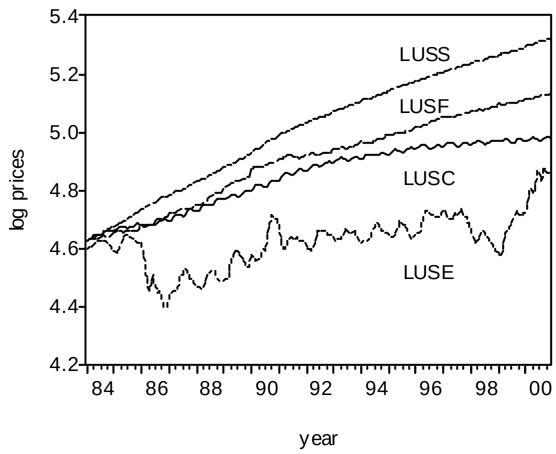

Figure 1: Natural logarithms of energy (LUSE), food (LUSF), rest of non-energy commodities (LUSC) and rest of services (LUSS).

the univariate models outperform the multivariate ones.

This paper is organized as follows. In section 2, we analyze the order of integration of the US CPI components. In sections 3 and 4, we investigate the sources of nonstationarity by means of the cointegration and factor analysis techniques, respectively. In section 5, we analyze the forecasting performance of the aggregated and disaggregated approaches to predict Non-Energy US CPI and compare their results. Finally, in section 6 we interpret the results and conclude.

2 Order of Integration of US CPI components

Four monthly U.S. consumer price index (CPI) series were analyzed over the sample from January 1983 to December 2000. The four monthly CPI components (in natural logarithms) are food (LUSF), non-energy commodities (LUSC), services (LUSS) and energy (LUSE). Figure 1 shows the four components.

In order to check the order of integration of the series, several unit root tests, that are all given in the appendix, were performed. The series exhibited seasonality and sometimes the need of intervention analysis. The unit root tests accounted for the possibility of these deterministic characteristics. Tables A.1 and A.3 of the appendix show the models estimated to test the null hypothesis of one and two unit roots, respectively, while tables A.2 and A.4 describe the characteristics of the series both under the null and alternative hypothesis. The results are given in tables A.5 to A.7.

All series show a trending behavior, so the unit root tests are designed to contemplate this feature, both, in the null and alternative hypothesis. The behavior of US energy CPI seems different as it can be seen in figure 1 and this is also confirmed by the unit root tests.

One unit root is not rejected in all cases, and when checking for the second one this is also clearly not rejected for all series but US energy CPI. This result shows that the energy CPI behavior is different to the rest of the components and for that reason it will not be analyzed jointly with the remaining series. For US food, services and non energy commodities CPIs we adopt the hypothesis that are I(2)[1]. Nevertheless, another possibility would be to contemplate I(1)[2] processes with segmentations on the means of the series; this would imply that inflation usually will be a stationary process. Still, in this case the long term equilibrium mean could not be estimated due to the incapacity of formulating a valid long term stochastic scheme of the breaks. For this reason, in this paper we proceed as if the series were I(2,0).

3 Cointegration Analysis

The different CPI components do not seem to follow a single common trend. This hypothesis is going to be tested using two different approaches: (1) cointegration analysis, in this section and (2) common trends in the next section.

As it has been shown in section 2, the variables can be characterized as I(2) processes. These

[1] More precisely, we would say that they are I(2,0) in the notation $I(d,m)$ of Espasa and Peña (1995), which we give in the appendix.

[2] or more precisely I(1,1) in the terminology $I(d,m)$ of Espasa and Peña (1995)

features complicate the analysis and requires the application of Johansen's I(2) techniques as developed in Johansen (1995), and Rahbek et al. (1999) amongst others.

The model we use is:

$$\begin{aligned} \Delta^2 \mathbf{y}_t &= \sum_{i=1}^{p} \Gamma_i \Delta^2 \mathbf{y}_{t-i} + \Gamma \Delta \mathbf{y}_{t-1} + \Pi \mathbf{y}_{t-2} + \mathbf{C}\mathbf{D}_t + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \qquad (1) \\ \boldsymbol{\varepsilon}_t &\sim N_n(\mathbf{0}, \Sigma), \qquad t = 1, \cdots, T \end{aligned}$$

where, $\mathbf{y}_t$ is an $n \times 1$ vector which collects the $n = 3$ CPI components, p is the order of the VAR used for serial correlation, $\mathbf{D}_t$ is a matrix (11×1) that accounts for eleven centered seasonal dummies, and T is the sample size. The model also restricts the constants so no quadratic trends are allowed in the data.

The existence of cointegration relationships is implied in the matrices associated to the levels (Π) and first differences (Γ) of the variables (otherwise the model would be a VAR(p) in second differences). Since the variables are I(2), we can find the following possibilities: (1) I(0) cointegration relations only in the levels of the variables; (2) I(0) polynomial cointegration between levels and first differences of the variables and (3) I(1) cointegration relations amongst the levels of the variables.

In the presence of cointegration relations in I(2) systems, there are two reduced rank conditions associated to the system matrices Π and Γ defined in (1):

1) the one associated with the I(1) analysis, that consists in checking the rank of $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$ to identify the number of I(0) relationships (r);

2) and the one that characterizes the I(2) analysis, that consists in checking the rank of $\boldsymbol{\alpha}'_{\perp}\Gamma\boldsymbol{\beta}'_{\perp}$ where $\boldsymbol{\alpha}_{\perp}$ and $\boldsymbol{\beta}_{\perp}$ are the orthogonal complements of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively. This step identifies the number of I(1) and I(2) components (s_1 and s_2 respectively).

Essentially, what the Johansen's method does it is to project the n variables collected in vector $\mathbf{y}_t$ in three mutually orthogonal subspaces with basis $\boldsymbol{\beta}$, $\boldsymbol{\beta}_{1\perp}$ and $\boldsymbol{\beta}_{2\perp}$ of dimensions $n \times r$, $n \times s_1$ and $n \times s_2$, respectively. This subspaces represent the stationary, I(1) and I(2) subspaces and are of ranks r,

s_1 and s_2, respectively. The stationary cointegration space spanned by $\boldsymbol{\beta}$ can be further decomposed in two subspaces with basis $\boldsymbol{\beta}_d$ and $\boldsymbol{\beta}_p$ of dimensions $n \times r_d$ and $n \times r_p$ such that $r_d + r_p = r$. The matrix $\boldsymbol{\beta}_d$ contains as columns the cointegrating vectors that generate linear combinations of the variables that are I(0) and $\boldsymbol{\beta}_p$ contains as columns the cointegrating vectors that generate linear combinations of the variables that are I(1) but that cointegrate to I(0) with the first differences of the variables. This later case is known in the literature as polynomial cointegration. (See, for instance, Johansen, 1995, pag. 39).

The first step is to fit an unrestricted (without imposing any cointegration restriction) VAR(p) model in levels to the data. Table 1 shows the misspecification residual tests for a VAR(3) model. We used both univariate and multivariate diagnostics tools. The multivariate diagnostic tools used were the Akaike Information Criterion to select the order p of the VAR, the multivariate portmanteau test of Hosking (1980) over the residuals of the VAR(p) which is distributed as a χ^2 with $n^2(l-p)$ degrees of freedom, where l is the lag length used to perform the test, and the Lagrange Multiplier (LM) test over the residuals whose distribution is χ^2 with n^2 degrees of freedom. The univariate diagnostic tools used were the usual portmanteau test of Ljung-Box-Pierce, the Jarque-Bera test of normality and the LM test of the squared of the residuals for conditional heterocedasticity. In the table e_{LUSC}, e_{LUSF} and e_{LUSS} are the residuals of the VAR(p) for commodities, food and services respectively. All the tests indicate that an unrestricted VAR(3) in levels is an appropriate model, except for the Jarque-Bera for the second series that is too high. This high value is due just to one observation in US food CPI in January 1990. The analysis including an impulse dummy variable on that date has been performed and the Jarque-Bera is downloaded to 2.45. All the remaining statistics remain approximately the same. We don't have information to consider this anomalous value as generated exogenously to the data generation process, so we have not included this dummy variable in further analysis.

The variance-covariance matrix for the residuals is given by

$$\widehat{\Sigma}=\begin{pmatrix} 4.11\times10^{-6} & 2.19\times10^{-8} & -2.96\times10^{-9} \\ 2.19\times10^{-8} & 8.01\times10^{-6} & -1.30\times10^{-7} \\ -2.96\times10^{-9} & -1.30\times10^{-7} & 1.29\times10^{-6} \end{pmatrix}.$$

The next step is to determine the number of I(0) cointegration relations (r), and the number of the nonstationary components (s_1 for the I(1) and s_2 for the I(2) components). We apply Johansen's (1995) likelihood ratio test. The null hypothesis is that conditioning to a given number r of cointegration relationships, there are 0, 1, 2,... I(1) components in the system.

Table 2 shows the results for the I(2) Johansen's two step procedure. In the first step, we determine the number r of cointegration relations as if the system was I(1). Based on this number r, in a second step we determine the number s_1 of I(1) components in the system and $s_2 = n - r - s_1$ is the number of I(2) components. Q(r) is the likelihood ratio statistic for the I(1) analysis and Q($s_1|r$) is the likelihood ratio test for s_1 given r. Looking at the Q(r) column we detect that the null hypothesis r □ 1 is not rejected at the 5% significance level (this is seen italics in the table). In the second step, we look at the Q($s_1|r$) statistic; the bold figures mean that the null hypothesis for $s_1 = 1$ is not rejected at the 5% critical value.

So the final result of the test is ($r = 1, s_1 = 1$). Since $s_2 = r$, this means that the only possibility for I(0) cointegrating relations is through polynomial cointegration[3]. This test implies two cointegration relations, one from I(2) to I(1) and the other with the levels and first differences of the variables, from I(2) to I(0). The test indicates the presence of one I(2) component and one I(1) component, in favor of our hypothesis of several sources of nonstationarity in the prices indexes and therefore the need for disaggregation.

The joint statistical tests of the null hypothesis of $r = 1, s_1 = 1$ of Paruolo (1996) does not reject this joint hypothesis at the 99% critical values. (See table 3). There are also three roots of the companion

[3] Also, as r is not greater than s_2, there are no cointegration relations from I(2) to I(0) directly, so the only cointegration relation involves levels and first differences of the variables. (See, for instance, Johansen, 1997).

matrix (see, table 4) very close to unity, what should confirm the results of the test. All these pieces of evidence taken together lead us to conclude that $r = 1$ and $s_1 = 1$.

Once we know the different number of stationary and nonstationary components, we proceed to estimate them.

$$0.06LUSC_t + LUSF_t - 0.93LUSS_t + 10.75\Delta LUSC_t + 16.74\Delta LUSF_t + 18.65\Delta LUSS_t - 0.3 \sim I(0)$$

$$1.79LUSC_t - 0.58LUSF_t - 0.51LUSS_t \sim I(1)$$

The existence of a unique multicointegrating I(0) relationship shows the existence of more than one source of nonstationarity. In fact there are $p - r = 2$ nonstationary variables associated to these three US CPI components. Also the existence of one I(1) cointegration relationship indicates that only one source of nonstationarity is I(2), being the other one I(1). These results are further confirmed in section 4.

The cointegration relationships could be interpreted as follows. The multicointegrating I(0) relationship indicates that the ratio between the indexes for food and services is explained by a linear combination of the rates of inflation of the three components (food, services and commodities). The coefficients that multiply $LUSF_t$ and $LUSS_t$ could be considered to be equal. The I(1) cointegration relation, if we consider that the coefficients of $LUSF_t$ and $LUSS_t$ are equal, indicates that the rate of inflation in commodities is approximately 60% of the mean rate of inflation of food and services. Nevertheless, it must be said that it is difficult to anticipate the type of cointegrating relationships between markets under a common monetary policy, but with different possibilities of technological innovations with diverse degrees in incorporating them and different patterns in the change of consumer preferences. This is a very different situation to the analysis of prices with other macro variables.

4 Common trend analysis

Let $n = 3$ be the number of series and $s = s_1 + s_2$ the number of common nonstationary factors or common trends. As it was shown by Escribano and Peña (1994) the number of cointegration relations $n - s$ is the number of series minus the number of common trends.

We will confirm the results of the cointegration analysis by a common factors analysis. We assume that each component of the vector of observed series, $\mathbf{y}_t$, can be written as a linear combination of common factors and specific components,

$$\underset{n\times 1}{\mathbf{y}_t} = \underset{n\times s}{\mathbf{P}}\ \underset{s\times 1}{\mathbf{f}_t} + \underset{n\times 12}{\mathbf{C}}\ \underset{12\times 1}{\mathbf{D}_t} + \underset{n\times 1}{\mathbf{n}_t},$$

where $\mathbf{f}_t$ is the s-dimensional vector of common trends or nonstationary factors, $\mathbf{P}$ is the factor loading matrix, $\mathbf{D}_t$ is a matrix (12×1) that accounts for twelve seasonal dummies[4], and $\mathbf{n}_t$ is the vector of specific components. After extracting the common factors, we will fit an univariate model for the specific components of each one of the series of the CPI, if it is needed. In this model, the common factors are non-stationary non-observed variables which determine the long run behavior of the series. Contrary to the cointegration analysis, where we focus on the stationary relations, with factor models we explicitly model the sources of nonstationarity; so both techniques should be considered as complementary.

The key point is to determine the number of common trends. To do that, we build the 'generalized covariance matrices', $\mathbf{C}_y(k) = \frac{1}{T^{2d}} \sum (\mathbf{y_{t-k}} - \bar{\mathbf{y}})(\mathbf{y_t} - \bar{\mathbf{y}})'$, for lags $k = 0, 1, \cdots, 5$, that is sample auto-covariance matrices of the series but using $\frac{1}{T^{2d}}$, where d is the order of integration of the series instead of the usual $\frac{1}{T}$ normalization factor for the sample quantities. In Peña and Poncela (2000) it is shown that (i) the generalized covariance matrices converge weakly in the sense of Billingsley (1968) to the random matrix $\mathbf{\Psi}_y$, for $k = 0, 1, \cdots, K$, k small enough to T, the limits are taken as T goes to infinity and $\mathbf{\Psi}_y$ is a functional of the *integrated Brownian motion,* where the d times integrated Brownian motion is

[4] To keep the parallelism with the cointegration analysis, we introduce dummies to take into account seasonality.

defined as $\mathbf{V}_d(\tau) = \mathbf{F}_d(\tau) - \int_0^1 \mathbf{F}_d(\tau)d\tau$ and $\mathbf{F}_d(\tau)$ is defined recursively by $\mathbf{F}_d(\tau) = \int_0^\tau \mathbf{F}_{d-1}(s)ds$, for $d = 1, 2, ...$ with $\mathbf{F}_0(\tau) = \mathbf{W}(\tau)$, the standard Brownian motion[5] and (ii) there are exactly s eigenvectors (the same as the number of common trends) of these matrices that are common for the different lags $k = 0, 1, \cdots, K$.

Tables 5 and 6 show the first and second eigenvectors for the components of the CPI. In both tables, the first column shows the name of the variables and columns 2 through 6 show the eigenvectors for the indicated lag in the first row. The first two eigenvectors are very stable. They clearly indicate the existence of two common trends. After extracting the first two factors, the series became stationary. A first estimation of the factors is obtained by the linear combinations given by the eigenvectors. We perform an ADF unit root test to check the integration order of each of the factors. Table 7 confirms that there are one I(2) nonstationary factor and one I(1) factor.

A nice interpretation can also be given to these first two common nonstationary factors. The first common factor is I(2) and can be interpreted as a weighted mean driving all the CPI components, giving more weight to services CPI. The second factor is I(1) and can be interpreted as separating commodities from food and services, since the associated eigenvector has a negative weight in the first component and a positive one in the remaining ones.

We have estimated the factor model by maximum likelihood using the EM algorithm, modelling the I(2) common trend as a local linear trend as in Harvey (1989) and the I(1) common trend as a random walk,

$$\begin{aligned} f_{1t} &= \mu_t + \beta_t + \varepsilon_{1t} \\ \beta_t &= \beta_{t-1} + \eta_t \\ f_{2t} &= f_{2t-1} + \varepsilon_{2t} \end{aligned}$$

where f_{it}, $i = 1, 2$ are the common factors. The variance of the three uncorrelated white noises (ε_{1t}, ε_{2t} η_t) are set equal to 1 to identify the model. The specific components were modelled as pure autoregressive

[5] This can be proven using the results in Phillips and Durlauf (1986), Tiao and Tsay (1990) and Tanaka (1998).

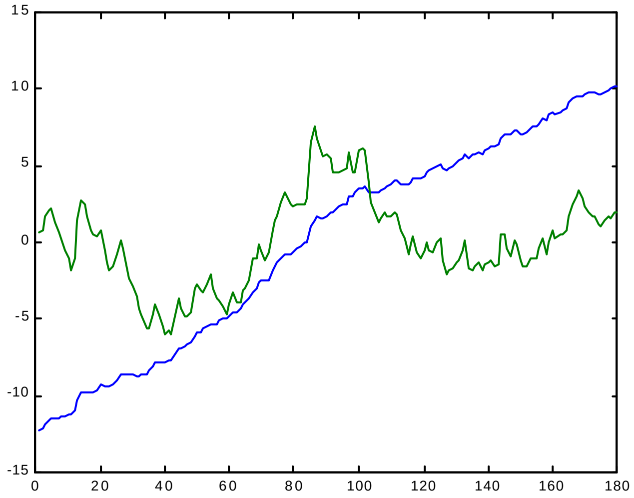

Figure 2: Graphs of the two nonstationary common factors of the food, rest of non-energy commodities and rest of services.

processes. Figure 2 shows the estimated nonstationary common factors.

5 Forecasting results

In this section, we forecast food, rest of non-energy commodities and rest of services US CPI and then aggregate them to forecast the non-energy US CPI. This is compared with the forecasts obtained directly from the non-energy US CPI.

Univariate ARIMA models are used both to forecast the non-energy CPI and its components. All the models are in second differences and used centered seasonal dummies. The multivariate forecasts obtained by either of the two procedures analyzed (cointegration and dynamic factor models) are not shown because they did not outperform the forecasts obtained by the univariate approach[6]. This is in the line with Christoffersen and Diebold (1998) who advocate that the use of multivariate models does

[6] They are available from the authors upon request.

not need to improve the ARIMA univariate forecasts. These authors point out the following paradox: it seems unlikely that the cointegrated systems provide information about the long-horizon evolution of the variables because the long-horizon forecast of the error-correction term is always 0. This could be a hint why the cointegration restrictions do not necessarily need to improve the univariate forecasts.

The sample considered for estimation is 1983:01 to 1997:12 and we generate one to twelve-step-ahead forecasts for the years 1998 through 2000, which have been reserved to evaluate the forecasting performance of both approaches. In consequence, we obtain 36 one-step-ahead forecast errors, 35 two-steps-ahead forecast errors and so on up to 24 twelve-steps-ahead forecast errors. The root mean squared error (RMSE) is employed as a measure of forecasting performance.

Also, to check if the differences between the two forecasting procedures are statistically significant we applied the Diebold and Mariano (1995) test statistic with the finite sample corrections suggested by Harvey et al. (1997). The null hypothesis states the equality of the two forecast procedures in terms of the quadratic loss as a function of their errors, $g(e)$, that is, $E(d_t) = 0$, where $d_t = [g(e_{At}) - g(e_{Dt})]$ and e_{At} and e_{Dt} are the forecast errors obtained with the aggregated and disaggregated approaches respectively and, in our case, g is the quadratic loss function. The Diebold and Mariano statistic is $S = \frac{\overline{d}}{\sqrt{\widehat{var}(\overline{d})}}$ where $\overline{d}$ is the sample mean of the d_t series and $\widehat{var}(\overline{d})$ is calculated taking into account the serial correlation in d_t. The S statistic follows an asymptotic standard normal distribution under the null hypothesis and its correction for finite sample follows a Student t with $l - 1$ degrees of freedom, where l is the number of forecasts. The computed S statistics for h=1 to 12 horizons of prediction are given in table 8. If the computed S is positive and statistically significant, it would indicate that the disaggregated model improves the aggregated one.

Table 8 shows the results on forecasting accuracy from the aggregated and disaggregated approaches. In the first column, we show the horizon of prediction, in the second and third columns, we present the RMSE of the aggregated and disaggregated approaches ($RMSE_A$ and $RMSE_D$, respectively), in the fourth column we measure the percentage of change of the RMSE of the aggregated approach over the disaggregated one and in the fifth and sixth columns we present the Diebold and Mariano test statistic and its p-value.

Table 8 shows that for the medium and long run we can obtain improvements up to 23% in the RMSE with the disaggregated approach. Only for the first three horizons of prediction, the univariate aggregated model slightly outperforms the disaggregated approach. This is in agreement with the fact that the diverse trending behavior is better captured modelling the different components separately. So, the major gains in forecasting are expected to happen in the medium and long run terms. A closer look to the data reveals that several extreme observations in the estimation sample in the CPI components cancel out when aggregating to form the non-energy CPI. Including intervention analysis in the components improves short term forecasts, while it keeps the important difference in the medium and long term forecasts. Regarding the Diebold and Mariano test, we can find statistically significant differences between both approaches in favor of the disaggregated methodology from the fifth step ahead forecasts onwards, while for the shorter run forecasts the differences between the two approaches are not statistically significant.

6 Conclusions

Four components of monthly US CPI have been analyzed: energy, food, rest of commodities, and rest of services. Several unit root tests performed indicated that the first three components are better described as I(2) processes, while the energy component exhibits a lower order of integration and was discarded from a joint multivariate analysis.

A cointegration analysis has been performed over the remaining three components (food, rest of non-energy commodities and rest of services) in order to identify the number and nature of sources of nonstationarity in the US CPI. We have obtained that there are at least one I(2) and one I(1) nonstationary trends driving the consumer price indexes.

According with the above mentioned results the data support the existence of two cointegrating relations: an I(0) multicointegrating relation in which the ratio between food and services consumer price indexes is explained by a linear combination of the inflation rates of all the components jointly analyzed (food, commodities and services); and an I(1) cointegrating relation that shows that the rate of inflation in commodities is about 60% of the mean rate of inflation of food and services.

The number and nature of the sources of nonstationarity has been corroborated through a common factors analysis that confirmed the existence of an I(2) common trend, which could be interpreted as a weighted mean of the three variables jointly analyzed, and an I(1) nonstationary factor.

All of this calls for a disaggregated analysis of the US non-energy CPI in order to be able to capture the different movements of prices in markets of different characteristics. This has been illustrated to be useful in forecasting the medium and long run terms. Gains of a disaggregated approach based on univariate time series models for the components, over an aggregated one become statistically significant from five months ahead forecasts and reach 23% for the twelve months ahead root mean squared error forecasts. Short term forecasts are improved by the disaggregated analysis when including specific intervention analysis for the components. The intervention analysis was not needed in the global US non-energy CPI due to the cancellation of extreme observations in the components. The conclusions in any case are the same with or without intervention analysis. The multivariate techniques were useful to identify the different sources of nonstationarity present in the data and improve the forecast accuracy of the aggregate.

Questions for further research are the following. First, the forecasts accuracy could be further improved by including specific leading indicators for each one of the CPI components. Second, it could be studied if the I(2) common trend is related to monetary policy and the I(1) to technology and changes in consumer habits.

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7 Appendix: Unit root tests

In the context of testing unit roots we shall use the terminology proposed by Espasa and Peña (1995) $I(d,m)$ for integrated processed, where d means the number of positive unit roots in the process and m equals the maximum possible number of deterministic elements in the mean of the differenced process. Thus, $m = 1$ indicates the presence of a constant, $m = 2$ the presence of a linear trend (constant plus slope), $m = 3$ the presence of a quadratic trend, etc. With this notation $h = d + m - 1$ gives the order of the trend polynomial in the forecasting function. This trend will be deterministic if $d = 0$ and will have parameters depending on the initial conditions if $d \neq 0$. All parameters will depend on the initial conditions if $m = 0$.

For the data we are analyzing we will have as maintained hypothesis that the maximum value of h is 1, which excludes the presence of quadratic terms (stochastic or deterministic) in the trend. The maintained hypothesis can be fulfilled without unit roots, I(0,2), with one unit root I(1,1) or with two unit roots I(2,0). We will first test the presence of one unit root and if this hypothesis is not rejected we will test the presence of two unit roots.

The models used for the test of one unit root are collected in table A.1 and they differ on the inclusion or not of the deterministic seasonality and deterministic intervention factors. The null, I(1,1), and the possible alternatives I(0,2), I(0,1) and I(1,2), are reflected in table A.2. The alternative I(0,1) would imply that the trend, just a constant, in the data is much simpler that the maximum allowed in the maintained hypothesis, but the alternative I(1,2) would imply that the maintained hypothesis is wrong, that the trend has quadratic terms and that the models used for these tests were not appropriate. They should allow for a quadratic (deterministic) trend in order to capture quadratic evolution in the long run without forcing the presence of a unit root.

The test of two unit roots, once it has not been rejected the presence of one, can be done with models in table A.3 and the same maintained hypothesis, considering different deterministic formulations. The null, I(2,0), and possible alternatives in these models, I(1,1), I(1,0) and I(2,1) are represented in table A.4. The first alternative implies the rejection of a second unit root within the context of $h = 1$, but the rejection of the third alternative could imply that the maintained hypothesis is wrong and the models used were inappropriate. They should include a linear deterministic trend. The alternative I(1,0) implies a simple trend but within the maintained hypothesis. From the three possibilities of the alternative, only the first one would fit our data. The joint hypothesis is considered in order to avoid the identification problems suggested by Franses (2001), Dickey and Fuller (1981) and Harris (1995) among others. In order to rule out the alternatives we are not interested in, we also compute Dickey- Fuller -statistics.

Tables A.5 and A.7 show the results for unit root testing. In all tables, τ is the augmented Dickey-Fuller (ADF) statistic for an unit root, F_1 and F_2 are the statistics for the joint null hypothesis described in tables A.2 and A.4 respectively, SCR is the sum of squared residuals and Q(24) is the Ljung-Box statistic at lag 24. The models where described in tables A.1 and A.3, the autoregressive order is 12 for the differenced process, but the results are robust to the lag length. The critical values can be found in Dickey and Fuller (1979), an * on the right side of the statistics indicates rejection at the usual 5% significance level, and ** indicates rejection at 1% significance level.

8 References

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9 Tables

Table 1: Misspecification residual tests for a VAR(3) model

Multivariate tests		Statistic		
Akaike		-28.52		
Portmanteau	Lag			
	4	12.25		
	5	22.98		
	6	30.90		
	7	52.54		
	12	113.65		
	24	232.42		
LM	Lag			
	1	7.73		
	12	14.89		
Univariate tests		e_{LUSC}	e_{LUSF}	e_{LUSS}
Q(12)		12.213	6.739	20.582
Q(24)		24.471	16.227	48.469
ARCH		1.256	0.0002	1.095
Jarque-Bera		8.62	72.99	5.12

Table 2: Two step I(2) Johansen's cointegration analysis

n-r	r	$Q(s_1 \mid r)$			Q(r)	5%
3	0	172.04	81.56	3.12	109.01	34.80
2	1		84.63	**7.05**	*18.06*	19.99
1	2			24.76	2.86	9.13
5%		34.80	19.99	9.13		
$s_2 = n-r-s_1$		3	2	1		

Table 3: Paruolo's joint statistical test for s_1 and r.

$n-r$	r	$Q(s_1, r)$			$Q(r)$
3	0	281.05	190.57	112.13	109.01
2	1		102.7	**25.11**	*18.06*
1	2			27.63	2.86
$s_2 = n - r - s_1$		3	2	1	

Table 4: Eigenvalues of the companion matrix.

Real	Complex	Modulus
0.1463	0.4256	0.4501
0.1463	-0.4256	0.4501
0.02915	0.2853	0.2867
0.02915	-0.2853	0.2867
0.1439	0.06221	0.1568
0.1439	-0.06221	0.1568
0.9974	0	0.9974
0.9752	0.01818	0.9753
0.9752	-0.01818	0.9753

Table 5 First eigenvector of the generalized covariance matrices, for lags 0 through 5.

Lag	0	1	2	3	4	5
LUSC	0.40	0.40	0.40	0.40	0.40	0.40
LUSF	0.55	0.55	0.55	0.55	0.55	0.55
LUSS	0.73	0.73	0.73	0.73	0.73	0.73

Table 6 Second eigenvector of the generalized covariance matrices, for lags 0 through 5.

Lag	0	1	2	3	4	5
LUSC	-0.90	-0.90	-0.91	-0.92	-0.92	-0.91
LUSF	0.39	0.37	0.33	0.26	0.23	0.32
LUSS	0.20	0.22	0.25	0.31	0.33	0.26

Table 7: Unit root tests on factors

	$H_0 : I(1)$	$H_0 : I(2)$
Factor 1	-1.36	-1.51
Factor 2	-0.87	-2.60*

Note: An asterisk on the table indicates that the null hypothesis is rejected at the usual 5% level of significance.

Table 8: Forecasting accuracy with aggregated and disaggregated approaches.

Horizon of Prediction	$RMSE_A$	$RMSE_D$	$\left(\frac{RMSE_A}{RMSE_D} - 1\right) \times 100$	DM	$p - val$
1	0.137	0.148	-7.05	-0.98	0.33
2	0.239	0.253	-5.45	-1.56	0.13
3	0.307	0.318	-3.71	-1.29	0.21
4	0.335	0.333	0.48	0.05	0.96
5	0.334	0.311	7.43	2.21	0.03
6	0.350	0.305	14.70	1.54	0.13
7	0.400	0.342	16.81	3.34	0.002
8	0.468	0.413	13.53	2.02	0.054
9	0.532	0.479	11.03	2.33	0.03
10	0.545	0.484	12.78	1.90	0.07
11	0.517	0.446	15.87	1.76	0.09
12	0.482	0.392	23.00	1.59	0.13

9.1 Tables for the Appendix

Table A.1: Models for the null hypothesis of a unit root. $H_0 : \gamma = 0, \psi = 0$. H_1 : no H_0.

[1.1]	$\triangle y_t = c + \gamma t + \psi y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle y_{t-i}$
[1.2]	$\triangle y_t = c + \gamma t + \psi y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle y_{t-i} + \sum_{i=1}^{11} c_i D_{it}$
[1.3]	$\triangle y_t = c + \gamma t + \psi y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle y_{t-i} + \sum_{i=1}^{11} c_i D_{it} + \sum_{i=1}^{s} b_i I_{it}$

Notes: $\Delta = 1 - L$ is the difference operator, where L is the lag operator such that $Ly_t = y_{t-1}$, c is a constant, t is a deterministic trend, $D_{it}, i = 1, \ldots 11$ are 11 centered seasonal dummies, and I_{it} represent the intervention dummies required in each case.

Table A.2: Characteristics of the processes under the null and the three possible alternatives.

H_0	$\gamma = 0,\ \psi = 0$	I(1) with constant ($c \neq 0$) or I(1,1)	
H_1	$\gamma \neq 0,\ \psi \neq 0$ **trend stationary process** I(0,2)	$\gamma = 0,\ \psi \neq 0$ pure stationary process I(0,1)	$\gamma \neq 0,\ \psi = 0$ quadratic linear trend plus an I(1) process I(1,2)

Table A.3: Models for the null of I(2).

[2.1]	$\triangle^2 y_t = c + \psi \triangle y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle^2 y_{t-i}$
[2.2]	$\triangle^2 y_t = c + \psi \triangle y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle^2 y_{t-i} + \sum_{i=1}^{11} c_i D_{it}$
[2.3]	$\triangle^2 y_t = c + \psi \triangle y_{t-1} + \sum_{i=1}^{p} \psi_i \triangle^2 y_{t-i} + \sum_{i=1}^{11} c_i D_{it} + \sum_{i=1}^{s} b_i I_{it}$

Notes: $\Delta = 1 - L$ is the difference operator, where L is the lag operator such that $Ly_t = y_{t-1}$, c is a constant, t is a deterministic trend, $D_{it}, i = 1,...11$ are 11 centered seasonal dummies, and I_{it} represent the intervention dummies required in each case.

Table A.4: Characteristics of the processes under the null and the three possible alternatives.

H_0	$c = 0,\ \psi = 0$	I(2) without constant or I(2,0)	
H_1	$c \neq 0,\ \psi \neq 0$ **I(1) with constant process** I(1,1)	$c = 0,\ \psi \neq 0$ I(1) process I(1,0)	$c \neq 0,\ \psi = 0$ I(2) with constant process I(2,1)

Table A.5: I(1) tests for the logarithm of US CPI components

		Components		
Model	Stats	LUSC	LUSC	LUSS
[1.1]	τ	0.12	-1.92	-0.20
	F_1	1.32	2.06	2.88
	SCR	0.000987	0.001820	0.000266
	Q(24)	20.08	23.04	24.21
[1.2]	τ	-0.37	-1.59	0.15
	F_1	0.83	1.62	2.76
	SCR	0.000705	0.001386	0.000195
	Q(24)	15.75	10.31	16.90
[1.3]	τ	-0.28	-1.90	0.30
	F_1	0.68	2.23	6.21
	SCR	0.000606	0.001183	0.000146
	Q(24)	15.98	18.76	20.41

Table A.6: I(2) tests for the logarithm of US CPI components

		Components		
Model	Stats	LUSC	LUSF	LUSS
[2.1]	τ	-1.75	-2.47	-0.63
	F_2	1.96	3.10	0.70
	SCR	0.0010	0.001808	0.000274
	Q(24)	19.16	19.38	20.22
[2.2]	τ	-1.88	-2.46	-0.57
	F_2	1.95	3.09	0.63
	SCR	0.000702	0.001366	0.000195
	Q(24)	14.21	10.44	9.18
[2.3]	τ	-1.81	-2.37	-0.46
	F_2	1.81	2.88	0.55
	SCR	0.000664	0.001300	0.000171
	Q(24)	14.65	12.82	13.66

Table A.7: Unit root tests for the logarithm of US Energy CPI.

	Models for I(1)		
Stats	[1.1], $\gamma = 0$	[1.2], $\gamma = 0$	[1.3], $\gamma = 0$
τ	-1.88	-1.43	-1.14
F_1	1.78	1.02	0.68
SCR	0.035782	0.025017	0.0154
Q(24)	11.07	16.86	32.33
	Models for I(2)		
Stats	[2.1], $c = 0$	[2.2], $c = 0$	[2.3], $c = 0$
τ	-3.72**	-3.77**	-3.49**
F_2	13.87**	14.20**	12.19**
SCR	0.035843	0.024952	0.01953
Q(24)	8.84	13.98	21.06

10 Captions for figures

Figure 1: Natural logarithms of energy (LUSE), food (LUSF), rest of non-energy commodities (LUSC) and rest of services (LUSS).

Figure 2: Graphs of the two nonstationary common factors of the food, rest of non-energy commodities and rest of services.