Non Newtonian Particle Transport Model For Haemorheology

SCHENKEL, Torsten <http://orcid.org/0000-0001-5560-1872>, SPENCER, Tim and HALLIDAY, Ian

NON NEWTONIAN PARTICLE TRANSPORT MODEL FOR HAEMORHEOLOGY

Torsten Schenkel (1), Tim Spencer (2), Ian Halliday (2)

1. Department of Engineering and Mathematics, Sheffield Hallam University, UK;
2. Materials and Engineering Research Institute, Sheffield Hallam University, UK

Introduction

Blood is a non-Newtonian fluid with very complex behaviour deriving from a mesoscopic composition which -minimally described- is a dense, mono-disperse suspension of deformable vesicles suspended in incompressible plasma. Accordingly, blood rheology is dominated by the interaction of cells, with a multitude of models having been proposed to account for such meso-scale effects as deformation, aggregation, and rouleaux formation which underline emergent macroscopic flow properties like concentration dependant viscosity and shear thinning.

The authors are currently developing a multi-scale approach, explicitly modelling meso-scale effects using Lattice Boltzmann Models (LBM), in which erythrocyte mechanics are fully resolved, while describing the macro-scale rheology using particle transport modelling and quasi-mechanistic non-Newtonian rheology models. The latter will eventually be parameterised using LBM data. Here, we present the continuum mechanical part of the modelling approach, which allows the simulation of realistic vessel geometries and complex flow patterns.

Methodology

Our macroscopic model is based on [1] and describes the particle migration based on the gradients of shear strain, concentration and viscosity. The scalar transport equation for haematocrit, H, is

$$\frac{DH}{Dt} = \nabla \cdot (a^2 \, K_c \, H \, \dot{\gamma} \, \nabla H) + a^2 \, K_c \, \nabla \cdot (H^2 \, \nabla \dot{\gamma}) + $$
$$a^2 \, K_\mu \, \nabla \cdot \left(\dot{\gamma} \, H^2 \, \frac{1}{\mu} \, \nabla \mu \right).$$

with a, particle radius, $\dot{\gamma}$, shear strain rate magnitude, μ, dynamic viscosity, K_c and K_μ, collision parameters. Typically, the viscosity is $\mu = f(\dot{\gamma}, H)$, which makes the last source term non-linear, which can, in turn, make the solution of this transport equation difficult. Linearisation of this source term involves the derivative of μ in both $\dot{\gamma}$ and H, which limits the model to the viscosity model, for which it has been implemented [2, 3].

Our current implementation deals with the non-linear, viscosity source term in a way that leaves the viscosity gradient term intact and is thus agnostic to the rheology model used. It has been successfully tested with the classical Krieger-Dougherty model

$$\mu = \mu_P \left(1 - \frac{H}{H_{crit}} \right)^{-n},$$

a modified Krieger model with shear thinning behaviour modelled in the exponent, n, of the classical model [4], and, also, the classical Quemada model

$$\mu = \mu_P \left(1 - 0.5 \left(k_0 + k_\infty \sqrt{\frac{\dot{\gamma}}{\dot{\gamma}_c}} \right) \left(1 + \sqrt{\frac{\dot{\gamma}}{\dot{\gamma}_c}} \right)^{-1} \right)^{-2}.$$

with parameters after Das [5, 6]. Using an under-relaxed, iterative time-stepping method (PIMPLE) a stable solution is achieved for an under-relaxation factor that scales with the ratio of vessel radius to particle collision radius.

Results

Comparison to the analytical benchmark is favourable, compared to previous implementations without application limitations with respect to mesh type. In particular, satisfactory stability and accuracy has been shown for 1D axisymmetric cases, as well as structured hexahedral, but also polyhedral hybrid meshes. The model currently allows e.g. detailed studies of wall shear stress behaviour in micro vessels, but also analysis of the influence of secondary flows in larger vessels on local haematocrit starvation which can lead to atherosclerosis due to local hypoxia in the vessel wall. In the longer term, it will serve as the vehicle for meso-scale LBM data to address temporal and spatial scales of clinical significance.
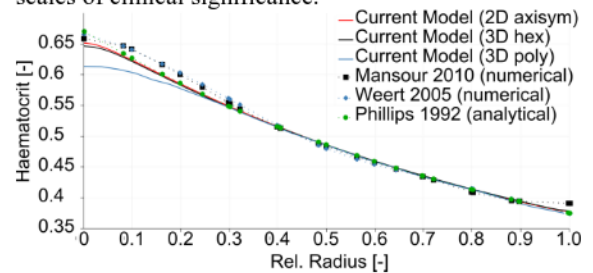

Figure 1: Haematocrit concentration in a 100 μm diameter pipe, Krieger-Dougherty non-Newtonian model (n=1.82, Hcrit = 0.67), Phillips particle transport model (a=3.5μm, Kc = 0.41, Kμ = 0.62). Compared to analytical solution [1] and numerical solutions from [7] and [2].

References

1. Phillips et al, *Physics of Fluids A: Fluid Dynamics,* 4(1), pp. 30-40, 1992
2. Mansour et al. *Biorheology,* 47(1), pp. 73., 2010
3. Chebbie et al, *Journal of Biological Physics,* **44**(4), pp. 591-603, 2018
4. Hund et al, *Fluids,* 2(1), pp. 10, 2017
5. Quemada et al. *Rheologica Acta,* 17(6), pp. 643-653, 1978
6. Das et al, *Biorheology,* 35(1), pp. 69-87, 1998
7. Weert, Master Thesis, Eindhoven University, 2005