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IMAGINING CIRCLES – EMPIRICAL DATA AND A PERCEPTUAL MODEL FOR THE ARC-SIZE ILLUSION

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Abstract

An essential part of visual object recognition is the evaluation of the curvature of both an object's outline as well as the contours on its surface. We studied a striking illusion of visual curvature – the arc-size illusion (ASI) – to gain insight into the visual coding of curvature. In the ASI, short circular arcs appear less curved than full circles. We investigated if and how the ASI depends on (i) the physical size of the stimulus and (ii) on the length of the arc. Our results show that perceived curvature monotonically increases with arc length up to an arc angle of about 60˚, thereafter remaining constant and equal to the perceived curvature of a full circle. We investigated if the misjudgment of curvature in the ASI translates into predictable biases for three other perceptual tasks: (i) judging the position of the centre of circular arcs; (ii) judging if two circular arcs fall on the circumference of the same (invisible) circle and (iii) interpolating the position of a point on the circumference of a circle defined by two circular arcs. We found that the biases in all the above tasks were reliably predicted by the same bias mediating the ASI. We present a simple model, based on the central angle subtended by an arc, that captures the data for all tasks. Importantly, we argue that the ASI and related biases are a

consequence of the fact that an object's curvature is perceived as constant with viewing distance, in other words is perceptually scale invariant.

INTRODUCTION

Curvature is an important feature of objects that is ubiquitous in natural scenes. Evidence for the existence of specialized detectors for curvature in the visual system (Watt, 1984; Watt & Andrews, 1982; Wilson & Richards, 1989) is supported by the observation that curvature is an adaptable feature (Arguin & Saumier, 2000; Gheorghiu & Kingdom, 2007; 2008; 2009; Hancock & Peirce, 2008). Furthermore, curvature has been hypothesized to play an important role in building object representations (Loffler, 2008; Wilson & Wilkinson, 2015). Many studies investigating curvature perception have focused on circles or circular segments, which are a special class of curves. Circularity has been the subject of many studies (see Loffler, 2008 for review) and it has been suggested that it plays a special role in contour detection (Achtman, Hess, & Wang, 2003), texture detection (Motoyoshi & Kingdom, 2010) and Glass pattern detection (Wilkinson, Wilson, & Habak, 1998; Wilson, Wilkinson, & Asaad, 1997), *cf* (Dakin & Bex, 2002 and Schmidtmann, Jennings, Bell & Kingdom 2015).

Given the importance of curvature for object detection and recognition, it may be surprising that curvature is misperceived in certain circumstances. Some studies find evidence for an overestimation of curvature (Coren & Festinger, 1967; Piaget & Vurpillot, 1956) - in this case subjects tend to perceive circular arcs as more curved than circles. Other studies have found an underestimation of curvature, at least for short arcs (Virsu, 1971b; 1971a; Virsu & Weintraub, 1971). Virsu (1971b) asked observers to compare the curvature of drawn arcs with a set of reference circles of varying radius, and found a consistent underestimation of curvature for arcs up to about 72˚. For longer arcs, curvature estimation became veridical. This underestimation of curvature for short arcs is convincingly demonstrated in the "Arc-size Illusion" (ASI), shown in Figure 1. In this simple geometric illusion, short arcs are perceived as flatter (less curved) compared to longer arcs of the same radius (Virsu, 1971b; Virsu & Weintraub, 1971).

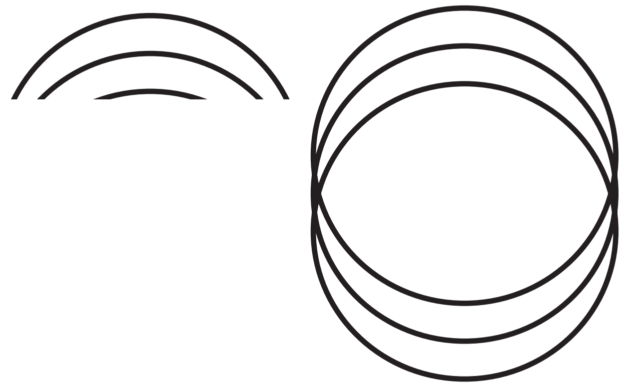

Figure 1 The Arc-size Illusion. In this illusion, arcs of the same radius (i.e. curvature) are perceived as flatter the shorter the size of the arc. The arcs on the left all have the same radius and therefore the same curvature. They are segments of the circles on the right. Observers typically describe shorter (e.g. innermost) arc as flatter than longer ones (e.g. outermost).

According to Virsu (1971a) this underestimation of curvature is caused by the observers' tendency to produce straight eye movements (see Discussion for details).

Here we employ a novel experimental method to measure and quantify the ASI. We then consider whether the misperception of curvature in the ASI underpins three other tasks that involve curvature judgments: judgments of the centre of a circular arcs, alignment judgments of two circular arcs, and interpolation judgements of curvature. Based on the results, we suggest a model for curvature perception and offer a functional explanation of the ASI in terms of perceptual scale invariance.

Methods

Subjects

Four subjects participated in this study. Two of the observers (IE and MO) were naïve as to the purpose of the experiments. All observers had normal or corrected-to-normal visual acuity. Experiments were carried out under binocular viewing conditions. No feedback was provided during practice or during the experiments. Informed consent was obtained from each observer; and all experiments were conducted in accordance with the Declaration of Helsinki.

Apparatus

The stimuli were generated within the MatLab (MatLab R 2013a, MathWorks) environment and presented on a calibrated, gamma-corrected "Iiyama Vision Master Pro 513" CRT monitor with a resolution of 1024x768 pixels and a frame rate of 85 Hz (mean luminance 38 cd/m^2) under the control of an Apple Mac Pro (3.33 GHz). Observers viewed the stimuli at distance of 120 cm. At this distance one pixel subtends 0.018˚. Experiments were carried out under dim room illumination. Routines from the Psychophysics Toolbox were employed to present the stimuli (Brainard, 1997; Pelli, 1997).

Stimuli

Stimuli were circles and circular arcs with radii of r = 1, 2 and 3˚ of visual angle. Curvature was defined as 1/r. Circular arcs were created by applying a pie-wedge shaped mask to the circles. In Experiment 1, where observers had to match the curvature of a test arc to that of a reference circle, the curvature of the circular arcs could be varied by altering their radii. In Experiments 2 to 4, observers had to judge the position of the centre of a circular arc (Exp 2), the position of a second arc so that it fell on the (invisible) circle given by a first arc (Exp 3), or the position of an interpolated point on the circumference of an (invisible) circle given two arcs. In these tasks, the circular arc remained fixed and the position of a reference dot (Exp 2 and 4) or the position of one of the arcs could be altered.

To create circular arcs of variable length, the contrast of the circle along their circumferences was ramped down by half a Gaussian either side of the arc centre (Schmidtmann, Kennedy, Orbach, & Loffler, 2012):

$$C(\partial) = \begin{cases} C_{nominal} \cdot e^{\frac{-(\partial-\theta/2)^2}{(\sigma/2)^2}}, & \partial > \partial + {}^{\partial}/_{2} \\ C_{nominal}, & \partial - {}^{\partial}/_{2} \leq \partial \leq \partial + {}^{\partial}/_{2} \\ C_{nominal} \cdot e^{\frac{-(\partial+\theta/2)^2}{(\sigma/2)^2}}, & \partial < \partial - {}^{\partial}/_{2} \end{cases} \qquad \text{(Eq. 1)}$$

where C is the contrast as a function of polar angle (∂), $C_{nominal}$ refers to the contrast of arc (100 % in all conditions), θ refers to the central angle (angular extent), σ is the space constant of the Gaussian (set to 15˚) that was used to ramp down the contrast on either end of the segment. The cross-sectional luminance profile of all stimuli was defined by a fourth derivative of a Gaussian with a peak spatial frequency of 8 c/˚ (Wilkinson et al., 1998).

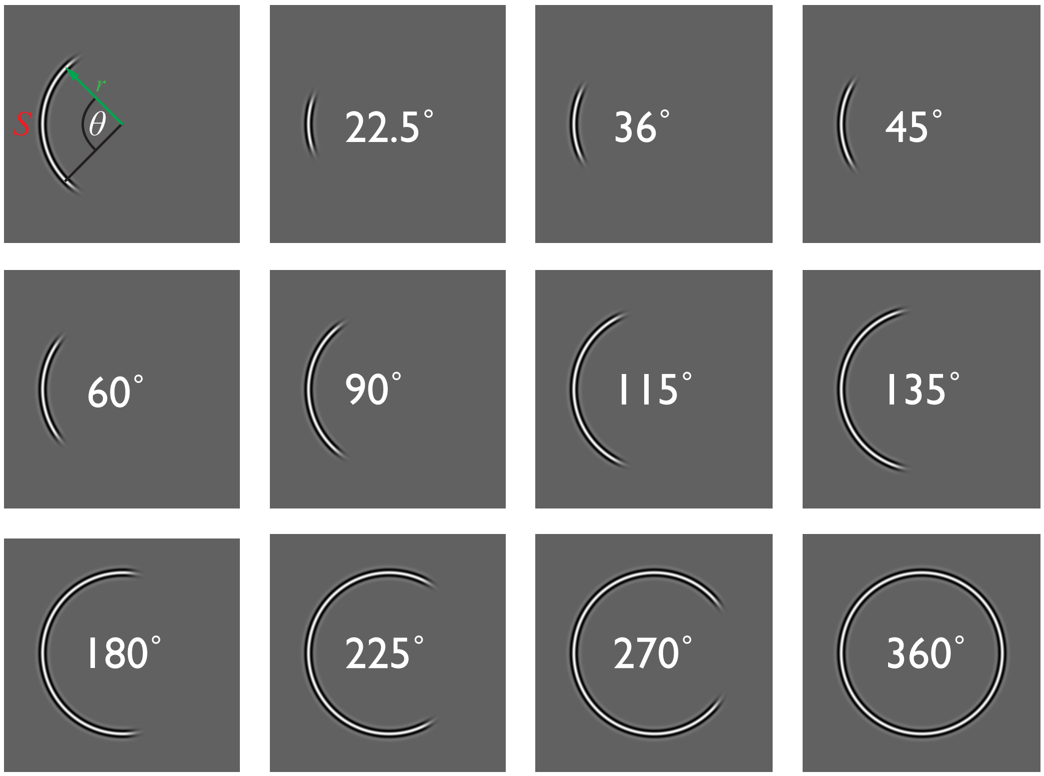

Figure 2 Sample circular arcs. The arcs used in this study were segments of circles with a D4 cross-sectional luminance profile with a peak spatial frequency of 8 c/°. The polar angle θ describes the central angle or angular extent of the arc (excluding the ramp; see text) and ranged from 22.5° to 360° (full circle).

Procedure

Experiment 1 - Arc-size Illusion

Using the Method of Adjustment (MOA), observers were asked to adjust the curvature of a test arc of fixed arc length to the curvature of a complete reference circle of given radius. There were three different reference radii R_{ref} of 1°, 2° and 3° (visual angle), and these were interleaved in each experimental session.

The reference circle was presented in the top half of the display (Fig. 3A), the test arc in the bottom half. The horizontal position of both stimuli was varied randomly and independently on each trial within the range ±0.18° (100 pixels) from the centre of the screen. The arcs were presented vertically and to the left of their centres. The initial radius of the test arc was randomly determined within the range ±50% of the radius of the reference circle. Subjects adjusted the curvature of the test arcs by increasing or decreasing their radius until it matched that of the reference circle. They indicated their point of subjective equality (PSE) by pressing a key on a numeric keypad. Coarse (3 pixels steps=0.0054˚) or fine changes (1 pixel steps=0.0018˚) could be applied to adjust the radius, using different keys on a numeric keypad. Eleven different arc lengths, ranging between an angular extent of θ = 22.5˚ (16th of a circle) and 360˚ (full circle) were tested. Each of the 11 different arc lengths was tested 20 times in an experimental block. The stimulus design is illustrated in Figure 3A. Observers completed three blocks for each experiment and the results from the blocks were averaged.

Experiment 2 – Estimation of the centre of an arc's circle

Using the MOA, the observers' task was to estimate the centre of the underlying circle of the arc, termed here the 'centre-point' (Fig. 3B). Each arc was positioned at the centre of the screen with a vertical and horizontal positional jitter of (±0.18˚). The arcs were always presented on the left side (at 9 o-clock) of the centre of the screen. Observers positioned a white dot (2x2 pixels) where they estimated the centre-point. The white test dot was initially presented with a random horizontal offset of ±0.072˚ from the true centre-point. The dot was always positioned with zero vertical offset and observers only had to adjust the horizontal position of the dot (Figure 3B). In all of the following experiments, coarse (0.0054˚) or fine adjustments (0.0018˚) of the centre-point could be applied by pressing different keys on a numeric keypad. As in Experiment 1, 11 different arc lengths ranging from θ = 22.5˚ to 360˚ were tested. Each arc length was tested 20 times.

Experiment 3 – Aligning two circular arcs

Observers were presented with two opposing arcs of the same arc lengths, placed at 3 and 9 o-clock (Fig. 3C). The arc pair was positioned at the centre of the screen with a random vertical and horizontal offset of ±0.18˚. One arc (9 o-clock) remained fixed while observers adjusted the position of the other arc so that it appeared to fall on the circumference of the (invisible) circle given by the fixed arc. The second arc was initially positioned at a random location relative respect to its veridical position within ±0.072˚. In order to avoid overlap of the two opposing arcs only seven different arc lengths, ranging from θ = 22.5˚ to 135˚ were tested (Figure 3C). Each arc length was tested 20 times within an experimental block.

Experiment 4 – Interpolation of a circle

Subjects were presented with two opposing vertical arcs (3 and 9 o-'clock) of the same length, which were positioned on the circumference of the same circle. Again, seven different arc lengths, ranging from θ = 22.5˚ to 135˚ were tested. As in Experiments 2 and 3, the stimulus was presented with a random vertical and horizontal positional jitter within ±0.18˚ from the centre of the screen. Observers adjusted the vertical position of a white circular dot (2x2 pixels) to indicate the position of the mid-point of the virtual arc that was part of the circle (Fig. 3D). The dot was positioned midway between the two vertical arcs with a random vertical positional jitter within ±0.072˚, either close to the upper or lower gap. As with Experiment 3, each of the seven arc lengths was tested 20 times within an experimental block.

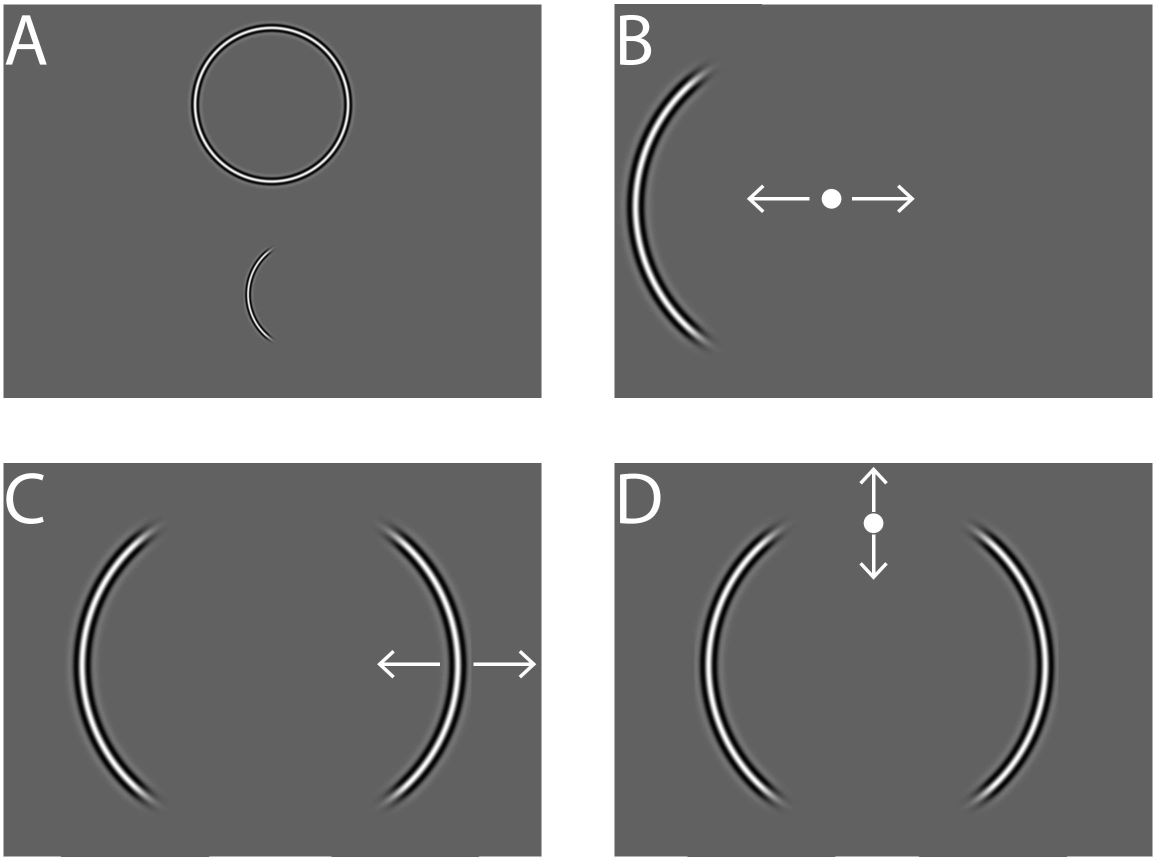

Figure 3 **(**A) Measuring the Arc-size illusion: the task was to adjust the curvature of a test arc with a specific fixed arc length (bottom) to match the curvature of a reference circle (top). (B) Estimation of the centre-point of an arc: subjects positioned a randomly located test dot to the perceived centre-point of the arc. (C) Aligning two circular arcs: subjects were asked to align two opposing arcs to form a circle. (D) Interpolation of a circle: subjects were presented with two opposite arcs of a circle and positioned a dot in the mid-point of the virtual arc that was part of the circle.

Results

The black circular data points in Figure 4 show the results from Experiment 1 (the Arc-size illusion) averaged across subjects. The graphs show the radius of the test arc, expressed as a proportion of the radius of the reference circle, at the PSE (point of subjective equality). If subjects judged the test curvature veridically, the resulting values would be 1. Test arcs judged to be flatter than that of the reference circle would result in smaller test arc radii at the PSE, resulting in values less than 1. Conversely, test arcs judged more curved than the reference would result in PSEs greater than 1. As the figure shows, nearly all values for the short test arc portion of the plots are less than 1 indicating that short arcs were perceived to be flatter than the reference circle. The bias however declines rapidly in magnitude up to an arc length about a sixth of a circle (60˚), at which point the bias disappears and judgments are near veridical.

A repeated measures ANOVA with size of reference circle (1˚, 2˚, 3˚) and arc length as factors revealed a significant main effect of arc length ($F_{10,30} = 26.774$; $p<.0001$), but no statistically significant difference for size ($F_{2,6} = 14.91$; $p>.05$). This demonstrates that the ASI is independent of pattern size.

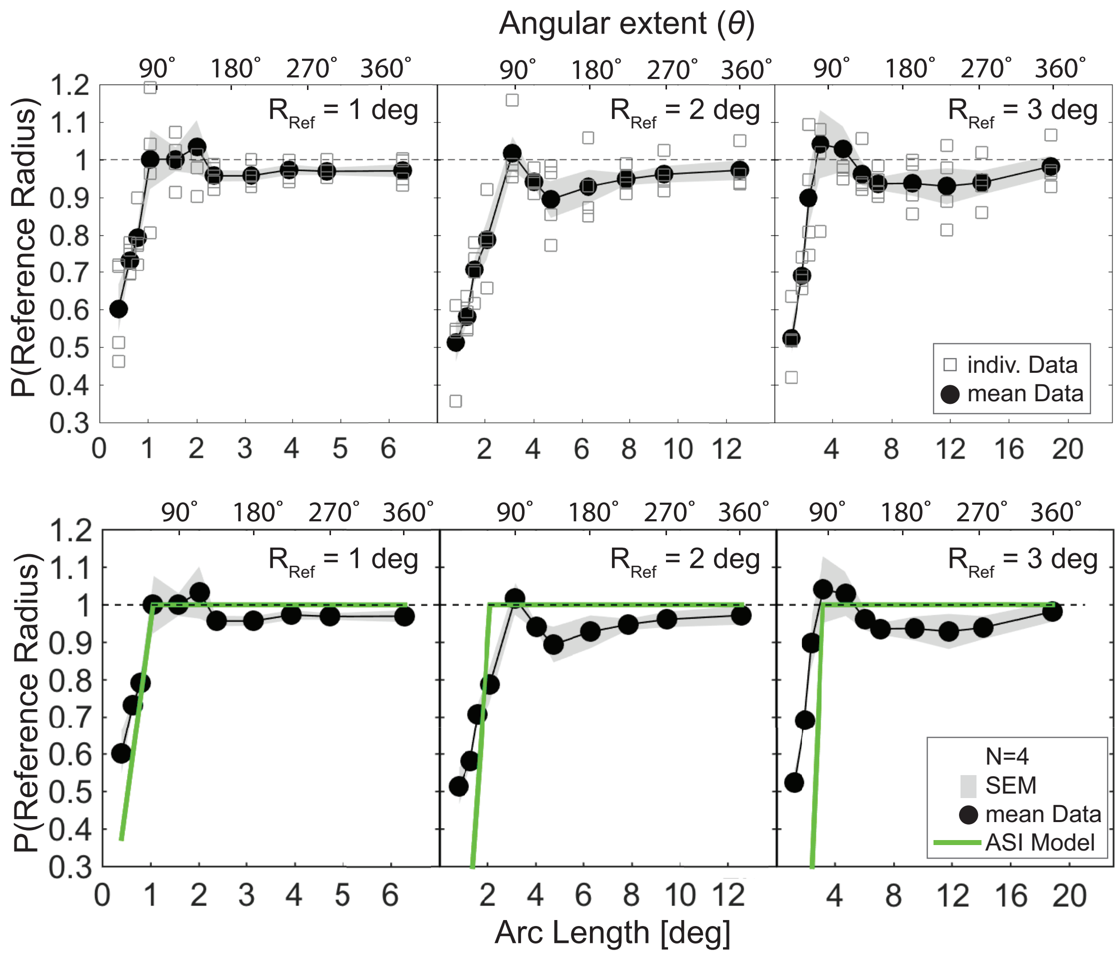

Figure 4. Arc size illusion (ASI) data. The graphs show the radius of the test arc, as a function of arc length, at which the curvature of the arc was perceived identical to that of a reference circle. The ordinate shows the test radius expressed as a proportion of the reference radius R_{ref} and the three graphs are for three different reference radii (left: $R_{ref} = 1^{\circ}$, middle: $R_{ref} = 2^{\circ}$, right: $R_{ref} = 3^{\circ}$). **Top row:** The grey squares in the graphs show individual data for four subjects averaged across blocks. Subjects completed three blocks for each Arc Length. The black circular points represent the mean data averaged across subjects. The grey-shaded regions represent ±standard error of the means. **Bottom row:** The black circular points are the mean data re-plotted from the top row and the solid green line the model (see text for details).

Results: Experiments 2-4

One can make the following predictions if the bias in curvature judgment revealed in the ASI translates to the other tasks. If the curvature of a short arc is perceived as flatter than that of a circle (the ASI result), one would expect an observer to judge the centre of an arc to be further away from the arc than the true distance (Experiment 2). By the same token, one would expect observers to position two arcs either side of the centre-point further apart to make a circle than the true distance (Experiment 3). Finally, one would expect observers to position a point between two arcs in order to make a circle further from the centre-point than the true distance (Experiment 4).

In order to compare the results of Experiments 2-4 with the ASI data, the data were transformed into equivalent perceived curvatures. The results of all experiments are shown in Figure 5 (Experiment 1, ASI, black; Experiment 2, judging centre of circle, red; Experiment 3, positioning a second circular arc to fall on circumference of circle given by reference arc, blue; Experiment 4, interpolating mid-point between two circular arcs, magenta). It is evident from Figure 5 that the bias seen for the ASI translates to the other conditions.

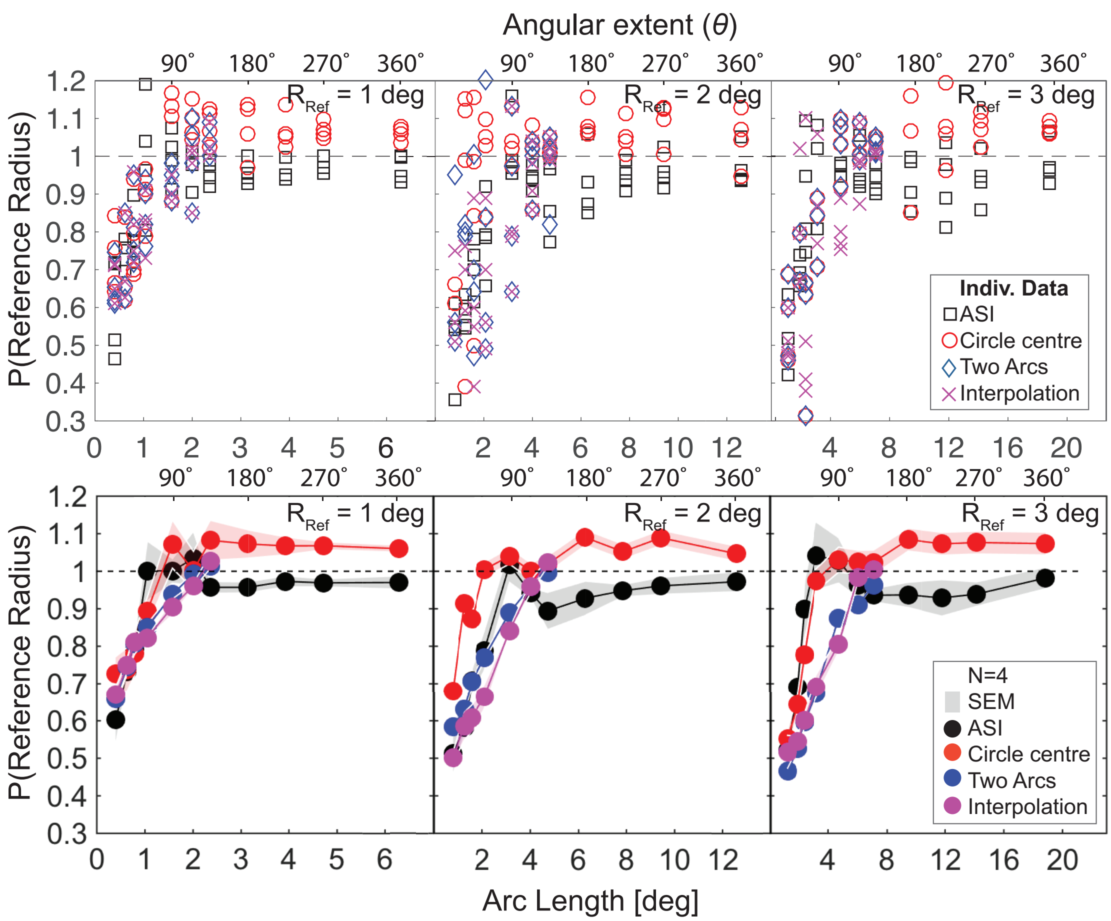

Figure 5. (A) shows the results for the Experiment 1 (black), Experiment 2 (red), Experiment 3 (blue) and Experiment 4 (magenta) for reference radii of 1˚ (leftmost), 2˚ (middle) and 3˚ (rightmost). **Top row:** The

graphs show the individual results (averaged across blocks) for four subjects for each experiment. **Bottom row:** Data are averaged across subjects. The shaded regions represent ±standard error of the means.

To test whether the results in the four experiments were different, a three factor within-subjects ANOVA was performed (Experiment (4) x Radii (3) x Arc length (7)). This analysis revealed a statistically significant interaction between experiment and radius ($F_{6,18}$=4.09, *p*=.009) as well as between radius and arc length ($F_{12,36}$=3.25, *p*=.003). Given the dramatic increase in perceived curvature with arc length for short arcs, the latter interaction is expected and is not important for this analysis. A simple main effects test between Experiments at each Radius only showed a significant effect of Experiment for the second radius (2˚) ($F_{3,15}$=4.56, *p*=.018). Subsequent post hoc tests (Bonferroni corrected T-test) revealed that a significant difference only occurred with Experiment 2 (centre-point judgment) and Experiment 4 (Interpolation of curvature) (t(15)=3.48, *p*=.003) and only for Arc lengths 2 (t(147)=3.09, *p*=.002) and 4 (t(147)=3.04, *p*=0.003). In summary, despite these significant differences between a few of the conditions this statistical analysis allows us to conclude that performance is very similar in all experiments.

ASI Model

One aim of the study was to develop a perceptual model that predicts the observed bias in the judgments of arc curvature. A number of geometrical features are potentially available for constructing a metric that encodes curvature. These include: 1. the chord (*CL*), defined as the line connecting the two endpoints of an arc; 2. the sagitta or sag (*S*), which refers to the perpendicular distance between the arc's midpoint and the chord; 3. the arc length; 4. the area enclosed by the chord and the arc; and 5. the central angle subtended by the test arc (θ). These features are illustrated in Figure 6. The successful metric needs to predict the relatively large underestimation of curvature for short arc lengths and the monotonic decrease in curvature misjudgment with increasing arc length up to 60˚ but not beyond. The sharp transition in behavior at around 60˚ suggest that there are two regimes, one producing bias the other not. Therefore our model only deals with the first, bias regime. Altering the radius of the arc while holding arc-length constant changes θ (arc length/r). We suppose that at the PSE the difference between the test θ and a 60˚ segment of the reference circle is minimized. In other words, when presented with an arc of a specific length, the observer adjusts the test arc radius in order to set θ to 60˚. This is illustrated in Figure 7A. The green solid line in each graph in Figure 5 shows the model prediction. The model involves no free parameters.

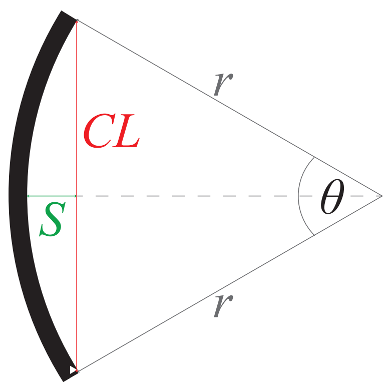

Figure 6. The Figure illustrates a circular arc of a specific arc length and some of the potential geometrical features and metrics available for modeling the ASI: S = sag (sagittal); CL = Chord length, r = radius and the θ = central angle .

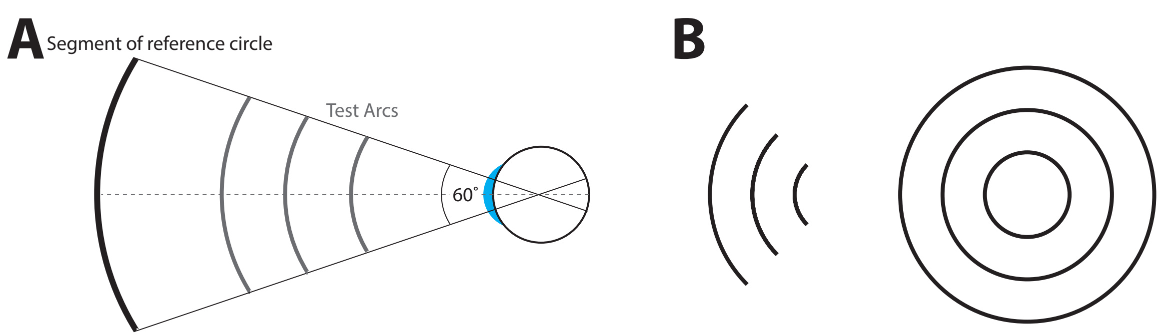

Figure 7. (A) Illustration of the ASI-Model. At the PSE the difference between the test θ and a 60˚ segment of the reference circle is minimized. The observer adjusts the test arc radius in order to equalize the test and reference θ. (B) demonstrates the scale invariant appearance of curvature. The curves on the left are equal central angle arcs taken from the circles on the right. They appear equally curved even though their curvatures are very different.

Given the similar performance between experiments, results for all four experiments were averaged. These averaged results are shown in Figure 8. The black data points show the average results and the shaded error bars represent 95% confidence interval. The green solid line shows the ASI model. The goodness of fit between the ASI Model and the data was evaluated by calculating the coefficient of determination R^2, which is provided in each graph of Figure 8. It is clearly evident that the ASI model gives a reasonable account for the data.

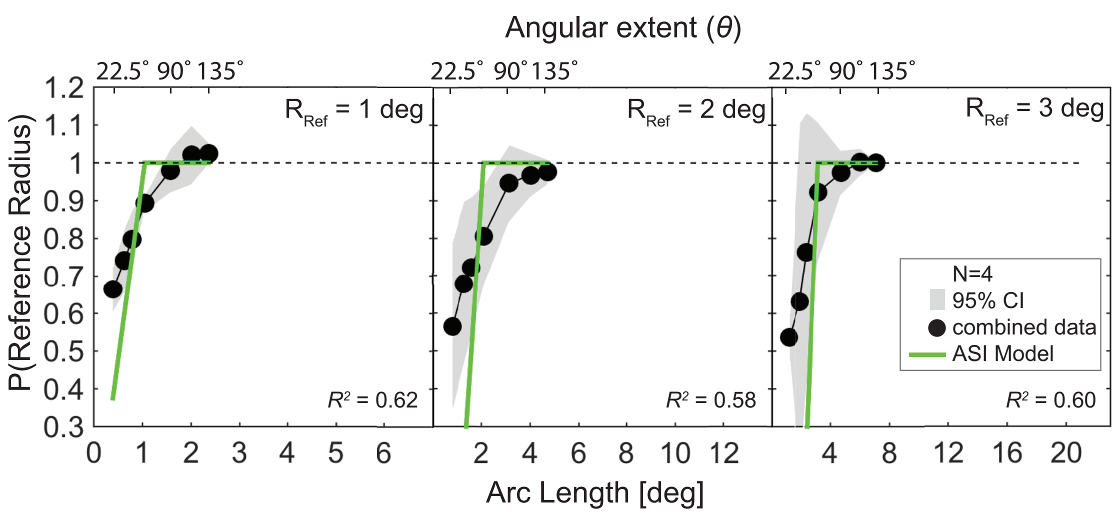

Figure 8. The black circular data points show the combined results for the first seven arc lengths averaged across Experiments 1-4. The grey-shaded error bars represent 95% CI. The green solid line in each graph shows the ASI Model prediction.

Discussion

Previous studies investigating the appearance of curvature reported opposing findings. Piaget and Vurpillot (1956) and Coren and Festinger (1967) measured the chord length and sag of circular arcs. Their results indicated an overestimation of the curvature of short arcs. In contrast, Virsu (1971b) used an experimental paradigm similar to the one employed here, whereby the apparent curvature of single arcs of varying length was compared with that of complete circles. However, Virsu (1971b) suggested that inferring the perceived curvature from judgments of linear features such as the sag and chord, as in Piaget and Vurpillot (1956) and Coren and Feistinger (1967), was an unreliable method of measuring the perception of curvature.

The experiments reported here produced similar results to Virsu's (1971a, b; Virsu & Weintraub, 1971). A comparison of Virsu's data with vertical arcs (1971b, his Table 1) and our results from Experiment 1 for a reference radius of 1˚ are illustrated in Figure 9, with arc lengths expressed as the angular extents of the circular arcs (central angle θ). Despite the fact that the radius of the reference arc used by Virsu (1971b) was larger (4.76˚ vs. 1˚), the overall pattern of results is remarkably similar. This underscores the size-invariant nature of curvature misjudgment found in our experiments. Further investigations by Virsu (1971b) showed that a similar pattern of curvature underestimation also occurs if the apparent continuations of arcs are measured. In Experiment 1 we present a much more accurate measurement of curvature judgment with better controlled stimuli and methods that were possible in these previous.

In addition, the results from our Experiments 2-4 add that the underestimation of curvature for short arcs and the subsequent decrease of curvature misjudgment is a general visual phenomenon, at least for the curvature judgment tasks tested here.

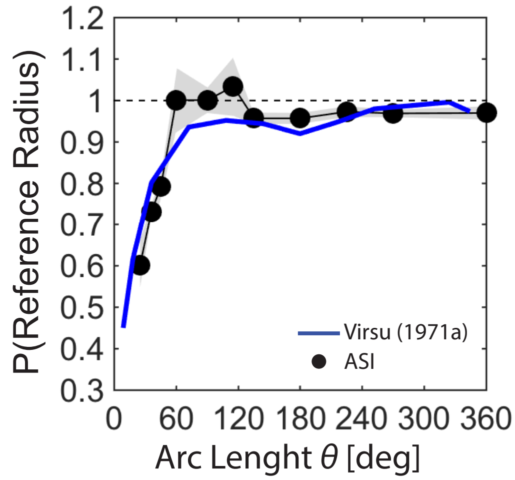

Figure 9. Comparison between Virsu (1971a) (continuous blue line) and the R_{ref} = 1˚ condition in Experiment 1 (filled circles). In order to compare the results with those of Virsu (1971a) the arc length is defined as the angular extent of the circular arcs (central angle *θ)*. Note, that the reference radius used by Virsu (1971a) was much larger (4.76˚).

What causes the misperception of curvature for short arcs? Various possible explanations for the underestimation of curvature have been put forward (Virsu, 1971a; 1971b; Virsu & Weintraub, 1971). Virsu (1979a) attributed the explanation to the tendency for rectilinear (straight line) eye movements. However, despite its potential for explaining some of his results, Virsu considered this explanation not very satisfactory. Another possibility is that the underestimation of curvature represents an initial stage of the "Gibson normalization effect", in which a curved line becomes perceptually straightened with prolonged inspection (Gibson, 1933). In other words neural adaptation might be the explanation. However, Virsu and Weintraub (1971) pointed out that the Gibson effect typically occurs with very long radii and large arcs and their results and the results presented here clearly demonstrate that the underestimation of curvature only occurs for short arcs. Furthermore, in our experiment subjects were allowed free eye movements. Hence, neural adaptation is an unlikely explanation.

Here we present an alternative explanation for the misperception of curvature: curvature constancy (scale invariance of curvature). A circular arc appears similarly curved irrespective of viewing distance, even though its curvature in the retinal image changes. This scale invariant property of curvature appearance is demonstrated in Figure 7B, where each of the circles on the right has a different radius and, therefore, different curvature. However, arcs from these circles with the same central angle θ (left) appear equally curved.

Several mechanisms have been suggested to explain sensitivity to curvature detection (deviation from straightness) and curvature discrimination (discrimination between two curves) experiments. For instance, Foster et al. (1993) found that the sag and the mean deviation (area enclosed by the arc divided by the chord length) best predicted discrimination performance. Kramer and Fahle (1996), on the other hand, measured detection thresholds for various stimuli including arcs, sinusoids, trapezoids and chevrons as a function of stimulus length. They suggested that at least for slight curvatures, detection might be realized by mechanisms detecting the differences in orientation between parts of the curve, rather than differences in sag. Wilson (1985) and Wilson and Richards (1989) suggested a similar mechanism for curvature discrimination, i.e. discrimination is mediated by mechanisms comparing orientation differences. Other studies have suggested that the aspect ratio (*CL*/*S*) could form the basis of curvature discrimination (Whitaker & McGraw, 1998). However, it is important to emphasize that curvature detection and curvature discrimination, both performance measures, are different to the task employed in this study, which measured appearance. In our experiments the arc length was kept constant and the subject had to adjust the curvature (or radius) to match the curvature of the test arc to that of the reference circle.

Importantly, we argue that the ASI and related biases in other tasks of curvature judgment are a consequence of the fact that curvature is perceived as constant with viewing distance, in other words is perceptually scale invariant. The importance of the scale invariant property of curvature has previously been demonstrated for curvature discrimination experiments (Foster, Simmons, & Cook; 1993, Whitaker & McGraw; 1998). Either of the aforementioned features could form part of a curvature metric that is scale invariant. Indeed, we are not tied to the idea that our observers computed θ when matching the test arc to the reference circle. Any scale-invariant metric of curvature could have sufficed: for example, a sag-to-chord ratio of 0.134 produces a θ of 60˚.

How exactly does this explain the ASI and related phenomena? Consider the situation in which one compares the curvatures of two short arcs of different length – remember the shorter of the two arcs is perceived as flatter. If the short arc were the same object as the long arc but viewed from further away, it would have a smaller retinal radius of curvature. It follows that if it were to have the same retinal radius of curvature it must be from a different object, one with a larger radius of curvature. Hence to make it the same object as the one with the longer retinal arc length one would need to decrease its radius of curvature accordingly, such that the central angle θ of the two curves were the same (see Figure 7). This is exactly what the observers did.

If this explanation is correct, then why does curvature constancy only operate with curves up to a sixth of a circle in length? One speculation could be that curves in the natural environment peak at angular extents of around a sixth of a circle or typically do not exceed these. To our knowledge, no such analysis of natural scenes has been carried out and so might usefully be a subject for future investigations.

Finally, we suggest that the curvature judgment strategy proposed in this paper is not only restricted to circular arcs, but might also be applicable to non-circular curves (parabolic, hyperbolic, elliptical etc.). With non-circular curves there is no single value of curvature and hence no single value of θ. However, other metrics, such as the chord-to-sag ratio, are applicable. Future research will be required to investigate this hypothesis.

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References

Achtman, R. L., Hess, R. F., & Wang, Y. (2003). Sensitivity for global shape detection. *Journal of Vision*, *3*(10), 616–624.

Arguin, M., & Saumier, D. (2000). Conjunction and linear non-separability effects in visual shape encoding. *Vision Research*, *40*(22), 3099–3115.

Brainard, D. H. (1997). The Psychophysics Toolbox. *Spatial Vision*, *10*(4), 433–436.

Coren, S., & Festinger, L. (1967). An alternative view of the "Gibson normalization effect." *Perception and Psychophysics*, *2*(12), 621–626.

Dakin, S. C., & Bex, P. J. (2002). Summation of concentric orientation structure: seeing the Glass or the window? *Vision Research*, *42*(16), 2013–2020.

Foster, D. H., Simmons, D. R., & Cook, M. J. (1993). The cue for contour-curvature discrimination. *Vision Research*, *33*(3), 329–341.

Gheorghiu, E., & Kingdom, F. A. A. (2007). The spatial feature underlying the shape-frequency and shape-amplitude after-effects. *Vision Research*, *47*(6), 834–844.

Gheorghiu, E., & Kingdom, F. A. A. (2008). Spatial properties of curvature-encoding mechanisms revealed through the shape-frequency and shape-amplitude after-effects. *Vision Research*, *48*(9), 1107–1124.

Gheorghiu, E., & Kingdom, F. A. A. (2009). Multiplication in curvature processing. *Journal of Vision*, *9*(2), 23 1–17.

Gibson, J. J. (1933). Adaptation, after-effect and contrast in the perception of curved lines. *Journal of Experimental Psychology*, *16*(1), 1-31

Hancock, S., & Peirce, J. W. (2008). Selective mechanisms for simple contours revealed by compound adaptation. *Journal of Vision*, *8*(7), 11–11. http://doi.org/10.1167/8.7.11

Kramer, D., & Fahle, M. (1996). A simple mechanism for detecting low curvatures. *Vision Research*, *36*(10), 1411–1419.

Loffler, G. (2008). Perception of contours and shapes: low and intermediate stage mechanisms. *Vision Research*, *48*(20), 2106–2127.

Motoyoshi, I., & Kingdom, F. A. A. (2010). The role of co-circularity of local elements in texture perception. *Journal of Vision*, *10*(1). 1-8

Pelli, D. G. (1997). The VideoToolbox software for visual psychophysics: transforming numbers into movies. *Spatial Vision*, *10*(4), 437–442.

Piaget, J., & Vurpillot, E. (1956). La surestimation de la courbure des arcs de cercle (Vol. 35, pp. 215–232). Delachaux et Niestlé.

Schmidtmann, G., Kennedy, G. J., Orbach, H. S., & Loffler, G. (2012). Non-linear global pooling in the discrimination of circular and non-circular shapes. *Vision Research*, *62*, 44–56.

Virsu, V. (1971a). Tendencies to eye movement, and misperception of curvature, direction, and length. *Perception and Psychophysics*, *9*(1), 65–72.

Virsu, V. (1971b). Underestimation of curvature and task dependence in visual perception of form. *Perception and Psychophysics*, *9*(3), 339–342.

Virsu, V., & Weintraub, D. (1971). Perceived curvature of arcs and dot patterns as a

function of curvature, arc length, and instructions. *Quarterly Journal of Experimental Psychology*, *23*(4), 373–380.
Watt, R. J. (1984). Further evidence concerning the analysis of curvature in human foveal vision. *Vision Research*, *24*(3), 251–253.
Watt, R. J., & Andrews, D. P. (1982). Contour curvature analysis: hyperacuities in the discrimination of detailed shape. *Vision Research*, *22*(4), 449–460.
Whitaker, D., & McGraw, P. V. (1998). Geometric representation of the mechanisms underlying human curvature detection. *Vision Research*, *38*(24), 3843–3848.
Wilkinson, F., Wilson, H. R., & Habak, C. (1998). Detection and recognition of radial frequency patterns. *Vision Research*, *38*(22), 3555–3568.
Wilson, H. (1985). Discrimination of contour curvature: data and theory. *Journal of the Optical Society of America A*, *2*(7), 1191–1199.
Wilson, H. R., & Richards, W. A. (1989). Mechanisms of contour curvature discrimination. *J Opt Soc Am A*, *6*(1), 106–115.
Wilson, H. R., & Wilkinson, F. (2015). From orientations to objects: Configural processing in the ventral stream. *Journal of Vision*, *15*(7), 4.
Wilson, H., Wilkinson, F., & Asaad, W. (1997). Concentric orientation summation in human form vision. *Vision Research*, *37*(17), 2325–2330.