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ASYMMETRIC LONG MEMORY GARCH: A REPLY TO HWANG'S MODEL

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Keywords: EGARCH, FGARCH, FIGARCH, FIEGARCH

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Asymmetric long memory GARCH: A reply to Hwang's model

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1 Introduction

The GARCH model was originally proposed by Engle (1982) and Bollerslev (1986) to represent the dynamic evolution of conditional variances. One of the most useful applications of GARCH models is to represent high frequency financial returns. The GARCH model has been extended in many directions. Two of the most popular generalizations deal with the asymmetric response of the conditional variance, σ_t^2, to positive and negative returns and with the persistence often observed in the autocorrelations of squared returns.

The first asymmetric model proposed in the financial econometrics literature was the Exponential GARCH (EGARCH) model of Nelson (1991). To simplify the exposition, we will focus on the simplest EGARCH(1,1) model and their equivalents. In the EGARCH(1,1) model, the series of returns is given by $\varepsilon_t = z_t\sigma_t$ where z_t is an independent white noise process with unit variance and symmetric distribution and the conditional variance of returns is given by:

$$\log(\sigma_t^2) = \omega + (1-\rho L)^{-1}(1+\psi L)g(z_{t-1}) \tag{1}$$

where $g(z_t) = \theta z_t + \gamma\left[|z_t| - E(|z_t|)\right]$ and L is the lag operator such that $L^j x_t = x_{t-j}$. The parameter θ measures the asymmetric response of volatility. Later, Henstchel (1995) proposed the Family GARCH (FGARCH) model that nests many previous heteroscedastic models. The conditional variance is given by:

$$\begin{aligned} \frac{\sigma_t^\lambda - 1}{\lambda} &= \omega' + \alpha\sigma_{t-1}^\lambda f^\nu(z_{t-1}) + \delta\frac{\sigma_{t-1}^\lambda - 1}{\lambda} \\ f(z_t) &= |z_t - b| - c(z_t - b) \end{aligned} \tag{2}$$

The asymmetry is introduced by shifting and rotating the absolute value of the shock through the parameters b and c respectively. The parameter λ represents the Box and Cox (1964) transformation for the conditional variance so that the limit when λ goes to zero is the logarithmic transformation. The parameter ν serves to transform the function $f(\cdot)$. Henstchel (1995) shows that model (2) encompasses, among many others, the GARCH model, the EGARCH model, the threshold GARCH (TGARCH) model of Zakoian (1994) and the Nonlinear GARCH (NGARCH) model of Higgins and Bera (1992).

With respect to the persistence of volatility, Baillie *et al.* (1996) propose the Fractionally Integrated GARCH (FIGARCH) model where squared returns have long memory. The conditional variance of the FIGARCH(1,d,1) model is given by

$$\sigma_t^2 = \omega^* + \left[1 - (1-\beta L)^{-1}(1-\phi L)(1-L)^d\right]\varepsilon_t^2 \tag{3}$$

where d is the long memory parameter such that $0 \leq d < 1$. The FIGARCH(1,d,1) model in (3) has been derived by introducing a fractional root in the autoregressive polynomial of the ARMA representation of squared observations of a GARCH model. However, in the appendix we show how it is possible to obtain the following alternative representation of the FIGARCH(1,d,1) model:

$$\sigma_t^2 = \omega'' + \alpha(1-\phi L)^{-1}(1-L)^{-d}(1-\delta L)\sigma_{t-1}^2(z_{t-1}^2 - 1) \tag{4}$$

At the moment, the only long memory asymmetric model of the GARCH family is the Fractionally Integrated EGARCH (FIEGARCH) model proposed by Bollerslev and Mikkelsen (1996). The conditional variance of the FIEGARCH(1,d,1) model is given by

$$\log(\sigma_t^2) = \omega + (1-\rho L)^{-1}(1-L)^{-d}(1+\psi L)g(z_{t-1}) \tag{5}$$

Hwang (2001) proposes a new asymmetric long memory model, the asymmetric FIFGARCH model, and claims that it nests the FIGARCH and FIEGARCH models. In section 2, we will show that this model is badly specified and does not encompass the previously mentioned models. We propose an alternative specification, based on the asymmetric FIFGARCH model, that seems to fullfil the objectives stated by Hwang (2001).

2 The asymmetric long memory GARCH model.

Hwang (2001) proposes the asymmetric FIFGARCH(1,d,1) model to represent both the long memory and asymmetry properties of conditional standard deviations. The specification of the conditional variance is given by:

$$\sigma_t^\lambda = \kappa^* + \left[1 - (1-\delta L)^{-1}(1-\phi L)(1-L)^d\right] f^\nu(z_t)\sigma_t^\lambda \tag{6}$$

Looking at equation (6), it seems that Hwang (2001) has tried to generalize the FGARCH model in equation (2) to allow for long memory, by introducing the fractional root, $(1-L)^d$, in the volatility equation. Alternatively, the FIFGARCH(1,d,1) model could be seen as a generalization of the FIGARCH model in (3) with the conditional variance depending on a transformation similar to the one proposed by Henstchel (1995), instead of depending on squared returns, so that the model could deal with asymmetry. In any case, Hwang (2001) does not refer to any of the previous related papers in this area.

Notice that model (6) is not able to represent adequately the simultaneous presence of long memory and asymmetry in the conditional variance. First of all, the long memory property is somehow defined in a strange way because the factor $(1-L)^d$ does not affect the corresponding transformation of the conditional standard deviation, but to the function $f(\cdot)$. The way that the long memory is introduced in equation (6) seems to be similar to the way that the GARCH model is generalized to the FIGARCH model in (3). However, this latter generalization is based on the ARMA representation of squared returns implied by the GARCH model; see, Baillie *et al.* (1996) and the appendix.

Secondly, Hwang (2001) is not considering a Box-Cox transformation of the conditional standard deviation and, as a consequence, model (6) is badly misspecified when $\lambda = 0$. In this case, the model becomes:

$$1 = \kappa^* + \left[1 - (1-\delta L)^{-1}(1-\phi L)(1-L)^d\right] f^{\nu}(z_t) \tag{7}$$

and the conditional standard deviation has disappeared. Therefore, the FIEGARCH model cannot be obtained from model (6) when $\lambda = 0$ as Hwang (2001) claims. But even if the asymmetric FIFGARCH(1,d,1) model were defined in such a way that, when $\lambda = 0$, the logarithmic transformation would be obtained and considering $\nu = 1$ and $b = 0$, the model would become:

$$\log(\sigma_t^2) = \kappa^* + \left[1 - (1-\delta L)^{-1}(1-\phi L)(1-L)^d\right] (|z_t| - cz_t)\log(\sigma_t^2) \tag{8}$$

which is not a FIEGARCH model.

Finally, notice that the FIGARCH model in (3) can be obtained as a particular case of model (6) when $\lambda = \nu = 2$ and $c = b = 0$. However, this model is not able to represent asymmetries in the response of conditional volatilities to negative and positive returns. On the other hand, if $d = 0$ in equation (6) and the Box-Cox transformation is applied to σ_t in the left hand side of such equation, then the short memory Hentchel's FGARCH model will be obtain. However, it is not obvious that the asymmetric FIFGARCH model is able to encompass adequately models with conditional variances that simultaneously are asymmetric and have long memory.

Alternatively, we propose the following model based on a direct generalization of the FGARCH model introducing a fractional unit root and subtracting the unconditional mean of $f^{\nu}(z_t)$ from the function $f^{\nu}(z_t)$:

$$\frac{\sigma_t^{\lambda}-1}{\lambda} = \omega' + \alpha \left[(1-\delta L)^{-1}(1-L)^{-d}(1-\phi L)\right] \sigma_{t-1}^{\lambda} \{f^{\nu}(z_{t-1}) - \mu E(f^{\nu}(z_{t-1}))\} \tag{9}$$

If $\lambda = 0$, $\nu = 1$, $b = 0$ and $\mu = 1$, we obtain the FIEGARCH model. Also, if $\lambda = \nu = 2$, $b = c = 0$ and $\mu = 1$, the FIGARCH model in (4) is obtained. Finally, if $d = \phi = \mu = 0$, the FGARCH model in (2) would be obtained. Remember that, as previously mentioned, the FGARCH model nests the short memory GARCH and EGARCH models.

However, it will be necessary to analyze in detail the statistical properties of this model as well as the properties of the estimators of the parameters before the model could be applied to the analysis of real time series.

3 Conclusions

The asymmetric FIFGARCH model has been proposed by Hwang (2001) to represent adequately the asymmetry and persistence often observed in the conditional variances of high frequency series of returns. It seems that the FIFGARCH model is trying to generalize the model proposed by Henstchel (1995) in the same way the GARCH model has been generalized to the FIGARCH model. However, contrary to the claims made by Hwang (2001), we have shown that the asymmetric FIFGARCH is not able to encompass previous models proposed in the literature with the exception of the FIGARCH model. We think that the new asymmetric FIFGARCH should be deeply revised before it can be safely applied to the analysis of real financial time series. We propose an alternative specification that encompasses the most popular long memory and asymmetric models for conditional variances.

4 Appendix

The GARCH(2,1) model is given by:

$$\begin{aligned} \varepsilon_t &= z_t \sigma_t \\ \sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{10}$$

Alternatively, model (10) can be written as an ARMA(2,1) for squared observations as follows:

$$\varepsilon_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \beta(\varepsilon_{t-1}^2 - \nu_{t-1}) + \nu_t \tag{11}$$

where $\nu_t = \varepsilon_t^2 - \sigma_t^2$. Reorganizing terms in (11), it is possible to obtain the following expression:

$$[1 - (\alpha_1 + \beta)L - \alpha_2 L^2]\varepsilon_t^2 = \omega^* + (1 - \beta L)\nu_t \tag{12}$$

If there is a unit root, the autoregressive polynomial in (12) can be factorized as $[1 - (\alpha_1 + \beta)L - \alpha_2 L^2] = (1 - \phi L)(1 - L)$ and the following expression is obtained for ε_t^2 :

$$(1 - \phi L)(1 - L)\varepsilon_t^2 = \omega^* + (1 - \beta L)\nu_t \tag{13}$$

However, if there is a fractional root instead of a unit root, the following model for squared observations is obtained

$$(1 - \phi L)(1 - L)^d \varepsilon_t^2 = \omega^* + (1 - \beta L)\nu_t \tag{14}$$

Finally, substituting $\nu_t = \varepsilon_t^2 - \sigma_t^2$ in (14), the FIGARCH(1,d,1) model in (3) is directly obtained. However, it is possible to obtain an alternative expression of the FIGARCH(1,d,1) model by considering directly the expression of the variance in (10):

$$\sigma_t^2 = \omega + (\alpha_1 + \alpha_2 L)\varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + (\alpha_1 + \alpha_2 L)\sigma_{t-1}^2 - (\alpha_1 + \alpha_2 L)\sigma_{t-1}^2 \tag{15}$$

$$= \omega + (\alpha_1 + \alpha_2 L)\sigma_{t-1}^2 (z_{t-1}^2 - 1) + (\beta + \alpha_1 + \alpha_2 L)\sigma_{t-1}^2 \tag{16}$$

Reorganizing terms in (15), the following expression is obtained:

$$\textbf{??}\sigma_t^2 = \omega^* + [1 - (\beta + \alpha_1)L - \alpha_2 L^2]^{-1}(\alpha_1 + \alpha_2 L)\sigma_{t-1}^2 (z_{t-1}^2 - 1) \tag{17}$$

If similarly to what we have done in expression (14), we factorize the autoregressive polynomial allowing for a fractional root, then the FIGARCH(1,d,1) model in (4) is obtained:

$$\sigma_t^2 = \omega^* + \alpha(1-\phi L)^{-1}(1-L)^{-d}(1-\delta L)\sigma_{t-1}^2(z_{t-1}^2 - 1) \tag{18}$$

where $\alpha = \alpha_1$ and $\delta = -\frac{\alpha_2}{\alpha_1}$.

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