UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 03-71
Business Economics Series 17
January 2004

Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249608

A COMPARISON BETWEEN CORRESPONDENCE ANALYSIS AND CATEGORICAL CONJOINT MEASUREMENT

Anna Torres-Lacomba[1]

Abstract

We show the equivalence of using correspondence analysis of concatenated tables and a particular algorithm of conjoint analysis named categorical conjoint measurement. The connection is made using canonical correlation. However, although we have proved that equivalence, the standard practice of conjoint analyses to focus in one dimension (the optimal solution) has some shortcomings once we introduce interaction effects. In that case, the use of visual techniques, like correspondence analysis, provides a faster and easier way to compile the preference structure. Finally, we provide an application of our setting making use of an experiment of perfumes where interaction effects between type of essences and strength of essences are shown.

Key words: Correspondence analysis, conjoint analysis, canonical correlation, categorical data

[1]Departamento de Economia de la Empresa, Universidad Carlos III de Madrid, C/ Madrid, 126, 28903 Getafe (Madrid), Spain.Telf: 34-91 6249322. Fax: 34-916249608, E-mail: atorres@emp.uc3m.es .

I am grateful to Michael Greenacre, Tammo Bijmolt and Josep A. Tribo for useful comments on previous versions of this paper. I acknowledge the FPU pre-doctoral grant of the Spanish Ministry of Education, Culture and Sport as well as the grant SEC2001-1169 directed by Clara Cardone Riportella, and financial suport by the Universidad Carlos III de Madrid and Universitat Pompeu Fabra.

A comparison between correspondence analysis and categorical conjoint measurement.

INTRODUCTION

Carroll and Green (1995) recognize the active role of the psychometric methods in the advancement of marketing research techniques. However they alert about a lack of critical comparison among competing techniques. Some works try to solve these limitations. For example, Torres and Greenacre (2002) make the extension of correspondence analysis, to treat preference data (rank order, paired comparisons and rating data) and establish the equivalence with the results of dual scaling. Also, the former authors state that researchers should work to give clear positions about the best technique in each particular problem situation. Also they should give a critical view of the "added value" of each approach from a practical point of view. For example, in the aforementioned work of Torres and Greenacre (2002), correspondence analysis is shown to be a superior technique due to the map it offers.

The present work follows this line of research and tries to clarify the relationship between correspondence analysis (Benzécri, J.P. et al 1973) and a special algorithm of conjoint analysis named categorical conjoint measurement (Carroll 1969, Rao; V.R. 1977; Green, P.E. & Rao, V. 1971). This analysis corresponds to a *full profile* data collection and a *rating scale* measurement of respondent judgments. Furthermore, following Green & Wind (1972) we also make the extension of the techniques to include interaction effects. They are relevant, specially in marketing research studies that involve sensory phenomena (Carmone & Green, 1981).

We present correspondence analysis as better alternative for managers to see in an easy and faster way the preference structure in comparison with the preference scheme obtained through conjoint analysis, which is based on comparing optimal values. This idea relies in the superiority of perceptual mapping techniques (see for example, Hauser & Koppelman, 1979). The improvement is specially important when interaction effects are included. We will discuss it in following sections.

Our paper introduces a brief description of canonical correlation analysis (CC). This is useful as an intermediate stage, since the equivalence between CCM and CC is already shown (Carroll 1969). The equivalence between CC and CA has been shown for the particular case where there is one attribute being related to preference (see, for example, Greenacre

1984, chap.4). We inspect what happens when two or more attributes are being related to preference and finally we compare the results obtained from the analysis of the data using CCM, CC and CA. Then, we introduce the objective function of CCM followed by the CA as well as the way to code the data so that CA can treat interactions effects. We repeat the operation with CC as well as with CCM to demonstrate the equivalence empirically.

The results of our study are illustrated with an application. Green & Wind (1972) point out that a relevant interaction effect between type of fragrance and the intensity of fragrance, for soaps, could exist. We borrow this idea and we repeat the analysis for perfumes. Marketing researchers[2], shop assistants[3] and other sources[4] suggest that the size of the bottle should be the third factor to consider in this analysis, where price and brand are excluded since they are more related with image than to the evaluation of an essence. Considering this information as well as relevant conjoint literature (Wittink, 1999) we decide to include the three following attributes with their particular levels. Type of fragrance (4 levels), intensity of fragrance (2 levels) and size of the bottle (3 levels). The Dictionary of Essences, that contains the description of the composition of each perfume, allows us to construct a sample of brands according to the pure essences (floral, citric, leather and oriental) and to differentiate between high and low intensity fragrances.

For collecting the data, we used face-to-face interviewing, differentiating by age, randomly presenting to each woman one brand for each combination of attributes (type and intensity on fragrance and size of bottle). They evaluated the item on a scale that goes from A: very high worth, to D: very low worth. To ensure independency in each choice, between each presentation some coffee was smelt. We start with the analysis of one subject who is 30 years old to demonstrate the equivalence between the different techniques with and without interaction effects. Since age determines preferences over perfumes, we also report the results for a group of three women, the 30 years old as well as a 17 and 55 years old.

DATA MATRIX

To describe the data matrix to be analyzed, we take the same notation than Carroll (1969) and Greenacre (1984). The data matrix is made up of a matrix Z_1 of dummy variables representing the *full profile* (i.e.,

[2] Millward Brown and Hamilton Consulting.

[3] Xaloc in Blanes, Gala Perfumeries in Mataró Parc and Body Bell, Getafe.

[4] "Diccionario de las fragancias, 2002", www.perfumedia.com and www.todofragancias.com

the matrix has M rows, where $M \equiv \prod_{q=1}^{Q} m_q$: number of all the possible combinations of the attribute levels, and $\sum_q m_q$ columns, where m_q: stands for the number of levels for the attribute q where $q = 1, ..., Q$) and another matrix Z_2 of dummy variables indicating one subject's preferences for each combination (i.e., the matrix has M rows and K columns, where K : number of response categories where $k = 1, ..., K$).

The description of our application is the following. We have three attributes ($Q = 3$), which are:

1.Type of fragrance, where $m_1 = 4$, with levels: C: citric (orange, lemon, mandarin), F: floral (petals, blade), O: oriental (balsam, oriental essence, vanilla) and L: leather (leather, smoked, wood, virgina tabacco).

2. Intensity of fragrance, where $m_2 = 2$, with levels: Hi: high intensity and Li: low intensity.

3. Size of the bottle, where $m_3 = 3$, with levels: S1: small (30cl), S2: medium (50cl) and S3: large (100cl).

The number of attribute combinations is $M = 4 \times 2 \times 3 = 24$. The response has $K = 4$ categories: A: very high worth; B: just high worth; C: just low worth; D: very low worth. The matrices Z_1 and Z_2 are 24×9 and 24×4 respectively.

METHODS

Canonical Correlation (CC)

The geometry of canonical correlation is given by Greenacre (1984, section 4.4) and also its relationship to the geometry of the correspondence analysis of an indicator matrix, for the classical case where two categorical variables are treated. As we noted before, since categorical conjoint analysis can be applied to more than two attributes, the equivalence between this technique and the correspondence analysis of an indicator matrix, via canonical correlation, is not obvious. We will describe the basic geometry of CC and we will introduce the new definitions and operations that will let us establish the connection.

The objective of CC is to find the strongest linear relationships between two sets of variables. If Z_1 and Z_2 are the data matrices corresponding to the two sets of variables, this objective can be expressed formally as finding linear combinations $\mathsf{Z}_1\mathsf{a}$ and $\mathsf{Z}_2\mathsf{b}$, which have maximum correlation ρ :

$$\rho = (\mathsf{a}^T \mathsf{S}_{12} \mathsf{b})/((\mathsf{a}^T \mathsf{S}_{11} \mathsf{a})(\mathsf{b}^T \mathsf{S}_{22} \mathsf{b}))^{1/2} \tag{1}$$

where S_{12} is the covariance matrix between Z_1 and Z_2, and S_{11} and S_{22} are the covariance matrices of Z_1 and Z_2 respectively.

The vectors a_k and b_k of canonical weights can be obtained from the left and right singular vectors of the matrix $\mathsf{S}_{11}^{-1/2}\mathsf{S}_{12}\mathsf{S}_{22}^{-1/2}$ (see, for example, Greenacre 1984). The singular value decomposition (SVD) (Eckart & Young, 1936) of the matrix is:

$$\mathsf{S}_{11}^{-1/2}\mathsf{S}_{12}\mathsf{S}_{22}^{-1/2} = \mathsf{U}\mathsf{D}_\rho\mathsf{V}^T \quad \text{with} \quad \mathsf{U}^T\mathsf{U} = \mathsf{V}^T\mathsf{V} = \mathsf{I}. \tag{2}$$

where D_ρ is a diagonal matrix with the canonical correlations in the diagonal, U and V are the matrices of left and right singular vectors.

The matrices of canonical weights are:

$$\mathsf{A} = \mathsf{S}_{11}^{-1/2}\mathsf{U} \qquad \text{and} \qquad \mathsf{B} = \mathsf{S}_{22}^{-1/2}\mathsf{V} \tag{3}$$

The standardization of the singular vectors of U and V to be orthonormal as in (2) implies that A and B are standardized as follows:

$$\mathsf{A}^T\mathsf{S}_{11}\mathsf{A} = \mathsf{B}^T\mathsf{S}_{22}\mathsf{B} = \mathsf{I} \tag{4}$$

Centering is not a necessary condition in canonical correlation but we apply it to be able to compare the results with the correspondence analysis ones (see Greenacre, 1984 for the case of 2 variables). We will develop it deeper in following sections.

The centering condition, for each one of the attributes, and for each dimension, takes the form:

$$\mathsf{r}^T\mathsf{a} = 0 \tag{5}$$

The centering condition for the response category variable, in each dimension is:

$$\mathsf{c}^T\mathsf{b} = 0 \tag{6}$$

where r and c are the *masses* (weights) for the attributes and the response categories respectively. They are defined as the total frequencies for the particular level, with respect to the grand total of the data matrix (Greenacre, 1984).

When an indicator matrix is analyzed, S_{11} and S_{22} are singular matrices, which implies that they cannot be inverted. To be able to do our computations, one level for each attribute and one response category have to be omitted, usually the last level in each case. This operation lets us estimate the canonical weights, as we describe in the next section. The notation for the estimated vector of canonical weights without the last level will be: $\mathsf{a}^*, \mathsf{b}^*$.

Categorical conjoint measurement (CCM)

Carroll (1969) shows that the problem of CCM could be fitted via a special case of canonical correlation analysis in which one set of variables corresponds to a factorial design matrix representing the conjoint structure (these variables are encoded as binary variables that indicate the attribute levels possessed by each stimulus) and the second set of variables provides an analogous dummy variable encoding of the dependent variable (a variable defining preferences for the particular stimulus) (Lattin, Carroll, and Green (2003)). Then the CCM objective is to find an additive combination of scale values for the attribute levels $\mathsf{Z}_1\mathsf{a}$ maximally correlated with the response category values $\mathsf{Z}_2\mathsf{b}$ assigned to each combination, where b is the vector that collects the optimal scale values for the categories of the response variable, and a is the vector with the optimal scale values for all $\sum_q m_q$ levels of the attributes. Further, Carroll (1969) shows the equivalence between applying canonical correlation analysis in this special case and the following formulation which is the one we will use to show the equivalence between CCM and CA. He defines:

$$s_{q,j_q,k} \equiv \sqrt{\frac{m_q}{n_k M}}\left(n_{q,j_q,k} - \frac{n_k}{m_q}\right) \tag{7}$$

where $n_{q,j_q,k} \equiv$ number of times k^{th} response category occurs with j_q^{th} level of attribute q and $n_k \equiv$ total number of times k^{th} response category value occurs ($k = 1, ..., K$).

Let S_q be the $m_q \times K$ matrix whose general entry is $s_{q,j_q,k}$. Define the $K \times K$ matrix R as:

$$\mathsf{R} = \sum_{q=1}^{Q} \mathsf{S}_q^T \mathsf{S}_q \tag{8}$$

The eigenvalue decomposition of this squared data matrix takes the form:

$$\mathsf{R} = \mathsf{V}\mathsf{D}_\lambda \mathsf{V}^T \qquad \text{with} \qquad \mathsf{V}^T\mathsf{V} = \mathsf{I} \tag{9}$$

where D_λ is a diagonal matrix with the eigenvalues in the main diagonal. Then the optimal scale values for the category responses b, are obtained from the first eigenvector as follows:

$$\mathsf{b} = \frac{\mathsf{v}_1}{\sqrt{n_k}} \tag{10}$$

Carroll (1969) does not specify the standardization condition, to define the solution, for the optimal response scale values, but we can deduce it from the previous expressions (9) and (10). It is,

$$\mathsf{b}^T \mathsf{D}_{n_k} \mathsf{b} = 1 \tag{11}$$

where D_{n_k} is a diagonal matrix with the category level totals in the main diagonal.

Notice that each attribute is initially treated separately, then combined in the matrix R, which is decomposed in order to find scale values for the response categories. He does not expose explicitly how to recover the optimal solution for the levels of the different attributes, which is not obvious from the particular formulation he is using.

The centering condition is not specified in Carroll (1969) but it is applied in the Rao's (1977) application. It is:

Category response:

$$M\mathsf{c}^T\mathsf{b} = 0 \tag{12}$$

Correspondence analysis (CA)

Correspondence analysis is a descriptive statistical technique that explains the association between the levels of different categorical variables. The overall association is quantified by the chi-squared statistic divided by n_{++} (the total number of cases), i.e. χ^2/n_{++}, called the *total inertia:*

$$\frac{\chi^2}{n_{++}} = \frac{1}{n_{++}} \sum_{j=1} \sum_{k=1} \frac{(n_{jk} - n_{j+}n_{+k}/n_{++})^2}{(n_{j+}n_{+k}/n_{++})} \tag{13}$$

where n_{jk} is the number of cases in a particular cell, n_{j+} the row total, n_{+k} the column total and n_{++} is the grand total.

The row and column coordinates, with respect to their respective principal axes, may be obtained from the singular value decomposition (SVD) of the matrix $\mathsf{N} = (n_{jk})$, transformed by double-centering and standardizing:

$$\mathsf{D}_r^{-1/2}[(1/n_{++})\mathsf{N} - \mathsf{r}\mathsf{c}^T]\mathsf{D}_c^{-1/2} = \mathsf{U}\mathsf{D}_\alpha\mathsf{V}^T, \qquad \text{with} \ \ \mathsf{U}^T\mathsf{U} = \mathsf{V}^T\mathsf{V} = \mathsf{I}. \tag{14}$$

where D_r and D_c are diagonal matrices with the row and column *masses*, respectively, in their main diagonal (see, for example, Greenacre (1984)). The squares of the singular values are the principal inertias or eigenvalues: $\mathsf{D}_\alpha^2 = \mathsf{D}_\lambda$.

Greenacre (1984) shows that, for the particular case where $Q = 1$, the CC analysis of the data matrix $[\mathsf{Z}_1|\mathsf{Z}_2]$ gives the same results than the CA of a contingency table, recovered from the transformation: $\mathsf{Z}_1^T\mathsf{Z}_2$. In our particular case, where $Q > 1$, the previous transformation gives a *concatenated table* (Greenacre & Blasius, 1994). It is composed by Q

stacked tables, one for each attribute, where the number of rows are the levels of the particular attribute, m_q, and the number of columns, K, corresponds to the levels of the category response variable.

Given the duality that characterizes correspondence analysis (Greenacre, 1984), the coordinates for rows and columns, are related in the following way:

1. Row problem. The rows will be the points projected in a map interpreted in terms of the columns as reference points. Row *profiles* (Greenacre, 1984), will be represented by *principal coordinates*, which take the form:

$$\mathsf{F} = \mathsf{D}_r^{-1/2}\mathsf{U}\,\mathsf{D}_\alpha \tag{15}$$

and will be expressed with respect to the column *vertices* (Greenacre, 1984) or *standard coordinates*, which take the form:

$$\mathsf{B} = \mathsf{D}_c^{-1/2}\mathsf{V} \tag{16}$$

2. Column problem. The columns will be the points projected in a map interpreted in terms of the rows as reference points. Column *profiles* will be represented by the *principal coordinates*, which take the form:

$$\mathsf{G} = \mathsf{D}_c^{-1/2}\mathsf{V}\,\mathsf{D}_\alpha \tag{17}$$

and will be expressed with respect to the row *vertices* or *standard coordinates*, which take the form:

$$\mathsf{A} = \mathsf{D}_r^{-1/2}\mathsf{U} \tag{18}$$

In our illustration, since rows (levels of the attributes) are projected in the map and expressed with respect to the columns (levels of the category response variable), the relevant expressions to consider are the (15) and the (16).

Centering is a necessary condition given the geometry of correspondence analysis (Greenacre, 1984). It takes the following form:

$$\mathsf{r}^T\mathsf{A} = \mathsf{c}^T\mathsf{B} = 0 \tag{19}$$

The standardization conditions, which determine the solutions, are the following:

Standard coordinates:

$$\mathsf{A}^T\mathsf{D}_r\mathsf{A} = \mathsf{B}^T\mathsf{D}_c\mathsf{B} = \mathsf{I}. \tag{20}$$

Principal coordinates:

$$\mathsf{F}^T\mathsf{D}_r\mathsf{F} = \mathsf{G}^T\mathsf{D}_c\mathsf{G} = \mathsf{D}_\lambda. \tag{21}$$

Once we have described the main idea behind the three techniques, the relationship between them is determined by the data matrices analyzed in each case as well as their centering and standardization conditions to define the solutions.

RELATIONSHIPS BETWEEN THE TECHNIQUES

CA of a concatenated table and its connection with CC

In the simpler case when $Q = 1$, one level is omitted from the first set of dummy variables and one from the second set in order to estimate the canonical weights. Greenacre (1984) then shows how the CA standard coordinates can be obtained from the canonical weights by imposing the centering conditions of CA, using row and column masses. In the present case where $Q > 1$, the matrix $\mathbf{Z}_1^T\mathbf{Z}_2$ is not a single crosstabulation but a *concatenated* set of crosstabulations. We again impose the CA conditions on the canonical weights to recover the standard coordinates for the categories and we impose the CA condition to each one of the attributes, to recover the principal coordinates for their levels. We shall check that the coordinates thus obtained are identical to the ones obtained in the correspondence analysis of the *concatenated table* $\mathbf{Z}_1^T\mathbf{Z}_2$.

If we have omitted the last attribute level dummy and last response category dummy, then canonical correlation analysis gives solutions, for each dimension, of the following form. For the attributes, we have, $\mathbf{a}_q^* = [a_{q,1}^*....a_{q,m_q-1}^* \; 0]^T$ for $q = 1, ..., Q$ and for the category response variable, we have, $\mathbf{b}^* = [b_1^*...b_{K-1}^* \; 0]^T$.

The CA results, for each dimension, are obtained from these, as follows (see Greenacre 1984, chap. 4, p. 122). For the attributes $q = 1, ..., Q$, we recover the coordinate of the omitted level and add it to the others as follows:

$$\begin{array}{ll} a_{m_q} = -\sum_{j=1}^{m_q-1} r_{q,j_q} a_{q,j_q}^* & \\ a_{q,j_q} = a_{q,j_q}^* + a_{q,m_q} & for \quad j = 1, ..., m_q \end{array} \tag{22}$$

where r_{q,j_q} is the proportion of cases for level j_q of attribute q. In this particular case, because data on the full profile are collected, $r_{q,j_q} = \dfrac{1}{m_q \times Q}$, which is constant for all j_q. For the response category variable, we have:

$$b_K = -\sum_{k=1}^{K-1} c_k b_k^* \qquad b_k = b_k^* + b_K \tag{23}$$

for $k = 1, ..., K$, where c_k is the proportion of cases for response category k, in this case, $c_k = \dfrac{n_k}{M}$.

To illustrate this equivalence between CC and CA, we analyze the perfumes data by CC obtaining the following optimal canonical weights for the response category (we use the STATA software):

$b^*_A = 2.8248$, $b^*_B = 1.9173$ and $b^*_C = 1.5765$.

From (23), the recovered values are the following:

$b_A = 1.5468$, $b_B = 0.6392$, $b_C = 0.2985$ and $b_D = -1.2780$

To find the relationship between them, we ran the SimCA program (Greenacre 1986) to get the CA standard coordinates. The values for the first principal axis are: $b_A : 1.580$, $b_B : 0.652$, $b_C : 0.306$, $b_D : -1.306$. The values agree with those recovered from CC once the correction factor $\sqrt{\dfrac{M-1}{M}} = \sqrt{\dfrac{23}{24}}$ is applied, due to the computation of unbiased variances in CC.

The canonical weights for the attributes are the following:

Type of essences: $a^*_C = 1.0362$, $a^*_F = 2.26044$, $a^*_O = 1.6831$, strength: $a^*_{Hi} = -0.4149$, and size of the bottle: $a^*_{S1} = 1.1076$, $a^*_{S2} = 0.3807$.

From (22), we recover the values to be compared with CA: Type of essences: $a_L = -1.2449$, $a_C = -0.2087$, $a_F = 1.0155$, $a_O = 0.4382$, strength: $a_{Li} = 0.2075$, $a_{Hi} = -0.2075$ and size of the bottle: $a_{S3} = -0.4961$, $a_{S2} = -0.1154$, $a_{S1} = 0.6115$.

The standard coordinates in CA for the attributes are the following: Type of essences: $a_C = -0.3695$, $a_F = 1.7967$, $a_O = 0.7752$, $a_L = -2.2024$, strength: $a_{Hi} = -0.3674$, $a_{Li} = 0.3674$ and size of the bottle: $a_{S1} = 1.0810$, $a_{S2} = -0.2039$, $a_{S3} = -0.8771$.

The values agree with those recovered from CC once the correction factor $\sqrt{\dfrac{M-1}{M \times Q}} = \sqrt{\dfrac{23}{24 \times 3}}$ is applied.

Then, CC (CCM when it is treated as CC) and CA are equivalent, offering the same preference structure, even when more than one attribute is included in the analysis.

Equivalence between CCM and CA

So far, we have established the relationship between CA of concatenated tables and canonical correlation analysis where one set of variables is composed of several categorical variables. Since Carroll (1969) proposes an alternative formulation for CCM, with respect to CC analysis, we now look at the relationship between Carroll's CCM and CA, showing that there are simple scaling factor differences in the eigenvalues (principal inertias in CA) and the response category scores (standard coordinates in CA). We show these relationships by detailing the CCM and CA theory in parallel.

1. Relationship between the eigenvalues.

In categorical conjoint measurement, Carroll (1969) defines the general element of the data matrix S as,

$$s_{q,j_q,k} \equiv \sqrt{\frac{m_q}{n_k M}}\,(n_{q,j_q,k} - \frac{n_k}{m_q})$$

In correspondence analysis, from (14) the centered and standardized matrix T has as general element,

$$\begin{aligned} t_{q,j_q,k} &= \frac{(\frac{n_{q,j_q,k}}{QM} - \frac{1}{Qm_q} \times \frac{n_k}{M})}{\sqrt{\frac{1}{Qm_q}} \times \sqrt{\frac{n_k}{M}}} \\ &= \frac{1}{QM} \times \frac{\sqrt{Q}\sqrt{m_q}\sqrt{M}}{\sqrt{n_k}} \times (n_{q,j_q,k} - \frac{n_k}{m_q}) \\ &= \frac{\sqrt{m_q}}{\sqrt{Q}\sqrt{M}\sqrt{n_k}} \times (n_{q,j_q,k} - \frac{n_k}{m_q}) \\ &= \frac{1}{\sqrt{Q}} \times \sqrt{\frac{m_q}{Mn_k}} \times (n_{q,j_q,k} - \frac{n_k}{m_q}) \end{aligned}$$

Thus, there is only a scaling factor equal to $\frac{1}{\sqrt{Q}}$, linking the two approaches, where Q is the number of attributes.

In both cases we calculate a square symmetric matrix (notice that in order to calculate the SVD of the rectangular form of the CA matrix, T, a square matrix $\mathsf{T}^T\mathsf{T}$ is computed and decomposed using eigenvalues and eigenvectors). For the case of CCM, the data matrix is $\mathsf{R} = \sum_{q=1}^{Q} \mathsf{S}_q^T\mathsf{S}_q$, while in the case of CA, it is $\mathsf{T}^T\mathsf{T} = \frac{1}{Q}\mathsf{R}$, which give the following relationship:

$$\frac{1}{Q}\boldsymbol{\lambda}_{CCM} = \boldsymbol{\lambda}_{CA} \tag{24}$$

where $\boldsymbol{\lambda}_{CCM}$ and $\boldsymbol{\lambda}_{CA}$ are the vectors of eigenvalues obtained applying CCM and CA respectively. Thus the principal inertias in CA are equal to the eigenvalues from CCM divided by Q.

Since the inertia of a concatenated table is the average of the inertias of the individual tables (Greenacre 1994), the total variance in CCM is just the sum of the inertias of the Q tables.

2. Relationship between the coordinates.

Since the column *mass* is $c_k = \frac{n_k}{M}$, it follows from (16) that the l-th column of B in CA is $\mathsf{b}_l = \frac{\sqrt{M}}{\sqrt{n_k}}\mathsf{v}_l$. If we rearrange the terms, it can be expressed as $\sqrt{n_k}\frac{\mathsf{b}_l}{\sqrt{M}} = \mathsf{v}_l$. Hence from (10), $\frac{\mathsf{b}_l}{\sqrt{M}}$ are the response category scores in CCM.

Thus the response category scores obtained by CCM are the same as the standard coordinates obtained when we apply CA to the concatenated table but rescaled by the factor $\frac{1}{\sqrt{M}}$.

We corroborate the described equivalences with an illustration. Since conjoint algorithms are focus in the optimal dimension, we compare the values for this first dimension.

Response categories.

CA standard coordinates: $b_A : 1.580$, $b_B : 0.6520$, $b_C : 0.3058$, $b_D : -1.3061$

CCM optimal scores: $b_A : 0.3225$, $b_B : 0.1331$, $b_C : 0.0624$, $b_D : -0.2666$, where the scaling factor is equal to $\frac{1}{\sqrt{M}} = \frac{1}{24}$.

As we indicate previously, Carroll (1969) does not offer explicitly the formulation to find the optimal solution for the levels of the attributes. To solve this limitation, we make reference to Rao's (1977) application, where we realize that the CCM coordinates for the levels of the attributes are the same than the CA principal coordinates, once the previously derived rescaling factor, $\frac{1}{\sqrt{M}}$, is applied. We should expect to find a relationship equal to $\frac{1}{\sqrt{M \times Q}}$, due to differences in the grand totals of the analyzed data matrices, but the relationship between the eigenvalues provokes this small change. We also have to say that this finding is quite surprising since, CCM does not define or differentiate between standard and principal coordinates. At the same time, the introduction of this concept suppose an improvement since, it makes the values of the attributes and the category responses, directly comparable.

For the eigenvalues, the values obtained are the following (we take the first dimension, which is the one CCM considers): CA: 0.22169 and CCM: 0.6650. It let us to corroborate that the two sets of eigenvalues differ by a factor of 3, which is the number of attributes, Q.

INTERACTION EFFECTS

At this point, the analysis offer, for a particular subject, the evaluation for each attribute separately. But when a subject has to evaluate attributes like type of fragrance and intensity of fragrance referring to a perfume, it may be possible that a combination of two variables generates a value, which differs from the expected if interaction effects do not exist, changing its position with respect to the category response variable. An example could be a subject who normally prefers perfumes with low intensity but, for a particular fragrance, she can prefer perfumes with a high intensity. The described effect can be collected with the inclusion of interaction effects.

As noted previously, Green & Wind (1972), in a similar applied context than the one we are treating, point out as future research the possibility to introduce interaction terms explicitly in categorical conjoint measurement. In one hand, relevant literature recognizes that CCM can be understood as a special case of CC, and CC is able to treat interaction effects. Then, one alternative to estimate interaction effects can be applying CC, and keeping the first dimension as the optimal solution. In that cases, the researchers check the relevance of interaction effects comparing the estimated utility levels for one attribute with respect the preferences of the levels for the other attribute, such that changes in patterns indicate the relevance of interaction effects (Green & Wind, 1972). On the other hand, since we have already prooved that CCM is equivalent to CA, we can also codify the data in CA such that the technique is able to treat interaction effects. This alternative is presented as a better option, for the treatment of the interacction effects, since CA is a visual technique, which let us to visualize the "total effects" (main plus differentials) as levels, in the same space than the "main" effects. It lets to the researcher to compare the values and to conclude if interactions are improving the description of the data, or not. The other attributes included in the analysis are also displayed in the same space, making the interpretation, with respect to the preference variable, more complete.

CA offers other benefits and it is the level of information we can ask to it. For this purpose, we offer two different formats of concatenated tables. Firstly, the description of preferences for a single subject and secondly, the description of preferences of a small set of subjects, where all the preferences related to each level for each one of the attributes are plotted (e.g., three subjects, represented by three stacked concatenated tables plus one more collecting the mean).

Study design

From the cited interviews, we realize that the variable age determines preferences over perfumes. Then, we elaborate two analysis, which differ from the number of women analyzed as well as their ages. The first study describes preferences of a woman who is 30 years old. The data corresponds to the one previously used to check the equivalence between the techniques. The second one includes the previous woman and two women more who are 17 and 55 years old.

The new variable, which collects the joint effect of type and intensity of fragrance, has the following levels: Ch: citric fragrance with a high intensity, Cl: citric fragrance with a low intensity, Fh: floral fragrance with a high intensity, Fl: floral fragrance with a low intensity, Oh: oriental fragrance with a high intensity, Ol: oriental fragrance with a low intensity, Lh: leather fragrance with high intensity and Ll: leather fragrance with low intensity.

The "Diccionario de las fragancias, 2002", which specifies the composition for all perfumes, let us to choose a sample of brands for our experiment. The particular brands are the following: Citric-low essence: Emporio-armani white, citric-high essence: O de Lancôme, leather-low essence: Nu of YvesSainLaurent, leather-high essence: Truth of Calvin Klein, floral-high essence: Trésor of Lancôme, floral-low essence: Gucci Envy, oriental-high essence: Opium of YvesSainLaurent, and oriental-low essence: EmporioElla of Armani.

Correspondence analysis results

Analysis including interaction effects: one subject.

The results of this analysis appear in the appendix. The new variable, "type of fragrance $\times$ intensity of fragrance" has 8 levels, labelled in the previous section. Once more we codify the data as a concatenated table, that includes two active variables with their levels: the interaction variable (type of fragrance $\times$ intensity of fragrance) and the size of the bottle. Original variables (type of fragrance and intensity of fragrance) are added as *supplementary* points. Geometrically, these supplementary points are centroids of the interactions and have zero mass so as not to repeat information.

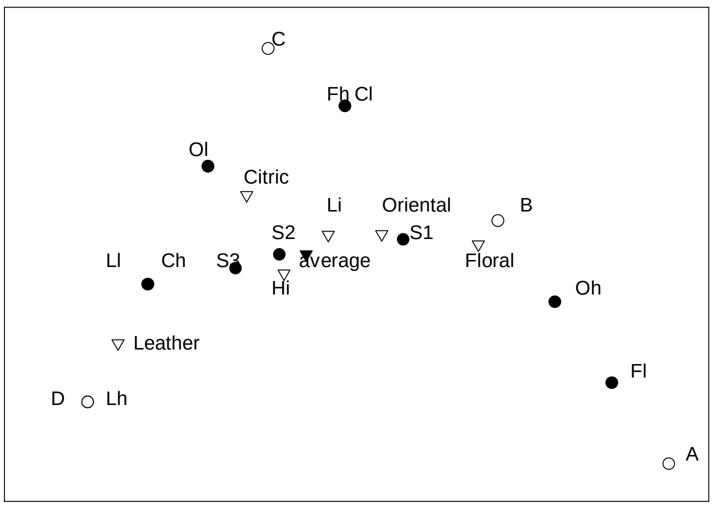
We are going to compare the total inertia with and without interactions, to be able to justify the inclusion of the interaction terms: Inertia without interactions $= 0.2750$ and the inertia with interactions $= 0.7694$. Then, with the introduction of the interaction effects, we get an inertia more than doubled. We can interpret the results from figure 1.

When we analyze data characterized to have a response variable which is ordinal, is usual to find the "arch" or "horseshoe" effect (Greenacre 1984). Some authors, like Hill, M.O. & Grauch, H.G. Jr. (1980) and Peet, R.K.; Knox, R.G.; Case, J.S. & Allen, R.B. (1988) consider that this effect is a mathematical artifact, which should be eliminated applying an alternative method named detrended correspondence analysis. Other authors, like Wartengerg, et al. (1987), which are cited in Oksanen, J. (1988), are very sceptical about the value of detrending. They suggest that it can hide the real data structure and as it cannot improve the order of the points in the first axis, it can confound the real pattern, and even introduce new distortions. Greenacre (1984), criticized detrending because the control of the geometry is lost and he exposes that it is better to add a second dimension which helps to discriminate between answers in the intermediate levels. Following the lasts authors, we will use both dimensions to describe preferences.

The interpretation of the map is the following. The first principal axis differentiates between the most and the least preferred levels. The inertia increases and it is distributed between both, the first and second principal axes. For the women who is 30 years old, the most preferred essences are, the floral-low intensity fragrance followed by the oriental-high intensity one. The floral-high intensity essence and the citric-low intensity essences appear one next to the other and related with the "just low worth" level of utility. Then, in the preference map of this subject, both essence become indifferent. The citric-high intensity, and the leather-low intensity essences appear next to D and finally, the leather-high intensity is the essence which dislikes more. The small bottle is the most preferred followed by the medium one which appears in the centroid. The biggest one is situated between the C and D levels of utility.

An ordinal-scaled response variable reduces the type of interactions to described, to be *crossover.* Then, with the introduction of interaction effects, we will be able to capture situations where the preference ordering for the levels of an attribute are dependent of the levels of another attribute (Vriens, 1995). We would not appreciate interaction effects if the revealed order of preferences for the levels of the attributes (essences or strength) remained unchanged, with respect to the analysis without interactions. We can corroborate it in the map. The supplementary points, which corresponds to the levels of the attributes related to the "main" effects, reveal preferences over essences and strength. They are the following: floral $\succ$ oriental $\succ$ citric $\succ$ leather, and high intensity $\succ$ low intensity. Then, if interaction effects were not significant and a strong preferences over essences was given, we could expect to find the following order: Fl$\succ$Fh$\succ$Ol$\succ$Oh$\succ$Cl$\succ$Ch$\succ$Ll$\succ$Lh. On the other hand, a strong preferences over the strength, when interaction effects were not significant, would imply this other order: Fl$\succ$Ol$\succ$Cl$\succ$Ll$\succ$Fh$\succ$Oh$\succ$Ch$\succ$Lh. In our analysis, Oh becomes the second most preferred level, changing the expected order. Then, interaction effects are relevant.

Figure 1
Asymmetric map once interaction effects are included.



● : attributes, ○ : category responses, ▽ : supplementary points and H : average.

The horizontal axis is dimension 1, with inertia = 0.4726 (61.4%). The vertical axis is dimension 2 with, inertia = 0.2196 (28.5%). 90.0% of total inertia is represented in the above map.

We calculate the standard coordinates for the response categories. Later on, we will compare them with the ones obtained in CC to check the equivalence. The values are the following: First dimension: b_A : 1802, b_B : 954, b_C : -188 and b_D : -1085. Second dimension: b_A : -1272, b_B : 203, b_C : 1248 and b_D : -898. The values for the attributes appear in the appendix.

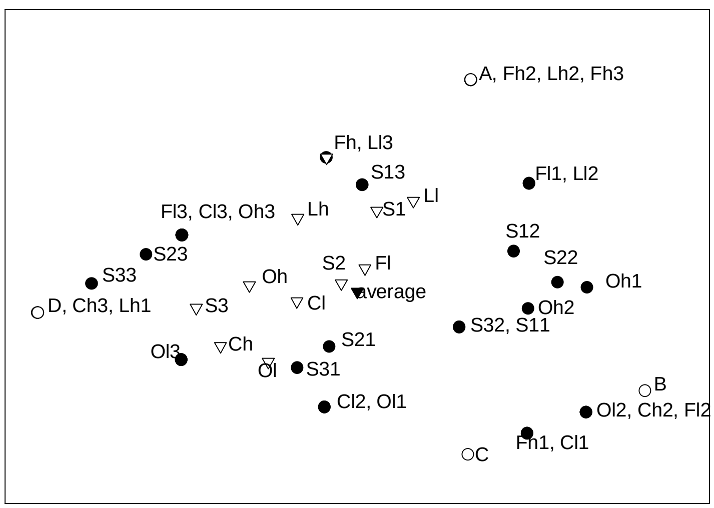
Analysis including interaction effects: three subjects.

In this analysis we add two new subjects to the one previously analyzed, who are 17 and 55 years old. We present the data as three stacked tables, with one subject bellow to the other. Each subject will have her own points for each attribute level. The centroids of these "clouds of points" will be the supplementary points which, aggregates all the individual level matrices. Since for the previous case we already made the comparison between the analysis with and without interaction variables, in this case, we are going directly to the analysis with the interaction effects showing the utility from an applied point of view, of this particular way of coding data.

The resulting map is the following:

Figure 2

Asymmetric map for the three women, once interaction effects are included.



• : attributes, ○ : category responses, ∇ : supplementary points and H : average. The third character identifies the women, such that woman 1 is 29 years old; woman 2 is 17 years old; and woman 3 is 55 years old. The horizontal axis is dimension 1, with inertia = 0.5423 (50.2%) and the vertical axis is dimension 2, with inertia = 0.3982 (36.8%). 87.0% of total inertia (1.081357) is represented in the above map

In terms of utility, the first principal axis is mainly contributed by the lowest level utility D, followed by the B one. The second principal axis is mainly contributed by the highest level of utility A, followed by the C one.

We are not going to describe the preferences for the woman who is 30 years old, since it was already made. Instead, we follow describing the 17 and the 55 years old woman's preferences. From the map, we observe differences in preferences between high and low intensity fragrances, depending on the type of essences. For example, the 17 years old women prefers high intensity perfumes for leather and floral essences, but leather-low essence receives also an important level of utility. The woman who is 55 years, neither presents a clear preference for low or high intensity perfumes, instead it also depends on the particular essences. Her most preferred essence is the floral one with high intensity, but followed by the leather one with low intensity. Then, for all three women, interaction effects are adding relevant information and improving in the description of the data.

Concluding, the map let us to describe preferences which remain hiden in the analysis of main effects. For example, for the third woman analyzed, floral and citric low intensity receive the same position, in terms of utility, than the oriental-high essences. Further more, the analysis of different subjects at time, let us to capture similarities and differences, based in some variables, in this case, the age. While the utility path followed by the three women is quite different, some agrees are reflected. For example, women 2 and 3 have as first preference the floral essence with high intensity.

Coding interactions in canonical correlation analysis

When we include interaction effects, the relation between CA and CC is less obvious than in the previous case. To show the connection we will first describe the way to code the interaction in CC as a single dummy variable, in order to obtain identical results to those of CA. Secondly, we will show the equivalence between the results obtained with this approach and the results obtained with the more customary way of handling interactions of categorical variables in linear models.

When we introduce an interaction variable, the type of data matrix to analyze in CA is still a concatenated table, in this case composed of 2 variables, one with 8 levels (type of fragrance$\times$intensity of fragrance) and the others with 3 levels (size of the bottle). It suggests immediately that CC has to be computed as before and that we have to omit one of the levels of the interaction variable. In other words, the interaction is treated just like a categorical variable with 8 levels, so 7 dummy variables are introduced into the CC analysis to estimate the interactions. The results of CA are recovered just as before by imposing the usual centering condition inherent in CA. Notice that in this case, the interaction effects include the main effects for type of fragrance and intensity of fragrance.

We point out that the usual way of handling categorical variables plus their interactions in linear models would be to omit one category of each variable and all category combinations of the interactions involving these omitted categories. In this case the model would still have 7 parameters; 3 for type of fragrance, 1 for intensity of fragrance and $3 \times 1 = 3$ for the interactions. Here the interaction effects do not include main effects and can be called "differentials" from main effects. The problem is how to recover all the coefficients, especially those of the interaction terms. The key point to realize is that in the calculations all the interactions belong to the same variable and so all the omitted coefficients take the same value. We illustrate the equivalence using the data on the 30-years old subject. A more general demonstration is given in Torres (2001).

Canonical Correlation. New way of coding.

Linear combinations for first canonical correlation. Number of observations = 24.

(a) Attribute levels.

The canonical weights for the interaction variable are the following (7 coefficients):

d^*_{Cl}: 0.985, d^*_{Ch}: 0, d^*_{Fl}: 2.322, d^*_{Fh}: 0.985, d^*_{Ol}: 0.301, d^*_{Oh}: 2.037, d^*_{Lh}: -0.301

where d^* represents the canonical weighs for the "new way" of coding.

The canonical weights for the variable "size of the bottle", are the following (2 coefficients):

a^*_{S1}: 0.840, a^*_{S2}: 0.220.

(b) Response category values.

The canonical weights for the response variable are the following:

b^*_A: 2.826, b^*_B: 1.997, b^*_C: 0.877.

The canonical correlations are: 0.9722, 0.6627, and 0.3932.

We recover the missing coefficients by centering condition:

Size of the bottle:

$$r_{S1}(a^*_{S1} + a_{S3}) + r_{S2}(a^*_{S2} + a_{S3}) + r_{S3}a_{S3} = 0$$

since $r_{S1} = r_{S2} = r_{S3} = \dfrac{1}{Q \times m_q}$, we simplify the expression, taking out this term. Then,

$$(a^*_{S1} + a_{S3}) + (a^*_{S2} + a_{S3}) + a_{S3} = 0$$

where $a_{S1} = 0.486$, $a_{S2} = -0.133$ and $a_{S3} = -0.353$.

Interaction variable "type of fragrance $\times$ Intensity of fragrance":

$r_{Cl}(d^*_{Cl} + d_{Ll}) + r_{Ch}(d^*_{Ch} + d_{Ll}) + r_{Fl}(d^*_{Fl} + d_{Ll}) + r_{Fh}(d^*_{Fh} + d_{Ll}) +$
$+ r_{Ol}(d^*_{Ol} + d_{Ll}) + r_{Oh}(d^*_{Oh} + d_{Ll}) + r_{Lh}(d^*_{Lh} + d_{Ll}) + r_{Ll}d_{Ll} = 0$

since $r_{Cl} = r_{Ch} = r_{Fl} = r_{Fh} = r_{Ol} = r_{Oh} = r_{Ll} = r_{Lh} = \dfrac{1}{m_q \times m_{q'} \times Q}$,

then the expression is simplified as follows:

$(d^*_{Cl} + d_{Ll}) + (d^*_{Ch} + d_{Ll}) + (d^*_{Fl} + d_{Ll}) + (d^*_{Fh} + d_{Ll}) + (d^*_{Ol} + d_{Ll}) +$
$+ (d^*_{Oh} + d_{Ll}) + (d^*_{Oh} + d_{Ll}) + (d^*_{Lh} + d_{Ll}) + d_{Ll} = 0$

where $d_{Ll} = -0.791$. Then, we recover the true values: $d_{Cl} = 0.194$, $d_{Ch} = -0.791$, $d_{Fl} = 1.531$, $d_{Fh} = 0.194$, $d_{Ol} = -0.490$, $d_{Oh} = 1.246$, $d_{Ll} = -0.791$ and $d_{Lh} = -1.092$.

Canonical Correlation. Traditional way of coding.

Linear combinations for first canonical correlation. Number of observations = 24.

(a) Attribute levels.

The canonical weights for the essences are the following (3 coefficients):

a^*_C: 0.985, a^*_F: 2.322, a^*_O: 0.301,

the canonical weight for the strength of the fragrance is (1 coefficient)

a^*_{Hi}: -0.301,

the canonical weights for the levels of the variable "size of the bottle" are (2 coefficients)

a^*_{S1}: 0.840, a^*_{S2}: 0.219,

the canonical weights for the levels of the interaction effects are (3 coefficients)

e^*_{Ch}: -0.685, e^*_{Fh}: -1.036, e^*_{Oh}: 2.037

where e^* is used to denominate the canonical weights of the interaction variable in the "traditional way" of coding.

(b) Response categories.

The canonical weights are the following:

b^*_A: 2.826, b^*_B: 1.997, b^*_C: 0.877.

The canonical correlations are: 0.9722, 0.6627, 0.3932.

From the results we corroborate that the canonical correlation coefficients are the same if we code the data in the new way and if we do it in the traditional form. Further, the coefficients for the main effects are also the same.

The remaining work is to find the values for the interactions in the new form.

The restriction to be applied in this case is the following:

$$r_{Ch}(e^*_{Ch}+c) + r_{Fh}(e^*_{Fh}+c) + r_{Oh}(e^*_{Oh}+c) + r_{Lh}c + r_{Cl}c + r_{Fl}c + r_{Ol}c + r_{Ll}c = 0$$

where $r_{jj^\theta} = \dfrac{1}{m_q \times m_{q^\theta} \times Q}$, where j represents the essence and j^θ the strength of the essence. Then, the previous expression can be simplified as

$$(e^*_{Ch}+c) + (e^*_{Fh}+c) + (e^*_{Oh}+c) + 5c = 0$$

where $c = -0.0396$.

The true coefficients are recovered in the following way:

$$f_{Ch} = a_C + a_{H_i} + (e^*_{Ch} + c)$$

$$f_{Ch} = 0.08335 + (-0.1504) + (-0.6884) = -0.7555$$

where f represents the coefficients which collects main and differential effect due to the interactions, in the traditional way of centering.

The operation is repeated for all the other coefficients obtaining the following solutions: $f_{Ch} = -0.7555$, $f_{Cl} = 0.1941$, $f_{Fh} = 0.1941$, $f_{Fl} = 1.5305$, $f_{Oh} = 1.2461$, $f_{Ol} = -0.4904$, $f_{Lh} = -1.0920$, $f_{Ll} = -0.7912$.

Equivalence between CC/CA once interaction effects are included.

To show the equivalence between the different techniques once the interaction effects are included, we corroborate the relationships previously found with the data analysis corresponding to one subject.

(a) Eigenvalues relationship: $\frac{\lambda_{1CC} = \lambda_{1CCM}}{\lambda_{1CA}} = \frac{0.9452}{0.4726} \approx 2$, where $Q = 2$.

(b) The standard coordinates for the response categories recovered from CC are the following: First dimension: $d_A = f_A$: 1764.58, $d_B = f_B$: 934.86, $d_C = f_C$: -184.3, $d_D = f_D$: -1061.6 and second dimension: $d_A = f_A$: 1245, $d_B = f_B$: -197.6, $d_C = f_C$: -1222.97, $d_D = f_D$: 879.59, which have a relation equal to $\sqrt{\frac{M}{M-1}} = \sqrt{\frac{24}{23}}$, with respect to the CA ones.

Then, the relationships are kept.

CONCLUSIONS AND DISCUSSION

The present research tries to go one step further, in the aim of giving some lights about the relationship, as well as the profits, of using some psychometric techniques versus another ones, in marketing research. We have prooved, analitically and also with an application, that correspondence analysis offers the same results than categorical conjoint measurement, for the optimal solution. We have also shown that, the main difference between both techniques is the map that correspondence analysis offers and then, the inclusion of a second dimension in the recovering of the preference structure.

Once the equivalence is shown we can introduce interaction effects in this type of analysis via correspondence analysis. The improvement of the new treatment becomes evident from the exposed application for perfumes. Further, correspondence analysis let to the researchers to make the analysis at different levels of aggregation (one, three or one thousand subjects). We propose as future research to include more subjects in the description of preference and get potential clusters, in terms of different variables.

APPENDIX

I) Analysis of the perfume data for one subject by CA.

Correspondence analysis (CA)

(a) CA output without interaction effects

Row contributions

Table 1

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
C	1000	83	66	-174	140	11	-268	332	213
F	1000	83	229	846	947	269	127	21	48
O	1000	83	52	365	774	50	159	147	75
L	1000	83	354	-1037	922	404	-18	0	1
Hi	1000	167	39	-173	468	22	182	518	196
Li	1000	167	39	173	468	22	-182	518	196
S1	1000	111	109	509	966	130	-27	3	3
S2	1000	111	18	-96	211	5	-171	665	115
S3	1000	111	96	-413	719	86	198	165	154

where QLT: quality of the display, MAS: mass, INR: inertia of the point, k=1 and k=2 are principal coordinates for the first and second principal axes, COR: correlation of the point with the axis and CTR: contribution of the point to the axis (Greenacre, 1986).

The principal inertias are the following: 1. 0.221690, 2. 0.028171, 3. 0.025140.

II) Analysis with the inclusion of interaction effects

(a) CC output when we include interaction effects: One subject.

- New way of coding

Table 2

Name	k=1	k=2
Ch	0.3008	1.0576
Cl	1.2861	2.6571
Fh	1.2861	2.6571
Fl	2.6225	0.1742
Oh	2.3381	0.8999
Ol	0.6016	2.1153
Ll	0.3008	1.0576
S1	0.8395	0.2587
S2	0.2195	0.1245

- Traditional way of coding

Table 3

Name	k=1	k=2
C	0.9853	1.5995
F	2.3217	-0.8834
O	0.3008	1.0575
Hi	-0.3008	-1.0576
Ch	-0.6845	-0.5418
Fh	-1.0356	.5406
Oh	2.0373	-0.1577
S1	0.8395	0.2587
S2	0.2195	0.1245

The canonical correlations are: 0.9722, 0.6627, 0.3932.

(b) CA output with interaction effects: One subject.

The previous attributes, type of fragrance and intensity of fragrance, as well as their levels, appear in the analysis as supplementary points.

Table 4
Row Contributions

Name	QLT	MAS	INT	k=1	COR	CTR	k=2	COR	CTR
Ch	1000	63	54	-786	926	82	-183	50	9
Cl	1000	63	70	193	43	5	900	935	231
Fh	1000	63	70	193	43	5	900	935	231
Fl	1000	63	251	1520	748	306	-781	197	173
Oh	1000	63	164	1237	757	203	-289	41	24
Ol	1000	63	54	-487	356	31	533	427	81
Lh	1000	63	162	-1085	588	156	-899	404	230
Ll	1000	63	54	-786	926	82	-183	50	9
S1	1000	167	58	483	869	82	89	29	6
S2	1000	167	9	-133	402	6	-2	0	0
S3	1000	167	51	-351	518	43	-86	31	6

Supplementary rows:

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
C	1000	125	35	-296	406	23	359	594	73
F	1000	125	123	856	971	194	60	5	2
O	1000	125	28	375	818	37	122	86	8
L	1000	125	190	-935	750	231	-541	250	166
Hi	1000	250	21	-110	189	6	-118	216	16
Li	1000	250	21	110	189	6	118	216	16

Table 5

Column Contributions

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
A	1000	125	334	1239	747	406	-596	173	202
B	1000	208	179	656	653	190	95	14	8
C	1000	333	169	-129	43	12	585	877	520
D	1000	333	318	-746	757	392	-421	241	269

The principal inertias are the following: 0.472602, 0.219554, 0.077288.

The standard coordinates for the category response variable are the following:

First principal axis: y_A: 1802, y_B: 954, y_C: -188, y_D: -1085.

Second principal axis: y_A: -1272, y_B: 203, y_C: 1248, y_D: -898.

(c) CA output with interaction effects: Three subject.

Subject 1:

Table 6

Row Contributions

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
Ch1	1000	21	17	-750	650	22	-483	270	12
Cl1	1000	21	34	723	299	20	-1004	576	53
Fh1	1000	21	34	723	299	20	-1004	576	53
Fl1	1000	21	23	731	440	21	766	483	31
Oh1	1000	21	36	978	514	37	31	1	0
Ol1	1000	21	28	-140	13	1	-818	457	35
Lh1	1000	21	39	-1361	926	71	-149	11	1
Ll1	1000	21	17	-750	650	22	-483	270	12
S11	1000	56	15	434	642	19	-252	217	9
S21	1000	56	12	-120	60	1	-389	630	21
S31	1000	56	20	-256	166	7	-539	733	40

Subject 2:

Table 7
Row Contributions

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
Ch2	1000	21	38	974	476	36	-854	366	38
Cl2	1000	21	28	-140	13	1	-818	457	35
Fh2	1000	21	50	484	90	9	1501	867	118
Fl2	1000	21	38	974	476	36	-854	366	38
Oh2	1000	21	11	727	962	20	-119	26	1
Ol2	1000	21	38	974	476	36	-854	366	38
Lh2	1000	21	50	484	90	9	1501	867	118
Ll2	1000	21	23	731	440	21	766	483	31
S12	1000	56	28	666	812	45	286	150	11
S22	1000	56	50	853	753	75	67	5	1
S32	1000	56	15	434	642	19	-252	217	9

Subject 3:

Table 8

Row Contributions

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
Ch3	1000	21	39	-1361	926	71	-149	11	1
Cl3	1000	21	14	-746	759	21	401	220	8
Fh3	1000	21	50	484	90	9	1501	867	118
Fl3	1000	21	14	-746	759	21	401	220	8
Oh3	1000	21	14	-746	759	21	401	220	8
Ol3	1000	21	17	-750	650	22	-483	270	12
Lh3	1000	21	39	-1361	926	71	-149	11	1
Ll3	1000	21	18	-131	18	1	951	969	47
S13	1000	56	34	21	1	0	757	857	80
S23	1000	56	47	-900	888	83	264	76	10
S33	1000	56	70	-1131	945	131	58	2	0

The supplementary rows:

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
Ch	1000	63	38	-583	510	39	-384	221	23
Cl	1000	63	4	-257	869	8	-67	59	1
Fh	1000	63	54	-131	18	2	951	969	142
Fl	1000	63	4	32	16	0	166	418	4
Oh	1000	63	13	-460	916	24	45	9	0
Ol	1000	63	25	-379	329	17	-495	562	39
Lh	1000	63	22	-254	173	7	523	735	43
Ll	1000	63	31	239	107	7	644	779	65
S1	1000	167	55	83	19	2	573	919	137
S2	1000	167	35	-69	21	1	60	16	2
S3	1000	167	78	-686	936	145	-114	26	5

Table 9
Column Contributions

Name	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR
A	1000	278	267	356	122	65	974	863	626
B	1000	181	229	902	595	271	-444	144	90
C	1000	208	186	347	124	46	-728	548	277
D	1000	333	318	-1002	974	618	-94	9	7

The principal inertias are the following: 0.542312, 0.398208, 0.140837.
The total inertia is equal to 1.081357.

References

[1] Benzécri, J-P & collaborators (1973), "L'Analyse des Données. Tome 2: l'Analyse des Correspondances". Paris: Dunod.

[2] Carmone, F.J. & P. E. Green (1981), "Model Misspecification in Multiattribute Parameter Estimation", Journal of Marketing Research, Vol. XVIII (February), 87-93.

[3] Carroll, J.D. (1969), "Categorical Conjoint Measurement", paper presented at the Mathematical psychology Meeting, Ann Arbor, Michigan.

[4] Carroll, J.D. & P.E. Green (1995), "Psychometric Methods in Marketing Research: Part I, Conjoint Analysis", Journal of Marketing Research, **32** (Nov), 385-391.

[5] Eckart, C. & G. Young (1936), "The Approximation of one matrix by another of lower rank", Psychometrika, **1**, 211-218.

[6] Green, P.E. (1973), "On the Analysis of Interactions in Marketing Research Data", Journal of Marketing Research, **10**, 410-420.

[7] - - - - & Rao, V. (1971), "Conjoint measurement for quantifying judgement data", Journal of Marketing Research, **8**, 355-363.

[8] - - - - & Wind, Y. (1972), "Recent approaches to the modeling of individuals' subjective evaluations", paper presented at the Attitude Research Conference.

[9] - - - - & - - - - (1973), "Multiattribute Decisions in Marketing: a Measurement Approach", eds. Illinois: The Dryden Press.

[10] Greenacre, M.J. (1984), "Theory and Applications of Correspondance Analsyis". London: Academic Press.

[11] - - - - (1986), "SimCA: a program to perform simple correspondence analysis", Psychometrika, **51**, 172-173.

[12] - - - - (1993), "Correspondence Analysis in Practice". London: Academic Press.

[13] - - - - (2002), "Scaling: Correspondence Analysis". International Encyclopedia of the Social & Behavioral Sciences.

[14] - - - - and Blasius, J. (1994), "Correspondence Analysis in the Social Sciences". London: Academic Press.

[15] - - - - and Hastie, T. (1987), "The geometric interpretation of correspondence analysis", Journal of the American Statistical Association, **82**, 437-444.

[16] Hauser, J.R. & F. S. Koppelman (1979), "Alternative Perceptual Mapping Techniques: Relative Accuracy and Usefulness", Journal of Marketing Research, Vol. XVI (November), 495-506.

[17] Hill, M.O. & Grauch, H.G. Jr. (1980), "Detrended correspondence analysis: an improved ordination technique", Vegetatio, **42**, 47-58.

[18] Lattin, J., Carroll, J.D. and Green, P.E. (2003), "Analyzing Multi-

variate Data". London: Thomson Learning.

[19] Oksanen, J. (1988), "A note on the occasional instability of detrending in correspondence analysis", Vegetatio, **74**, 29-32.

[20] Peet, R.K, Knox, R.G., Case, J.S. , Allen, R.B. (1988), "Putting Things in Order: The Advantages of Detrended Correspondence Analysis", The American Naturalist, **131(6)**, 924-934.

[21] Rao, V.R. (1977), "Conjoint Measurement in Marketing Analysis, in Multivariate Methods for Market and Survey Research", Jagdish Sheth, eds. University of Illinois: American Marketing Association.

[22] "Stata reference manual"l (1993), Release 3.1, vol. 2. 6th ed., 100-104. Texas: College Station.

[23] Torres, A., "Correspondence analysis and categorical conjoint measurement", Working Paper, **569**, at Universitat Pompeu Fabra.

[24] Torres, A. and Greenacre, M.J. (2002), "Dual scaling and correspondence analysis of preferences, paired comparisons and ratings", International Journal of Research in Marketing, **19**, 401-405.

[25] Vriens, M. (1995), "Conjoint Analysis in Marketing", Theses on Systems, Organizations and Management. The Netherlands: Labyring Publication.

[26] Wartengerg, D., Ferson, S., Rohlf, F.J. (1987), "Putting things in order: A Critique of Detrended Correspondence Analysis", The American Naturalist, **129(3)**, 434-448.

[27] Wittink, D.R. (1999). "An Introduction to Conjoint Analysis" Course notes of Applied Statistics Week, Institut d'Educació Continua.

[28] - - - - , Huber J., Zandan, P. and Johnson, R. M. (1992), "The number of levels effect in conjoint: where does it come from, and can it be eliminated?", in Sawtooth Software Conference Proceedings, Ketchum, Idaho, 355-364.