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Departamento de Estadística y Econometría  
Universidad Carlos III de Madrid  
Calle Madrid, 126  
28903 Getafe (Spain)  
Fax (34) 91 624-98-49

## ESTIMATION OF A DYNAMIC DISCRETE CHOICE MODEL OF IRREVERSIBLE INVESTMENT

Rocío Sánchez-Mangas\*

### Abstract

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In this paper we propose and estimate a dynamic structural model of fixed capital investment at the firm level. Our dataset consists of an unbalanced panel of Spanish manufacturing firms. Two important features are present in this dataset. There are periods in which firms decide not to invest and periods of large investment episodes. These empirical evidence of infrequent and lumpy investment provides evidence in favour of irreversibilities and nonconvex capital adjustment costs. We consider a dynamic discrete choice model of irreversible investment with a general specification of adjustment costs including convex and nonconvex components. We use a two stage estimation procedure. In a first stage, we obtain GMM estimates of technological parameters. In the second stage, we obtain partial maximum likelihood estimates for the adjustment cost parameters. The estimation strategy builds on the representation of conditional value functions as a computable function of conditional choice probabilities. It is in the line of structural estimation techniques which avoid the solution of the dynamic programming problem.

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**Keywords:** Structural estimation, Irreversible investment, Capital adjustment costs.

\*Sánchez-Mangas, Dept. of Statistics and Econometrics, Universidad Carlos III de Madrid, C/Madrid, 126, 28903 Getafe (Madrid). Spain. E-mail: [rsman@est-econ.uc3m.es](mailto:rsman@est-econ.uc3m.es). Tel: +34 916249826. I am very grateful to my thesis advisor César Alonso. I would also like to thank Victor Aguirregabiria, Pedro Mira, Alfonso R. Sánchez and seminar participants at Universidad Carlos III de Madrid, Universitat Autònoma de Barcelona and Universidad de Vigo for very helpful comments. Financial support from Spanish DGI, grant BEC2000-0170, is acknowledged. All remaining errors are my own.

# Estimation of a Dynamic Discrete Choice Model of Irreversible Investment

Rocio Sanchez-Mangas\* †  
Universidad Carlos III de Madrid

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## Abstract

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*JEL Codes:* C25, C23, C14, D21

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†Contact address: Universidad Carlos III de Madrid. Dept. of Statistics and Econometrics. C/Madrid, 126. 28903 Getafe-Madrid (Spain). Tel: +34 916249826 Fax: +34 91624984. E-mail: rsman@est-econ.uc3m.es

# 1 Introduction

Firms' investment is an economic decision that plays a crucial role in both the cyclical and long run performance of any economy. As one of the main components of the aggregate demand, it has a direct impact on the business cycle. Furthermore, capital accumulation, as well as technology adoption, constitute the leading sources of economic growth in the long run. In the real world governments are interested on moderating both the impact and duration of the recessions and, at the same time, on fostering the economic growth of the countries. Implementation of public policies aimed at stimulating firms' investment appears as a primary instrument to enhance economic growth. Thus, the importance of investment to macroeconomics is obvious. A previous step to understand macro investment is the characterization of the investment process taking place at the firm level. Our goal in this paper is to get a better understanding of microeconomic investment decisions and the nature of capital adjustment costs firms face when they decide to undertake an investment project.

Although there exists a huge literature concerning investment at macro level, it has been in the last years when investment literature has shown an increasing concern about the modelling of microeconomic investment decisions. A strong evolution has taken place from the initial frictionless neoclassical model to the actual nonconvex adjustment cost models. The Jorgenson's neoclassical model (1963), characterized by the absence of capital adjustment costs, yielded a static decision rule for capital stock. Empirical evidence has shown the failure of this model to explain investment behaviour, because of the observation that capital adjustment takes time to complete. Given this limitation, it was necessary to introduce dynamics in investment models. Generalizations of the structural approach of Jorgenson's model appeared in the seventies. Treadway (1969), Lucas (1971) and Abel (1980) proposed models introducing explicitly capital adjustment costs. Initially, as a matter of analytical convenience, these costs were assumed to be strictly convex and differentiable. This structure yielded a closed-form expression for the investment decision, which generated a smooth adjustment of capital consistent with the observed aggregated data. But descriptive evidence on the time series behaviour of firm-level investment data

has questioned the validity of such assumption, because this convex structure can not match the periods of inaction and lumpy investment observed at micro level. There is an increasing literature highlighting the importance of these phenomena. Doms and Dunne (1994), using data of US manufacturing firms showed that more than half of them experienced a year in which the capital stock increases more than 35%. Nilsen and Schiantarelli (1998), using information of Norwegian plants, find that around 30% of them have zero investment in an average year. Barnett and Sakellaris (1995), Caballero, Engel and Haltiwanger (1995), and Abel and Eberly (1996) obtain similar results for US firm-level data.

The evidence about infrequent and lumpy adjustment have given rise to a new generation of investment models which take into account irreversibilities and nonconvex adjustment costs. Among them, we can cite Bertola and Caballero (1994), Abel and Eberly (1996), and Caballero and Leahy (1996), Cooper, Haltiwanger and Power (1999) and Cooper and Haltiwanger (2000). In some of these papers it is stressed the convenience of consider convex and nonconvex costs rather than either convex or nonconvex costs to best fit the data.

In this paper we propose and estimate a dynamic structural model of fixed capital investment at the firm level. Our dataset consists on an unbalanced panel of Spanish manufacturing firms. The evidence of infrequent and lumpy investment is also present in these data. Based on this empirical facts we consider a dynamic discrete choice model of irreversible investment with a general specification of adjustment costs including convex and nonconvex components. We use a two stage estimation procedure. In a first stage, we obtain GMM estimates of technological parameters. In the second stage, we obtain partial maximum likelihood estimates for the adjustment cost parameters. The estimation strategy builds on the representation of conditional value functions as a computable function of conditional choice probabilities. It is in the line of structural estimation techniques which avoid the solution of the dynamic programming problem. More specifically, it is based on a new representation of conditional value function proposed by Hotz and Miller (1993)

The rest of the paper is organized as follows. In Section 2 we describe the dataset used in this study. Section 3 formulates the dynamic structural model of investment.

In Section 4 we describe the estimation method we implement. Section 5 reports the estimation results and Section 6 concludes.

## 2 The data. Preliminary analysis

The dataset we use in this study has been taken from the Encuesta de Estrategias Empresariales (ESEE) conducted by the Spanish Ministry of Industry and Energy. It contains annual information of the balance sheet and other economic variables for a large number of Spanish manufacturing firms. Our sample is an unbalanced panel of 1592 firms between 1990 and 1997. We concentrate on capital stock and gross expenditure on capital goods. The investment rate for period  $t$  has been constructed as the ratio between gross expenditure in that period and the capital stock at the beginning of the period.

There exists a huge empirical literature highlighting the importance of inaction and lumpiness in microeconomic investment datasets. Doms and Dunne (1998) use data on american firms from 1972 to 1989. They find that more than half of them increase their capital stock over than 35% in some of the years considered. Nielsen and Schiantarelli (1998), using information on Norwegian plants, find that about 30% of them present zero investment in an average year. Similar findings are reported for different countries in Barnett and Sakellaris (1995), Caballero, Engel and Haltiwanger (1995), Abel and Eberly (1996) or Eberly (1997).

These empirical features are also present in our dataset. Figure 1 depicts a histogram of annual firm-level gross investment rates. The distribution is strongly skewed to the right. Around 30% of the observations have investment rates that are zero or close to zero (less than 0.033 gross investment rate), which reflects the fact that many firm-year observations involve little or no investment. The long right tail illustrates the fact that a fraction of plants experiment a large investment episode in any given year. The last bar accounts for the observations having an investment

rate greater than 0.98.

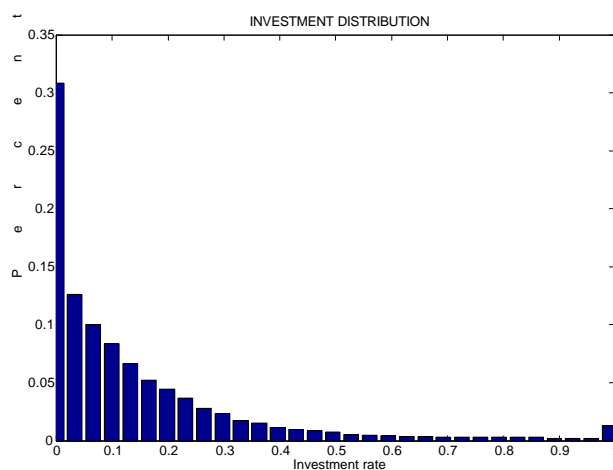


Figure 1

In the following table we can see the evidence of inaction and lumpiness by year. The first column shows the percentage of observations that experiment zero investment in a given year, while the second one shows the percentage that experiment an investment rate greater than 20%, where  $i$  stands for investment rate.

Year	Inaction (% obs. with $i = 0$ )	Lumpiness (% obs. with $i > 0.2$ )
1991	17.46	30.81
1992	18.61	26.59
1993	23.46	18.52
1994	20.28	21.54
1995	18.39	26.65
1996	16.52	23.75
1997	16.24	26.19

Table 1: Evidence of inaction and lumpiness.

Even in the year of lowest percentage of observations with zero investment, this percentage is quite high, above 16%. On the other hand, investment rates higher than 20% arise in more than 20% of the observations in almost every year, reaching 30% in one of them. We also report this evidence of infrequent and lumpy capital adjustment

distinguishing three categories of firms: small, medium and large firms. We follow the classification criterion established by the European Commission. According to this criterion, small firms are those with no more than 50 employees and no more than 7 million euro of annual turnover. Medium firms are those with more than 50 and no more than 250 employees and an annual turnover greater than 7 million euro and lesser than 40 million euro. Large firms are those with more than 250 employees and an annual turnover greater than 40 million euro. The following table shows in the first column the distribution of firms in the sample, the percentage of observations with no investment (inaction) in the second column and the percentage of observations with an investment rate greater than 20% of the installed capital (lumpiness) in the third column.

Type of firm	% obs.	Inaction (% obs. with $i = 0$ )	Lumpiness (% obs. with $i > 0.2$ )
Small firms	57.22	29.24	24.52
Medium firms	25.67	4.51	23.78
Large firms	17.12	0.93	26.65
Total	100	18.05	24.70

Table 2: Evidence of inaction and lumpiness by categories of firms

As we can see, more than half of the firms in our dataset are small firms. The percentage of observations with no investment is very different for small, medium and large firms. While there are around 30% of observations accounting for zero investment in the group of small firms, this percentage is only 4% for medium firms and almost insignificant for large firms. However, in the three categories considered, the percentage of observations with investment rates greater than 20% of installed capital is quite similar, around 23%.

Figure 2 mimics Figure 2a in Doms and Dunne (1998). For each firm in our dataset we have ranked its annual investment rate in descending order. The figure shows the mean and the median investment rate in each rank.

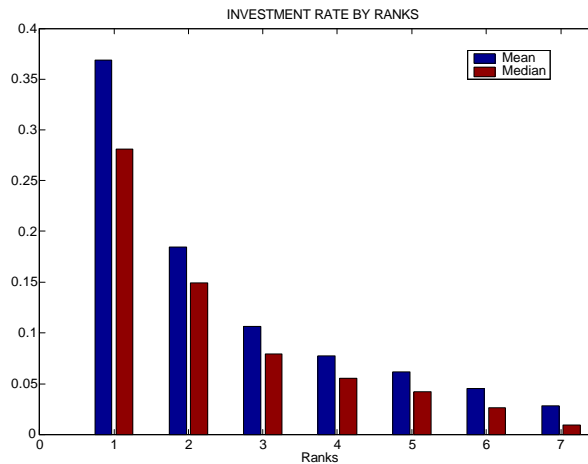


Figure 2

As we can see, the first bar, corresponding to the highest mean investment rate, exceeds 35%, while for the second rank is below 20%, and for subsequent ranks is even less than 10%. That is, the means drop off significantly after ranks 1 and 2, meaning that many firms experiment one or two periods of intense investment, while the rest of the periods are characterized by moderate investment. The median is always below the mean, reflecting the skewness to the right of the investment rate distribution.

Figure 3 gives an insight about the importance of large investment episodes on the time series fluctuations of investment.

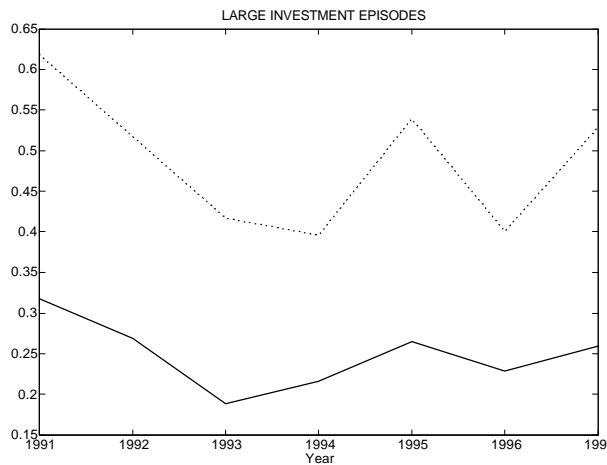


Figure 3



The solid line reflects the percentage of observations with an investment rate greater than 20%. The dotted line represents the percentage of investment accounted by observations having these large investment episodes. Observations with large investment episodes constitute around 25% of the total, but account for approximately 50% of gross investment. That is, around half of the total gross investment is related with lumpiness and half of it with smooth adjustments. Similar evidence has been reported in Cooper, Haltiwanger and Plant (1999) for a large set of US manufacturing firms.

The empirical evidence reported in this Section stresses two important features present in our dataset: high frequency of inaction and lumpiness. These empirical findings clearly support the convenience of an investment model which accounts for irreversibilities and nonconvex capital adjustment costs.

### **3 A model of firm equipment investment**

#### **3.1 Framework and basic assumptions**

Consider a risk neutral firm that produces an homogeneous good using as inputs labor and capital equipment which has some firm-specific characteristics. At each period the firm decides hirings and dismissals of workers and purchases of new capital in order to maximize the expected discounted stream of current and future profits over an infinite time horizon. The firm operates in competitive product and input markets, and its profit at period  $t$ , in output units is given by:

$$\Pi_t = Y_t(K_t, L_t, a_t) - w_t L_t - p_t I_t - AC(K_t, I_t, p_t) \quad (1)$$

where  $Y_t$  is real output,  $K_t$  is the capital stock installed at the beginning of period  $t$ ,  $L_t$  represents labor in physical units,  $I_t$  represents new capital purchases in physical units and  $w_t$  and  $p_t$  are input prices relative to product price. We assume that labor can be adjusted costlessly, so the decision on employment is static. However, when the firm decides to adjust its capital stock it faces some adjustment costs represented by the function  $AC(K_t, I_t, p_t)$ .

Output depends on labor and installed capital at the beginning of the period and a productivity shock  $a_t$ , according to the Cobb-Douglas production function:

$$Y_t = a_t K_t^{\alpha_K} L_t^{\alpha_L} \quad (2)$$

where  $\alpha_K, \alpha_L \in (0, 1]$ . We assume there is one period time-to-build, i.e, the new equipment is productive one period after its acquisition. The productivity shock is exogenous and follows a first order Markov process with transition density  $\phi_a(a_{t+1} | a_t)$ .

We assume that adjustment costs faced by the firm when it decides to invest can be variable or fixed costs:

$$AC(K_t, I_t, p_t) = VC(K_t, I_t, p_t) + FC(K_t) \quad (3)$$

Variable costs  $VC(\cdot)$  include costs associated with the installation of the capital stock. We assume a convex structure for these costs, similar to the specification of adjustment costs in the traditional investment models. More specifically, we use the following quadratic function:

$$VC_t = VC(K_t, i_t, p_t) = \frac{\theta_Q}{2} \left( \frac{I_t}{K_t} \right)^2 p_t K_t \quad (4)$$

where  $\theta_Q$  is a constant parameter.

Fixed adjustment costs  $FC(\cdot)$  are internal costs related to the reorganization of the productive process and retraining of employees in the handling of the new equipment. We assume that these costs are proportional to the installed capital stock:

$$FC = FC(K_t) = 1(I_t > 0) \theta_F K_t \quad (5)$$

where  $1(\cdot)$  is the indicator function and  $\theta_F$  is a constant parameter.

Since the firm operates in competitive markets, input prices are exogenous to the firm. We assume that capital price and wages follow a Markov process with transitional densities  $\phi_p(p_{t+1} | p_t)$  and  $\phi_w(w_{t+1} | w_t)$ , respectively. Capital retirement and physical depreciation are exogenously given to the firm. The capital stock follows a transition rule given by

$$K_{t+1} = (1 - \delta_i) K_t + I_t \quad (6)$$

where  $\delta_t \in (0, 1)$  is the depreciation rate, which includes not only the economic depreciation of the capital stock but also the capital retirements due to obsolescence.

At the beginning of each period, the firm knows its level of capital stock and labor, the input prices in the industry where it operates and the value of productivity and cost shocks. The one-period profit function can then be written as

$$\Pi_t = Y_t(K_t, L_t^*, a_t) - w_t L_t^* - p_t I_t - AC(K_t, I_t, p_t), \quad (7)$$

where  $L_t$  have been optimally chosen. The optimal condition for employment, under the assumption of a Cobb-Douglas production function with constant returns to scale, is given by

$$L_t^* = \left( \frac{a_t(1 - \alpha_K)}{w_t} \right)^{\frac{1}{\alpha_K}} K_t \quad (8)$$

Thus, the profit function in terms of capital stock can be written as:<sup>1</sup>

$$\Pi_t = R_t K_t - p_t I_t - AC(K_t, I_t, p_t) \quad (9)$$

where  $R_t$  is a profitability shock in terms of the productivity shock, wages and technological parameters according to the following expression:

$$R_t = R(a_t, w_t, \alpha_K) = \left( \frac{a_t(1 - \alpha_K)}{w_t^{1 - \alpha_K}} \right)^{1/\alpha_K} \frac{\alpha_K}{1 - \alpha_K} \quad (10)$$

A well-known evidence that arises in any empirical study of firms' behavior is the large amount of heterogeneity in firms size, productivity and behaviour in general, even after controlling for location, industry or product characteristics. It will be convenient to represent a firm's decision using investment rate  $i_t \equiv I_t/K_t$  as the decision variable instead of investment in physical units. Using  $i_t$  as argument, the transition rule for the capital stock becomes  $K_{t+1} = K_t [(1 - \delta_t) + i_t]$ , and the one-period profit function, where employment has been optimally chosen, can be written as:

$$\Pi_t = R_t K_t - p_t K_t i_t - AC(i_t, K_t, p_t) \quad (11)$$

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<sup>1</sup>Note that under Cobb-Douglas production with constant returns to scale, and our specification of capital adjustment costs, the one-period profit function is linear in the capital stock.

where

$$AC(i_t, K_t, p_t) = \frac{\theta_Q}{2} p_t K_t i_t^2 + 1(i_t > 0) \theta_F K_t \quad (12)$$

We assume that investment decision is completely irreversible, i.e, the firm decides purchases of capital stock and once a new equipment has been acquired, it cannot be sold. The firm chooses either not to invest or to face a strictly positive investment. The decision variable in this problem is  $i_t \geq 0$ . Let  $s_t$  be the vector of state variables, which can be observed by the firm and the econometrician, or only by the firm. The firm's problem can be written as:

$$\max_{\{i_t \geq 0\}} \sum_{t=0}^{\infty} \beta^t E[\Pi(i_t, s_t)] \quad (13)$$

where  $\beta \in (0, 1)$  is the discount factor, related to the interest rate of the economy. The Bellman's equation for this problem is given by:

$$V(s_t) = \max_{\{i_t \geq 0\}} \Pi(i_t, s_t) + \beta EV(s_{t+1} | s_t, i_t) \quad (14)$$

where  $EV(s_{t+1} | s_t, i_t)$  is the expected conditional value function

$$EV(s_{t+1} | s_t, i_t) = \int V(s_{t+1}) \phi(ds_{t+1} | s_t, i_t) \quad (15)$$

and  $\phi(ds_{t+1} | s_t, i_t)$  is the transition probability of the state variables.

### 3.2 Optimal decision rule

As it is explained above, firms choose between not to invest or to undertake an investment project. If they decide not to invest, it can be due to two different reasons: irreversibility and fixed adjustment costs.

When there are no fixed costs associated with capital adjustment, the value function is continuous and concave. However, the introduction of fixed adjustment costs makes the value function nonconcave. Aguirregabiria (1999) and Slade (1998) have characterized the optimal decision rule for these type of problems with nonconcave value functions.

The optimal decision rule for this problem is given by:

$$i(s_t, \theta) = \begin{cases} i^*(s_t, \theta) & \text{if } i^*(s_t, \theta) > 0 \text{ and } \gamma(s_t, \theta) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

where  $i^*(s_t, \theta)$  is the optimal interior solution characterized by

$$\tilde{\pi}_i(s, i^*(s, \theta), \theta) + \beta EV_i(s, i^*(s, \theta), \theta) = 0, \quad (17)$$

with  $\tilde{\pi}_i \equiv \partial \tilde{\pi} / \partial i$  and  $EV_i = \partial EV / \partial i$  and the function  $\gamma(s, \theta)$  is given by

$$\tilde{\pi}(s, i^*(s, \theta), \theta) - FC(s, \theta) - \tilde{\pi}(s, 0, \theta) + \beta [EV(s, i^*(s, \theta), \theta) - EV(s, 0, \theta)]. \quad (18)$$

That is, there is a first order condition of optimality for the interior solution, given by (17), and there are two conditions for the discrete choice between interior and corner solution. The first one concerns  $i^*(s, \theta)$ , which is related with the non-negativity constraint, so that the interior solution will be optimal only if it is positive. If condition (17) holds for a negative value, we will choose  $i(s, \theta) = 0$ , due to total irreversibility. The second condition is given in terms of  $\gamma(s, \theta)$ , which is related with the presence of fixed costs. If  $\gamma(s, \theta) > 0$ , it means that the fixed costs are not high enough to lead the firm to decide not to invest.

Our model is a dynamic choice model in which the decision variable is censored at zero as a consequence of inaction. As it is explained above, there are two sources of censoring indistinguishable for the econometrician: irreversibility and fixed adjustment costs. When the intertemporal profit, gross of fixed adjustment costs, is maximized for a negative value of investment, the optimal decision is inaction due to irreversibility. When it is maximized for a positive level of investment, but the value obtained with this level is lower than the value obtained with zero investment, the optimal decision is inaction due to the presence of fixed adjustment costs.

Although the optimal decision rule (16) involves marginal conditions of optimality and optimal discrete choices, in this paper we obtain estimates of the structural parameters which only exploit conditions associated to the optimal discrete choice between interior and corner solution.<sup>2</sup>

<sup>2</sup>Since corner solutions are very frequent in our dataset, the subsample of observations that we can use to exploit moment conditions associated to marginal conditions of optimality (i.e, Euler equations) is relatively small. Besides, parameters associated with fixed costs can only be identified by exploiting the discrete decision between interior and corner solution.

## 4 Estimation method

Consider a panel of firms with information on output, capital, labor, investment and input prices.

$$\{Y_{nt}, K_{nt}, I_{nt}, p_{nt}, w_{nt}; \quad n = 1, \dots, N; \quad t = 1, \dots, T_n\}$$

We are interested in exploiting this sample to estimate the structural parameters. According to the estimation procedure that we describe here, we can classify the structural parameters in four groups: a) the parameters entering the production function; b) the parameters that describe the transition probabilities of input prices and profitability shock; c) the adjustment costs parameters:  $\theta_Q$  and  $\theta_F$ ; and d) the parameters of the distribution of the state variables which are unobservable for the econometrician.

For estimation purposes, we proceed in two stages. In a first stage we estimate the parameters of the production function and the transition probabilities of the state variables. Once we have estimates of the parameters entering the production function, we can obtain estimates of the productivity shock  $a_{nt}$ , and construct the profitability shocks  $R_{nt}$  as in (10). In a second stage we estimate the rest of the structural parameters. In order to do this, we exploit the optimal discrete choice “to invest vs. not to invest” to obtain partial maximum likelihood estimates of the adjustment costs parameters  $\theta_Q$  and  $\theta_F$  and the parameters in the distribution of the unobservable state variables.

### 4.1 Estimation of the production function

We begin by considering a Cobb-Couglas production function without imposing constant returns to scale:

$$y_{nt} = \alpha_K k_{nt} + \alpha_L l_{nt} + u_{nt} \tag{19}$$

where  $y_{nt} = \ln(Y_{nt})$ ,  $k_{nt} = \ln(K_{nt})$ ,  $l_{nt} = \ln(L_{nt})$  and  $u_{nt} = \ln(a_{nt})$ . We allow the following structure for the productivity shock:

$$\begin{aligned} u_{nt} &= A_t + \eta_n + v_{nt} \\ v_{nt} &= \rho v_{n,t-1} + \xi_{nt} \end{aligned} \tag{20}$$

where  $A_t$  is an aggregate effect,  $\eta_n$  is a time invariant firm-specific effect,  $v_{nt}$  is an  $AR(1)$  idiosyncratic shock and  $\xi_{nt}$  is *iid*  $N(0, \sigma_\xi^2)$ .

In order to estimate the parameters  $(\alpha_K, \alpha_L, \rho)$ , we formulate the dynamic representation of (19):

$$y_{nt} = \alpha_K k_{nt} - \alpha_K \rho k_{n,t-1} + \alpha_L l_{nt} - \alpha_L \rho l_{n,t-1} + \rho y_{i,t-1} + (A_t - \rho A_{t-1}) + (1 - \rho) \eta_n + \xi_{nt}$$

or

$$y_{nt} = \pi_1 k_{nt} + \pi_2 k_{n,t-1} + \pi_3 l_{nt} + \pi_4 l_{n,t-1} + \pi_5 y_{i,t-1} + A_t^* + \eta_n^* + \xi_{nt}$$

subject to two non-linear restrictions:  $\pi_2 = -\pi_1 \pi_5$  and  $\pi_4 = -\pi_2 \pi_5$ , and where  $A_t^* = A_t - \rho A_{t-1}$  and  $\eta_n^* = (1 - \rho) \eta_n$ .

Given consistent estimates of the unrestricted parameter vector  $\pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)'$  and its variance-covariance matrix, the restrictions can be tested and imposed by minimum distance to obtain estimates for the restricted parameter vector  $(\alpha_K, \alpha_L, \rho)'$ .

We apply a GMM estimation method to obtain estimates of the unrestricted parameter vector. In the estimation of the Cobb-Douglas production function from panel data, the application of standard GMM estimators which take first differences to eliminate unobserved firm-specific effects and use as instruments lagged levels has produced unsatisfactory results (Mairesse and Hall, 1996). More specifically, it yields a low and statistical insignificant capital coefficient and suggest decreasing returns to scale.

These problems in the GMM estimation are due to weak instruments. The series of firm sales, capital and employment are highly persistent, so their lagged levels are only weakly correlated with the first differences to which they must instrument. Arellano and Bover (1995) proposed and extended a GMM estimator which is based on a system including not only differenced equations with lagged levels as instruments, but also level equations with lagged differences as instruments. They show that the shortcomings of the first difference estimation is dramatically reduced in a context with highly persistent variables. Blundell and Bond (1998) and Blundell, Bond and Windmeijer (2000) apply this “system GMM” to the estimation of production functions from company panel data. They find that the system GMM greatly improves

the performance of the first differences GMM estimator. They obtain a strongly significant capital coefficient and confirm that the lagged differences are informative instruments for the endogenous variables in levels.

In the next section we obtain GMM estimates for the parameters in the production function, using both the standard first differences and the system estimator. Our results confirm the findings in Blundell and Bond (1998) and Blundell, Bond and Windmeijer (2000). We also test the nonlinear restrictions by minimum distance, obtaining estimates for the restricted parameters, and show the validity of the additional instruments used in this extended GMM.

Since our specification of the profit function as a linear function of the capital stock is based on the constant returns to scale hypothesis, we also test the validity of this hypothesis and obtain estimates imposing constant returns to scale.

## 4.2 Estimation of adjustment costs parameters

Once we have estimated the technological parameters and the productivity shock, we can obtain  $R_{nt}$  as in (10). This profitability shock will be treated as an observable state variable in the estimation of the remaining structural parameters.

Let  $x_{nt} = (p_{nt}, K_{nt}, R_{nt})'$  be the vector of state variables which are observable for both the firm and the econometrician, and consider a panel dataset with the following information:  $\{i_{nt}, x_{nt}; \quad n = 1, \dots, N; \quad t = 1, \dots, T_n\}$ .

Let  $d = \{0, 1\}$  be the index for the optimal discrete choice, where  $d = 0$  means that the optimal decision is not to invest, i.e,  $i(s_t) = 0$ , and  $d = 1$  means that the optimal decision is to undertake an investment project, i.e,  $i(s_t) > 0$ .

Let us consider the following assumptions:

AS: *Additive separability* assumption (Rust, 1987):

$$\Pi^d(s_t, i_t, \theta) = \Pi^d(x_t, I_t, \theta) + \varepsilon_t^d \quad \text{for } d = 0, 1 \quad (21)$$

There are unobservable state variables, one associated with each choice, that are independent and enter additively the one-period profit function. The unobservable state variables  $\varepsilon^d$ , for  $d = \{0, 1\}$ , represent the uncertainty of the researcher about the



actual expected profit that is observable to the firm. We assume they are identically distributed with zero mean and variance  $\sigma_\varepsilon^2$ .

MS: *Multiplicative separability*:

$$E [\Pi^d(x_t, i_t, \theta)] = \Pi^d(x_t)' \mu(\theta) \quad \text{for } d = 0, 1 \quad (22)$$

Under this assumption, the expected profit function can be factorized as a function depending only on observable state variables and a function of the structural parameter vector. If the parameters enter the one-period profit function in a linear fashion, this assumption trivially holds. In our model, these factorization is given by the following functions:

$$\Pi^0(x_t) = \begin{pmatrix} R_t K_t \\ 0 \\ 0 \end{pmatrix} \quad \Pi^1(x_t) = \begin{pmatrix} R_t K_t - p_t K_t E [i_t | x_t, d_t = 1] \\ -\frac{1}{2} p_t K_t E [(i_t)^2 | x_t, d_t = 1] \\ -K_t \end{pmatrix} \quad \mu(\theta) = \begin{pmatrix} 1 \\ \theta_Q \\ \theta_F \end{pmatrix}$$

The first component of  $\Pi^1(x_t)$  is related to the revenues realized by the firm net of the acquisition price of the new capital stock. The second and third components are related, respectively, to the quadratic and fixed adjustment costs.

CI: *Conditional independence assumption* (Rust, 1987)

$$pdf(x_{t+1}, \varepsilon_{t+1} | x_t, \varepsilon_t, d_t) = pdf(\varepsilon_{t+1} | x_{t+1}) pdf(x_{t+1} | x_t, d_t) \quad (23)$$

This assumption implies, on one hand, that conditional on the discrete choice and the current value of the observable state variables, the future observable state variables do not depend on unobservables. On the other hand, this assumption rules out the existence of autocorrelated unobservable state variables that difficult extremely the estimation of the decision problem.

Under these assumptions, the optimal discrete choice can be written as:

$$d^* = d \iff d = \arg \max_{j=0,1} \{ \Pi^j(x)' \mu(\theta) + \varepsilon^j + \beta EV^j(x; \theta) \}$$

The log-likelihood function for this problem is

$$\ln L = \sum_{n=1}^N \sum_{t=1}^{T_n} \sum_{d=0,1} \ln (\Pr(d_{nt}^* = d | x_{nt})) \quad (24)$$

where , for  $d = \{0, 1\}$  ,

$$P^d(x_{nt}) = \Pr(d_{nt}^* = d | x_{nt}) = \\ = \Pr \left\{ d = \arg \max_{j=0,1} \left\{ \Pi^j(x_{nt})' \mu(\theta) + \varepsilon_{nt}^j + \beta EV^j(x_{nt}; \theta) \right\} \middle| x_{nt} \right\}$$

These probabilities entering the log-likelihood function are expressed in terms of unknown conditional value functions  $EV^d(x_{nt}; \theta)$ . An obvious approach to estimate the structural parameters is a solution method consisting in some nested algorithm in the spirit of Rust's Nested Fixed Point (1987). This technique consists in an outer algorithm that maximizes the likelihood function and an inner algorithm which solves the dynamic programming problem at each iteration in the search for the parameter estimates. The main drawback of this kind of techniques that solve the dynamic programming problem is its high computational cost.

The estimation strategy we use in this paper is in the line of the estimation techniques recently appeared in the literature on the estimation of dynamic discrete choice models, that avoid the solution of the dynamic programming problem. More specifically, our method is in the line of Hotz and Miller (1993) and their Conditional Choice Probability estimator (CCP). They show that there is a representation of the difference in conditional value functions as a computable function of state variables, structural parameters and conditional choice probabilities. This representation have been used, among others, by Aguirregabiria (1999) for the estimation of a model of price and inventory decisions and Slade (1998) for a model of price decisions.

Under assumptions *AS – CI*, Aguirregabiria (1999), following the strategy by Hotz and Miller (1993), showed that the expected conditional valuation function can be written, for  $d \in \{0, 1\}$  , as:

$$EV^d(x_t; \theta) = W^d(x_t)' \lambda(\theta)$$

where

$$W^d(x_t) = \hat{F}^d(x_t) \left( I - \beta \hat{F}(x_t) \right)^{-1} \left( \sum_{d=0,1} \hat{P}^d(x_t) * \Pi^d(x_t) \quad \sum_{d=0,1} \hat{P}^d(x_t) * g^d(x_t) \right) \quad (25)$$

$\lambda(\theta) = (\mu(\theta)' \quad 1)'$ ,  $*$  is the element-by-element product, the functions  $g^d(x_t)$  are given by:

$$g^d(x_t) = E [\varepsilon_t^d | x_t, d_t^* = d]$$

and  $\hat{P}^d(x_t)$ ,  $\hat{F}^d(x_t)$  and  $\hat{F}(x_t)$  are nonparametric estimates of the conditional choice probabilities, conditional transition probabilities and unconditional transition probabilities of the state variables respectively.

The vector  $W^d(x_t)$  is related to the expected and discounted stream of the future components associated with the corresponding components of the one period profit function  $\Pi^d(x_t)$ . The conditional expectation of unobservable state variables,  $g^d(x_t)$ , can be written in terms of conditional choice probabilities. For instance, if  $\varepsilon^d$  are independent with extreme value distribution, that expectation becomes  $E [\varepsilon_t^d | x_t, d_t^* = d] = 0.57721 - \ln [P^d(x_t)]$ , where 0.57721 is the Euler's constant.

From expression (25) it is straightforward to obtain a closed expression for the conditional valuation function  $EV^d(x_t, \theta)$ .

The estimation method consists in two stages: in a first stage we obtain nonparametric estimates of the conditional choice probabilities  $\hat{P}^d(x_t)$  and the conditional transition probabilities of the state variables,  $\hat{F}^d(x_t)$ . From these estimates we can obtain an estimate for the unconditional transition probability matrix,  $\hat{F}(x_t) = \sum_{d=0,1} \hat{P}(x_t) * \hat{F}^d(x_t)$ . In the second stage, these estimates can be used to construct the values  $W^d(x_t)$ . Given extreme value distribution for the unobservable state variables, we can obtain structural parameter estimates by partial maximum likelihood, where the probability of choosing alternative  $d$  is given by the well-known logit formula:

$$P^d(x_t, \theta) = \frac{\exp \{ \Pi^d(x_t)' \mu(\theta) + \beta W^d(x_t)' \lambda(\theta) \}}{\sum_{j=0,1} \exp \{ \Pi^j(x_t)' \mu(\theta) + \beta W^j(x_t)' \lambda(\theta) \}}$$

Hotz and Miller (1993) prove the consistency and asymptotic normality of this kind of estimators.<sup>3</sup>

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<sup>3</sup>Hotz and Miller estimator (CCP) is not based on the maximum likelihood method. It is a GMM estimator that exploits the following moment conditions:

$$E (Z_{nt} [1(d_{nt} = d) - P^d(x_t, \theta)]) = 0 \quad \text{for } d = 0, 1$$

## 5 Estimation results

### 5.1 Estimation of technological parameters

We have obtained estimates of the production function (19). The results are reported in Table 3. We report results for the two-step GMM estimator for both the first-differenced equations and the system. We take as instruments the lagged levels dated  $t - 2$  and earlier in the first-differenced equations. As additional instruments in the system GMM estimation, we take the lagged differences dated  $t - 1$ . Year dummies have been included in both models. The non-linear restrictions have been tested and imposed by minimum distance.

Table 3 reports the production function estimates without imposing constant returns to scale constraint. As we can see, in the first differences GMM estimation we obtain a nonsignificant capital coefficient. Although the shortcomings of the standard first differenced GMM estimation in our case does not seem to be as dramatic as in Mairesse and Hall (1996) or Blundell and Bond (1998), the gains of the system GMM estimation are important. The precision of all the estimates improves considerably, specially in the estimation of the capital coefficient.

The autocorrelation tests are consistent with the  $AR(1)$  error specification. The validity of lagged levels dated  $t - 2$  and earlier as instruments in the first-differenced equations are not rejected. The Dif. Sargan test clearly does not reject the additional instruments in the system estimation. We can see that constant returns to scale and non-linear restrictions are not rejected in none of the models.

Given that our model specification is based on the constant returns to scale hypothesis, we have also estimated the parameters in the production function imposing this restriction. The results are reported in Table 4. Neither the Sargan test nor the first and second order autocorrelation tests provide any evidence against the specification. Again, we observe the gains obtained with the system GMM estimation in terms of the precision of the estimates.

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where  $Z_{nt}$  is a vector of instrumental variables (e.g., functions of  $x_{nt}$ ). However it is straightforward to show that if we define the vector of instrumental variables  $Z_{nt}$  as the gradient of the log-likelihood function, the CCP estimator is equivalent to the partial maximum likelihood estimator that we describe in this paper.

GMM estimates		
	First differences $t - 2$	System $t - 2$
$k_{nt}$	0.332 (0.313)	0.444 (0.154)
$k_{n,t-1}$	-0.348 (0.203)	-0.378 (0.132)
$l_{nt}$	0.754 (0.310)	0.568 (0.199)
$l_{n,t-1}$	-0.609 (0.212)	-0.255 (0.235)
$y_{n,t-1}$	0.849 (0.136)	0.724 (0.097)
$m1$	-5.695	-7.572
p-value	0.000	0.000
$m2$	0.073	-0.563
p-value	0.942	0.574
Sargan	29.474	31.684
p-value	0.245	0.629
Dif. Sargan	—	2.210
p-value	—	0.980
Minimum distance estimates		
$\alpha_k$	0.435 (0.225)	0.456 (0.121)
$\alpha_L$	0.722 (0.231)	0.576 (0.195)
$\rho$	0.874 (0.082)	0.819 (0.074)
p-value MD test	0.658	0.314
p-value CRS test	0.660	0.880

Table 3: Production function estimates without imposing constant returns to scale.

Standard errors in parenthesis

$m1, m2$ : tests for first and second order correlation in first differenced residuals

Sargan: Sargan test of overidentifying restrictions

Dif Sargan: test of the validity of the additional instruments in system estimation

MD: minimum distance

CRS: constant returns to scale

GMM estimates		
	First differences $t - 2$	System $t - 2$
$k_{nt}$	0.289 (0.154)	0.436 (0.113)
$k_{n,t-1}$	-0.301 (0.117)	-0.368 (0.092)
$y_{n,t-1}$	0.864 (0.090)	0.844 (0.061)
$m1$	-7.446	-10.659
p-value	0.000	0.000
$m2$	0.036	-0.245
p-value	0.971	0.807
Sargan	25.811	39.539
p-value	0.529	0.357
Dif. Sargan	—	13.728
p-value	—	0.800
Minimum distance estimates		
$\alpha_k$	0.357 (0.133)	0.436 (0.109)
$\alpha_L$	0.643 —	0.564 —
$\rho$	0.878 (0.088)	0.844 (0.058)
p-value MD test	0.382	0.993

Table 4: Production function estimates imposing constant returns to scale. Standard errors in parenthesis. See notes on Table 3

Once we have estimates of the parameters entering the production function and the productivity shock, we can construct the profitability shock  $R_{nt}$  which will be used as an observable state variable in the estimation of the rest of the structural parameters, those related with capital adjustment costs.

## 5.2 Estimation of adjustment cost parameters

As we said in the previous Section, the estimation of the adjustment cost parameters involves two steps. First, we estimate the conditional choice probabilities and the transition probabilities of the state variables. In a second step, we obtain an expression for the conditional valuation functions in terms of the structural parameters, state variables and previous nonparametric estimators and use this expression to estimate the discrete choice model.

This estimation method requires a discretization of the space of observable state variables. The observable state variable we are considering are the capital stock  $K_{nt}$  and the profitability shock  $R_{nt}$ .<sup>4</sup> The capital stock is measured in thousand euros. Since the range of variation of  $K_{nt}$  is very different for different firms, we have considered the variable  $\tilde{K}_{nt} = \frac{K_{nt}}{K_n}$ , which represents the capital stock of firm  $n$  in period  $t$  over the average capital stock of firm  $n$  in the sample period. The profitability shock  $R_{nt}$  have been considered in logarithms and decomposed in aggregate and idiosyncratic components. The aggregate component is the yearly mean of the log-profitability shocks and the idiosyncratic component is the deviation of the log-profitability shock from that mean. We have discretized the observable state variables using a uniform grid in the space of their empirical probability distributions. The nonparametric estimators for the conditional choice probabilities have been obtained using multivariate kernel estimators. The bandwidth parameter has been chosen according to Silverman's rule. The conditional transition probabilities have been estimated using multinomial logits on the space of the discretized variables.

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<sup>4</sup>We have not discretized capital price  $p_t$  in the set of state variables. Since there is very low variability of this variable in our dataset, we have considered it as constant.

The amount of investment when a firm decides to invest,  $E[i_t|x_t, d = 1]$ , which enter the functions  $\Pi^1(x_t)$  and  $W^1(x_t)$ , has been estimated nonparametrically using kernel estimators.

The structural parameter estimates are reported in Table 5. Since one of the components of the one-period profit function has a parameter restricted to be 1 (the component associated with the revenue function) it is possible to identify the standard deviation of the unobservable state variables,  $\sigma_\varepsilon$ . The discount factor  $\beta$  has been fixed at 0.975. We have also estimated the model with different values of  $\beta$  ( from 0.95 to 0.99) obtaining similar results.

	Structural parameter estimates
$\theta_Q$	115.6 (26.10)
$\theta_F$	30.78 (4.513)
$\sigma_\varepsilon$	25.34 (5.082 )
LogL	-3162.01
Obs.	7519

Table 5: Structural parameter estimates. Standard errors in parenthesis

We have obtained very precise parameter estimates. Since the capital stock has been considered as the ratio  $\tilde{K}_{nt} = \frac{K_{nt}}{K_n}$ , our estimate of the fixed adjustment cost parameter implies that a firm with, for example, an average capital stock of one million euro that decides to undertake an investment project, must face a fixed cost between 2.19% and 3.96% of its capital stock (95% confidence interval). The following table shows the proportion that variable and fixed adjustment costs implied by our estimates represent on average over the installed capital and the sales of the firms in three categories: small, medium and large firms, according to the classification criterion established by the European Commission.



	VC/Cap. stock	FC/Cap. stock	VC/Sales	FC/Sales
Small firms	0.12545	0.23303	0.011902	0.03461
Medium firms	0.00327	0.01148	0.000362	0.00199
Large firms	0.00051	0.00239	0.000067	0.00041

Table 6: Proportion of capital adjustment costs over installed capital and sales. VC: Variable adjustment costs. FC: Fixed adjustment costs.

On average, fixed adjustment costs are much more relevant than convex costs in each category. It is worthwhile to emphasize the importance of fixed adjustment costs in small firms, representing around 23% of the installed capital stock and around 3% of the total sales. These proportion decreases a lot in the other categories: medium and large firms. In the group of medium firms, fixed adjustment costs represent 1.11% of the installed capital and 0.19% of the firm sales. In the group of large firms, fixed costs represent 0.23% of the installed capital and only 0.04% of the sales. The very different magnitude of fixed adjustment costs for the categories considered can explain the very different importance of inaction found in each of them. As we saw in Table 2, the percentage of observations accounting for zero investment was very high in small firms, 29.24%, while in medium and large firms was much smaller, 4.51% and 0.93% respectively.

## 6 Conclusions

In this paper we have estimated a dynamic structural model of irreversible investment for Spanish manufacturing firms. The dataset we use exhibit some of the characteristics reported in the recent microeconomic investment literature. More specifically, we have found strong evidence of inaction and lumpy investment. Based on these empirical features, we have proposed a dynamic structural investment model in which irreversibilities and nonconvex adjustment costs have been included. The adjustment cost function we consider includes quadratic and fixed components.

We have estimated the model in two stages. In a first stage we have obtained GMM estimates of the parameters entering the production function. We have applied the standard first-differenced GMM estimator to eliminate the unobservable

individual time invariant effect. We have also obtained the extended GMM estimator proposed by Arellano and Bover (1995) which consist on a system of equations including not only first differenced equations but also level equations. This system GMM seems to work well in our dataset, improving the precision of the estimates and reducing some of the defficiencies found in the estimation of the first differenced equations. Once we have estimated the production function, in the second stage we have proposed a dynamic discrete choice model, in which the firm chooses between not to invest and to undertake an investment project. We have obtained partial maximum likelihood estimates of the adjustment cost parameters, using an estimation strategy that avoids the solution of the dynamic programming problem. It builds on the Hotz and Miller's CCP estimator, which is based on the representation of the conditional value functions as computable functions of conditional choice probabilities, state variables and structural parameters. Given nonparametric estimates of the conditional choice probabilities and transition probabilities of the state variables, the conditional value functions have a closed form expression in terms of the structural parameters. Our estimation results reflect the importance of fixed adjustment costs, which can represent a considerable proportion of installed capital and sales. The magnitude of these costs varies a lot depending on the firm size. For small firms they can represent around 20% of the installed capital and 3% of the firm sales, while for medium and large firms these proportions are much smaller. This can explain the observed investment behavior in our dataset. In the group of small firms, around 29% of observations accounted for zero investment, while this percentage was much smaller for medium and large firms.

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