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Published version

OLORUNTOBA, O and KARA, Fuat (2019). Computationally efficient stratified flow wet angle correlation for high resolution simulations. International Journal of Scientific & Engineering Research, 10 (1), 1228-1233.

Computationally Efficient Stratified Flow Wet Angle Correlation for High Resolution Simulations

Olusola Oloruntoba, Fuat Kara

Abstract— In high resolution two-phase pipe flow simulations, such as slug capturing simulation for liquid-gas pipe flow, explicit calculation of stratified flow wet angle has been proposed to improve computational speed of simulations. Most phenomenological and approximate models for obtaining reliable predictions for stratified flow wet angle employ iterative methods or contain long explicit equations which reduce computational efficiency of these models in high-resolution simulations. Therefore, the aim of this study is to adapt a simple mathematical model for predicting stratified flow wet angle to achieve computationally efficient high-resolution liquid-gas pipe flow simulations. The proposed model for predicting stratified flow wet angle is obtained by fitting the generic regression model, Hoerl power law, to analytical stratified flow wet angle data. The proposed model is compared with existing prediction model. Results obtained show that the prediction model proposed gives up to 25.9% savings in computational time over the existing prediction model.

Index Terms— Stratified Flow, Wet Angle, High Resolution Simulations, Prediction Model, Computationally Efficient, multiphase flow, liquid-gas pipe flow

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1 INTRODUCTION

Multiphase pipe flow is a common occurrence in several industrial applications, such as: petroleum, nuclear, chemical and process industries [1], [2]. The design of these applications generally require transient analysis in order to understand system characteristics necessary for developing safe operational envelops [2]–[4]. Transient multiphase flow is governed by complex interactions between phases and pipe walls. Furthermore, the governing equations feature simultaneous variations in pressure and velocity of flow, resulting in stiff differential equations which must be solved. However, exact solutions are practically unavailable for most applications; therefore, numerical solutions are generally applied to obtain realistic solutions to practical multiphase pipe flow problems. In order to solve stiff equations which represent multiphase pipe flow, numerical solutions require small time steps to accurately capture transient phenomena such as slug flow [5], [6].

Slug capturing phenomenon has been demonstrated using high resolution in time and space [7]. However, high resolution in time and space is computationally intensive and limits its application to short pipelines and operational period [8]. Gourma et al. [9] employed adaptive mesh slug capturing simulations in order to reduce requirements on computational resources and lower computational time; yet, the computational overhead demands in calculating stratified flow wet angle needs to be addressed. Issa et al. [10] utilised massively parallelised computations to capture intermittent flows in long pipeline. The approach of Issa et al., however, relies on the availability of high performance computer.

Later, Pasqualette and Nieckele [8] showed that application of explicit expression for stratified wet angle significantly reduced computational time in slug capturing simulations. In the work of Pasqualette and Nieckele, Biberg's explicit model for estimating stratified wet angle was applied with computational improvement of up to 70%. Further improvement in computational efficiency can be achieved if the long explicit expression of the Biberg approximation model is replaced with a simplified expression. Therefore, this study aims to present a simple mathematical model, for predicting stratified wet angle to achieve computationally efficient high resolution two-phase pipe flow simulations. Thereafter, the proposed model would be compared with existing approximation model.

2 MODEL DESCRIPTION

2.1 Existing Model

The existing model that is widely used in computing stratified wet angle is the Biberg equation [11]–[17], given in Equation (1). The equation is an approximation with claimed prediction accuracy of ±0.002 rad [5].

$$\beta = \pi\alpha_L + \left(\frac{3\pi}{2}\right)^{\frac{1}{3}}\left[1 - 2\alpha_L + \alpha_L^{\frac{1}{3}} - (1-\alpha_L)^{\frac{1}{3}}\right] \qquad (1)$$

where, $\beta =$ stratified flow wet angle, and $\alpha_L =$ liquid holdup. Figure 1 shows schematic illustration of stratified flow and wet angle for horizontal flow.

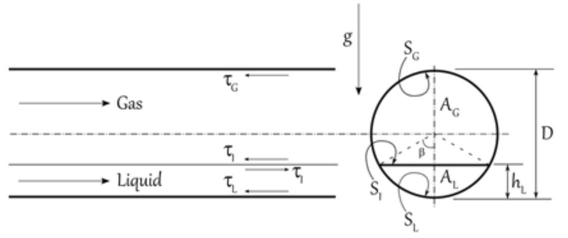

Fig. 1. Schematic illustration of stratified flow and wet angle.

Equation (1) shows that the Biberg approximation of stratified wet angle consists of at least ten numerical computations. For high resolution two-phase pipe flow simulations, such as slug capturing, the demand on computational resources and resultant effect on computational time require approximation equations with reduced numerical computations. This need necessitated the development of Proposed Prediction Model 1 for stratified wet angle and it is provided presently.

2.2 Proposed Prediction Model 1

In order to obtain suitable prediction model 1, several standard regression models (Figure 2) are tested against analytical liquid holdup values. The analytical liquid holdup values are obtained using methods described by Shoham [2]. The results of the test show that Hoerl power law gives the minimum average standard error of 0.001570 and correlation coefficient which is approximately unity. Therefore, Hoerl power law is employed in proposed model 1 to predict stratified wet angle.

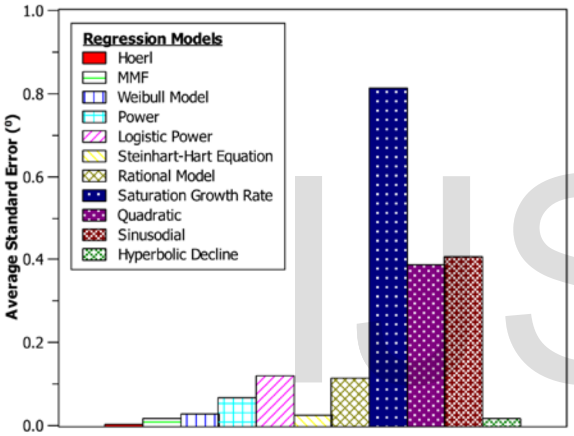

Fig. 2. Average standard error of regression models applied to estimate stratified flow wet angle.

Hoerl power law consists of four arithmetic operators, which are six less arithmetic operators in Biberg's approximation equation. It should be noted that the effect of different arithmetic operators on execution time is not explicitly studied. Therefore, the proposed prediction model 1 for stratified flow wet angle is given in Equation (2). Gas phase fraction is defined as $\alpha_G = 1 - \alpha_L$ [18].

$$\beta = \begin{cases} a_j b_j^{(\alpha_L)} (\alpha_L)^{c_j} \ldots 0.0 \leq \alpha_L \leq 0.5 \\ \pi - \; a_j b_j^{(\alpha_G)} (\alpha_G)^{c_j} \ldots 0.5 \leq \alpha_L \leq 1.0 \end{cases} \quad (2)$$

Coefficients a_j, b_j, and c_j are described in Table 1. The choice of these coefficients is determined, via IF-ELSE logical statements, by the value of liquid holdup (H_L) or void fraction (H_G).

Proposed prediction model 1 for stratified wet angle is compared with Biberg's approximation as shown in Figure 3. In terms of prediction accuracy, the maximum value of absolute percentage prediction error for proposed prediction model 1 is 0.094% which is lower than Biberg's approximation error at 0.187%. Despite the overall prediction improvement of proposed prediction model 1 over Biberg's approximation, proposed model 1 exhibits higher prediction error at wet angle less than 10^0. Therefore, improvement to proposed prediction model 1 at wet angle less than 10^0 is required and is described as proposed prediction model 2 below.

TABLE 1
DESCRIPTION OF COEFFICIENTS AJ, BJ, AND CJ FOR PROPOSED MODEL TO PREDICT STRATIFIED FLOW WET ANGLE; H_L: LIQUID HOLDUP, H_G: VOID FRACTION.

j	H_L or H_G range (-)	a_j	b_j	c_j
1	0.0000 – 0.0038	96.347743	2.355092	0.333620
2	0.0038 – 0.0288	97.466323	1.470465	0.335398
3	0.0288 – 0.1955	99.060031	1.305013	0.339054
4	0.1955 – 0.3371	97.309247	1.347514	0.331962
5	0.3371 – 0.5000	90.268823	1.496925	0.295398

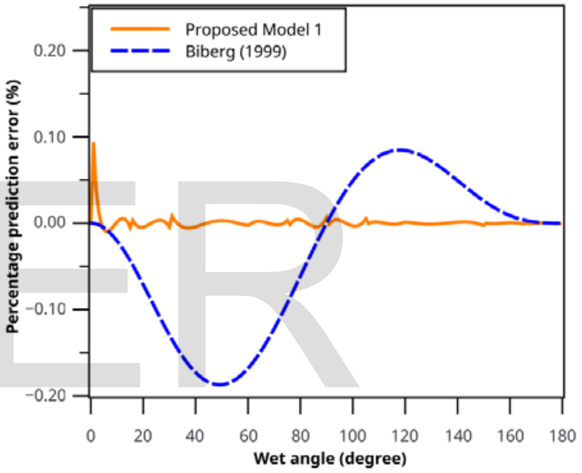

Fig. 3. Comparison of proposed model 1 and Biberg (1999) – percentage prediction error of wet angle (β) as a function of wet angle β.

2.3 Proposed Prediction Model 2

In this study, it has been demonstrated that the proposed prediction model 1 only gives high prediction error at wet angle less than 10^0. It was also shown that Biberg's approximation gives lower prediction error than proposed model 1 at wet angle less than 10^0. Therefore, in order to reduce the prediction error of proposed prediction model 1, a modification to the model is introduced. The modification is achieved by combining Biberg's approximation (for $\beta < 10^0$ and $\beta > 170^0$) with proposed prediction model 1 (for $10^0 \leq \beta \leq 170^0$). Since $\beta = 10^0$ is equivalent to $\alpha_L = 0.0011$, therefore, proposed prediction model 2 is expressed as shown in Equation (3), where $\eta = \; 180^0/\pi$ represents conversion factor from radians to degrees. It should be noted that Biberg's approximation is also applied to the range $\beta > 170^0$. This is done to ensure accurate prediction in the case where void fraction is required.

$$\beta = \begin{cases} \eta\left(\pi\alpha_L + \left(\frac{3\pi}{2}\right)^{\frac{1}{3}}\left[1 - 2\alpha_L + \alpha_L^{\frac{1}{3}} - (1-\alpha_L)^{\frac{1}{3}}\right]\right) \ldots 0 \le \alpha_L < \frac{11}{10^4} \\ a_j b_j^{(\alpha_L)}(\alpha_L)^{c_j} \ldots \frac{11}{10^4} \le \alpha_L \le 0.5 \\ 180^0 - a_j b_j^{(\alpha_G)}(\alpha_G)^{c_j} \ldots 0.5 < \alpha_L \le \frac{9989}{10^4} \\ \eta\left(\pi - \pi\alpha_G + \left(\frac{3\pi}{2}\right)^{\frac{1}{3}}\left[1 - 2\alpha_G + \alpha_G^{\frac{1}{3}} - (1-\alpha_G)^{\frac{1}{3}}\right]\right) \ldots \frac{9989}{10^4} < \alpha_L \le 1 \end{cases} \tag{3}$$

Proposed prediction model 2 for stratified wet angle gives similar prediction for $\alpha_L < 0.0011$ when compared with Biberg's approximation (Figure 4). This is because proposed model 2 adopts the Biberg's approximation for this range.

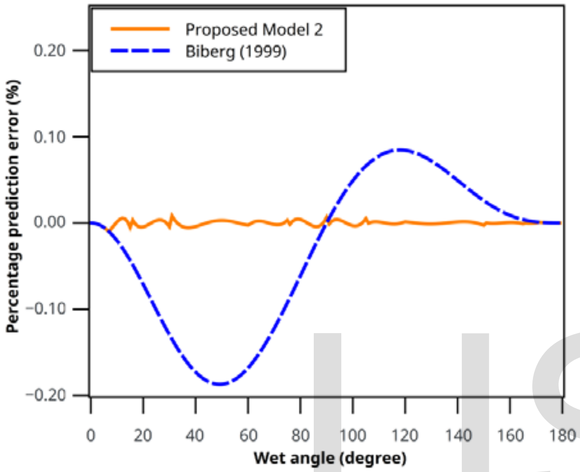

Fig. 4. Comparison of proposed model 2 and Biberg (1999) – percentage prediction error of wet angle (β) as a function of wet angle β.

2.3 Model evaluation

Experimental cases for validation of proposed prediction model 2

The performance of proposed prediction model 2, with respect to Biberg's approximation, is examined using two-phase experimental cases from existing literature. Source and average liquid holdup of the experimental cases are given in Table 2.

Percentage error

$$\epsilon_R = \frac{\beta_{analytical} - \beta_{computed}}{\beta_{analytical}} \times 100\% \tag{4}$$

Average percentage error

$$\varepsilon_{ave} = \left(\frac{1}{n}\sum_{i=1}^{n}\epsilon_R\right) \tag{5}$$

Savings in computational time

TABLE 2

TWO-PHASE PIPE FLOW EXPERIMENTAL CASES FOR VALIDATION OF PROPOSED PREDICTION MODEL 2.

Data Set	Source	Experiment	Mean H_L
1	Kempf [19]	V-Section	0.5074
2	Vigneron et al. [20]	1-C	0.0705
3	Vigneron et al. [20]	1-D	0.5433
4	Vigneron et al. [20]	2-B	0.2921
5	Vigneron et al. [20]	2-D	0.2795
6	Vigneron et al. [20]	3-A	0.0894
7	Vigneron et al. [20]	3-B	0.0877
8	Vigneron et al. [20]	4-B0	0.4171

$$t_{savings} = \frac{t_{Biberg} - t_{model\,2}}{t_{Biberg}} \times 100\% \tag{6}$$

3 RESULTS AND DISCUSSION

Figure 5 compares the performance of proposed model 2 with Biberg's approximation, in terms of computational time in simulating slug capturing in two-phase pipe flow. Results are presented for a total of eight datasets provided in Table 2, ranging between 1 and 8 inclusive. The results show that the proposed prediction model 2 gives savings in computational time over Biberg's approximation equation. Minimum (17.2%) and maximum (25.9%) savings in computational time are observed for datasets 5 and 1 respectively. Average value and standard deviation of computational savings in time are 21.2% and 2.96% respectively for the eight datasets considered in this study. Savings in computational time by proposed model 2 is due to fact that it has four arithmetic operators which are six less arithmetic operators in Biberg's approximation equation. Variation in computational savings for the dataset is attributed to the logical implementation (i.e. IF-ELSE statements) of Equation (3), which is dependent on instantaneous value of liquid holdup.

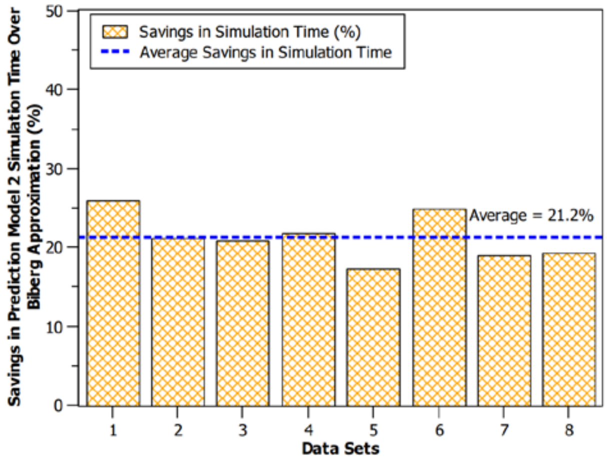

Fig. 5. Savings in prediction computational time of proposed model 2 over Biberg (1999); plot of t_savings for experimental cases in Table 1.

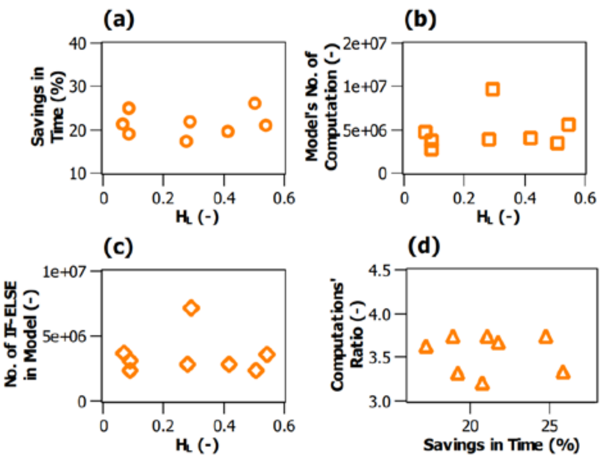

Fig. 6. Correlation plots between liquid holdup and (a) savings in computational time of proposed model 2 with respect to Biberg's approximation (b) number of computations in proposed model 2 and (c) number of IF-ELSE statements in proposed model 2. (d) Correlation plot of computation ratio and savings in computational time. Computation ratio is the ratio of number of computations in Biberg's approximation to number of computations in proposed model 2.

However, there is no direct correlation between average liquid holdup (Table 2) and savings in computational time of proposed prediction model 2 over Biberg's approximation. This observation is illustrated in Figure 6(a). Figures 6(b and c) show that only average values of number of computations and number of IF-ELSE statements respectively are best expression for the range of average liquid holdup considered in this study. Figure 6(d), illustrates similar trend where average ratio of number of computations of Biberg's approximation to number of computations of proposed prediction model 2 is obtained for the observed range of savings in computational time for the eight data studied.

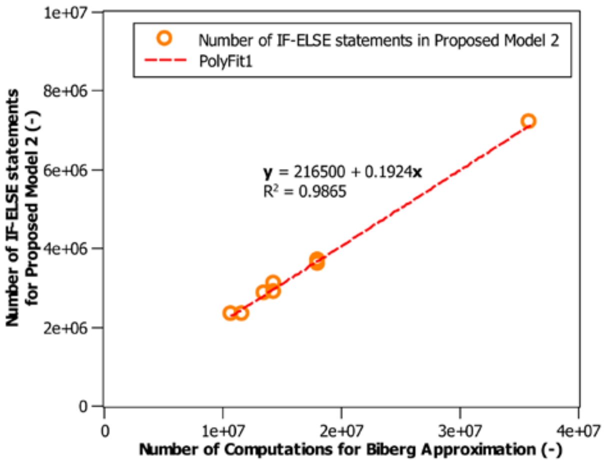

Fig. 7. Number of IF-ELSE statements in Proposed Model 2 is compared with number of computations for Biberg's approximation; data are presented for fixed location in pipeline where flow variation is observed.

Figure 7 shows that the number of IF-ELSE statements used in the implementation of proposed prediction model 2 for the eight datasets are linearly related to the number of computations for the case of Biberg's approximation. The corresponding correlation coefficient is 0.9865. This implies that for transient two-phase or liquid-gas pipe flow high resolution simulations, once the number of instantaneous liquid holdup is known based on temporal discretisation of the flow governing equations, the number of IF-ELSE statements can be estimated. The ratio of number of IF-ELSE statements in the application of proposed prediction model 2 to number of computations in the application of Biberg's approximation is found to follow a polynomial pattern with correlation coefficient of 0.9526 (Figure 8). Thus, these would be important to future statistical application of liquid holdup distribution and verification of justifications for variations in computational savings in time observed in Figure 5.

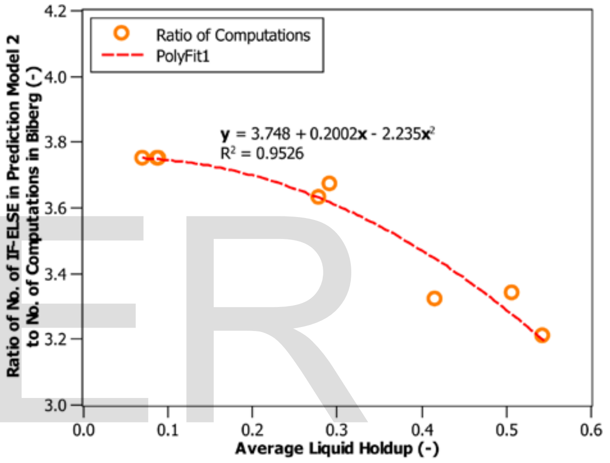

Fig.8. Computational ratio compared with average liquid holdup; computational ratio is the ratio of the number of IF-ELSE statements in proposed model 2 to the number of computations in Biberg's approximation.

5 CONCLUSIONS

This study aimed to develop computationally efficient stratified flow wet angle correlation for high resolution simulations. Specific objectives are (a) to develop stratified flow wet angle correlation, and (b) to compare the proposed model with existing model and experimental data.

Proposed prediction model 2 is developed for predicting stratified flow wet angle. The model is achieved by fitting Hoerl power law equation to analytical stratified flow wet angle data, and adopting Biberg's approximation for $\beta < 10^0$ and $\beta > 170^0$. The resulting model is compared with existing model (i.e. Biberg 1999). The results show that the proposed model 2 have minimum (17.2%) and maximum (25.9%) savings in computational time for datasets 5 and 1 respectively. The variation in computational time savings is attributed due to logical implementation of proposed model, which is dependent on instantaneous value of liquid holdup.

Though, no correlation is observed between average liquid holdup and savings in computational time of proposed prediction model 2 over Biberg's approximation. However, the observed fairly accurate linear and polynomial pattern observed for logical IF-ELSE statements as a function of Biberg's approximation and ratio of computations as a function of liquid holdup respectively would provide insight to future statistical application of liquid holdup distribution and verification of justifications for variations in observed computational savings in time.

Therefore, in order to achieve savings in computational time in high resolution simulations, such as high-resolution slug capturing simulations, proposed prediction model 2 gives improved performance over existing method.

NOMENCLATURE

α_G = Gas fraction [-]

α_L, H_L = Liquid fraction or liquid holdup [-]

β = stratified flow wet angle [degree]

$\beta_{analytical}$ = β computed from Shoham's method [degree] (Shoham, 2005)

β_{model2} = β computed from proposed model 2 [degree]

η = Conversion from radians to degrees [degree/π]

D = Internal diameter of pipe [m]

ϵ_R = Percentage error [%]

ε_{ave} = Average percentage error [%]

g = Acceleration due to gravity [m/s^2]

h_L = Stratified flow film height [-]

n = number of data [-]

t_{Biberg} = Simulation time for Biberg's approximation [s]

t_{model2} = Simulation time for proposed model 2 [s]

$t_{savings}$ = Savings in computational time over Biberg [%]

S_G = Perimeter of gas [m]

S_L = Perimeter of liquid [m]

S_I = Perimeter of gas-liquid interface [m]

A_G = Cross-sectional area of gas flow [m^2]

A_L = Cross-sectional area of liquid flow [m^2]

τ_G = Gas-wall shear stress [Pa]

τ_L = Liquid-wall shear stress [Pa]

τ_I = Gas-liquid interface shear stress [Pa]

Subscript

G = Gas

L = Liquid

I = Gas-liquid interface

REFERENCES

[1] G. Yadigaroglu and G. Hetsroni, "Nature of Multiphase Flows and Basic Concepts," in *Introduction to Multiphase Flow*, Cham: Springer International Publishing, 2018, pp. 1–37.

[2] O. Shoham, *Mechanistic modeling of gas-liquid two-phase flow in pipes*. Society of Petroleum, 2005.

[3] Y. Bai and Q. Bai, *Subsea Pipelines and Risers*. Elsevier Science Ltd, 2005.

[4] M. Jerez-Carrizales, J. E. Jaramillo, and D. Fuentes, "Prediction of Multiphase Flow in Pipelines: Literature Review," *Ing. y Cienc.*, vol. 11, no. 22, pp. 213–233, 2015.

[5] O. Bratland, "Pipe Flow 2: Multiphase Flow Assurance," Bratland, 2010.

[6] V. T. Luan and D. L. Michels, "Explicit Exponential Rosenbrock Methods and their Application in Visual Computing," May 2018.

[7] R. I. Issa and M. H. W. Kempf, "Simulation of slug flow in horizontal and nearly horizontal pipes with the two-fluid model," *Int. J. Multiph. Flow*, vol. 29, no. 1, pp. 69–95, Jan. 2003.

[8] M. Pasqualette and A. Nieckele, "Numerical Simulation of Horizonal Slug Flows with the Two Fluid Model Using Explicit Geometrical Approximations TWO-FLUID MODEL USING EXPLICIT GEOMETRICAL," no. November, 2014.

[9] M. Gourma, N. Jia, and C. Thompson, "Adaptive mesh refinement for two-phase slug flows with an a priori indicator," *Int. J. Multiph. Flow*, vol. 49, pp. 83–98, Mar. 2013.

[10] R. I. Issa, J. Castagna, and A. Sheikh, "Accurate Simulation of Intermittent/Slug Flow in Oil and Gas Pipelines," Jun. 2011.

[11] O. Al-saif, "Slugging in Large Diameter Pipelines : Field Measurements , Experiments and Simulation," Norwegian University of Science and Technology (NTNU - Trondheim), 2015.

[12] S. Shen, Y. Wang, and D. Yuan, "Circumferential distribution of local heat transfer coefficient during steam stratified flow condensation in vacuum horizontal tube," *International Journal of Heat and Mass Transfer*, vol. 114. pp. 816–825, 2017.

[13] A. H. Akselsen, "Characteristic methods and Roe's method for the incompressible two-fluid model for stratified pipe flow," *Int. J. Multiph. Flow*, vol. 89, pp. 81–91, 2017.

[14] O. E. Turgut, M. T. Çoban, and M. Asker, "Comparison of Flow Boiling Pressure Drop Correlations for Smooth Macrotubes," *Heat Transf. Eng.*, vol. 37, no. 6, pp. 487–506, 2016.

[15] R. Szijártó, J. Freixa, and H. M. Prasser, "Simulation of condensation in a closed, slightly inclined horizontal pipe with a modified RELAP5 code," *Nucl. Eng. Des.*, vol. 273, pp. 288–297, 2014.

[16] L. E. Zerpa *et al.*, "Multiphase flow modeling of gas hydrates with a simple hydrodynamic slug flow model," *Chem. Eng. Sci.*, vol. 99, pp. 298–304, 2013.

[17] A. M. Elsafi, "On thermo-hydraulic modeling of direct steam generation," *Sol. Energy*, vol. 120, pp. 636–650, 2015.

[18] J. N. E. Carneiro, R. Fonseca Jr., A. J. Ortega, R. C. Chucuya, A. O. Nieckele, and L. F. A. Azevedo, "Statistical characterization of two-phase slug flow

in a horizontal pipe," *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 33, no. spe1, pp. 251–258, 2011.

[19] M. Kempf, "Simulation of a slug flow in a 'v'-section," London, 1999.

[20] F. Vigneron, C. Sarica, and J. P. Brill, "Experimental analysis of imposed two-phase flow transients in horizontal pipelines," in *7th International Conference*, 1995, pp. 199–217.

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