Macroeconomic fundamentals, jump dynamics and expected volatility*

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Abstract

In this paper, we develop a new volatility model capturing the effects of macroeconomic variables and jump dynamics on the stock volatility. The proposed GARCH-Jump-MIDAS model is applied to the S&P 500 index. Our in-sample results indicate that macroeconomic activities have important impacts on aggregate market volatility. Out-of-sample evidence suggests that our model with macroeconomic variables significantly outperform a wide range of competitors including the original GARCH(1,1), GARCH-MIDAS and GJR-A-MIDAS models. The volatility timing results also show that the information from jumps and macroeconomic activity is helpful for improving the portfolio performance.

Keywords: Macroeconomic activity; Jump; Volatility; Portfolio; Loss function

JEL Classifications: C58; G17

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1 Introduction

Modeling and forecasting volatility is of great interest by academics and market participants. The reason is that volatility plays a key role in many areas, such as risk management, option pricing and portfolio allocation. Both macroeconomic reason and jump are considered as the important sources of high volatility in stock market. However, these two effects may exist at the same time. For example, the global financial crisis in 2008-2009 introduced great macroeconomic uncertainty and led to high volatility of stock returns. Prominent jumps were also observed during that period. In this paper, we contribute to the literature by differentiating these two effects. Our empirical results indicate that disentangling the roles of macroeconomic and jump effects are helpful for improving volatility forecasting accuracy.

We develop a GARCH-type model that accounts for both the macroeconomic effect and the jump effect. Our model is actually an extension of the GARCH-MIDAS model proposed by Engle, Ghysels, and Sohn (2013). The original GARCH-MIDAS model decomposes volatility into two components: a short-term component following the GARCH process and a long-term component following a mixed data sampling (MIDAS) process[1]. The outstanding advantage of the model is that it uses a MIDAS specification to capture the influences of low-frequency macroeconomic variables on high-frequency volatility. We modify the short-term GARCH process using the mixed GARCH-Jump model with the autoregressive jump intensity of Maheu and McCurdy (2004). In this way, our new model, called the GARCH-Jump-MIDAS model, accommodates the roles of macroeconomic variables and jumps.

We consider the level and volatility of industrial production (IP) and producer price index (PPI) as the potential explanatory variables for long-term component of stock volatility. Using the developed GARCH-Jump-MIDAS model, the predictive content of the level and uncertainty variables and the jump for the volatility of the S&P 500 index is investigated. Our in-sample results suggest the existence of significant jump dynamics in the aggregate market volatility. The contribution of return jumps to the total stock return

[1] For the detailed specification about the MIDAS process, one can refer to the pioneering work by Ghysels, Santa-Clara, and Valkanov (2004, 2006).

variance is roughly 10%. We compute the jump probability around the days for which large innovations to returns are associated with significant news events. The ex-post probabilities of a jump for some occasional event such as Black Monday (October 19, 1987), Black Friday (October 13, 1989), and the Monday after the 9/11 attacks (September 17, 2001) are rather close to 1.

During the full sample period from 1928 to 2018, we find the significant effects of macroeconomic variables of interest on market volatility. We analyze the out-of-sample performance based on the rolling window scheme. GARCH-Jump-MIDAS models with macroeconomic levels and uncertainties are used to generate volatility forecasts during the period from 1978 through 2018. For comparison, we also consider several competing models: the simple GARCH (Bollerslev, 1986), original GARCH-MIDAS with and without macroeconomic variables (Engle, Ghysels, and Sohn, 2013) and the GJR-A-MIDAS model (Amendola, Candila, and Gallo, 2019). The forecasting performance is evaluated under five loss criteria. We use the popular model confidence set (MCS) (Hansen, Lunde, and Nason, 2011) to test whether the loss functions of these volatility models are significantly different. Our evidence indicates that during the whole out-of-sample period, accounting roles of jumps and economic activity can significantly improve volatility forecasting accuracy.

We further investigate the portfolio implications of marcoeconomic activity and jumps, that is, the economic value of volatility predictability. For this purpose, the out-of-sample performance of volatility models is also assessed under an environment of portfolio exercise. In this framework, an investor with mean-variance preference is assumed to allocate her wealth between the stock index and the risk-free Treasury bill. Because we use the same return forecasts of the historical mean, the portfolio performance is uniquely determined by the accuracy of the volatility forecasts. The evaluation results indicate that the portfolio formed by the GARCH-Jump-MIDAS model has the greatest certainty equivalent return (CER) of all of the volatility models. Accounting for the roles of jumps and economic activities in volatility models can improve portfolio performance. This finding is robust to changes in the risk aversion coefficient.

This paper contributes to the literature on the sources of stock volatility. A strand of the literature shows that macroeconomic uncertainty is the source of stock market volatility (Andrei, Carlin, and Hasler, 2015; Greenwald, Lettau, and Ludvigson, 2014; Engle and Rangel, 2008; Engle, Ghysels, and Sohn, 2013; Bansal and Yaron, 2004; Zhou and Zhu, 2014). Another strand of literature explains volatility from the perspective of market microstructure. These studies document the essential effects of jumps on aggregate market volatility (Christoffersen, Jacobs, and Ornthanalai, 2012; Andersen, Bollerslev, and Diebold, 2007; Huang and Tauchen, 2005; Santa-Clara and Yan, 2010; Eraker, 2004; Maheu and McCurdy, 2004). We reconcile these findings by developing a new model accounting for both macroeconomic and jump effects.

This paper is related to quite a large number of papers on forecasting stock market volatility. We contribute to the literature by developing a GARCH-type model that uses macroeconomic and jump information. Notably, Paye (2012) and Christoffersen, Jacobs, and Ornthanalai (2012) use predictive regressions for monthly realized volatility. They find that although many economic variables can predict aggregate market volatility in sample, they lose predictive ability out of sample. By contrast, we use a modified GARCH-type specification and show that macroeconomic variables can predict market volatility out of sample. This predictability is significant from statistical and economic perspectives.

The volatility timing strategy can be seen in many studies (e.g., Fleming, Kirby, and Ostdiek, 2001, 2003; Chou and Liu, 2010; Moreira and Muir, 2017; Bollerslev, Hood, Huss, and Pedersen, 2018). These studies only use the past information of stock volatility for investment decisions, whereas our paper takes economic uncertainty and jumps into account. We find that, based on the appropriate mode specification, the incorporation of macroeconomic effects and jumps in the GARCH model can improve portfolio performance.

The remainder of the paper is organized as follows: Section 2 describes the specification of our GARCH-Jump-MIDAS model. Section 3 provides the model estimation and inference methodology. Section 4 briefly shows the specifications of competing models. The in-sample and out-of-sample empirical results are reported in Section 5. The last section concludes the paper.

2 Model specification

Let $R_{i,t}$ be the logarithmic return on day i in month t, which is modeled as

$$R_{i,t} = \mu + \sqrt{g_{i,t}\tau_t}\epsilon_{i,t}^{(1)} + \epsilon_{i,t}^{(2)}, \quad \text{for} \quad i = 1, \cdots, M_t \quad \text{and} \quad t = 1, \cdots, T, \tag{1}$$

where $\epsilon_{i,t}^{(1)}|\mathcal{I}_{i-1,t} \sim \mathcal{N}(0,1)$ with $\mathcal{I}_{i-1,t}$ is the information set available up to time $i-1$ of period t. M_t and T denote the number of observations in month t and the number of months in total, respectively. $\epsilon_{i,t}^{(2)}$ is the jump innovation with a conditional mean of zero and is assumed to be independent of $\epsilon_{i,t}^{(1)}$, which is defined as

$$\epsilon_{i,t}^{(2)} = J_{i,t} - E(J_{i,t}|\mathcal{I}_{i-1,t}) = \sum_{j=1}^{N_{i,t}} Y_{i,t}^{(j)} - \theta\lambda_{i,t}, \tag{2}$$

where $Y_{i,t}^{(j)}$ is the jump size with the normal independent distribution, $Y_{i,t}^{(j)} \sim \mathcal{NID}(\theta, \delta^2)$. $N_{i,t}$ follows the Poisson distribution that captures the likelihood of jumps occurring over the interval from $(i-1,t)$ to (i,t), which is expressed as

$$P(N_{i,t} = j|\mathcal{I}_{i-1,t}) = \frac{\exp(-\lambda_{i,t})\lambda_{i,t}^j}{j!}, \quad j = 0, 1, 2, \cdots, \tag{3}$$

where $\lambda_{i,t}$ refers to the conditional jump intensity, which is allowed to be time-varying. Just as in Maheu and McCurdy (2004), $\lambda_{i,t}$ is assumed to follow a simple AR(1) process:

$$\lambda_{i,t} = \rho_0 + \rho_1\lambda_{i-1,t} + \sigma z_{i-1,t}, \tag{4}$$

where $z_{i,t}$ is the jump intensity residual, which is given by

$$z_{i,t} = E(N_{i,t}|\mathcal{I}_{i,t}) - \lambda_{i,t} \tag{5}$$

Obviously, $z_{i,t}$ is the conditional martingale difference sequence according to its definition; that is, $E(z_{i,t}|\mathcal{I}_{i-1,t}) = 0$. We thus re-write equation (4) as

$$\lambda_{i,t} = \rho_0 + (\rho_1 - \sigma)\lambda_{i-1,t} + \sigma E(N_{i,t}|\mathcal{I}_{i,t}), \tag{6}$$

To ensure the stationarity in model (4), we impose a restriction $|\rho_1| < 1$. Further, some sufficient conditions for $\lambda_{i,t} > 0$ are required, i.e., $\rho_0 > 0$, $\rho_1 \geq \sigma$ and $\sigma \geq 0$. By tedious but straightforward algebra, we have $E(J_{i,t}|\mathcal{I}_{i-1,t}) = \theta\lambda_{i,t}$. From the definition in equation

(2), we therefore have the conditional mean and variance of $\epsilon_{i,t}^{(2)}$, which are equal to 0 and $(\theta^2+\delta^2)\lambda_{i,t}$, respectively. Note that the jump intensity becomes constant when $\rho_1=\sigma=0$, which is the special case of our specification.

Following the framework in Engle, Ghysels, and Sohn (2013), we assume that the long-run component, τ_t, is driven as

$$\tau_t = \exp[m + \theta_0 \sum_{i=1}^{K} \varphi_i(\omega_0) BV_{t-i} + \theta_1 \sum_{i=1}^{K} \varphi_i(\omega_1) X_{t-i}], \tag{7}$$

where BV_t is the bipower variation proposed by Barndorff-Nielsen and Shephard (2006) in which the jump component is excluded from the realized volatility used in Engle, Ghysels, and Sohn (2013).

$$BV_t = \frac{\pi}{2} \sum_{i=2}^{M_t} |R_{i-1,t}||R_{i,t}|. \tag{8}$$

X_t represents the macroeconomic factors of interest, such as industrial production and inflation. The one-parameter Beta polynomial is chosen as the weighting scheme because it is flexible and common,

$$\varphi_i(\omega_d) = \frac{[1-i/(K+1)]^{\omega_d-1}}{\sum_{j=1}^{K}[1-j/(K+1)]^{\omega_d-1}}, \quad d=0,1. \tag{9}$$

The weights $\varphi_i(\omega_d)$ sum up to one such that the parameters $\{\theta_0,\theta_1,\omega_0,\omega_1\}$ are identified. Taking exp guarantees that the long-run variance is always positive.

We assume that the short-run component $g_{i,t}$ has the GARCH(1,1) form as Engle and Rangel (2008) and Engle, Ghysels, and Sohn (2013):

$$g_{i,t} = (1-\alpha-\beta) + \alpha \frac{(R_{i-1,t}-\mu)^2 - (\theta^2+\delta^2)\lambda_{i-1,t}}{\tau_t} + \beta g_{i-1,t}. \tag{10}$$

Note that $(\theta^2+\delta^2)\lambda_{i,t}$ is the adjusted term, so the conditional expectation of the short-run component, $E(g_{i,t}|\mathcal{I}_{i-1,t})$, is guaranteed to be its unconditional expectation, $E(g_{i,t}|\mathcal{I}_{i-1,t})=1$. We refer to the mixed GARCH-MIDAS model with jump dynamics as the GARCH-Jump-MIDAS model.

3 Estimation and inference

We use the conventional quasi-maximum likelihood (QML) method to estimate the parameters because it is popular and easy to carry out. Given the number of jumps that occur, $N_{i,t}=j$, we have the conditional density of $R_{i,t}$ as

$$f(R_{i,t}|N_{i,t}=j,\mathcal{I}_{i-1,t};\Theta)=\frac{1}{\sqrt{2\pi(g_{i,t}\tau_t+j\delta^2)}}\exp[-\frac{(R_{i,t}-\mu+\theta\lambda_{i,t}-j\theta)^2}{2(g_{i,t}\tau_t+j\delta^2)}], \tag{11}$$

where the parameter space $\Theta=[\mu,\alpha,\beta,m,\theta_0,\omega_0,\theta_1,\omega_1,\theta,\delta,\rho_0,\rho_1,\sigma]'$. To obtain the conditional density of $R_{i,t}$ exclusively given the information $\mathcal{I}_{i-1,t}$, we utilize the law of total probability to integrate out the number of jumps:

$$f(R_{i,t}|\mathcal{I}_{i-1,t};\Theta)=\sum_{j=0}^{jMax}f(R_{i,t}|N_{i,t}=j,\mathcal{I}_{i-1,t};\Theta)P(N_{i,t}=j|\mathcal{I}_{i-1,t};\Theta), \tag{12}$$

where $jMax\to\infty$. However, $jMax$ is pre-determined when the estimation is less sensitive to the choice itself in practice. To make the estimation feasible in equation (4), we need the filter $P(N_{i,t}|\mathcal{I}_{i,t};\Theta)$[2], which can be derived by the Bayes's theorem,

$$P(N_{i,t}=j|\mathcal{I}_{i,t};\Theta)=\frac{f(R_{i,t}|N_{i,t}=j,\mathcal{I}_{i-1,t};\Theta)P(N_{i,t}=j|\mathcal{I}_{i-1,t};\Theta)}{f(R_{i,t}|\mathcal{I}_{i-1,t};\Theta)} \tag{13}$$

Finally, the log-likelihood function can be expressed as

$$\ell(\Theta)=\sum_{t=1}^{T}\sum_{i=1}^{M_t}\log f(R_{i,t}|\mathcal{I}_{i-1,t};\Theta), \tag{14}$$

and can be maximized using the optimization toolbox in MATLAB to yield the estimates of $\hat{\Theta}$.

Next, we sketch how to obtain the standard errors of the estimates in our proposed model. According to the substantial progress on the asymptotic properties for the quasi maximum likelihood estimator (QMLE) (see, e.g., Bollerslev and Wooldridge, 1992; Wooldridge, 1994; Straumann and Mikosch, 2006, among others), it is reasonable to conjecture that the estimators of the GARCH-Jump-MIDAS model have an asymptotic normal distribution under some standard regularity conditions,

$$\sqrt{T}(\hat{\Theta}-\Theta_0)\xrightarrow{d}\mathcal{N}(0,J^{-1}IJ^{-1}), \tag{15}$$

where $\hat{\Theta}$ is the estimates of Θ and Θ_0 denotes the true values. J and I are the expected Hessian and the covariance of the scores of the log-likelihood function (14), respectively. Both

[2] For convenience, we make $P(N_{i,t}|\mathcal{I}_{i,t};\Theta)=P(N_{i,t}|\mathcal{I}_{i,t})$, $f(R_{i,t}|\mathcal{I}_{i-1,t};\Theta)=f(R_{i,t}|\mathcal{I}_{i-1,t})$ and so on.

are numerically calculated as

$$\hat{J}_{i,j} \approx \frac{1}{T}\frac{\ell(\hat{\Theta}+e_i s_i+e_j s_j)-\ell(\hat{\Theta}+e_i s_i)-\ell(\hat{\Theta}+e_j s_j)+\ell(\hat{\Theta})}{s_i s_j}, \tag{16}$$

and

$$\hat{I} = \frac{1}{T}\frac{\partial\ell(\hat{\Theta})}{\partial\Theta}'\frac{\partial\ell(\hat{\Theta})}{\partial\Theta}, \tag{17}$$

where

$$\frac{\partial\ell(\hat{\Theta})}{\partial\Theta} \approx \frac{\ell(\hat{\Theta}+e_i s_i)-\ell(\hat{\Theta})}{s_i} \tag{18}$$

s_i is a scalar step size and e_i denotes a vector of zeros except for element i. See Flannery, Press, Teukolsky, and Vetterling (1992) for more details on the numerical derivative. We therefore have the standard errors for the estimators directly as

$$s.e.(\hat{\Theta}) = \sqrt{diag(\hat{J}^{-1}\hat{I}\hat{J}^{-1}/T)} \tag{19}$$

where the symbol *diag* denotes the diagonal elements of the matrix.

4 Competing models

For comparison, we consider three alternative volatility model specifications. The first is the traditional GARCH(1,1) of Bollerslev (1986), which can be taken as the special case of our GARCH-Jump-MIDAS model after removing the long-term component and jump component.

$$\begin{aligned} R_{i,t} &= \mu + \sqrt{g_{i,t}}\epsilon_{i,t}^{(1)}, \\ g_{i,t} &= m + \alpha(R_{i-1,t}-\mu)^2 + \beta g_{i-1,t}. \end{aligned} \tag{20}$$

As usual, we impose $\alpha + \beta < 1$ to make the conditional variance itself mean-reverting and $m, \alpha, \beta > 0$ to the positive conditional variance.

The second specification that we use is the GARCH-MIDAS model proposed by Engle, Ghysels, and Sohn (2013), which is extended in this paper. Specifically, our GARCH-Jump-MIDAS model becomes the GARCH-MIDAS model if we remove the jump component.

$$\begin{aligned} R_{i,t} &= \mu + \sqrt{g_{i,t}\tau_t}\epsilon_{i,t}^{(1)}, \\ g_{i,t} &= (1-\alpha-\beta) + \alpha\frac{(R_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t}. \end{aligned} \tag{21}$$

The long-run component τ_t has the same specification as in equation (7), and is thus omitted.

The last specification is the recent GJR-A-MIDAS model introduced by Amendola, Candila, and Gallo (2019), which allows the asymmetric impact on both short-term component and long-term component.

$$R_{i,t} = \mu + \sqrt{g_{i,t}\tau_t}\epsilon_{i,t}^{(1)},$$
$$g_{i,t} = (1-\alpha-\beta-\gamma/2) + (\alpha + \gamma 1(R_{i,t}-\mu<0))\frac{(R_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t},$$
$$\tau_t = \exp[m + \theta_0 \sum_{i=1}^{K}\varphi_i(\omega_0)BV_{t-i} + \theta_1^{+}\sum_{i=1}^{K}\varphi_i(\omega_1^{+})X_{t-i}^{+} + \theta_1^{-}\sum_{i=1}^{K}\varphi_i(\omega_1^{-})X_{t-i}^{-}] \tag{22}$$

where $1(\cdot)$ is an indicator function that takes value one if the argument holds, and zero otherwise. As usual, $X_t^{+} \equiv X_t \cdot 1(X_t \geq 0)$ and $X_t^{-} \equiv X_t \cdot 1(X_t < 0)$. Note that X_t is just set to be macroeconomic variable IP level or PPI level. The uncertainty variable where IP variance or PPI variance is as the proxy is excluded in GJR-A-MIDAS model since the variance is always positive.

5 Empirical analysis

5.1 The Data

We use the daily S&P 500 index data over the period from January 2, 1928 through December 31, 2018, resulting in 22, 858 observations. Our data is collected from Bloomberg. The starting data we choose is the earliest day available at the Bloomberg. The monthly data for industrial production (IP) and producer price index (PPI) are obtained from the Federal Reserve Bank of Saint Louis[3]. Figure 1 plots the stock prices, stock returns and the growth rate of IP and PPI measured by the log changes. Table 1 reports the summary statistics of stock returns and the growth rate of two economic variables. We can see that all three series are stationary according to the augmented Dickey and Fuller test. The Jarque-Bera statistics indicate the fat-tailed distribution of stock returns and changes in log IP and log PPI. The ARCH test suggests the significant time-variation property in the volatility of stock returns and two macroeconomic variables.

Table 1 and Figure 1 about here

[3] https://fred.stlouisfed.org.

Following Engle, Ghysels, and Sohn (2013), we estimate the macroeconomic volatility using a simple regression:

$$X_t = \sum_{i=1}^{12} \beta_{x,i} X_{t-i} + \sum_{i=1}^{12} \alpha_{x,i} D_{t-i} + \epsilon_{x,t}, \quad t = 1, \cdots, T, \tag{23}$$

where $X_t \in \{IP, PPI\}$, and D_t refers to the 12 monthly dummy variables. The squared residuals $\epsilon_{x,t}^2$ is taken as the proxy of the volatility for the macroeconomic variable X_t. As the summary, we consider four macroeconomic variables including the levels and uncertainties of industry production and producer price index.

5.2 In-sample performance

This subsection discusses the in-sample performance for our GARCH-Jump-MIDAS model. GARCH-MIDAS models use a MIDAS regression which takes the lagged macroeconomic variables as the explanatory variables for long-term component of volatility. Therefore, it is of key importance to determine the optimal lags in the MIDAS regression. For this purpose, we allow the lag lengths to change from 12 to 120 with a fixed step of 12. The likelihood values plotted in Figure 2 indicate that the best in-sample fitting performance is related to the lag length of 48 months (i.e., 4 years). Therefore, we choose $K = 48$ in empirical analysis.

Figure 2 about here

Figure 3 plots the autocorrelations of the squared series of original returns and standardized returns. We find that the squared returns display significant and strong autocorrelations until the 50 lags. As the sharp contrast, the autocorrelations of the squared standard residuals from the four GARCH-Jump-MIDAS models with macroeconomic variables decay quickly with the lag order increases and become insignificant for the lags greater than 1. This evidence indicates that our GARCH-Jump-MIDAS can well accommodate the ARCH effect in stock index returns.

Figure 3 about here

Figure 4 and Figure 5 illustrate the conditional volatility and jump dynamics implied by our GARCH-Jump-MIDAS with the levels of IP and PPI, respectively. The corresponding volatilities and jumps of the model with economic uncertainty variables are plotted in Figure 6 and Figure 7. We find that three largest volatilities occurred because of the great recession during 1929-1933, the stock market crash in 1987 and the financial crisis during mid-2008-2009. By observing the jump densities, we can conclude that the jumps play the important role in forming the extreme volatilities during these three periods. The long-term components show that the macroeconomic effects can also partly explain the volatilities during the 1929-1933 recession and the recent financial crisis, whereas its explanatory power for the volatility during the 1987 market crash is rather weak.

Figure 4-Figure 7 about here

Table 2 reports the parameter estimation results of the GARCH-Jump-MIDAS, as well as those of the competing models. We find that the parameter estimates $\alpha + \beta$ is greater than 0.93 in all specifications, suggesting the stylized fact of strong volatility persistence. For the 11 GARCH-MIDAS specifications, we can see that the parameter θ_0 is significant highlighting the important role of long-term volatility in affecting future short-term volatility. The estimates of θ_1 in the volatility models of interest differ depending on the macroeconomic variables included. We find that θ_1 for IP level is significantly negative, regardless of which specification is considered. That is, higher industrial production leads to lower long-term volatility in the future. The estimate of θ_1 for PPI is significantly negative, indicating that higher inflation always causes higher stock volatility in the long-term. As the stagflation is a typical characteristic of economic recession, our results are consistent with the consensus that stock volatility increases during the recession period (e.g., Schwert, 1989; Hamilton and Lin, 1996). The θ_1's for IP and PPI volatility are always significantly positive, implying that higher economic uncertainty results in higher stock market volatility. This result is consistent with Engle, Ghysels, and Sohn (2013) who use data covering the sample period from 1890 through 2010.

Table 2 about here

The jump size mean θ in two GARCH-Jump-MIDAS models is significantly negative and is close to -6e-3. This implies a negative conditional correlation between jump innovations and squared return innovations. Maheu and McCurdy (2004) document that the insignificant θ does not implies that the jumps have no impacts on the return distribution. They clearly show that the jump dynamics affect conditional variance, conditional kurtosis and tail distribution even if $\theta = 0$. Based on the parameter estimates, the contributions of return jumps to the total stock return variance are about 11%[4]. These values are close to the numbers reported in Christoffersen, Jacobs, and Ornthanalai (2012) (12%∼15%) and Andersen, Bollerslev, and Diebold (2007) (14.6%) although they use different model specifications.

The estimate of θ_1 is significantly positive which indicates the time variation in the arrival of jump events. Furthermore, the coefficient is 0.98, close to unity, implying the strong persistence for the arrival of jumps (jump clustering). The evidence from the parameter σ which measures the effect of the most recent intensity residual is not significant across different GARCH-Jump-MIDAS model specifications. The unconditional jump density, computed as $E[\epsilon_{2,t}^2] = (\theta^2 + \delta^2)\rho_0/(1 - \rho_1)$, is about 1.63e-4.

We compute the probability of jumps around the days that some extreme events occurred to see the association between the jump and significant news innovations. These events are the Great Crash (October 28, 1929), the Oil Crisis (October 9, 1979), the Black Monday (October 19, 1987), the Black Friday (October 13, 1989), 9/11 attack (September 17, 2001), and the Lehman Brothers failure (September 15, 2008). The results are shown in Table 3. On the days when these events broke out, we observe large negative returns between -4.828% and -22.90%. The probability of jumps related to Lehman Brothers failure is as high as 0.653, and the probability associated with the other events is rather close to 1.

Table 3 about here

5.3 Out-of-sample performance

We use the technique of the rolling estimation window to generate 1-day-ahead forecasts of volatility. In detail, the whole sample period is divided into two subsample periods. The first subsample is used for initial parameter estimation covering the period from January 1928 through December 1977. The second subsample for forecast evaluation covers the period

[4] As in Christoffersen, Jacobs, and Ornthanalai (2012), the total unconditional variance is defined by $\sigma^2_{total} \equiv \overline{g\tau} + (\theta^2 + \delta^2)\bar{\lambda}$, where $\overline{g\tau}$ and $\bar{\lambda}$ are calculated as the time series averages of $g_{i,t}\tau_t$ and $\lambda_{i,t}$, respectively. Thus, the proportion of the contribution of the jump component to the total variance is given as $(\theta^2 + \delta^2)\bar{\lambda}/\sigma^2_{total}$.

from January 1978 to December 31, 2018. During the out-of-sample period, a global financial crisis broke out in mid-2008 and end in the end of 2009 which caused large fluctuations in the US stock market. Five occasional events listed in Table 3 occurred during such period. This provides us a good opportunity to evaluate the performance of volatility forecasting. As the popular way, the estimation window is updated by adding a new observation and discarding the most distant observation at the same time. In this way, the length of the estimation sample is fixed. However, in our case, thousands of volatility forecasts are generated by each model, which takes too much time in parameter estimation. For the sake of computational convenience, we re-estimate the model parameter every year using the rolling window method. Thus, the parameters are fixed within each year, and only the data are updated[5]. This fixed parameter and rolling window forecast scheme follows the work of Laurent, Rombouts, and Violante (2012) to satisfy the assumptions of the MCS test (Hansen, Lunde, and Nason, 2011) for comparing the forecasting abilities of nested models. With the estimation window rolls forward, we generate a series of volatility forecasts. These forecasts are plotted in Figure 8.

Figure 8 about here

To guarantee that the forecasts have a reasonable range, we adopt the "insanity filter". That is, the forecasts are restricted between the smallest realization and the largest realization observed in the estimation window. The similar treatment is also found in recent volatility forecasting literature (see, e.g., Patton and Sheppard, 2015; Bollerslev, Hood, Huss, and Pedersen, 2018).

In order to evaluate the forecasting performance, the loss functions are always employed. It is likely that the evaluation results are dependent of the selection of loss function. Because of this consideration, we use a total of 5 loss functions rather than make a single choice. These loss criteria are given as follows:

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(h_t - \hat{h}_t)^2$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|h_t - \hat{h}_t|$$

[5]For robustness check, we also update parameter every 1.5 and 2 years and find highly consistent results.

$$MSD = \frac{1}{N}\sum_{t=1}^{N}(\sqrt{h_t} - \sqrt{\hat{h}_t})^2$$
$$MAD = \frac{1}{N}\sum_{t=1}^{N}|\sqrt{h_t} - \sqrt{\hat{h}_t}|$$
$$QLIKE = \frac{1}{N}\sum_{t=1}^{N}\big(\frac{h_t}{\hat{h}_t} - \log(\frac{h_t}{\hat{h}_t})\big) \tag{24}$$

where N is the number of out-of-sample forecasts; h_t and $\hat{h}_t$ are the true value and forecasts of stock volatility. Because the true volatility is unobservable, the inappropriate proxy of true volatility is possible to cause the spurious results of predictability. To address this issue, we use both squared daily return and realized volatility as the proxies of true volatility[6]. The realized variance (RV) is obtained from the Oxford-Man Institute's "realized library". Among these loss functions, MSE and QLIKE are more robust to the biased proxy of true volatility according to Patton (2011), which is constructed based on 5-min intraday high-frequency data.

A limitation of the loss functions is that they cannot tell us whether the difference of forecasting performance is statistically significant. To address this problem, we use a model confidence set (MCS) test recently developed by Hansen, Lunde, and Nason (2011). The idea behind this test is that the data available may be not informative enough to yield a single model that can dominate all of its competitors significantly. In this situation, one can only have a smaller set of the models, called the model confidence set, which have the beast forecasting models at a pre-specified level of confidence. Thus, the volatility models included in the MCS work equally well in the statistical sense at this given confidence level. The detailed description of this test is given in Hansen, Lunde, and Nason (2011).

We produce volatility forecasts during the period from January 1978 through December 2018. The evaluation results for GARCH-Jump-MIDAS with economic level variables and those for competing models are reported in Table 4. Since the realized volatility from Oxford-Man Institute's "realized library" is available only after January 2000, we use the squared daily return as the proxy of true volatility in this evaluation sample. We find that the GARCH-

[6] A large number of studies documents that the realized volatility based on intraday high-frequency data is a better measure of volatility (e.g., Andersen, Bollerslev, Diebold, and Labys, 2001, 2003). We also use the squared daily returns because all GARCH-type models are on the basis of daily returns.

Jump-MIDAS with PPI level has the lowest loss functions under four out of five criteria. The MCS results indicate that at the 90% confidence level most of the competing models are excluded from the model confidence set under these loss criteria. Under the remaining criterion of QLIKE, Amendola, Candila, and Gallo (2019) GJR-A-MIDAS displays the best forecasting performance.

We also assess the out-of-sample performance in three subsamples to investigate the forecasting performance over time. During the sample period from January 1985 to December 2018 and the period from January 1995 to December 2018, the GARCH-Jump-MIDAS with PPI level has the lowest forecasting losses under the criteria of MSD, MAE and MAD. The MCS test results show that this model significantly outperforms all competing models. Under the MSE criterion, the GARCH-Jump-MIDAS models perform as well as the others according to the MCS tests. Under the QLIKE criterion, the GJR-A-MIDAS on average generates the most accurate volatility forecasts. During the subsample period from January 2005 to December 2018, the relative forecasting performance of volatility models is consistent with those during the other subsample periods when using squared returns. When using RV as the proxy of true volatility, GARCH-Jump-MIDAS with PPI level performs significantly better than the others under four out of five criteria.

Table 4 about here

Table 5 reports the forecasting results of GARCH-MIDAS models when the economic uncertainty variables are incorporated. When the squared return is employed as the proxy of true volatility, the subsample and full sample results are generally consistent. Under the criterion of MSE, the GARCH-MIDAS with PPI uncertainty has the lowest forecasting loss. However, all volatility models are included in the MCS suggesting that they perform equally well in the statistical sense. Under the other three loss criteria except QLIKE, the GARCH-Jump-MIDAS with PPI uncertainty produces the most accurate volatility forecasts, and its superiority over the other volatility models of interest is significant. Overall, our GARCH-Jump-MIDAS specification generally has more reliable out-of-sample performance than the other models of interest.

Table 5 about here

We can find some more interesting results by observing the forecasting performance. First, GARCH-MIDAS leads to lower forecasting loss than GARCH in most cases, suggesting the improvement of forecasting accuracy by the incorporation of macroeconomic variables. Second, two macroeconomic uncertainty variables provide different predictive content regarding future aggregate market volatility. GARCH-Jump-MIDAS with IP volatility has higher loss functions than the GARCH-Jump model with PPI volatility. The similar result holds for the comparison with the volatility models with economic level variables. That is, IP uncertainty (level) predicts stock volatility in a worse way than the PPI uncertainty (level). Third, we find that the GARCH-Jump-MIDAS performs better than the GARCH-MIDAS with the same macroeconomic variables under all loss criteria. This finding highlights the importance of jump in improving forecasting performance.

5.4 Portfolio exercise

To examine the economic value of variance forecast based on various volatility models, we use a volatility timing strategy which is popular in forecasting literature (Campbell and Thompson, 2007; Ferreira and Santa-Clara, 2011; Neely, Rapach, Tu, and Zhou, 2014). We assume an investor with mean-variance preferences who allocates wealth between stocks and risk-free bills. The optimal weight of the stock in the portfolio is ex-ante determined by the variance forecasts. In detail, at the end of day t, the investor calculate the optimal weight of the stock index according the following formula for the next day $t+1$,

$$\omega_t = \frac{1}{\gamma}\frac{\hat{R}_{t+1}}{\hat{h}_{t+1}} \tag{25}$$

where γ is the risk aversion coefficient, $\hat{R}_{t+1}$ denotes the forecasts of stock returns in excess of the risk-free rate $R_{f,t}$. We use the 3-month Treasury bill rate as the proxy of risk-free rate. Naturally, the remainder weight $1-\omega_t$ is assigned to the risk free asset. Certainly, the optimal weight of stock is affected by the value of risk coefficient γ. For robustness check, we use four different γ's of 5, 10, 15 and 20.

The realized portfolio return is given by,

$$R_{p,t+1} = \omega_t R_{t+1} + R_{f,t}. \tag{26}$$

To assess the portfolio performance, we use the measure of certainty equivalent return (CER) as follows,

$$CER_p = \mu_p - \frac{1}{2}\gamma\sigma_p^2. \tag{27}$$

where μ_p and σ_p^2 are the mean and variance of the portfolio returns, respectively.

We use the same return forecasts of historical average and the volatility forecasts from different models to do portfolio exercise. Although the return forecasting model is simple, recent studies show that the economic models are difficult to produce more accurate forecasts than the historical average benchmark (Rapach, Strauss, and Zhou, 2010). In this way, the portfolio performance is uniquely dependent of the accuracy of volatility forecasts.

We compute the CERs of portfolios formed by volatility models. These numbers are multiplied by 25, 000 to denote the annualized percentage values since we use the daily data. The results for economic level variables are reported in Table 6. We can find that the GARCH-Jump-MIDAS with IP level has the annualized CER as high as 2.078% when $\gamma = 5$. This economic gain is greater than all the other volatility models. By comparing the CERs of GARCH-MIDAS models with economic level variables and those of the GARCH model, we find that both IP and PPI levels are useful for improving portfolio performance. The IP level performs better than the PPI in portfolio allocation. The results for economic uncertainty variables are shown in Table 7. The optimal model is GARCH-Jump-MIDAS with PPT volatility. The economic implications of macroeconomic variables and jump for portfolio exercise are also consistent across different values of risk aversion coefficients.

Table 6 and Table 7 about here

5.5 Sensitivity analysis

This subsection executes a series of sensitivity analysis. First, we analyze the effects of lag length K in MIDAS equation on estimation results. For this purpose, the lag lengths of 36 and 60 months are employed. Second, we investigate the impacts of parameter re-estimation window on out-of-sample forecasting results and portfolio allocation performance. Overall, our evidence indicates that the main finding on the superiority of GARCH-Jump-MIDAS in modeling and forecasting volatility is very robust. To save space, we show these results in the appendix.

6 Conclusions

We extend the existing GARCH-MIDAS model by accounting for the role of jumps and develop the GARCH-Jump-MIDAS accordingly. The proposed model is then employed to capture the effects of macroeconomic variables and jumps on the stock market volatility. Our in-sample results indicate that jumps have important impacts on future market volatility. The economic variables reflecting economic activity and uncertainty can significantly affect the market volatility. Out-of-sample findings suggest that the information from jumps and macroeconomic variables can improve the forecasting accuracy significantly. Our GARCH-Jump-MIDAS model also has better performance in the sense of portfolio allocation than its competitors including the traditional GARCH, GJR-A-MIDAS and the GARCH-MIDAS models.

We would like to conclude the paper by outlining some directions for future research. First, our model can be further modified by accommodate more stylized facts such as fat-tailed distribution, structural break and long memory in volatility (Cont, 2001). Second, it is highly possible that the relationship between macroeconomic variables and volatility changes over time. The forecasting accuracy may be improved by taking the dynamic linkages into account (Dangl and Halling, 2012). Third, we can obtain more accurate volatility forecasts using the combination of information from a wide range of macroeconomic indicators, rather than any single variables (Paye, 2012). Finally, in the literature on volatility forecasting, daily jump is always defined as a short-term behavior (e.g., Becker, Clements, and McClelland, 2009; Maheu and McCurdy, 2004; Andersen, Bollerslev, and Diebold, 2007). Andersen, Bollerslev, and Diebold (2007) define the jump over the horizon of h days as the average of h daily jump. Therefore, we mainly focus on the effects of jump on short-term volatility component instead of on the long-term component. Indeed, macroeconomic news may have fundamentally impacts on volatility, and cause the phenomenon of structural break. Our model does not account for the role of structural break. We leave it in the future work.

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Table 1. Descriptive statistics.

	Returns on SPX	IP growth rates	PPI growth rates
Mean	0.000	0.003	0.002
Variance	0.000	0.000	0.000
Min	-0.229	-0.104	-0.053
Max	0.154	0.166	0.108
Skewness	-0.435	0.847	1.456
Kurtosis	21.679	20.328	18.727
JB Stat.	328639.408***	13944.457***	11767.221***
ADF Stat.	-31.714***	-6.408***	-5.098***
Q(20) Stat.	81.806***	641.979***	462.833***
$Q^2(20)$ Stat.	10594.934***	902.455***	238.591***
ARCH(20) Stat.	3133.413***	476.862***	162.317***

Notes: This table provides the summary statistics of S&P 500 returns (SPX) and the growth rate of industrial production (IP) and producer price index (PPI). The Jarque and Bera (JB) statistic tests the null hypothesis of normal distribution. The Augmented Dickey and Fuller (ADF) test statistic is for the null hypothesis of a unit root in the time series. $Q(\ell)$ denotes the Ljung and Box statistic for the null hypothesis of no series correlation until the order of ℓ. The ARCH(ℓ) is the test for the null hypothesis of no heteroscedasticity for the lag of ℓ. The asterisks *, ** and *** denote rejections of null hypothesis at the 10%, 5% and 1% significance levels, respectively. Sample period: January 2, 1928-December 31, 2018.

Table 2. Estimation Results (Full sample)

	GARCH	GARCH-MIDAS (BV)	GARCH-MIDAS (BV+IP level)	GARCH-MIDAS (BV+PPI level)	GJR-A-MIDAS (BV+level IP)	GJR-A-MIDAS (BV+level IP)	GARCH-Jump-MIDAS (BV+IP level)	GARCH-Jump-MIDAS (BV+PPI level)	GARCH-MIDAS (BV+IP variance)	GARCH-MIDAS (BV+PPI variance)	GARCH-Jump-MIDAS (BV+IP variance)	GARCH-Jump-MIDAS (BV+PPI variance)
μ	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α	0.094***	0.094***	0.097***	0.093***	0.036***	0.036***	0.070***	0.070***	0.098***	0.098***	0.070***	0.070***
	(0.010)	(0.010)	(0.009)	(0.009)	(0.007)	(0.008)	(0.005)	(0.005)	(0.011)	(0.010)	(0.005)	(0.005)
β	0.901***	0.892***	0.894***	0.898***	0.899***	0.899***	0.912***	0.913***	0.893***	0.893***	0.912***	0.913***
	(0.010)	(0.012)	(0.010)	(0.009)	(0.017)	(0.014)	(0.006)	(0.006)	(0.011)	(0.010)	(0.006)	(0.005)
γ/ω	0.000***				0.101***	0.101***						
	(0.000)				(0.018)	(0.012)						
m		-9.236***	-8.962***	-9.037***	-9.265***	-9.263***	-9.431***	-9.432***	-8.891***	-8.892***	-9.438***	-9.431***
		(0.146)	(0.198)	(0.237)	(0.116)	(0.149)	(0.120)	(0.137)	(0.236)	(0.232)	(0.103)	(0.087)
θ_0		48.542***	48.573***	48.575	48.557**	48.556	48.578***	48.587***	48.558***	48.558***	48.561***	48.586
		(16.753)	(10.174)	(43.513)	(21.841)	(32.105)	(8.451)	(15.719)	(3.194)	(12.566)	(5.276)	(38.985)
ω_0		3.396	1.011***	1.010***	2.324	2.322	1.010***	1.010***	1.678***	1.669***	1.356***	1.010***
		(16.785)	(0.049)	(0.083)	(1.652)	(1.453)	(0.028)	(0.026)	(0.202)	(0.076)	(0.260)	(0.058)
θ_1/θ_1^+			-0.321***	0.250**	0.684***	0.691***	-0.324***	0.314***	0.075***	0.269***	0.075**	0.267
			(0.049)	(0.117)	(0.159)	(0.158)	(0.052)	(0.083)	(0.010)	(0.067)	(0.035)	(0.576)
ω_1/ω_1^+			5.351***	20.729	5.014**	4.990	5.344***	20.677***	5.390***	20.830***	5.390**	20.826***
			(0.723)	(13.522)	(2.109)	(4.306)	(0.808)	(4.935)	(1.245)	(2.678)	(2.578)	(7.106)
θ_1^-					-0.163*	-0.158***						
					(0.095)	(0.027)						
ω_1^-					4.214***	4.219***						
					(1.550)	(1.086)						
θ							-0.006***	-0.006***			-0.006***	-0.006***
							(0.001)	(0.001)			(0.001)	(0.001)
δ							0.017***	0.017***			0.016***	0.017***
							(0.003)	(0.003)			(0.002)	(0.005)
ρ_0							0.001	0.001*			0.001	0.001*
							(0.001)	(0.001)			(0.001)	(0.001)
ρ_1							0.979***	0.980***			0.980***	0.980***
							(0.020)	(0.016)			(0.021)	(0.019)
σ							0.001	0.001			0.001	0.001
							(0.001)	(0.001)			(0.001)	(0.002)
Loglik	74237.68	74236.99	74246.52	74247.04	74448.10	74448.25	74715.45	74715.75	74244.15	74244.16	74714.31	74715.69
AIC	-148467.35	-148461.98	-148477.05	-148478.09	-148874.19	-148874.51	-149404.90	-149405.49	-148472.30	-148472.33	-149402.63	-149405.38
BIC	-148435.26	-148413.84	-148412.86	-148413.90	-148785.93	-148786.24	-149300.59	-149301.18	-148408.10	-148408.14	-149298.32	-149301.07

Notes: This table reports the parameter estimates of the proposed GARCH-Jump-MIDAS and the competing volatility models. The parameters are obtained via the standard MLE using daily returns data of S&P 500 index from January 2, 1928-December 31, 2018. We also provide the standard errors accompanied by parameter estimates. The asterisks *, ** and *** denote rejections of null hypothesis at 10%, 5% and 1% significance levels, respectively.

Table 3. Events and jumps (Full Sample)

Events	Date	Return (%)	$P(N_{i,t} >= 1 \vert \mathcal{I}_{i,t})$ (BV+IP level)	$P(N_{i,t} >= 1 \vert \mathcal{I}_{i,t})$ (BV+IP level)	$P(N_{i,t} >= 1 \vert \mathcal{I}_{i,t})$ (BV+IP variance)	$P(N_{i,t} >= 1 \vert \mathcal{I}_{i,t})$ (BV+IP variance)
	Oct. 25, 1929	1.4267	0.0359	0.0359	0.0371	0.0359
The great crash	Oct. 28, 1929	-13.8576	0.9295	0.9313	0.9266	0.9310
	Oct. 29, 1929	-10.7121	0.0806	0.0811	0.0819	0.0812
	Oct. 08, 1979	-1.2571	0.0907	0.0900	0.0938	0.0903
The 1979 oil crisis	Oct. 09, 1979	-3.0024	0.9688	0.9686	0.9694	0.9689
	Oct. 10, 1979	-1.2551	0.0479	0.0477	0.0495	0.0478
	Oct. 16, 1987	-5.2976	0.7790	0.7804	0.7778	0.7801
Black Monday	Oct. 19, 1987	-22.8997	1.0000	1.0000	1.0000	1.0000
	Oct. 20, 1987	5.1954	0.0429	0.0432	0.0442	0.0433
	Oct. 12, 1989	-0.4492	0.0218	0.0216	0.0227	0.0216
Black Friday	Oct. 13, 1989	-6.3213	1.0000	1.0000	1.0000	1.0000
	Oct. 16, 1989	2.7290	0.0410	0.0409	0.0422	0.0409
	Sept. 10, 2001	0.6207	0.0245	0.0243	0.0254	0.0244
The September 11 attacked	Sept. 17, 2001	-5.0468	0.9861	0.9863	0.9861	0.9863
	Sept. 18, 2001	-0.5822	0.0348	0.0347	0.0360	0.0347
	Sept. 12, 2008	0.2119	0.0284	0.0283	0.0295	0.0283
Lehman Brothers failed	Sept. 15, 2008	-4.8283	0.6455	0.6484	0.6472	0.6466
	Sept. 16, 2008	1.7363	0.0334	0.0333	0.0345	0.0334
	Nov. 4, 2016	-0.1668	0.0145	0.0143	0.0151	0.0144
U.S. presidential election of 2016	Nov. 7, 2016	2.1980	0.9680	0.9680	0.9687	0.9680
	Nov. 8, 2016	0.3765	0.0179	0.0178	0.0186	0.0178

Notes: $P(N_{i,t} >= 1|\mathcal{I}_{i,t})$ is the ex-post probability of at least one jump based on GARCH-Jump-MIDAS model with BV and IP (PPI) level or BV and IP (PPI) variance. Period: January 2, 1928-December 31, 2018

Table 4. Out-of-sample forecasting performance based on level variables (12 months).

	GARCH	GARCH-MIDAS (BV)	GARCH-MIDAS (BV+IP level)	GARCH-MIDAS (BV+PPI level)	GJR-A-MIDAS (BV+IP level)	GJR-A-MIDAS (BV+PPI level)	GARCH-Jump-MIDAS (BV+IP level)	GARCH-Jump-MIDAS (BV+PPI level)
Panel A: Squared return (R^2) as a proxy of the true variance (period: January 1978 - December 2018)								
MSE($\times 10^7$)	3.571	3.568	3.568	3.567	3.610	3.611	3.544	**3.544**
p-value	(0.146)	(0.146)	(0.146)	(0.588)	(0.146)	(0.146)	(0.852)	(1.000)
MSD($\times 10^5$)	6.077	6.019	6.020	6.018	6.007	6.012	5.804	**5.783**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)
MAE($\times 10^4$)	1.290	1.280	1.280	1.280	1.282	1.283	1.240	**1.235**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)
MAD($\times 10^3$)	5.682	5.650	5.651	5.650	5.595	5.598	5.566	**5.549**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)
QLIKE	1.527	1.526	1.526	1.526	**1.506**	1.506	1.516	1.517
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.912)	(0.040)	(0.037)
Panel B: Squared return (R^2) as a proxy of the true variance (period: January 1985 - December 2018)								
MSE($\times 10^7$)	4.268	4.265	4.265	4.265	4.317	4.317	4.237	**4.237**
p-value	(0.140)	(0.140)	(0.140)	(0.594)	(0.140)	(0.140)	(0.822)	(1.000)
MSD($\times 10^5$)	6.539	6.478	6.479	6.477	6.481	6.487	6.223	**6.210**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
MAE($\times 10^4$)	1.382	1.372	1.372	1.372	1.377	1.378	1.323	**1.320**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(1.000)
MAD($\times 10^3$)	5.814	5.784	5.785	5.784	5.731	5.734	5.678	**5.672**
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
QLIKE	1.548	1.547	1.547	1.547	**1.525**	1.525	1.538	1.538
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.498)	(0.034)	(0.034)
Panel C: Squared return (R^2) as a proxy of the true variance (period: January 1995 - December 2018)								
MSE($\times 10^7$)	1.524	1.523	1.523	1.523	**1.482**	1.482	1.538	1.537
p-value	(0.165)	(0.165)	(0.165)	(0.165)	(1.000)	(0.353)	(0.165)	(0.302)
MSD($\times 10^5$)	6.442	6.378	6.379	6.378	6.207	6.213	6.094	**6.078**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
MAE($\times 10^4$)	1.438	1.427	1.427	1.427	1.408	1.409	1.369	**1.366**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
MAD($\times 10^3$)	5.996	5.964	5.965	5.965	5.871	5.874	5.821	**5.814**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)
QLIKE	1.511	1.510	1.510	1.510	**1.480**	1.480	1.501	1.501
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.922)	(0.000)	(0.000)
Panel D: Squared return (R^2) as a proxy of the true variance (period: January 2005 - December 2018)								
MSE($\times 10^7$)	2.077	2.076	2.076	2.076	**2.024**	2.024	2.106	2.106
p-value	(0.371)	(0.371)	(0.371)	(0.371)	(1.000)	(0.386)	(0.371)	(0.386)
MSD($\times 10^5$)	6.775	6.715	6.716	6.714	6.583	6.593	6.434	**6.412**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.012)	(0.000)	(0.012)	(1.000)
MAE($\times 10^4$)	1.506	1.496	1.496	1.496	1.483	1.484	1.432	**1.427**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.006)	(0.000)	(0.006)	(1.000)
MAD($\times 10^3$)	5.952	5.930	5.930	5.929	5.855	5.859	5.792	**5.784**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.000)	(0.003)	(1.000)
QLIKE	1.586	1.584	1.584	1.584	**1.552**	1.552	1.571	1.571
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.980)	(0.004)	(0.004)
Panel E: Realized Variance (RV) as a proxy of the true variance (period: January 2005 - December 2018)								
MSE($\times 10^7$)	0.473	0.461	0.461	0.460	0.436	0.438	0.413	**0.404**
p-value	(0.002)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)	(1.000)
MSD($\times 10^5$)	2.040	1.984	1.985	1.982	1.870	1.878	1.761	**1.737**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.002)	(1.000)
MAE($\times 10^4$)	0.804	0.789	0.789	0.788	0.768	0.771	0.713	**0.706**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.000)	(0.003)	(1.000)
MAD($\times 10^3$)	3.158	3.123	3.124	3.123	3.036	3.042	2.941	**2.929**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
QLIKE	0.325	0.323	0.323	0.323	**0.305**	0.306	0.310	0.309
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.000)	(0.000)	(0.000)

Notes: This table reports the loss functions of the volatility models. The model parameters are re-estimated after 12 months (1 year). The numbers in parentheses are the p-value of MCS test of Hansen, Lunde, and Nason (2011) with 10,000 bootstrap replications and an average block length of 5. The numbers with underlines denote that the corresponding models are included in MCS at the 90% confidence level.

Table 5. Out-of-sample forecasting performance based on variance variables (12 months).

	GARCH	GARCH-MIDAS (BV)	GARCH-MIDAS (BV+IP variance)	GARCH-MIDAS (BV+PPI variance)	GJR-A-MIDAS (BV+IP level)	GJR-A-MIDAS (BV+PPI level)	GARCH-Jump-MIDAS (BV+IP variance)	GARCH-Jump-MIDAS (BV+PPI variance)
Panel A: Squared return (R^2) as a proxy of the true variance (period: January 1978 - December 2018)								
MSE($\times 10^7$)	3.571	3.568	3.568	3.568	3.610	3.611	3.552	**3.544**
p-value	(0.327)	(0.687)	(0.327)	(0.687)	(0.327)	(0.327)	(0.687)	(1.000)
MSD($\times 10^5$)	6.077	6.019	6.019	6.019	6.010	6.013	5.806	**5.801**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.012)	(0.000)	(0.709)	(1.000)
MAE($\times 10^4$)	1.290	1.280	1.280	1.280	1.283	1.283	1.240	**1.239**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.750)	(1.000)
MAD($\times 10^3$)	5.682	5.650	5.650	5.650	5.598	5.598	5.573	**5.564**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.010)	(0.000)	(0.010)	(1.000)
QLIKE	1.527	1.526	1.526	1.526	1.506	**1.506**	1.517	1.516
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.041)	(1.000)	(0.041)	(0.041)
Panel B: Squared return (R^2) as a proxy of the true variance (period: January 1985 - December 2018)								
MSE($\times 10^7$)	4.268	4.265	4.265	4.265	4.317	4.318	4.246	**4.237**
p-value	(0.335)	(0.690)	(0.335)	(0.690)	(0.335)	(0.335)	(0.690)	(1.000)
MSD($\times 10^5$)	6.539	6.478	6.479	6.477	6.485	6.487	6.221	**6.220**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.000)	(0.950)	(1.000)
MAE($\times 10^4$)	1.382	1.372	1.372	1.372	1.377	1.378	**1.322**	1.322
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.896)
MAD($\times 10^3$)	5.814	5.784	5.784	5.784	5.734	5.735	5.679	**5.675**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.193)	(1.000)
QLIKE	1.548	1.547	1.547	1.547	1.525	**1.525**	1.539	1.537
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.050)	(1.000)	(0.050)	(0.038)
Panel C: Squared return (R^2) as a proxy of the true variance (period: January 1995 - December 2018)								
MSE($\times 10^7$)	1.524	1.523	1.523	1.523	**1.482**	1.482	1.537	1.537
p-value	(0.155)	(0.155)	(0.155)	(0.155)	(1.000)	(0.520)	(0.372)	(0.372)
MSD($\times 10^5$)	6.442	6.378	6.377	6.377	6.210	6.214	6.092	**6.091**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.047)	(0.000)	(0.047)	(1.000)
MAE($\times 10^4$)	1.438	1.427	1.427	1.427	1.408	1.409	1.369	**1.369**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.005)	(1.000)
MAD($\times 10^3$)	5.996	5.964	5.964	5.964	5.873	5.875	5.819	**5.818**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)	(0.000)	(0.005)	(1.000)
QLIKE	1.511	1.510	1.510	1.510	1.480	**1.480**	1.501	1.501
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.050)	(1.000)	(0.000)	(0.000)
Panel D: Squared return (R^2) as a proxy of the true variance (period: January 2005 - December 2018)								
MSE($\times 10^7$)	2.077	2.076	2.076	2.076	**2.024**	2.025	2.106	2.106
p-value	(0.409)	(0.409)	(0.409)	(0.409)	(1.000)	(0.551)	(0.409)	(0.409)
MSD($\times 10^5$)	6.775	6.715	6.714	6.714	6.588	6.594	6.433	**6.429**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	(0.000)	(0.018)	(1.000)
MAE($\times 10^4$)	1.506	1.496	1.496	1.496	1.483	1.485	1.432	**1.431**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.002)	(1.000)
MAD($\times 10^3$)	5.952	5.930	5.929	5.929	5.857	5.859	5.791	**5.789**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.000)	(0.003)	(1.000)
QLIKE	1.586	1.584	1.584	1.584	1.552	**1.552**	1.571	1.571
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.083)	(1.000)	(0.005)	(0.005)
Panel E: Realized Variance (RV) as a proxy of the true variance (period: January 2005 - December 2018)								
MSE($\times 10^7$)	0.473	0.461	0.460	0.460	0.436	0.438	0.413	**0.412**
p-value	(0.002)	(0.004)	(0.022)	(0.022)	(0.063)	(0.022)	(0.063)	(1.000)
MSD($\times 10^5$)	2.040	1.984	1.983	1.983	1.870	1.878	1.761	**1.756**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
MAE($\times 10^4$)	0.804	0.789	0.789	0.789	0.768	0.771	0.713	**0.712**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(1.000)
MAD($\times 10^3$)	3.158	3.123	3.123	3.123	3.036	3.042	2.939	**2.937**
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)
QLIKE	0.325	0.323	0.323	0.323	**0.305**	0.306	0.309	0.309
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.000)	(0.000)	(0.000)

Notes: This table reports the loss functions of the volatility models. The model parameters are re-estimated after 12 months (1 year). The numbers in parentheses are the p-value of MCS test of Hansen, Lunde, and Nason (2011) with 10,000 bootstrap replications and an average block length of 5. The numbers with underlines denote that the corresponding models are included in MCS at the 90% confidence level.

Table 6. Portfolio performance of volatility models based on level variables (12 months)

	GARCH	GARCH-MIDAS (BV)	GARCH-MIDAS (BV+IP level)	GARCH-MIDAS (BV+PPI level)	GJR-A-MIDAS (BV+IP level)	GJR-A-MIDAS (BV+PPI level)	GARCH-Jump-MIDAS (BV+IP level)	GARCH-Jump-MIDAS (BV+PPI level)
$\lambda = 5$	2.073	2.074	2.074	2.074	2.003	2.003	**2.078**	2.073
$\lambda = 10$	1.949	1.951	1.951	1.951	1.914	1.915	**1.955**	1.953
$\lambda = 15$	1.910	1.911	1.911	1.911	1.887	1.887	**1.914**	1.913
$\lambda = 20$	1.891	1.892	1.892	1.892	1.873	1.874	**1.894**	1.893

Notes: This table shows the performance of volatility models in portfolio exercise. The model parameters are re-estimated after 12 months (1 year) and out-of-sample period is from January 1978 to December 2018. At the end of day t, the investor with mean-variance preference calculate the optimal weight of the stock index for the next day $t+1$ as, $\omega_t = \frac{1}{\gamma}\frac{\hat{R}_{t+1}-R_{f,t}}{\hat{h}_{t+1}}$, where γ is the risk aversion coefficient, $\hat{R}_{t+1}$ denotes the forecasts of stock returns in excess of the risk-free rate $R_{f,t}$, $\hat{h}_{t+1}$ is the volatility forecasts. We compute the certainty equivalent returns of portfolios formed by volatility models. These numbers are multiplied by 25, 000 to denote the annualized percentage values since we use the daily data.

Table 7. Portfolio performance of volatility models based on variance variables (12 months)

	GARCH	GARCH-MIDAS (BV)	GARCH-MIDAS (BV+IP variance)	GARCH-MIDAS (BV+PPI variance)	GJR-A-MIDAS (BV+IP level)	GJR-A-MIDAS (BV+PPI level)	GARCH-Jump-MIDAS (BV+IP variance)	GARCH-Jump-MIDAS (BV+PPI variance)
$\lambda = 5$	2.073	2.074	2.075	2.074	2.004	2.003	2.056	**2.076**
$\lambda = 10$	1.949	1.951	1.951	1.951	1.915	1.914	1.944	**1.954**
$\lambda = 15$	1.910	1.911	1.911	1.911	1.887	1.887	1.907	**1.914**
$\lambda = 20$	1.891	1.892	1.892	1.892	1.874	1.873	1.888	**1.893**

Notes: This table shows the performance of volatility models in portfolio exercise. The model parameters are re-estimated after 12 months (1 year) and out-of-sample period is from January 1978 to December 2018. At the end of day t, the investor with mean-variance preference calculate the optimal weight of the stock index for the next day $t+1$ as, $\omega_t = \frac{1}{\gamma}\frac{\hat{R}_{t+1}-R_{f,t}}{\hat{h}_{t+1}}$, where γ is the risk aversion coefficient, $\hat{R}_{t+1}$ denotes the forecasts of stock returns in excess of the risk-free rate $R_{f,t}$, $\hat{h}_{t+1}$ is the volatility forecasts. We compute the certainty equivalent returns of portfolios formed by volatility models. These numbers are multiplied by 25, 000 to denote the annualized percentage values since we use the daily data.

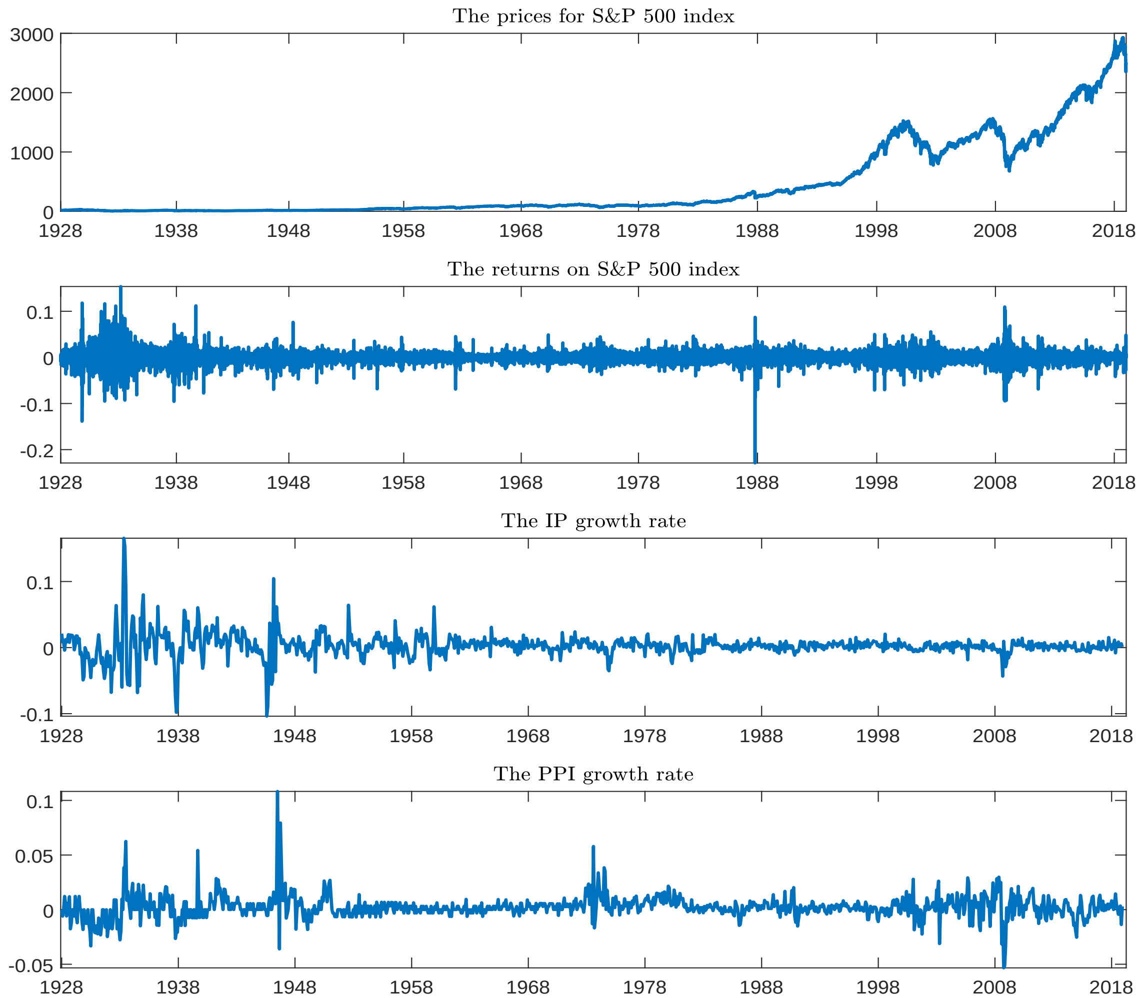


Figure 1. Stock prices, returns and economic variables. This figure plots the daily prices and returns of S&P 500 index, and the monthly growth rates of industrial production (IP) and producer price index (PPI). The sample period is from January 1928 through December 2018. The shadowed area denote the financial crisis period from September 2008 to March 2009.

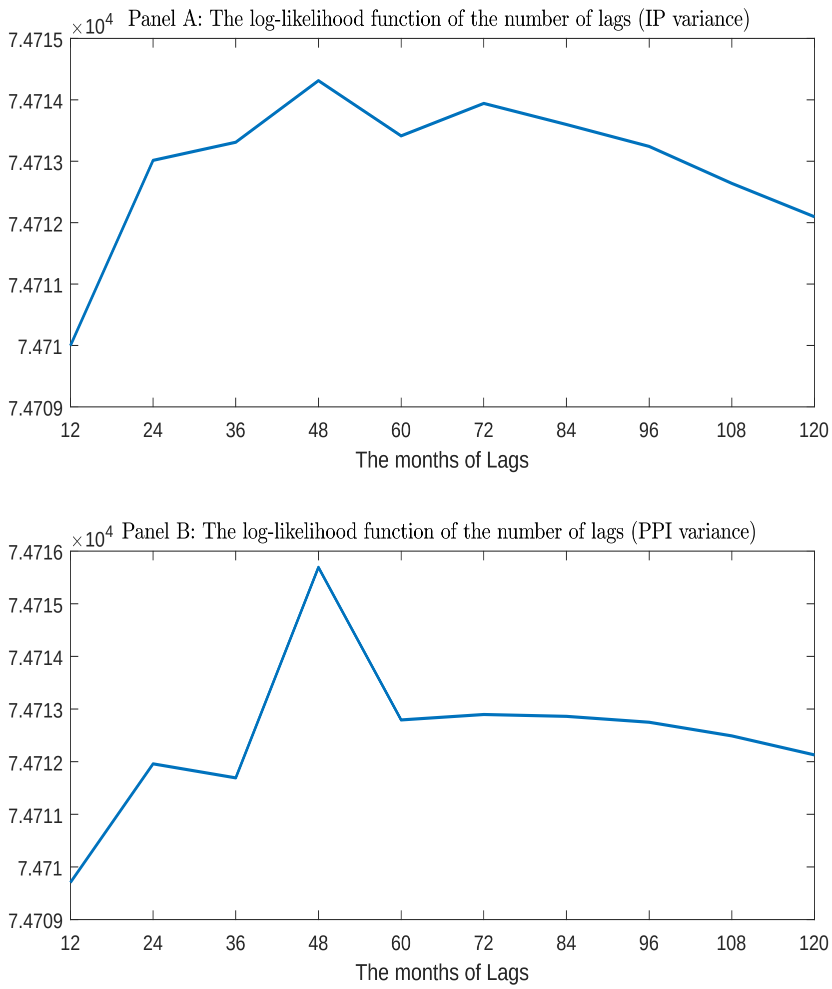


Figure 2. The value of the log-likelihood function. This figure plots the value of the log-likelihood function with various lags (K) based on GARCH-Jump-MIDAS model with BV and IP/PPI variance. Sample period: January 1928-December 2018.

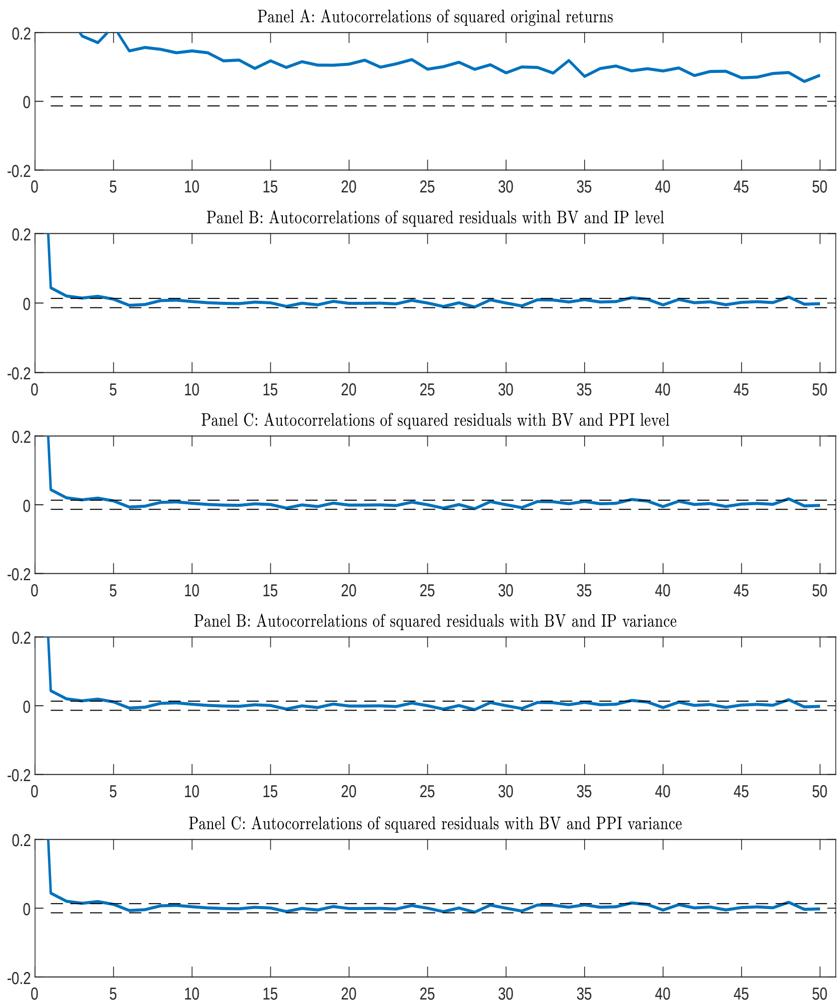


Figure 3. Autocorrelations. This figure plots autocorrelations for squared returns, and those for squared standardized residuals based on GARCH-Jump-MIDAS model with BV and a macroeconomic variable. Sample period: January 1928-December 2018.

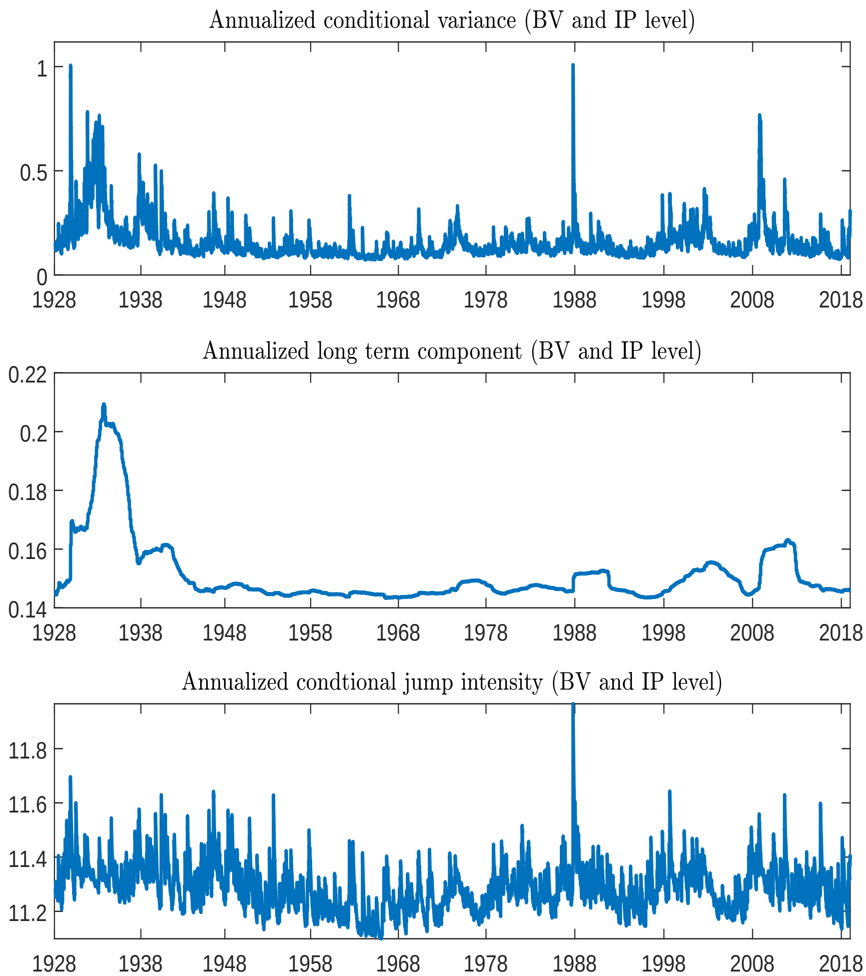


Figure 4. Conditional variance, long-term component and conditional jump density (BV and IP level). This figure shows the conditional variance, the long term component, and the conditional jump intensity based on GARCH-Jump-MIDAS model with BV and IP level. All of them are annualized scale. Sample period: January 1928-December 2018.

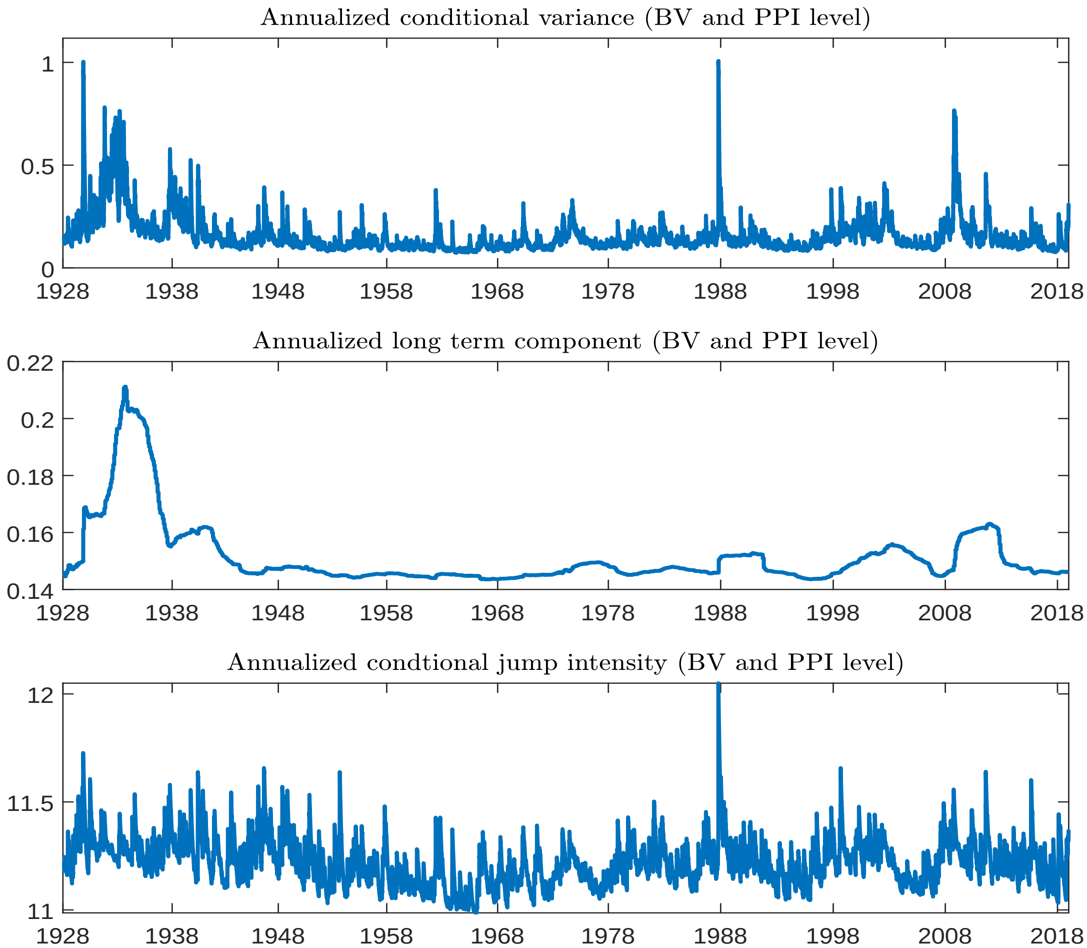


Figure 5. Conditional variance, long-term component and conditional jump density (BV and PPI level). This figure shows the conditional variance, the long term component, and the conditional jump intensity based on GARCH-Jump-MIDAS model with BV and PPI level. All of them are annualized scale. Sample period: January 1928-December 2018.

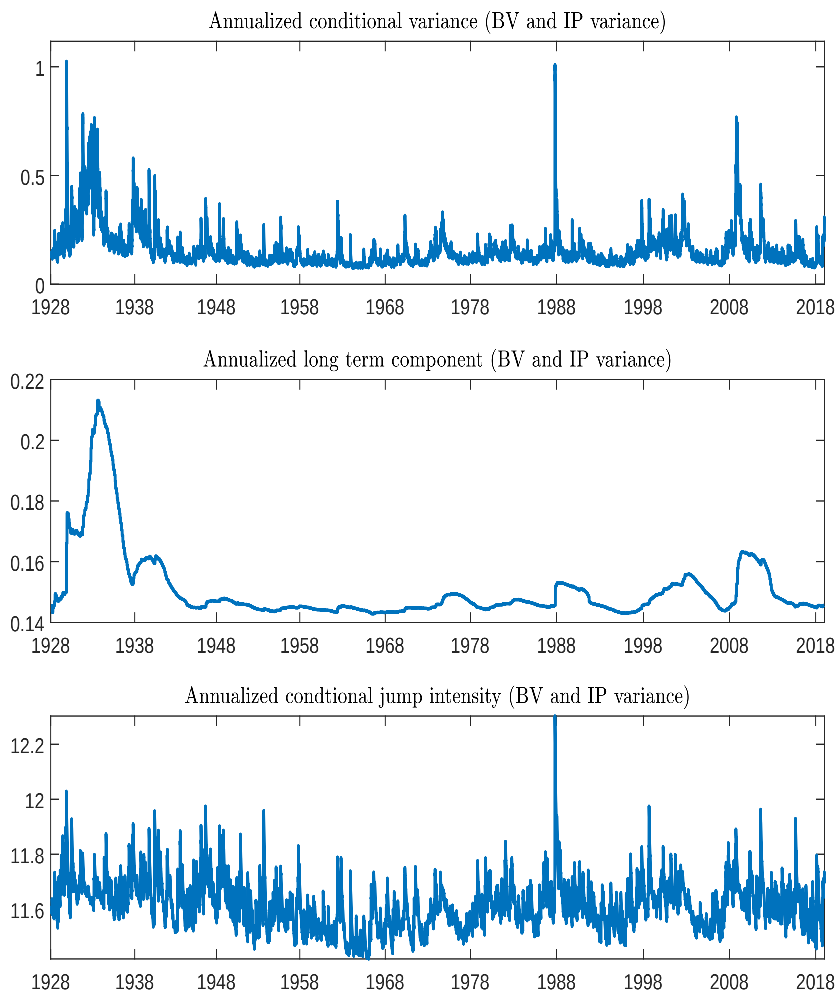


Figure 6. Conditional variance, long-term component and conditional jump density (BV and IP variance). This figure shows the conditional variance, the long term component, and the conditional jump intensity based on GARCH-Jump-MIDAS model with BV and IP variance. All of them are annualized scale. Sample period: January 1928-December 2018.

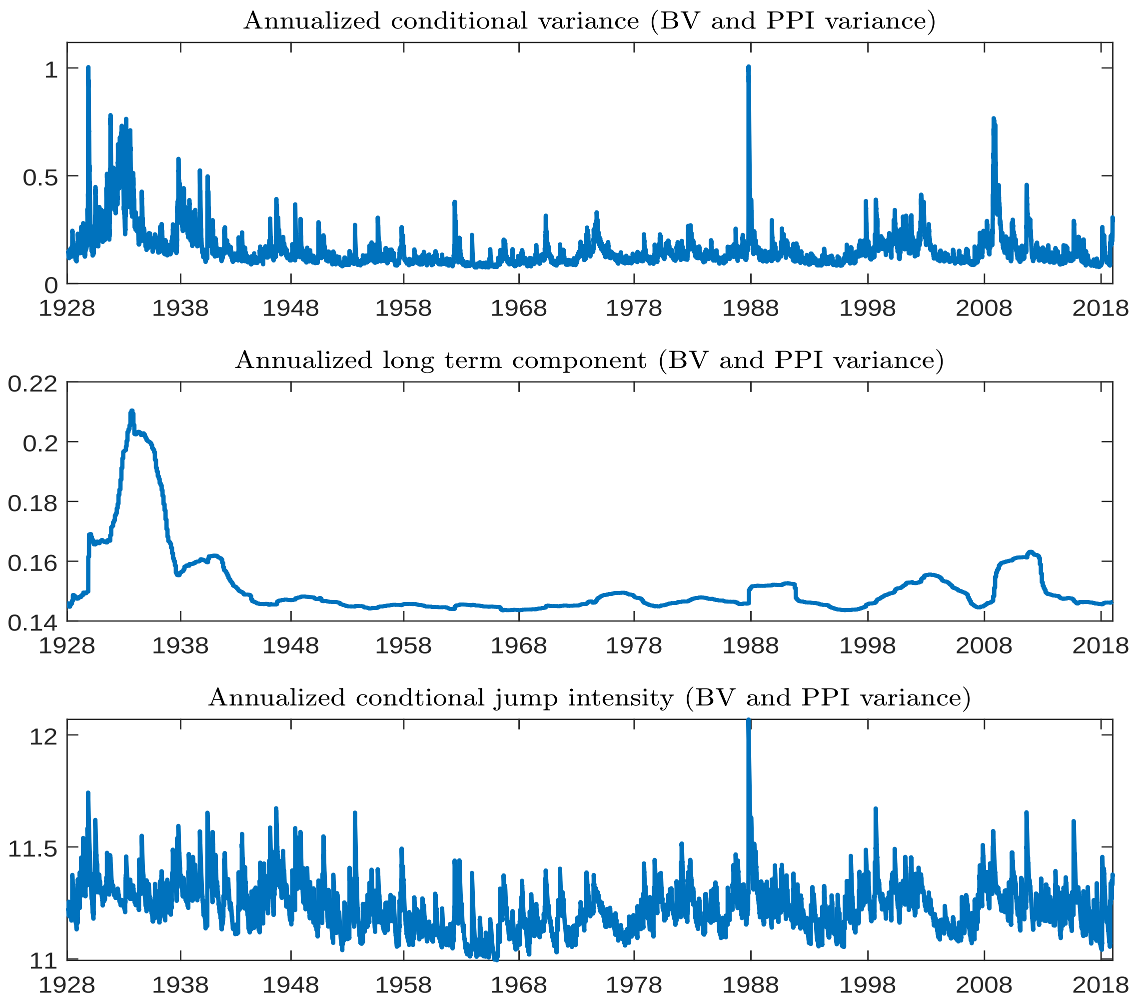
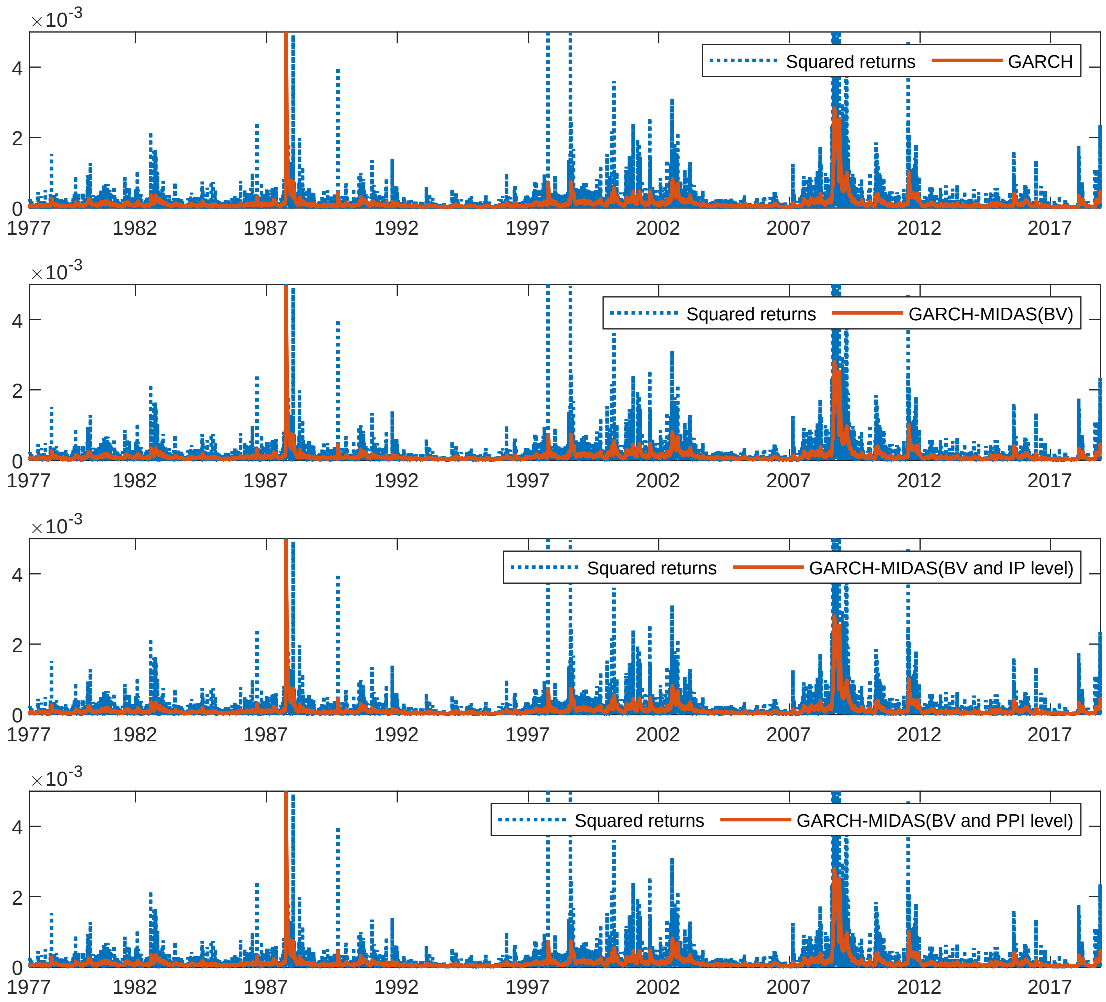


Figure 7. Conditional variance, long-term component and conditional jump density (BV and PPI variance). This figure shows the conditional variance, the long term component, and the conditional jump intensity based on GARCH-Jump-MIDAS model with BV and PPI variance. All of them are annualized scale. Sample period: January 1928-December 2018.



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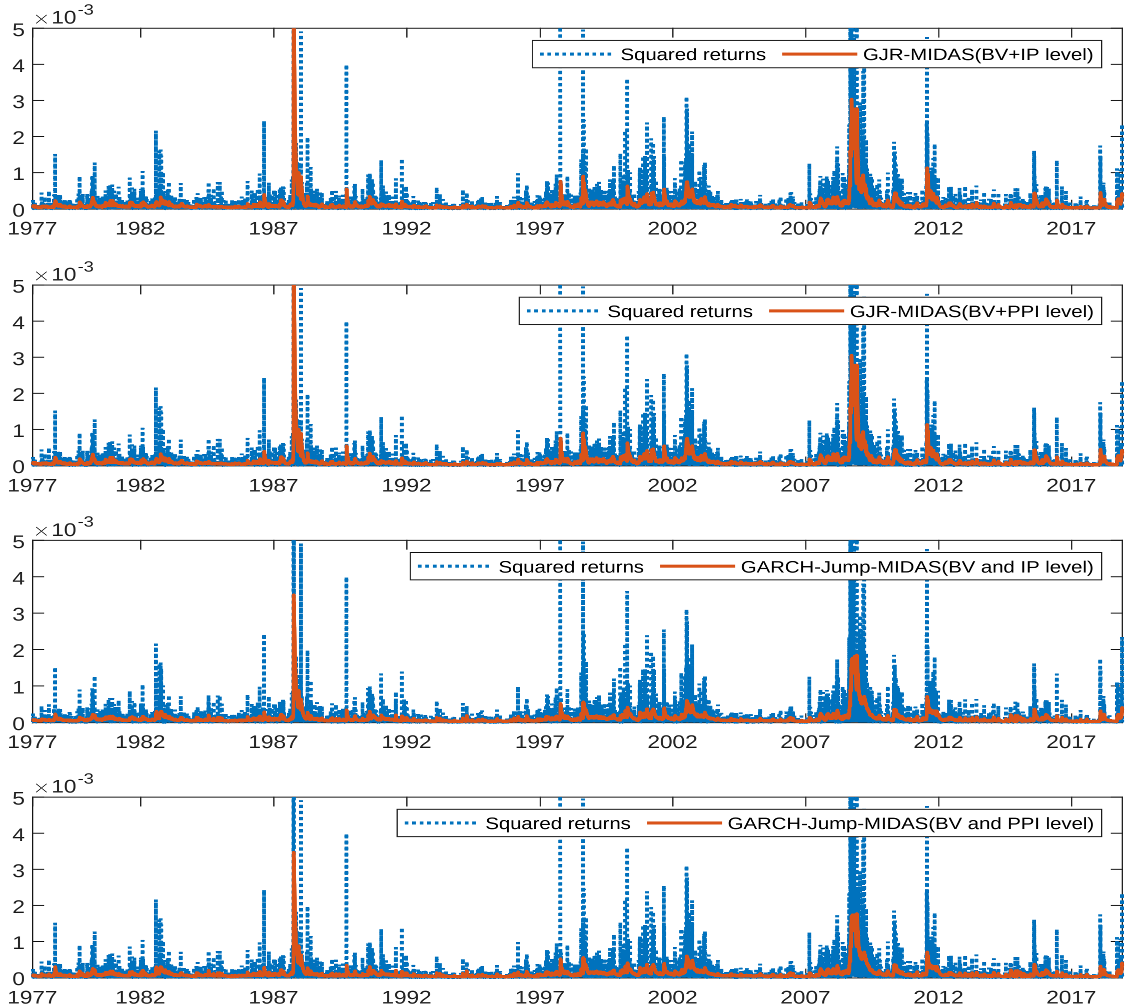

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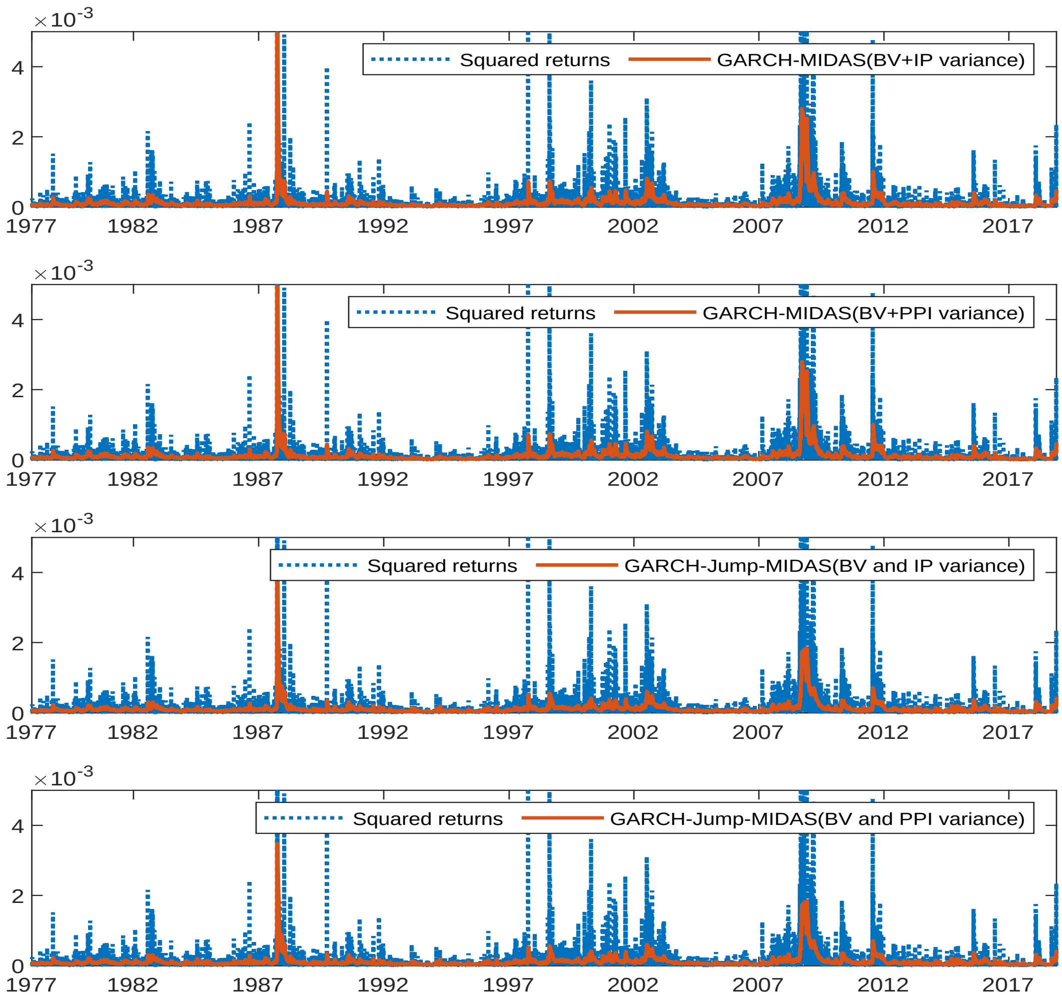

Figure 8. Volatility forecasts. The figure plots the out-of-sample variance forecasts generated from volatility models of interest, together with the squared returns. Out-of-sample period: January 1978-December 2018.