A E R O D Y N A M I C A L   I N V E S T I G A T I O N S

with special reference to

the Flow in the Rear of a Rotating Sphere.

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C O N T E N T S .

AERODYNAMICAL INVESTIGATIONS

with special reference to the Flow in the Rear of a Rotating Sphere.

Introduction.

The full equations of motion for the flow of a viscous fluid have proved too complicated for any general solution to be obtained. It is to be doubted if, in the near future, the problem of fluid motion will be solved in a general manner, although solutions for a few special cases which have specified conditions may be found. In view of this it is of some value to investigate experimentally certain cases which are likely to prove of mathematical interest at a later date, although these examples may be beyond the range of analysis for the present. This paper deals with such a problem. A solution for this case is known provided the velocity be small, a restriction usual to many examples of viscous flow. It may be regarded as possible to extend this solution to apply to higher velocities and in doing so some experimental evidence as to the nature of the flow to be ultimately arrived at is invaluable as, by this means, the mathematical analysis may be guided along the right lines.

No such analytical extension is attempted here but the present position of the investigation marks a definite step

towards

towards the solution of the fuller problem which it is hoped to analyse later. It also marks a suitable point for the collection and presentation of the results obtained so far, which results may prove of some general hydro- and aero-dynamical interest.

The instrument used in exploring the fluid motion has a special interest of its own as it may prove of use in further aerodynamical investigations where the flow is of three dimensional character. The developement of this instrument , and its performance when tested, are described in Part I. The theoretical investigations (§§ 1.0 to 1.2) formed the subject of a thesis awarded the James Thomson Centenary Prize of Glasgow University in 1926.

The experiments for the measurement of the forces on a rotating sphere, as described in Part II, were the first of the tests to be performed. It was from these that a suitable wind speed and speed of rotation were arrived at for the main experiments described in Part III. These experiments deal with the exploration of three sections behind a 6" dia. sphere placed in the wind channel and rotating at a constant speed about an axis at right angles to the wind direction. Part III gives a description of these tests and the method of reproducing the results on suitable diagrams.

The investigations of Part II were deemed necessary because of the extremely unstable condition of the flow that is present when a stationary sphere is placed in a stream. The

flow

flow past a rotating sphere can hardly be expected to resemble the flow past a stationary sphere, except for low values of spin, because of the very different boundary conditions. It is most desirable, however, that the flow investigated be of a steady nature and the results of Part II suggested suitable conditions for the investigations of Part III.

In Part IV a short analysis is given for the fluid motion past a rotating sphere, the solution being applicable when the inertia forces of the fluid may be neglected in comparison with the viscous forces. Lines of investigation are indicated for extending this solution, so as to apply in the more general case when inertia cannot be neglected.

All the experimental work described in this paper was performed in the 2 ft. wind channel of the James Watt Engineering Laboratories, Glasgow University.

PART I.

The Spherical Velocity Gauge.

1.0 Many types of velocity gauge have been designed from time to time, of which most, if not all, of the pressure type have been "null reading" instruments. These gauges have to be capable of rotation until certain pressure differences are made zero, the angles of rotation giving the pitch and yaw of the wind, while the air speed is obtained from a separate pressure reading. The three dimensional velocity gauge designed and in use at the N.P.L. is of this class.* It is a delicate, and hence costly, instrument and the supports are relatively large for use in experimental work where the space is restricted. The gauge described here does not require rotation. It gives the direction and speed at a point from simultaneous readings of three pressure differences. If the flow be steady, one manometer gives the three readings in a short period of time by taking the observations consecutively.

Two sets of experiments were performed, the first on a 2" diameter model and the second set on a ½" diameter gauge, which instrument was used later in the investigations of Part III.

1.1 Theoretical.

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* Report of the Advisory Committee for Aeronautics, 1914-5 p. 178 and Report T.1761.

1.1 Theoretical. Consider the sphere shown in Fig. 1. Let A,B,C,X be small pressure holes on the surface. The holes A,B,C are equidistant from X and equidistant from each other. A is taken to lie on the vertical great circle through X. The arcs AX, BX, CX each subtend an angle θ at the centre of the sphere.

Let OP lie along the wind direction; then P is the point on the sphere where the pressure is $\frac{1}{2}\rho V^2$ above the static pressure and the pressures at A, B, C, X will depend on the angles subtended by PA, PB, PC and PX at the centre of the sphere.

By an application of the theory of dimensions it is found that the pressures at A, B, C, X may be written in the form

$$p_A = \tfrac{1}{2}\rho V^2 . f(VL/\nu , PA)$$

with three similar equations for p_B , p_C , p_X . Here f denotes some function of the non-dimensional quantities VL/ν and PA, and may be found from hydrodynamical theory or from experiment.

Suppose P_{AX} , P_{BX} , P_{CX} denote the pressure differences recorded on gauges connecting A and X, B and X, C and X. Then, considering VL/ν to have no effect on f for the present, we have

$$\begin{aligned} P_{AX} &= p_X - p_A \\ &= \tfrac{1}{2}\rho V^2 . \left\{ f(PX) - f(PA) \right\} \quad . \quad . \quad . \quad (1) \end{aligned}$$

with two similar equations for P_{BX} , P_{CX} .

Let α and β denote the angles of pitch and yaw through

which

which the sphere has to be rotated to make X coincide with P. The arcs PA, PB, PC and PX of great circles on the sphere are functions of $\underline{\alpha}$, $\underline{\beta}$, $\underline{\theta}$. Their relationships, as found by spherical trigonometry, are

$$PX = \cos^{-1}(\cos\alpha \cos\beta)$$
$$PA = \cos^{-1}(\sin\theta \sin\alpha + \cos\theta \cos\alpha \cos\beta)$$
$$PB = \cos^{-1}(-\tfrac{1}{2}.\sin\theta \sin\alpha + \cos\theta \cos\alpha \cos\beta + \tfrac{\sqrt{3}}{2} \sin\theta \cos\alpha \sin\beta)$$
$$PC = \cos^{-1}(-\tfrac{1}{2}.\sin\theta \sin\alpha + \cos\theta \cos\alpha \cos\beta - \tfrac{\sqrt{3}}{2} \sin\theta \cos\alpha \sin\beta) \quad . \quad . \quad . \ (2)$$

These four equations express PX, PA, PB and PC as functions of $\underline{\alpha}$, $\underline{\beta}$, $\underline{\theta}$ and the values obtained might be substituted in equations (1) thereby expressing the recorded pressure differences in terms of $\underline{\alpha}$, $\underline{\beta}$, $\underline{V}$ and $\underline{\theta}$, the form of the function $\underline{f}$ being known.

Introduce at this stage two auxiliary functions $\underline{\Phi}$ and $\underline{\Psi}$, involving the pressure differences P_{AX} , P_{BX} , P_{CX}, these functions being chosen in such a way as to be independent of the velocity if, as was assumed above, change in VL/ν does not affect the function $\underline{f}$. It is necessary too that they should have no singularity in the region of (α , β) considered.

Forms of $\underline{\Phi}$ and $\underline{\Psi}$ found to be satisfactory are :

$$\Phi = \frac{P_{AX} - P_{BX}}{P_{AX} + P_{BX} + P_{CX}} = \frac{f(PB) - f(PA)}{3.f(PX) - \{f(PA) + f(PB) + f(PC)\}}$$

$$\Psi = \frac{P_{AX} - P_{CX}}{P_{AX} + P_{BX} + P_{CX}} = \frac{f(PC) - f(PA)}{3.f(PX) - \{f(PA) + f(PB) + f(PC)\}} \quad . \quad . \quad . \ (3)$$

$\underline{\Phi}$ and $\underline{\Psi}$ may be regarded as calculable from two standpoints, viz., (a) from the pressure differences observed, or (b) knowing $\underline{\alpha}$, $\underline{\beta}$, $\underline{\theta}$ and the form of $\underline{f}$, the expressions on the right of equations (3) give $\underline{\Phi}$ and $\underline{\Psi}$, equations (2) being used to find PX, PA, &c.

A third function $\underline{F}$ is defined by any of the three equations for P_{AX}, P_{BX} or P_{CX}, say

$$P_{AX} = \tfrac{1}{2}\rho V^2 . \left\{ f(PX) - f(PA) \right\} = \tfrac{1}{2}\rho V^2 . F$$

If the form of $\underline{f}$ be known (either from theory or from experiment) then it is seen that the three functions $\underline{\Phi}$, $\underline{\Psi}$ and $\underline{F}$ may be calculated in the case of a special instrument for various values of $\underline{\alpha}$ and $\underline{\beta}$. The results may be embodied on two suitable charts, the first giving $\underline{\alpha}$ and $\underline{\beta}$ for varying $\underline{\Phi}$, $\underline{\Psi}$ and the second chart giving $\underline{F}$ for $\underline{\alpha}$ and $\underline{\beta}$.

It will be realised, however, that the most desirable means of obtaining these charts would be to run a series of tests on the instrument when the wind speed is known and kept constant. By giving the gauge suitable angles of pitch and yaw, a set of pressure difference readings may be obtained from which the charts can be derived since the velocity is known.

The method of procedure for the determination of $\underline{\alpha}$, $\underline{\beta}$, $\underline{V}$ at a point is :

(1) From the pressure readings obtain $\underline{\Phi}$ and $\underline{\Psi}$ as given by equations (3),

(2) Read off $\underline{\alpha}$ and $\underline{\beta}$ from the first chart for these values of

of $\underline{\Phi}$ and $\underline{\Psi}$,

(3) From the equation $V^2 = P_{Ax}/\frac{1}{2}\rho F_{Ax}$ or either of the two similar equations obtain the value of V, F_{Ax} being obtained from the second chart as $\underline{\alpha}$, $\underline{\beta}$ are known.

1.2 In order to give some idea of the working of the instrument a preliminary investigation was made to find the probable form of the $\underline{\Phi}$ and $\underline{\Psi}$ chart. The pressure distributions over a 4" diameter sphere, when placed in an air current of various speeds up to 45 f./s., were obtained and the results are embodied in Fig.3 which shows the function $\underline{f} = p/\frac{1}{2}\rho V^2$. It will be seen that the effect of VL/ν on the form of $\underline{f}$ over the front portion of the sphere is not very great for at least the range of speeds tested, and part of the effect noted may be attributed to the interference of the channel walls.

The flow round the sphere appears to develope turbulence at a point about 60° or 65° from the nose, so readings beyond this are not to be relied upon. It may be remarked in passing that the curves would indicate that from 140° or 150° to 180° the conditions appear to be more steady as shown by the consistency of the pressure readings in this region.

Taking the curve shown in Fig. 4 as the form of the function $\underline{f}$ which holds up to 80° from the nose of the sphere, this curve was made the basis of a preliminary investigation in which $\underline{\Phi}$ and $\underline{\Psi}$ were calculated for sets of $\underline{\alpha}$, $\underline{\beta}$ as indicated by equations (3). The value of $\underline{\theta}$ assumed was 45°, which

value

value placed the holes A, B, C near to static pressure when the wind was in the direction BX. The results of these calculations are given in Tables 1 to 4.

The chart for these values of $\underline{\varphi}$ and $\underline{\psi}$ is shown in Fig. 2. As the form was considered to be satisfactory it was deemed unnecessary to calculate the forms of the function $\underline{F}$.

1.3 A model gauge for which the sphere was 2" diameter was constructed along the lines shown in Fig. 5. The sphere was of box-wood turned to give a smooth and accurately spherical surface. The ¼" diameter supporting tube was of copper and was screwed firmly into the sphere along a radial hole. The small holes in the sphere were bored as accurately as possible with $\underline{\theta} = 45^\circ$, and aluminium tubing of 0.036" external diameter was run through these holes and continued along the copper tube. One end of the tubing was cut off flush with the surface of the sphere while at the other end connection was made to pieces of glass tubing, the joints being made air tight by sealing with wax. Fig. 6 shows the model mounted in the channel, and indicates how the pressure tubes of the gauge were connected by rubber tubing to the two sides of the Chattock gauge, through a set of stop-cocks fixed on the side of the channel. Any of the pressure differences P_{AX}, P_{BX}, P_{CX} could be applied to the Chattock manometer by manipulation of the stop-cocks.

The gauge was set up in the channel with the holes A and X approximately in a vertical plane. The gauge could be given

yaw

yaw by rotating the supporting spindle, while pitch was given by rotation of the instrument in a vertical plane about the pin which attached it to the spindle. The angles were measured by suitable scales fixed on the top of the channel and on the spindle respectively, the latter one being removed when the channel was running. The zero angles of pitch and yaw were obtained by bringing the three pressure differences to the same value. The pressures were noted when the gauge was given specified angles of pitch and yaw between $+30^\circ$ and -30°. The channel velocity was kept constant at 14.9 f./s.

$\underline{\Phi}$ and $\underline{\Psi}$ for the different values of $\underline{\alpha}$ and $\underline{\beta}$ were calculated and the chart obtained is shown in Fig.7 where it is compared with the chart calculated by the methods of § 1.2 . The experimental chart is not quite symmetrical due to some of the pressure holes being displaced slightly. As was to be expected from the pressure distribution curves of Fig. 7 , the values of $\underline{\Phi}$ and $\underline{\Psi}$ deviate from the calculated values for angles greater than about 15°, but it was not reasonable to expect too great a correspondence beyond this range.

The channel velocity being known for these tests, the function $\underline{F}$ was calculated and a chart of $\underline{F}$ is shown in Fig. 8. It was considered more accurate to use three different $\underline{F}$ functions according to the (α, β) dealt with. The pressure $P_{A\alpha}$ was used when (α, β) lay nearest to the hole A, and so on.

A further test was carried out in which the pitch and yaw of the gauge and the channel speed were unknown to the

observer

observer who determined $\underline{\alpha}$, β, $\underline{V}$ from the pressure readings. Within the pitch and yaw limits of $+15^\circ$ and -15° the angles were correct to within 1° and the velocity was read to about 2% at 27 f./s.

1.4 <u>The Final Instrument.</u> The tests on the 2" diameter model indicated that the instrument was a feasible proposition and that, moreover, with a properly designed gauge the results could be satisfactorily accurate. Therefore, a gauge was designed as shown in Fig. 9, and was manufactured from this design by the Bar Knight Model Engineering Co. of Glasgow, E.

The ½" diameter sphere and screwed cylindrical part of 3/16" length were of brass. Steel tubing of ¼" external diameter was screwed on to the cylinder firmly. Along this tubing there passed four steel tubes of 1/16" diameter which were connected to the fine pressure holes in the sphere in the manner indicated in the figure. A brass plug in the end of the ¼" dia. tube kept the smaller tubes in position. The value of $\underline{\theta}$ taken for this instrument was $32\frac{1}{2}^\circ$ as the tests of § 1.3 indicated that 45° was too great, the range of (α, β) measurable with any great accuracy being rather small to be of much practical value. With $\underline{\theta} = 32\frac{1}{2}^\circ$, the pitch and yaw were considered to be fairly correct when $\underline{\alpha}$ and β were within the limits of about $\pm 25^\circ$ or $\pm 30^\circ$.

Fig. 10 is a photograph of the instrument and its supporting rod. The figures on the rod mark inches whereby the instrument could be set at any position along the spindle. One

side

side of the rod was milled plane in order to give the same yaw at any position. A special brass attachment made the instrument capable of rotation about the main $\frac{1}{4}$" dia. tube, a screw fixing it in position when the holes A and X were nearly in the same vertical plane. ~~If~~ Pitch could be given by rotation about a short horizontal spindle connecting two parts of the brass attachment. The photograph shows the gauge with an upward pitch.

The instrument was mounted in the channel and was tested under conditions similar to those under which the model had been tested. The channel velocity at which the gauge was calibrated was 27.0 f./s. Tables 5 give the observations and the functions $\underline{\phi}$, $\underline{\Psi}$ and $\underline{G}$ derived from these readings. Throughout all the tables the pressure differences $\underline{P_{AX}}$, &c, are recorded as the number of revolutions of the disc on the Chattock gauge. If these differences be desired in lbs./ft.^{2} the transformation be achieved easily as 1 Turn = 0.1017 lb/ft.^{2}. Fig. 11 shows a reduced form of the ϕ and Ψ chart obtained. On the full scale chart it was possible to interpolate and draw lines at intervals of 1°.

The G-functions are a modified form of the F-functions. Previously, we defined the F-function by the equation

$$P_{AX} = \tfrac{1}{2}\rho V^2 . \left\{ f(PX) - f(PA) \right\} = \tfrac{1}{2}\rho V^2 . F$$

The G-function is now defined by the equation

$$P_{AX} = V^2 . G/10^2 ,$$

this being a more convenient system for finding the velocity

in

in practice. Fig.12 gives the G-functions as derived from the tests on the gauge.

As in the case of the model, the instrument underwent a test designed to find its accuracy. Table 6 records the observations and the values of $\underline{\alpha}$, $\underline{\beta}$, $\underline{V}$ derived from them.

A comparison of the actual settings and the observed readings indicates that the instrument is decidedly better than the model with regard to the measurement of the angles but that the speed determined ~~is~~ has not improved greatly in accuracy. This might have beem expected as the angle $\underline{\theta} = 32\frac{1}{2}^\circ$ in the instrument reduces the pressure differences considerably under those obtained with $\underline{\theta} = 45^\circ$. Table 6 shows that the angles are correct to $\pm 0^\circ.5$ and this may be taken as very satisfactory as the setting of the instbument can be guaranteed only to about $\pm 0^\circ.2$. The speed appears to be correct to about 2.0%, although five of the eight readings are correct to less than 1.0%.

In conclusion, a few remarks on the final instrument may be appropriate. As pointed out at the beginning, the main advantages are its compactness and independence of any need of rotation provided the angles dealt with are not too great. It is a decided disadvantage that the range is confined to $\pm 25^\circ$ or $\pm 30^\circ$ but there are many applications in aerodynamics to which such an instrument may be useful. Of course the range may be extended by giving the gauge initially a certain amount of pitch or yaw.

With regard to scale effect , the results tabulated in

Table

Table 6 indicate that no great variation is noticeable in the range dealt with in these tests. In cases where the speed range is very much greater, methods can be devised easily for allowing for such an effect.

The ideal method of using the instrument would be to derive the function <u>f</u> from the stream function ψ, for for the flow past a stationary sphere, were this latter function known. This function <u>f</u> would be made the basis of both the charts and the necessary corrections for large ranges of VL/ν.

As will be shown by the results of Part III, the gauge in its present form has proved quite a useful and reliable instrument.

PART II.

The Aerodynamical Forces on a Rotating Sphere in an Air Current.

2.0 A sphere in translational motion and spinning about an axis at right angles to its direction of flight presents certain interesting aerodynamical features which have been known well for a considerable time. The resultant force on this sphere does not act opposite to the direction of motion but is such that there is a component at right angles to the flight path. Hence the path differs from that of a non-rotating sphere.

As far as records exist, Newton appears to have had a remarkably clear conception of the phenomenon considering the early date on which he wrote. In a letter describing his investigations on dispersion in 1666, he refers to his observation of the phenomenon and gives what he conceived to be the cause. He says that he had "often seen a tennis ball, struck by an oblique racket, describe a curve line. For, a circular as well as a progressive motion being communicated to it by that stroke, its parts, on that side where the motions conspire, must press and beat the contiguous air more violently than on the other ; and there excite a reluctancy and reaction of the air proportionally greater." Newton's idea lacks only in the insight of the fluid mechanism which has been made possible by recent advances in hydro- and aero-dynamics.

About 1747, Robins experimented by firing bullets from

a

a gun whose barrel was bent slightly so that the shot was given spin before leaving the muzzle. He experimented also with a pendulum rotating about its supporting chord. With a physical view very similar to that of Newton, Robins found it easy to predict the direction of displacement before performing any experiment.

It is of interest that both Euler and Poisson appear to have had an erroneous physical conception of the phenomenon for they regard the side force, or lift, as a consequence of the compressibility of the air. They consider that the density on the front hemisphere must exceed that on the rear so that the frictional force is greater on the front than on the rear. This certainly gives a side thrust but the direction is opposite to that observed. Poisson, in his analysis, showed the deflecting force to be small.

Rayleigh deals with the problem of the path of a spinning tennis ball in one of his papers*, but, in his analytical investigations, he confines himself to the more straight-forward case of the flow past a rotating cylinder. He considers the fluid to be inviscid and that there exists a circulation round the body. The motion is taken also as irrotational.

Tait appears to have spent a great amount of time and thought on considerations of the path of a rotating sphere. In two memoirs† he deals with the problem, in attempting to

explain

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* Rayleigh, Mess. of Maths. vii, 14, (1878).

† Tait, Trans. Roy. Soc. Edin. V.37, p.427 ; V.39, p. 491.

explain the observed flight of a golf ball and to find the essentials of a good drive. His papers are of considerable interest from the dynamical, if not from the hydrodynamical, standpoint.

In reviewing the literature of this subject, it is to be remarked that very little advance has been made in explaining the motion of the fluid over that given by Newton. It was evidently Newton's conception that the spinning sphere produces a swirl of air round it, and that this swirl modifies the uniform streaming so that, in consequence, the pressure is diminished on one side while increased on the other, thus giving a side thrust. As will be shown in Part III, this is only the first feature of the phenomenon. The difference of conditions on the two sides of the sphere produce a type of motion which none of the investigators mentioned above seem to have conceived.

As no experimental determination of the forces experienced by a spinning sphere in an air current appear to have been made previously, it was considered that such an investigation would be of some aerodynamical interest and that the results might indicate also some of the features of the fluid motion.

2.1 The Experiments. The balance used to find the forces on the rotating sphere was that designed by Dr. A. Thom, and is described by him in a paper on the forces on rotating cylinders*. Fig. 3 of that paper shows the balance as used in the tests described here, with the modification that a 6" dia.

wooden

* Thom, Aeronautical Research Committee, R. and M. 1018.

wooden sphere was mounted in the centre of the channel.

Throughout any test, the wind speed was maintained constant by observing the pressure in the throat of a Venturi tube, placed some distance ahead of the working section and about 4.5" from the channel wall. This pressure was recorded on a U-tube containing water, it being arranged by a system of mirrors that both meniscuses were visible in the field of the microscope. Temperature effects were thus eliminated and the channel velocity was obtained from a previous calibration. This manometer was developed by Dr. Thom and is described fully by him in his thesis for Ph. D. (Glasgow University, 1926).

For a constant channel speed, the moments M_1 and M_2 were observed when the sphere was spinning first in one sense and then in the opposite sense. If $\underline{T}$ be the air torque acting on the sphere, then, for a definate rate of spin,

$$\text{Lift} = L' = M_1 - M_2 - 2T / \sqrt{2}d$$

$$\text{Drag} = D' = M_1 + M_2 / \sqrt{2}d$$

where $\underline{d}$ is the length of the supporting arm of the balance.

The balance was not adaptable for the measurement of air torque on the sphere except when the channel was stopped. However, running down tests indicated that the difference in the air torque for the wind on and for the wind off was very slight. This was borne out also by the fact that when the wind was put on with the sphere spinning, the rate of spin was but very slightly affected. The torque was found to be about 2 oz.ins. at 4000 r.p.m. and a linear law was

assumed

assumed whereby the torque was taken as 0.5 oz.in. per 1000 r.p.m. This was the value assumed in both preliminary and final tests.

In the preliminary experiments the spindle carrying the sphere was of 7/8" diameter, and it was estimated that the force on the spindle itself accounted for a large part of the force measured on the balance. The spindle was corrected for in the following manner. The velocity was taken at several points on a vertical line between the sphere and the top of the channel. The root-mean-square velocity for this section was found to be 26 f./s. when the channel velocity was 23 f./s. It was assumed that the velocity of the air in the region of the spindle had an increase of the same proportion. The lift and drag of the exposed spindle for this air velocity were obtained from previous experiments on rotating cylinders and these were subtracted from the observed forces.

By the Principle of Dynamical Similarity it may be shown that the lift and drag are of the form

$$K(VL/\nu,\ v/V)\,\rho L^2 V^2$$

where V = channel speed, v = equatorial speed of the sphere, L = diameter of the sphere, ρ = density and ν = kinematic viscosity of the air.

Hence K_L, the lift coefficient, and K_D, the drag coefficient, are some functions of the non-dimensional quantities VL/ν and v/V. Fig. 13 shows the coefficients obtained from the preliminary tests plotted on a v/V base for constant channel speed,

i.e.,

i.e., constant VL.

In the later tests, the spindle used was 0.45" diameter and it was shielded from the air stream by 7/8" diameter guards which extended from the channel walls to about 5/8" from the surface of the sphere. Fig.14 gives the results of these later experiments.

2.2 Remarks on the Results. In Fig. 14, the curves for V = 10 f./s. cannot be regarded as altogether satisfactory, for the balance moments measured at this wind speed were very small and the curves may be somewhat in error. The curves obtained for the other four wind speeds appear satisfactory.

The preliminary curves for K_L on Fig. 13 agree with those on Fig. 14 for low values of v/V ; they reach a slightly greater maximum and thereafter swing about the values obtained in the later tests.

One special feature to be noted is the appearance of negative lift at low values of v/V and high values of VL. No satisfactory explanation has been postulated so far. It may be due to turbulent flow at small rotations or to some effect of channel wall interference. On the other hand, there may be a tendency for some type of flow to develop and produce negative lift. The experiments on rotating cylinders show a similar feature. As it is shown in Part IV that there is no lift if VL/ν be small and $0 < v/V < 1$, it seems most feasible that the turbulence of the fluid motion is the cause.

It will be noticed that the K_L curve rises steeply from

v/V

$v/V \doteqdot 0.5$. This is contrary to what several investigators have predicted. In a letter to Tait, Stokes said that he considered that, provided v/V was small (<1), then the lift component would be proportional to $V\omega$, where ω = angular speed of the sphere. This would make $K_L \propto v/V$, but these experiments indicate that it would be more correct to take $K_L \propto (v/V - 0.5)$ for a short range of v/V above 0.5 .

In the final tests, the drag coefficients are grouped together for $v/V > 0.5$. If it were considered legitimate to extend these curves backwards for $v/V \to 0$, values of the drag coefficient might be obtained for a stationary sphere in an air stream, the flow being considered steady and not turbulent.

PART III.

The Exploration of the Flow in the Rear of a Rotating Sphere.

3.0 Of recent years, the physical conception of air-flow has received several modifications due to the advancement of the theory associated with Prandtl. This theory, which claims to be only a first approximination in analysis, deals chiefly with the flow past aerofoils -- an aerofoil being defined as a surface which gives a large value of lift/drag. In some respects the rotating sphere may be regarded as a limiting form of a finite aerofoil -- the aspect ratio decreasing to unity and also the section getting thicker. To this extent the problem of the rotating sphere may be considered as a step towards the more general investigation in which the surfaces are not necessarly of aerofoil character. However, it is obvious that there are many wide differences, which are due mainly to the different boundary conditions.

In the Prandtl theory of the flow past a monoplane, one is given the picture of a vortex sheet extending in the rear of the body from the trailing edge, and being so modified at the wing tips to form two vortices. It was considered of some interest to investigate whether trailing vortices were formed in the rear of a spinning sphere and in what manner they modified the flow.

From past experience, aerodynamicians are rather shy of problems concerning the flow past a stationary sphere since the motion in the rear is extremely turbulent and drag determinations

minations have never been satisfactory. It must be noted at the outset that the problem of the flow past a rotating sphere must not necessarly have the disadvantages of that past a stationary sphere. The boundary conditions in the two examples are entirely different so that it is not advisable to connect the two problems except when the speed of rotation is low. In the tests described in this part, the rotation was great, thus giving an intense circulation round the equatorial section of the sphere and adding, it was considered, to the stability of the flow.

3.1 The Axes of Co-ordinates. Diagram 3a gives the set of co-ordinate axes adopted in these tests. For an observer looking upstream from the sphere, the positive x-direction was taken forward while y and z were measured towards the right and vertically downward respectively.

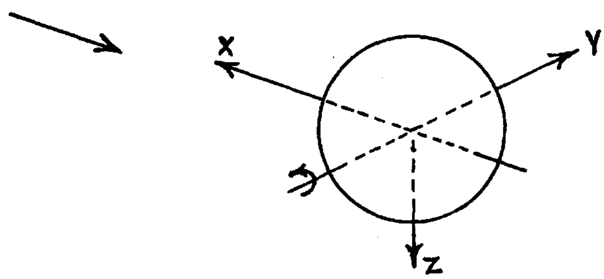

Diagram 3a.

In measuring pitch and yaw, the convention was the same as that used when testing the velocity instrument. The wind was considered to have positive pitch and yaw when it had components along the positive z-axis and the negative y-axis respectively.

The direction of rotation of the sphere was such as to

give

give lift along the positive z-direction, i.e., vertically downward.

In representing the results by contour diagrams of pitch, yaw and velocity (Figs. 19 to 27) it was considered desirable that these figures should be drawn similar to the contour diagrams given in aerofoil work. Hence z has been drawn vertically upward. In discussing these diagrams it must be remembered that the top part of the sphere is approaching the observer while the bottom part is receeding. The sphere is shown by a broken line in each figure. Here, positive pitch represents an up-current along the positive direction of z.

In practice, it was found convenient to use the scale on the supporting rod of the velocity gauge for measurement in the z-direction, and all references to z have been made with respect to this scale. This displaced the centre of the sphere by -9/16" along the z-axis. By measurement, it was found that the sphere had a slight displacement along the y-axis relative to the gauge, when this instrument was placed at $y = 0$ by the scale on the top of the channel. This was evident again when plotting the pitch readings. Hence, the y-axis was displaced by 0.10" ; the pitch readings on different sides of $y = 0$ were found then to correspond very closely. The centre of the sphere has to be taken as 3", 6" and 9" ahead of the point marked on the z-axis in the contour diagrams.

3.2 The Apparatus. A wooden sphere of 6" diameter was mounted on a 0.45" diameter spindle which passed through holes in the side walls of the channel. The housings for the ball-races on

which

which the spindle ran were fixed rigidly to steel strips mounted horizontally on the channel. These strips had holes bored suitably for mounting the sphere at three positions 3" apart down the channel. Thus, throughout these tests, the velocity instrument was kept in the same section of the channel and the sphere itself was moved to change the section of investigation behind the body.

A small electric motor, fixed on the roof of the channel, was connected to one end of the spindle by a belt drive. A Bonniksen cyclometer, mounted to the channel wall by a frame, was attached by a piece of flexible rubber to the other end of the spindle. This cyclometer proved to be most valuable, for, though only registered to read 1000 r.p.m., it was found to act very satisfactorily at much higher speeds, care being taken that the revs. recorded were of the correct order, as no record of the thousands was given on the dial. This was easily achieved by observing the revs. as they increased steadily.

In order to eliminate the effect of the rotating spindle on the air pattern, the spindle was enclosed in thin tubes of sheet tin, thus making the external diameter approximately ½". These tubes extended from close to the surface of the sphere to the channel walls and were fixed so that they did not rotate with the spindle. Friction was diminished by inserting thin metal rings at the ends of the tubes.

It was found that vibration of the rotating system was aggravated at certain rates of revolution and wind speeds by slight vibrations of the channel as a whole. This was remedied

by

by attaching a wire to one of the spindle tubes, about 2" from the surface of the sphere, and running it along the channel and through the honey-comb to the outside, where it was fixed under a small tension.

Throughout these tests, the channel velocity was maintained at V = 27 f./s. by means of the U-gauge referred to in Part II. This gauge was connected to the static hole in the channel wall and Fig. 15 shows its initial and final calibrations. The speed of rotation of the sphere was 4130 r.p.m. which gave v/V a value of 4.0 . Care was taken that the revolutions did not vary by more than ±30 r.p.m.

The investigations of Part II indicated that, with V = 27 f./s. and v/V = 4.0 , the lift and drag coefficients had become constant so that the flow in this instance should be expected to be steady and not to vary greatly with slight variations of V and v/V . In addition, the velocity gauge had been calibrated initially for V = 27 f./s., so this instrument should be expected to be functioning under the most favourable conditions.

3.3 The Experiments. The three sections explored were X = -3", X = -6" and X = -9". It was considered that these three sections should show the main character of the flow behind a rotating sphere when v/V is large.

In exploring the section at X = -3", it was found impossible to use the velocity gauge immediately behind the sphere. The moving surface produced large values of pitch and yaw,

making

making it difficult to manipulate the instrument close to the body.

By means of the sliding shutter on the roof of the channel, the gauge was made to traverse a series of lines behind the sphere for which $\underline{z}$ was constant. It was found necessary to give the gauge an initial pitch of 20° for the central readings at $z = -4''$. This kept the angles within the range of the instrument, which was moved down the supporting spindle in order to bring the recording part of the gauge into its former position. Similar angles were given in other regions where it was obvious that such a step was necessary. The angle of bias was kept small and never exceeded 20° of pitch. Yaw did not require any initial setting although there were indications of this being necessary had it been possible to get immediately behind the sphere.

Observations were taken at over 150 points in this section. Only a very few of these were found to lie too far out of the range of the instrument to be valueless.

The sphere was then moved forward by 3" and part of the section at $X = -6''$ was explored in a similar manner, starting with $z = -6''$. At $z = -2''$, the instrument had to be compensated for both pitch and yaw, the bias being 15° in each case. Even then, the central readings were beyond the range of the instrument and tests with smoke jets, etc., were carried out to get an idea of the magnitude of the angles. These tests will be referred to later. It was discovered from them that the angles were very large . Hence the sphere was moved back

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by 3" and the gauge was given a downward tilt of $54\frac{3}{4}°$, thus bringing the instrument into its correct position relative to the sphere. Readings were obtained immediately behind the sphere by these means.

The majority of the 220 observations taken over this section were within the range of the instrument. Care had to be taken, however, in the manipulation of the gauge as great changes often occured in very short distances and, in traversing some of the lines, the magnitude of the angles had to be anticipated to a certain extent.

Over the section at X = -9", very little trouble was experienced. Changes in the angles were more gradual than in the other sections. About 150 points were observed.

3.4 Reduction of the Observations. All the observations were tabulated, and the values of the Φ, Ψ and $\underline{G}$ functions were obtained for each point. The pitch, yaw and velocity were calculated by the methods given in Part I.

Readings of pitch for negative values of $\underline{y}$ were superimposed upon those for positive values of $\underline{y}$ and curves were drawn through the points.

Yaw readings for negative values of $\underline{y}$ were changed in sign and plotted with the readings obtained when $\underline{y}$ was positive.

It was considered desirable that the ratio V/U be plotted rather than $\underline{V}$, where $\underline{V}$ = speed at point and $\underline{U}$ = channel speed = 27 f./s. This gives a more general indication of the conditions over the field. The ratio V/U was found at all the points observed and was plotted after the manner of the pitch readings.

The

The readings were usually of the same order for points beyond $y = 4''$; it was considered better to tabulate the mean reading at these points rather than plot all the observations in the diagrams of α , β and V/U.

The contour diagrams were obtained from the curves of observations and the cross-plotted curves of these quantities.

Figs. 16-17-18 give the observations of α , β , V/U at the three sections while Figs. 19 to 27 give the contours for the same sections.

3.5 Discussion on the Observation Curves. It will be noticed that all pitch readings are very consistent and that there are no excessive divergences from the curves. The yaw and V/U observations are not quite so good, but it was found usually that a curve could be drawn through the points in a satisfactory manner.

Lines shown broken in Sections $X = -3''$, $X = -6''$ are indications only of the curves, the number of observations not being sufficient to establish the forms definately. No great error in these curves is anticipated.

The observations obtained by tilting the gauge steeply in Section $X = -6''$ are satisfactory from the standpoint of consistency.

The velocity along $z = 0$, between $y = 4$ and $y = 8$, is found to be greatly reduced due to the presence of the spindle some distance ahead. Readings were taken along this line in the $X = -6$ and $X = -9$ sections. In the section $X = -3$, readings were taken along $z = -1$ and $z = 1$, the line $z = 0$ being omitted.

It

It was not realised at the time of the tests that the spindle would have so great an effect ; otherwise, some means of eliminating this effect would have been devised.

The line z = 0 has not been considered beyond y = 3, in plotting the contours of V/U in section X = -6. At X = -9, however, the effect of the spindle is shown, though it might have been preferable to have omitted readings beyond y = 3.

3.6 <u>The Contour Diagrams.</u> These diagrams give the best general view of conditions over each section. There are several details which stand out clearly in these figures.

Perhaps the most important result of these experiments is the establishment of the vortices behind the sphere. Figs. 25 and 26 show the vortex very clearly. In the diagram of pitch, positive contours indicate an up-current while negative contours show a downward tendancy of the wind. In the diagram of yaw, positive contours show that the wind has a component towards the left and negative contours show a motion towards the right. It will be seen, by superimposing the two diagrams, that there is a general rotary motion ; the centre of rotation may be taken approximately as the point of intersection of $\alpha = 0$, $\beta = 0$, viz. (y = 1.45, z = -2.4) . Comparison of the figs. for sections X = -6, X = -9 show that the vortex does not travel horizontally down-wind, but is inclined at an angle. This was to be expected as one vortex must influence the other. Towards the core of the vortex, the speed of the air is low but appears to rise slightly as it travels down-stream.

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In seeking the means by which the vortex forms, Figs. 19 - 21, which show the conditions at X = -3, may be considered. The speed of the air below the sphere is seen to be extremely low due to the under surface of the body being in rapid motion against the main stream. On the other hand, the speed above the sphere seems to be increased only slightly above the channel speed. This might be taken to indicate that there is considerable dissipation in the fluid around the body. The pressure difference above and below the sphere, brought about by the great difference in the speed of the air, gives rise to the lift, as conceived by Newton and other investigators. This pressure difference must be responsible also for the forming of the vortices. Figs. 19-20 indicate that the air under the sphere tends to curl round the lower quarter of the body, while the air from above descends steeply in a contracting band. The rotation must be set up in this manner. It would be necessary to know the conditions immediately behind the sphere to get a clearer view of the motion.

Several other points may be noted in the diagrams. At X =-6, the angles of pitch are seen to vary very rapidly in the region where the up-current meets the descending current. There appears to be a slight in-flow along the y-axis some distance behind the body. This is seen in the yaw diagrams.

The low velocity near (y = 2, z = 0) at X = -6, -9 may be due in part to the spindle.

3.7 <u>The Flow ahead of the Sphere.</u> It has been mentioned above that some difficulty was experienced in obtaining an indication of of the angles in certain regions behind the sphere. Hence, an attempt was made to trace the flow by means of smoke jets. The apparatus was similar to that used at the N.P.L. in investigating the discontinuous flow past a bluff obstacle. The jets were of ammonium chloride.

Very little knowledge of the conditions in the rear of the sphere was gained by these tests. The jets spread almost immediately on leaving the mouth of the tube.

An indication of the flow in front of the sphere was quite easily obtained, however. It was found that the flow appeared to be little affected by the rotation of the body a short distance ahead. The velocity gradient over the forward part of the sphere seems to be great,and it appears that the flow not very far from the body is little different from that obtained without rotation. These statements would require to be confirmed as no observations were taken in this region and the jets merely give an indication of the flow.

PART IV.

The Slow Viscous Flow past a Slowly Rotating Sphere.

4.0 The notation used throughout this part will be the same as that used in Lamb's "Hydrodynamics" and will require no explanation.

The equations of motion of a viscous fluid, as found by Navier, Saint-Venant, Poisson and Stokes, may be written

$$Du/Dt = X - \frac{1}{\rho}.\partial p/\partial x + \nu.\nabla^2 u + \frac{\mu}{3}.\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \quad . \quad . \quad (1)$$

with two similar equations, where the operator D/Dt denotes $\partial/\partial t + u.\partial/\partial x + v.\partial/\partial y + w.\partial/\partial z$.

It being permissible for all ordinary motions to neglect compressability, the equation of continuity in this case becomes

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0 \quad . \quad . \quad . \quad (2)$$

These four equations have to be solved for u, v, w and p . So far they have proved to be unmanageable in their full form and, in consequence, have had to be modified to allow for the application of known means of analysis.

A common modification consists of neglecting the terms on the left of (1). This amounts to taking the motion as steady and neglecting the inertia forces of the fluid ; hence, any solution obtained from these new equations must be regarded as being valid only when the inertia forces are small in

comparison

comparison with the viscous forces. For this to be the case the flow must be exceedingly slow or the fluid very viscous.

Accordingly, for slow motion and in the absence of extraneous forces, the equations (1) reduce to

$$\nu.\nabla^2 u \quad = \quad \frac{1}{\rho} \cdot \frac{\partial p}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

with two similar equations.

It will be seen that if u_1, v_1, w_1, p_1 and u_2, v_2, w_2, p_2 are two independent solutions of equations (2) and (3), then the flow represented by $u_1 + u_2$, $v_1 + v_2$, $w_1 + w_2$, $p_1 + p_2$ is also a solution. The boundary conditions of u,&c, satisfied in this latter case will be the sum of the two conditions satisfied in the ~~satisfied in the~~ initial cases.

If the first motion be considered as a slow steady streaming past a fixed sphere, and the second motion be that obtained by slowly rotating a sphere in a fluid at rest at infinity, then the motion obtained by summing these two independent motions will represent the flow past a slowly rotating sphere.

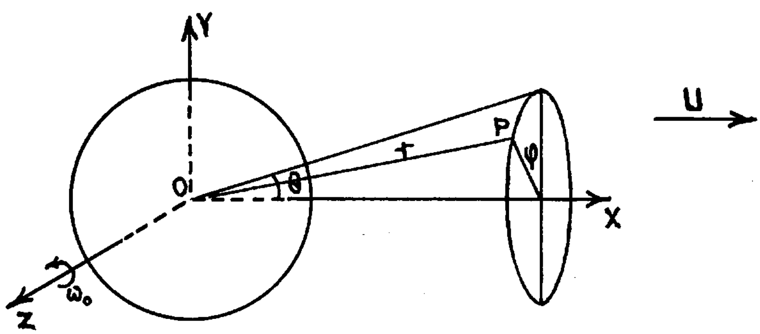

DIAGRAM 4a.

4.1 Let Diagram 4a represent the system under consideration.

X,

X, Y, Z are rectangular axes having their origin at the centre of the sphere of radius a and the flow at any point P is (u, v, w). It will be convenient to express some of the functions in terms of the spherical polar coordinates (r, θ, ϕ) shown.

The uniform slow flow past a stationary sphere was investigated by Stokes and is usually expressed in terms of the current function ψ. The fluid has velocity (U, 0, 0) at infinity and the flow is considered to take place in planes which intersect along the axis OX.

The stream function takes the form

$$\psi = -\tfrac{1}{2}U\left(1 - \frac{3}{2}\cdot\frac{a}{r} + \tfrac{1}{2}\cdot\frac{a^3}{r^3}\right).r^2 \sin^2\theta \quad . \quad . \quad . \quad . \ (4)$$

and the components of velocity along and at right angles to the radius vector are given by the expressions

$$R_1 = -\frac{1}{r^2 \sin\theta}\cdot\frac{\partial\psi}{\partial\theta} \qquad = U\cos\theta\left(1 - \frac{3}{2}\cdot\frac{a}{r} + \tfrac{1}{2}\cdot\frac{a^3}{r^3}\right)$$

$$\theta_1 = \frac{1}{r.\sin}\cdot\frac{\partial\psi}{\partial r} \qquad = -U\sin\theta\left(1 - \frac{3}{4}\cdot\frac{a}{r} - \tfrac{1}{4}\cdot\frac{a^3}{r^3}\right)$$

$$. \quad . \quad . \ (5)$$

When u_1, v_1, w_1 are the velocity components with respect to the axes x, y, z then

$$\begin{aligned} u_1 &= R_1\cos\theta - \theta_1\sin\theta \\ &= U.\cos^2\theta\left(1 - \frac{3}{2}\cdot\frac{a}{r} + \frac{1}{2}\cdot\frac{a^3}{r^3}\right) + U.\sin^2\theta\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right) \\ &= U.\left(1 - \tfrac{3}{4}\cdot\frac{a}{r} - \tfrac{1}{4}\cdot\frac{a^3}{r^3}\right) - \tfrac{3}{4}.U.\cos^2\theta\left(\frac{a}{r} - \frac{a^3}{r^3}\right) \\ &= U.\left[\left(1 - \frac{a}{r}\right) + \tfrac{1}{4}\cdot\frac{a}{r^3}\cdot(r^2 - a^2)(1 - 3.\cos^2\theta)\right. \end{aligned}$$

$v_1 =$

$$v_1 = (R.\sin\theta + \Theta\cos\theta).\cos\phi$$
$$= -\tfrac{3}{4}.U.\tfrac{a}{r^3}.(r^2 - a^2).\cos\theta\sin\theta\cos\phi$$

$$w_1 = (R.\sin\theta + \Theta\cos\theta).\sin\phi$$
$$= -\tfrac{3}{4}.U.\tfrac{a}{r^3}.(r^2 - a^2).\cos\theta\sin\theta\sin\phi$$

. . . . . (6)

Consider now the flow produced in an infinite expanse of fluid by the slow rotation of the sphere about the axis Oz. The flow may be expressed in terms of the function $\chi = A.\frac{z}{r^3}$, the components of velocity being

$$u_2 = z.\frac{\partial\chi}{\partial y} - y.\frac{\partial\chi}{\partial z} = -\omega_0\frac{a^3}{r^3}.y = -\omega_0\frac{a^3}{r^2}.\sin\theta\cos\phi$$
$$v_2 = x.\frac{\partial\chi}{\partial z} - z.\frac{\partial\chi}{\partial x} = \omega_0\frac{a^3}{r^3}.x = \omega_0\frac{a^3}{r^2}.\cos\theta$$
$$w_2 = y.\frac{\partial\chi}{\partial x} - x.\frac{\partial\chi}{\partial y} = 0$$

. . . . (7)

since the boundary conditions make $A = \omega_0 a^3$, ω_0 being the angular velocity of the rotating sphere.

For the complete analytical processes for obtaining the above solutions reference may be made to the usual text-books on hydrodynamics.*

4.2 The slow viscous flow past a slowly rotating sphere is obtained by adding the two independent solutions given in the last paragraph.

For any particular value of U, the flow will depend on the

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* e.g., Lamb, "Hydrodynamics", 5th. edn. §§ 335-8, pp.562-71.

the rate of rotation of the sphere. The theory of similarity indicates that the non-dimensional variable $\lambda \equiv \omega_0 a/U$ will be one of the factors which determines the flow in the general case, the other being Ua/ν.

Inserting the factor λ into the set of equations (7) :

$$u = u_1 + u_2 = U\left\{\left(1 - \frac{a}{r}\right) + \frac{1}{4}\cdot\frac{a}{r^3}\cdot(r^2 - a^2)(1 - 3.\cos^2\theta) - \lambda.\frac{a^2}{r^2}. \sin\theta \cos\phi\right\}$$

$$v = v_1 + v_2 = U\left\{\lambda.\frac{a^2}{r^2}.\cos\theta - \frac{3}{4}.\frac{a}{r^3}.(r^2 - a^2).\cos\theta \sin\theta \cos\phi\right.$$

$$w = w_1 + w_2 = U\left\{-\frac{3}{4}.\frac{a}{r^3}.(r^2 - a^2).\cos\theta \sin\theta \sin\phi\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . (8)$$

These equations give the components of velocity at any point in the fluid when a sphere is rotated about an axis at right angles to the main stream, provided the inertia forces are small in comparison with the viscous forces. Stokes's solution, as given in equations (6), is valid when Ua/ν is small and the equations (7) are valid if $\omega_0 a^2/\nu$ or $\lambda.Ua/\nu$ be small.

Thus, for equations (8) to hold for the flow past a rotating sphere, both Ua/ν and $\lambda.Ua/\nu$ must be small. If λ be of unit order, Ua has to be small compared with ν which, for air at atmospheric pressure, is 0.132 c.g.s. units. Hence, for the equations to be valid for a sphere of 1 cm. radius in an air stream, U has to be small compared with 0.132 cm./sec. and $0 < \lambda < 1$.

From the above considerations it will be realised that the solution given above holds only when the motions are minute.

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A method of extending the above solution to higher values of VL/ν is advanced in § 4.4 below and is based upon some suggestions put forward a few years ago by certain investigators. In the paragraph mentioned the line of attack is outlined and a number of suggestions are made with regard to the method of solution and possible limitations.

4.3 When the solution given by equations (8) holds, the force and torque experienced by the rotating sphere may be derived very simply if the additive quality of the primary solutions be applied.

It may be shown* that the x- and y-components of stress across the surface of a sphere of radius r are given by

$$p_{rx} = -\frac{x}{r}.p + \frac{\mu}{r}.(r.\frac{\partial}{\partial r} - 1)u + \frac{\mu}{r}\frac{\partial}{\partial x}(xu + yv + zw) = p'_{rx} + p''_{rx}$$

$$p_{ry} = -\frac{y}{r}.p + \frac{\mu}{r}.(r.\frac{\partial}{\partial r} - 1)v + \frac{\mu}{r}.\frac{\partial}{\partial y}(xu + yv + zw) = p'_{rx} + p''_{rx}$$

Here p'_{rx}, p'_{ry} represent the terms involving u_1, v_1, w_1 and p''_{rx}, p''_{ry} represent those involving u_2, v_2, w_2 when $u_1 + u_2$, &c., are substituted for u, v, w.

Since $\nabla^2 u_2 = \nabla^2 v_2 = 0$, then p_2 is constant by equations (3), and p, the mean pressure at any point in the fluid, is given by $p = p_1 + p_2 = p_1 +$ constant. It will be seen also that $xu_2 + yv_2 + zw_2$, which represents r.(radial velocity), is zero, the flow in the second of the initial cases being in circles about Oz.

The stresses p_{rx} and p_{ry}, when integrated over the sphere

of

* Lamb, § 336, pp. 563-4.

of radius a, give the drag and lift respectively. If dS be an element of the surface of the sphere, then $\iint p''_{rx}.dS$ and $\iint p''_{ry}.dS$ are easily seen to be zero. Since p_2 is constant and $xu_2 + yv_2 + zw_2 = 0$ then the first and third terms of these integrals vanish for the reasons given above. The second terms $\mu\iint(\frac{\partial}{\partial r} - \frac{1}{r})u_2 dS$ and $\mu\iint(\frac{\partial}{\partial r} - \frac{1}{r})v_2 dS$ are also zero, the integrand of the first, viz. $(3\omega_0 a^3 y/r^4)_{r=a}$, being proportional to y and that of the second to x.

Hence, the drag of a slowly rotating sphere in a slowly moving stream is the same as if it had no rotation and is that found by Stokes, viz. $6\pi\mu aU$. The sphere experiences no lift component.

The torque acting on the sphere about Oz, and in the positive direction, will be seen to be $\mu\iint(x.p_{ry} - y.p_{rx}).dS$. As the integral $\iint(x.p'_{ry} - y.p'_{rx}).dS$ vanishes, this becomes

$$\text{Torque} = \mu\iint(x.p''_{ry} - y.p''_{rx}).dS$$

$$= \mu\iint\left\{(x.\frac{\partial}{\partial r} - \frac{x}{r})v_2 - (y.\frac{\partial}{\partial r} - \frac{y}{r})u_2\right\}_{r=a}.dS$$

$$= -3\mu\frac{\omega_0}{a}\iint(x^2+y^2)dS = -3\mu\frac{\omega_0}{a}\iint a^2(\cos^2\theta + \sin^2\theta\cos^2\phi).a^2\sin\theta\,.d\theta.d\phi$$

The value of the double integral can be easily shown to be $\frac{8}{3}\cdot\pi a^4$ so that the torque becomes $-8\pi\mu\omega_0 a^3$ and tends to stop the rotation.

It is now generally accepted in aerofoil theory that when a body experiences a force component in the cross-stream direction then this component, or lift, is proportional to the circulation. At first sight it appears strange that the rotating sphere does not have lift when there is clearly a

circulation

circulation present.

In the case dealt with above, the circulation produced is due entirely to viscosity. Any fluid reaction which may be brought into play on one hemisphere due to the spin is annulled by the equal but opposite reaction on the other hemisphere. Thus, there is no addition to the force experienced by a non-rotating sphere, but there does come into existence a torque tending to stop the spin.

However, it must not be concluded from this that the fluid inertia is wholly responsible for the production of lift and that the fluid may be taken as non-viscous when calculating the cross-wind force. The inertia forces of themselves will not give lift.

In the theory advanced by Prandtl and his colleagues, viscosity is dispensed with almost entirely. The circulation which produces lift is considered as that in existence on the outside of the boundary layer. Its magnitude is fixed as sufficient to bring the rear stagnation point to the trailing edge, it being supposed that eddies formed in the rear are carried off by the fluid, leaving a circulation round the body which gradually builds up to this value. This cannot be considered as an altogether satisfactory basis. The boundary layer is the region where the viscosity is of greatest importance, the shear in this part of the fluid being great. Hence, it would be decidedly more satisfactory if the mechanism of this thin layer were more clearly

understood

understood so that the growth or disappearance of the circulation in passing through the boundary could be analysed, instead of simply treating the layer as an auxiliary part of the solid body.

The method suggested below indicates a scheme, whereby, starting from the purely viscous flow,-- such as that represented by equations (8) -- the inertia of the fluid may be brought gradually into the problem, and a solution obtained which promises to be of some value as it would deal with the whole expanse of fluid instead of only that part where the viscous forces are small.

4.4 From time to time, a number of attempts have been made to obtain solutions for particular cases of fluid motion by inserting solutions which are valid only for low speeds into the dynamical equations, and then re-solving these equations. It was hoped by this step-by-step process to obtain a solution which would be valid for higher values of VL/ν, but none of these attacks were advanced very far.*

A few years ago Cowley and Levy† advanced the important idea, that, since the quantity VL/ν forms the determining factor in all usual problems in fluid motion, then the stream function ψ, in any example of steady two-dimensional flow, may be expansible in some series of the variable VL/ν. The authors do not indicate any method whereby the form of the series may be determined but they suggest the power series

as

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* Whitehead, Quart. Jnl. Maths. 1889 p. 143.
Williams, Phil. Mag. xxix. (1915), p. 526.

† Phil. Mag. (1921) p 584.

as perhaps a suitable form.

Thus they consider that ψ may be put into the form

$$\psi = \psi_0 + \psi_1 C + \psi_2 C^2 + \psi_3 C^3 + \ldots + \ldots$$

where $C = VL/\nu$.

If this expansion of ψ be substituted in the equation

$$\nabla^4\psi + C\left(\frac{\partial\psi}{\partial y}\cdot\frac{\partial}{\partial x}\nabla^2\psi - \frac{\partial\psi}{\partial x}\cdot\frac{\partial}{\partial y}\nabla^2\psi\right) = 0 ,$$

which has to be satisfied for steady two-dimensional flow, then the equation may be broken up into an infinite series of differential equations. This is permissible since the equation is an identity in C, and, therefore, the coefficients of powers of C may be equated to zero. Each differential equation depends on the preceeding one, and the first equation is $\nabla^4\psi_0 = 0$, simply that for slow motion.

Thus, starting with ψ_0 -- the solution for slow flow -- it is conceivable that $\psi_1, \psi_2, \ldots\ldots$ may be determined successively and thus a function ψ built up which will satisfy the dynamical equations and the boundary conditions completely. Above all, it should be valid for a greater range of VL/ν than that satisfied by the solutions for slow motion.

There are some problems in two-dimensional flow where it is fairly obvious that ψ cannot be expanded in a power series. In these examples a different series should be sought. The flow past any cylinder in an infinite fluid is such a case, for here no steady slow viscous motion is possible, it being impossible to satisfy all the boundary conditions.* R.A.Frazer† attempted

* Lamb, 5th. edn. p.581, § 343.

† Frazer, Phil.Trans.Roy.Soc.,A.225,(1926), p.93.

attempted the problem of a circular cylinder, using the power series. As might have been expected, the results of the first stages were not too encouraging, it being found that eddies were formed ahead of the cylinder as well as in the rear. Professor Bairstow has suggested that one might start initially with Oseen's solution as the basis of some expansion. This would avoid certain of the difficulties which exist in dealing with the slow motion solution, and would partly take account of inertia at the beginning.

If, in an example of two-dimensional flow, we can assume Ψ to be expansible in a power series, then it is evident that $\underline{u}$ and $\underline{v}$, the velocity components at a point, are expansible also in a power series.

In the case of any general three-dimensional flow, the equations of motion are not so compact as those for two-dimensional examples and consequently analysis is more laborious. As all flows depend on VL/ν, it still may be considered that the actual solution in a three-dimensional case will depend fundamentally on the variable VL/ν . In fact, the conditions at any point in the fluid must be functions of this variable.

Hence, the suggestion may be advanced that the velocity components at any point may be expansible in some series of VL/ν. The form of the series for any particular case cannot be stated off-hand but there may be some hope that an examination of the dynamical equations might suggest a suitable form.

Hence, as a method of solution, we suggest to start by carefully examining the equations of motion for an incompressible fluid and find the expansions of u, v, w which promise most satisfactorily. This done, these expansions would be substituted in the dynamical equations, when written in their non-dimensional form, and an attempt made to solve for the coefficients in the expansion assumed. Having thus determined the series, the flow represented by this solution may be examined to find the forces experienced by the solid bodies in the stream, to find by what means the circulation is produced, to establish the criterion for instability, and many such fundamental problems.

In the example dealt with above, equations (8) give values of u, v, w from which the general method of solution might be commenced. It will be realised that all will depend on the validity of these equations. Rayleigh* has pointed out that the kinetic energy of the fluid is infinite, when the flow produced by the motion of a sphere is that represented by Stokes's solution. Thus equations (8) give the flow which is established on the elapse of some time after the motion starts. This suggests that it might be necessary to take time into consideration in the initial solution, so that equations (8) are arrived at asymptotically as the time increases. However, it is not deemed advisable to lay too much stress on this point though it is conceivable that such a modification might be necessary when other examples come to be considered.

* Rayleigh, Phil. Mag. (5), xxi. p.374 (1886) ; Papers, ii. p.480.

Conclusion and Acknowledgments.

The results of the experiments described in Parts II and III seem sufficient to check any analytical work which may be embarked upon in investigating the flow past a rotating sphere. Hence, the next necessary step in this problem is the extension of the analysis. The method indicated in Part IV appears to be a very natural means of attack and seems to have many hopeful possibilities.

Finally, the author desires to express his indebtedness to Professor J.D.Cormack, Director of the James Watt Engineering Laboratories, for placing at his disposal the facilities for carrying out the work involved in these investigations. To Dr. A.Thom, he is under a great debt for guidance and encouragement in these aerodynamical studies.

The reduction of the observations and the construction of the diagrams of Part III were carried out during the tenure of a Beit Fellowship at the Imperial College of Science and Technology. The author's thanks are due to Professor L.Bairstow for this privilege.

List of Diagrams and Figures.

List of Tables.

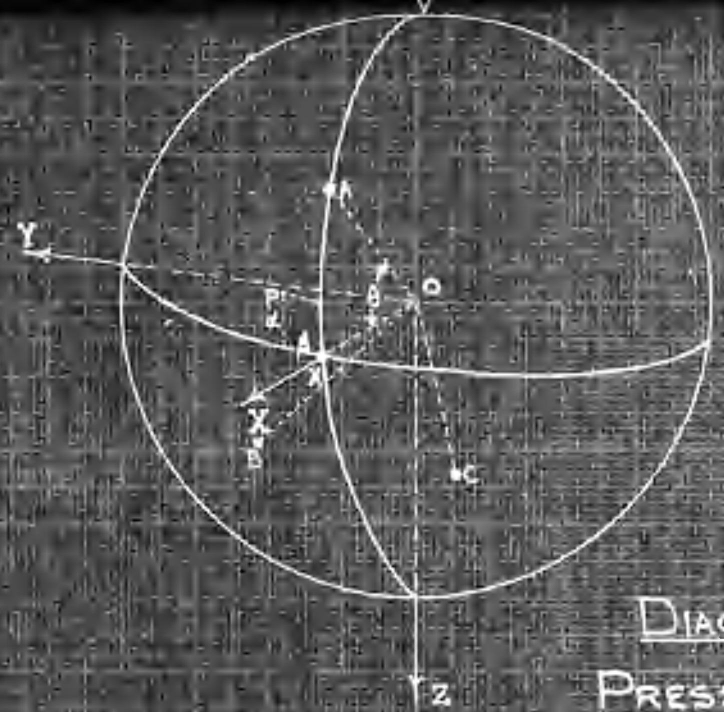
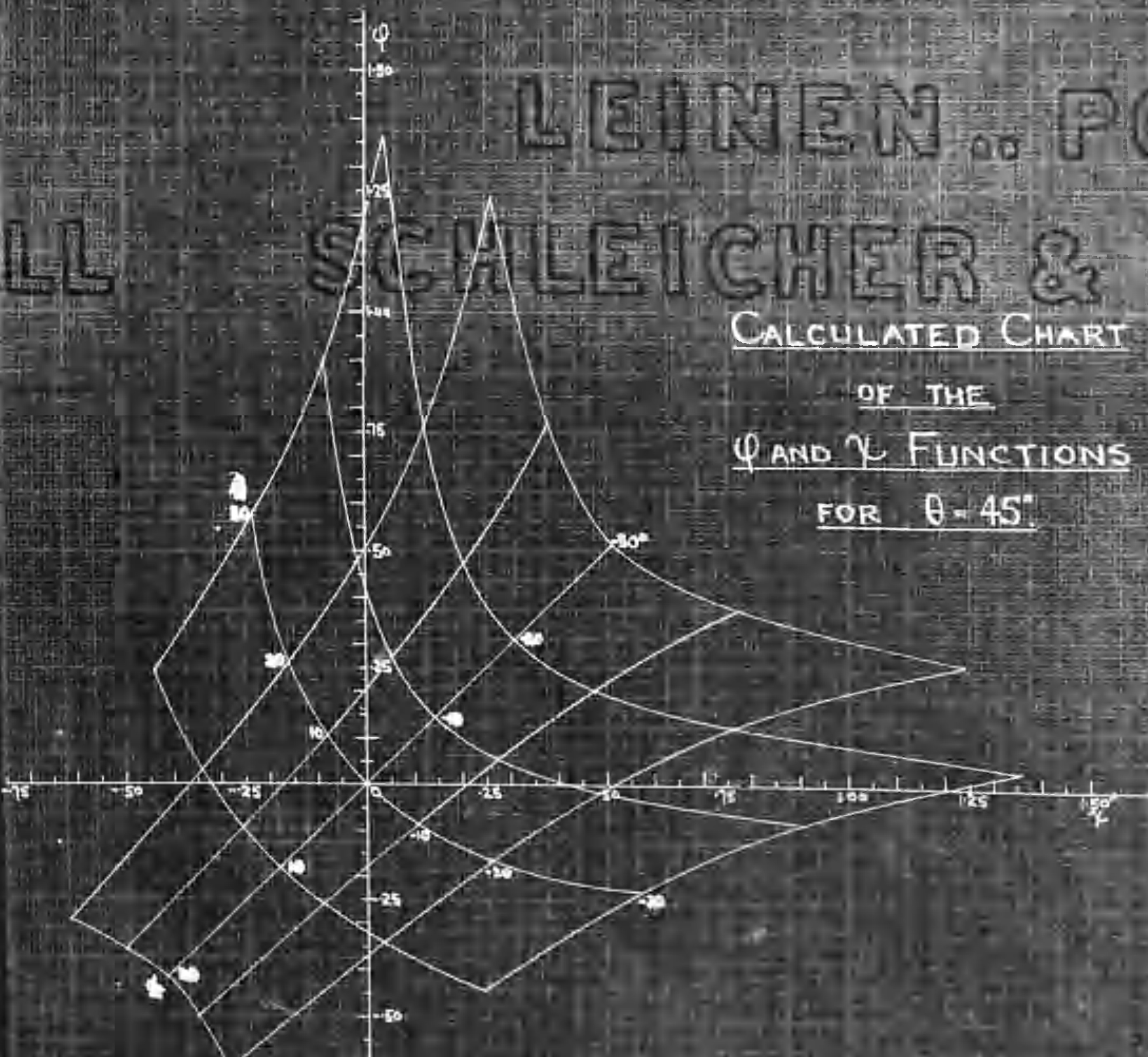
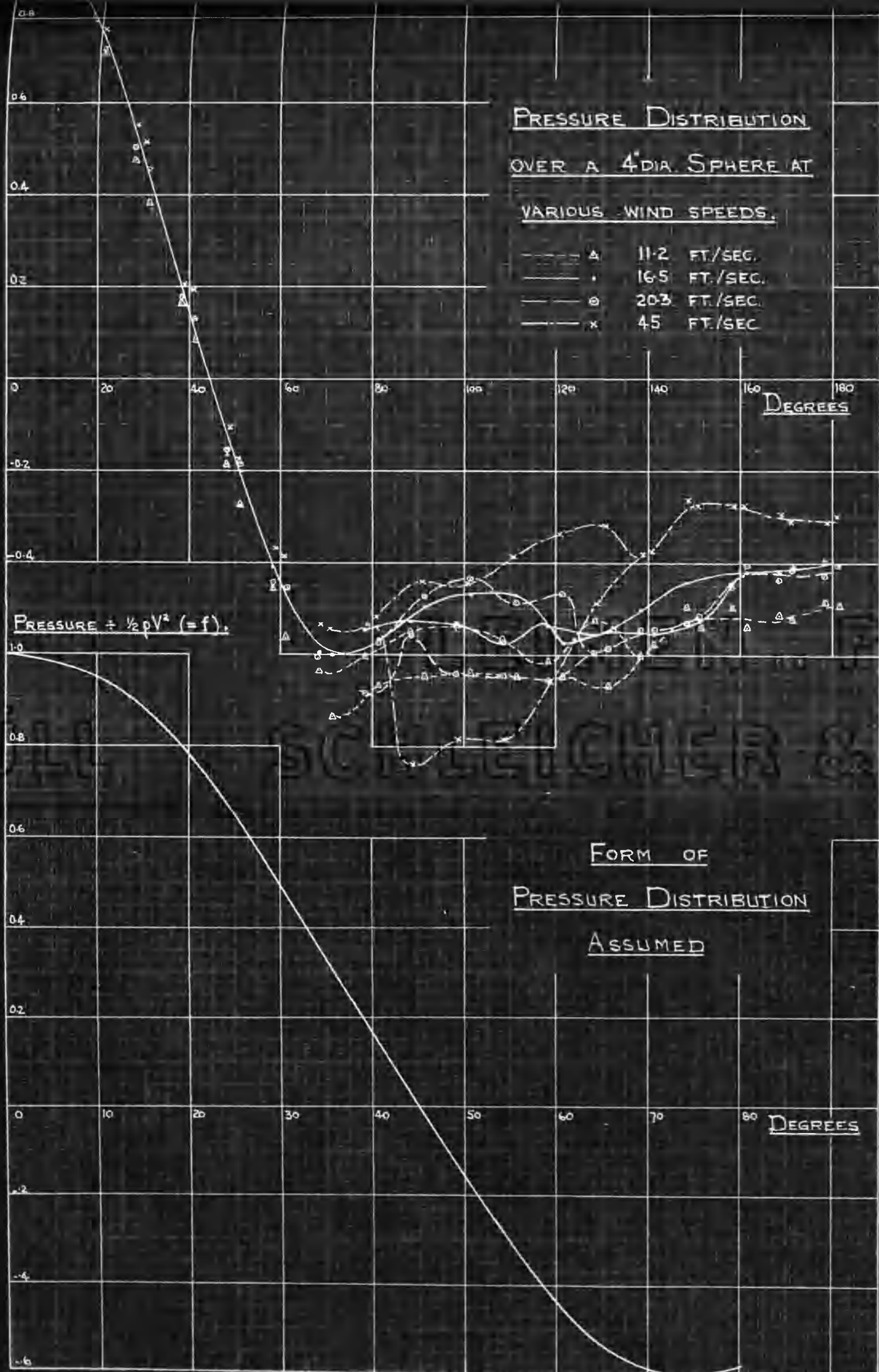


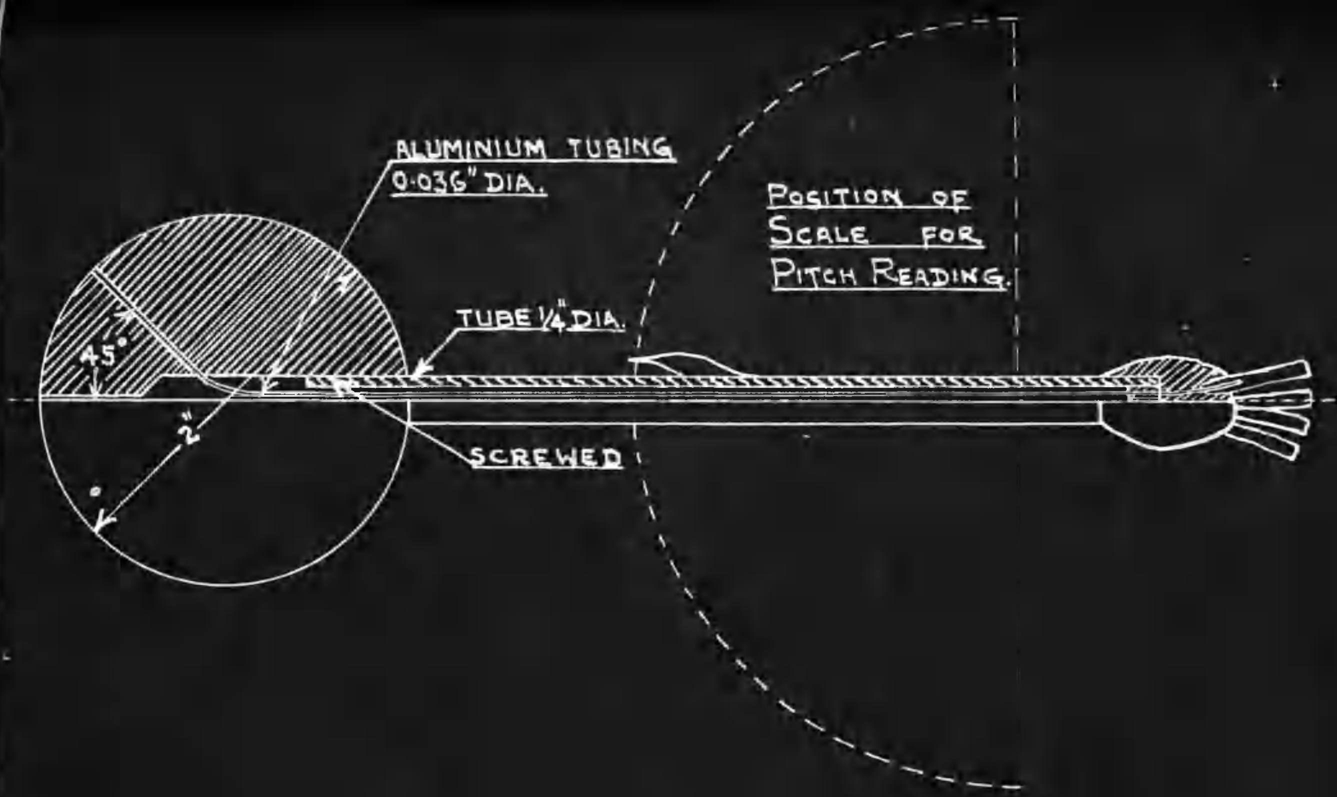
Diagram showing Pressure Holes on Spherical Velocity Gauge



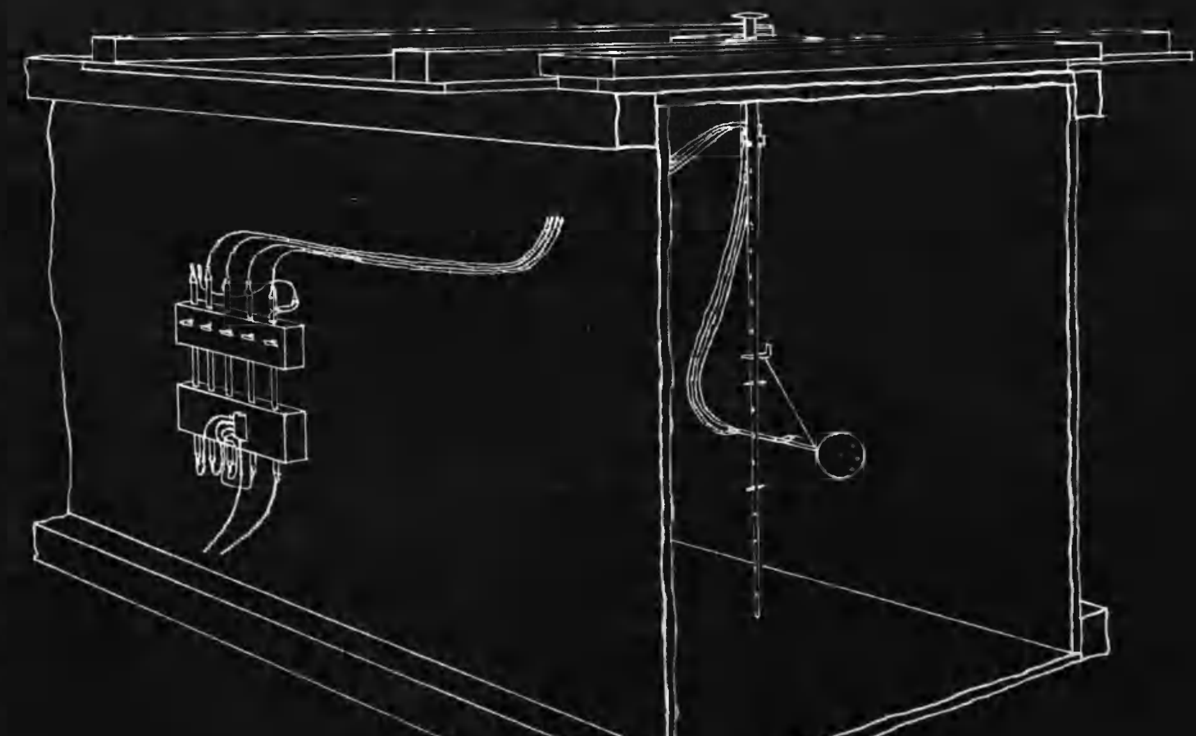
Calculated Chart of the φ and ψ Functions for $\theta = 45°$



PRESSURE DISTRIBUTION
OVER A 4" DIA. SPHERE AT
VARIOUS WIND SPEEDS.
11·2 FT./SEC.
16·5 FT./SEC.
20·3 FT./SEC.
45 FT./SEC.
DEGREES
PRESSURE ÷ ½ρV² (= f).
FORM OF
PRESSURE DISTRIBUTION
ASSUMED
DEGREES



MODEL SPHERICAL GAUGE



ARRANGEMENT FOR TESTING MODEL.

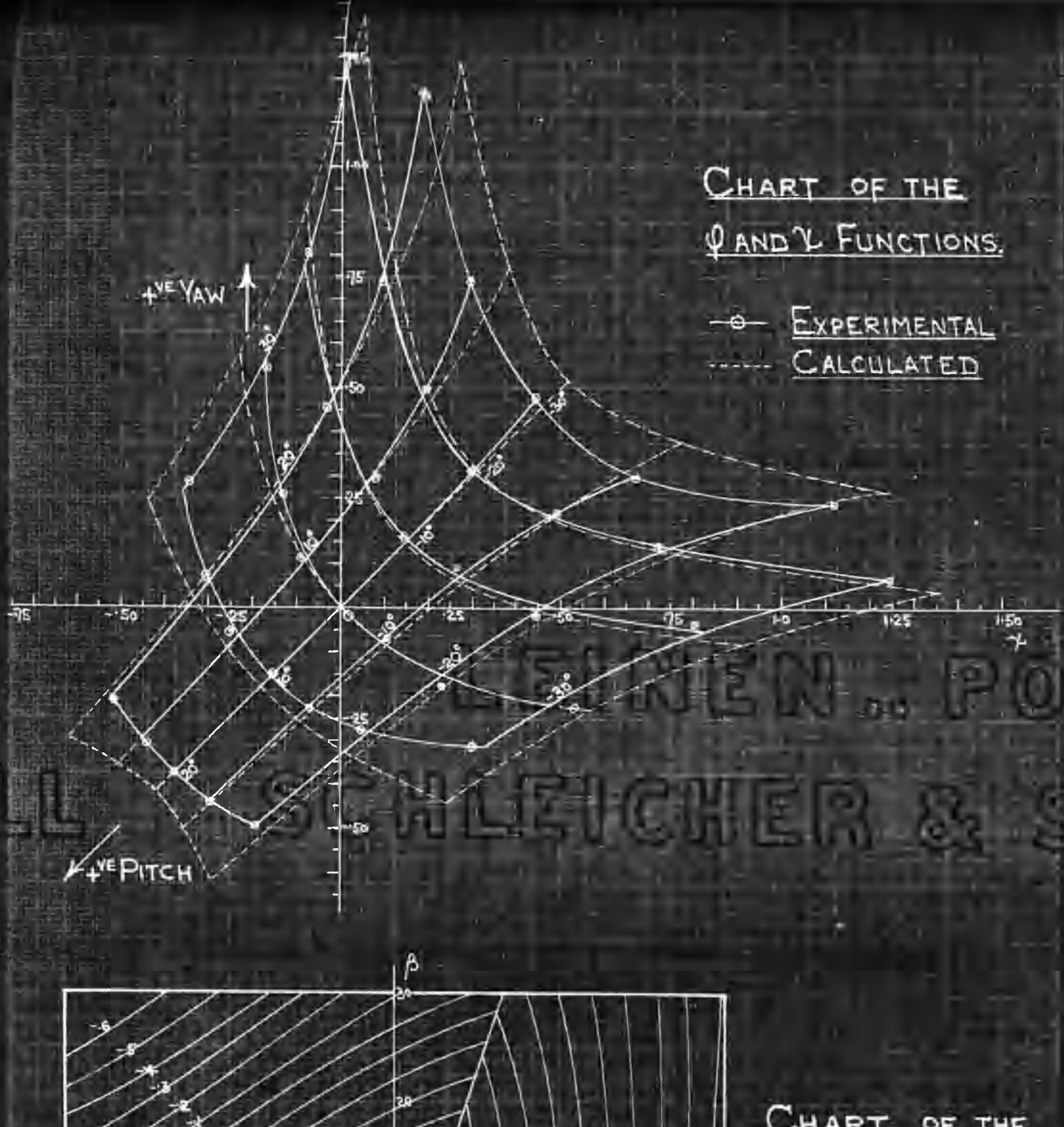


CHART OF THE
φ AND ψ FUNCTIONS.
EXPERIMENTAL
CALCULATED
+VE YAW
+VE PITCH
ψ

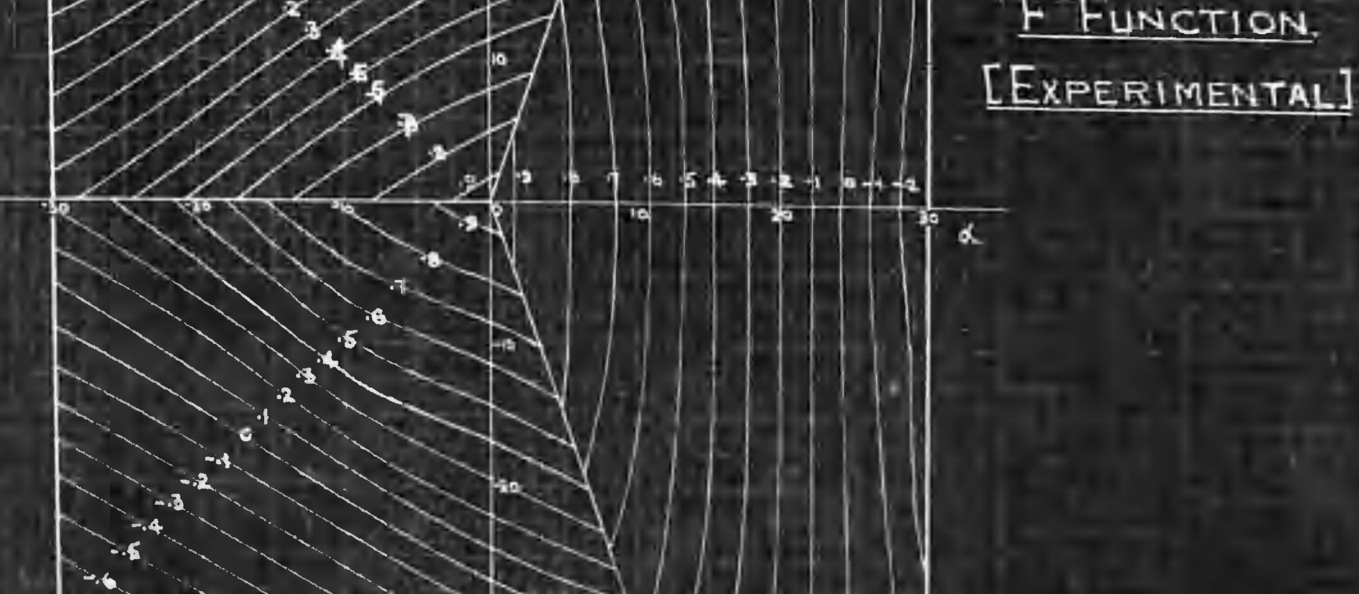
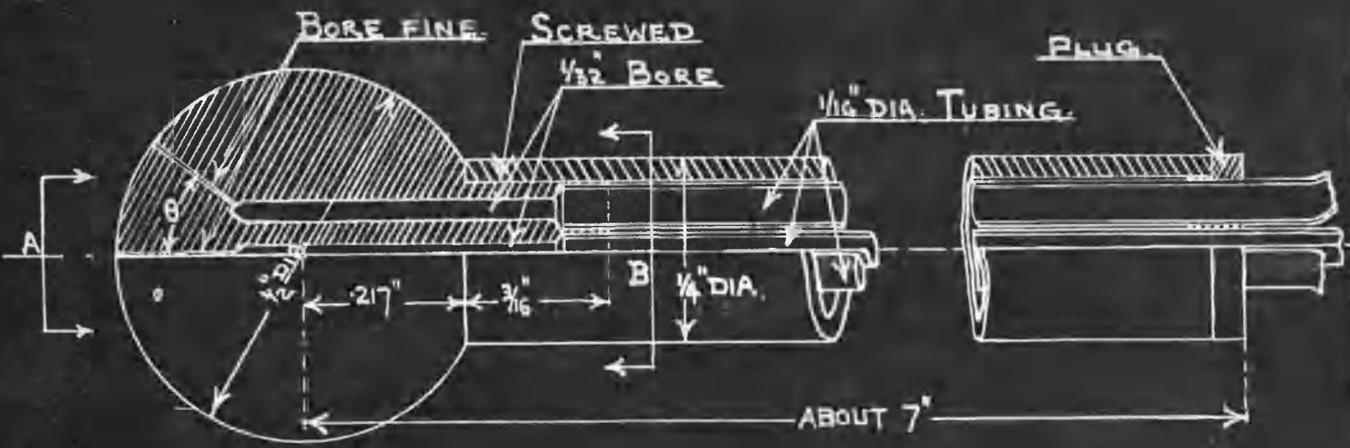
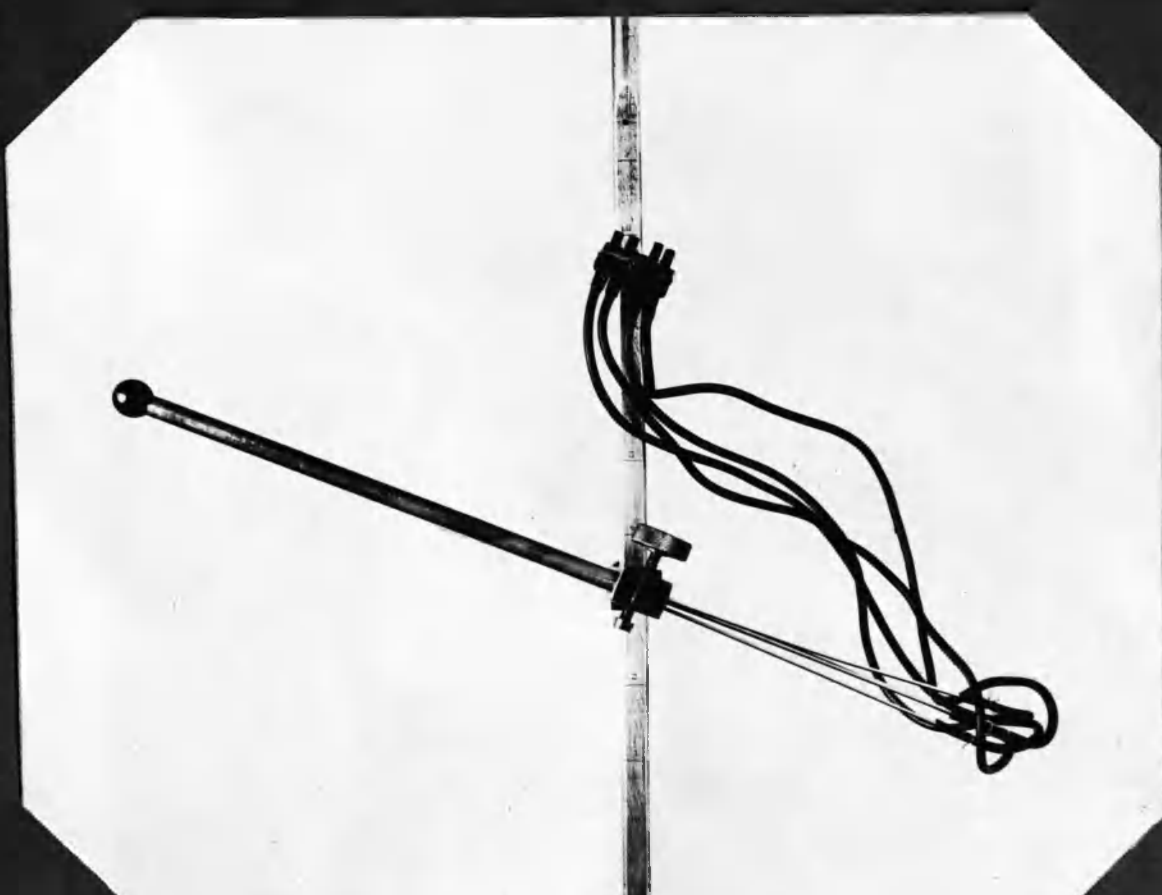


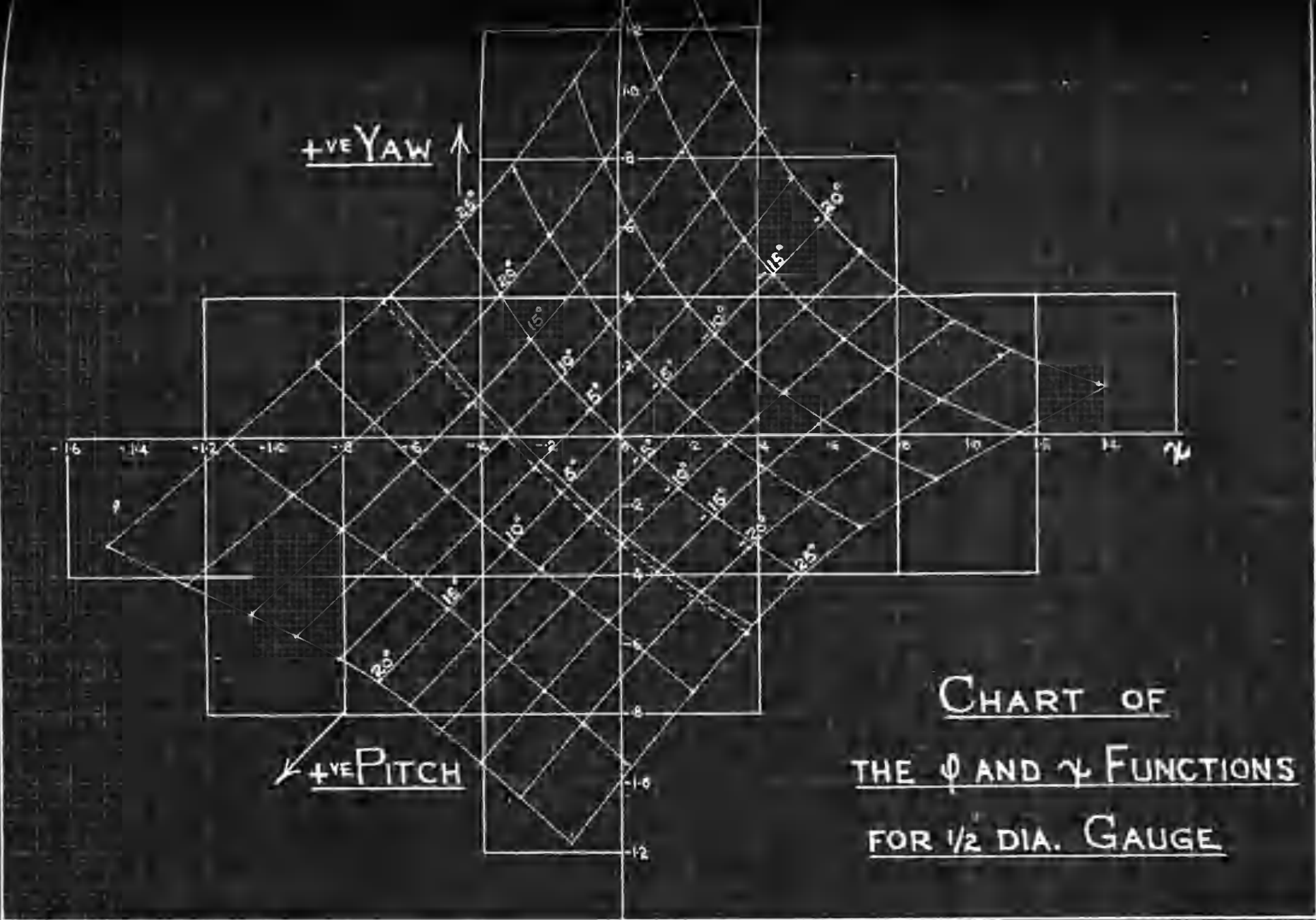
CHART OF THE
"F" FUNCTION.
[EXPERIMENTAL]
β
α



VIEW FROM "A".

SECTION AT "B".




+ve Yaw
+ve Pitch
Chart of
the φ and ψ Functions
for 1/2" dia. Gauge

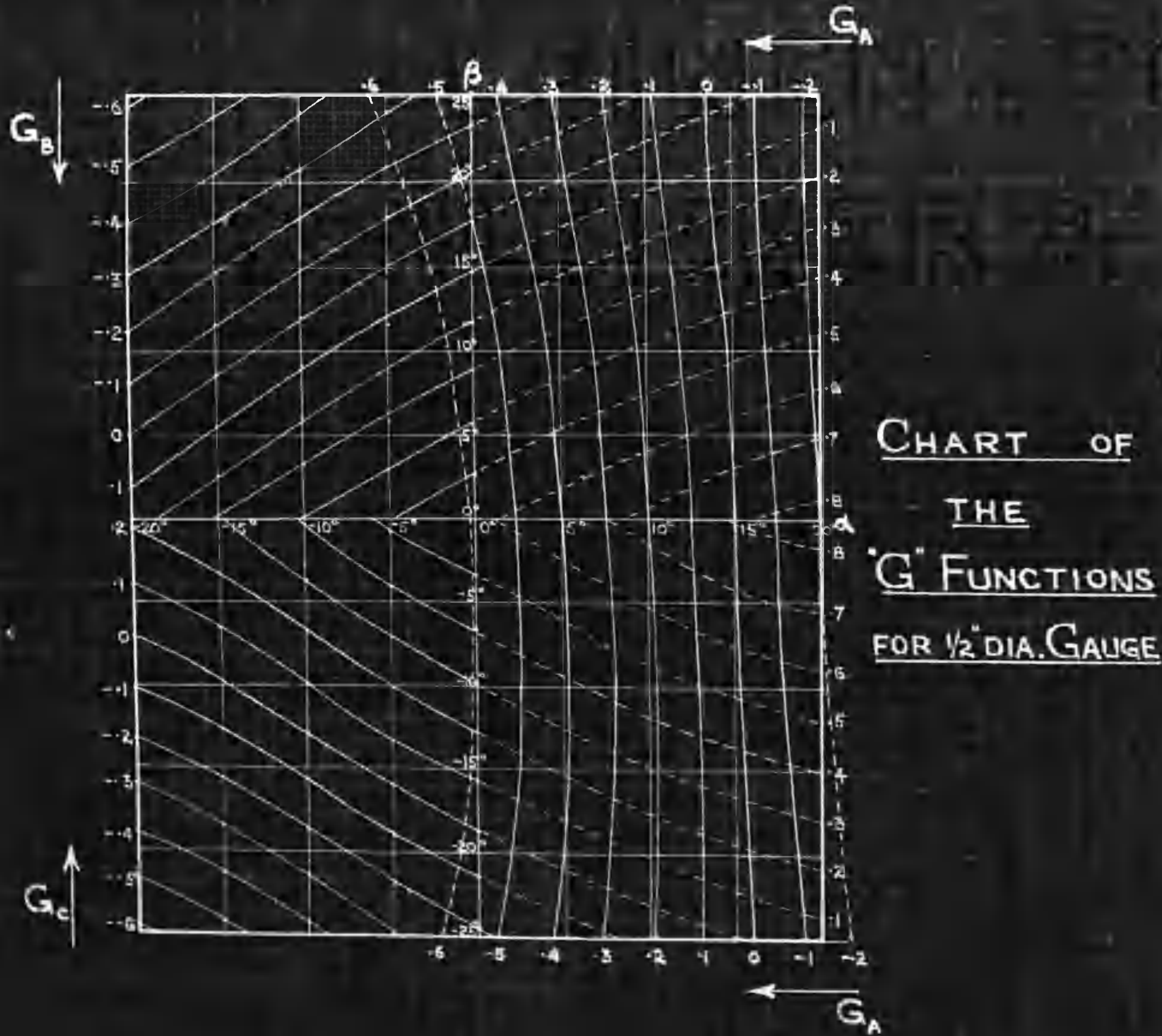
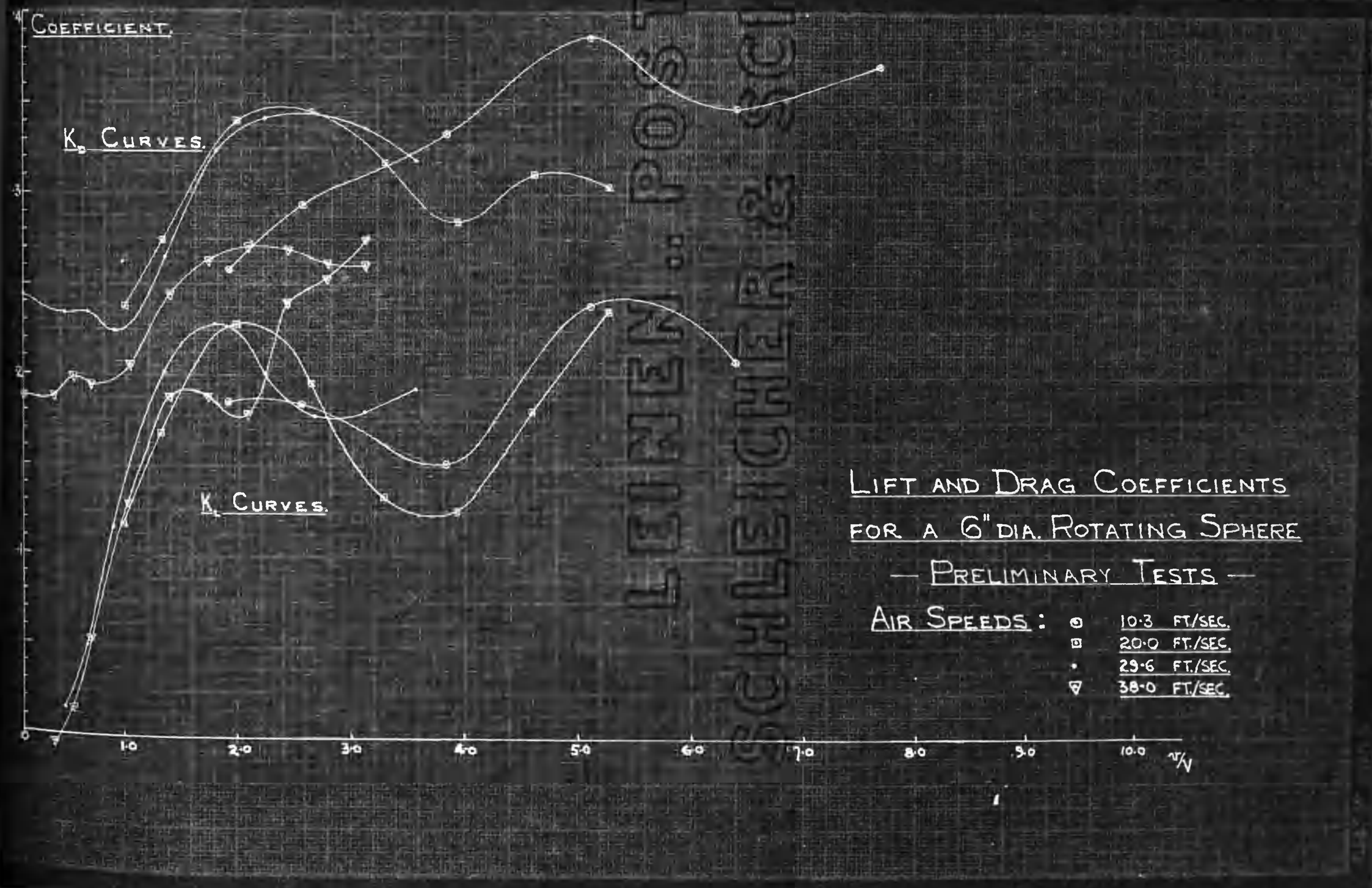
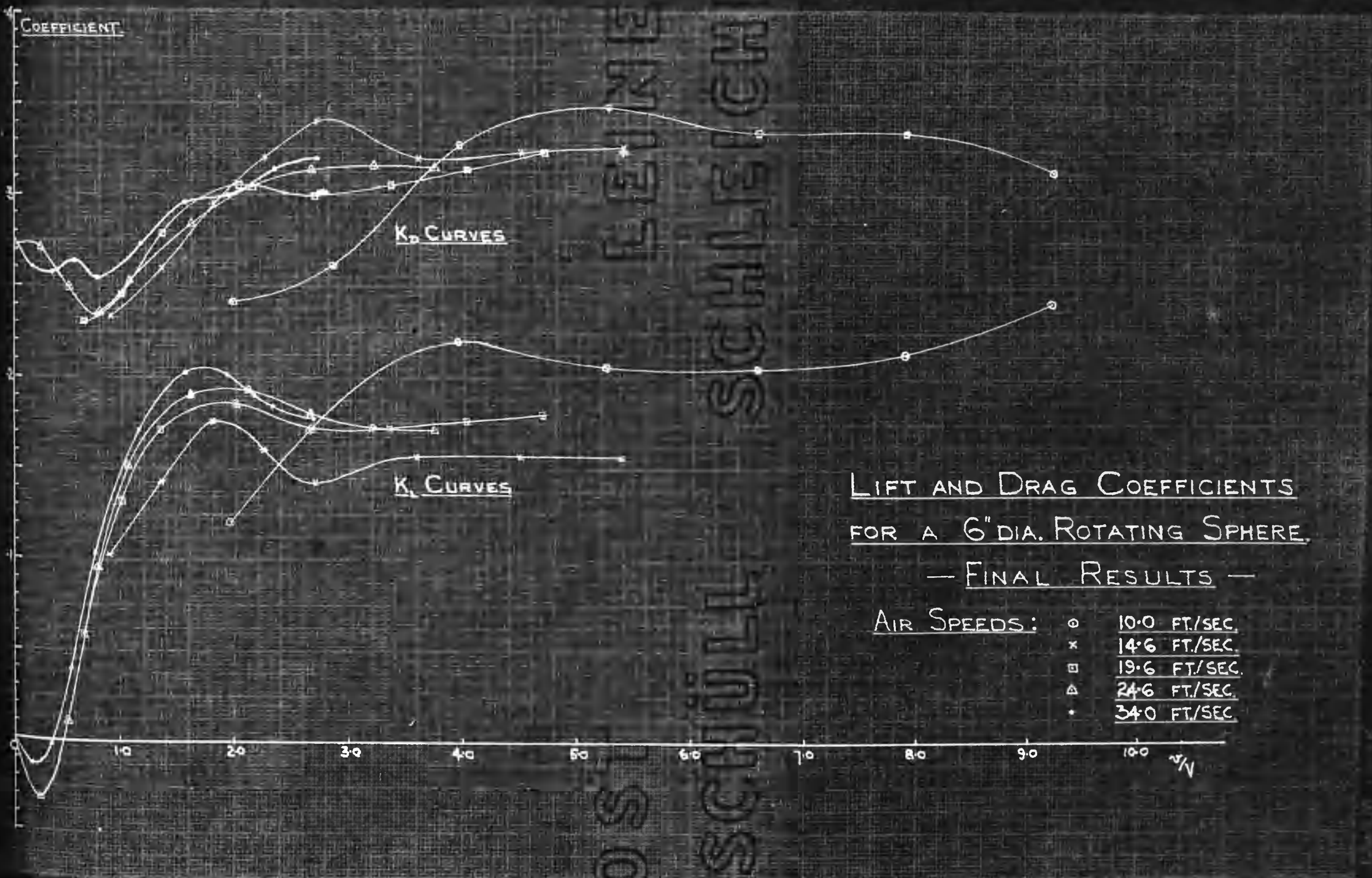
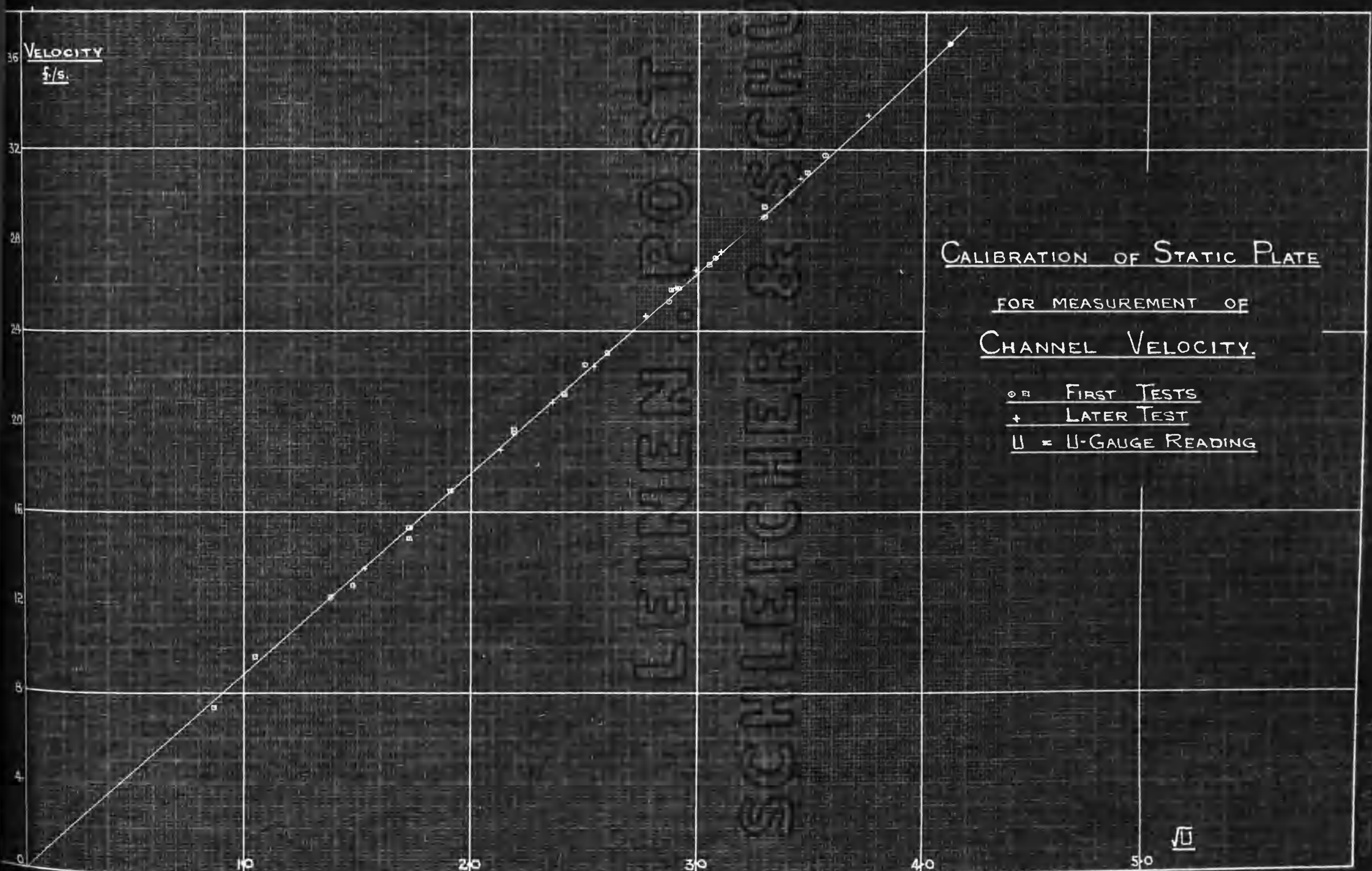


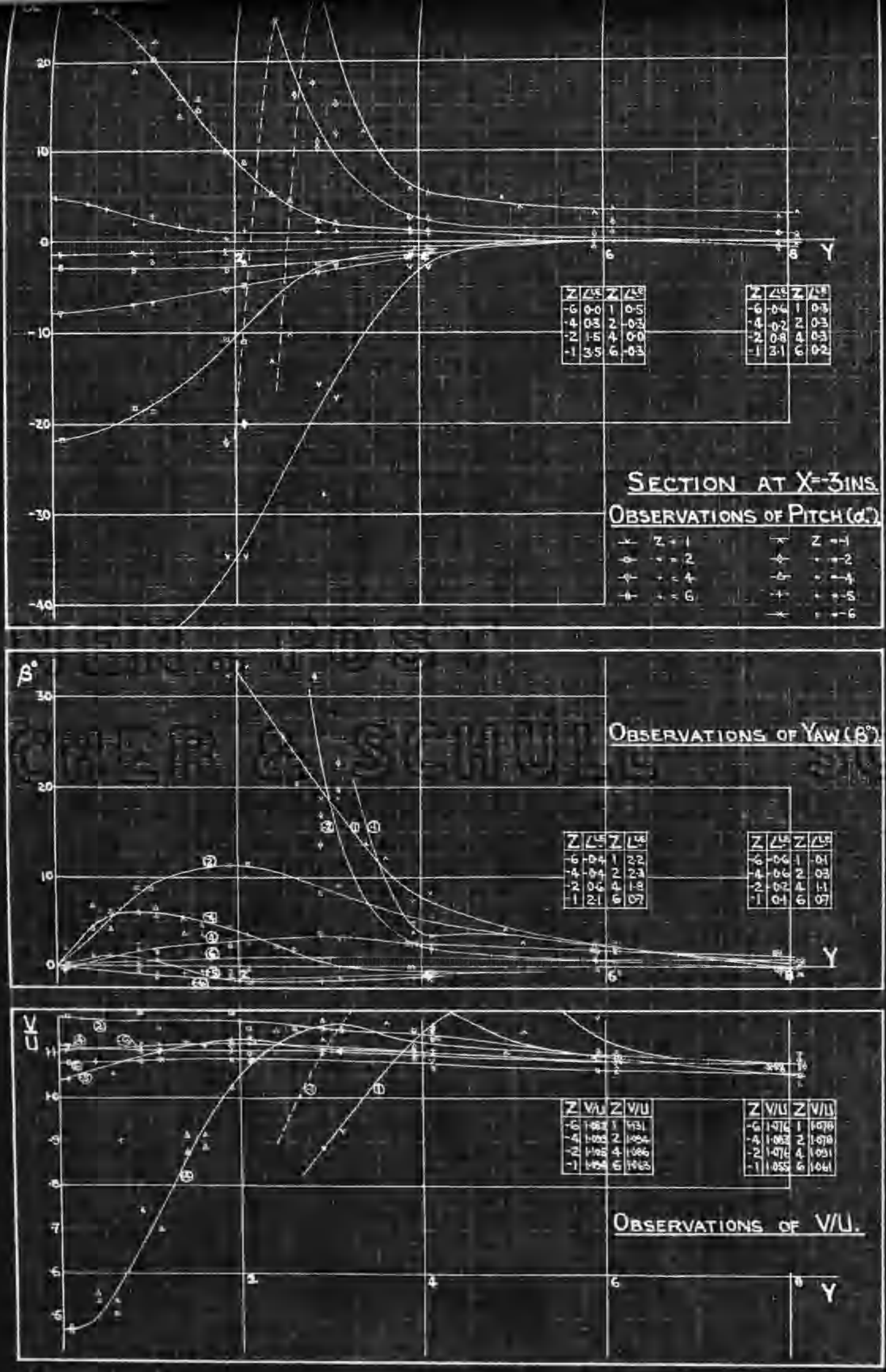
Chart of
the
"G" Functions
for 1/2" dia. Gauge
G_A
G_B
G_C
β
α

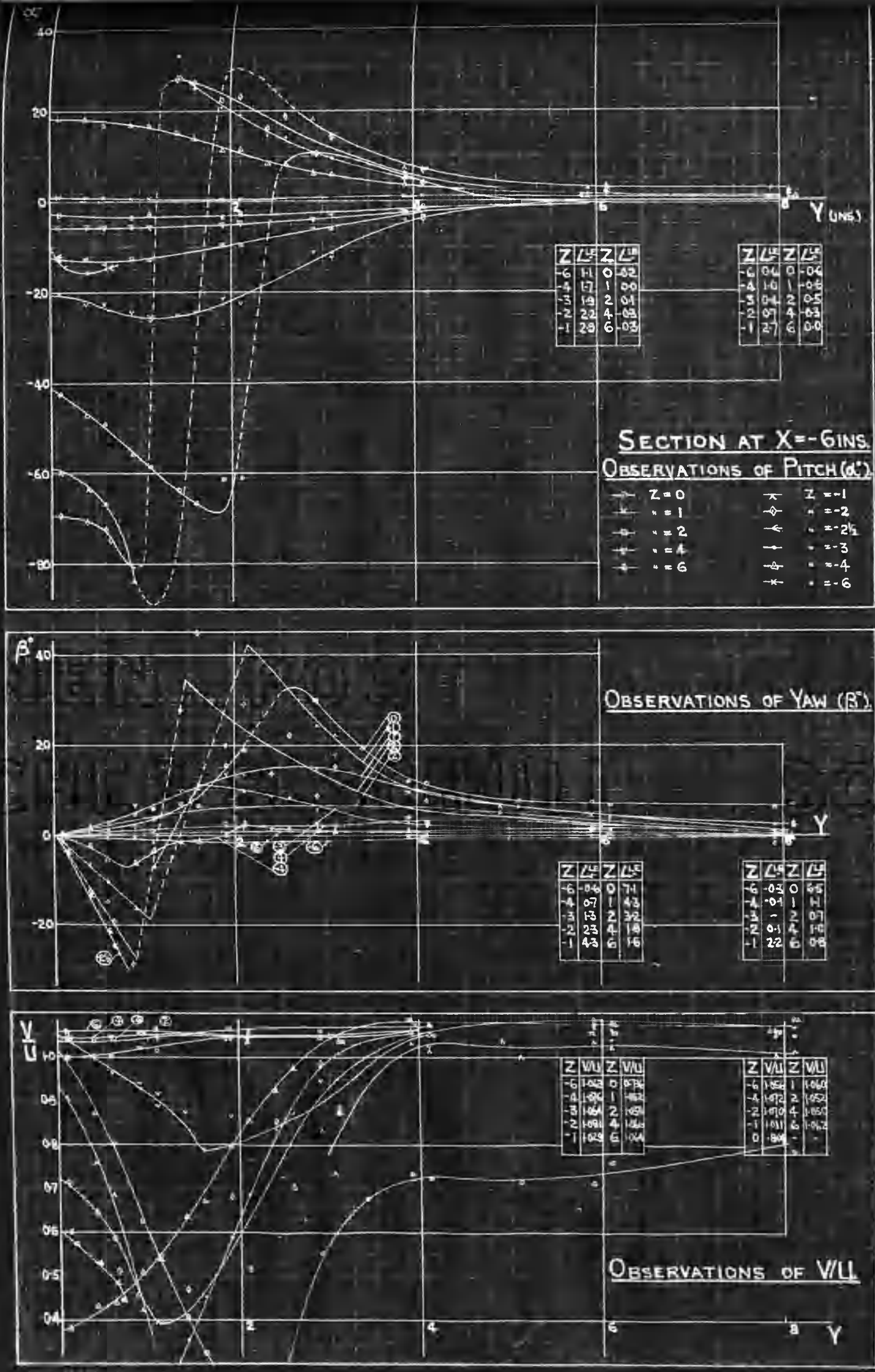

COEFFICIENT.
K_D CURVES.
K_L CURVES.
·4
·3
·2
·1
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
v/V
LIFT AND DRAG COEFFICIENTS
FOR A 6" DIA. ROTATING SPHERE
— PRELIMINARY TESTS —
AIR SPEEDS :
10·3 FT/SEC.
20·0 FT./SEC.
29·6 FT./SEC.
38·0 FT./SEC.

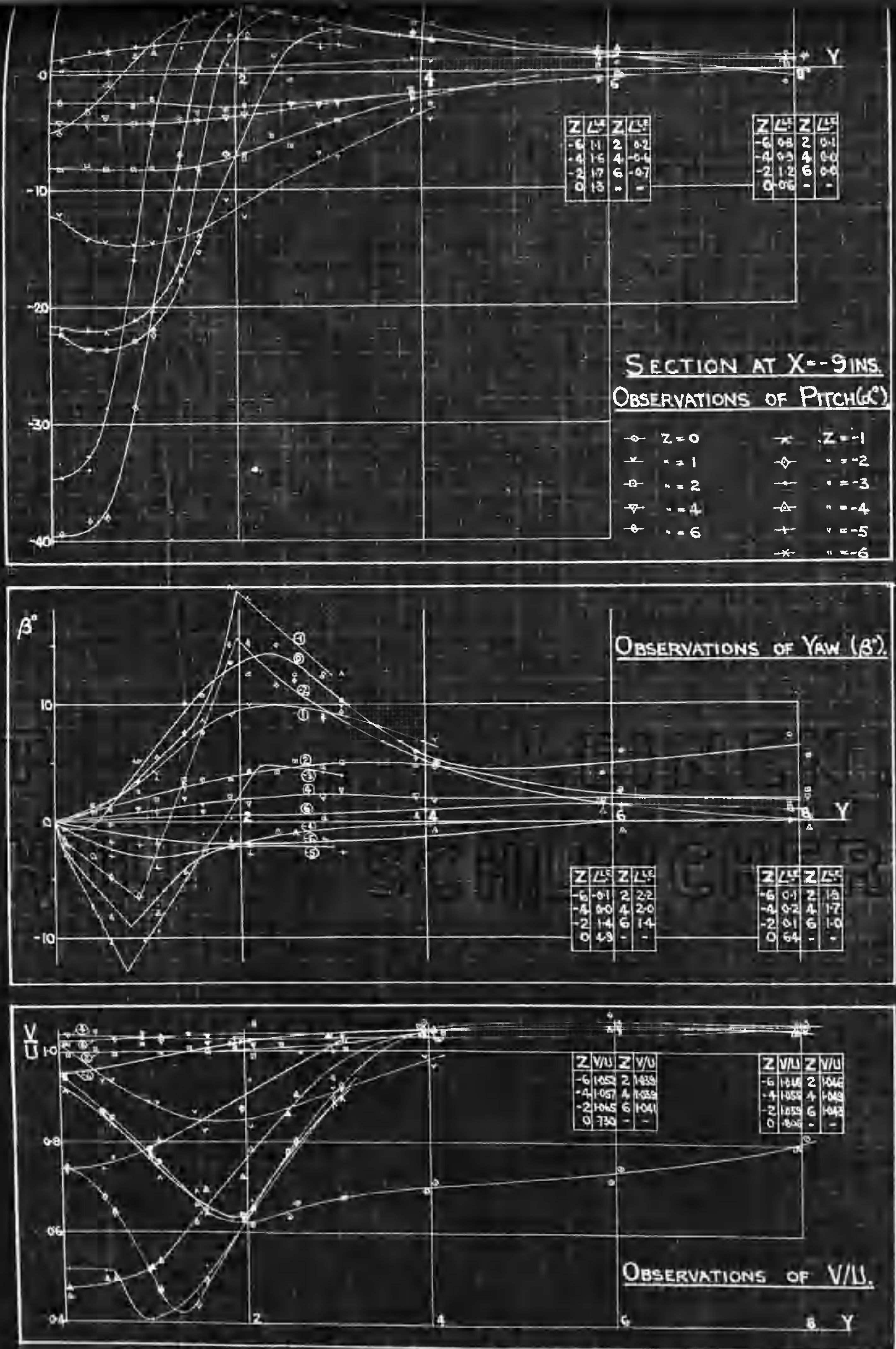

COEFFICIENT
K_D CURVES
K_L CURVES
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
v/V
LIFT AND DRAG COEFFICIENTS
FOR A 6" DIA. ROTATING SPHERE.
— FINAL RESULTS —
AIR SPEEDS:
10·0 FT./SEC.
14·6 FT./SEC.
19·6 FT./SEC.
24·6 FT./SEC.
34·0 FT./SEC.



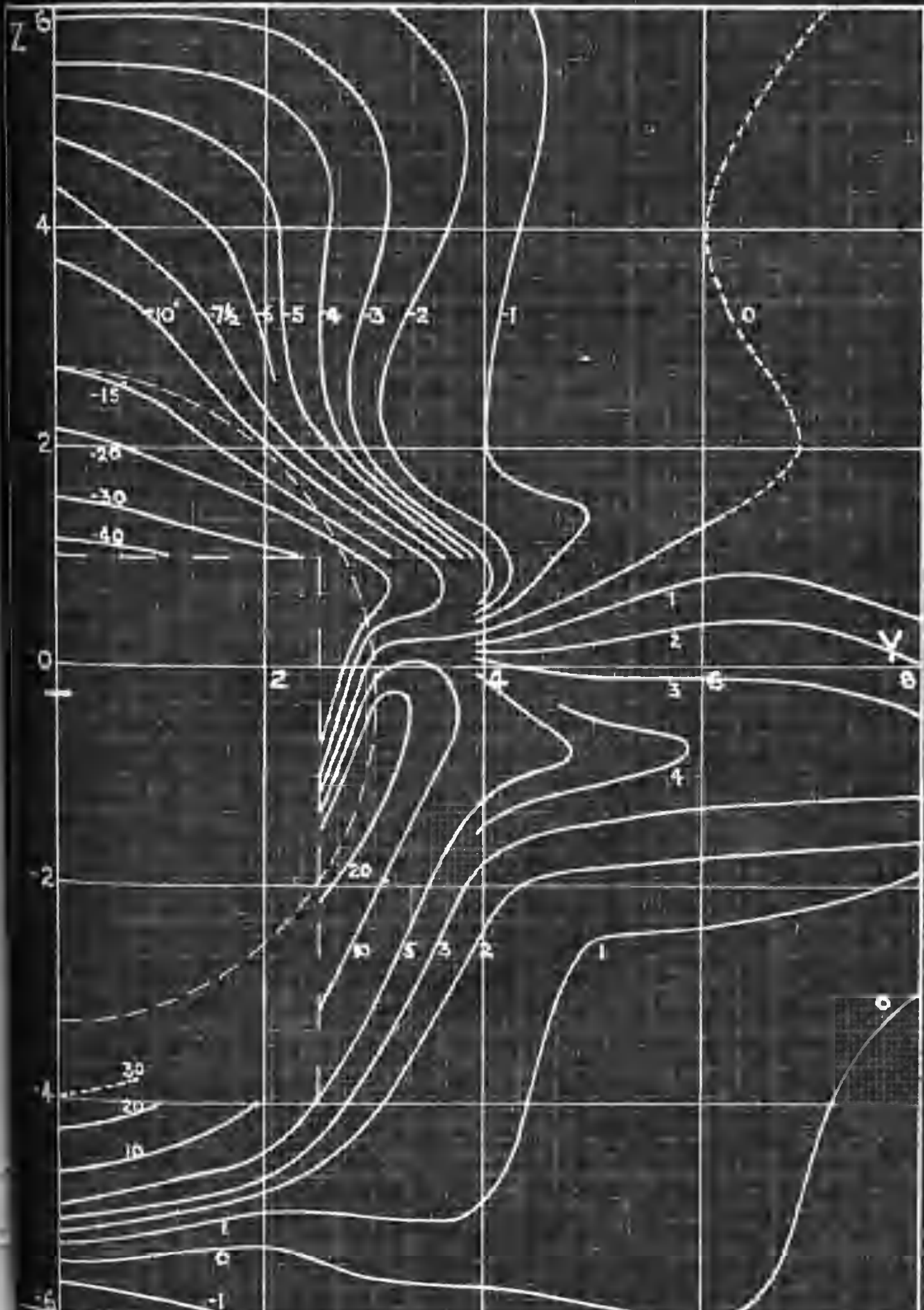
Velocity
f./s.
36
32
28
24
20
16
12
8
4
0
1·0
2·0
3·0
4·0
5·0
√U
Calibration of Static Plate
for measurement of
Channel Velocity.
⊙ ⊡ First Tests
+ Later Test
U = U-Gauge Reading


SECTION AT X=-3INS.
OBSERVATIONS OF PITCH (α°)
OBSERVATIONS OF YAW (β°)
OBSERVATIONS OF V/U.

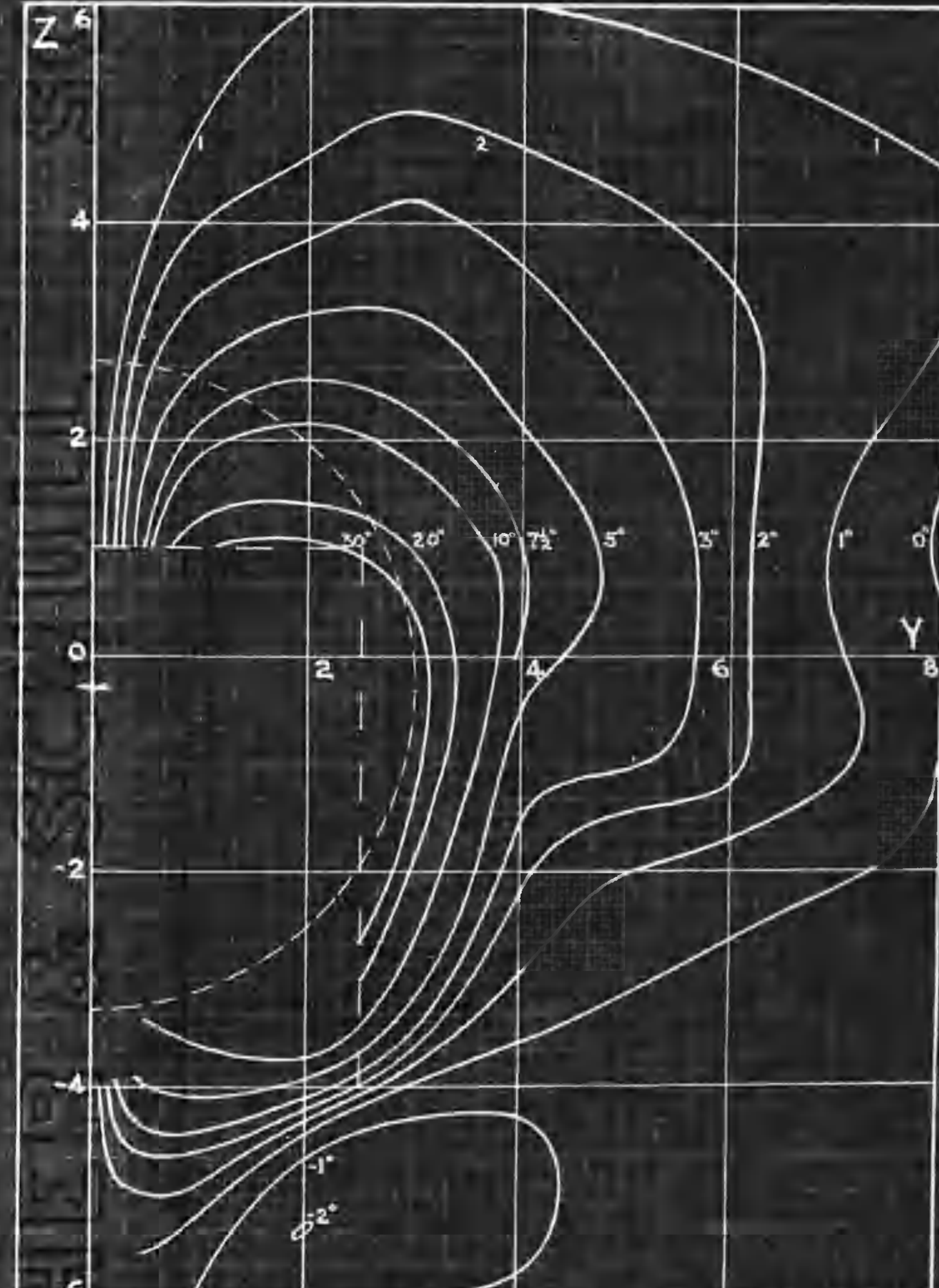

SECTION AT X=-6INS.
OBSERVATIONS OF PITCH (α°).
Z = 0
" = 1
" = 2
" = 4
" = 6
Z = -1
" = -2
" = -2½
" = -3
" = -4
" = -6
α°
Y (INS.)
OBSERVATIONS OF YAW (β°).
β°
Y
OBSERVATIONS OF V/U.
V/U
Y



SECTION AT X=-9INS.
OBSERVATIONS OF PITCH(α°)
Z = 0
" = 1
" = 2
" = 4
" = 6
Z = -1
" = -2
" = -3
" = -4
" = -5
" = -6
OBSERVATIONS OF YAW (β°).
β°
OBSERVATIONS OF V/U.
V/U
Y



SECTION AT X = -3 INS :: CONTOURS OF PITCH (α°)



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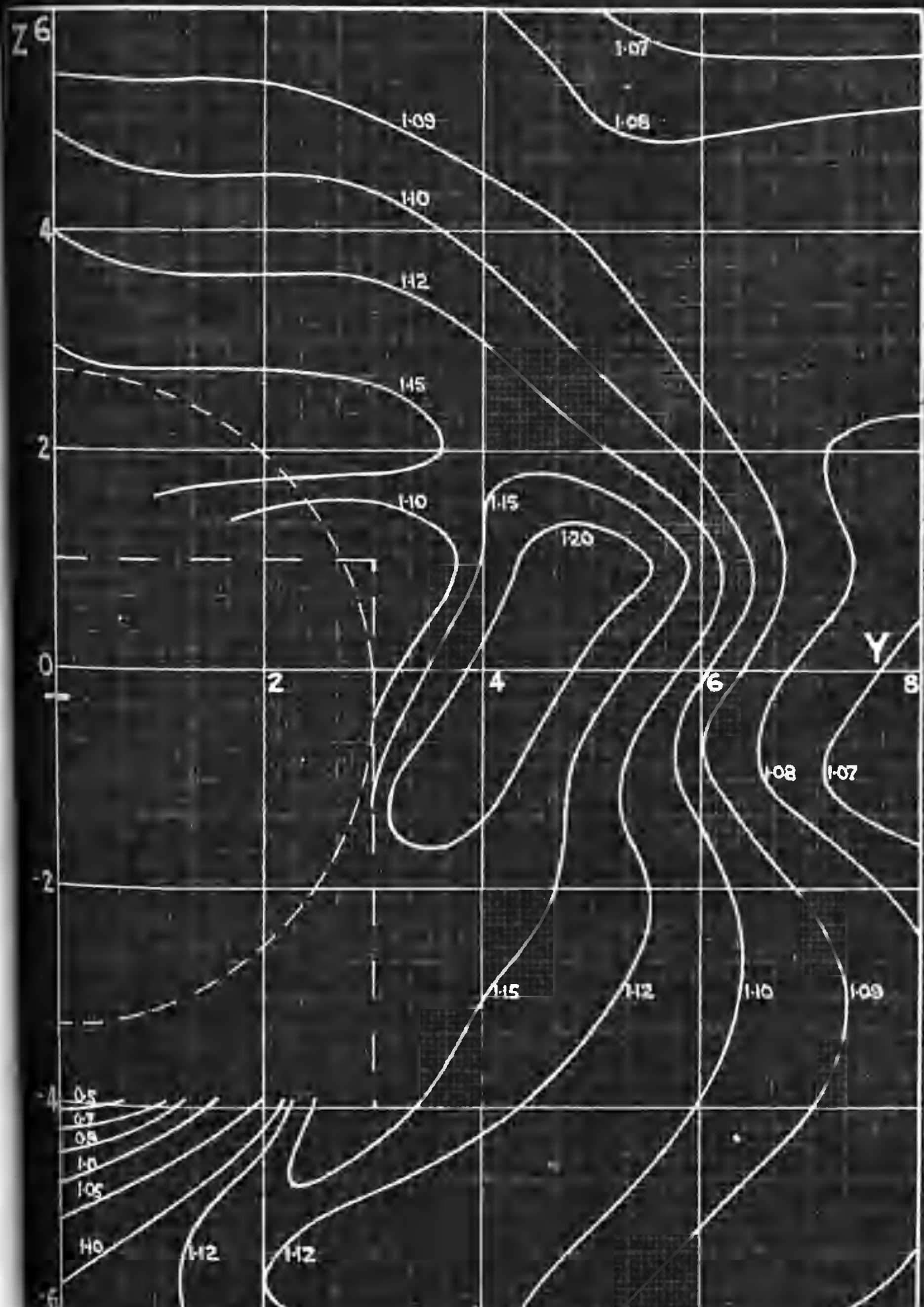

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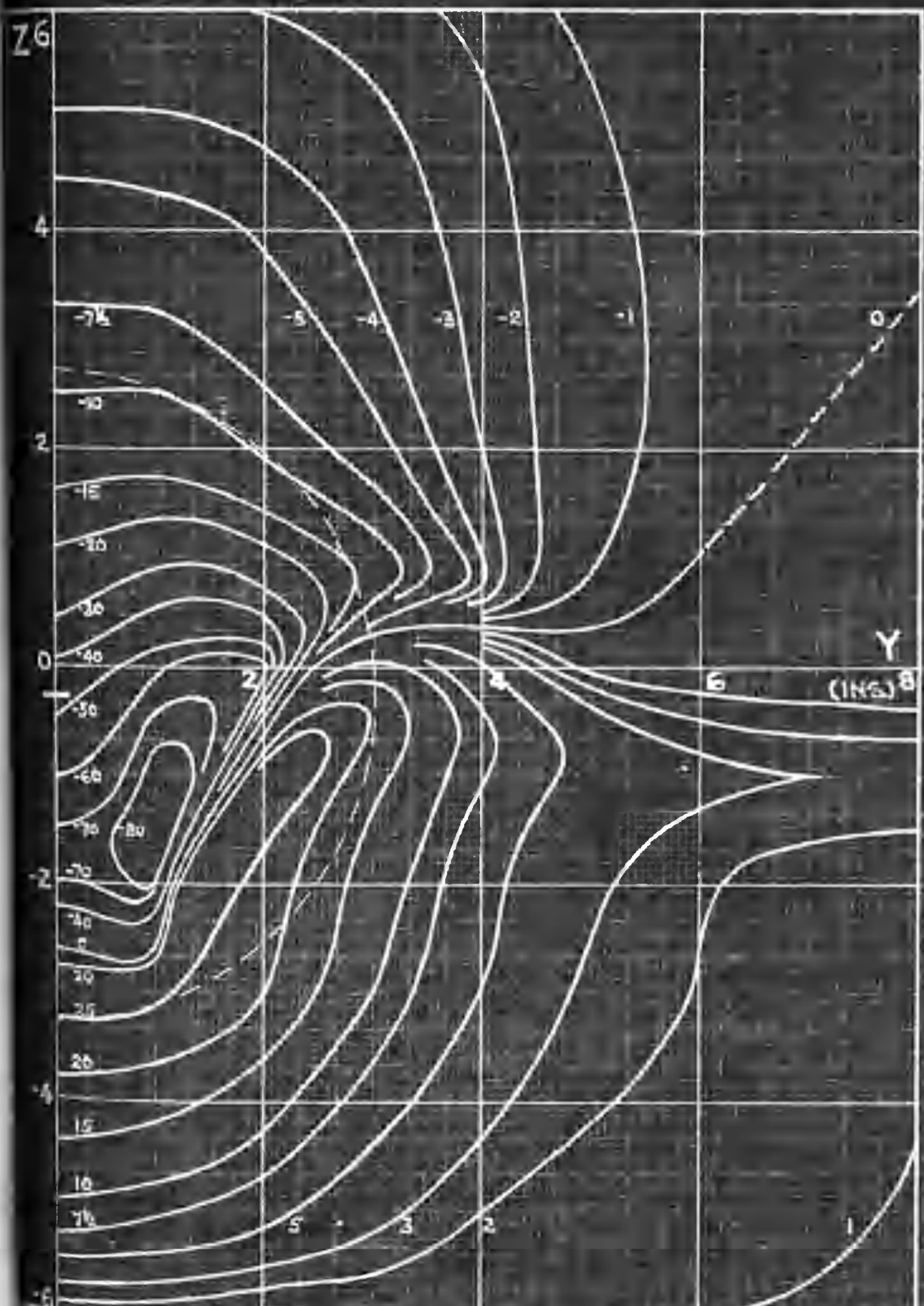
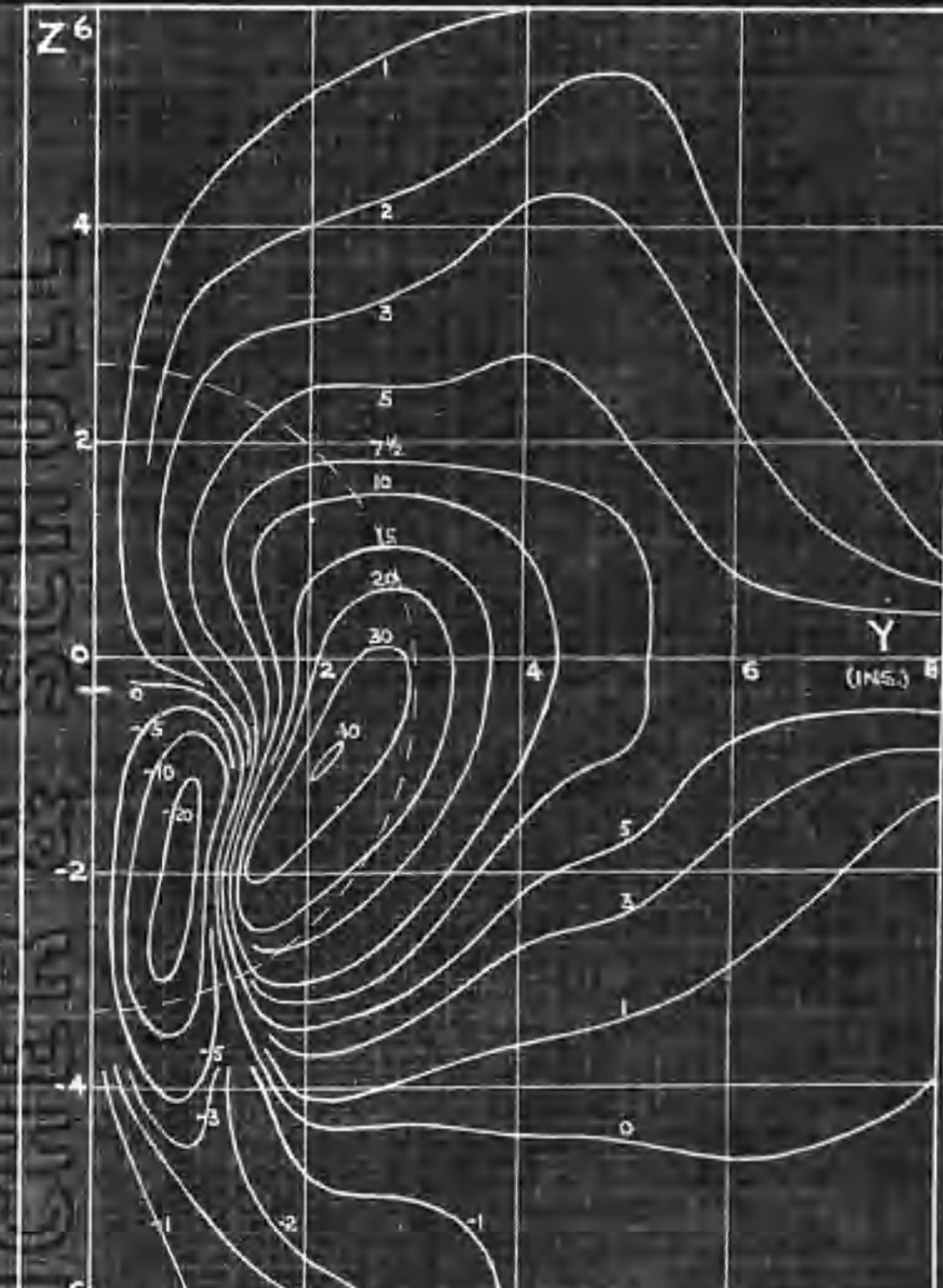
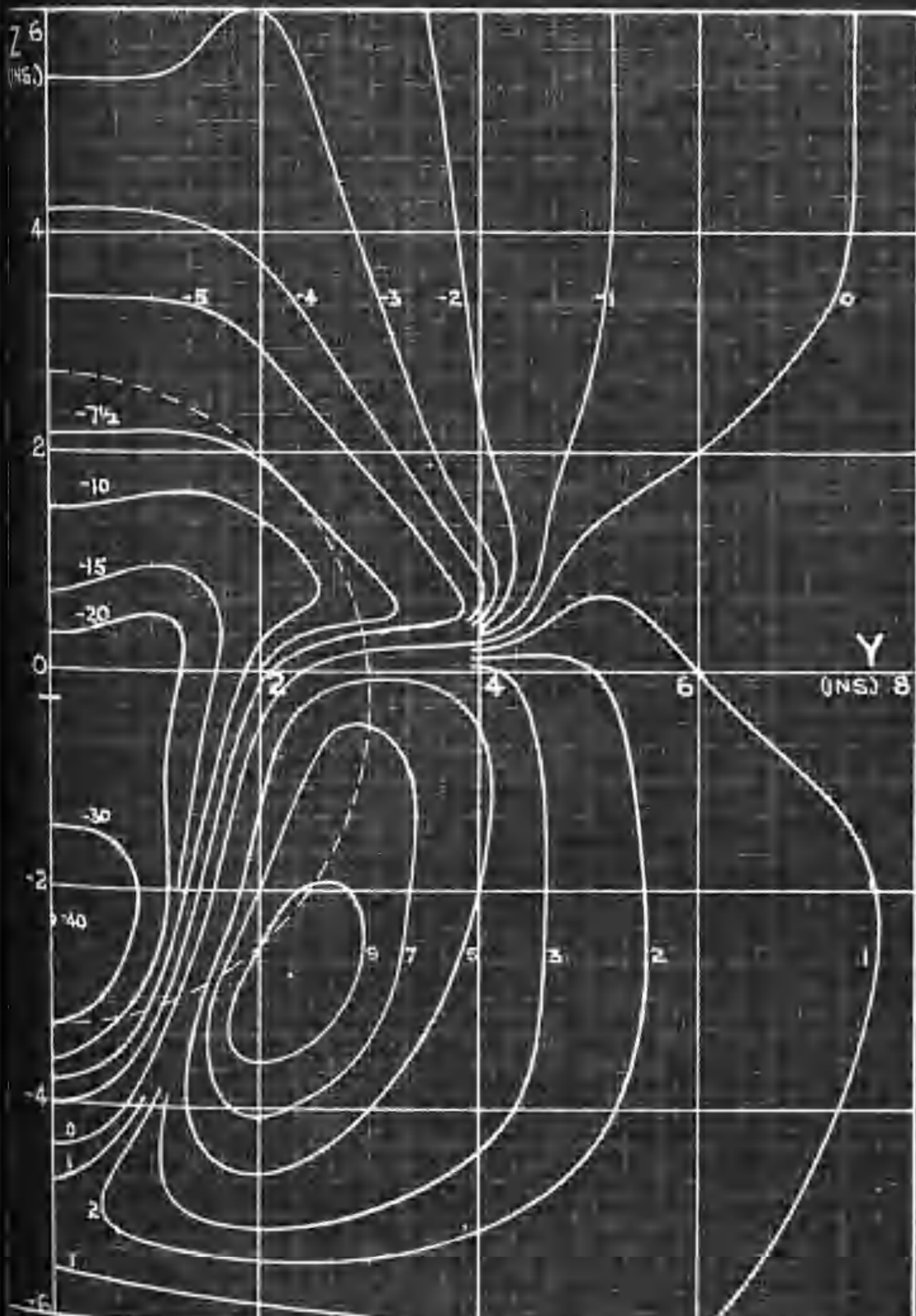


SECTION AT X=-6INS :: CONTOURS OF PITCH (α°)



SECTION AT X=-6INS :: CONTOURS OF YAW (β°)

SECTION AT X=-6INS. :: CONTOURS OF V/U.



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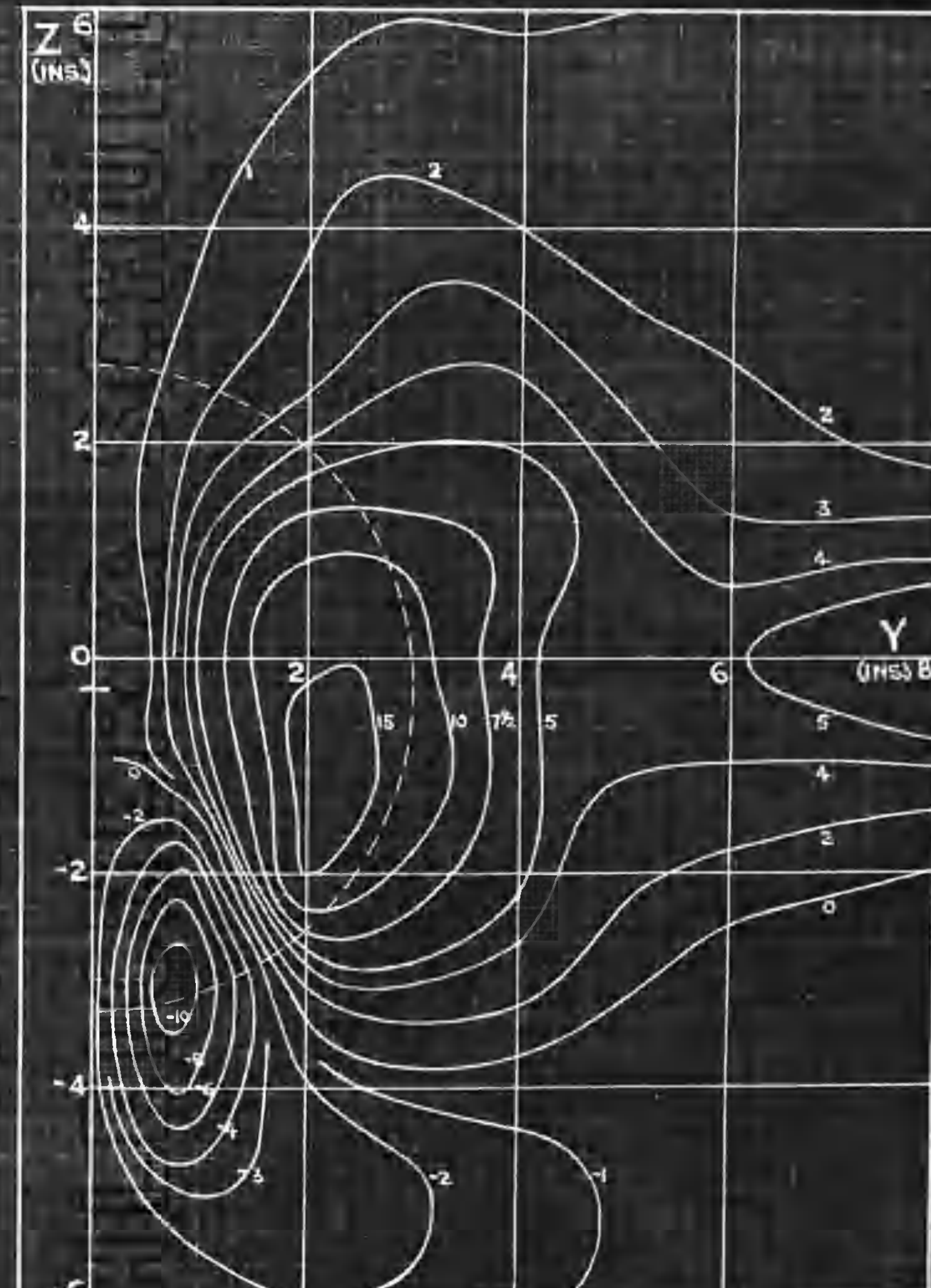

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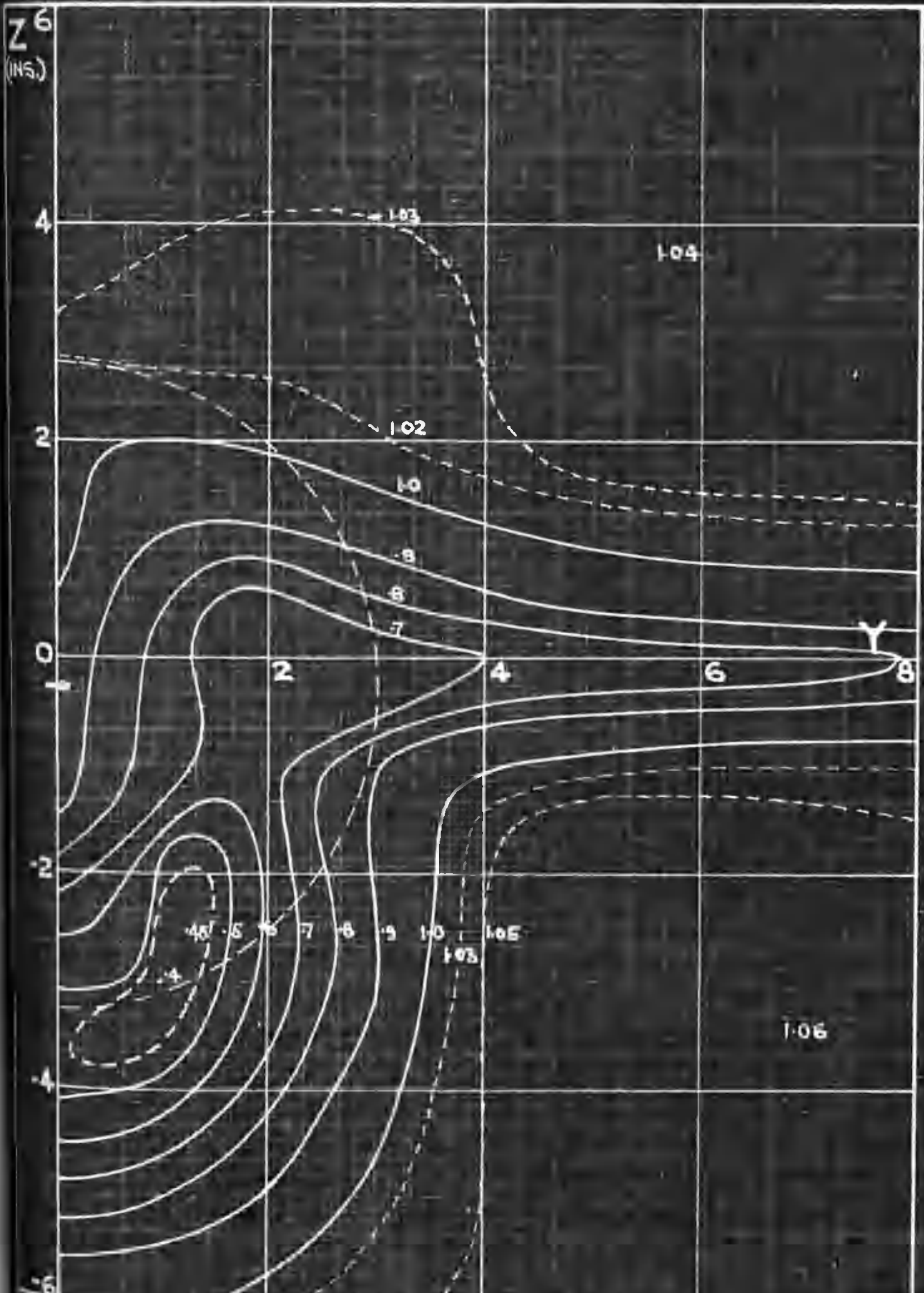

SECTION AT X=-9INS. :: CONTOURS OF V/U.

	α°	β°	$\log(\cos\alpha)$	$\log(\cos\alpha\cos\beta)$	$\log(\cos\theta\cos\alpha\cos\beta)$	$\cos\theta\cos\alpha\cos\beta$	$\pi - X$	$\log(\frac{\sqrt{3}}{2}\sin\theta\sin\alpha\sin\beta)$	$\frac{\sqrt{3}}{2}\sin\theta\sin\alpha\sin\beta$
	0		0.0000		$\bar{1}.8495$	.7071	0°		
	5		$\bar{1}.9983$		$\bar{1}.8478$	.7044	5°		
	10		$\bar{1}.9934$		$\bar{1}.8429$	.6965	10°		
$\log(\cos\theta)$	15		$\bar{1}.9849$		$\bar{1}.8344$	.6829	15°		
$= \bar{1}.8495$	20	0	$\bar{1}.9730$	SAME AS	$\bar{1}.8225$	.6645	20°	—	ZERO
	25		$\bar{1}.9573$	COLUMN (B)	$\bar{1}.8068$	.6409	25°		THROUGHOUT
	30		$\bar{1}.9375$		$\bar{1}.7870$	.6124	30°		
	35		$\bar{1}.9134$		$\bar{1}.7629$	.5793	35°		
	40		$\bar{1}.8843$		$\bar{1}.7338$	.5418	40°		
	45		$\bar{1}.8495$		$\bar{1}.6990$	.5000	45°		
	0		0.0000	$\bar{1}.9983$	$\bar{1}.8478$	.7044	5.0	$\bar{2}.7273$	.05337
$\log(\cos 5^\circ)$	5		$\bar{1}.9983$	$\bar{1}.9966$	$\bar{1}.8461$	.7017	7.2	$\bar{2}.7256$	.05316
$= \bar{1}.9983$	10		$\bar{1}.9934$	$\bar{1}.9917$	$\bar{1}.8412$	.6937	11.15	$\bar{2}.7207$	.05257
$\log(\cos\theta)$	15		$\bar{1}.9849$	$\bar{1}.9832$	$\bar{1}.8327$	.6803	15.85	$\bar{2}.7122$	.05154
$= \bar{1}.8495$	20	5	$\bar{1}.9730$	$\bar{1}.9713$	$\bar{1}.8208$	.6619	20.6	$\bar{2}.7003$	.05016
$\log(\cos 5^\circ\cos\theta)$	25		$\bar{1}.9573$	$\bar{1}.9556$	$\bar{1}.8051$	.6384	25.5	$\bar{2}.6846$	.04838
$= \bar{1}.8478$	30		$\bar{1}.9375$	$\bar{1}.9358$	$\bar{1}.7853$	.6099	30.4	$\bar{2}.6648$	.04622
$\log(\frac{\sqrt{3}}{2}\sin\theta\sin 5^\circ)$	35		$\bar{1}.9134$	$\bar{1}.9117$	$\bar{1}.7612$	.5771	35.3	$\bar{2}.6407$	.04372
$= \bar{2}.7273$	40		$\bar{1}.8843$	$\bar{1}.8826$	$\bar{1}.7321$	.5396	40.3	$\bar{2}.6116$	.04089
	45		$\bar{1}.8495$	$\bar{1}.8478$	$\bar{1}.6973$	.4980	45.2	$\bar{2}.5768$	.03774
	0		0.0000	$\bar{1}.9934$	$\bar{1}.8429$	.6965	10.0	$\bar{1}.0257$	.1064
$\log(\cos 10^\circ)$	5		$\bar{1}.9983$	$\bar{1}.9917$	$\bar{1}.8412$	.6937	11.4	$\bar{1}.0250$	.1059
$= \bar{1}.9934$	10		$\bar{1}.9934$	$\bar{1}.9868$	$\bar{1}.8363$	.6860	14.0	$\bar{1}.0201$	.1047
$\log(\cos\theta)$	15		$\bar{1}.9849$	$\bar{1}.9783$	$\bar{1}.8278$	.6726	18.0	$\bar{1}.0116$	.1027
$= \bar{1}.8495$	20	10	$\bar{1}.9730$	$\bar{1}.9664$	$\bar{1}.8156$	.6545	22.25	$\bar{2}.9997$	.0999
$\log(\cos\theta\cos 10^\circ)$	25		$\bar{1}.9573$	$\bar{1}.9507$	$\bar{1}.8002$	.6313	26.8	$\bar{2}.9840$	.0964
$= \bar{1}.8429$	30		$\bar{1}.9375$	$\bar{1}.9309$	$\bar{1}.7804$	.6032	31.5	$\bar{2}.9642$	.0921
$\log(\frac{\sqrt{3}}{2}\sin\theta\sin 10^\circ)$	35		$\bar{1}.9134$	$\bar{1}.9068$	$\bar{1}.7563$	.5706	36.2	$\bar{2}.9401$	.0871
$= \bar{1}.0257$	40		$\bar{1}.8843$	$\bar{1}.8777$	$\bar{1}.7272$	.5335	41.0	$\bar{2}.9110$	.0815
	45		$\bar{1}.8495$	$\bar{1}.8429$	$\bar{1}.6924$	.4925	45.9	$\bar{2}.8762$	.0752
	0		0.0000	$\bar{1}.9849$	$\bar{1}.8344$	.6829	15.0	$\bar{1}.2000$	.1585
$\log(\cos 15^\circ)$	5		$\bar{1}.9983$	$\bar{1}.9832$	$\bar{1}.8327$	.6803	15.85	$\bar{1}.1983$	.1579
$= \bar{1}.9849$	10		$\bar{1}.9934$	$\bar{1}.9783$	$\bar{1}.8278$	.6726	18.0	$\bar{1}.1934$	.1561
$\log(\cos\theta)$	15		$\bar{1}.9849$	$\bar{1}.9698$	$\bar{1}.8193$	.6597	21.1	$\bar{1}.1849$	.1531
$= \bar{1}.8495$	20	15	$\bar{1}.9730$	$\bar{1}.9579$	$\bar{1}.8074$	.6418	24.8	$\bar{1}.1730$	.1489
$\log(\cos\theta\cos 15^\circ)$	25		$\bar{1}.9573$	$\bar{1}.9422$	$\bar{1}.7917$	.6190	28.9	$\bar{1}.1573$	.1435
$= \bar{1}.8344$	30		$\bar{1}.9375$	$\bar{1}.9224$	$\bar{1}.7719$	.5914	33.2	$\bar{1}.1375$	.1373
$\log(\frac{\sqrt{3}}{2}\sin\theta\sin 15^\circ)$	35		$\bar{1}.9134$	$\bar{1}.8983$	$\bar{1}.7478$	.5595	37.7	$\bar{1}.1134$	.1298
$= \bar{1}.2000$	40		$\bar{1}.8843$	$\bar{1}.8692$	$\bar{1}.7187$	.5232	42.3	$\bar{1}.0843$	.1214
	45		$\bar{1}.8495$	$\bar{1}.8344$	$\bar{1}.6839$	.4829	46.5	$\bar{1}.0495$	.1120
	0		0.0000	$\bar{1}.9730$	$\bar{1}.8225$	.6645	20.0	$\bar{1}.3211$	.2094
$\log(\cos 20^\circ)$	5		$\bar{1}.9983$	$\bar{1}.9713$	$\bar{1}.8208$	.6619	20.6	$\bar{1}.3194$	.2086
$= \bar{1}.9730$	10		$\bar{1}.9934$	$\bar{1}.9664$	$\bar{1}.8159$	.6545	22.25	$\bar{1}.3145$	.2063
$\log(\cos\theta)$	15		$\bar{1}.9849$	$\bar{1}.9579$	$\bar{1}.8074$	.6418	24.8	$\bar{1}.3060$	.2023
$= \bar{1}.8495$	20	20	$\bar{1}.9730$	$\bar{1}.9460$	$\bar{1}.7955$	.6244	28.0	$\bar{1}.2941$	.1968
$\log(\cos\theta\cos 20^\circ)$	25		$\bar{1}.9573$	$\bar{1}.9303$	$\bar{1}.7798$	.6023	31.6	$\bar{1}.2784$	.1899
$= \bar{1}.8225$	30		$\bar{1}.9375$	$\bar{1}.9105$	$\bar{1}.7600$	.5754	35.5	$\bar{1}.2586$	.1813
$\log(\frac{\sqrt{3}}{2}\sin\theta\sin 20^\circ)$	35		$\bar{1}.9134$	$\bar{1}.8864$	$\bar{1}.7359$	.5444	39.7	$\bar{1}.2345$	.1716
$= \bar{1}.3211$	40		$\bar{1}.8843$	$\bar{1}.8573$	$\bar{1}.7068$	.5091	43.95	$\bar{1}.2054$	.1604
	45		$\bar{1}.8495$	$\bar{1}.8225$	$\bar{1}.6720$	.4699	48.4	$\bar{1}.1706$	.1481
	0		0.0000	$\bar{1}.9573$	$\bar{1}.8068$	.6409	25.0	$\bar{1}.4129$	.2587
$\log(\cos 25^\circ)$	5		$\bar{1}.9983$	$\bar{1}.9556$	$\bar{1}.8051$	.6384	25.5	$\bar{1}.4112$	.2577
$= \bar{1}.9573$	10		$\bar{1}.9934$	$\bar{1}.9507$	$\bar{1}.8002$	.6313	26.8	$\bar{1}.4063$	.2549
$\log(\cos\theta)$	15		$\bar{1}.9849$	$\bar{1}.9422$	$\bar{1}.7917$	.6190	28.9	$\bar{1}.3978$	.2500
$= \bar{1}.8495$	20	25	$\bar{1}.9730$	$\bar{1}.9303$	$\bar{1}.7798$	.6023	31.6	$\bar{1}.3859$	.2432
$\log(\cos\theta\cos 25^\circ)$	25		$\bar{1}.9573$	$\bar{1}.9146$	$\bar{1}.7641$	.5809	34.8	$\bar{1}.3702$	.2345
$= \bar{1}.8068$	30		$\bar{1}.9375$	$\bar{1}.8948$	$\bar{1}.7443$	.5550	38.3	$\bar{1}.3504$	.2241
$\log(\frac{\sqrt{3}}{2}\sin\theta\sin 25^\circ)$	35		$\bar{1}.9134$	$\bar{1}.8707$	$\bar{1}.7202$	.5250	42.1	$\bar{1}.3263$	.2119
$= \bar{1}.4129$	40		$\bar{1}.8843$	$\bar{1}.8416$	$\bar{1}.6911$	.4910	46.0	$\bar{1}.2972$	.1983
	45		$\bar{1}.8495$	$\bar{1}.8068$	$\bar{1}.6563$	.4532	50.1	$\bar{1}.2624$	.1830

PA

α° \ β°	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	β° / α°
-45	98.4	96.7	95.2	93.8	92.7	91.7	91.0	90.4	90.1	90	90.1	90.4	91.0	91.7	92.7	93.8	95.2	96.7	98.4	-45
-40	94.1	92.3	90.6	89.2	87.9	86.9	86.1	85.5	85.1	85	85.1	85.5	86.1	86.9	87.9	89.2	90.6	92.3	94.1	-40
-35	89.8	87.8	86.05	84.5	83.1	82.0	81.15	80.5	80.1	80	80.1	80.5	81.15	82.0	83.1	84.5	86.05	87.8	89.8	-35
-30	85.45	83.4	81.5	79.8	78.4	77.2	76.2	75.55	75.15	75	75.15	75.55	76.2	77.2	78.4	79.8	81.5	83.4	85.45	-30
-25	81.1	78.9	76.9	75.15	73.6	72.3	71.3	70.6	70.15	70	70.15	70.6	71.3	72.3	73.6	75.15	76.9	78.9	81.1	-25
-20	76.8	74.5	72.4	70.5	68.9	67.5	66.4	65.6	65.2	65	65.2	65.6	66.4	67.5	68.9	70.5	72.4	74.5	76.8	-20
-15	72.55	70.1	67.9	65.9	64.15	62.7	61.5	60.7	60.2	60	60.2	60.7	61.5	62.7	64.15	65.9	67.9	70.1	72.55	-15
-10	68.3	65.75	63.4	61.3	59.4	57.9	56.6	55.7	55.2	55	55.2	55.7	56.6	57.9	59.4	61.3	63.4	65.75	68.3	-10
-5	64.1	61.45	59.0	56.75	54.8	53.1	51.8	50.8	50.2	50	50.2	50.8	51.8	53.1	54.8	56.75	59.0	61.45	64.1	-5
0	60.0	57.2	54.6	52.2	50.1	48.35	46.9	45.85	45.2	45	45.2	45.85	46.9	48.35	50.1	52.2	54.6	57.2	60.0	0
5	56.0	53.0	50.3	47.8	45.6	43.7	42.1	40.95	40.2	40	40.2	40.95	42.1	43.7	45.6	47.8	50.3	53.0	56.0	5
10	52.0	49.0	46.1	43.45	41.05	39.0	37.3	36.0	35.3	35	35.3	36.0	37.3	39.0	41.05	43.45	46.1	49.0	52.0	10
15	48.25	45.1	42.05	39.25	36.7	34.4	32.6	31.2	30.3	30	30.3	31.2	32.6	34.4	36.7	39.25	42.05	45.1	48.25	15
20	44.6	41.3	38.2	35.2	32.4	30.0	27.9	26.3	25.35	25	25.35	26.3	27.9	30.0	32.4	35.2	38.2	41.3	44.6	20
25	41.2	37.8	34.5	31.4	28.4	25.7	23.4	21.55	20.4	20	20.4	21.55	23.4	25.7	28.4	31.4	34.5	37.8	41.2	25
30	38.1	34.6	31.2	27.9	24.7	21.7	19.1	16.9	15.5	15	15.5	16.9	19.1	21.7	24.7	27.9	31.2	34.6	38.1	30
35	35.4	31.85	28.35	24.9	21.5	18.2	15.2	12.5	10.7	10	10.7	12.5	15.2	18.2	21.5	24.9	28.35	31.85	35.4	35
40	33.1	29.6	26.1	22.55	19.0	15.5	12.1	8.9	6.25	5	6.25	8.9	12.1	15.5	19.0	22.55	26.1	29.6	33.1	40
45	31.4	28.0	24.5	21.1	17.6	14.1	10.6	7.0	3.6	0	3.6	7.0	10.6	14.1	17.6	21.1	24.5	28.0	31.4	45

β° / α°	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	β° / α°
-45	72.7	69.2	65.7	62.2	58.7	55.1	51.6	48.1	44.7	41.4	38.2	35.1	32.3	29.8	27.6	25.9	24.8	24.3	24.5	-45
-40	73.8	70.1	66.3	62.5	58.7	54.8	51.0	47.2	43.45	39.7	36.1	32.6	29.3	26.3	23.6	21.4	19.9	19.3	19.6	-40
-35	75.1	71.1	67.1	63.0	58.9	54.9	50.8	46.7	42.6	38.5	34.55	30.6	26.9	23.25	20.0	17.2	15.2	14.3	14.7	-35
-30	76.4	72.2	68.0	63.8	59.5	55.2	50.9	46.5	42.2	37.9	33.6	29.3	25.1	21.0	17.1	13.5	10.7	9.3	10.0	-30
-25	77.9	73.5	69.1	64.7	60.3	55.8	51.3	46.8	42.3	37.8	33.3	28.7	24.2	19.7	15.3	10.9	6.9	4.3	5.7	-25
-20	79.4	74.9	70.4	65.9	61.3	56.7	52.1	47.5	42.9	38.25	33.6	28.9	24.3	19.6	14.9	10.3	5.5	0.7	3.9	-20
-15	81.0	76.4	71.8	67.2	62.6	57.9	53.3	48.6	43.9	39.25	34.6	29.9	25.3	20.7	16.2	11.9	8.0	5.8	7.0	-15
-10	82.7	78.0	73.4	68.7	64.0	59.4	54.7	50.0	45.4	40.7	36.1	31.55	27.1	22.75	18.6	14.9	12.1	10.7	11.4	-10
-5	84.4	79.7	75.0	70.4	65.7	61.0	56.4	51.8	47.2	42.7	38.2	33.9	29.7	25.7	22.1	19.0	16.7	15.7	16.2	-5
0	86.2	81.5	76.8	72.2	67.5	62.9	58.4	53.8	49.6	45.0	40.7	36.6	32.7	29.1	25.9	23.3	21.5	20.7	21.1	0
5	87.9	83.3	78.7	74.1	65.5	65.0	60.55	56.15	51.85	47.65	43.6	39.75	36.2	32.9	30.1	27.9	26.35	25.7	26.0	5
10	89.7	85.2	80.6	76.1	71.6	67.2	62.9	58.7	54.6	50.6	46.8	43.2	39.9	36.9	34.4	32.5	31.2	30.7	31.0	10
15	91.5	87.0	82.6	78.2	73.9	69.6	65.5	61.4	57.5	53.7	50.2	46.9	43.8	41.2	39.0	37.3	36.2	35.7	35.9	15
20	93.3	88.95	84.6	80.4	76.2	72.1	68.2	64.3	60.6	57.1	53.8	50.7	47.95	45.55	43.6	42.1	41.1	40.7	40.9	20
25	95.1	90.9	86.7	82.6	78.6	74.75	71.0	67.3	63.85	60.6	57.5	54.7	52.2	50.0	48.2	46.9	46.05	45.7	45.9	25
30	96.8	92.8	88.8	84.9	81.1	77.45	73.9	70.5	67.2	64.2	61.4	58.8	56.5	54.6	53.0	51.8	51.0	50.7	50.9	30
35	98.5	94.7	90.9	87.2	83.7	80.2	76.9	73.7	70.7	67.9	65.3	62.9	60.9	59.1	57.7	56.65	56.0	55.7	55.8	35
40	100.1	96.5	93.0	89.6	86.25	83.0	79.95	77.0	74.25	71.7	69.3	67.2	65.3	63.8	62.5	61.55	60.95	60.7	60.8	40
45	101.7	98.35	95.1	91.9	88.8	85.9	83.1	80.4	77.9	75.5	73.4	71.5	69.8	68.4	67.3	66.45	65.9	65.7	65.8	45
α° / β°	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	β° / α°

TABLE 5.

PITCH DEG.	YAW DEG.	CHATTOCK READINGS A→X	B→X	C→X	ZERO	PRESSURE DIFFS P_{AX}	P_{BX}	P_{CX}	$P_{AX}-P_{BX}$	$P_{AX}-P_{CX}$	$\sum_A^C P_{AX}$	AUXILIARY FNS. ϕ	ψ		"G" FUNCTIONS G_{AX}	G_{BX}	G_{CX}
	0	9.28	9.45	9.36	13.66 13.69	4.38	4.23	4.31	0.15	0.07	12.92	·012	·005		·600	·580	·591
	-5	9.34	8.47	10.28	13.66	4.32	5.19	3.39	-0.87	0.93	12.90	-·067	·072		·592	·711	·466
	5	9.54	10.49	8.39	13.64 13.60	4.08	3.15	5.25	0.93	-1.17	12.48	·074	-·034		·559	·431	·720
	5	9.47	10.42	8.42	13.61	4.13	3.18	5.19	0.95	-1.06	12.50	·076	-·085		·566	·436	·711
	-10	9.32	7.34	11.46	13.63	4.31	6.28	2.17	-1.97	2.14	12.76	-·154	·168		·591	·860	·297
0°	10	9.59	11.72	7.71	13.63	4.04	1.91	5.92	2.13	-1.88	11.87	·180	-·158		·554	·262	·831
	-15	9.39	6.55	12.77	13.62	4.23	7.08	0.86	-2.85	3.37	12.17	-·234	·277		·580	·970	·118
	15	9.79	12.95	6.85	13.59	3.80	0.66	6.75	3.14	-2.95	11.21	·280	-·263		·521	·090	·926
	-20	9.71	6.02	13.86	13.60	3.89	7.58	-0.26	-3.69	4.15	11.21	-·329	·370		·533	1.039	-·036
	20	10.09	14.16	6.67	13.59	3.50	-0.56	6.92	4.06	-3.42	9.86	·412	-·347		·480	-·077	·949
	-25	9.72	5.63	15.12	13.58	3.86	7.96	-1.54	-4.10	5.40	10.28	-·399	·525		·529	1.091	-·211
	25	10.36	15.54	6.47	13.58	3.22	-1.96	7.11	5.18	-3.88	8.38	·619	-·463		·442	-·269	·975
	0	8.15	10.00	9.78	13.64 13.61	5.48	3.62	3.83	1.86	1.65	12.93	·144	·128		·750	·496	·525
	-5	8.11	8.95	10.86	13.61	5.50	4.66	2.75	0.84	2.75	12.91	·065	·213		·754	·638	·376
	5	8.25	11.12	8.88	13.58	5.34	2.48	4.76	2.86	0.60	12.60	·229	·048		·734	·340	·652
	-10	8.25	7.88	12.04	13.59	5.33	5.70	1.53	-0.37	3.80	12.56	-·029	·308		·730	·781	·210
	10	8.57	12.43	8.09	13.55	5.00	1.13	5.46	3.87	-0.46	11.59	·334	-·040		·685	·155	·798
-5°	-15	8.15	7.10	13.16	13.54	5.40	6.45	0.38	-1.05	5.02	12.23	-·086	·410		·740	·884	·062
	15	8.58	13.63	7.53	13.55	4.96	-0.08	6.02	5.04	-1.06	10.90	·462	-·097		·680	-·011	·825
	-20	8.34	6.51	14.55	13.56	5.21	7.04	-0.99	-1.83	6.20	11.26	-·162	·555		·714	·965	-·136
	20	8.98	14.73	6.95	13.52	4.57	-1.19	6.57	5.76	-2.01	9.96	·575	-·202		·626	-·163	·901
	-25	8.63	5.89	15.75	13.54	4.90	7.65	-2.21	-2.75	7.11	10.34	-·266	·659		·671	1.089	-·303
	25	9.31	16.00	6.70	13.54	4.23	-2.46	6.84	6.69	-2.61	8.61	·774	-·308		·579	-·237	·937
	0	11.06	9.05	8.83	13.85 13.81	2.78	4.78	4.93	-2.00	-2.15	12.49	-·160	-·172		·381	·655	·676
	-5	10.97	7.94	9.84	13.75	2.83	5.84	3.92	-3.01	-1.09	12.59	-·239	-·087		·368	·800	·537
	5	11.04	9.86	8.03	13.71	2.70	3.87	5.69	-1.17	-2.99	12.26	-·095	-·244		·370	·530	·780
	-10	10.80	6.89	10.83	13.68	2.91	6.81	2.86	-3.90	0.05	12.58	-·310	·004		·399	·934	·392
	10	11.10	11.05	7.25	13.67	2.58	2.63	6.42	-0.05	-3.84	11.63	-·004	-·330		·354	·360	·880
-5½°	-15	10.82	6.15	12.03	13.67	2.85	7.52	1.64	-4.67	1.21	12.01	-·389	·101		·390	1.030	·225
	15	11.25	12.22	6.67	13.64	2.42	1.44	6.97	0.98	-4.55	10.83	·090	-·420		·332	·197	·955
	-20	10.87	5.67	13.33	13.63	2.77	7.97	0.30	-5.20	2.47	11.04	-·471	·224		·380	1.092	-·041
	20	11.50	13.55	6.25	13.62	2.13	0.08	7.37	2.05	-5.24	9.58	·214	-·546		·292	·011	1.010
	-25	11.02	5.39	14.63	13.63	2.60	8.23	-1.00	-5.63	3.60	8.83	-·573	·366		·356	1.127	-·137
	25	11.65	14.88	6.13	13.59	1.93	-1.27	7.47	3.20	-5.54	8.13	·314	-·680		·264	-·174	1.024
	0	7.37	10.85	10.73	13.83 13.72	6.43	2.92	3.01	3.51	3.42	12.36	·284	·277		·881	·400	·412
	-5	7.21	9.74	11.73	13.68	6.50	3.96	1.96	2.54	4.54	12.42	·204	·366		·891	·543	·269
	5	7.34	11.89	9.69	13.65	6.33	1.77	3.96	4.56	2.37	12.06	·378	·196		·867	·242	·543
	-10	7.12	8.67	12.97	13.65	6.53	4.98	0.68	1.55	5.85	12.19	·127	·480		·895	·682	·093
	10	7.43	13.10	8.85	13.62	6.22	0.54	4.78	5.68	1.44	11.54	·492	·125		·853	·074	·655
-10°	-15	7.31	7.68	14.05	13.61	6.31	5.93	-0.44	0.38	6.75	11.80	·032	·572		·865	·813	-·060
	15	7.68	14.25	8.00	13.59	5.93	-0.75	5.59	6.68	0.34	10.77	·620	·032		·813	-·103	·766
	-20	7.54	7.08	15.42	13.59	6.05	6.51	-1.83	-0.46	7.88	10.73	-·043	·734		·829	·892	-·251
	20	8.01	15.58	7.60	13.58	5.58	-2.00	5.98	7.58	-0.40	9.56	·793	-·042		·765	-·274	·820
	-25	7.91	6.66	16.65	13.59	5.67	6.92	-3.06	-1.25	8.73	9.53	-·131	·916		·777	·949	-·419
	25	8.56	16.76	7.48	13.58	5.01	-3.17	6.09	8.18	-1.08	7.93	1.031	-·136		·686	-·435	·835

TABLE 5 (CONT.)

Pitch	Yaw	Chattock Readings				Pressure Diffs			$P_{AX}-P_{BX}$	$P_{AX}-P_{CX}$	$\sum_{A}^{C} P_{RX}$	Auxiliary Fns.			"G" Functions		
DEG	DEG	A→X	B→X	C→X	ZERO	P_{AX}	P_{BX}	P_{CX}				φ	ψ		G_{AX}	G_{BX}	G_{CX}
10	0	12.28	8.46	8.37	13.76 13.73	1.47	5.25	5.36	-3.81	-3.89	12.11	-.314	-.321		.201	.724	.735
	-5	12.13	7.45	9.27	13.69	1.59	6.25	4.42	-4.66	-2.83	12.26	-.381	-.231		.219	.856	.606
	5	12.34	9.56	7.47	13.67	1.34	4.11	6.20	-2.77	-4.86	11.65	-.238	-.417		.184	.564	.850
	-10	12.08	6.57	10.31	13.67	1.62	7.10	3.36	-5.48	-1.74	12.08	-.453	-.144		.222	.974	.461
	10	12.34	10.66	6.70	13.64	1.30	2.99	6.94	-1.69	-5.64	11.23	-.150	-.501		.178	.410	.951
	-15	11.97	5.87	11.56	13.63	1.67	7.76	2.48	-6.09	-0.81	11.51	-.529	-.036		.229	1.063	.785
	15	12.49	4.71	6.27	13.60	1.13	1.90	7.33	-0.77	-6.20	10.36	-.074	-.599		.155	.260	1.005
	-20	12.01	5.81	12.73	13.60	1.59	8.19	0.87	-6.60	0.72	10.65	-.619	.068		.219	1.122	.119
	20	12.57	13.00	5.88	13.60	1.03	0.60	7.72	0.43	-6.69	9.35	.046	-.715		.141	.072	1.058
	-25	12.14	5.11	14.04	13.59	1.46	8.48	-0.45	-7.02	1.91	9.49	-.740	.201		.200	1.162	-.062
	25	12.72	14.36	5.89	13.59	0.87	-0.77	7.70	1.64	-6.83	7.80	.210	-.875		.119	-.106	1.055
-15	0	6.27	11.51	11.37	13.55 13.53	7.28	2.03	2.16	5.25	5.12	11.47	.458	.447		.988	.278	.296
	-5	6.24	10.47	12.42	13.53	7.29	3.06	1.11	4.23	6.18	11.46	.369	.539		.999	.420	.152
	5	6.34	12.64	10.37	13.53	7.19	1.89	3.16	6.30	4.03	11.24	.560	.358		.985	.172	.433
	-10	6.39	9.38	13.58	13.54	7.14	4.15	-0.04	2.99	7.18	11.25	.266	.638		.981	.569	.005
		6.47	9.40	13.69	13.70 13.66	7.22	4.07	-0.03	3.15	7.25	11.26	.280	.643		.990	.558	-.004
	10	6.66	14.02	9.56	13.64	6.99	-0.38	4.08	7.37	2.91	10.69	.690	.272		.956	-.042	.560
	-15	6.48	8.50	14.79	13.52	7.04	5.02	-1.27	2.02	8.31	10.79	.187	.770		.965	.488	-.174
	15	6.95	15.04	8.75	13.56	6.41	-1.48	4.81	8.09	1.80	9.94	.814	.181		.904	-.203	.651
	-20	6.82	7.76	16.03	13.55	6.76	5.78	-2.49	0.96	9.23	10.03	.096	.920		.924	.752	-.341
	20	7.17	14.33	8.20	13.53	6.36	-2.81	5.32	9.17	1.04	8.57	1.034	.117		.871	-.385	.729
	-25	7.37	7.35	17.47	13.64 13.58	6.25	6.25	-3.89	0.00	10.14	8.61	.000	1.180		.886	.866	-.533
	25	7.75	17.54	7.76	13.57	5.83	-3.97	5.80	9.80	0.03	7.66	1.280	.004		.795	-.544	.795
15	0	13.75	7.95	7.82	13.73 13.71	-0.02	5.77	5.89	-5.79	-5.91	11.64	-.497	-.508		-.013	.791	.806
	-5	13.69	7.02	8.74	13.70	0.02	6.69	4.96	-6.67	-4.94	11.67	-.572	-.424		.003	.917	.680
	5	13.72	8.96	7.15	13.70	-0.01	4.74	6.55	-4.76	-6.57	11.27	-.422	-.583		-.003	.649	.898
	-10	13.53	6.24	9.85	13.67	0.17	7.45	3.83	-7.28	-3.66	11.45	-.636	-.320		.002	1.021	.525
	10	13.75	10.08	6.48	13.65	-0.08	3.58	7.17	-3.66	-7.25	10.67	-.343	-.680		-.011	.490	.983
	-15	13.50	6.52	11.00	13.63	0.15	8.12	2.63	-7.97	-2.48	10.90	-.731	-.228		.021	1.112	.360
	15	13.89	11.30	6.04	13.62	-0.26	2.33	7.58	-2.59	-7.84	9.65	-.268	-.812		-.036	.319	1.039
	-20	13.43	5.11	12.35	13.60	0.19	8.50	1.25	-8.31	-1.06	9.94	-.836	-.107		.026	1.165	.171
	20	13.94	12.55	5.78	13.60	-0.34	1.05	7.82	-1.39	-8.16	8.53	-.163	-.956		-.047	.144	1.072
	-25	13.38	5.57	13.95	13.60	0.22	8.03	-0.35	-7.81	0.57	7.90	-.989	.072		.030	1.100	-.048
	25	13.93	13.79	6.01	13.60	-0.33	-0.19	7.59	-0.14	-7.92	7.07	-.020	-1.12		-.045	-.026	1.040
-20	0	13.24	19.59	19.62	21.16 21.18	7.92	1.18	1.56	6.74	6.36	10.66	.632	.596		1.086	.162	.214
	-5	13.04	18.88	20.65	21.18	8.14	1.30	0.53	6.84	7.61	10.97	.532	.694		1.115	.315	.073
	5	13.55	21.40	18.78	21.46 21.46	7.91	0.06	2.68	7.85	5.23	10.65	.737	.491		1.084	.008	.367
	-10	13.37	17.97	22.21	21.45	8.09	3.49	-0.76	4.60	8.85	10.82	.425	.818		1.109	.478	-.104
	10	13.78	22.63	17.94	21.48 21.43	7.69	-1.18	3.49	8.87	4.20	10.00	.887	.420		1.054	-.162	.478
	-15	13.60	16.89	23.48	21.43	7.83	4.54	-2.05	3.29	9.88	10.32	.319	.957		1.073	.621	-.281
	15	14.03	23.68	17.05	21.41	7.40	-2.26	4.36	9.66	3.04	9.50	1.017	.320		1.014	-.310	.598
	-20	13.93	16.08	24.50	21.38	7.48	5.32	-3.11	2.16	10.59	9.69	.223	1.091		1.025	.729	-.426
	20	14.63	24.88	16.43	21.35	6.75	-3.51	4.93	10.26	1.83	8.17	1.256	.223		.925	-.481	.675
	-25	14.45	15.69	25.77	21.37	6.90	5.71	-4.40	1.19	11.30	8.21	.145	1.376		.946	.782	-.603
	25	15.04	25.86	16.00	21.33	6.33	-4.51	5.33	10.84	1.00	7.15	1.517	.140		.866	-.618	.730
	-10	13.51	17.94	22.34	21.52 21.50	8.01	3.57	-0.84	4.44	8.85	10.74	.413	.824		1.098	.489	-.115
	-10	13.46	17.90	22.15	21.48	8.04	3.59	-0.67	4.45	8.71	10.96	.406	.795		1.101	.492	-.097

TABLE 5 (CONT.)

Pitch deg.	Yaw deg.	Chattock Readings A→X	B→X	C→X	Zero	Pressure Diffs P_{AX}	P_{BX}	P_{CX}	$P_{AX}-P_{BX}$	$P_{AX}-P_{CX}$	$\sum_A^C P_{AX}$	Auxiliary Fns. ϕ	ψ		"G" Functions G_{AX}	G_{BX}	G_{CX}
					20.91												
	0	22.37	14.74	14.78	20.93	-1.46	6.18	6.15	-7.64	-7.61	10.87	-.7[illegible]3	-.700		-.200	.846	.848
	-5	22.33	13.93	15.66	20.92	-1.40	7.00	5.26	-8.40	-6.66	10.86	-.773	-.614		-.192	.960	.721
	5	22.46	15.93	14.13	20.91	-1.54	4.99	6.78	-6.53	-8.32	10.23	-.638	-.813		-.211	.684	.929
20°	-10	22.25	13.33	16.73	20.91	-1.34	7.58	4.18	-8.92	-5.82	10.42	-.856	-.530		-.184	1.039	.573
	10	22.58	17.02	13.43	20.90	-1.67	3.89	7.47	-5.56	-9.14	9.69	-.574	-.943		-.229	.533	1.024
	-15	22.08	12.75	18.03	20.91	-1.18	8.15	2.88	-9.33	-4.06	9.85	-.946	-.412		-.162	1.117	.395
	15	22.57	18.16	13.27	20.90	-1.66	2.75	7.63	-4.41	-9.29	8.72	-.506	-1.064		-.227	.377	1.045
	-20	22.03	12.48	19.06	20.91	-1.13	8.42	1.85	-9.55	-2.98	9.14	-1.045	-.326		-.155	1.154	.254
	20	22.97	19.77	13.45	21.25 21.26	-1.72	1.48	7.81	-3.20	-9.53	7.57	-.423	-1.26		-.236	.203	1.070
	-25	22.04	12.92	20.87	21.25	-0.78	8.34	0.38	-9.12	-1.16	7.94	-1.15	-.146		-.107	1.142	.052
	25	22.91	20.99	13.60	21.26	-1.66	0.26	7.66	-1.92	-9.32	6.26	-.307	-1.49		-.227	.036	1.050

TABLE 6.

Gauge Settings Vel. V f/s	Pitch α deg	Yaw β deg	Chattock Readings A→X	B→X	C→X	Zero	Press. Diffs P_{AX}	P_{BX}	P_{CX}	$P_{AX}-P_{BX}$	$P_{AX}-P_{CX}$	$\sum_A^C P_{AX}$	Aux.y Fns. ϕ	ψ	"G" Functions G_{AX}	G_{BX}	G_{CX}	Obsd. Reads. V f/s	α"	β"
27.0	5.0	+11.0	18.22	18.54	14.25	20.86 20.87	2.64	2.32	6.62	0.32	-3.98	11.88	.028	-.344	.37	-	-	26.8	5.2	11.1
27.0	7.0	-12.0	18.42	13.88	18.40	20.86 20.88	2.44	6.99	2.48	-4.55	-0.04	11.91	-.382	-.063	.35	-	-	26.4	6.7	-11.5
27.0	-8.5	-7.0	14.62	16.45	19.13	20.86 20.89	6.24	4.42	1.76	1.81	4.48	12.43	.146	.360	-	-	.24	27.1	-8.7	-6.5
27.0	-8.5	+7.0	14.90	19.55	16.27	20.95 20.95	6.05	1.40	4.68	4.65	1.37	12.13	.383	.113	-	.19	-	27.2	-8.5	7.3
20.6	7.0	-14.0	19.41	16.60	19.70	20.85 20.86	1.44	4.25	1.16	2.81	0.28	6.85	-.410	.041	.35	-	-	20.3	6.7	-14.0
20.6	-8.5	-7.0	17.44	18.33	19.93	20.87 20.88	3.43	2.55	0.95	0.88	2.48	6.93	.127	.358	-	-	.225	20.5	-8.3	-7.0
20.6	-8.5	+7.0	17.58	20.22	18.40	20.94 20.94	3.36	0.72	2.54	2.64	0.82	6.62	.399	.124	-	.18	-	20.0	-8.9	7.3
33.0	-8.5	+7.0	11.65	18.87	13.98	20.97 20.95	9.32	2.09	6.97	7.23	2.35	18.38	.393	.128	-	.19	-	33.2	-8.9	7.0

of radius a, give the drag and lift respectively. If dS be an element of the surface of the sphere, then $\iint p''_{rx}.dS$ and $\iint p''_{ry}.dS$ are easily seen to be zero. Since p_2 is constant and $xu_2 + yv_2 + zw_2 = 0$ then the first and third terms of these integrals vanish for the reasons given above. The second terms $\mu\iint(\frac{\partial}{\partial r} - \frac{1}{r})u_2 dS$ and $\mu\iint(\frac{\partial}{\partial r} - \frac{1}{r})v_2 dS$ are also zero, the integrand of the first, viz. $(3\omega_0 a^3 y/r^4)_{r=a}$, being proportional to y and that of the second to x.

Hence, the drag of a slowly rotating sphere in a slowly moving stream is the same as if it had no rotation and is that found by Stokes, viz. $6\pi\mu aU$. The sphere experiences no lift component.

The torque acting on the sphere about Oz, and in the positive direction, will be seen to be $\mu\iint(x.p_{ry} - y.p_{rx}).dS$. As the integral $\iint(x.p'_{ry} - y.p'_{rx}).dS$ vanishes, this becomes

$$\text{Torque} = \mu\iint(x.p''_{ry} - y.p''_{rx}).dS$$
$$= \mu\iint\left\{(x.\frac{\partial}{\partial r} - \frac{x}{r})v_2 - (y.\frac{\partial}{\partial r} - \frac{y}{r})u_2\right\}_{r=a}.dS$$
$$= -3\mu\frac{\omega_0}{a}\iint(x^2+y^2)dS = -3\mu\frac{\omega_0}{a}\iint a^2(\cos^2\theta + \sin^2\theta\cos^2\phi).a^2\sin\theta\,.d\theta\,.d\phi$$

The value of the double integral can be easily shown to be $\frac{8}{3}\cdot\pi a^4$ so that the torque becomes $-8\pi\mu\omega_0 a^3$ and tends to stop the rotation.

It is now generally accepted in aerofoil theory that when a body experiences a force component in the cross-stream direction then this component, or lift, is proportional to the circulation. At first sight it appears strange that the rotating sphere does not have lift when there is clearly a

circulation