How Can Mathematical Objects Be Real but Mind-Dependent?

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Abstract Taking mathematics as a language based on empirical experience, I argue for an account of mathematics in which its objects are abstracta that describe and communicate the structure of reality based on some of our ancestral interactions with their environment. I argue that mathematics as a language is mostly invented, and it is mind-dependent in a specific sense. However, the bases of mathematics will characterize it as a real, non-fictional science of structures.

1. Introduction

I can sum up the thesis under discussion here to the idea that mathematics can be thought of as a very abstract theoretical physics, this is an idea that Quine welcomes (Linnebo 2017, 94). Quine (1986, 402) claims that epistemologically, mathematical objects and that of theoretical physics are the same.

I take mathematical objects as non-spatiotemporal. If we agree that space and time are a (transcendental or physical) basis of our reality, it's not so far from tautological to say that all spatiotemporal objects are necessarily a part of the physical reality. However, I argue that mathematical objects despite being non-spatiotemporal are also in a sense real. Regarding reality in mathematics, one interesting view is that of Wittgenstein in the *Philosophical Investigations*, where he sees this problem as an illness that we should get rid of (Wittgenstein 2009 §254; Bangu 2012, sec. 4). In the *Tractatus*, Wittgenstein thought that mathematical propositions are not real propositions (1961 6.2), for one reason because we can see that their forms are superficial (Bangu 2012, introduction). But even with their superficiality, it has a descriptive power that for example a game of chess doesn't have. The most important reason of this superficiality is because Wittgenstein thought that mathematics is not independent of human language and practices (Bangu 2012, sec. 4). Further, regarding the axioms being used and certain arbitrary practices in mathematics (the usage of symbols, the heuristics of proof and so on), Resnik (1997, 124–26) argues that we cannot hope for a discipline that is deprived of presuppositions. As MacBride (2004, 311) puts it "[n]o discipline can investigate a given subject matter without taking a substantial body of auxiliary principles for granted". Adhering to this argument, we can never hope for a discipline that describes reality purely.

2. Background

In philosophy of mathematics, the characterizing difference between nominalism and platonism is that in the latter we hold that mathematical objects exist. Roughly based on what Linnebo calls the Frege's argument, the singular terms of true arithmetical statements refer to numbers, these statements can be true only if there exist such things as numbers (Linnebo 2018, sec. 2.1). I stand for the existence of these objects but in a different sense. They exist as the necessary result of the interaction of the human

ematical *abilities* … e basic properties of numbers (multitudinous of things) are perceptual attributes has been established in cognitive science (Jones 2018, 148).

Mathematical objects are mind-dependent as much as the word "table", but a table is not. So, this claim doesn't deny the existence of a pile of particles that we dub as "table"; it just happens to be the case that we have an ability and a practical inclination to discern so much of these particles as a table, but not a handful of electrons and other elementary particles within this pile (Frege 1953, 34). Mathematics is a descriptive apparatus, a language. It describes reality, and maybe we can call it real in this thin sense: mathematical intuition works as a thin transparent film or wrap that bundles reality into understandable equivalence classes that show the structure of reality.

This thin wrap abstracts reality as to be comprehensible by our cognitive faculties. It packs and forces the components of our perception to fit into their place in a commensurable—let's say "mathematical"—structure that our mind can generate. The general working of this process is similar to what Lewis (1986, 84–85) calls "the Way of Abstraction" that is "abstract entities are abstractions from concrete entities. They result from somehow subtracting specificity, so that an incomplete description of the original concrete entity would be a complete description of the abstraction".

Wittgenstein thinks that we invent mathematics (1976, 22), but I think this is not to say that mathematics is just another formal game like chess. The rules that mathematics is based on are derived from the empirical experiences, which is regulated by the regularities of the world and as humans progress through history, some of these regularities are "hardened" into rules. However, when the rules are derived, then mathematics starts the life of its own and its truth and expansion becomes independent of the experience and physical world. So much so that mathematics becomes the paradigm on which reality itself is measured, not the other way around (Wittgenstein 1978 VI-22).

Wittgenstein mocks platonists by saying that "chess only had to be *discovered*, it was always there!" (1974, 374). However, I say that although it was not necessary to have chess in order to describe the world, it was necessary to have for example the notion of a geometric line in order to describe the world and communicate our intentions. So, I argue that the notion of a line—although it's not out there in the concrete world—is naturally attainable by the workings of our mind.

Wittgenstein also talks about some behavioural agreement that is not simply an agreement of opinion: it is the agreement on the whole natural procedure that leads to an agreement about a matter at hand. It's a consensus "of action" (Bangu 2012, sec. 5). For example, if we are to run a simulation, in which we populate a physically similar world with people that are mentally similar to us, these people should end up having roughly the same mathematics as we do—given enough time. In the *Philosophical Investigations*, Wittgenstein says, that these agreements are not "agreement[s] . . . in opinions, but rather in form of life" (2009 §241).

It is important to note that what we observe as some mathematical object, is always an object in a structure. To put this object outside the structure and to try to get to the essence of it is to eliminate all the ground on which the object comes to be meaningful. Resnik (2018, 2) argues that structuralism can enable us to avoid the necessity of explaining the nature of single mathematical objects, as they are just some points in a structure. But still, we have to explain what are the bases of these structures and why their construction is so. This is exactly where the empirical bases of mathematics become useful.

Resnik (2018, 3) narrate an oral account from Benacerraf that we acquire mathematical knowledge by somewhat abstracting physical reality into properties. Resnik himself refers to these properties as "patterns". As Linnebo (2017, 15) points out the focus on the empirical experience for commencing the epistemology of mathematics is not a new thing, e.g., Meno's slave boy needed some empirical trigger to activate his mathematical intuition. Kant also admits the importance of empirical experiences but eliminates the sort of Platonic basis. With Kant (1998, B1), the source of such knowledge comes built-in in our cognitive faculties.

Resnik (2018, 6) argues that incompleteness is the most important part of structural objects. This property is for an object to have never-ending implications when it fits in its place in the structure. That is the object can truthfully branch out on and on if the rules of the structure are the governing body of this growth. "[M]athematics is like fiction", it describes its objects but this fiction is based on reality (Resnik 2018, 8).

Similar to Wittgenstein, Resnik talks about the practices of our ancestors in sorting, arranging, fitting things and constructing geometric templates as the precursor to mathematics. What is new with Resnik is the importance of communication in this regard (Resnik 1997, 226–28; MacBride 2004, 312).

In psychologism, the existence of entities depends on the existence of minds. For example, if nobody has thought about a very large number then that number doesn't exist (Balaguer 2016, sec. 4.1). But in the case of mathematics, I argue that the structure of integers is based on empirical experience and it can account for very large numbers. The further development of this structure is based on a set of axioms and rules of logic that appealed to our intuition. If the structure is sound, so is its parts, such as a very large number that nobody has ever thought of.

3. A Comparison with Mathematical Platonism

Mathematical platonism constitutes of three theses of existence, abstractness, and independence. That is: there are mathematical objects, they are abstract, and they are mind-independent. Further, it is important in platonism that we account for the reality of mathematical objects (Linnebo 2017, 11). Generally, what I am arguing for here can be thought of as a version of what Linnebo (2018, sec. 1.3) calls "object realism". That is the conjunction of the existence and abstractness criteria.

I assume that we agree that there are things in the world that we specify by names, that these things interact in ways that we can describe them, and that there are laws that these objects are based on and rules that they follow. Regardless, we cannot point out to these laws or rules—they are abstract—but this doesn't mean that they do not exist and that they are not real. (This is comparable to Quine's argument about the difference between Parthenon with the Parthenon-idea (1961, 2).) Think for example of the second law of thermodynamic—we can only observe systems that exemplify its idea. Similarly, it's not correct to talk about the existence and the reality of mathematical objects in this sense. Therefore, although the rules and the laws that mathematics is based on are not tangible, their examples are concrete or measurable and they are also mind-independent. These examples are the doors that enable us to perceive the underlying abstract structure. We should also note that objects of mathematics are ontologically more abstract and general than that of physics.

Finally, what we call mathematical abstracta are defined in a language, thus bound by that language. They are attained by a method of thinking that is dependent on human cognitive abilities, and the way they evolve and presented is based on human practices. Therefore, the language in which we describe mathematics is mind-dependent. So, I conclude that without intelligent beings, truths of mathematics hold only vacuously, because they are not defined.

4. Real but Mind-Dependent

Mathematics starts with the discovery of the way our mind works regarding some of its interactions with the environment, such as comparing, measuring, counting, sorting, and arranging. It proceeds by coming up with a way to represent these interactions abstractly and to communicate them to others. Then we propose (invent) a set of intuitive axioms, and then we unfold (discover) the rest of mathematics. But the way we do mathematics and how it looks and sounds is invented. And without this invention mathematics would've remained very primitive.

The aura under which mathematical abstracta become real is society. Here it's helpful to think of reality as a paradigm for the communicability of mental activities. It's very important to note that what I call the invention of mathematics is like coming up with a description to communicate certain regularities and patterns that we observe. These regularities are discovered or recognized; but the means by which we are able to communicate and work with them abstractly, is invented, and it is the language of mathematics. So, a group of people who share the same basic mental faculties will reach similar conclusions about mathematical objects. Their agreement and their ability to convey one another solidify these objects as *real* objects as opposed to the *fictional* ones.

Our argument of the reality of mathematical objects boils down to the way we describe reality in terms of our cognitive abilities that is bounded by its psychology and neurobiology. Similarly, the basic sphere of our reality is our habitat on earth that is governed by the mind-independent laws of nature. Now, (by the indispensability argument) our most accurate description of the world is the scientific description which is inseparable from mathematics (Resnik 1995, 166).

5. Conclusion

The seeds of mathematical abstracta exist as physical codes, and human beings have the ability to decode, or abstract them from the concrete source of information they come from. After getting to our mathematical abstracta then again, we encode them into something concrete, as for example letters on a paper, to be able to work with them and to communicate them.

Further, I think that our cognitive abilities give the possibility for these objects to be a path going to a point in the horizon that we can asymptotically approach in our intellectual endeavours. This idea is very much the same as what Resnik (2018, 8) coins as the incompleteness of abstract objects. In my view, the incompleteness is the generative power of mathematical abstracta to become further refined as our mathematics is developed further.

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